Midterm 2 Review Sheet

Fractional Exponents

1. Evaluate:

(a) $4^{\frac{5}{2}}$ (b) $(-27)^{\frac{2}{3}}$

(c) $(-8)^{-\frac{1}{3}}$ (d) $16^{-\frac{1}{4}}$

⁕ It's worth looking back at your notes to see how we evaluated fractional exponents.

Radicals

2. Evaluate:

(a) $\sqrt{3^2}$ (b) $\sqrt{(-3)^2}$

3. If $n = 4$ and $x = 2$, evaluate $\sqrt[n]{4^x}$.

4. Fill in the blanks. *(Tip: you don't have to do any calculations. For some of these, the answer is a root.)*

(a) $\sqrt{17} \cdot \sqrt{17} = \boxed{}$ (b) $(\sqrt[3]{2})^3 = \boxed{}$

(c) $\left(\boxed{}\right)^2 = 9$ (d) $\left(\boxed{}\right)^2 = 7$

(e) $\left(\sqrt[3]{q}\right)^3 = \boxed{}$ (f) $\left(\sqrt[5]{m^2}\right)^5 = \boxed{}$

(g) $\sqrt[5]{x^5} = \boxed{}$ (h) $\sqrt[2]{z^2} = \boxed{}$

(i) $\sqrt{a} \cdot \boxed{} = a$

Simplifying Radical Expressions

5. Simplify these radical expressions. Watch out for absolute values and imaginary numbers!

(a) $\sqrt{50x^7}$ (b) $\sqrt[4]{-80}$

(c) $\sqrt[3]{-24x^7}$ (d) $\sqrt[3]{x^2y} \cdot \sqrt[3]{-x^2y^6}$

(e) $\frac{\sqrt{90t^5u^2}}{\sqrt{30t^4u}}$ (f) $\sqrt{\frac{81}{49m^3}}$

(g) $\frac{\sqrt[3]{2^5 \cdot y^3}}{\sqrt[3]{2y}}$ (h) $\sqrt[8]{(5xy)^{14}}$

(i) $\sqrt[2]{2 \cdot \sqrt[3]{8x^9}}$ (j) $\sqrt{12} \cdot \sqrt{8} \cdot \sqrt{14}$

(k) $\sqrt[3]{2m^3a^6}$ (l) $\sqrt[2]{(-2)^2m^3a^6}$

(m) $\sqrt{27}$ (n) $\sqrt[3]{-96}$

⁕ If there are multiple roots involved, try to combine them. You can use the rules

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}, \qquad \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}.$$

⁕ Conversely, if you have a fraction inside a root, try splitting it apart, then simplifying. For instance,

$$\sqrt{\frac{100x}{2y}} = \frac{\sqrt{100x}}{\sqrt{2y}} = \frac{10\sqrt{x}}{\sqrt{2y}}.$$

When you're done, combine any remaining roots together:

$$\frac{10\sqrt{x}}{\sqrt{2y}} = \frac{10}{1} \cdot \frac{\sqrt{x}}{\sqrt{2y}} = 10 \cdot \sqrt{\frac{x}{2y}}$$

⁕ Use absolute values when pulling a variable out of a root with **even index**:

$$\sqrt[2]{x^7} = |x|^3 \cdot \sqrt{x}.$$

When pulling out a complicated factor, absolute values go around the entire factor:

$$\sqrt[2]{(3x+1)^7} = |3x+1|^3 \cdot \sqrt{3x+1}$$

Radical Rules

6. Which of the following equations are true?

(a) $\sqrt{2+3} \stackrel{?}{=} \sqrt{2}+\sqrt{3}$ (b) $\sqrt{5^2+4^2} \stackrel{?}{=} 9$

(c) $\sqrt{2\cdot 3} \stackrel{?}{=} \sqrt{2}\cdot\sqrt{3}$ (d) $\sqrt{5^2\cdot 4^2} \stackrel{?}{=} 5\cdot 4$

⁕ There's a list of radical and exponent rules on the last page.

