Math 111 Final Exam *Name* ______________________

August 19, 2011

Honor Statement

"I affirm that my work upholds the highest standards of honesty and integrity, and that I have neither given nor received any unauthorized assistance on this exam."

Signature ______________________

1	13	
2	12	
3	13	
4	12	
total	50	

- Your test should have 4 problems on 4 pages (not including this one). Make sure that it does!
- If you can't solve part (a) of a problem, but need the answer to solve part (b), then write down *how* you would solve (b) if you did know the answer for (a).
- Cell phones and any electronic items that aren't calculators must be turned off and put away.
- **Important:** Always show your work, unless the problem says not to. Otherwise, you may not receive credit, even if your answer is correct. For graph problems, draw and label all lines you use. If you use your calculator to do the quadratic formula, you must indicate this.
- The good news: all problems can be solved algebraically or graphically.
- The bad news: as a result, you will not receive full credit for guess-and-check.
- You're allowed:
  - one double-sided $8\frac{1}{2}'' \times 11''$ sheet of notes
  - a ruler
  - a calculator
- Raise your hand if you have a question or can't understand a problem.
- If you'd like to pick up your final after grading, check the class website at the beginning of fall quarter.

1. *Note:* Each part of this problem is completely unrelated to the other parts. So, you can do them in any order.

(a) Claire has an investment earning 3% annually, compounded continuously. What is the APY for her investment?
You can use the APY formula for continuous compounding:

$$\mathrm{APY} = [e^{r} - 1] \cdot 100\% = [e^{0.03} - 1] \cdot 100\% = 3.0454\%.$$

(b) Your savings account earns 2% annually, compounded monthly. What's the present value of $1500 in 20 years?
The idea here is that you'll have $1500 in 20 years. There's two different time periods here (annually vs. monthly), so the 2% is an APR. Using the APR CAF, we get:

$$\begin{aligned} A &= P \cdot \left(1 + \frac{r}{n}\right)^{nt} \\ 1500 &= P \cdot \left(1 + \frac{0.02}{12}\right)^{12 \cdot 20} \\ 1500 &= P \cdot (1.0016667)^{240} \\ 1500 &= P \cdot (1.49132) \\ \$1005.81 &= P. \end{aligned}$$

P is the principal, or present value, so the answer is $1005.81.

(c) 10 years ago, Thurgood and Menander invested the same amount of money into two different accounts:

- Thurgood's account earns 2% annually, compounded annually;
- Menander's account earns 1.5% annually, compounded annually.

If Thurgood's account has $20 now, how much does Menander have?
We want to know how much Menander has now, but we don't know how much he invested 20 years ago. But we do know it was the same amount as Thurgood invested; let's call this amount $P. *We're given "real" annual interest rates (since the accounts are compounded annually), so we can use the CAF for Thurgood's account:*

$$\begin{aligned} A_{\text{Thurgood}} &= P(1 + r)^{k} \\ 20 &= P(1.02)^{10} \\ 20 &= P(1.21899) \\ 16.40697 &= P. \end{aligned}$$

So they both invested about $16.41. *Now we can use the CAF for Menander:*

$$\begin{aligned} A_{\text{Menander}} &= P(1 + r)^{k} \\ &= 16.40697 \cdot (1.015)^{10} \\ &= 19.04095 \end{aligned}$$

So the answer is $19.04.

2. You've been keeping a tab on the temperature inside and outside your room.

Let $I(t)$ stand for the inside temperature and $O(t)$ the outside temperature, where t is hours since midnight. You found the following equations for $I(t)$ and $O(t)$:

$$\begin{aligned} I(t) &= -\frac{1}{4}t^2 + 8t + 16 \\ O(t) &= -\frac{1}{2}t^2 + 12t + 8 \end{aligned}$$

(a) At what time(s) are the inside and outside temperatures equal?
We need to solve the equation $I(t) = O(t)$. *It's a quadratic equation, so the quadratic formula is the one to use:*

$$\begin{aligned} -\frac{1}{4}t^2 + 8t + 16 &= -\frac{1}{2}t^2 + 12t + 8 \\ (-\frac{1}{4}t^2 + 8t + 16) - (-\frac{1}{2}t^2 + 12t + 8) &= 0 \\ \frac{1}{4}t^2 - 4t + 8 &= 0 \\ \text{Q.F.} \qquad t &= \frac{4 \pm \sqrt{4^2 - 4(\frac{1}{4})(8)}}{\frac{1}{2}} \\ &= 2.3431,\ 13.6568 \end{aligned}$$

