

## Math 102 Final Exam Solutions

1. Factor as much as possible:

(a)  $2qr^2 - 8q$

*Answer:*  $2q(r+2)(r-2)$

(b)  $y(z+5) + a(z+5)$

*Answer:*  $(z+5)(y+a)$

(c)  $z^2 + 9$

*Answer:* *no factors*

(d)  $3m^3n - 3mn$

*Answer:*  $3mn(m-1)(m+1)$

2. If  $2y - kx = 3$  and  $6x - 2y = 2$  are parallel, what is  $k$ ?

*Answer:*  $k = 6$

3. Multiply and collect like terms:  $(x^2 - xy - 4y^2)(x^2 + 4y^2)$ .

*Answer:*  $x^4 - x^3y - 4xy^3 - 16y^4$

4. Solve the following equations. If the solution is imaginary, say so. Otherwise, check at least one of your solutions in the original equations.

(a)  $2x^2 - 1 = 4x$

*Answer:*  $1 \pm \frac{\sqrt{6}}{2}$

(b)  $x^2 + 3x = 9$

*Answer:*  $\frac{-3 \pm \sqrt{45}}{2}$

(c)  $x^2 - 32 = -4x$

*Answer:*  $x = -8, x = 4$

*Solve by completing the square:*

(d)  $x^2 - 3x + 1 = 0$

*Answer:*  $\frac{3 \pm \sqrt{5}}{2}$

(e)  $x^2 - 6x + 10 = 0$

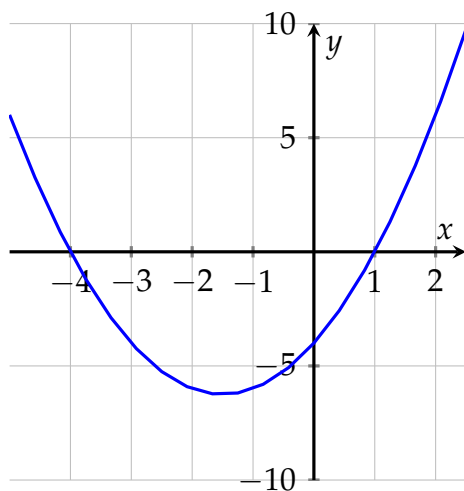
*Answer:* solutions are imaginary

5. Evaluate  $(-8)^{-\frac{4}{3}}$ .

Answer:  $\frac{1}{16}$

6. Graph  $y = x^2 + 3x - 4$ , showing the roots and vertex.

The roots are  $x = -4, x = 1$ ; the vertex is  $-\frac{3}{2}, -\frac{25}{4}$ . Graph:



7. Find the roots of:

(a)  $y = x^3 - 4x^2 + 3x$

Answer:  $x = 0, 1, 3$

(b)  $y = 3x^2 + x + 4$

Answer: no real roots (imaginary)

8. Solve the following equations. Check all solutions in the original equation.

If there are no solutions, or the answer is imaginary, say so.

(a)  $\sqrt[3]{4 + (m - 1)^2} + 1 = 3$

Answer:  $m = -1, 3$

(b)  $\sqrt{4z^2 - 3} = 3$

Answer:  $z = \pm\sqrt{3}$

(c)  $8 - |y^3 - 5| = 4$

Answer:  $z = 1, \sqrt[3]{9}$

9. Simplify the following radical expressions:

(a)  $\frac{\sqrt{48y^2z^5}}{\sqrt{2yz}}$

Answer:  $2|z|^2\sqrt{6y}$

(b)  $\sqrt[3]{16x^{14}y} \cdot \sqrt[3]{5y^4}$

Answer:  $2x^4y\sqrt[3]{10x^2y^2}$

10. Find the  $x$ -intercept(s) and  $y$ -intercept of  $y = 2x^2 - x - 1$ .

*Answer:  $x$ -intercepts are  $x = 1, -\frac{1}{2}$ ;  $y$ -intercept is  $-1$ .*

11. True or false: a quadratic equation can have three different solutions.

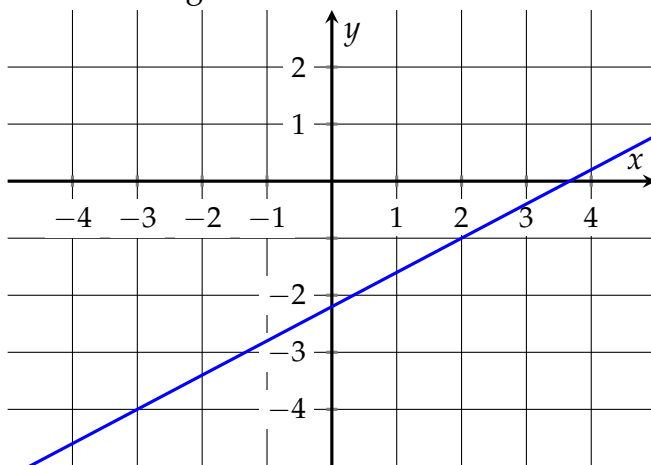
*Answer: false (a quadratic equation has at most two solutions)*

