Math 307 C - Spring 2011
Final Exam
June 6, 2011

Name: _____ Student number: _____

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
Total	70	

- Complete all questions.
- You may use a calculator during this examination. Other electronic devices are not allowed, and should be turned off for the duration of the exam.
- If you use a trial-and-error or guess-and-check method, or read a numerical solution from a graph on your calculator, when an algebraic method is available, you will not receive credit.
- You may use one hand-written 8.5 by 11 inch page of notes.
- Show all work for full credit.
- You have 110 minutes to complete the exam.

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1. (a) Solve the initial value problem

$$t^3y' = t^5 + 3t^2y, \qquad t > 0, \qquad y(1) = 1.$$

(5 points)

(b) Find the general solution of

$$\frac{dy}{dx} = \frac{y^2 - 3}{e^x y}, \qquad y > 0.$$

Leave in non-simplified form.
(5 points)

2. Suppose we have a tank containing 1/2 lb of salt mixed in 2 gal of water. You pour salt into the tank at a constant rate of γ lbs/min, and the well-stirred mixture leaves the tank at a rate in gal/min equal to half the square of the current volume of water in the tank. If the tank contains 2 lbs of salt after 1 min, what is γ?
(10 points)

3. (a) Find the general solution of

$$y'' - 6y' + 9y = 2e^{-3t}.$$

(5 points)

(b) For the following equation, y_1 is a solution. Use the method of reduction of order to find a second solution.

$$ty'' + (2+t)y' + y = 0, \quad t > 0, \qquad y_1 = 1/t.$$

(5 points)

4. An object hangs from a spring with spring constant 4 lb/ft. If the phase of the object's motion is $3\pi/4$ and the object first returns to equilibrium position after $\pi/16$ sec, what is the *weight* of the object? Use $g = 32$ ft/sec^2.

   (10 points)

5. Suppose the motion of an object on spring is described by

$$u(t) = e^{-t}\Big(\cos(2t) + 3\sin(2t)\Big)$$

where $u(t)$ is the displacement (in feet) of the object from equilibrium position after t seconds. Find the maximum displacement of the object from equilibrium position.

(10 points)

Laplace transforms:

$$\begin{aligned}
\mathcal{L}\{t^n e^{at}\} &= \frac{n!}{(s-a)^{n+1}} \\
\mathcal{L}\{e^{at}\sin(bt)\} &= \frac{b}{(s-a)^2+b^2} \\
\mathcal{L}\{e^{at}\cos(bt)\} &= \frac{s-a}{(s-a)^2+b^2} \\
\mathcal{L}\{u_c(t)\} &= \frac{e^{-cs}}{s} \\
\mathcal{L}\{y'\} &= s\mathcal{L}\{y\} - y(0) \\
\mathcal{L}\{y''\} &= s^2\mathcal{L}\{y\} - sy(0) - y'(0).
\end{aligned}$$

If $\mathcal{L}\{f(t)\} = F(s)$, then

$$\mathcal{L}\{u_c(t)f(t-c)\} = e^{-cs}F(s).$$

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6. Find the Laplace transform of

$$f(t) = \begin{cases} 5t & 0 \le t < 4 \\ e^{2t} & 4 \le t. \end{cases}$$

(10 points)

7. Solve the initial value problem

$$y'' + 6y' + 13y = g(t), \qquad y(0) = -2, \quad y'(0) = 3,$$

where

$$g(t) = \begin{cases} -2 & 0 \le t < \pi \\ 3/2 & \pi \le t < 2\pi \\ 1 & 2\pi \le t. \end{cases}$$

(10 points)