Solving Equations with Square Roots

In a typical square root problem, you have to solve an equation like

$$4t - \sqrt{t} - 6 = 2\sqrt{t}.$$

Fortunately, you can use a single method to solve almost all square root problems you'll see. Below, the steps are listed on the left side, and the right side shows how to solve the example equation.

Step 1. Move the square roots onto one side, and everything else onto the other side.

$$4t - \sqrt{t} - 6 = 2\sqrt{t}$$
$$4t - 6 = 2\sqrt{t} + \sqrt{t}$$
$$4t - 6 = 3\sqrt{t}$$

Step 2. Square both sides.

$$(4t - 6)^2 = (3\sqrt{t})^2$$
$$16t^2 - 48t + 36 = 9t$$

(using FOIL)

Step 3. Solve the equation you just got.
(Often, it will be a quadratic equation, so you'll use the quadratic formula.)

$$16t^2 - 48t + 36 = 9t$$
$$16t^2 - 57t + 36 = 0$$

$$Q.F.: \quad t = \frac{57 \pm \sqrt{(-57)^2 - 4 \cdot 16 \cdot 36}}{2 \cdot 16}$$
$$t \approx 0.821, \ 2.741$$

Step 4. Plug the values of t you found into your original equation and check whether they work.

$$4(2.74) - \sqrt{2.74} - 6 = 3.31 = 2\sqrt{2.74} \ \checkmark$$
$$4(0.82) - \sqrt{0.82} - 6 = -3.63$$
$$\text{but } 2\sqrt{0.82} = 1.81 \ \times \ \text{not a solution}$$
So t = 2.74 is the only solution.

Notice that only *one* of the "solutions" we found from the quadratic formula was really a solution. This is normal when square roots are involved, and doesn't mean we made a mistake. In fact, it's possible to get two answers from the quadratic formula, and find that neither of them solve the original equation. (But this situation shouldn't happen on the homework or on a test.)

Here's another example. We'd like to find the value(s) of t that solve the following equation.

$$2t - \sqrt{t} = 5t - 6$$

Following these steps, we get:

Move square roots to one side:	$2t - \sqrt{t} = 5t - 6$
	$-\sqrt{t} = 3t - 6$
Square both sides:	$t = (3t - 6)^2$
	$t = 9t^2 - 36t + 36$
Solve (using quadratic formula):	$0 = 9t^2 - 37t + 36$
	$t = \dfrac{37 \pm \sqrt{37^2 - 4 \cdot 9 \cdot 36}}{2 \cdot 9}$
	$t \approx 1.58, \ 2.53$
Check the solutions:	$2(1.58) - \sqrt{1.58} = 1.90 = 5(1.58) - 6 \ \checkmark$
	$2(2.53) - \sqrt{2.53} = 3.47$
	but $5(2.53) - 6 = 6.65 \ \times$ not a solution

Practice Problems

Try solving each of these equations:

1. $x + 5\sqrt{x} - 1 = 25 + 2\sqrt{x} - 3$

2. $2\sqrt{x} = 3 = 5x$

3. $x + 8\sqrt{x} - 2 = 15$

4. $7 + 5\sqrt{x} = 8 + \sqrt{x} + x$

5. $1 + \sqrt{x} = 6\sqrt{x} - 7$

6. $x = \sqrt{x + 6}$

7. $2 = \dfrac{1}{\sqrt{x}}$

Alternative Method

If you're feeling adventurous, an alternative method can sometimes be faster. The idea is replace $\sqrt{t}$ (or $\sqrt{x}$ or whatever square root you have) with a new variable. For our example, we'll call this variable w.

So we will say that $w = \sqrt{t}$. If this is true, then $w^2 = (\sqrt{t})^2 = t$. Going back to our first example, let's substitute w's for the t's:

$$4t - \sqrt{t} - 6 = 2\sqrt{t}$$
$$4w^2 - w - 6 = 2w$$

At this point, we have a quadratic equation, and we can use the quadratic formula. This tells us what w has to be:

$$4w^2 - 3w - 6 = 0$$
$$w = \frac{3 \pm \sqrt{3^2 - 4(4)(-6)}}{2 \cdot 4}$$
$$w \approx -0.91,\ 1.66$$

We're not done yet— these are values of w, not values of t. Now, w is supposed to be a square root, and square roots are never negative. So $w = 1.66$ is our only possibility. Then, remembering that $t = w^2$, we get $t = (1.66)^2 = 2.74$.