Powers and Roots

Refresher: the exponent rules

$x^a x^b = x^{a+b}$	multiplying powers $\rightarrow$ add exponents
$\dfrac{x^a}{x^b} = x^{a-b}$	dividing powers $\rightarrow$ subtract exponents
$(x^a)^b = x^{ab}$	combining powers $\rightarrow$ multiply exponents
$x^{-a} = \dfrac{1}{x^a}$	negative powers $\rightarrow$ fractions
$x^a y^a = (xy)^a$	bases with the same exponent merge

When tracking investments and other types of exponential growth, you'll need to be familiar with manipulating powers. Try these out:

- Simplify $\dfrac{x^5}{x^2} \cdot x^3$:
- Simplify $\dfrac{(xy)^3}{x^2 y}$:
- Simplify $\dfrac{2^k}{2}$:
- Write $\dfrac{1}{2x^6}$ as ax^b, for some numbers a and b:
- Simplify $\dfrac{x^7 - x^4}{x^4}$:
- Simplify $\dfrac{rK^3 - rK^2}{K^2}$:
- Solve $m^7 = 1.3m^5$ for m:
- Solve $(2t)^5 = t^4$ for t:
- Solve $(x^2)^{-1} = \dfrac{1}{9}$ for x:

Fractional powers and roots

Fractional powers will come in handy a lot. If the population of blackbirds in Seattle is 100 now and doubles every year, then the explicit formula for the blackbird population is $B(k) = 100 \cdot 2^k$. That makes sense: after 2 years, there should be $400 = 100 \cdot 2^2$ blackbirds.

What about the population after 2.25 years? We would want to plug in 2.25 for k, and that means dealing with $2^{2.25}$— a fractional power.

Roots are also fractional powers:

$$\sqrt{x} = x^{1/2} \qquad \sqrt{x^a} = x^{a/2}$$
$$\sqrt[b]{x} = x^{1/b} \qquad \sqrt[b]{x^a} = x^{a/b}$$

- By definition, $(\sqrt{x})^2 = x$. Check that $(x^{1/2})^2 = x$ also:

- Simplify $x^{1.3} \cdot \sqrt{x}$:

- Simplify $\dfrac{3x}{\sqrt{x}}$:

- Simplify $\sqrt{ab^2}$ as much as possible:

- Simplify $\dfrac{\sqrt{3^{k+1}}}{\sqrt{3^k}}$:

- Simplify $\dfrac{\sqrt{4 \cdot a^{k+2}}}{\sqrt{4 \cdot a^k}}$:

- Simplify $\sqrt[4]{x^3}$:

- Solve $\dfrac{t^{4.5}}{t} = 3$ for t:

- Solve $(u^3)^2 = 70u$ for u: