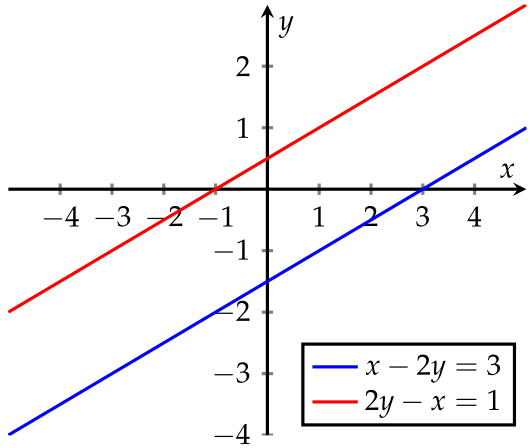
Solving systems of equations — the weird cases

Usually, a system of linear equations has just one solution. However, some systems have many solutions, and others have no solutions. Two examples showed up in exercise set 36 IV.

36 IV(e) In (e), you're given $\begin{cases} x - 2y = 3 \\ 2y - x = 1. \end{cases}$

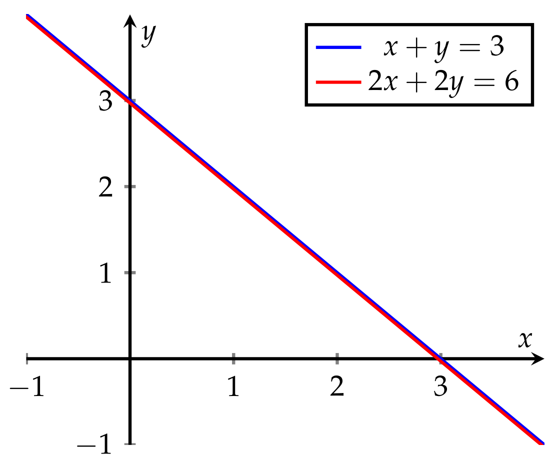


The graphs of these equations are parallel lines. The solution of a system is supposed to be where the lines cross— since they don't cross, that means there's no solution.

What happens if you try to solve it algebraically? The key is that **when you eliminate one variable, the other goes away, and and you get a false equation**. For instance, if we add the two equations, we can eliminate y, but x also goes away:

$$\begin{array}{r} x - 2y = 3 \\ +\ 2y - x = 1 \\ \hline 0 = 4 \end{array}$$

36 IV(f) Part (f) gives you $\begin{cases} x + y = 3 \\ 2x + 2y = 6. \end{cases}$



These equations turn out to have the same graph. Now, *every* point on the line is a solution to the system.

Now, let's what happens if you solve it algebraically.

$$\begin{array}{rcr} x + y = 3 & \xrightarrow{\times(-2)} & -2x - 2y = -6 \\ 2x + 2y = 6 & \rightarrow & 2x + 2y = 6 \\ \hline & & 0 = 0 \end{array}$$

The key here is that **when you eliminate one variable, the other goes away, and and you get a *true* equation**.