Group Project: Pharmacokinetics

Intro: The techniques we've used to compute interest apply to completely different fields as well: depreciation, radioactive decay, ... even the behavior of medicines in the body. Just as an investment might gain 5% every year, the concentration of a drug or toxin in the body might decrease by 15% each hour.

Part 1: Suppose a patient takes 500mg of acetaminophen (Tylenol) every 6 hours after a mild injury. Let $A(k)$ be the amount of acetaminophen in the bloodstream k hours after the injury, in milligrams. The first dose is taken at $k = 0$, so $A(0) = 500$. (For simplicity, we assume all the acetaminophen is absorbed at once.)

Suppose that $A(k)$ drops by r% each hour, as acetaminophen leaves the bloodstream. For the moment, assume $r = 20\%$— so 20% is lost every hour.

- What is $A(1)$? What is $A(2)$?
- Write down an explicit formula for $A(k)$ that works for $k < 6$ hours. (This is similar to the depreciation problems from the last activity.)

After 6 hours, the patient takes another dose. So $A(6)$ includes what's left over from the first dose, plus 500mg from the second dose. Six hours later, another dose is taken, so $A(12)$ includes what's left over from the first two doses, plus 500mg from the third dose. And so on.

- Still assuming $r = 20\%$, find $A(9)$.
- Graph $A(k)$ over the first 18 hours. You'll want to measure $A(k)$ every hour, or every two hours.

The exact value of r varies from person to person. For instance, it's smaller for infants, and larger for children[1]. To handle this, we have to leave r as a variable in our calculations.

- Leaving r as a variable, find $A(6)$.
- Also find $A(24)$.
  Hint: think about how much of the first dose is left after 24 hours. How much of the second dose is left? (And so on.)

Let's take a step back and look at what's happening.

- From your graph, do you think the amount of acetaminophen keeps building at a steady rate, or does it taper off after some time?
- In certain situations, physicians will start with a larger first dose (a *loading dose*[2]) so that the concentration of the drug rises more quickly. In our situation, if the first dose were increased to 1000mg, what do you think the short-term and long-term effects would be? Use algebra to support your answer.

[1] A Pharmacologic Overview of TYLENOL, `http://www.tylenolprofessional.com/pharmacology.html`.

[2] Loading dose definition, `http://pharmacologycorner.com/loading-dose-definition/`.

Part 2: Let's start over. Suppose a patient takes 50mcg of levothyroxine (a thyroid medication) daily. Let $L(t)$ be the amount of levothyroxine on day t, in micrograms. The first dose on day 1, and each day approximately 10% of the drug exits the body[3]. So for the first day, $L(1) = 50$. The second day, 10% of the first dose is gone, and there is another 50mcg dose, so $L(2) = 90\% \cdot 50 + 50 = 95$.

Ideally, we'd like a formula for $L(k)$. It might look tough, but you can actually find this formula using pretty much the same kind of algebra you've used so far in Math 111.

It turns out that the right formula is

$$L(k) = 50 \cdot \frac{1-(1-r)^k}{r}.$$

Here, r is the percentage of levothyroxine lost per day: $r = 0.10$, or 10%.

- Using this formula, graph $L(k)$ for $1 \le k \le 50$.
- You should be able to see from your graph that the level of levothyroxine eventually approaches a stable or "steady-state" value. Discuss how you can use the formula for $L(k)$ to find the steady-state value.
- Write a formula for the steady-state value in terms of r, the percentage of the drug lost per day, and d, the dosage. In the situation here, $r = 10\%$ and $d = 50$.

Postscript: In medicine, it's important to know how a certain dosage of a medication will affect its steady-state concentration in the bloodstream. This problem shows you a little bit about how the calculations can be done.

[3]Synthroid drug facts, `http://www.rxabbott.com/pdf/synthroid.pdf`.