Group Project: BECU Interest Rates and Buying in Bulk

Intro: BECU's savings accounts have a split rate. You earn 6% APR on the first \$500 of your balance, then 0.25% APR on the rest. Interest is compounded monthly, so your monthly interest rate on the first \$500 is $6\%/12 = 0.5\%$, and the monthly interest rate for the rest is $0.25\%/12 \approx 0.033\%$. So if your balance is \$700 now, in month you will earn 0.5% on the first \$500, and 0.033% on the remaining \$200.

Because of the split rate, we can't use the CAF or the APR CAF to analyze what's going on.

- If your account has \$1500 on January 1, how much does it have one month later?
- How much does it have two months later? (Again, only \$500 of February's balance is compounded at 6% APR.)
- Let $A(k)$ be the amount of money in your account after k months, and assume the balance is always at least \$500. Find a *recursive* formula for $A(k)$.
  Hint: Look back at worksheet #17 for the definition of "recursive." Your formula won't look exactly the same as the recursive formulas there, however.
- Expand and simplify your recursive formula as much as possible. When you're done, it should look like $A(k+1) = mA(k) + s$, where m and s are numbers.

If a sequence $A(k)$ has a recursive formula that looks like $A(k+1) = mA(k) + s$, then its *explicit* formula will be

$$A(k) = m^k \cdot A(0) + \frac{s}{m-1} \cdot (m^k - 1).$$

At first glance, this formula seems complicated, but once you plug in numbers for m and s, it will become simpler.

- Write down an explicit formula for $A(k)$ for this situation. Your formula will involve $A(0)$ and k.
- If you start with \$900 in the account, how much will you have after one year? What is your APY?
- Suppose that BECU kept the same APRs, but compounded annually instead of monthly. If you start with \$900 in the account now, how much will you have after one year?
- The difference between your answers for the last two questions should be very small. Try to explain why.

Now we're going to take a different tack.

- Using the APY equations, find the APY for 6% APR, compounded monthly. Also find the APY for 0.25% APR, compounded monthly.
- Suppose that BECU kept the same APYs, but compounded annually instead of monthly. Once again, you start with \$900 in your account. How much will you have after one year?
- Try to explain what's going on. When you kept the same APY, you should have found that compounding annually earned you *less* interest than compounding monthly. But when you kept the same APR, annual compounding earned slightly more than monthly compounding. Why?

Buying in Bulk

Part 2: Suppose your business produces and sells small water pumps. Usually, in Math 111 we've assumed that you could buy all the inputs to your business individually (the components you need to manufacture your pumps). But oftentimes you're forced to buy your inputs in bulk. In this situation, assume that motors come in packs of 100, at a cost of $800 per pack. The issue is that if you only produce 3 pumps, you still have to buy an entire pack of 100 motors. This makes analyzing the business a little trickier.

Your total revenue function for selling q pumps is $TR(q) = -.005q^2 + 14.5q$ (measured in dollars), and you pay $2800 in fixed costs.

To make things simpler, assume for the moment that your only variable costs are the price of the motor packs (the other parts are so much cheaper that you can focus on the motors).

- Graph TC vs. quantity, for $0 \le q \le 500$. Also, graph TR on the same axes.
- What is $MC(0)$? $MC(1)$? $MC(99)$? $MC(100)$?
- Make a graph of MC vs. quantity for $0 \le q \le 500$.
- What is the minimum quantity you must sell and not lose money? (Use algebra.)
  Note: This is tricky. Start by thinking about everything in terms of packs. If you also sold your pumps in packs of 100, how many packs would you have to sell to avoid losing money? Suppose it turned out that you have to sell 300 pumps to not lose money. Could you sell 299 pumps without losing money? Could you sell 250?