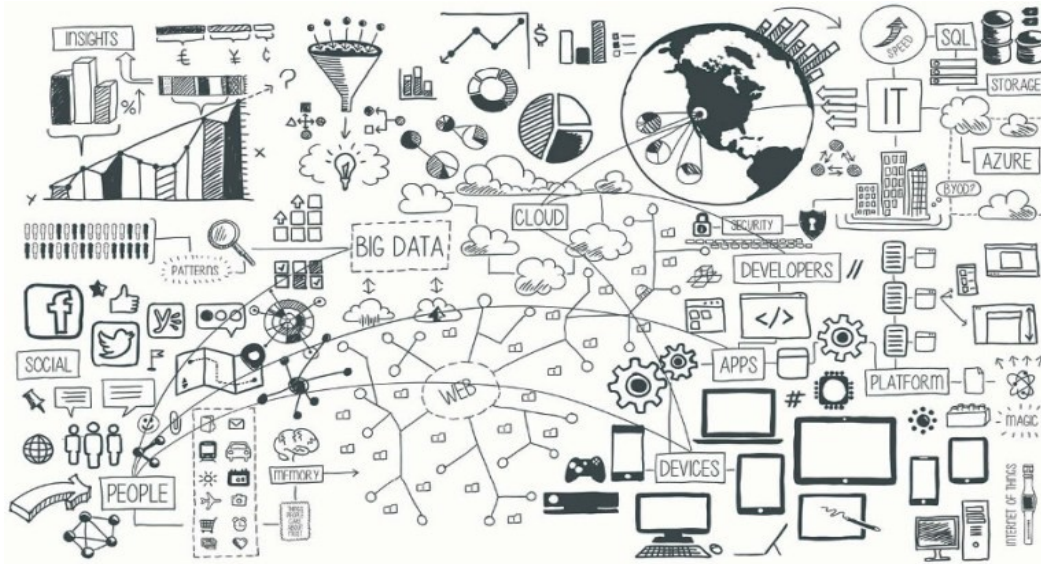


# Data Mining (Minería de Datos)

## Evaluación, sobreajuste y validación cruzada (cross-validation)



Sixto Herrera

Rodrigo G. Manzananas

Grupo de Meteorología

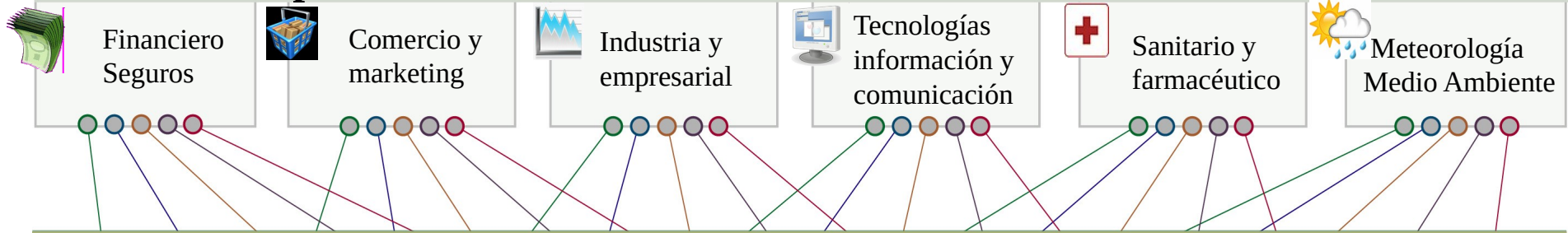
Univ. de Cantabria – CSIC  
MACC / IFCA



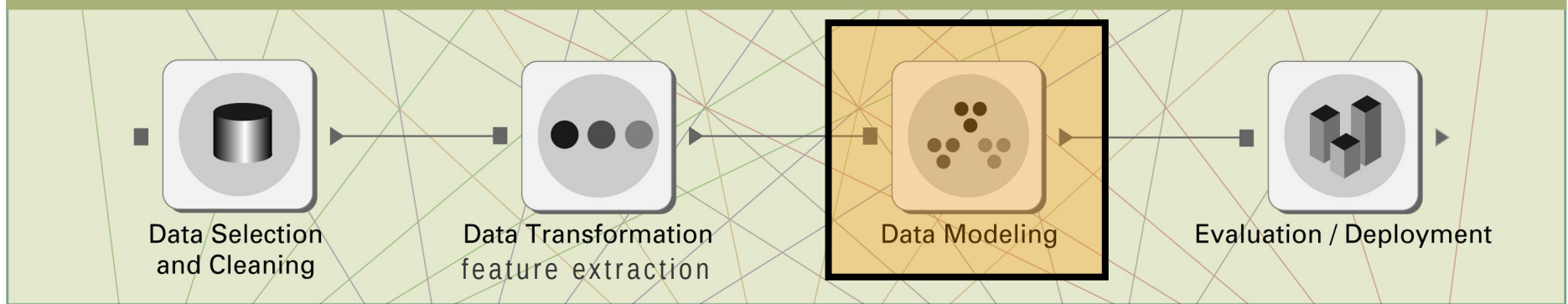
**NOTA:** Las líneas de código de R en esta presentación se muestran sobre un fondo gris.

Nov	2	Presentación, introducción y perspectiva histórica
	4	Paradigmas, problemas canonicos y data challenges
	9	Reglas de asociación
	11	Practica: Reglas de asociación
	16	<b>Evaluación, sobreajuste y crossvalidacion</b>
	18	<b>Practica: Crossvalidacion</b>
	23	Árboles de clasificacion y decision
	25	Practica: Árboles de clasificación
		<b>T01. Datos discretos</b>
	30	<b>Técnicas de vecinos cercano (k-NN)</b>
Dic	2	Práctica: Vecinos cercanos
	9	Comparación de Técnicas de Clasificación.
	14	Reducción de dimensión no lineal
	16	Reducción de dimensión no lineal
		<b>T02. Clasificación</b>
	17	Árboles de clasificación y regresion (CART)
	20	Práctica: Árboles de clasificación y regresion (CART)
	21	Practica: El paquete CARET
		<b>T03. Prediccion</b>
Ene	11	Ensembles: Bagging and Boosting
	13	Random Forests y <b>Gradient boosting</b>
	14	<b>Técnicas de agrupamiento</b>
	20	<b>Técnicas de agrupamiento</b>
	24	Predicción Condicionada
	26	Sesión de refuerzo/repaso.
Ene	27	Examen

