Homework #5 — due Tuesday, 2/26

To hand in:

3.7: $11^\dagger$, 14
3.8: 10^*, 12, 16^{**}
additional problems (below)

† *Note on 3.7 #11:* To find the damping coefficient γ, note that the problem says that the damping force is 3 N when $u' = 5$. In the standard equation for the motion a mass on a spring, $mu'' + \gamma u' + ku = F(t)$, the term $\gamma u'$ is the damping force, so you can these pieces of information to find γ.

* *Note on 3.8 #10:* When using standard units (instead of metric units), remember that pounds are a unit of force. To get mass from pounds, divide by the acceleration due to gravity, $g = 32\text{ft}/\text{s}^2$. You'll also need to convert the measurements in inches to feet.

** *Note on 3.8#16:* We haven't yet covered circuits like this one in class. If you want to get started on it, the book explains how to set up the differential equation for these circuits on pages 201–202.

To do (not to be handed in):

3.7: 13, 18
3.8: 11

Additional problems

1. Two pendulums are swinging from the ceiling. The angle between the first pendulum and vertical, $\theta(t)$, is governed by the equation

$$\theta'' + 25\theta = 0$$

The second pendulum is like the first but has some damping due to air resistance. The angle between it and vertical, $\alpha(t)$, is governed by the equation

$$\alpha'' + 8\alpha' + 25\alpha = 0$$

Both pendulums have the same initial conditions: $\theta(0) = 0$, $\alpha(0) = 0$ and $\theta'(0) = 2$, $\alpha'(0) = 2$.

(a) Solve each equation to find $\theta(t)$ and $\alpha(t)$.

(b) Find the *period* of the first pendulum and the *quasi-period* of the second.

2. Recall that a mass on a spring without any external force is governed by the equation

$$mu'' + \gamma u' + ku = 0,$$

where m is its mass, γ is the damping coefficient, and k is the spring constant. For this problem, all our objects will have mass 1, so $m = 1$. Each part of this problem is unrelated to the other parts.

(a) Find two choices for γ and k that yield the same quasi- frequency. In other words, find γ_1, γ_2, k_1, and k_2 so that the solutions to $u'' + \gamma_1 u' + k_1 u = 0$ and $u'' + \gamma_2 u' + k_2 u = 0$ have the same quasi-frequency.

(b) Next, suppose $\gamma = 2$ and $k = 2$. If you want to increase γ but keep the same quasi-frequency, what has to happen to k?

(c) If γ_1, k_1 are one choice of constants, and γ_2, k_2 is a different choice of constants, is it possible for the equations $u'' + \gamma_1 u' + k_1 u = 0$ and $u'' + \gamma_2 u' + k_2 u = 0$ to have the same general solution? Explain why or why not.