Homework 2

CAAM 335 • Matrix Analysis • Spring 2016

Due Date: January 29, 4pm

Submission Instructions: Homework submission will be on OWL-Space, as with Homework 1. You can take a look at the Homework 1 problems page for details on the process.

You are welcome to collaborate with other CAAM 335 students, consult the textbook, and get help from an instructor or TA. For this assignment, you *may* use MATLAB, Octave, or another program to do your matrix computations. There is a handout on OWL-Space illustrating how to solve linear systems of equations with MATLAB/Octave if you are new to it.

Problem 1 Suppose we have a circuit with current source(s) but no voltage sources. Which of the following is the correct interpretation of the action of $A^T GA$?

a. If $\vec{e}$ is the vector of voltage drops, $A^T GA\vec{e}$ is the vector of the sum of currents going into each node (including current sources).

b. If $\vec{v}$ is the vector of voltages, $A^T GA\vec{v}$ is the vector of the sum of currents going into each node (including current sources).

c. If $\vec{z}$ is the vector of the sum of currents going out of each node (including current sources), $A^T GA\vec{z}$ is the vector of voltages.

d. If $\vec{v}$ is the vector of voltages, $A^T GA\vec{v}$ is the vector of the sum of currents going into each node (excluding current sources).

e. If $\vec{v}$ is the vector of voltages, $A^T GA\vec{v}$ is the vector of the sum of currents going out of each node (excluding current sources).

f. If $\vec{v}$ is the vector of voltages, $A^T GA\vec{v}$ is the vector of currents.

Problem 2 In this problem, you will analyze the circuit shown in Figure 1, which has a current source I and a voltage source E as well as resistors $R_1, \ldots, R_5$. You'll consider the voltages at points $v_1, \ldots, v_3$ and currents $i_1, \ldots, i_5$ across resistors $R_1, \ldots, R_5$. The voltages at the lower ends of R_1, R_4, and the source E are all zero, as these ends are grounded; the voltage at the top of the voltage source is the value of E.

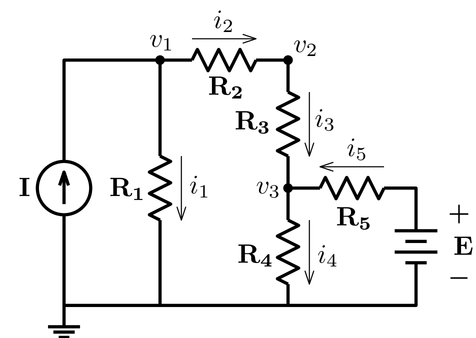

Figure 1: A circuit to analyze

Let $R_1 = 200\Omega$, $R_2 = 20\Omega$, $R_3 = 100\Omega$, $R_4 = 50\Omega$, and $R_5 = 100\Omega$. Let $I = 100\text{mA}$, and $E = 5\text{V}$.

i. As in class and the textbook, let $\vec{e} = -A\vec{v} + \vec{b}$, where $\vec{v} = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}^T$ is the vector of voltages, $\vec{e} = \begin{bmatrix} e_1 & \cdots & e_5 \end{bmatrix}^T$ is the vector of voltage drops, and $\vec{b}$ is the vector of voltage sources. Which of the following is A?

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$

ii. You should find the (1,1) entry of $A^T G A$ to be 0.055. What is the (3,3) entry of $A^T G A$? Express your answer as an exact decimal.

iii. Calculate i_4 and v_3. Then, calculate their values without the battery (i.e., let $E = 0\text{V}$). Express your answers in mA and V, rounded to the nearest integer.

Problem 3 Figure 2 shows a modified *Wheatstone bridge* (with a light R_5 replacing the voltmeter). The role of a Wheatstone bridge is to compare an unknown resistance R_4 against a known resistance R_3. Resistors R_1 and R_2 are also known and have equal values. With the light removed, it's possible to compare R_3 against R_4: if $R_3 > R_4$, then $v_1 > v_2$, and vice versa. Only if $R_3 = R_4$ will the two voltages be the same.

