Homework #4 — due Wednesday, 2/20

3.5: 12, 15, 18
3.7: 7
additional problems (below)

1. Two pendulums are swinging from the ceiling. The angle between the first pendulum and vertical, $\theta(t)$, is governed by the equation

$$\theta'' + 25\theta = 0$$

The second pendulum is like the first but has some damping due to air resistance. The angle between it and vertical, $\alpha(t)$, is governed by the equation

$$\alpha'' + 16\alpha' + 25\alpha = 0$$

Both pendulums have the same initial conditions: $\theta(0) = 0$, $\alpha(0) = 0$ and $\theta'(0) = 2$, $\alpha'(0) = 2$.

(a) Solve each equation to find $\theta(t)$ and $\alpha(t)$.

(b) Find the *period* of the first pendulum and the *quasi-period* of the second.

2. Recall that a mass on a spring is governed by the equation

$$mu'' + \gamma u' + ku = 0,$$

where m is its mass, γ is the damping coefficient, and k is the spring constant. For this problem, all our objects will have mass 1, so $m = 1$. Each part of this problem is unrelated to the other parts.

(a) Find two choices for γ and k that yield the same quasi- frequency. In other words, find γ_1, γ_2, k_1, and k_2 so that the solutions to $u'' + \gamma_1 u' + k_1 u = 0$ and $u'' + \gamma_2 u' + k_2 u = 0$ have the same quasi-frequency.

(b) Next, suppose $\gamma = 2$ and $k = 2$. If you want to increase γ but keep the same quasi-frequency, what has to happen to k?

(c) If γ_1, k_1 are one choice of constants, and γ_2, k_2 is a different choice of constants, is it possible for the equations $u'' + \gamma_1 u' + k_1 u = 0$ and $u'' + \gamma_2 u' + k_2 u = 0$ to have the same general solution? Explain why or why not.