

Disturbance Estimation  
And Cancellation  
for  
Linear Uncertain Systems

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# Summary

This thesis was born on the boundary of the theory of control and a particular application, namely a smart structural system. More specifically, the primary motivation of this study comes from the following questions. Is it really possible to use smart structural system in a practical situation? What is the problem from the view point of the theory of control? In this work, it is assumed that one of the most important problems is that of robustness of the controlled system, and it is supposed that one of the solution would be to estimate uncertainty and disturbance without *a priori* knowledge in order to cancel their effect on system behaviour. This study provides a method, including its theoretical foundation, to estimate and cancel out any bounded disturbance and/or uncertainty.

One of the most important problems in control systems is the robustness of the controlled system. It is known that almost all physical systems, such as mechanical or structural system, contain some form of uncertainty. Even smart structural systems cannot escape from this problem. Such systems consist of host materials, sensing and actuating layers, which are attached or embedded to the host materials. The modelling of smart structural systems gives rise to infinite dimensional models if it were modelled by ordinary differential equations. In practice, however, a model is obtained by using the Finite Element Method, and, hence, a high order finite dimensional model is obtained. Such approximate models will inevitably generate uncertainty, representing unmodelled dynamics. In addition, such system models would suffer from parametric uncertainty and external disturbances. Thus, this type of system model would include many types of uncertainties.

In past decades, much research has been done using a deterministic approach for the robust control problem. The majority of this work assumes a known upper bound to uncertainty and disturbance, and robust controllers are determined deterministically. However, for smart structural systems, it may be difficult, or even impossible, to obtain such *a priori* knowledge of any disturbance. If this is the case, what can be said about the robustness of controlled system without *a priori* knowledge of any disturbance? Part of the answer to this question can be found in this thesis.

This thesis considers a linear uncertain system in which the uncertainty and/or disturbance is known to be bounded, but its bound is unknown. The main contribution is that an adaptive feedback control law is designed to estimate the bounded disturbance. The design of the adaptive control algorithm is novel and the adaptive control algorithm is easy to implement. This information can then be used to cancel the effect of the disturbance in the system. This has the advantage that, if further design objectives are to be realized, the controls can be designed based on the information from the known nominal model only

and not on the model with uncertainty. This thesis is organized as follows. In Chapter 1, firstly, concept of stability of systems and deterministic approach of robust control are recalled. At the end of that chapter, it is implied that there is some limitation of that approach of robust control. In Chapter 2, motivated by the limitation discussed at Chapter 1, the method of disturbance estimation is introduced. Firstly, the statement of the problem is provided. It is followed by some preliminary works which are required for analysis. Then, for each class of systems, an adaptive algorithm, lemmas, theorem, and simulation examples are provided. The class of systems examined are second-order single-input linear systems,  $n^{\text{th}}$ -order single-input linear systems, and multi-input linear systems. In Chapter 3, based on the works of Chapter 2, some applications of the method of disturbance estimation are presented. In Section 3.2, an adaptive algorithm which guarantees robustness of a controlled system is presented. In Section 3.3, treatment of input uncertainty and unmodelled dynamics is discussed. In the following section, Section 3.4, it is shown that under appropriate assumptions, it is possible to treat residual disturbance by the method proposed. In Section 3.5, the method is extended so that the method can be used only by outputs of a system. At last, in Section 3.6, it is demonstrated that parameter variations can be extracted from estimated disturbances. In Chapter 4, conclusion remarks and suggestions for future works are provided.

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# Contents

1	Deterministic approach of robust control and Lyapunov stabilization	1
1	1.1 Introduction	1
1	1.2 Mathematical preliminaries	1
2	1.3 Stability of a system and Lyapunov function	2
3	1.3.1 Stability concepts	3
3	1.3.2 Lyapunov stability theorems	3
5	1.4 Deterministic robust controllers	5
7	1.5 Limitation of deterministic approach	7
7	1.6 Conclusions	7
9	2 Disturbance estimation and cancellation -the basis-	9
9	2.1 Introduction	9
11	2.2 Problem statement	11
13	2.3 Preliminaries	13
13	2.3.1 Definition of the error system	13
13	2.3.2 Assumptions	13
14	2.3.3 Diagonalisation of error system	14
16	2.4 Second order single-input system	16
16	2.4.1 Adaptive algorithm	16
18	2.4.2 Main Result	18
26	2.4.3 Simulation example	26
35	2.5 $n^{\text{th}}$ order single-input system	35
35	2.5.1 Main result	35
41	2.5.2 Simulation example	41
51	2.6 Multi-input system	51
51	2.6.1 Structure of the system and input matrices	51
51	2.6.2 Structure of a time-varying transformation matrix	51
53	2.6.3 Adaptive algorithm	53
54	2.6.4 Main result	54
59	2.6.5 Simulation example	59
73	2.7 Conclusions	73
75	3 Applications of disturbance estimation and cancellation	75
75	3.1 Introduction	75
75	3.2 Robustness of controlled system	75
76	3.2.1 An adaptive algorithm which guarantees robustness	76

