Final Review Sheet

General information for the final:

- Bring a ruler for graphing.
- If a question asks you to check your answer, do so *in the original equation given in the problem.*
- Roughly half of the final will cover new material (factoring, polynomial equations), and roughly half the material from Chapter 7 (linear equations) and Chapter 8 (roots and powers).
- The quadratic formula will be given on the exam, so you don't need to memorize it.

Chapter 7 — Linear Equations in Two Variables

Systems of Equations

1. Solve each system using both the graphical method and the algebraic method. Check that your solutions from the two methods agree.

(a) $\begin{cases} 2x + y = 1 \\ 3x + 2y = 1 \end{cases}$ (b) $\begin{cases} 3x + 2y = -1 \\ x + 3y = -5 \end{cases}$

2. Solve each system algebraically. If there is just one solution, check it in both equations. If there are many solutions or no solution, say so.

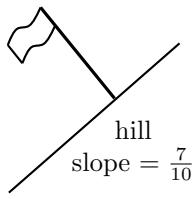
(a) $\begin{cases} 5x + 3y = -2 \\ 4x - 2y = 5 \end{cases}$ (b) $\begin{cases} 2x + y = 5 \\ 4x + 2y = 10 \end{cases}$

(c) $\begin{cases} 2x + y = 5 \\ 4x + 2y = 13 \end{cases}$ (d) $\begin{cases} 4x + 2y = 2 \\ x + 5y = 8 \end{cases}$

3. Find the point where the lines $3x - 2y = 5$ and $6x + 5y = -8$ intersect.

Parallel/Perpendicular

4. A hill has a slope of $\frac{7}{10}$. A flagpole is perpendicular to the hill. What is the slope of the flagpole?



5. If the lines $y = bx$ and $y = -bx$ are perpendicular, what could b be? What is the value of b if they are *parallel* instead?

6. If $5y = x + 3$ and $y - kx = 0$ are parallel, what is k?

Slopes

7. A ramp has a slope of 6% and is 10 feet long. How high is it?

8. Before being restored in 1990, the Leaning Tower of Pisa had a slope of approximately $\frac{21}{2}$, and was about 180 feet high. By how many feet did it lean? (i.e. find Δx).

9. Which line is steeper (has a larger slope): $3x - 5y = 0$ or $-2x + 3y = 0$?

Graphing

10. Graph the line with slope $\frac{2}{3}$ and y-intercept -1.

Finding Linear Equations

11. Find the equation of:

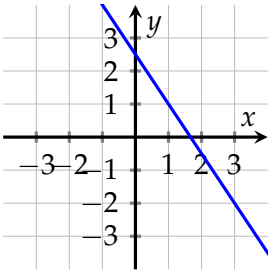
(a) A line through $(3,-1)$ and $(2,4)$

(b) A line parallel to $2x+3y=0$, passing through $(-2,2)$

(c) A line *perpendicular* to $2x+3y=0$, passing through $(-2,2)$

(d) A line with a y-intercept of 3 and the same x-intercept as $y=-3x+6$.

(e) This line:



✳ Our method for finding linear equations:

① find the slope

② write down partial equation

③ plug in one point on the line, and solve for the y-intercept

④ write full equation

Chapter 8 — Fractional Powers and Roots

Exponents

12. Evaluate the following:

(a) $(-64)^{\frac{2}{3}}$ (b) $(4)^{-\frac{3}{2}}$

(c) $(3\alpha)^0$ (d) $17^{\frac{1}{2}} \cdot 17^{\frac{1}{2}}$

Facts and Rules

13. Fill in the blanks with "positive," "negative," or "imaginary":

(a) $\sqrt[\text{even}]{\text{positive}} =$ ☐

(b) $\sqrt[\text{even}]{\text{negative}} =$ ☐

(c) $\sqrt[\text{odd}]{\text{negative}} =$ ☐

(d) $\sqrt[3]{-7} =$ ☐

(e) $\sqrt[4]{-7} =$ ☐

14. Fill in the blanks:

(a) $\sqrt{2^{\square}} = 2^3$ (b) $(\sqrt[5]{3})^{\square} = 3$

Simplifying Radicals

15. Simplify the following radical expressions:

(a) $\sqrt[3]{25x^2y^2} \cdot \sqrt[3]{40x^4y^2}$ (b) $\sqrt{18t^7m^4}$

(c) $\sqrt{(14)^2a} \cdot \sqrt{a}$ (d) $\sqrt[3]{160}$

(e) $\sqrt{12(s-3)^7}$ (f) $\dfrac{\sqrt[5]{2x} \cdot \sqrt[5]{3x^8}}{\sqrt[5]{x^2}}$

