Setting up Volume Integrals

For volume problems, setting up the integral is usually the hardest part of the problem. Here's the basic outline:

- Draw a sketch
- Decide which variable to integrate over
- Decide what method to use (washers vs. shells)
- Find all the information you need (radius, height,. . .)

Let's look at each part in more detail:

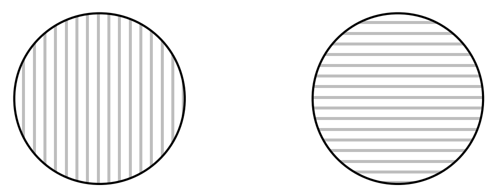
Draw a Sketch

For most problems, they'll give you fairly simple functions for the region: straight lines, parabolas, or circles. Otherwise, there's always the tried and true method of plugging in points to graph the functions you're given. Calculate any points of intersection that you don't know.

Deciding on the Integration Variable

Usually, you'll have to decide whether to integrate over x or over y.

It all depends on your region. Integrating over x means chopping up your region into vertical slices. Integrating over y gives you vertical slices.

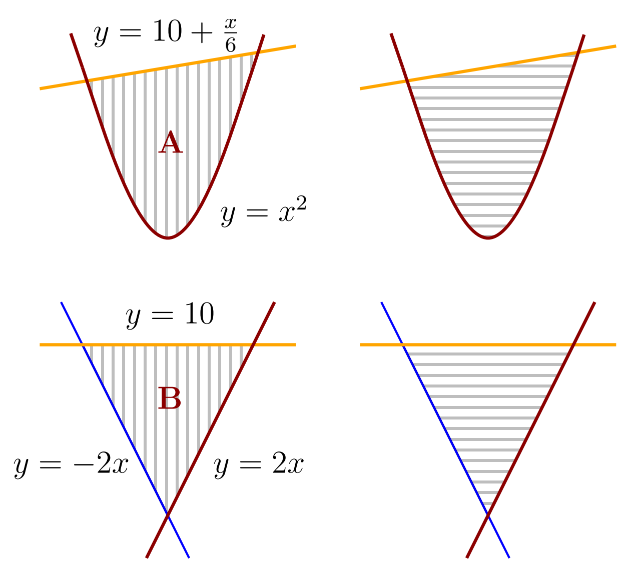


Integrating over x Integrating over y

Which one should you choose? For integrating over x, look at the *top* and *bottom* functions. For y, look at the *right* and *left* functions.

- Look at region A. If we integrated over x, the parabola is the top function and the line is the bottom function. If we chose y, the parabola is the right function, but there are two functions on the left: the parabola *and* the line. We'd have to break up the region into two pieces, one for each part.

- For region B, it's the opposite way around. Integrating over x would be harder because there are two bottom functions (blue and red). Integrating over y is much easier.



Once you choose your integration variable, *write everything in terms of it.*

Choosing Between Washers and Shells

For this part, look at the axis of rotation. If your slices are *parallel* to the axis of rotation, you'll use the shell method. If they are *perpendicular* to the axis of rotation, use washers.

Writing Down the Integral

Now we're getting closer to the end. To write down an integral for the volume of the solid, you can follow these recipes:

Shells

$$V = \int_a^b 2\pi(\text{radius})(\text{height})\, d\_\_$$

For the shell method, you need three pieces of information:

Radius: This is the distance from the *shell* to the *axis of rotation*. Often, the radius is just x or y, if you're rotating around the y- or x-axis.

If you're not rotating around the x- or y-axis, it will always be something different— for instance, $x-5$, $10-x$, or $y-3$.

Height: If you integrate over x, the height is

[top function] – [bottom function].

For y, the height is

[right function] – [left function].

Limits: You'll get these from your sketch. If you integrate over x, calculate the smallest and largest values of x in your region. Similarly for y. Notice that this *doesn't* depend on the axis of rotation!

Washers

$$V = \int_a^b \pi\left[(\text{outer radius})^2 - (\text{inner radius})^2\right] d\_\_$$

You'll also need three pieces of information for the washer method:

Radii: The inner radius is the function nearest to the axis of rotation. If there's only one function (no hole in the "washer") this is 0. The outer radius is the farther function.

Warning—You need to make one change if the rotation axis isn't the x- or y-axis. If you rotate around $x = a$, then

$$\text{inner radius} = \text{inner function} - a,$$
$$\text{outer radius} = \text{outer function} - a.$$

Same thing if you rotate around $y = a$.

Limits: Just like for shells. If you're integrating over x, calculate the smallest and largest values of x in the region. Similarly for y. Again this does not depend on the axis of rotation.