Math 307F Midterm 1 name_____
January 30, 2013

Honor Statement

I affirm that my work upholds the highest standards of honesty and integrity, and that I have neither given nor received any unauthorized assistance on this exam.

Signature _____

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total	

Instructions:

- Show all your work, and box your final answer.
- You may use one double-sided $8\frac{1}{2}$" by 11" sheet of notes.
- No calculators, cell phones, headphones, or other electronics are allowed,
- Your test should have 4 problems on 4 pages (not including this cover page)—double-check that it does!

1. Solve the initial value problem

$$ty' + 4y + \frac{3}{t^3} = 0, \quad y(1) = a.$$

2. A bucket contains 10 gallons of pure water at time $t = 0$. Water containing 2 pounds/gallon of minerals is flowing into a bucket at a rate of 4 gallons/second. The mixture in the bucket drains through a hole at a rate of 2 gallons/second.

Find a formula for the amount of minerals present in the tank (in pounds) after t seconds.

3. Suppose the percentage $P(t)$ of people having a certain gene at time t is described by the differential equation

$$\frac{dP}{dt} = 3(P - 30)(P - 75).$$

(a) Find the equilibrium solutions and classify them as stable, unstable, or semistable.

(b) Without solving the equation, draw the phase line and sketch the direction field. Also, sketch a few solutions.

4. A rocket is launched from the surface of a planet. Its velocity v has the equation

$$\frac{dv}{dt} = -c(x+R)^{-2},$$

where x is the distance from the rocket to the planet's surface, $R \geq 0$ is the planet's radius, and $c \geq 0$ is a constant.

(a) Eliminate t from the differential equation and solve it.

For parts (b) and (c), suppose $R = 1$ and $c = 4$, and that the rocket has velocity $v_0 = 2$ when it leaves the surface of the planet ($v = 2$ when $x = 0$).

In this case, the solution to the differential equation is $v(x) = \sqrt{\frac{8}{x+1} - 4}$. If you were unable to solve part (a), you may use this solution for parts (b) and (c).

(b) What is the domain of your solution in this case?

(c) How far does the rocket travel before it begins to return to earth?