Homework 3

CAAM 335 • Matrix Analysis • Spring 2016

Due Date: February 5, 4pm

Submission Instructions: Homework submission will be on OWL-Space, as with Homework 1. You can take a look at the Homework 1 problems page for details on the process.

You are welcome to collaborate with other CAAM 335 students, consult the textbook, and get help from an instructor or TA. For this assignment, you ***may not*** use MATLAB, Octave, or any another program to do your matrix computations.

Problem 1 Let $A, B \in \mathbb{R}^{n\times n}$ be square, nonsingular matrices. Which of the following is equal to $((AB)^{-1})^T$?

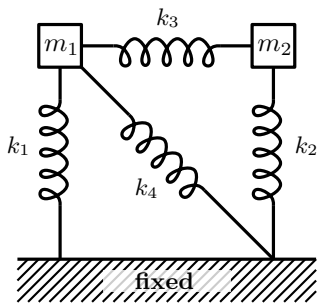
a. $(B^{-1})^T(A^{-1})^T$

b. $(A^T)^{-1}(B^{-1})^T$

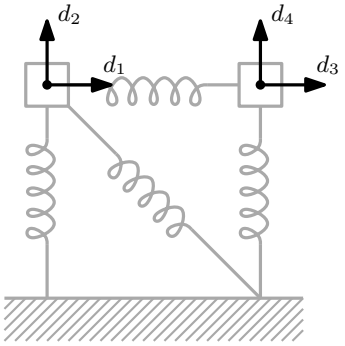
c. $(B^T)^{-1}(A^T)^{-1}$

d. $((BA)^T)^{-1}$

Problem 2 In this problem, we have a 2D frame with a diagonal brace, anchored to an immovable object. Two masses are at its corners.



(a) Masses and Springs



(b) Displacements

For parts (i–iii), let $k_1 = k_2 = k_3 = k_4 = 1$.

i. Calculate A^TKA. It will have the form:

$$\begin{bmatrix} \frac{3}{2} & \alpha & -1 & 0 \\ \alpha & \beta & \gamma & 0 \\ -1 & \gamma & \delta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where α, β, γ, δ represent some numbers. What are these numbers?

ii. Compute the LU factorization of A^TKA by hand as demonstrated in class (no pivoting). You should find the (3,1) entry of L is $-2/3$ and the $(2,3)$ entry of U is $-1/3$. Find the following entries:

   (a) the (3,2) entry of L;

   (b) the (4,3) entry of L;

   (c) the (1,2) entry of U.

iii. What are the pivots of A^TKA?

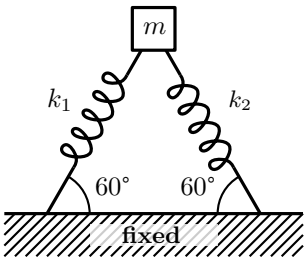
iv. If A^TKA is not invertible, find a vector $\vec{d}$ such that $A^TKA\vec{d} = 0$ and $d_1 = 2$.

For the next two questions, imagine removing the diagonal brace by letting $k_4 = 0$ (keeping $k_1 = k_2 = k_3 = 1$).

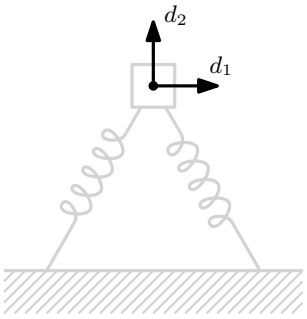
v. What is the determinant of A^TKA?

vi. If A^TKA is not invertible, find a vector $\vec{d}$ such that $A^TKA\vec{d} = 0$ and $d_1 = 2$.

Problem 3 In the following problem, two springs anchor a mass to a fixed object. At rest, the springs and the fixed object form an equilateral triangle.



(a) Masses and Springs



(b) Displacements

i. Let $k_1 = k_2 = 2\text{N/cm}$. Consider applying a force of 9N in two ways: first, horizontally ($f_1 = 9$, $f_2 = 0$) and second, vertically ($f_1 = 0$, $f_2 = 9$). In each case, find the magnitude of the mass's displacement, $\sqrt{d_1^2 + d_2^2}$.

   Which direction produces the greatest displacement, and what is the magnitude of that displacement (in cm)?

ii. Now, let $k_1 = 2\text{N/cm}$ and $k_2 = 6\text{N/cm}$. Apply the same 9N vertical force. Find the components d_1, d_2 of the mass's displacement (in cm). Round your answer to one place after the decimal, making sure to include the correct signs. Check that your answer makes sense physically.