Math 102 Quiz 1 — Solutions
January 6, 2012

Note: There's more than one way to solve a problem— I've just listed one way here. These aren't the only right methods.

1. (68pts) Solve each equation. Check your solutions.

(a)

$$\begin{aligned} 2a + 15 &= 5 \\ 2a &= -10 && \text{subtract 15 from both sides} \\ a &= -5 && \text{divide both sides by 2} \end{aligned}$$

(b)

$$\begin{aligned} -2a - 1 + 3 - 2a &= 7 \\ -4a + 2 &= 7 && \text{combine the } a\text{'s and use } -1+3=2 \\ -4a &= 5 && \text{subtract 2 from both sides} \\ a &= -\frac{5}{4} && \text{divide both sides by 4} \end{aligned}$$

(c)

$$\begin{aligned} \frac{1}{a} + 3 &= 5 \\ \frac{1}{a} &= 2 && \text{subtract 2 from both sides} \\ 1 &= 2a && \text{multiply both sides by } a \\ \frac{1}{2} &= a && \text{divide both sides by 2} \end{aligned}$$

(d)

$$\begin{aligned} \frac{2}{3}a + 10 &= -\frac{1}{3}a - 6 \\ \frac{2}{3}a &= -\frac{1}{3}a - 16 && \text{subtract 10 from both sides} \\ \frac{2}{3}a + \frac{1}{3}a &= -16 && \text{add } \tfrac{1}{3}a \text{ to both sides} \\ \frac{3}{3}a &= -16 && \text{add } \tfrac{1}{3} + \tfrac{2}{3} = \tfrac{3}{3} \\ a &= -16 && \tfrac{3}{3} = 1. \end{aligned}$$

2. (32pts) Solve each equation **for x**. Check your solutions.

(a)

$$\begin{aligned} 3x + y &= x + 2y \\ 2x + y &= 2y && \text{subtract } x \text{ from both sides} \\ 2x &= y && \text{subtract } y \text{ from both sides} \\ x &= \frac{y}{2} && \text{divide both sides by 2} \end{aligned}$$

(b)

$$\begin{aligned} \frac{1}{2}x + 3 &= x + y \\ \frac{1}{2}x &= x + y - 3 && \text{subtract 3 from both sides} \\ \frac{1}{2}x - x &= y - 3 && \text{subtract } x \text{ from both sides} \\ -\frac{1}{2}x &= y - 3 && \text{subtract } \frac{1}{2}x - x \text{ (more on this below)} \\ x &= -2(y-3) && \text{multiply both sides by } -2 \end{aligned}$$

To subtract $\frac{1}{2}x - x$:

$$\begin{aligned} &\text{remember there's an invisible "1":} && \frac{1}{2}x - 1x \\ &\text{convert the 1 into a fraction:} && \frac{1}{2}x - \frac{2}{2}x \\ &\text{now you can subtract (fractions have same number on the bottom):} && \frac{1-2}{2}x \\ &\text{simplify:} && -\frac{1}{2}x. \end{aligned}$$

Extra credit. (10pts) If you converted \$5 into euros, you would have three euros, with ninety U.S. cents left over.

How much is one euro worth, in dollars? Round to the nearest cent.

Let $e = 1$ euro, $d = 1$ dollar. We can write 90 cents as \$0.90. Then

$$\overbrace{5d}^{5 \text{ dollars}} = \overbrace{3e}^{3 \text{ euros}} + \overbrace{0.90d}^{\$0.90}$$

Now solve for e:

$$\begin{aligned} 5d &= 3e + 0.90d \\ 4.10d &= 3e && \text{subtract } 0.90d \text{ from both sides} \\ \frac{4.10d}{3} &= e && \text{divide by 3} \\ \frac{1.36}{d} &= e \end{aligned}$$

So one euro equals \$1.36.