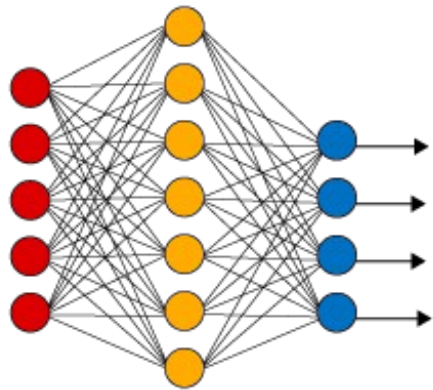
# Sectores de aplicación



## Proceso de Minería de Datos

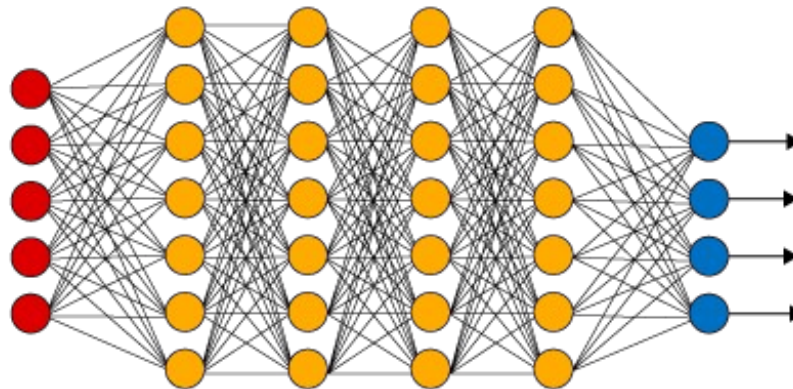


## Simple Neural Network



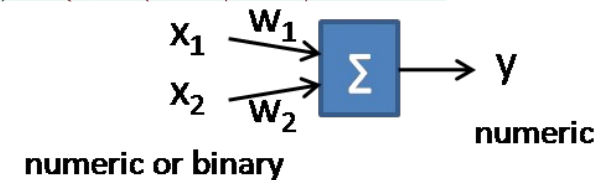
Input Layer

## Deep Learning Neural Network



Hidden Layer

Output Layer



$$y = w_0 + w_1x_1 + w_2x_2$$

$$y = f(\mathbf{x}, \mathbf{w}) = \mathbf{x}^T \cdot \mathbf{w}$$

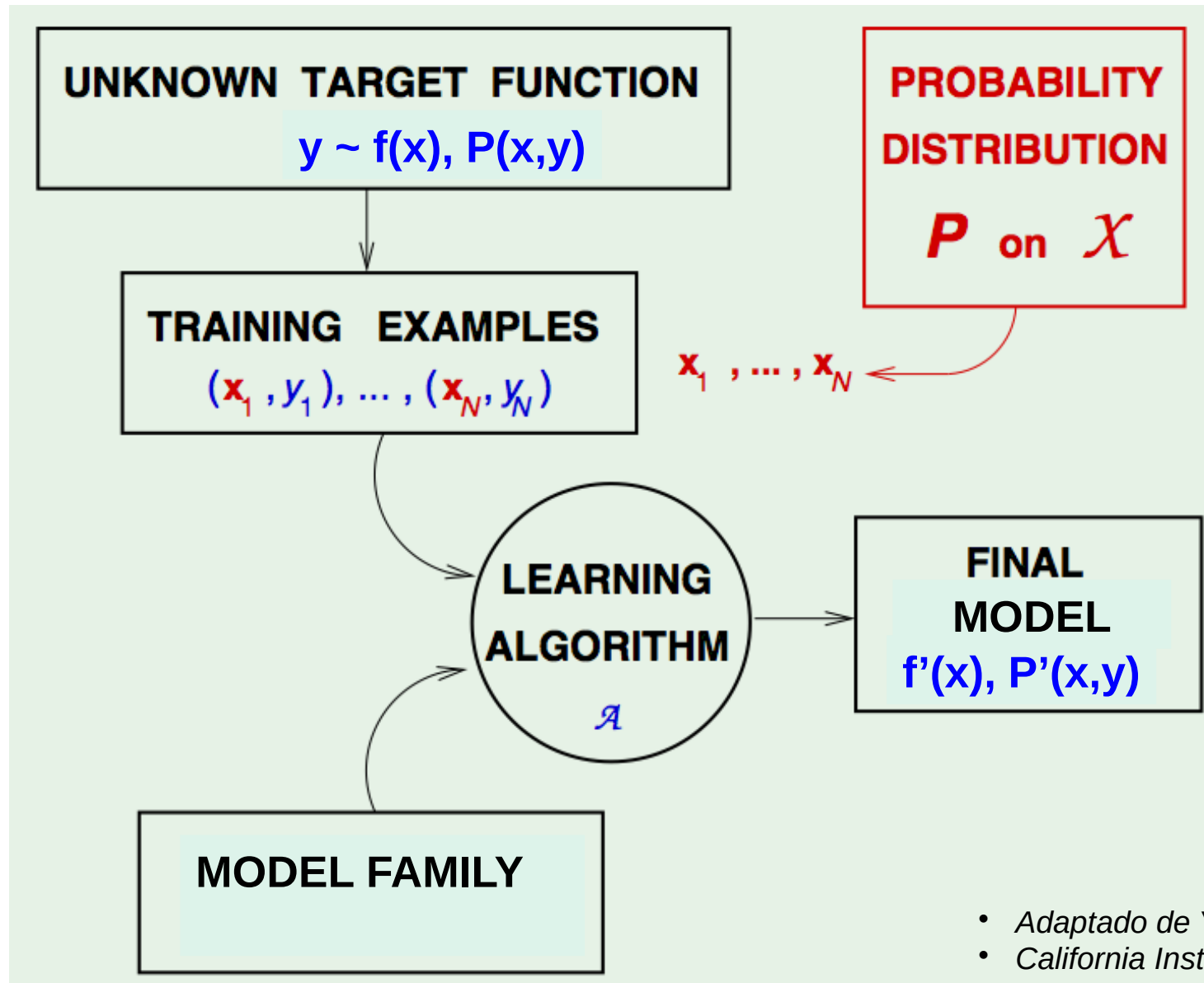
REGRESSION

$$\mathbf{w} = \begin{bmatrix} w_0 \\ w_1 \\ w_2 \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} 1 \\ x_1 \\ x_2 \end{bmatrix}$$

