Example word problems with multiple variables

Let's say $f(x, y) = 6xy^2 + 2xy - x^2$. Compute f_x and f_y, and use them to answer these questions:

(a) Which is bigger: $\dfrac{f(1.00001, 5) - f(1, 5)}{0.00001}$ or $\dfrac{f(1, 5.0001) - f(1, 5)}{0.0001}$?

(b) Consider the four functions of y: $f(3, y)$, $f(4, y)$, $f(5, y)$, and $f(6, y)$. Which function has the steepest slope at $y = 2$?

(c) Suppose $(x, y) = (1, 1)$. Which leads to a larger increase in $f(x, y)$:

(i) A small increase in x, holding y fixed

(ii) A small increase in y, holding x fixed.

(d) Consider these three functions of x: $f(x, 1)$, $f(x, 3)$, and $f(x, 5)$. Which function has the least steep slope at $x = 0$?

(e) Which function has a steeper slope:

(i) $g(x) = f(x, 1)$ at $x = 0$

(ii) $h(y) = f(1, y)$ at $y = 1$

(f) Suppose $x = -1$, $y = 3$. If we increase y slightly, does $f(x, y)$ increase or decrease?

More Practice

1. Let j be the function $j(x, y) = \dfrac{x}{y} + \ln(x) \cdot \ln(y)$. Compute j_x and j_y.

2. Find the derivative of the function $g(x) = \sqrt{3 - x} \cdot \sqrt{\ln x}$.