Power and Radical Equations

16. Solve the following equations:

(a) $\sqrt{3s^2+4} = \sqrt{2s^2+8}$ (b) $10 - q^2 = 1$

(c) $2(m^2+5) = 8$ (d) $3 - |2m-1| = 0$

(e) $(\sqrt{s+4}-2)^2 = 1$ (f) $\sqrt[3]{3(r+2)^2} = 3$

(g) $\sqrt[3]{|2x+1|} = 2$ (h) $4\sqrt{x^2+3} = 8$

Chapter 9 — Polynomials

Multiplying and Factoring

17. Multiply, and collect like terms:

(a) $(x^2-3)(x^2+4x+3)$ (b) $(c+2d)(-2c^2+cd-d^2)$

18. Factor the following expressions:

(a) $x^2-8x+12$ (b) $x^2-8x-48$

(c) r^2+4r+5 (d) a^3-25a

(e) a^2x-x (f) $18m^3n+12mn^2$

Roots

19. Is -4 a root of $y = x^2 - x - 20$?

20. Find the roots of:

(a) $y = x^3+4x^2+4x$ (b) $y = 2x^2-7x+5$

(c) $y = x^2+4x+5$

Quadratic Equations

21. Solve each quadratic equation, using any method. Write "imaginary" if the solutions are imaginary. Otherwise, check at least one solution.

(a) $x^2-13x = -40$ (b) $2x^2+5x-3 = 0$

(c) $a^2+3a-5 = 4a+7$ (d) $x^2+2x+3 = 0$

(e) $2x^2+3x-3 = 0$ (f) $2x^2+2x = 3x^2+1$

22. Solve by completing the square, and check at least one solution.

(a) $x^2-5x+2 = 0$ (b) $x^2-10 = -2x$

Graphing

23. Graph these quadratic equations by finding the roots and vertex:

(a) $y = -x^2+7x$ (b) $y = x^2-x-12$

(c) $y = 2x^2+6x-4$ (d) $y = -x^2-5x-4$

Falling/Rising Objects

24. A model rocket is launched from the ground. Its starting velocity is 160 feet per second.

(a) Fill in the equation for its height after t seconds:

$$h(t) = -16t^2 + \square\, t + \square.$$

(b) At what time does the rocket return to the ground?

(c) What is its maximum height?

(d) How high is the rocket 1 second after it is launched?

Misc

25. Find the x-intercepts (if any) and y-intercept of each equation:

(a) $y = 7x - 5$ (b) $y = x^2 + 1$

26. True or false?

(a) $\sqrt{7} - \sqrt{3} = \sqrt{4} = 2$ (b) $\sqrt{7} - 3 = \sqrt{4}$

(c) $\dfrac{\sqrt{6}}{\sqrt{3}} = \sqrt{2}$ (d) $(2+3)^2 = 2^2 + 3^2$

(e) $(\sqrt{a} + \sqrt{b})^2 = a + b$

Answers

1. (a) $(1, -1)$ (b) $(1, -2)$

2. (a) $(\frac{1}{2}, -\frac{3}{2})$ (b) many solutions (c) no solution (d) $(-\frac{1}{3}, \frac{5}{3})$

3. $(\frac{1}{3}, -2)$

4. $-\frac{10}{7}$

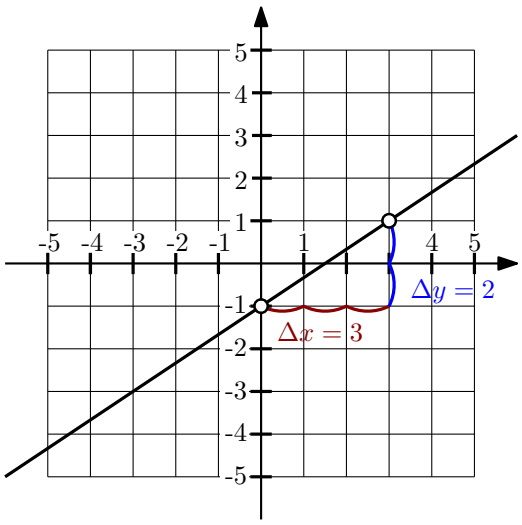
5. If they're perpendicular, then $b = 1$ or $b = -1$. If parallel, then $b = 0$.

