Homework 5

Due Friday, July 29

Make sure to start this assignment early—it's a long one!

Worksheet 13: Do 1–21 in the book, then 22–28 below.

The goal of WS 13 is for you to get familiar with solving problems with quadratics. Remember, the reason we're spending so much time with quadratic functions is that they often appear when math is applied to real-world situations (objects falling, vehicles accelerating and decelerating, areas of squares, circles, ...).

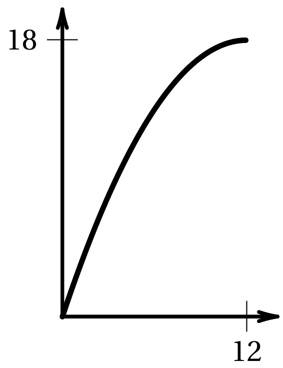
Worksheet 14: Do 1–4, 7–18 in the book.

Worksheet 15: Do 9–11 in the book.

Have questions? Drop in at office hours Tuesday through Thursday (see website for the info), or during Thursday's Q & A.

If you get stuck on an arrowed question, remember that your first step is to convert the question into functional notation.

Worksheet #13



→ $\boxed{22}$ The graph to the right is of distance *vs.* time for a car, Car F. Its formula is $F(t) = 3t - \frac{1}{8}t^2$ where t is in minutes and distance is in miles. We will call the place where Car F is at 0 minutes the Starting Place.

a) How long does it take the car to go the first 10 miles?

b) How long does it take the car to travel from a point 3 miles from the Starting Place to the point 4 miles from the Starting Place?

c) Another car, Car G, has a linear distance *vs.* time graph about which we have the following information:

- Car G is 4 miles ahead of Car F at time $t = 0$.
- Car F passes Car G at 8 minutes.

Give the formula for $G(t)$, the distance from the Starting Place for Car G at time t.

d) How far is Car G from the Starting Place at time $t = 12$?

e) How far apart are the two cars at time $t = 2$?

f) Find a time before $t = 8$ when the two cars are 2 miles apart.

g) Suppose another car, Car H, is always going the same speed as Car F, but starts 2 miles ahead of Car F.

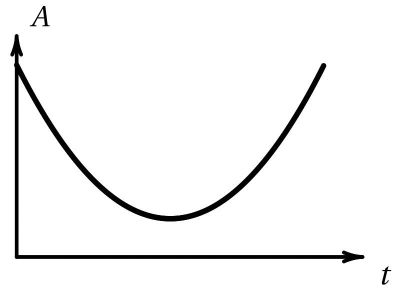
(i) Give the formula for $H(t)$, the distance that Car H is from the Starting Place at time t.

(ii) How far apart are Car H and Car F at time $t = 3.713$ minutes?

→ $\boxed{23}$ The graph to the right is of the amount of water in a vat at time t. It is given by the formula

$$A(t) = t^2 - 10t + 30,$$

where t is in minutes and A is in gallons.



a) At what time does the water reach its lowest level? How many gallons are in the vat at that time?

b) Give the two times when the vat has 11 gallons in it.

c) What is the change in water level from 2 to 8 minutes?

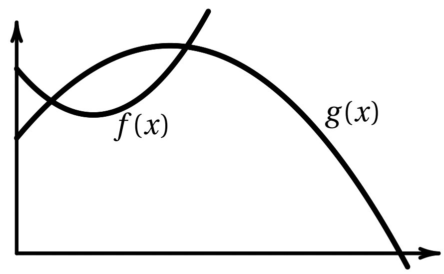
d) Another vat, Vat B, has a graph of amount *vs.* time which is a straight line. Water comes into Vat B at a constant rate of 1 gallon per minute. If Vat B were empty at time $t = 0$, would it ever have as much water in it as Vat A?

e) Suppose that Vat B actually starts with 3 gallons in it. At what time will the difference $B(t) - A(t)$ be greatest?

f) Name a time when the water level in Vat A is increasing and the difference $B(t) - A(t)$ is also increasing.

→ $\boxed{24}$ The graphs to the right are parabolas, with formulas

$$\begin{aligned} f(x) &= x^2 - 2x + 12 \\ g(x) &= -\tfrac{1}{2}x^2 + 6x + 10. \end{aligned}$$



a) Find the value of $g(1) - f(1)$.

b) Find all values of x at which the two graphs intersect. If g is marginal revenue and f is marginal cost, what is special about these values?

c) Find the values of x at which $g(x) - f(x) = 1$.

d) Find the longest interval you can, starting at $x = 1$, over which $g(x) - f(x)$ is increasing. [*Hint:* First look for where $g(x) - f(x)$ hits its peak.]

e) Find the longest interval over which both $f(x)$ and $g(x)$ are increasing. [*Hint:* Look for where the two functions peak or bottom out.]

f) For what values of x is $g(x)$ is greater than or equal to 15?

