Math 307C - Spring 2011
Mid-Term Exam 2
May 18, 2011

Name: _____   Student number: _____

1	10	
2	8	
3	10	
4	8	
5	14	
Total	50	

- Complete all questions.

- You may use a calculator during this examination. Other electronic devices are not allowed, and should be turned off for the duration of the exam.

- If you use a trial-and-error or guess-and-check method, or read a numerical solution from a graph on your calculator, when an algebraic method is available, you will not receive credit.

- You may use one hand-written 8.5 by 11 inch page of notes.

- Show all work for full credit.

- You have 50 minutes to complete the exam.

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1. Find the general solution of:

    (a)
    $$4y'' - 8y' + 5y = 0.$$

    (5 points)

    (b)
    $$y'' - y' - 2y = 4e^{2t}.$$

    (5 points)

2. For the following equation, y_1 is a solution. Use the method of reduction of order to find a second solution.

$$\cos(t)y'' + 3\sin(t)y' + \cos(t)\left(1 + 3\tan^2(t)\right)y = 0, \quad -\frac{\pi}{2} < t < \frac{\pi}{2}, \quad y_1 = \cos(t).$$

(8 points)

Hint: $\int \tan(x)\,dx = -\ln|\cos(x)| + c.$

3. An object weighing w lbs stretches a spring L feet. It is pushed up x ft and set into motion with initial downward velocity of $\sqrt{2}$ ft/sec. If the object's motion has phase $3\pi/4$ and the object reaches a maximum displacement of $1/2$ ft,

   (a) what is the frequency of motion?

   (b) What is L?

   Use $g = 32$ ft/sec^2.

   (10 points)

4. Suppose a spring system is governed by the equation

$$100u'' + u' + ku = 0.$$

(a) For which spring constant k is the system critically damped?
(4 points)

(b) Suppose you attach a motor to the above spring system which applies a force of $3\cos(2t)$ lbs to the object. Which of the following possible spring constants will give the steady state solution with largest amplitude, and why?

$$k = 122 \qquad\qquad k = 401 \qquad\qquad k = 750$$

(4 points)

5. Suppose the motion of an object on spring is described by

$$u(t) = 8e^{-t/2}\sin(t/2),$$

where $u(t)$ is the displacement (in feet) of the object from equilibrium position after t seconds.

(a) When does the object cross equilibrium position for the **fifth** time? (4 points)

(b) Find a time after which the object remains within 1/10 in of equilibrium position. (3 points)

(c) Find the maximum distance of the object from equilibrium position.
(4 points)

(d) Find a time after which the speed of the object is less than 1/10 in/sec.
(3 points)