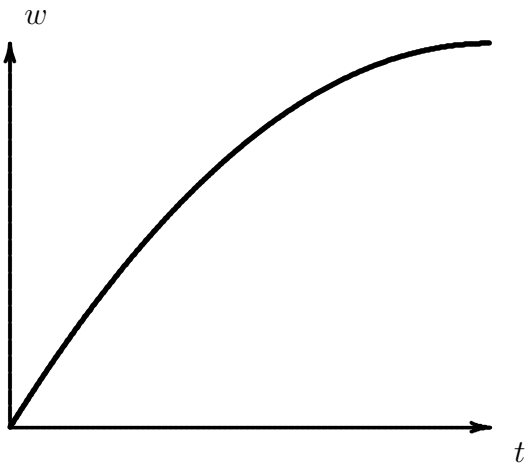
Homework 4

Due Friday, July 22

Worksheet 8: Do all problems in the book (1–15).
Worksheet 10: Do 1–3, 6, 8, 9b, 10.
Worksheet 11: Do 1, 2, 4, 6–8, 11–13, 15, and the versions of 16–17 below.
Worksheet 12: Do 1–12 and the versions of 13–14 below.
Have questions? Drop in at office hours Tuesday through Thursday (see website for the info), or get last-minute help during Thursday's Q & A.
Hint: If you get stuck on an arrowed question, start by trying to convert the question into functional notation.

Worksheet #11

→ 16 The graph to the right gives the amount of water that has flowed into a reservoir by time t. The formula for this graph is $w = I(t) = at^2 + bt$ (where w is in gallons and t is in hours).



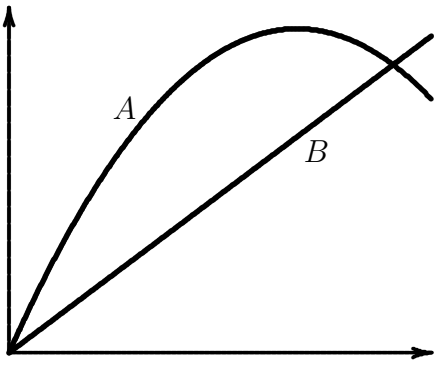
a) Write out a formula in terms of a, b, and t for the overall average rate of flow into the reservoir (i.e. $\frac{I(t)}{t}$).

b) Given that $\frac{I(5)}{5} = 30$ and $\frac{I(10)}{10} = 20$, determine the values of a and b.

c) Find the amount of water that flows into the reservoir from time $t = 5$ hours to time $t = 7$ hours.

d) Water flows out of the reservoir at a constant rate of 15 gallons per hour. The reservoir starts with 20 gallons in it at time $t = 0$. Write out a formula for the actual amount of water contained in the reservoir at time t.

→ 17 Two balloons are released at the same time. One goes up steadily, and the other goes up for a while and then begins to descend. The formulas for the heights h (feet) of the balloons at time t (minutes) are:



$$\text{Balloon } A: h = A(t) = -\tfrac{1}{2}t^2 + 8t$$
$$\text{Balloon } B: h = B(t) = 3t.$$

By the Average Rate of Ascent at time t, we mean the overall average vertical speed from time 0 to time t (i.e., $\frac{A(t)}{t}$ or $\frac{B(t)}{t}$).

a) Write out the formulas for Average Rate of Ascent of the two balloons at time t.

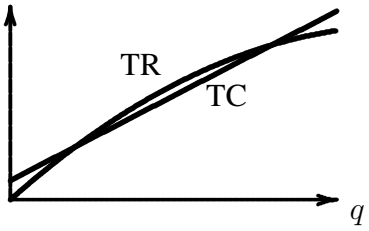
b) At what time will the two balloons have the same Average Rate of Ascent?

c) Find the time at which the Average Rate of Ascent of A exceeds the Average Rate of Ascent of B by 2 ft/min.

d) Find the Average Rate of Ascent of A when B is 12 ft high.

Worksheet #12

→ 13 The graphs to the right are of total cost and total revenue of manufacturing Trivets. Their formulas are:



$$\begin{aligned} TR(q) &= -\tfrac{1}{4}q^2 + 20q \\ TC(q) &= 10q + 50. \end{aligned}$$

Recall that

$$MR(q) = TR(q+1) - TR(q) \text{ and } MC(q) = TC(q+1) - TC(q).$$

a) Find formulas for MR and MC.

b) Find the quantity q at which profits are maximized (*Hint:* It's easiest to use MR and MC.)

c) Find the quantities q at which profit is zero.

d) Find the quantity q at which Average Revenue is \$10 per Trivet.

e) Find the quantity q at which Average Cost is \$12 per Trivet.

→ 14 The total revenue from manufacturing and selling q Blivets is given by the formula:

$$\mathrm{TR}(q) = 10\sqrt{q}$$

where q is given in thousands of Blivets, and $TR(q)$ is given in thousands of dollars.

a) Write out a formula in terms of q for the marginal revenue of producing q thousand Blivets: (No need to simplify!)

b) The total cost of manufacturing q thousand Blivets has the formula $TC(q) = q + 10$ (thousands of dollars). Write the equation you would solve to answer the following question:

> *What is the largest quantity of Blivets you can manufacture without taking a loss?*

You don't have to solve the equation.

c) Find the value of q at which the average cost of manufacturing Blivets is \$5 per Blivet.

d) Find the value of q at which the average revenue for manufacturing Blivets is \$2.50 per Blivet.

e) Write out the equation that you would solve to answer the following question:

> *For what quantities q is the profit of manufacturing q thousand Blivets going to be \$5,000?*

Again, you don't have to solve your equation.