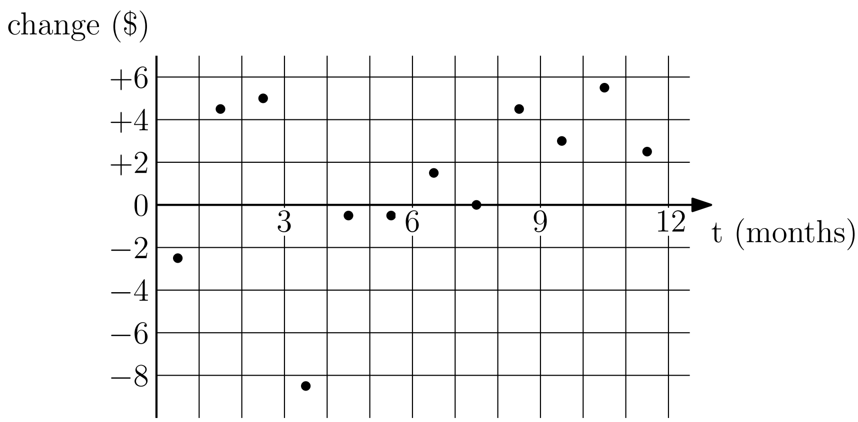
1. The graph on the right shows the approximate monthly **change** in crude oil prices, starting in January 2010. Each dot represents the change in crude oil price during one month. (The first dot represents the change in price from $t = 0$ months to $t = 1$ months, the second dot represents the change from $t = 1$ to $t = 2$, and so on.)



(a) If oil costs \$76/barrel at $t = 9$ months, what is its price at $t = 6$ months? At $t = 11$ months?

Each dot represents the change over that month. To get the price at $t = 11$ *months, add the changes for the two months from* $t = 9$ *to* $t = 11$ *to \$76. To get the price at* $t = 6$ *months, subtract the changes for the three months from* $t = 6$ *to* $t = 9$*. Basically, you're undoing the price changes for those three months.*

$t = 6$ months: $\$76 - 4.5 - 0 - 1.5 = \$70$

$t = 11$ months: $\$76 + 3 + 5.5 = \$84.5$

price at $t = 6$: ________________

price at $t = 11$: ________________

(b) Find the longest period of time over which oil prices are falling.

From $t = 3$ to $t = 6$.

This is the longest period of time where all monthly changes are negative. Oil prices are decreasing from $t = 3$ *to* $t = 4$ *(dot is below* x*-axis), from* $t = 4$ *to* $t = 5$*, and from* $t = 5$ *to* $t = 6$*.*

from $t =$ ________ to $t =$ ________

2. You take a road trip to Calgary, leaving at midnight. Let $D(t)$ represent the distance you have travelled after t hours.

(a) Each row in the table contains a phrase in one of our three "languages." Fill the blanks with the correct translations of each phrase. You don't need to do any calculations or show your work.

Intuitive Description	*Functional Notation*	*Graphical Notation*
Distance travelled between 8:00 and 10:00	$D(10) - D(8)$	Change in height of the graph from $t = 8$ to $t = 10$
Average speed during the time period from 3:00 to 7:00	$\dfrac{D(7) - D(3)}{4}$	slope of secant line through $t = 3$ and $t = 7$
At 5:00, average trip speed is more than 30 miles/hour.	$\dfrac{D(5)}{5} > 30$	The slope of the diagonal line through $t = 5$ is greater than 30.
From t to $t + 2$, the average speed is 10 miles/hour	$\dfrac{D(t+2) - D(t)}{2} = 10$	Slope of secant line through t and $t + 2$ is 10

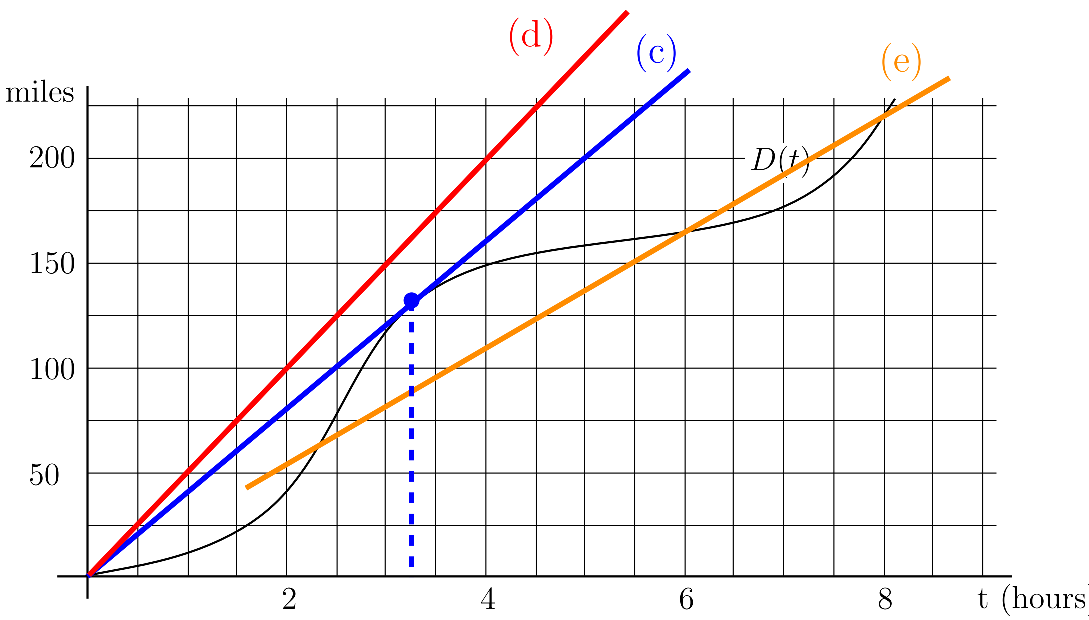
(b) The rest of your family is ahead of you, in another car. Let $F(t)$ represent the distance they have travelled after t hours.