Cross-Validation

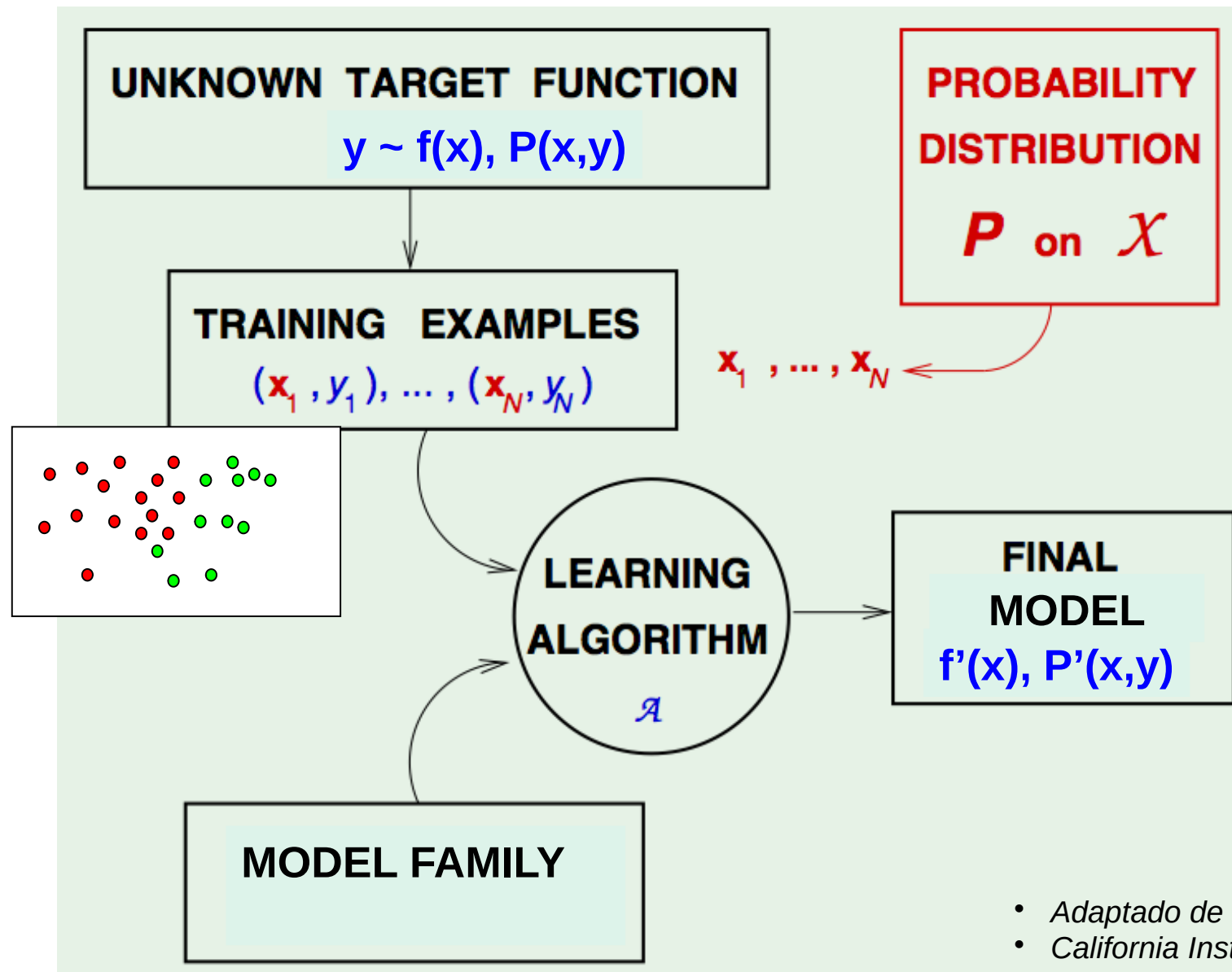
Data Mining: Data Modeling

**Learning** is the automatic process of building (adjusting) a model from a data set which is representative from the full population.



- Adaptado de Y.S. Abu-Mostafa
- California Institute of Technology

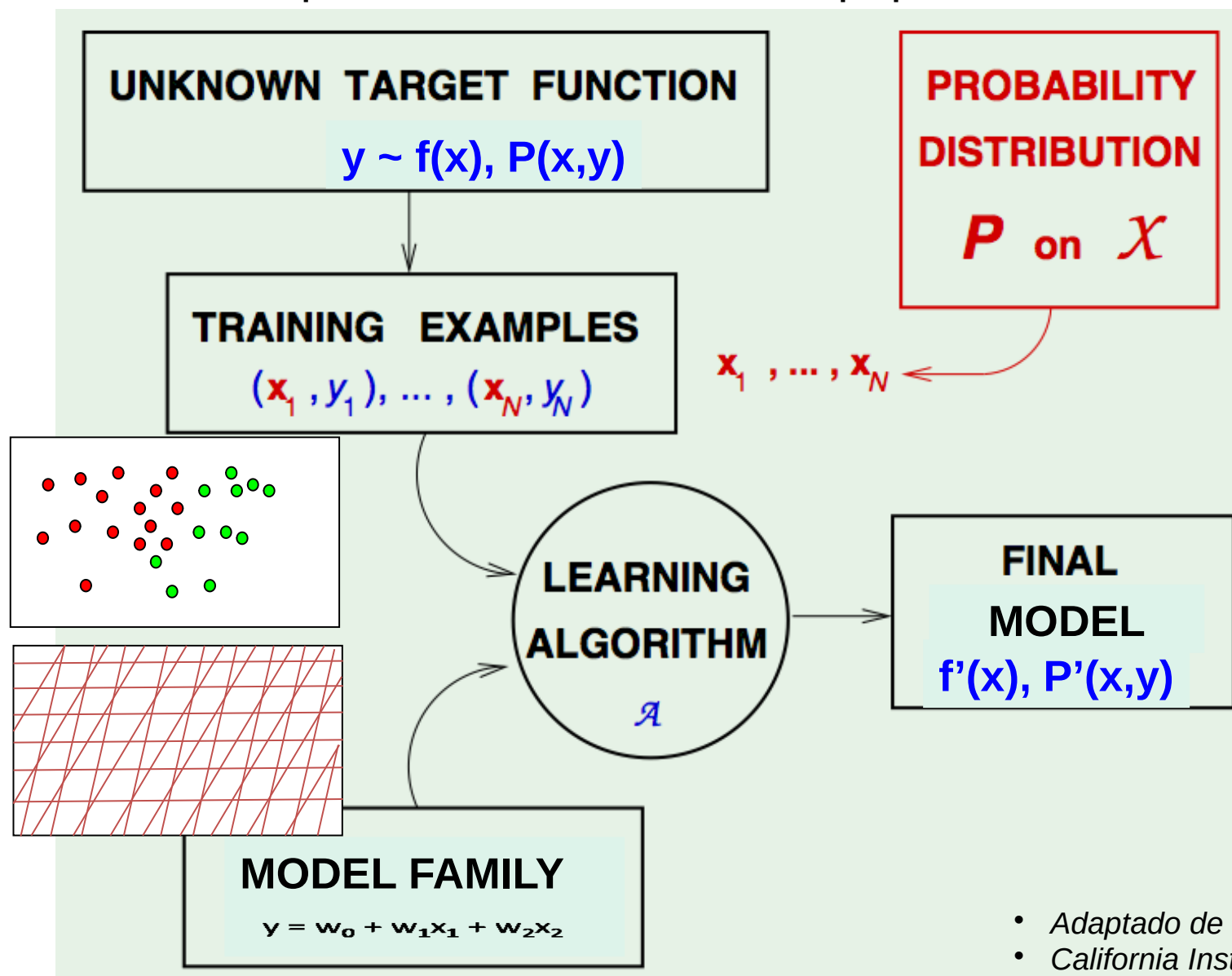
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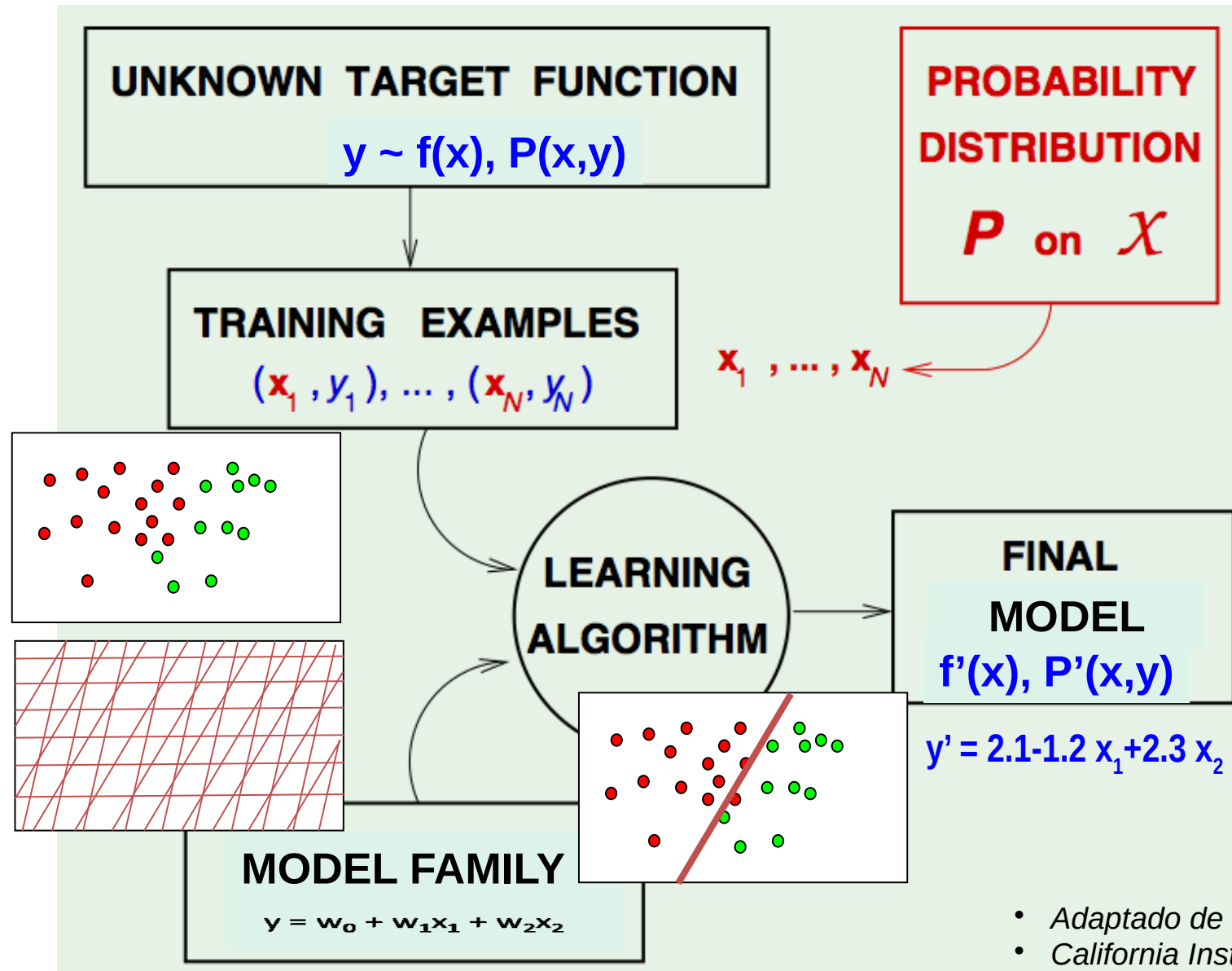


