Homework 4

CAAM 335 • Matrix Analysis • Spring 2016

Due Date: February 19, 4pm

Submission Instructions: Homework submission will be on OWL-Space, as with Homework 1. You can take a look at the Homework 1 problems page for details on the process.

You are welcome to collaborate with other CAAM 335 students, consult the textbook, and get help from an instructor or TA. For this assignment, you ***may not*** use MATLAB, Octave, or any another program to do your matrix computations.

Problem 1 For each of the following *subsets* of $\mathbb{R}^3$, decide whether they are *subspaces* or not. If they are, find their dimension.

i. $\left\{ \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T \;\middle|\; x_1 + x_2 \neq 0 \text{ and } x_3 = 0 \right\}$

ii. $\left\{ \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T \;\middle|\; x_1 + x_2 = 0 \text{ and } x_3 = 0 \right\}$

iii. $\left\{ \begin{bmatrix} a & a+b & b \end{bmatrix}^T \;\middle|\; a, b \in \mathbb{R} \right\}$

iv. $\left\{ \begin{bmatrix} a & ab & b \end{bmatrix}^T \;\middle|\; a, b \in \mathbb{R} \right\}$

Problem 2 Consider the following matrix:

$$A = \begin{bmatrix} 1 & 1 & 3 & 0 & 2 \\ 0 & 1 & 2 & -4 & 0 \\ 2 & 0 & 2 & 1 & 4 \\ -1 & 1 & 1 & -1 & -2 \end{bmatrix}.$$

i. Find the row-reduced echelon form of A; call it A_{rref}. You should find that its (2,5) entry is 0 and its (1,3) entry is 1. What are the (1,3), (2,4), and (3,5) entries?

ii. What are the free columns of A according to the row-reduced echelon form?

iii. What is the rank of A^T?

iv. If $\vec{n} = \begin{bmatrix} n_1 & n_2 & 0 & n_4 & 1 \end{bmatrix}$ is in the null space of A, find the missing entries n_1, n_2, n_4.

v. If $\vec{n} = \begin{bmatrix} n_1 & 6 & n_3 & n_4 & 1 \end{bmatrix}$ is in the null space of A, find the missing entries n_1, n_3, n_4.

vi. Find the dimension of the null space and left null space of LA, where L is the invertible matrix

$$L = \begin{bmatrix} 3 & 1 & 3 & 2 \\ 1 & 1 & 1 & 3 \\ 3 & 2 & 3 & 1 \\ 2 & 3 & 3 & 2 \end{bmatrix}.$$

vii. Write $\vec{b} = \begin{bmatrix}-2 & 1 & 2 & -4\end{bmatrix}^T$ as a sum of vectors $\vec{b}_r + \vec{b}_n$, where $\vec{b}_r \in \mathcal{R}(A)$ and $\vec{b}_n \in \mathcal{N}(A^T)$.

viii. Write $\vec{c} = \begin{bmatrix}-1 & 2 & -1 & 8 & 1\end{bmatrix}^T$ as a sum of vectors $\vec{c}_r + \vec{c}_n$, where $\vec{c}_r \in \mathcal{R}(A^T)$ and $\vec{c}_n \in \mathcal{N}(A)$.

Problem 3 Suppose S_1, S_2, and S_3 are subspaces of $\mathbb{R}^n$, and we have

$$\mathbb{R}^n = S_1 \oplus S_2, \qquad \mathbb{R}^n = S_1 \oplus S_3.$$

i. Decide which of the following statements must always be true. (More than one may be true; all could be false.)

   (a) S_2 is a subset of S_3.
   (b) S_3 is a subset of S_2.
   (c) S_2 and S_3 have the same dimension.

ii. Suppose we also know $S_1 \perp S_2$ and $S_1 \perp S_3$. Which of the following statements must always be true? (Again, more than one may be true; all could be false.)

   (a) S_2 is a subset of S_3.
   (b) S_3 is a subset of S_2.
   (c) S_2 and S_3 have the same dimension.

iii. Now, combine part (ii) with the Fundamental Theorem of Linear Algebra. Suppose we don't know A but we do know the null space $\mathcal{N}(A)$. With this knowledge, how many possible choices are there for the row space of A?

   (a) Exactly one.
   (b) A finite number.
   (c) Infinitely many.
   (d) Either exactly one or infinitely many, depending on $\mathcal{N}(A)$.