Midterm 1 Review Sheet

Solving linear equations

Solve each of these equations:

(a) $-x = 4$ (b) $-3a - 2a = 5$

(c) $3k = \frac{1}{9}$ (d) $\frac{2}{9}m = \frac{5}{10}$

(e) $\frac{7}{2}m = -3$ (f) $-r - (-4) = -6$

(g) $\frac{1}{r} = 2$ (h) $-\frac{b}{3} = \frac{5}{6}$

Tips:

- Say your variable is in the denominator: $\frac{2}{z} = 1$, for instance. To solve for z, first multiply both sides by z to get it out of the denominator:

$$z\left(\frac{2}{z}\right) = z(1)$$
$$2 = z$$

Standard and functional form

Make sure you know the difference between standard form ($ax + by = c$) and functional form ($y =$ *something*).

Convert to standard form:

(a) $2x + 1 = y$ (b) $\frac{2}{3}x + y = 2y - 1$

(c) $-3x + 2y = -3x$

Convert to functional form:

(a) $2x - 2y = 1$ (b) $8x + 2y + 3 = 6x + y$

(c) $\frac{1}{2}x + \frac{1}{2}y = 4$

Function Graphing

Try graphing the following equations:

(a) $y = 2$ (b) $y = \frac{1}{3}x + 1$

(c) $2y + 4x = 0$ (d) $3y - 5x = 1$

(e) $x = -4$ (f) $y = \frac{1+x^2}{2}$

- For more complicated equations (like $5y - x = 2$), you can usually save time by converting to functional form first.
- Include a *scale* on every graph (that means putting numbers on the axes).

Systems of Linear Equations

Summary:

- A system of linear equations is just a collection of equations, like $\begin{cases} x + 2y = 1 \\ 3x - 2y = 3 \end{cases}$.
- Solving a linear system means finding values for x and y that make both equations true. Here you can check that $x = 1$, $y = 0$ works: so $(1, 0)$ is a *solution*.
- We have two methods for solving systems of equations: graphically and algebraically.
  - With the graphical method, graph both equations carefully. Locate the point where they cross and find the x and y coordinates of this point (sometimes this can take some guessing).
  - For the algebraic method, you multiply and add the two equations to eliminate one variable, then solve for the other.
- *Always* check your answer by plugging the x- and y-values into both of the original equations.

- Most systems have exactly one solution. However, some systems have many solutions, and some have none at all (see the last page for more on this).

Example problems: Solve these systems, both algebraically and graphically. You should find the same solution with both methods.

(a) $\begin{cases} x - y = 0 \\ x + y = 4 \end{cases}$ (b) $\begin{cases} -x + 2y = 8 \\ 2x + y = -2 \end{cases}$

(c) $\begin{cases} y = 1 \\ -2x + 3y = -7 \end{cases}$ (d) $\begin{cases} x - y = 0 \\ -2x + 2y = 3 \end{cases}$

(e) $\begin{cases} -2x + y = 1 \\ 4x - 2y = -2 \end{cases}$

✳ When solving graphically, be as precise as possible. Try to use points that are whole numbers when graphing, and use a ruler. You'll avoid a lot of frustration this way!

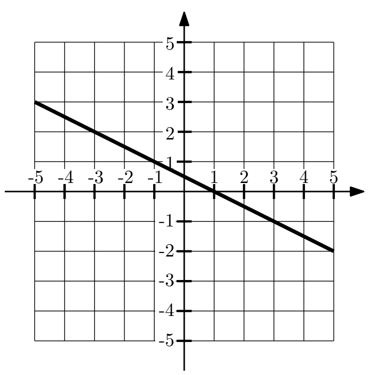
✳ When graphing, if one of your points looks out of line, check through your calculations.

Slopes and Intercepts

Find the slopes and y-intercepts of these lines:

(a) $y = -2x$ (b) $2x + 5y = 10$

(c) $x = 4 - 3y$ (d) $-\frac{3}{2}x + \frac{1}{4}y = 1$

(e)


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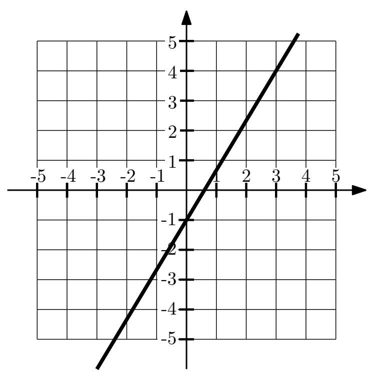


Find the x-intercepts of: (a) $x + y = 1$ (b) $y = -2x + 7$

Parallel and Perpendicular Lines

1. For each pair of lines below, decide if the two lines are parallel, perpendicular, or neither.

(a) $y = -\frac{1}{2}x + 3$, $2y + x = 4$

(b) $2y + 8x = 1$, $y = -4x + 3$

(c) $y - \frac{1}{3}x = 0$, $2y = 8 - 6x$

2. Find a value for j so that $2x + 3y = 0$ and $5x + jy = 1$ are parallel.

3. Find k so that $kx + 2y = 0$ and $4x + y = 5$ are perpendicular.

✳ For parallel/perpendicular problems, start by finding the slopes of all the lines involved.

