Midterm 2 Topic Sheet

Equation-Solving Methods

- ◇ Know how to find the general solution for the *homogeneous equation* $ay'' + by' + cy$, for any numbers a, b, c.
  - Remember the three cases: distinct real roots (§3.1), repeated (single) root (§3.4), and complex roots (§3.3), and their general solutions.
  - Once you've found the roots of the characteristic equation, you can just write down the general solution. When we first discussed the general solutions in class (the lectures from 2/1–2/6) we went through some of the details of where the general solutions came from, but you don't need to include these details on the midterm. For instance, if you've found the roots of the characteristic equation are $3 \pm 2i$, you can immediately write down that $u(t) = c_1 e^{3t}\cos(2t) + c_2 e^{3t}\sin(2t)$ is the general solution, without explaining where it came from (but it's good for you to know!)
- ◇ Know how to use *reduction of order* to find a second solution to a linear 2nd-order ODE (not necessarily constant coefficient) once you've already found one solution. (§3.4 p.170)
- ◇ Know how to find one solution to the *nonhomogeneous equation* $ay'' + by' + cy = g(t)$ using the *method of undetermined coefficients* (§3.5):
  - Know how to use $g(t)$ to write down a template for the particular solution $Y(t)$.
  - When do you have to multiply terms in $Y(t)$ by t?
  - Know how to solve for the undetermined coefficients (tip: this part often takes a lot of algebra and is not as important as other parts, so leave it for last on the exam.)
- ◇ Know how to find the general solution to a nonhomogeneous equation once you have a particular solution. (§3.5)
- ◇ Know how to use the initial conditions to solve for the constants in the general solution (both homogeneous & nonhomogeneous equations). (§3.1 ex. 1, 3, §3.8 ex. 1)

Applications

- ◇ Spring-mass systems: Know how to set up an ODE for the motion of a mass on a spring from a description of it: (§3.7 ex. 1)
  - Know the meaning of the unknown function $u(t)$.
  - Know how to find k given the distance the mass stretches the spring.
  - Know how to find the damping coefficient γ if you are given the damping force when the mass is traveling at a certain velocity.
  - Know the differences between using the metric (kilograms, meters) and U.S. (pounds, feet) unit systems. Remember you can get mass from a measurement in pounds by dividing by $g = 32\text{ft/s}^2$.

Oscillating Systems

- ◇ Know the terms *phase*, *amplitude*, *frequency*, *period*, *quasi-frequency*, *quasi-period*, *Q factor*, and know how to find them for a given problem. (§3.7, except Q factor)
  - Know which of these terms apply to damped systems, and which to undamped systems.
- ◇ Know what the *transient solution* and *steady-state solution* are, and how to find them. (§3.8 ex. 1)
- ◇ Know the terms *critical damping* and *overdamped*. (§3.7 p.199)
- ◇ Know how to rewrite a sum of sines and cosines with the same frequency into a single cosine term of the form $A\cos(\omega t - \delta)$. (For instance, know how to rewrite $4\sin 3t - 8\cos 3t$ in the form $A\cos(3t - \delta)$.) (§3.7 ex. 2)
- ◇ Know how to find the time when a system passes the equilibrium position. (If $u(t)$ is the solution, this is the t value where $u(t) = 0$.) (§3.7 ex. 3)