Math 307 C - Spring 2011
Mid-Term Exam 1
April 20, 2011

Name: _____ Student number: _____

1	10	
2	10	
3	12	
4	8	
5	10	
Total	50	

- Complete all questions.
- You may use a calculator during this examination. Other electronic devices are not allowed, and should be turned off for the duration of the exam.
- You may use one hand-written 8.5 by 11 inch page of notes.
- Show all work for full credit.
- You have 50 minutes to complete the exam.

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1. (a) Find the general solution to

$$xf' = x^4 + 2f, \quad x > 0.$$

(5 points)

(b) Solve the initial value problem

$$e^t \frac{df}{dt} = f^3 e^{2t}, \qquad f(1) = 5.$$

(5 points)

2. (a) Find the general solution to

$$\frac{dy}{dx} = (2 - e^y)/e^{y-x}.$$

(5 points)

(b) Solve the initial value problem

$$\frac{dy}{dt} + 2y = e^{4t}, \qquad y(0) = 1.$$

(5 points)

3. A population of organisms in your coffee, in the absence of other factors, grows at a rate proportional to the current population: specifically, it triples every minute. There is another factor though: you drink the coffee at a constant rate of 4 oz/min. If the organisms are evenly distributed in the coffee and there are 3 organisms in 12 oz of coffee initially, how many organisms are in your coffee after 1 minute?

   (12 points)

4. You pour yourself another cup of coffee at a constant rate of k oz/min. Your mug leaks the coffee at a rate in oz/min equal to half the cube root of the current volume of coffee in the mug. Does the volume of coffee in your mug converge? If so, to what?

   (8 points)

5. You dispose of the faulty mug (of mass m) by dropping it out the window (zero initial velocity). Two forces act on the mug: gravity, which acts down with magnitude mg (where g is the gravitational constant), and a resistive force which acts up with magnitude proportional to the velocity of the mug. Find the distance of the mug from the window at any given time.

   (10 points)