Finding Δx, Δy

(a) A mountain trail has a slope of 6% and climbs 500 feet ($\Delta y = 500$). How long is the trail, approximately?

(In other words, find Δx)

(b) A ramp has a slope of $\frac{1}{8}$, and is 100 feet long. How high is it?

Finding Linear Equations

Find equations for:

(a) A line through $(6, -1)$ and $(-3, -5)$

(b) A line parallel to $x = 4y$, passing through $(-6, 0)$.

(c) A line perpendicular to $4x + 2y = 0$, passing through $(1, 0)$.

(d) A line perpendicular to $3y + x = 0$, with the same x-intercept as $3x + y = -9$.

Graphing Linear Equations Using Slopes and Intercepts

(a) Graph the line with y-intercept 3 and slope $2/3$.

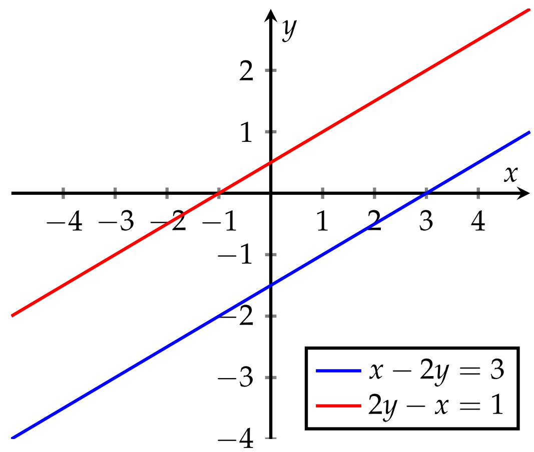
(b) Graph the line $3y = -4x + 3$ without making a table (First, find the slope, and find one point on the line. Then, try to use that information draw the rest of the line).

(c) Graph the line with slope -4, passing through (1,1).

Solving systems of equations — the weird cases

Usually, a system of linear equations has just one solution. However, some systems have many solutions, and others have no solutions. Two examples showed up in exercise set 36 IV.

36 IV(e) In (e), you're given $\begin{cases} x - 2y = 3 \\ 2y - x = 1. \end{cases}$

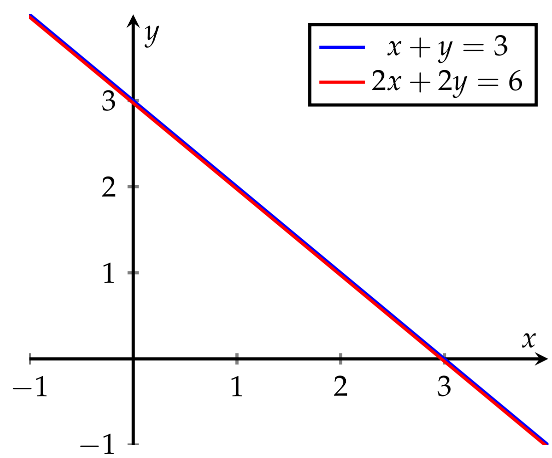


The graphs of these equations are parallel lines. The solution of a system is supposed to be where the lines cross— since they don't cross, that means there's no solution.

What happens if you try to solve it algebraically? The key is that **when you eliminate one variable, the other goes away, and and you get a false equation**. For instance, if we add the two equations, we can eliminate y, but x also goes away:

$$\begin{array}{r} x - 2y = 3 \\ +\ 2y - x = 1 \\ \hline 0 = 4 \end{array}$$

36 IV(f) Part (f) gives you $\begin{cases} x + y = 3 \\ 2x + 2y = 6. \end{cases}$



These equations turn out to have the same graph. Now, *every* point on the line is a solution to the system.

Now, let's what happens if you solve it algebraically.

$$\begin{array}{rcr} x + y = 3 & \xrightarrow{\times(-2)} & -2x - 2y = -6 \\ 2x + 2y = 6 & \rightarrow & 2x + 2y = 6 \\ \hline & & 0 = 0 \end{array}$$

The key here is that **when you eliminate one variable, the other goes away, and and you get a *true* equation**.