Math 111 Week 10 Review

This review is not all inclusive. You are expected to know how to do all the problems in the homework.

Worksheet 23: The Natural Logarithm (Solving for time)

1. The natural logarithm, $x = \ln(y)$, is the inverse of the exponential function, $y = e^x$. So

$$\ln\left(e^x\right) = x \quad \text{and} \quad e^{\ln(x)} = x.$$

2. You need to use $\ln(x)$ when you are solving for a variable that is in the exponent of an equation. For example, when you are solving for **time** in any compounding problem and when you are solving for r in continuous compounding problems. Here are two examples to help you recognize when to use logarithms and when to use roots:

   (a) Solve $10(3)^x = 40$.
   (Variable in the exponent; You will have to use logarithms)

   - *SOLUTION:*

$$\begin{aligned} 3^x &= 4 \\ \ln(3^x) &= \ln(4)) \\ x\ln(3) &= \ln(4) \\ x &= \frac{\ln(4)}{\ln(3)} \end{aligned}$$

   So $x \approx 1.26186$.

   (b) Solve $5x^3 = 25$.
   (Variable in the base; You will **NOT** use logarithms, you will have to use the cube root.)

   - *SOLUTION:*

$$\begin{aligned} x^3 &= 5 \\ x &= \sqrt[3]{5} \\ x &= 5^{1/3} \qquad (5^{1/3} \text{ is the same as } \sqrt[3]{5}) \end{aligned}$$

   So $x \approx 1.709976$.

3. Here are two typical examples that occur in bank problems (**expect problems like these on the final exam!**):

   (a) You deposit \$500 into an account that pays 6% annually, compounded continuously.
   How long does it take the account to double in size?

   - *FORMULA:* $A(t) = 500e^{0.06t}$
     TRANSLATION: Solve $1000 = 500e^{0.06t}$
     SOLUTION:

$$\begin{aligned} 2 &= e^{0.06t} \\ \ln(2) &= \ln(e^{0.06t}) \\ \ln(2) &= 0.06t \\ \frac{\ln(2)}{0.06} &= t \end{aligned}$$

   So $t \approx 11.5525$ years

   (b) You deposit \$200 into an account that pays 9% annually, compounded quarterly.
   How long does it take the account to triple in size?

   - *FORMULA:* $A(t) = 200\left(1 + \frac{0.09}{4}\right)^{4t}$
     TRANSLATION: Solve $600 = 200\left(1 + \frac{0.09}{4}\right)^{4t}$
     SOLUTION:

$$\begin{aligned} 600 &= 200\left(1.0225\right)^{4t} \\ 3 &= \left(1.0225\right)^{4t} \\ \ln(3) &= \ln\left(1.0225\right)^{4t} \\ \ln(3) &= 4t\ln(1.0225) \\ \frac{\ln(3)}{4\ln(1.0225)} &= t \end{aligned}$$

   So $t \approx 12.3436$ years