

Honor Statement

I affirm that my work upholds the highest standards of honesty and integrity, and that I have neither given nor received any unauthorized assistance on this exam.

Signature _____

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total	

Instructions:

- Show all your work, and box your final answer.
- You may use one double-sided $8\frac{1}{2}$ " by 11" sheet of notes.
- No calculators, cell phones, headphones, or other electronics are allowed,

1. Solve the equation

$$ty' = \frac{t+1}{y^2}$$

with the initial condition $y(1) = 2$.

2. Write down an equation that is linear but not separable. Don't solve it!

3. A pond initially contains 5 L of water, and no algae. Water containing $4e^{-t}$ grams/liter of algae enters the pond at a rate of 2 liters/second. It mixes with the contents of the pond, and drains at a rate of 1 liter/second. Find the amount of algae in the pond in grams after t hours.

4. Suppose that you are solving the equation $x \frac{dy}{dx} = \sin y$ approximately, using Euler's method. If you start at the point (x_0, y_0) , and take a step size of h , write down formulas for your new location, (x_1, y_1) . Your formulas should be in terms of x_0 , y_0 , and h :

$$x_1 = \underline{\hspace{2cm}}$$

$$y_1 = \underline{\hspace{2cm}}$$

5. For this problem, we have the differential equation

$$y' = -3y(1 - y^2).$$

- (a) Find the equilibrium solutions, and classify them as stable, unstable, or semistable.
- (b) Sketch the direction field.
- (c) If $y(0) = \frac{1}{2}$, what value does $y(t)$ tend to as $t \rightarrow \infty$?
- (d) Solve the differential equation with the initial condition $y(0) = 2$.

(Use the back if necessary)