Translate these statements into functional notation: (no need to show work)

Intuitive Description	*Functional Notation*
Your family is always 5 miles ahead of you.	$F(t) = D(t) + 5$ *or* $F(t) - 5 = D(t)$ *or* $F(t) - D(t) = 5$
After t hours, you are in the same place your family was 1 hour earlier.	$D(t) = F(t - 1)$

(problem continues on next page)

2 *(continued)*. Here is the graph of distance vs. time for your trip:



(c) At what time is your average trip speed the highest?
Use rolling ruler to find the highest diagonal line that crosses the graph. This line will just touch the graph at one point, and the question asks for the time (which is the x-coordinate of this point). The answer is $t \approx 3.25$.

at $t =$ ____________________

(d) Find a time t where $\dfrac{D(t+2)-D(t)}{2} = 50$.
This question is asking you to find a two-hour interval where average speed is 50 mph. To solve it, draw a reference line with slope 50 (see graph) and slide it up/down until your ruler crosses the graph at two points that are two hours apart. In this case, there's only interval that works, from $t = 1$ *to* $t = 3$. *The answer is the beginning of the interval,* $t = 1$.

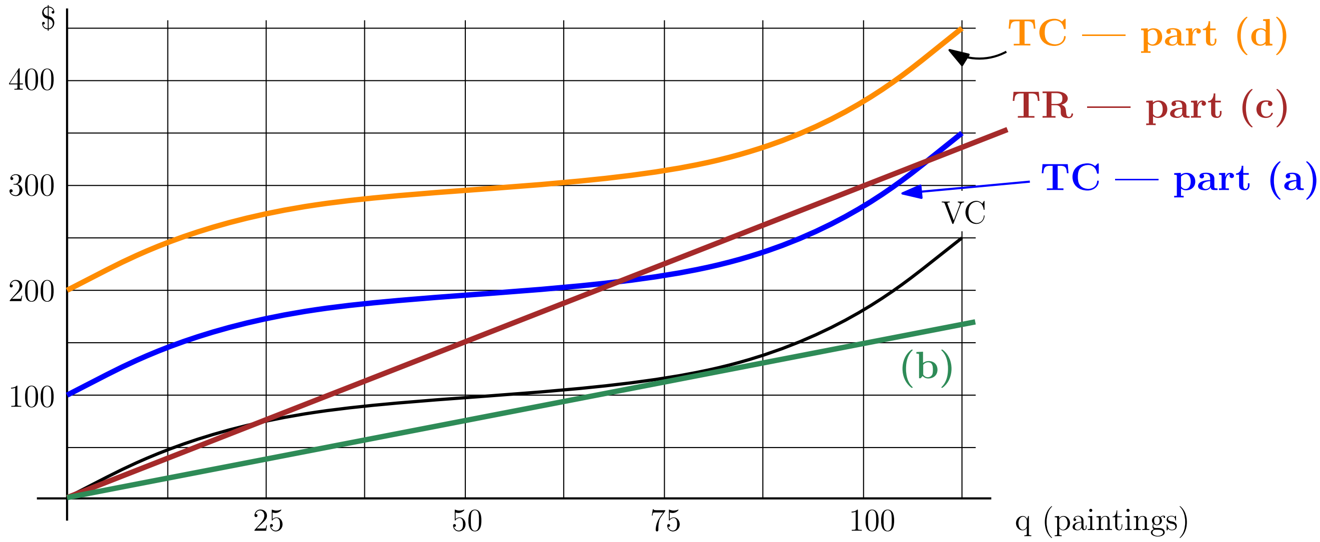
$t =$ ____________________

(e) How far do you travel from $t = 6$ to $t = 8$? Find another 2-hour interval where you travel the same distance, *using a reference line.*
To find the distance travelled, measure the change in height from $t = 6$ *to* $t = 8$, *approximately 55 miles.*
Here's one way to solve the second part: Draw the secant line through $t = 6$ *and* $t = 8$, *then slide it up/down until it crosses the graph at another two points that are two hours apart. There are two possibilities:* $t \approx 0.3$ *to* $t \approx 2.3$ *and* $t \approx 2.9$ *to* $t \approx 5.9$.

distance travelled: ____________________

$t =$ __________ to __________

3. You sell bear paintings. Below is the graph of your *variable cost*.



(a) Suppose your fixed cost is $100. Draw the graph of TC.

To get the TC *graph, shift the* VC *graph up by $100 (the fixed cost).*

(b) What is the shutdown price?

Use rolling ruler to find the lowest diagonal line through VC*. The shutdown price is the* slope *of this line (*$\approx$ 1.50 *per painting).*

SDP = ________________

(c) Suppose the market price is $3/painting. What is the maximum profit you can make?

First, draw the graph of TR*. Since the market price is $3,* TR *will have slope 3. The* TR *graph always goes through the origin since you don't make any money if you don't sell any items.*

Then, find the maximum vertical distance between TR *and* TC*, with* TR $>$ TC*. The max vertical distance you find is max profit, approximately $30.*

profit = ________________

(d) Now, suppose your fixed cost is $200.

If the market price is still $3/painting, you will always lose money, no matter how many paintings you sell. What *quantity* should you produce in order to minimize your loss?

Because fixed price has changed, first draw a new TC *graph, starting at $200 instead of $100. You want to minimize your loss, so find where the vertical distance between* TR *and* TC *is closest (around* q = 87.5*).*

q = ________________