Applications of 1st Order ODEs

1. Water containing 2 lb/gal of magnesium is entering a storage tank at a rate of 6 gal/sec. The magnesium mixes with the water and drains at a rate of 4 gal/sec. The storage tank originally contains 25 gallons of water, and 75 lb of magnesium.

   Find the quantity of magnesium present in the tank after t seconds. Find the limit of the *concentration* of magnesium as $t \to \infty$.

2. (2.3, #22a–b, modified) A ball with mass 0.1 kg is thrown upward with initial velocity 20m/s from a building 30 m high. There is a force due to air resistance of $v^2/4$, where the velocity v is measured in m/s. Use $g = 10$ m/s^2 for gravity.

   Find the velocity of the ball, $v(t)$, after t seconds have passed. Find the maximum height above the ground that the ball reaches.

3. An oil droplet of mass m falls through an electric field. There are three forces acting on it: gravity pulls it downward, the electric field exerts a constant force F downward, and there is a force due to air resistance of μv in the opposite direction of the droplet's velocity.

   Find the velocity of the droplet at time t, if the initial velocity is v_0.

4. The population of bacteria on a counter grows at a rate proportional to the current population. At time $t = 0$ minutes, there are 100 bacteria, while at $t = 1$ minute, there are 400 bacteria.

   (a) What is the population at $t = 3$?

   (b) Starting at $t = 3$ minutes, the bacteria begin to die at a rate of 50/minute. How many bacteria are there at $t = 5$?