→ $\boxed{25}$ While standing on a building, Charlie tosses a rock upwards and watches it fall into the alley below. At time t (seconds) the height h (feet) above *street level* is recorded. This height is given by the formula

$$h = F(t) = -16t^2 + 40t + 25.$$

a) How high is the rock at the start?
b) At what time does the rock hit the ground?
c) At what time does the rock reach its greatest height?
d) Sketch the graph of h *vs.* t, indicating the rock's high point and the time it hits the ground.
e) At what time has the rock first traveled to a point 5 feet higher than the height at which it was thrown?
f) Find the time does when the rock is 30 feet above street level, and is on its way down.

→ $\boxed{26}$ The formula for the **average trip speed** of a car is $s(t) = \frac{5}{t} + 10$ feet/sec.

a) The recipe for distance covered given average trip speed is

$$\text{Distance covered } = D(t) = t \times s(t).$$

Write out a formula for $D(t)$.
b) Find the distance covered from $t = 4$ to $t = 6$ seconds.
c) How long does it take the car to travel 20 feet?
d) Another car has the following formula for average trip speed *vs.* time:

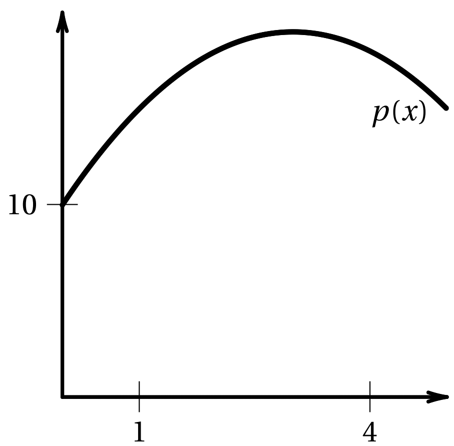
$$q(t) = 2t.$$

(i) Write down the equation you would solve in order to find the time t when the two cars have the same average trip speed.
(ii) Rewrite your equation from (i) in a form that can be solved using the quadratic formula.

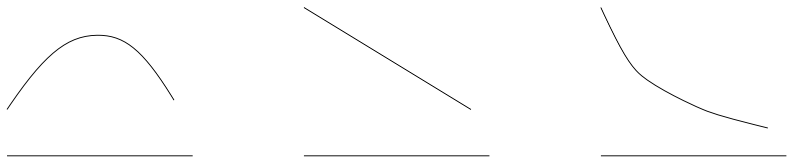
e) At what times are the two cars 10 feet apart? [*Note:* there are three answers!]

→ $\boxed{27}$ To the right is the graph of the quadratic function $p(x) = -x^2 + 6x + 10$. Let $q(x)$ be a different function defined by the recipe $q(x) = \dfrac{p(x)}{x}$.

a) Find the value of $q(3)$.
b) Find a formula for $q(x)$.



c) Which of the graphs below looks like it could be the graph of $q(x)$ from $x = 1$ to $x = 4$? Justify your answer.



NONE

d) Find the positive value of x at which the value of $q(x)$ is 5.

e) What is the largest value of $q(x)$ for values of x in the interval $x = 2$ to $x = 5$? Show your work and tell why your answer is correct.

f) (i) Set up the equation you would solve in order to answer the following question: For what positive value of x does the graph of $q(x)$ intersect the straight line graph $y = 3x + 1$?

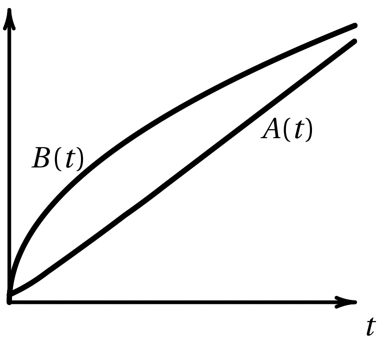
(ii) Rewrite the equation you gave in (i) into a form that can be solved by using the quadratic formula.

(iii) Answer the question given in (i).

→ $\boxed{28}$ The two graphs to the right are of the amounts of water in two vats, Vat A and Vat B. The formulas for these graphs are as follows:

$$\begin{aligned} \text{Vat A}: A(t) &= 2t - \sqrt{t} + 6 \\ \text{Vat B}: B(t) &= 7 \cdot \sqrt{t} \end{aligned}$$

(t is measured in hours, and amounts in gallons.)



a) What is the change in the amount of water in Vat A from time $t = 4$ to time $t = 9$?

b) What is the overall rate of change of water in Vat B at $t = 9$ hours? (Remember that the formula for overall rate of change is $\frac{B(t)}{t}$.)

c) Find *all* times at which the amounts of water in the two vats are equal.

d) Write an equation that *could* be solved directly by the quadratic formula to answer the following question:

At what time(s) are there 9 more gallons in Vat B than Vat A?

Don't solve this equation.

e) How much water is in Vat A when there are 28 gallons in Vat B?

f) Suppose we have a gauge which measures the difference

(Amount in Vat B) − (Amount in Vat A)

Write out a formula, in terms of t, for the reading on this gauge at time t.