Midterm 2 Practice Questions

1. We're given the equation

$$y'' + \left(2 - \frac{5}{t}\right) y' + \left(\frac{8}{t^2} - \frac{4}{t}\right) y = 0$$

Knowing that one solution to this equation is $y_1(t) = t^2$, find another solution.

2. Write down the trial solutions for the following equations: (You don't have to solve them!)

(a) $y'' + 3y' + 2y = t^2 e^{-t} + t$

(b) $y'' - 2y' + 1y = t^2 e^{t} + \cos t$

(c) $y'' + 4y = t \cos 2t.$

3. A 1kg mass is attached to a spring, stretching it 1m. There is a damping force, equal to 4 N when the mass is traveling at 2 m/s. There is also an external force of $2\sin\omega t$ on the system.

   The mass starts 1 m above its equilibrium position, with an initial velocity of 2 m/s downward.

   (For this problem, you may approximate g as 10 m/s^2 to make the numbers nicer.)

   (a) Find the position of the mass at time t (your answer may involve ω).

   (b) What is the amplitude and phase of the steady state solution if $\omega = 1$? (challenge: try finding the amplitude and phase without assuming $\omega = 1$; your answer will be in terms of ω).

   (c) Find the transient solution if $\omega = 1$.

   (d) Suppose the mass and spring constant were kept the same, but we could adjust the damping coefficient γ. What would γ have to be in order for the system to be critically damped?

4. A 2kg mass is attached to a spring with unknown spring constant k. Assume there is no damping, and the mass starts 1m below equilibrium and is released with initial velocity 1 m/s upward.

   (a) Suppose the ceiling is 2m above the equilibrium position of the mass. If the mass just touches the ceiling, what is k?

   (b) Now suppose $k = 8$, and the system is driven by an external force $\sin 2t - \cos 2t$. Find the position of the mass after t seconds.

5. Find the general solution to

$$y'' + 2y' = e^{3t}\cos 2t + 4.$$

6. Solve the initial value problem

$$\begin{aligned} y'' + 4y' + 4y &= 0 \\ y(1) &= 1 \\ y'(1) &= 1 \end{aligned}$$

If this equation represents a mass-spring system, is the system underdamped, critically damped, or overdamped?

7. (a) Find the general solution of

$$y'' + 2y' + ay = 0$$

assuming $a > 1$.

Note: if your solution has the form $ce^{\lambda t}\cos(\mu t) + de^{\lambda t}\sin(\mu t)$, we call μ the **quasifrequency**.

(b) If the quasifrequency is 8, find a.