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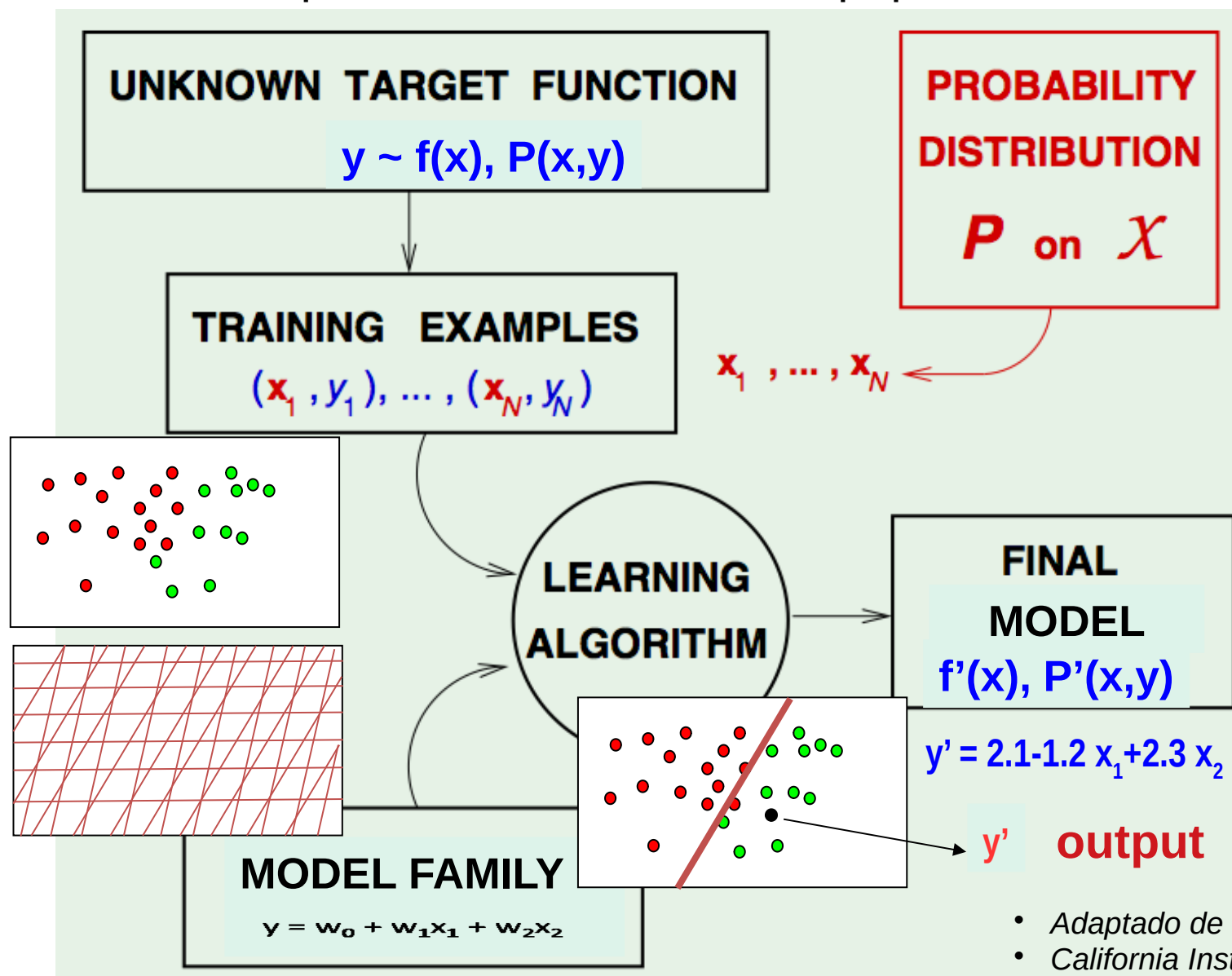
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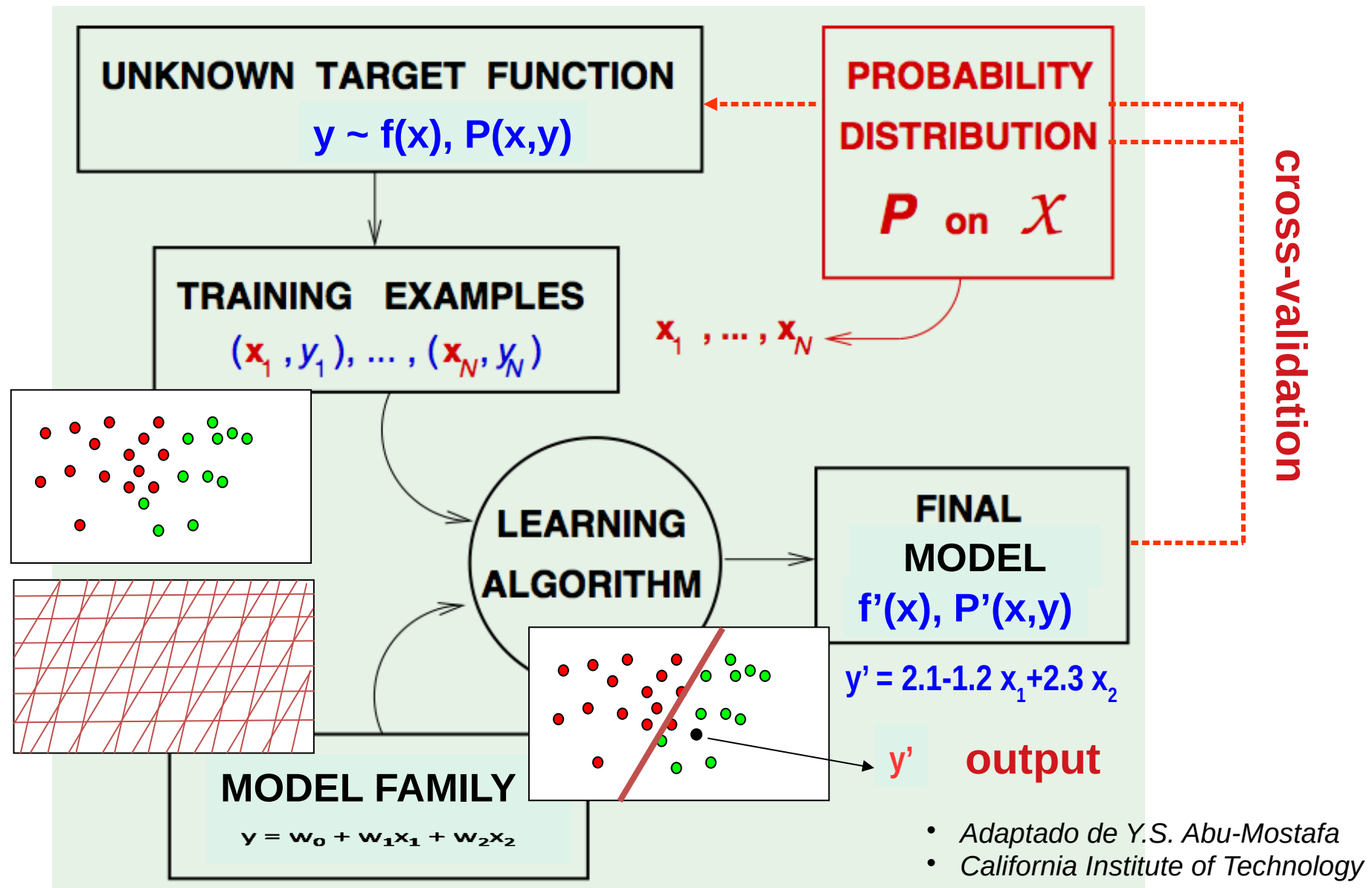
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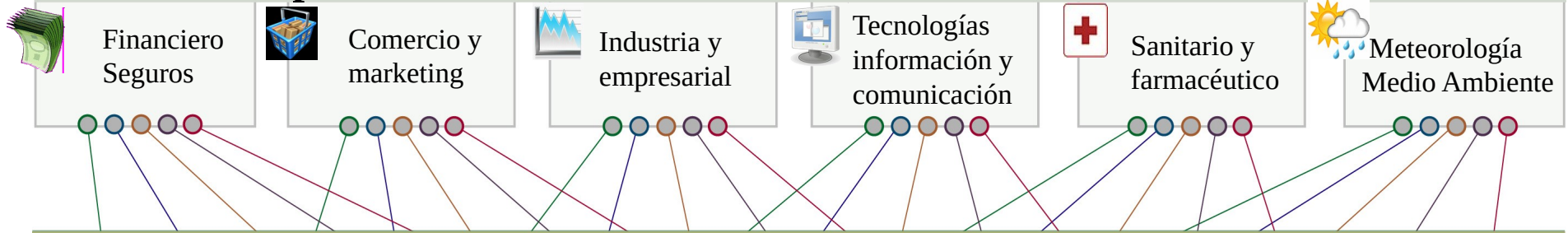
- Adaptado de Y.S. Abu-Mostafa
- California Institute of Technology