At least, that's what happens in the unmodified version of the Wheatstone bridge! In the following version, a light R_5, which acts as a resistor, has been placed across the bridge. How will the new circuit behave? As in problem 1, the voltages and currents of interest are marked out.

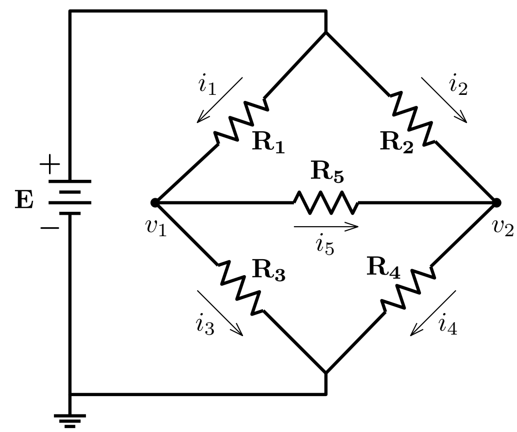

Figure 2: A Wheatstone bridge, with a light R_5

For questions (i–ii), let $R_1 = R_2 = 25\Omega$, $R_3 = 10\Omega$, $R_4 = 20\Omega$, and $R_5 = 50\Omega$. Let $E = 10\text{V}$.

i. You should find the (1,1) entry of $A^T GA$ to be 0.16. What is the (2,2) entry? Express your answer as an exact decimal.

ii. Compute the current and voltage drop across the light R_5. Your answers will be positive if (conventional) current is flowing with the arrow, and negative if current is flowing against it. Express your answers in mA and V, rounded to the nearest integer.

For questions (iii–vi), let $R_1 = R_2 = 10\Omega$, $R_5 = 50\Omega$, and $E = 10\text{V}$. Let R_3 be a fixed, but unknown value; R_4 will be variable.

iii. Which of the following matrices is $A^T GA$?

(a) $\begin{bmatrix} 0.02 + 1/R_3 & -0.02 \\ -0.02 & 0.02 + 1/R_4 \end{bmatrix}$

(b) $\begin{bmatrix} 0.08 + 1/R_4 & 0.02 \\ 0.02 & 0.08 + 1/R_3 \end{bmatrix}$

(c) $\begin{bmatrix} 0.08 - 1/R_3 & 0.02 \\ 0.02 & 0.08 - 1/R_4 \end{bmatrix}$

(d) $\begin{bmatrix} 0.12 + 1/R_3 & -0.02 \\ -0.02 & 0.12 + 1/R_4 \end{bmatrix}$

iv. If there is zero current across the light, what is the ratio R_4/R_3?

v. If $R_3 = 2R_4$, what is the voltage drop $v_1 - v_2$ in terms of R_4?

(a) $\dfrac{500R_4}{7R_4^2 + 180R_4 + 1000}$

(b) $\dfrac{500R_4}{14R_4^2 + 180R_4 + 500}$

(c) $\dfrac{250R_4}{14R_4^2 + 90R_4 + 500}$

(d) $\dfrac{200}{7R_4^2 + 180R_4 + 1000}$

vi. Suppose the light requires a voltage of at least $\pm 1\text{V}$ to illuminate it. Imagine varying the resistance R_4, starting with a very low value (much smaller than R_3) and then steadily increasing it. R_3, meanwhile, is fixed.

At what value of R_4 will the light turn off? At what value will it turn back on? The answers to both questions are in the following list:

(a) $\dfrac{440R_3 + 500}{560 - 10R_3}$

(b) $\dfrac{440R_3 + 500}{440 + 10R_3}$

(c) $\dfrac{560R_3 + 500}{440 - 7R_3}$

(d) $\dfrac{440R_3 - 500}{560 + 14R_3}$

(e) $\dfrac{440R_3 - 500}{7R_3 + 560}$

(f) $\dfrac{560R_3 - 500}{440 - 14R_3}$