Solving Radical and Power Equations

7. Solve the following equations and check your solutions.

(a) $2\sqrt{5+x}=4$ (b) $(3-z)^2=49$

(c) $3+\sqrt[3]{2r+6}=1$ (d) $\sqrt{2+(4c)^2}=\sqrt{18}$

(e) $\left|3+5x^3\right|=2$ (f) $-(2q)^2=-25$

(g) $\sqrt[3]{\frac{w^3}{8}}=2$ (h) $(x-5)^2=-3$

(i) $z^4=2^4$ (j) $(5y+2)^2-4=0$

(k) $\sqrt{(2-a)^3}=8$ (l) $\sqrt{5m}=\sqrt{2-3m}$

(m) $\sqrt{\frac{1}{s}}=4$

The Whys and Hows of Absolute Values

Sometimes absolute values come with an equation, and sometimes you need to include them yourself. Here's how to negotiate the situation.

When absolute values appear in an equation:

⁕ Think of absolute values as walls: you can't move anything past them until you break down the wall. When solving equations, this means you *must* deal with the absolute values before taking of care of what's inside them.

⁕ To deal with absolute values, split the equation in two, removing the absolute values:

$$|x+3|=5$$

$$\swarrow \qquad\qquad \searrow$$

$$x+3=5 \qquad\qquad x+3=-5$$

When you need to add absolute values:

⁕ When **you take a root (with even index)** of both sides of an equation, add absolute values:

$$(x+2)^4=16$$
$$\sqrt[4]{(x+2)^4}=\sqrt[4]{16}$$
$$|x+2|=2$$

Reason: An even root (like a square root) is always positive. We don't know if x is negative or positive, so make it positive by adding absolute values.

⁕ After adding absolute values, you must deal with them by splitting the equation.

Absolute Values

8. (a) Graph $y = 2|x-2| - 3$.

(b) Like most absolute value graphs, this graph has a "V" shape. Find the coordinates of the corner of the V. (If you don't see a V, plug in larger numbers for x.)

(c) Now solve $2|x-2| - 3 = 1$ for x. Locate these x coordinates on your graph. Where does the line $y = 1$ cross your graph?

9. Calculate:

(i) $\Big| -4 - |-7+2| \Big| + 1$ (ii) $2\Big|(1-4)^2\Big|$

Exponents

10. Find the natural form of:

(a) 7^{-2} (b) $\left(-\frac{2}{3}\right)^{-2}$

(c) $5^3 \cdot 5^{-2}$ (d) $((-2)^3)^2$

11. Fill in the blanks:

(a) $3^{\square} \cdot 3^2 = 1$ (b) $(y^3)^{\square} = y^6$

12. Evaluate each of these powers. Are any of them the same?

(a) 2^3 (b) 2^{-3}

(c) $(-2)^3$ (d) $(-2)^{-3}$

Combining Radicals

Note: We will talk about these problems in class on Tuesday. Until then, don't worry about them.

13. Simplify the following expressions, and combine like terms.

(a) $5\sqrt{2} + 3(\sqrt{7} - \sqrt{2}) - 4\sqrt{7}$ (b) $3\sqrt{12} + 2\sqrt{18} - \sqrt{3}$

(c) $\sqrt{3}(2 + \sqrt{3} + \sqrt{27})$ (d) $\sqrt{5}(\sqrt{3} - 4\sqrt{2}) + \sqrt{10}$

(e) $\frac{\sqrt{6}}{\sqrt{2}} + \sqrt{27}$ (f) $\sqrt{8x} - \sqrt{2x}$

Exponent Rules

Rule	*Example*
$n^0 = 1$ if n is not zero	$3^0 = 1$
$n^{-a} = \dfrac{1}{n^a}$	$x^{-2} = \frac{1}{x^2}$
additive law: $n^a n^b = n^{a+b}$	$13^2 \cdot 13^3 = 13^5$
multiplicative law: $(n^a)^b = n^{ab}$	$(y^{-\frac{2}{3}})^6 = y^{(-\frac{2}{3}\cdot 6)} = y^{-4}$
$(a \cdot b)^n = a^n \cdot b^n$	$(2x)^2 = 2^2x^2 = 4x^2$
$\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$	$\left(\dfrac{3}{4}\right)^3 = \dfrac{3^3}{4^3} = \dfrac{27}{64}$

