1. Solve the differential equation
$$(1+t^2)y' + y = 1.$$
Answer: $y = 1 + Ce^{-\arctan x}$

2. Suppose that you are solving the equation $y' = -xy$ approximately, using Euler's method. If you start at the point (x_0, y_0), and take a step size of h, write down formulas for your new location, (x_1, y_1). Your formulas should be in terms of x_0, y_0, and h:

answer: $x_1 = x_0 + h$

answer: $y_1 = y_0 - hx_0y_0$

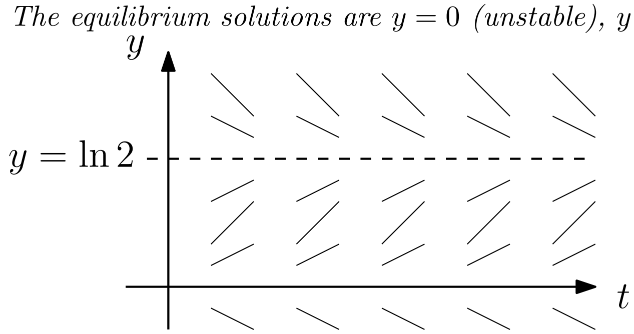
3. Solve the differential equation
$$-2ty' + (t+1)y = ty^3$$
Answer: $y = \pm\left(1 - \frac{1}{t} + \frac{C}{te^t}\right)^{-1/2}$

4. Initially, a tank contains 6 gal of water containing 1 lb of salt. There is water flowing into the tank through two pipes: Water containing salt is entering the tank through the first pipe at rate of 2 gal/min. Several measurements indicate that the amount of salt contained in one gallon of the incoming water is $e^{-3t/2}$ lb at time t. One gallon of fresh water per minute is entering the tank through the second pipe. Finally, the well-stirred mixture is draining the tank at a rate of 3 gal/min. Determine the amount of salt at any time $t \geq 0$.
Let $Q(t)$ be the amount of salt at time t, in pounds.
Differential equation: $\frac{dQ}{dt} = (2\frac{gal}{min})(e^{-3t/2}\frac{lb}{gal}) - (3\frac{gal}{min})(\frac{Q\ lb}{6\ gal})$, *so* $\frac{dQ}{dt} = 2e^{-3t/2} - \frac{Q}{2}$
Answer: $Q(t) = -2e^{-3t/2} + 3e^{-t/2}$ *grams*

5. You're waving a flag in the air horizontally. Let $v(t)$ be the velocity of the flag (in meters/second) at time t, There is air resistance opposing the flag's motion, with a *magnitude* of $3|v|$, and you apply a force of $2\sin t$ (both measured in newtons). Find $v(t)$, if the starting velocity of the flag is 1 m/s.
Differential equation: $m\frac{dv}{dt} = 2\sin t - 3v$
Solution: It's messy: $v(t) = \left(\frac{2m}{m^2+9} + 1\right)e^{-3t/m} + \frac{1}{m^2+9}(6\sin(t) - 2m\cos(t))$

6. A bird population $y(t)$ (measured in millions of birds) has the differential equation
$$y' = y\left(e^{-y} - \frac{1}{2}\right).$$
   (a) Find the equilibrium solutions and classify each of them as stable, unstable, or semistable.
   (b) Sketch the direction field.
   (c) If the starting population is $y = 1$, find the limit of the population as $t \to \infty$.

(a): The equilibrium solutions are $y = 0$ *(unstable),* $y = \ln 2$ *(stable)*



(b):
(c): The limiting population is $\ln 2$.