(b) When is the temperature difference $O(t) - I(t)$ greatest?
First, simplify the formula for $O(t) - I(t)$*:*

$$O(t) - I(t) = (-\frac{1}{2}t^2 + 12t + 8) - (-\frac{1}{4}t^2 + 8t + 16) = -\frac{1}{4}t^2 + 4t - 8$$

This is quadratic, so we can use the vertex formula to find when it is largest:

$$t = \frac{-b}{2a} = \frac{-4}{-\frac{1}{2}} = 8.$$

(c) Translate the following statements into functional notation. You don't have to check if they're true or not.

- "It's always three degrees colder inside than it is outside."

  translation: $I(t) = O(t) - 3$

- "The inside temperature after t hours is the same as the outside temperature three hours earlier."

  translation: $I(t) = O(t - 3)$

3. More interest problems:

(a) The flu has just begun to spread through campus. Every 2 days, the number of infected people doubles. What is the percentage increase in the number of infections over any 5-day period?
There's more than one way to do this problem. One approach is to think in two-day periods. Thinking this way, let $F(k)$ *represent the number of people infected after* k *two-day periods. Since the number of infections is doubling, we can write an explicit formula for* $F(k)$*:*

$$F(k) = F(0) \cdot 2^k$$

Now, we can think about the percentage change from $t = 0$ to five days later, which is $t = 5/2 = 2.5$:

$$\begin{aligned}
\text{old} &= F(0) \\
\text{new} &= F(2.5) = F(0) \cdot 2^{2.5} \approx 5.6569 \cdot F(0) \\
\text{\% change} &= \frac{\text{new} - \text{old}}{\text{old}} \\
&= \frac{(5.6569)F(0) - F(0)}{F(0)} \\
&= \frac{5.6569 - 1}{1} \qquad \text{(dividing both sides by } F(0)\text{)} \\
&= 4.6569.
\end{aligned}$$

To finish up, convert this to a percentage: 465.69%.

(b) You put \$50 in an account. It's compounded continuously, but you don't remember the APR. In $1\frac{1}{2}$ years you have \$60 in the account. At what time does your balance to reach \$100?
To figure out when you'd have \$100, you would use Pe^{rt} *and solve for* t*:*

$$\begin{aligned} A &= Pe^{rt} \\ 100 &= 50e^{rt}. \end{aligned}$$

Without knowing r*, we can't go further. But, you do know that you have* $\$60$ *after* 1.5 *years, so you can find* r*:*

$$\begin{aligned} 60 &= 50e^{r\cdot 1.5} \\ 1.2 &= e^{1.5r} \qquad \text{(dividing both sides by 50)} \\ \ln 1.2 &= 1.5r \\ r &= \frac{\ln 1.2}{5} = 0.12155 \end{aligned}$$

Now we can finish up:

$$\begin{aligned} 100 &= 50e^{rt} \\ 100 &= 50e^{0.12155t} \\ 2 &= e^{0.12155t} \\ \ln 2 &= 0.12155t \\ \frac{\ln 2}{0.12155} &= t. \end{aligned}$$

This gives $t \approx 5.703$ *years.*

(c) You put \$100 into Account U, which pays 1% interest, compounded continuously. Five years later, you move all your money to Account Q, which is compounded monthly. After another five years, you have \$125. What is Account Q's APR?
After five years in Account U, your balance will be:

$$A = Pe^{rt} = 100e^{0.01\cdot 5} = 105.1271.$$

This is the principal for Account Q. For account Q, we need to use the APR CAF, since we want to find its APR:

