Let's say $f(x,y) = 6xy^2 + 2xy - x^2$. Compute f_x and f_y, and use them to answer these questions:

$$\boxed{\begin{aligned} f_x(x,y) &= 6y^2 + 2y - 2x \\ f_y(x,y) &= 12xy + 2x \end{aligned}}$$

(a) Which is bigger: $\dfrac{f(1.00001,5) - f(1,5)}{0.00001}$ or $\dfrac{f(1,5.0001) - f(1,5)}{0.0001}$?

These are slope fractions. Look for the variable that's changing:

1st: x changing $\quad \dfrac{f(\overset{x}{1.00001}, \overset{y}{5}) - f(\overset{x}{1}, \overset{y}{5})}{0.00001} = \dfrac{\text{change in } f}{\text{change in } x} \approx \dfrac{\partial f}{\partial x}$ or f_x

Base point is $(x,y) = (1,5)$, so plug in $x=1, y=5$ into f_x formula: $f_x(1,5) = 6(5)^2 + 2(5) - 2(1) = \boxed{158}$

2nd: y changing $\quad \dfrac{f(\overset{x}{1}, \overset{y}{5.0001}) - f(\overset{x}{1}, \overset{y}{5})}{0.0001} \approx f_y$

Base point is still $(x,y) = (1,5)$. $\rightarrow$ $f_y(1,5) = 12(1)(5) + 2(5) = \boxed{70}$

(b) Consider the four functions of y: $f(3,y)$, $f(4,y)$, $f(5,y)$, and $f(6,y)$. Which function has the steepest slope at $y = 2$?

Focus on one function at a time. These are functions of y, so think of y as changing and x as fixed. Slope = partial derivative with respect to y.

$$\begin{aligned} f_y(3,2) &= 12(3)(2) + 2(3) = 78 \\ f_y(4,2) &= 12(4)(2) + 2(4) = 104 \\ f_y(5,2) &= 12(5)(2) + 2(5) = 130 \\ f_y(6,2) &= 12(6)(2) + 2(6) = \boxed{156} \leftarrow \text{steepest} \end{aligned}$$

(c) Suppose $(x,y) = (1,1)$. Which leads to a larger increase in $z = f(x,y)$:

(i) A small increase in x, holding y fixed

(ii) A small increase in y, holding x fixed.

f_x tells us what happens to $f(x,y)$ when x increases slightly (& y stays the same)
Vice versa, f_y tells us what happens to $f(x,y)$ when y increases (& x stays the same)

$$\begin{aligned} f_x(1,1) &= 6(1)^2 + 2(1) - 2(1) = 6 \\ f_y(1,1) &= 12(1)(1) + 2(1) = 14 \end{aligned}$$

So increasing y by 1 increases $f(x,y)$ by about 14,
while increasing x by 1 increases $f(x,y)$ by only 6 (approximately). $\Big\} \rightarrow$ (ii) wins

(d) Consider these three functions of x: $f(x,1)$, $f(x,3)$, and $f(x,5)$. Which function has the least steep slope at $x=0$?

Similar to (b) except this time we think of x as the variable, and y fixed
(Think about the functions one at a time. In each function, y is a constant.)

$f_x(0,1) = 6(1)^2+2(1)-2(0) = \boxed{8}$ ← least steep (smallest) slope

$f_x(0,3) = 6(3)^2+2(3)-2(0) = 60$

$f_x(0,5) = 6(5)^2+2(5)-2(0) = 160$

(e) Which function has a steeper slope:

(i) $g(x) = f(x,1)$ at $x=0$

(ii) $h(\cancel{x}) = f(1,y)$ at $y=1$

x should be y.

Slope is measured by partial derivatives:

(i) x is the variable, so f_x measures slope

Plug in $x=0$ and $y=1$ (because $g(x)=f(x,1)$)

slope $= f_x(0,1) = 6(1)^2+2(1)-2(0) = \boxed{8}$

(ii) y is the variable, so f_y measures slope

Plug in $x=1$, $y=1$

slope $= f_y(1,1) = 12(1)(1)+2(1) = \boxed{14}$ ← steeper slope

(f) Suppose $x=-1$, $y=3$. If we increase y slightly, does $f(x,y)$ increase or decrease?

The y partial derivative can tell us this.

$f_y(-1,3) = 12(-1)(3)+2(-1) = -38$

Since f_y is negative, increasing y slightly <u>decreases</u> $f(x,y)$.

(If we increase y by 0.01, for instance, $f(x,y)$ will change by $(0.01)(-38) = -0.38$ approximately.)

1. Let j be the function $j(x,y) = \dfrac{x}{y} + \ln(x)\cdot\ln(y)$. Compute j_x and j_y.

$\boxed{j_x}$ y is a constant (just another number), and $\ln(y)$ is too.
(Imagine how you would take the derivative if we plugged in some number for y, like 2:

$$j(x,y) = \frac{x}{\text{number}} + \ln(x)\cdot(\text{number})$$

$$j_x(x,y) = \frac{1}{\text{number}} + \frac{1}{x}\cdot(\text{number})$$

$$= \boxed{\frac{1}{y} + \frac{1}{x}\cdot\ln(y)}$$

$\boxed{j_y}$ x & $\ln(x)$ are constants this time

$$j(x,y) = \frac{\text{number}}{y} + (\text{number})\cdot\ln(y)$$

$$= (\text{number})\cdot y^{-1} + (\text{number})\cdot\ln(y)$$

$$j_y(x,y) = (\text{number})\cdot(-y^{-2}) + (\text{number})\cdot\frac{1}{y}$$

$$= \boxed{-xy^{-2} + (\ln(x))\cdot\frac{1}{y}}.$$

2. Find the derivative of the function $g(x) = \sqrt{3-x}\cdot\sqrt{\ln x}$.

$$g(x) = (3-x)^{\frac{1}{2}}\cdot(\ln x)^{\frac{1}{2}}$$

$$g'(x) = (3-x)^{\frac{1}{2}}\cdot\frac{1}{2}(\ln x)^{-\frac{1}{2}}\left(\frac{1}{x}\right) + (\ln x)^{\frac{1}{2}}\cdot\frac{1}{2}(3-x)^{-\frac{1}{2}}\cdot(-1)$$