Group Project: Modelling Real-Life Problems

Intro: Often in Math 111, we're given nice equations for total revenue, or distance travelled, or bacteria populations. But real-world problems aren't so considerate and usually don't come with a nice equation printed on the side. But that's not to say that we can't apply algebraic techniques. In this project, you'll look at three examples of how data from real situations can be processed into algebraic formulas.

Part 1: A flood sweeps through your town in mid-February. You take a few measurements of the water level at three different times. Let $F(t)$ stand for the river level in feet t hours after midnight on February 12. Your measurements are:

$$F(10) = 20.7 \text{ ft}$$
$$F(50) = 21.5 \text{ ft}$$
$$F(100) = 18.0 \text{ ft}$$

You'd like to get a reasonable equation for $F(t)$ that matches your data, so you can fill in the gaps in your data, and predict the water level in the near future.

Of course, there's lots of possible equations for $F(t)$. But the water level first rises, then falls, so it looks like the graph of $F(t)$ might roughly resemble a parabola. In that case, $F(t)$ would be a quadratic function, and would have the form $F(t) = at^2 + bt + c$.

- Using the data, find a, b, and c.
  Hint: Start with the template formula $F(t) = at^2 + bt + c$. If you plug 10 into this formula, you should get 20.7. That gives you one equation relating a, b, and c. The other two data points will give you two more equations.
- Graph $F(t)$ and the data points: $(10, 20.7)$, $(50, 21.5)$, $(100, 18.0)$.

Now that we have a formula for $F(t)$, we can use it to estimate the flood levels at other times. Estimating levels between $t = 10$ and $t = 100$ (inside of the time range where data was collected) is called *interpolation*. On the other hand, estimating levels for $t < 10$ (in the past) or $t > 100$ (in the future) is called *extrapolation*. Both are extremely useful concepts, and have lots of applications. For instance, if we determined the average cost at a few different quantities, we could use interpolation to estimate AC between those quantities.

- Suppose that "major flood stage" occurs when the water is higher than 21 feet. At what time did major flood stage start? (Here, you're interpolating.)
- Suppose that the river level is normally below 15 feet. When will the water level return to normal, approximately? (Here, you're extrapolating from your data.)
- Of course quadratic functions aren't your only possibilities. Try to think of a few ways you could get a more accurate equation. Would your methods require more data?

Part 2: In real life, measurements are never exact, so to compensate, we often take a number of measurements, to get the best idea of what's going on. Here's some hypothetical data for temperature ($T(t)$) and humidity ($H(t)$) t hours after midnight:

	$T(t)$	$H(t)$
0	59	51
5	66	46
10	71	36
15	72	30
20	62	25

- Graph the temperature and humidity data. One set of data should look roughly linear, and the other should look more like a parabola.
- Try to find equations for $T(t)$ and $H(t)$ that match the data well. Your equations will probably not match the data exactly, but that's to be expected. After all, not even the data is perfect.
- In situations like this, where the equations are only approximate, it might be hard to decide whether one equation matches the data better than another. Think of a method which would allow you to objectively measure the discrepancy between your equation and the data.

Part 3: In some ways, analyzing radioactive decay is similar to analyzing interest. For instance, the radioactive isotope iodine-131 has a half-life of around 8 days[1]so the amount of iodine-131 in a sample will decrease by 50% every 8 days. The following data, obtained online[2], shows radiation levels, $R(t)$, in Kadobe Naka City, Japan on March 17, 2011, following the Fukushima I nuclear accident:

t (hours after midnight, UTC)	$R(t)$, in nGy/h
0	583.7
1	577.8
2	573.5
3	570.0
4	565.7
5	562.0

- Is the sequence $R(t)$ multiplicative? Is it even close to being multiplicative? Explain your answer, using algebra.
- If conditions were perfect and there were only one isotope decaying, the radiation sequence $R(t)$ would be exactly multiplicative. Find a multiplicative sequence $F(t)$ that matches $R(t)$ as well as possible.
  Hint: start by deciding what the multiplier of this sequence should be.
- Find an explicit formula for the multiplicative sequence $F(t)$. Use it to estimate the radiation level one day later ($t = 24$).
- Doing any long-term extrapolation from such limited data will be very inaccurate, for a number of reasons. If it were accurate, how many days would it be before radiation levels return to normal (0.15 nGy/h)?

[1] Iodine | Radiation Protection, `http://epa.gov/radiation/radionuclides/iodine.html`.

[2] Obtained from `www.sendung.de/japan-radiation-open-data/`.