

## Group Project: Supply and Demand

*Intro:* You grow and sell organic wheat. As a grower, you have to sell your wheat at the current price (market price). Typically, we're told what the market prices are in Math 111. In lecture, we briefly talked about using the law of supply and demand to determine what the market price for an item will be. This is an interesting question, since it gives us insight into the different factors affecting food prices.

For simplicity, we'll assume that you're the only organic wheat grower (alternatively, you represent all organic wheat growers). Your total cost for producing  $q$  thousand bushels is:

$$TC(q) = 42 + 4q + 0.1q^2 \quad (\text{thousands of dollars})$$

The law of supply and demand says that the equilibrium price for an item occurs when the supply and demand for that item are equal. The supply and demand both depend on the market price  $p$ : for instance, if  $p$  increases, then supply will increase since producers are willing to make more, and demand will decrease, since consumers are less willing to buy.

Write  $S(p)$  for the quantity (in thousands of bushels) that you would produce if the market price were  $\$p/\text{bushel}$ . Suppose that if the market price is  $\$p/\text{bushel}$ , then the quantity demanded is

$$D(p) = 44 - 2p \quad (\text{thousands of bushels})$$

The law of supply and demand says that if  $p$  is the equilibrium price, then  $S(p) = D(p)$ .

Your first task is to calculate supply. If your only concern is maximizing profit, then  $S(p)$  will be the quantity that gives you maximum profit when the market price is  $\$p/\text{bushel}$ .

- Write out the equation for profit—it will depend on market price,  $p$ .
- Find the quantity that maximizes profit, in terms of  $p$ . This quantity will be the supply,  $S(p)$ .
- Now you have equations for  $S(p)$  and  $D(p)$ . Solve  $S(p) = D(p)$  to find the equilibrium price.
- Graph  $S(p)$  and  $D(p)$ . It's customary to put  $p$  on the vertical axis and  $S(p)$  and  $D(p)$  on the horizontal axis.

Now, imagine your business becomes non-profit, and now your goal is to maximize the amount of wheat you produce (without losing money), instead of maximizing profit.

- For this question only, suppose  $p$  happens to be  $10\$/\text{bushel}$ . How much can you produce?
- More generally, if  $p$  is a variable, what is the most you can you produce without losing money? This quantity is the supply; call it  $S_{\text{np}}(p)$  ("np" for non-profit.)  
*Hint:* You can still use the quadratic formula, although you can't simplify your answer completely. It will be ugly.
- Solve  $S_{\text{np}}(p) = D(p)$  to find the equilibrium price.
- Make a new graph, containing  $S(p)$ ,  $S_{\text{np}}(p)$ , and  $D(p)$ . How much has the equilibrium price changed, and how do you see that on the graph? If the demand curve were steeper,

would the price difference between the for-profit business and non-profit business be bigger, or smaller?

Imagine that in the future, the U.S. government decides to subsidize organic wheat (currently, regular wheat is subsidized). For each bushel you produce, they give you  $\$a$  extra. What this means for you is that if the market price is  $p$ , your total revenue for producing  $q$  thousand bushels is now  $TR(q) = pq + aq$ . Suppose you are still a non-profit.

- Find the equation for  $S_a(p)$ , the supply with a subsidy of  $\$a$ /bushel.  
*Hint:* There's a hard way to solve this, and an easy way. You don't have to redo your computations from the previous part.
- Find the equilibrium price. Now the equilibrium price depends on  $a$ , the subsidy. Graph equilibrium price vs. the subsidy.

A few last questions:

- Currently, the price of organic wheat is  $\$8.88$  per bushel. Change the formula for the demand curve so that the equilibrium price becomes  $\$8.88$ .
- If you become a non-profit, your fixed costs might rise, since your salary would now be considered part of fixed cost. How does this affect the equilibrium price? Do you think that this increase in fixed cost will cancel out the increase in supply that came when you switched from for-profit to non-profit?