**Math 307F Final** name_____
March 21, 2013

## Honor Statement

I affirm that my work upholds the highest standards of honesty and integrity, and that I have neither given nor received any unauthorized assistance on this exam.

*Signature* _____

| | |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| total | |

*Instructions:*

- Show all your work, and box your final answer.
- You may use one handwritten, double-sided $8\frac{1}{2}$" by 11" sheet of notes.
- No calculators, cell phones, headphones, or other electronics are allowed. You can include square roots and trigonometric functions in your answers.
- Your test should have 7 problems on 7 pages (not including this cover page or the back page)—
  double-check that it does!
- Raise your hand if you have any questions.
- You can use the table of Laplace transforms for any question unless it specifically says otherwise.

A table of Laplace transforms and the values of $g$ are on the back.

**1.** (a) Solve the equation

$$y' = te^{2t} + \frac{y}{t}.$$

(b) A barrel contains 20 gallons of pure water. Water containing $\frac{1}{2}$ lb/gal of calcium enters at a rate of 10 gal/hour, and mixes with the contents of the barrel. The mixture drains at a rate of 12 gal/hour.

Write, but **do not solve** a differential equation for $C(t)$, the amount of calcium in the barrel, in pounds, after $t$ hours.

*Answer:* $\frac{dC}{dt} =$_____

**2.** The population $P$ of insects (in thousands) in a given area has the following equation:

$$\frac{dP}{dt} = (P - 10)^2(-2P + 20).$$

where $t$ is measured in days.

(a) Without solving the equation, find all equilibrium solution(s) and classify them as stable, semistable, or unstable.

(b) Solve the differential equation, with the initial condition $P(0) = 14$.

*Note: you can do part (b) without doing part (a), and vice-versa.*

**3.** A 2kg object is attached to a spring with spring constant $k = 4$. The damping force is 20 N when the object is traveling at 5 m/s, and there is an external force of $-10 \sin t$. At $t = 0$, you pull the object downward by 1m and release it with initial velocity 1 m/s upward.

(a) Find the quasifrequency of the system.

(b) Find the position of the object at time $t$. Identify the transient solution and steady-state solutions.

**4.** Given that $y_1(t) = t^3$ is one solution to the ODE

$$t^2y'' - ty' - 3y = 0,$$

find another solution that is not a multiple of $y_1(t)$.

**5.** Find the Laplace transform of $f(t) = \sin bt$, using only the definition of the Laplace transform (not using the table).

**6.** Solve the IVP

$$\begin{aligned} y''' + 4y' &= \begin{cases} 0, & t < 3 \\ 5e^{-t}, & t \geq 3 \end{cases} \\ y(0) &= 3 \\ y'(0) &= 2 \\ y''(0) &= -9 \end{aligned}$$

You do not need to solve for the coefficients $A, B, \ldots$ in the partial fractions decomposition – you may leave those in your answer. You may also leave your answer in terms of step functions.

**7.** Find the Laplace transform of $f(t) = \begin{cases} 3t, & t < \pi \\ e^{-2t}\cos 3t, & t \geq \pi \end{cases}$.

## Table of Laplace Transforms

| $f$ | $\mathcal{L}\{f\}$ | $f$ | $\mathcal{L}\{f\}$ |
|---|---|---|---|
| $1$ | $\frac{1}{s}$ | $\cos bt$ | $\frac{s}{s^2+b^2}$ |
| $e^{at}$ | $\frac{1}{s-a}$ | $\sin bt$ | $\frac{b}{s^2+b^2}$ |
| $t^n$ | $\frac{n!}{s^{n+1}}$ | $e^{at}\cos bt$ | $\frac{(s-a)}{(s-a)^2+b^2}$ |
| $t^n e^{at}$ | $\frac{n!}{(s-a)^{n+1}}$ | $e^{at}\sin bt$ | $\frac{b}{(s-a)^2+b^2}$ |

## Acceleration Due to Gravity

standard: $g = 32.2$ ft/s$^2$ (you can use $g = 32$)

metric: $g = 9.8$ m/s$^2$ (you can use $g = 10$)