Test Prep Solutions

Autumn 2006 (Taggart), #3

The amount of water (in gallons) that has flowed into a vat after t minutes is given by the formula

$$I(t) = -2t^2 + 14t.$$

(a) The incremental rate of flow into the vat ovr the five-minute interval starting at time t is

$$R(t) = \frac{I(t+5) - I(t)}{5}$$

Write out the formula for $R(t)$ and simplify as much as you can.

Solution:

$$\begin{aligned} I(t+5) &= -2(t+5)^2 + 14(t+5) \\ &= -2(t^2 + 10t + 25) + 14(t+5) \\ &= -2t^2 - 20t - 50 + 14t + 70 \\ &= -2t^2 - 6t + 20 \end{aligned}$$

$$\begin{aligned} R(t) &= \frac{I(t+5) - I(t)}{5} \\ &= \frac{[-2t^2 - 6t + 20] - [-2t^2 + 14t]}{5} \\ &= \frac{-20t + 20}{5} \\ &= -4t + 4. \end{aligned}$$

(b) Water flows out of the vat at a constant rate of 0.25 gallons per minute. At $t = 0$, there are 50 gallons in the vat. Give a formula for $A(t)$, the amount of water that the vat contains after t minutes. Simplify as much as possible.

Solution:
First, we need to find an equation for the amount of water that has flowed out after t minutes. Let's call that $O(t)$.
Water flows out at a constant rate, so the graph of $O(t)$ will be a straight line. That tells you that $O(t)$ will have a linear equation, looking like $O(t) = mt + b$. At time 0, no water has flowed out yet, so $b = 0$. The slope is the rate of water flow (a rate is a slope), so $m = 0.25$.
That tells us that $O(t) = 0.25t$.
To find the total amount of water at time t, we have to subtract water going out from water coming in, including the 50 gallons we started with:

$$A(t) = I(t) - O(t) + 50 = -2t^2 + 14t - 0.25t + 50 = -2t^2 + 13.75t + 50.$$

(c) What is the highest level that the water in the vat reaches?

Solution: The water level is measured by $A(t)$, which is a quadratic equation. To find when $A(t)$ is highest, use the vertex formula:

$$\text{vertex at}\, t = \frac{-b}{2a} = \frac{-13.75}{2(-2)} = 3.4375.$$

This is the *time* when water level is highest. We want the water level itself, so we plug $t = 3.4375$ into $A(t)$:

$$A(3.4375) = -2(3.4375)^2 + 13.75(3.4375) + 50 \approx 73.63.$$