1. Solve the initial value problem

$$\frac{1}{t}y' + y - 3 = 0.$$

2. Determine explicitly all the solutions to the differential equation

$$(1 + t^2)y' + y = 1.$$

3. Let $P(t)$ be the population of fish in Green Lake at time t. Suppose that fish are harvested at a constant rate E from the total population, so that the population is given by the differential equation:

$$\frac{dP}{dt} = r\left(1 - \frac{P}{K}\right)P - EP$$

where K and r are positive constants, and $E \geq 0$ is a nonnegative constant.

(a) Assume that $E < r$. Determine all the equilibrium solutions to this equation and classify them as stable, unstable, or semistable.

(b) Sketch the direction field without solving the differential equation.

(c) How does your answer to part (a) change if instead we assume $E > r$?

(d) Solve the differential equation if $K = 1$, $r = 4$, and $E = 2$, with the initial condition $P(0) = 100$.

4. Initially, a tank contains 6 gal of water containing 1 lb of salt. There is water flowing into the tank through two pipes: Water containing salt is entering the tank through the first pipe at rate of 2 gal/min. Several measurements indicate that the amount of salt contained in one gallon of the incoming water is $e^{-3t/2}$ lb at time t. One gallon of fresh water per minute is entering the tank through the second pipe. Finally, the well-stirred mixture is draining the tank at a rate of 3 gal/min.

Determine the amount of salt at any time $t \geq 0$.

5. A mass is attached to a spring. Its velocity v is given by the initial value problem

$$\frac{dv}{dt} = -2(x-3), \quad v = 4 \text{ when } x = 4$$

where x is the position of the mass. Eliminate t from the differential equation so it only involves v and x, and solve it.

What is the domain of $v(x)$?