Homework 6

Due Friday, August 12

Note: We skipped worksheet 16, so you don't have to do it. It won't be on the final, either.

Worksheet 17: Do 2, 3, 6–8, 10–15 in the book, and the versions of 16–18 below.
Worksheet 18: Do 3–11 in the book.
Worksheet 19: Do 1–13 in the book, and the version of 14 below.

Have questions? Drop in at office hours Tuesday through Thursday (see website for the info), or during Thursday's Q & A.

General hint: For sequence questions, it's often helpful to start by drawing a timeline like the ones in the book. This can help you get an idea of what's going. For some examples of timelines, see WS 18 #2 and WS 19 #7–9.

Worksheet #17

→ 16 (a) Suppose $A(k)$ is an additive sequence with increment 3.

i. Give a recursive formula for $A(k)$.

ii. $B(k)$ is a sequence with the following property:

$$B(0) = 5A(0) + 7, \quad B(1) = 5A(1) + 7, \quad B(2) = 5A(2) + 7, \ldots$$

Is $B(k)$ an additive sequence? If so, find its increment. If not, explain why not. *Hint: check whether $B(k+1) - B(k)$ is always the same number.*

iii. $B(2) = 32$. Compute the value of $B(50)$.

(b) Suppose $C(k)$ is a multiplicative sequence with multiplier 4.

i. Give a recursive formula for $C(k)$.

ii. $D(k)$ is a sequence with the following property:

$$D(0) = 3\sqrt{C(0)}, \quad D(1) = 3\sqrt{C(1)}, \quad D(2) = 3\sqrt{C(2)}, \ldots$$

Is $D(k)$ a multiplicative sequence? If so, find its multiplier. If not, explain why not. *Hint: check whether $\frac{D(k+1)}{D(k)}$ is always the same number.*

iii. $D(3) = 192$. Give an explicit formula for $D(k)$.

→ 17 We have two sequences: $F(0), F(1), F(2), \ldots$ and $G(0), G(1), G(2), \ldots$. The two sequences have the property that: The k^{th} term of the G-sequence is equal to the $(4k)^{\text{th}}$ term of the F-sequence. This can be restated as $G(k) = F(4k)$, or, equivalently,

$$\begin{array}{cccccccccc} F(0), & F(1), & F(2), & F(3), & F(4), & F(5), & F(6), & F(7), & F(8), & F(9), \ldots \\ \| & & & & \| & & & & \| & \\ G(0) & & & & G(1) & & & & G(2) & \end{array}$$

The multiplier of the F-sequence is m (where $m > 1$).

The multiplier of the G-sequence is n.

a) Write out an equation that relates the letters m and n. *Hint: to get from* $F(0)$ *to* $F(4)$, *what do you have to multiply by?*

b) In terms of the letter m, what number must you multiply $G(2)$ by to get $G(6)$?

c) In terms of the letter n, what must you multiply $F(4)$ by to get $F(7)$?

d) Let j be an integer. In terms of the letters m and j, tell what you must multiply $F(4)$ by to get $F(4+j)$. *Hint: try* $j=1$, $j=2$, *and* $j=3$. *What's the pattern?*

e) Put the symbol $<$, $>$, or $=$ in the boxes below:

$$F(12)/F(11) \quad \square \quad F(13)/F(12)$$
$$F(12)-F(11) \quad \square \quad F(13)-F(12)$$
$$F(16)/F(12) \quad \square \quad G(3)/G(2)$$

→ 18 Suppose we have a function $f(t) = K\cdot(1.4)^t$. The value of K varies in each of the questions below. However, in each case, you can either deduce the value of K, or you don't need to know K, and your answers should not involve K.

a) Suppose $f(1) = 10$. What is the change in the value of $f(t)$ as t goes from 1 to 3?

b) Suppose $f(3) = 200$. Find $f(0)$ and $f(5)$.

c) Suppose $f(0) = 20$. Find $f(7) - f(5)$.

d) By what factor is $f(1)$ multiplied to get $f(6)$?

e) What is the percentage change in the value of $f(t)$ as t goes from 3 to 5?

f) What is the proportionate change in $f(t)$ as t goes from 3 to 4?

Worksheet #19

→ 14 Mary and Charlie both have bank accounts. Mary's earns 5% per year and Charlie's earns 4% per year.

a) If Mary puts P dollars in her account now, how much must Charlie put in his in order for them to have identical balances in 6 years?

b) If Mary puts P dollars into her account now, how much should Charlie put into his account so that at the end of the 10^{th} year, his balance will be identical to the balance Mary has at the end of the 8^{th} year?

c) Suppose that Mary and Charlie put P dollars in each of their accounts now, but then in 3 years they switch, with Mary putting her money into what used to be Charlie's account and Charlie putting his money into what used to be Mary's account. Is it possible to tell who would have more money 6 years from now? Is it possible to tell who would have more money 7 years from now? Explain.

Hint: Start by making a timeline of the situation. Using the timeline, try to write down formulas for Charlie's and Mary's balance after 6 years and after 7 years.