3.2.2	Robust property of the controlled system for the stab-	77
3.2.3	Robust property of the controlled system for the tracking	82
3.2.4	Simulation example	84
3.3	Input uncertainty and unmodelled	99
3.3.1	Analysis	99
3.3.2	Simulation example	100
3.4	Residual uncertainty	109
3.4.1	Estimation of residual uncertainty	109
3.4.2	Exact cancellation of residual uncertainty	110
3.4.3	Simulation example	112
3.5	Output control	125
3.5.1	Preliminaries	125
3.5.2	Estimation of disturbance by output measurement	125
3.5.3	Estimation with measurement noise	137
3.6	Estimation of the parameters generating	146
3.6.1	Constant parametric uncertainty without external distur-	146
3.6.2	Constant parametric uncertainty in the presence of peri-	146
3.6.3	Constant parametric uncertainty in the presence of con-	148
3.6.4	stant and periodic disturbance	149
3.7	Conclusions	150
4	Conclusions and further research	157
4.1	Concluding remarks	157
4.2	Recommendations for further work	158
	Bibliography	158

## Chapter 4

# Conclusions and further research

### 4.1 Concluding remarks

As it is shown in previous chapters, in this study, the following topics are investigated.

1. For both single-input and multi-input systems, estimation and cancellation of bounded disturbance/uncertainty can be performed without *a priori* knowledge of bounded disturbance/uncertainty (see Theorem 3, 4, 5, and related remarks).

2. Estimation and cancellation can be achieved, even in the presence of residual uncertainty/disturbance under appropriate conditions (see Section 3.4).

3. Using the disturbance estimation method, a tracking controller can be designed with respect to the nominal model (see Theorem 7).

4. Estimation and cancellation of unknown bounded disturbance/uncertainty can, also, be achieved by using only output measurement, even in the presence of sensor noise (see Section 3.5).

5. For both the stabilization and tracking problems, the system to be controlled is robust against parametric uncertainty, input uncertainty, unmodelled dynamics, external disturbance and/or sensor noise using the disturbance estimation/cancellation method (see Theorem 7, 8, and Section 3.5.3).

6. The parameter variations for the nominal model can be estimated from the estimated disturbance under certain assumptions (see Section 3.6).

In addition, numerical simulations are presented to demonstrate the methods developed.



## 4.2 Recommendations for further work

Topics which are not studied, but are suggested for further work, are listed as follows.

1. Implementation of the methods in some practical application.
2. Treatment of the case when there are input constraints.

The author believes that theory is enhanced as a result of interaction with various applications. Although the primary motivation of this study comes from one specific area of application and the assumptions developed are constructed so that these assumptions are realistic in the specific practical situation, it is uncertain what kind of problems exist when the theory is implemented for applications. The author believes that almost exact estimation of such uncertainty can be achieved but perfect estimation of such uncertainty is not possible. Thus, to make clear and resolve such problems, it is recommended that the methods be implemented in a number of applications.

In relation to *Treatment of input constraint*, improvements of the adaptive algorithms and a better understanding of the nominal model and modelling is encouraged. The method proposed gives an engineering solution for the worst case situation regarding uncertainty and disturbance, as well as control of a nonlinear and time-varying system, by cancelling out their effect. However, due to input constraints, there is a limit on the amount of estimation error of uncertainty and uncertainty that can be tolerated as modelling errors. Thus, improvements of the adaptive algorithms and a better understanding of modelling and the characteristics of the nominal model, which are fundamental characteristics of nonlinear system and time-varying system, are required.

Since the methods developed do not require any *a priori* knowledge of disturbance and the methods can be applied using only output measurement, the methods will be relatively easy to implement in a practical situation. The author believes that the method proposed will contribute to the development of high performance/reliable systems.