Chapter 4 Practice Problems
CAAM 335 • Matrix Analysis • Spring 2016

Here are some practice problems from the Chapter 4 material: column space, nullspace, and LU factorization.

Problem 1   Let A be the 4×6 matrix

$$A = \begin{bmatrix} 2 & 1 & 2 & 2 & 1 & 1 \\ -2 & 0 & -3 & -3 & 0 & 0 \\ 0 & -1 & -1 & 1 & 3 & 5 \\ 2 & 1 & 0 & 2 & 5 & 7 \end{bmatrix}.$$

i. Find an LU factorization of A.

ii. What is A's rank?

iii. Find a basis for the column space of A.

iv. Find a basis for the null space of A.

Problem 2   Let A be the following 3×3 matrix, with an unknown third column $\vec{a}_3$:

$$A = \begin{bmatrix} 1 & 0 & \\ 1 & 1 & \vec{a}_3 \\ 0 & 1 & \end{bmatrix}$$

i. What is the rank of A if the three columns ($\begin{bmatrix} 1 & 1 & 0 \end{bmatrix}^T$, $\begin{bmatrix} 0 & 1 & 1 \end{bmatrix}^T$, and $\vec{a}_3$) are linearly *independent*?

ii. What is the rank of A if they are linearly *dependent*?

iii. Find a vector $\vec{a}_3$ that makes $\begin{bmatrix} 1 & 1 & 0 \end{bmatrix}^T$, $\begin{bmatrix} 0 & 1 & 1 \end{bmatrix}^T$, and $\vec{a}_3$ linearly *independent*.