6. $k = \frac{1}{5}$

7. 0.6 feet or $\frac{3}{5}$ feet

8. $\frac{360}{21}$ feet (about 17.1 feet!)

9. $-2x + 3y = 0$ is steeper (its slope is $\frac{2}{3} \approx 0.66$; the other line has slope $\frac{3}{5} = 0.6$).

10. 

11. (a) $y = -5x + 14$ (b) $y = -\frac{2}{3}x + \frac{2}{3}$ (c) $y = \frac{3}{2}x + 5$ (d) $y = -\frac{3}{2}x + 3$ (e) $y = \frac{5}{2} - \frac{3}{2}x$

12. (a) 16 (b) $\frac{1}{8}$ (c) 1 (d) 17

13. (a) positive (b) imaginary (c) negative (d) negative (e) imaginary

14. (a) 6 (b) 5

15. (a) $10x^2y\sqrt[3]{y}$ (b) $3\left|t\right|^3\left|m\right|^2\sqrt{2t}$ (c) $14\left|a\right|$ (d) $2\sqrt[3]{20}$ (e) $2\left|s-3\right|^3\sqrt{3(s-3)}$ (f) $x\sqrt[5]{6x^2}$

16. (a) $s = 2$, $s = -2$ (b) $q = 3$, $q = -3$ (c) imaginary (d) $m = 2$, $m = -1$ (e) $s = 5$, $s = -3$ (f) $r = 1$, $r = -5$ (g) $x = \frac{7}{2}$, $x = -\frac{9}{2}$ (h) $x = 1$, $x = -1$

17. (a) $x^4 + 4x^3 - 12x - 9$ (b) $-2c^3 - 3c^2d + cd^2 - 2d^3$.

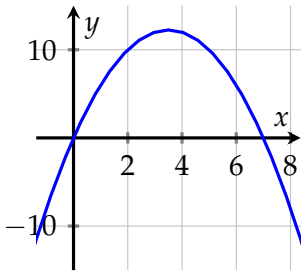
18. (a) $(x-6)(x-2)$ (b) $(x-12)(x+4)$ (c) $r^2 + 4r + 5$ (can't be factored further) (d) $a(a-5)(a+5)$ (e) $x(a-1)(a+1)$ (f) $6mn(3m^2 + 2n)$

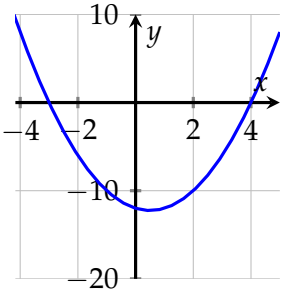
19. yes

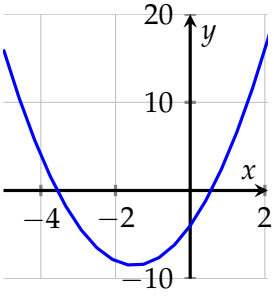
20. (a) $x = 0$, $x = -2$ (b) $x = 1$, $x = \frac{5}{2}$ (c) no roots (imaginary)

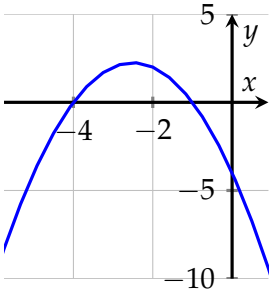
21. (a) $x = 5$, $x = 8$ (b) $x = -3$, $x = \frac{1}{2}$ (c) $a = -3$, $a = 4$ (d) imaginary (e) $x = \frac{-3 \pm \sqrt{33}}{4}$ (f) $x = 1$.

22. (a) $x = \dfrac{5 \pm \sqrt{17}}{2}$ (b) $x = -1 \pm \sqrt{11}$

23. (a) 

(b) 

(c) 

(d) 

24. (a) $-16t^2 + 160t$ (b) $t = 10$ seconds (c) 400 feet (d) 144 feet.

25. (a) x-intercept: $\frac{5}{7}$; y-intercept: -5 (b) no x-intercepts, y-intercept: 1

26. Only (c) is true.