12. Find the equations of:

(a) The line perpendicular to  $4x + 5y = 3$  and passing through  $(-2, 3)$

*Answer:  $y = \frac{5}{4}x + \frac{11}{2}$*

(b) The following line:



*Answer:  $y = \frac{3}{5}x - \frac{11}{5}$*

13. A cat jumps off a 4-foot table, with a starting velocity of 0.

(a) Fill in the equation for its height  $h$  after  $t$  seconds have passed:

$$h(t) = -16t^2 + \boxed{0}t + \boxed{4}.$$

(b) How high is it after  $\frac{1}{4}$  second?

*Answer: 3 feet*

(c) When does it reach the ground?

*Answer: after  $\frac{1}{2}$  second*

14. Fill in the blanks:

(a)  $\sqrt{3} \cdot \boxed{\phantom{00}} = 9$

*Answer:  $\sqrt{27}$*

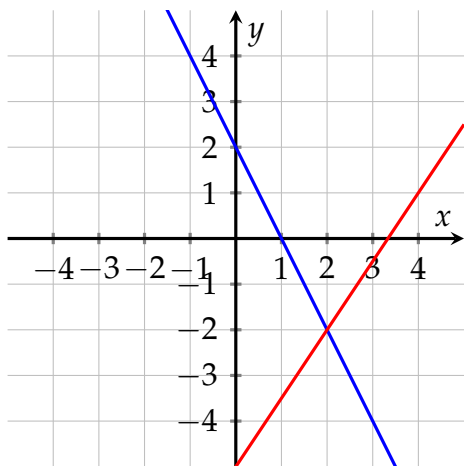
(b)  $\sqrt[3]{r^6} = r^2$

*Answer: 3*

15. Solve the following system of equations, using *both* the graphical and algebraic methods. Check that your solutions from the two methods agree.

$$\begin{cases} 2x + y = 2 \\ -3x + 2y = -10 \end{cases}$$

Answer: (2,-2)



16. Solve each of the following systems of equations algebraically. Check your solutions (if any) in both of the original equations.

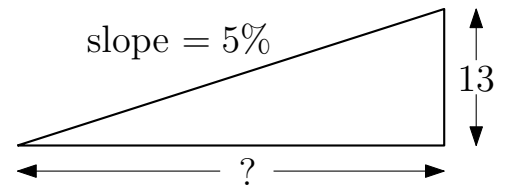
(a) 
$$\begin{cases} -2x + 6y = 0 \\ -x + 9y = 1 \end{cases}$$

Answer:  $x = \frac{1}{2}, y = \frac{1}{6}$

(b) 
$$\begin{cases} 2x - 2y = -4 \\ -3x + 3y = 6 \end{cases}$$

Answer: *infinitely many solutions*

17. The road outside the IC has a slope of 5%. One end of the block is 13 feet higher than the other. How long is the block?

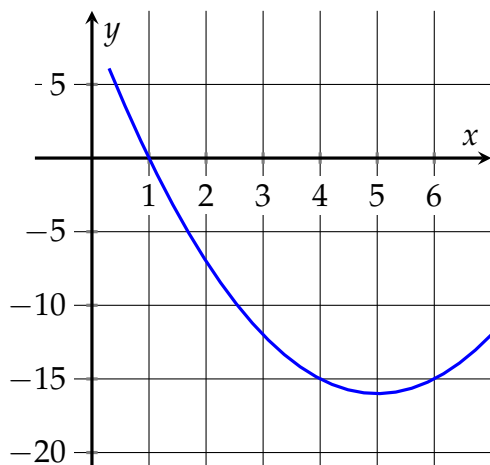


*Answer: 260 feet*

**Extra credit.**

(a) Fill in the equation for the parabola pictured below:

*Answer:  $y = x^2 + \boxed{-10}x + \boxed{9}$  (roots are  $x = 1, x = 9$ )*



(b) You are buying snacks for the Math 102 post-final party. Bagels cost 55¢ each and bananas cost 35¢ each. You need to purchase 18 items total, but don't want to spend more than \$8.50.

How much of each item should you buy if you want to spend exactly \$8.50?

*Solve the system  $\begin{cases} x + y = 18 \\ 0.55x + 0.35y = 8.50 \end{cases}$ .*

*Answer: 11 bagels, 7 bananas.*