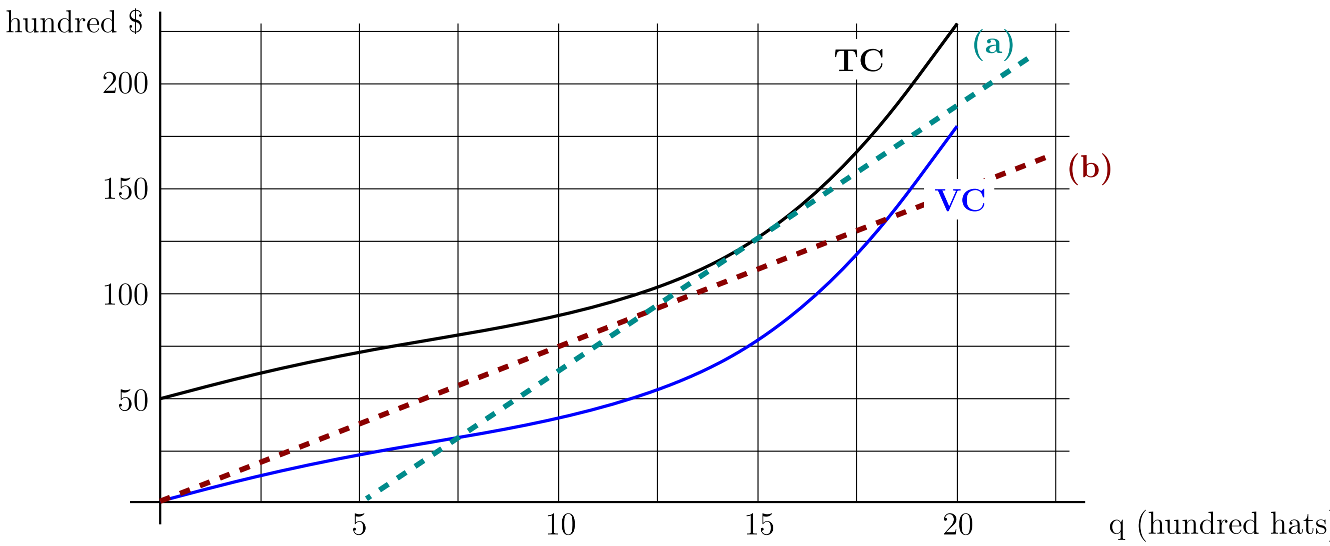
$$\begin{aligned} A &= P\left(1+\frac{r}{n}\right)^{nt} \\ 125 &= (105.1271)\left(1+\frac{r}{12}\right)^{12\cdot 5} \\ \frac{125}{105.1271} &= \left(1+\frac{r}{12}\right)^{60} \\ 1.189036781 &= \left(1+\frac{r}{12}\right)^{60}. \end{aligned}$$

There's a 60^{th} power, so take the 60^{th} root of both sides:

$$\begin{aligned}
\sqrt[60]{1.189036781} &= 1 + \frac{r}{12} \\
1.0028899 &= 1 + \frac{r}{12} \\
0.0028899 &= \frac{r}{12} \\
0.034679 &= r
\end{aligned}$$

Since we used the APR, the rate we got is the APR, 3.4679%.

4. You produce and sell rubber hats. Here's the graph of your total cost:
(Note that q is measured in hundreds.)



(a) What is the cost of producing the 1500th hat? *(careful with the units!)*
This is asking for marginal cost— how much extra it costs to produce that one hat. Marginal cost is the cost of producing one more *item, so we want to find* MC *at 1499 hats. Because the scale on the* x*-axis is so small, use a tangent line instead of a secant line to measure* MC *(see graph), and find its slope, which is about* 12.5.

Depending on how you drew the tangent line, your answer could have varied a fair amount in either direction. The units of MC *are hundred dollars per hundred items, or in other words, dollars per item.*

(b) Draw the VC graph. At what quantity q is *average variable cost* equal to $7.50/hat?
The VC *graph is the same as the* TC *graph, but shifted down to start at the origin.*

Average variable cost is an overall rate of change, so it's the slope of a diagonal line through VC. *We have a value for* AVC, *so draw a diagonal reference line with a slope of* 7.5, *and see where it intersects the graph, around* $q = 18$.

(c) Find the longest range of quantities where average cost is decreasing.
Be sure to explain how you found your answer.
Average cost is the slope of a diagonal line through TC. *Using the rolling ruler technique, measure the slopes of diagonal lines, going from* $q = 0$ *through* $q = 20$. *The slope decreases up to* $q \approx 12.5$, *and then increases again (this is related to the breakeven price. . .). So the answer is* $q = 0$ *to* $q = 12.5$.