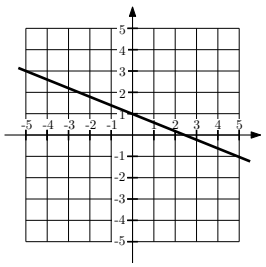
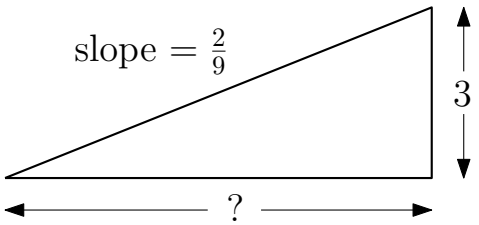
Math 102 Midterm 1 — Answers

1. Find the slope and y-intercept of the line below:



slope: $-\frac{2}{5}$
y-intercept: 1

2. A section of road has a slope of $\frac{2}{9}$. The change in height (Δy) is 3 yards. Find Δx.



Answer: $\Delta x = \frac{27}{2}$

3. Find a value for n so that $-7x + 3y = 5$ and $nx + 2y = 0$ are perpendicular.

First, convert both equations to functional form so you can read off the slopes. In functional form, the two lines are $y = \frac{7}{3}x + \frac{5}{3}$ *and* $y = -\frac{n}{2}x$. *So the slope of the first line is* $\frac{7}{3}$ *and the slope of the second is* $-\frac{n}{2}$.

For these lines to be perpendicular, they need to be negative reciprocals. The negative reciprocal of $\frac{7}{3}$ *is* $-\frac{3}{7}$. *So, write down the equation* $-\frac{3}{7} = -\frac{n}{2}$.

We want to know what value for n will make this equation true. So, solve this equation:

$$\begin{aligned} -\frac{3}{7} &= -\frac{n}{2} \\ 2\left(-\frac{3}{7}\right) &= -n \\ -\frac{6}{7} &= n \\ \frac{6}{7} &= n. \end{aligned}$$

4. Find equations for the following lines:

(a) The line parallel to $9x + 3y = 6$, passing through $(-2, 2)$.
Answer: $y = -3x - 4$

(b) The line through $(0, 0)$ and $(-3, -4)$.
Answer: $y = \frac{4}{3}x$

(c) The line perpendicular to $3x - 2y = 5$, with the same x-intercept as $2x + y = -8$.

Here's the steps:

1. *Find the slope: The functional form of* $3x - 2y = 5$ *is* $y = \frac{3}{2}x - \frac{5}{2}$*, so this line has the slope* $\frac{3}{2}$*. Our new line is perpendicular to this, so its slope is the negative reciprocal,* $-\frac{2}{3}$*.*

2. *Write down the partial equation:* $y = -\frac{2}{3}x + b$

3. *Solve for b: Here's where we will use the x-intercept of* $2x + y = -8$*. To find the x-intercept for* $2x + y = -8$*, plug in* $y = 0$ *and solve for* x*. That should give you* $x = -4$ *as the x-intercept. You use* $y = 0$ *because the x-intercept always has a y-coordinate of zero.*

Our new line also is supposed to have -4 *as its x-intercept. So, if we plug in* $x = -4$*, we should get* $y = 0$*:*

$$\begin{aligned} y &= -\frac{2}{3}x + b \\ 0 &= -\frac{2}{3}(-4) + b \\ 0 &= \frac{8}{3} + b \\ -\frac{8}{3} &= b \end{aligned}$$

So for -4 *to be the x-intercept, b must be* $-\frac{8}{3}$*.*

4. *Write down the full equation:* $y = -\frac{2}{3}x - \frac{8}{3}$*.*

5. Solve the following systems of equations, and check your answers if possible. If there is no solution, write "no solution." Similarly, write "many solutions" if there are infintely many solutions.

(a) $\begin{cases} 2x + 2y = 4 \\ 3x + 4y = 5 \end{cases}$
Answer: $(x, y) = (3, -1)$

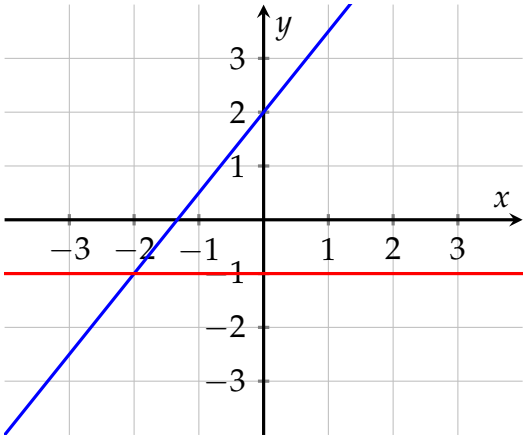
(b) $\begin{cases} x - 5y = -3 \\ -2x + 10y = 6 \end{cases}$
Answer: many solutions

(c) $\begin{cases} x - y = 1 \\ 3x - y = 2 \end{cases}$ *Answer:* $(x, y) = (\frac{1}{2}, -\frac{1}{2})$

6. (a) Solve this system of equations graphically:

$$\begin{cases} 3x - 2y = -4 \\ y = -1 \end{cases}$$

You don't have to check your answer.



Answer: $(x, y) = (-2, -1)$

Extra credit. This system of equations has three equations and three variables: x, y, and z:

$$\begin{cases} x + y + z = 2 \\ x - y + z = 4 \\ x - y - z = 0 \end{cases}$$

A solution to this system is a set of values for x, y and z that make all three equations true. Using the same methods we used for systems with two equations, try to solve this system.

Answer: $(x, y, z) = (1, -1, 2)$