Undetermined Coefficients Worksheet

- 1 We'll start out by solving the ODE $y'' + 3y' + 2y = e^{t}$.
 - (a) Find the general solution, $y_c(t)$, of the corresponding homogeneous equation, y'' + 3y' + 2y = 0.
 - (b) Write down the trial solution Y(t) for the nonhomogeneous equation.
 - (c) Plug Y(t) into the nonhomogeneous equation and solve for the unknown coefficient(s) A, B, C,.... (*Side note:* the solution Y(t) you get after plugging in the coefficients is usually called a *particular solution*.)

- (*d*) Write down the general solution of the ODE by adding the particular solution Y(t) and the homogeneous solution $y_c(t)$.
- 2 Now let's solve a similar looking ODE: $y'' 3y' + 2y = e^{t}$.
 - (a) Find $y_c(t)$, the general solution of the corresponding homogeneous equation.
 - (*b*) The trial solution Y(t) for the nonhomogeneous equation should be the same as for the first problem, because the trial solution only depends on the nonhomogeneous part of the equation, which hasn't changed.
 - $Plug \ Y(t) \ into \ the \ nonhomogeneous \ equation \ and \ solve \ for \ the \ unknown \ coefficient (s). \ What \ happens?$

3 Write down the trial solutions for each of these nonhomogeneous equations. (You don't have to solve for the coefficients)

(a)
$$y'' - 3y = t \cos t$$

(b)
$$y'' + 2y' + 5y = e^{-t} \cos t + 4t$$

4 Write down the trial solutions for each of these nonhomogeneous equations — some may need multiplying by t.

(a)
$$y'' + 4y = 3 \sin 2t$$

(b)
$$y'' - y = 3te^t + t\cos t$$

(c)
$$y'' + y = 3te^t + t\cos t$$

(d)
$$y'' + y = e^t \cos t$$