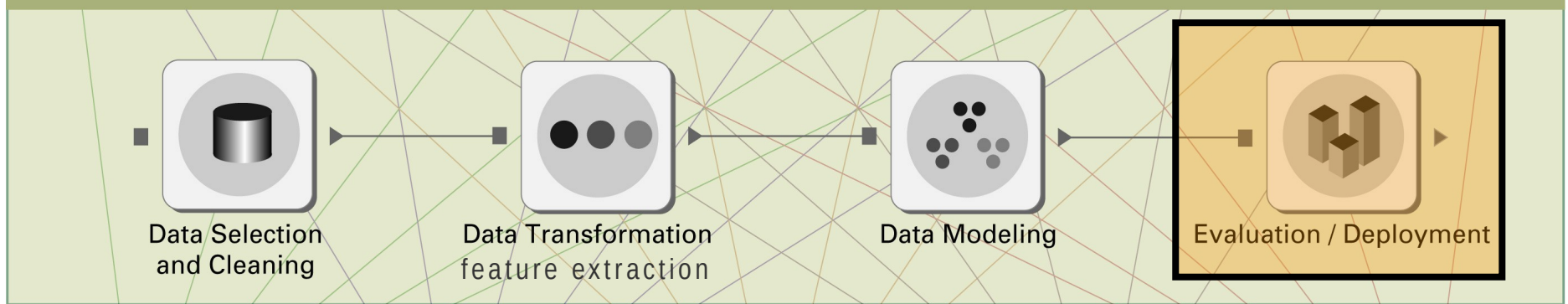
**Generalization** is the most important feature for data driven systems:  
They must perform “well” when applied to new data (**cross-validation**).