Radical Rules

Rule	*Example*
multiplicative law: $\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$	$\sqrt[2]{9 \cdot 5} = \sqrt[2]{9} = \sqrt[2]{5} = 3 \cdot \sqrt[2]{5}$
	$\sqrt[3]{4} \cdot \sqrt[3]{16} = \sqrt[3]{4 \cdot 16} = \sqrt[3]{2^2 \cdot 2^4} = \sqrt[3]{2^6} = 2^2.$

Converting Radicals/Exponents

Rule	*Example*
$\sqrt[n]{a} = a^{\frac{1}{n}}$	$\sqrt[7]{11} = 11^{\frac{1}{7}}$
$\sqrt[n]{a^b} = a^{\frac{b}{n}}$	$(-15)^{\frac{2}{3}} = \sqrt[3]{(-15)^2}$

Answers

1. (a) 32 (b) 9 (c) $-\frac{1}{2}$ (d) $\frac{1}{2}$

2. (a) 3 (b) 3

3. 2

4. (a) 17 (b) 2 (c) 3 or $\sqrt{9}$ (d) $\sqrt{7}$ (e) q (f) m^2 (g) x (h) z (i) $\sqrt{a}$

5. (a) $5|x|^3 \cdot \sqrt{2x}$ (b) imaginary! (c) $-2x^2 \cdot \sqrt[3]{3x}$ or $2x^2 \cdot \sqrt[3]{-3x}$ (d) $xy^2 \cdot \sqrt[3]{-xy}$ or $-xy^2 \cdot \sqrt[3]{xy}$ (e) $\sqrt{3tu}$ (f) $\frac{9}{7m\sqrt{m}}$ (g) $2\sqrt[3]{2y^2}$ (h) $|5xy| \cdot \sqrt[8]{(5xy)^6}$ (i) $2|x| \cdot \sqrt{x}$ (j) $8\sqrt{21}$ (k) $ma^2 \cdot \sqrt[3]{2}$ (l) $2|m|\,|a^3| \cdot \sqrt{m}$ (m) $3\sqrt{3}$ (n) $2\sqrt[3]{-12}$ or $-2\sqrt[3]{12}$.

6. Only (c) and (d) are true.

7. (a) $x = -1$ (b) $z = -4$, $z = 10$ (c) $r = -7$ (d) $c = -1$, $c = 1$ (e) $x = -1$, $x = \sqrt[3]{-\frac{1}{5}}$ (f) $q = \frac{5}{2}$, $q = -\frac{5}{2}$ (g) $w = 4$ (h) no solutions (imaginary) (i) $z = 2$, $z = -2$ (j) $y = 0$, $y = -\frac{4}{5}$ (k) $a = 0$ (l) $m = \frac{1}{4}$ (m) $s = \frac{1}{16}$

8. (b) $(2, -3)$ (c) solutions: $x = 0$, $x = 4$. The line $y = 1$ crosses the graph at these x-coordinates.

9. (i) 10 (ii) 18

10. (a) $\frac{1}{49}$ (b) $\frac{9}{4}$ (c) 5 (d) 64

11. (a) -2 (b) 2

12. (a) 8 (b) $\frac{1}{8}$ (c) -8 (d) $-\frac{1}{8}$

13. (a) $2\sqrt{2} - \sqrt{7}$ (b) $5\sqrt{3} + 6\sqrt{2}$ (c) $2\sqrt{3} + 12$ (d) $\sqrt{15} - 3\sqrt{10}$ (e) $4\sqrt{3}$ (f) $\sqrt{2x}$