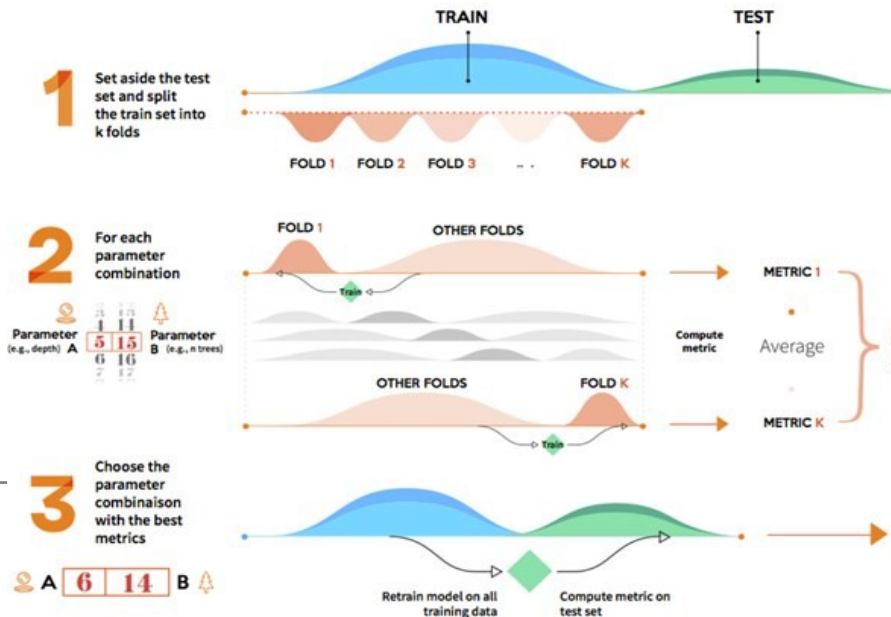
# Sectores de aplicación



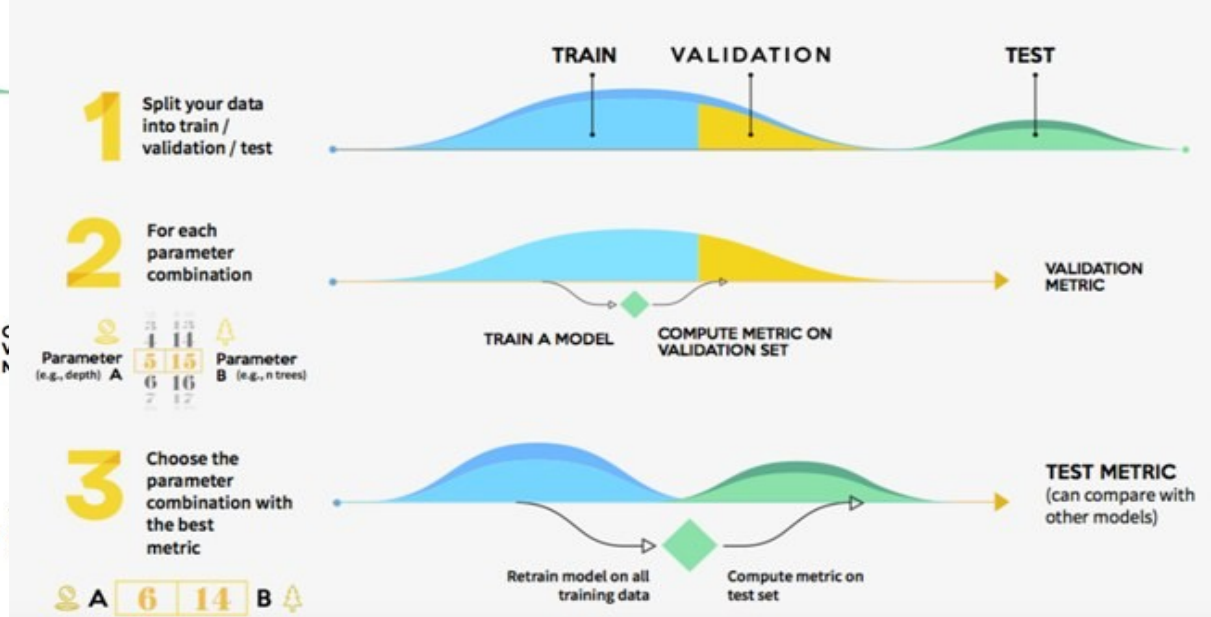
## Proceso de Minería de Datos



### K-FOLD STRATEGY



### HOLDOUT STRATEGY



UNKNOWN TARGET FUNCTION

$$y \sim f(x), P(x,y)$$

Finite/Incomplete sample

TRAINING EXAMPLES

$$(\mathbf{x}_1, y_1), \dots, (\mathbf{x}_N, y_N)$$

$$\mathbf{x}_1, \dots, \mathbf{x}_N$$

PROBABILITY  
DISTRIBUTION

$P$  on  $\mathcal{X}$

LEARNING  
ALGORITHM

$\mathcal{A}$

FINAL  
MODEL

$$f'(x), P'(x,y)$$

$$y' \quad P'(y|x)$$

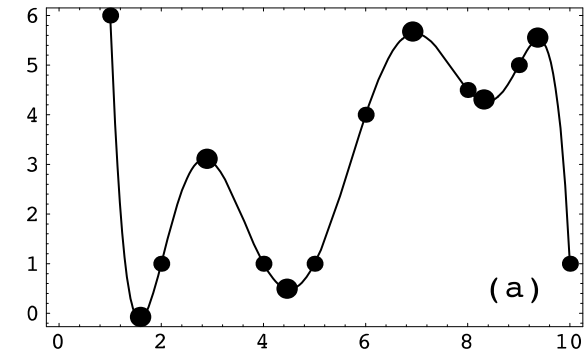
OUTPUT

MODEL FAMILY

$\mathcal{H}$

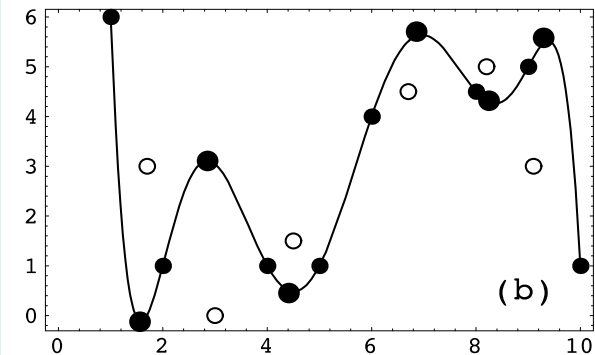
Adaptado de Y.S. Abu-Mostafa  
California Institute of Technology

$$y = \sum_{i=0}^N (a_i * x^i)$$



New sample

Overfitting



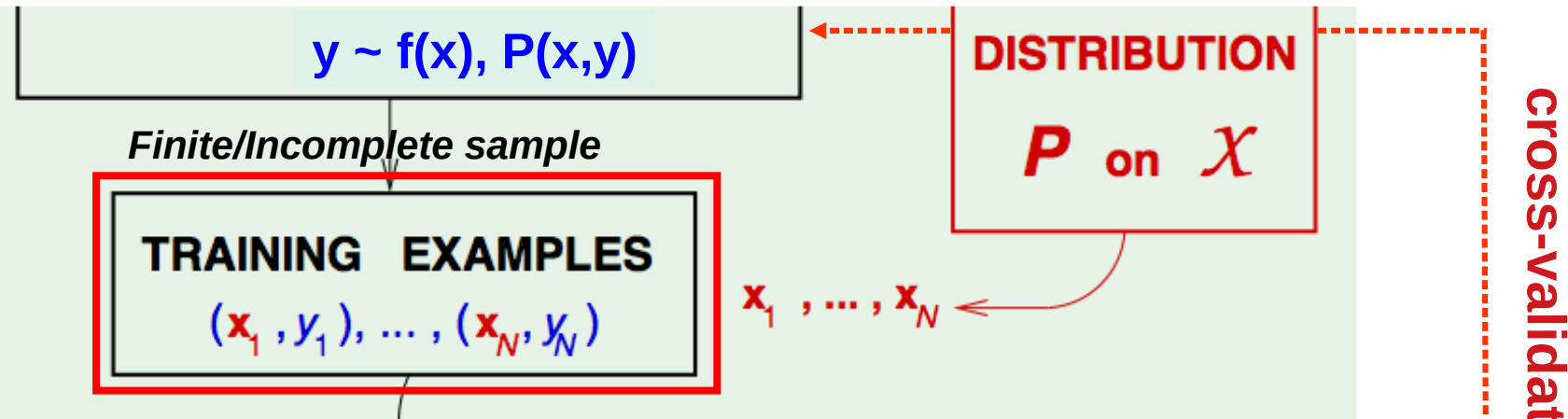
Trade-off between  
bias and variance

Model complexity  $\leftrightarrow$  degrees of freedom

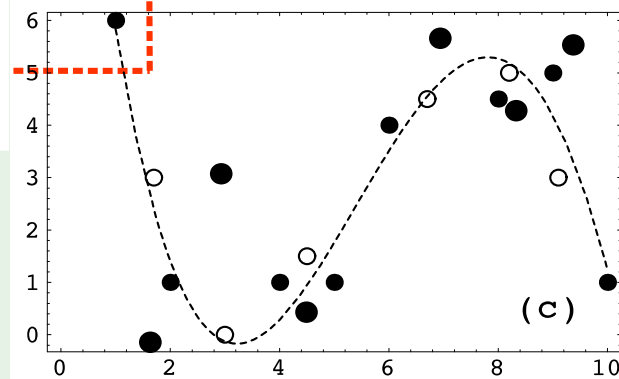
Cross-  
Validation

LEARNING FROM DATA

**Generalization** is the most important feature for data driven systems: They must perform “well” when applied to new data (**cross-validation**).



Simplest models have better generalization properties and avoid the **overfitting** removing parameters/degree of freedom.



**Trade-off between bias and variance**

**Generalization** is the most important feature for data driven systems:  
They must perform “well” when applied to new data (**cross-validation**).

1. Can we make sure that  $E_{out,1}(g)$  is close enough to  $E_{in}(g)$ ?
2. Can we make  $E_{in}(g)$  small enough?

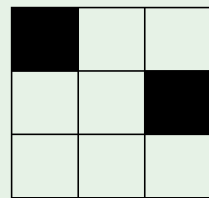
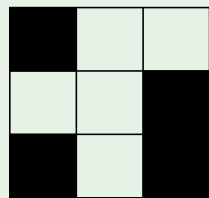
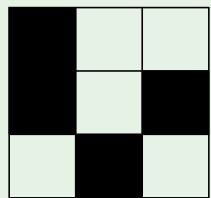


**Generalization** is the most important feature for data driven systems:  
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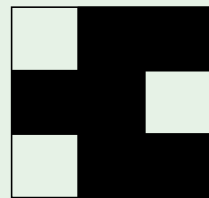
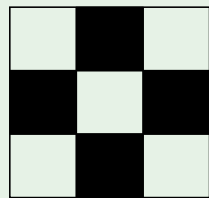
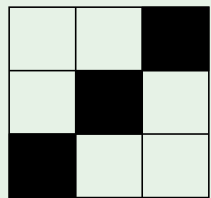
1. Can we make sure that  $E_{out}^1(g)$  is close enough to  $E_{in}(g)$ ?
2. Can we make  $E_{in}(g)$  small enough?

The (*in-sample*) error is the unique which can be estimated:

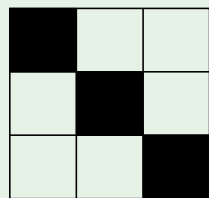
$$E_{in}(h) = \frac{1}{N} \sum_{n=1}^N (h(x_n) - y_n)^2$$



$f = -1$



$f = +1$



$f = ?$

		$f$	
		+ 1	- 1
$h$	+ 1	no error	false accept
	- 1	false reject	no error

$$E_{out}(h) = E(f, h)$$

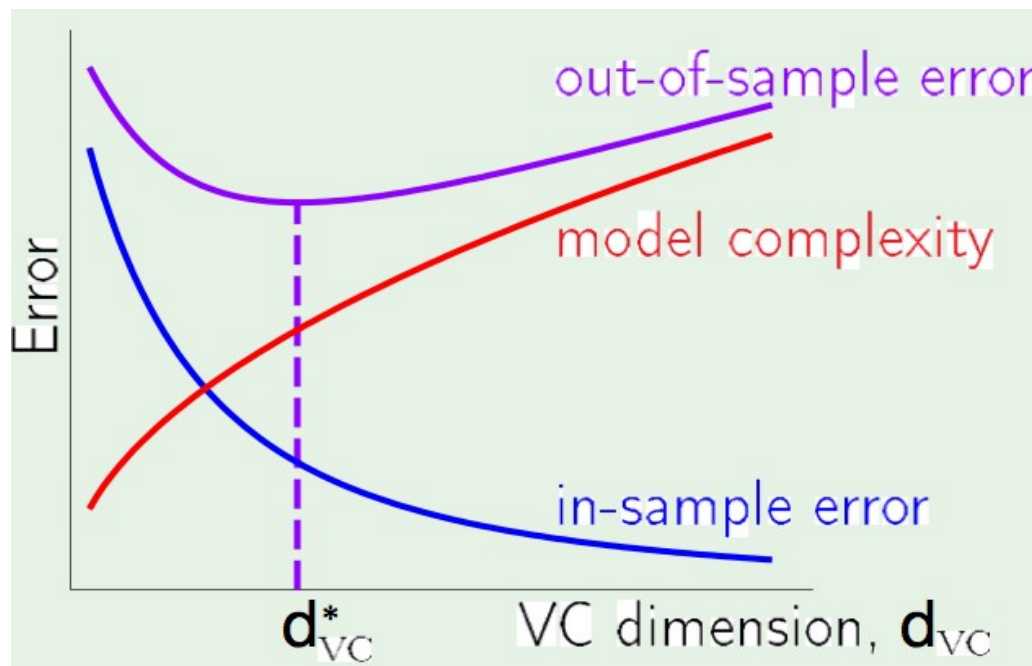
**Generalization** is the most important feature for data driven systems:  
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1. Can we make sure that  $E_{out,1}(g)$  is close enough to  $E_{in}(g)$ ?
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The (*in-sample*) error is the unique which can be estimated:

$$E_{in}(h) = \frac{1}{N} \sum_{n=1}^N (h(x_n) - y_n)^2$$

## Vapnik-Chervonenkis (VC) Dimension



		$f$	
		+ 1	- 1
$h$	+ 1	no error	false accept
	- 1	false reject	no error

$$E_{out}(h) = E(f, h)$$

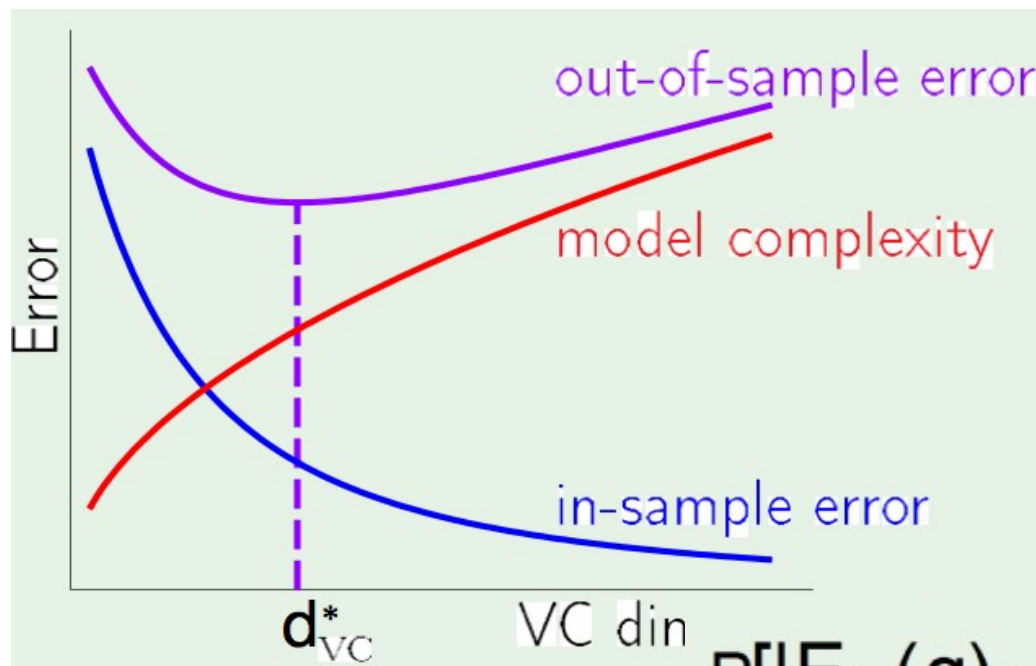
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The (**in-sample**) error is the unique which can be estimated:

$$E_{in}(h) = \frac{1}{N} \sum_{n=1}^N (h(x_n) - y_n)^2$$

## Vapnik-Chervonenkis (VC) Dimension



$$E_{out}(h) = E(f, h)$$

$$\mathbb{P}[|v - \mu| > \epsilon] \leq 2e^{-2\epsilon^2 N}$$

$N$ =sample size

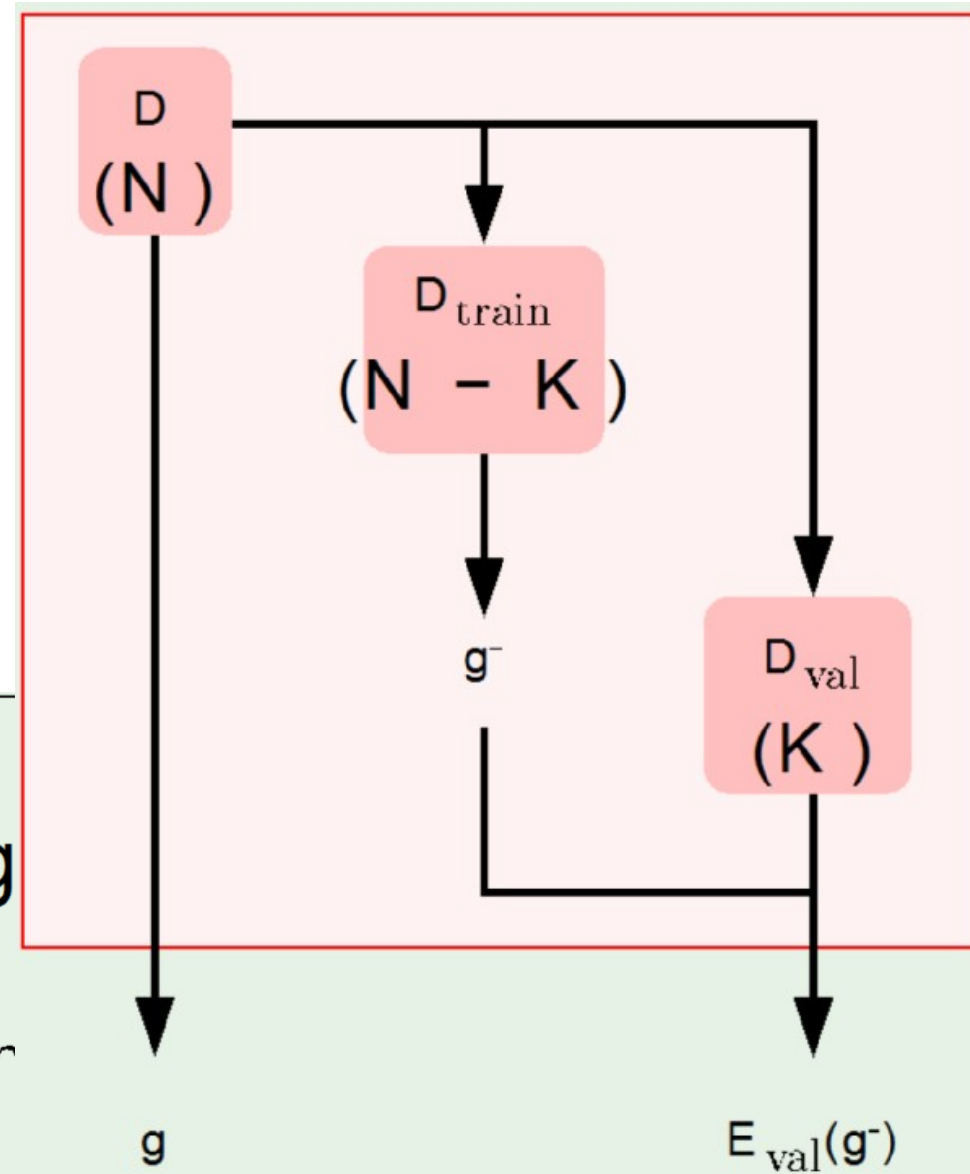
$M$ =model complexity

$$P[|E_{in}(g) - E_{out}(g)| > \epsilon] \leq 2M e^{-2\epsilon^2 N}$$

**Generalization** is the most important feature for data driven systems:  
They must perform “well” when applied to new data (**cross-validation**).

The sample is divided in two subsets:  
**train** and **test**.

- Hold-out
- Leave-one-out
- K-fold



1. Can we make sure that  $E_{\text{out}}(g)$

2. Can we make  $E_{\text{in}}(g)$  smaller

# HOLDOUT STRATEGY

**1** Split your data into train / validation



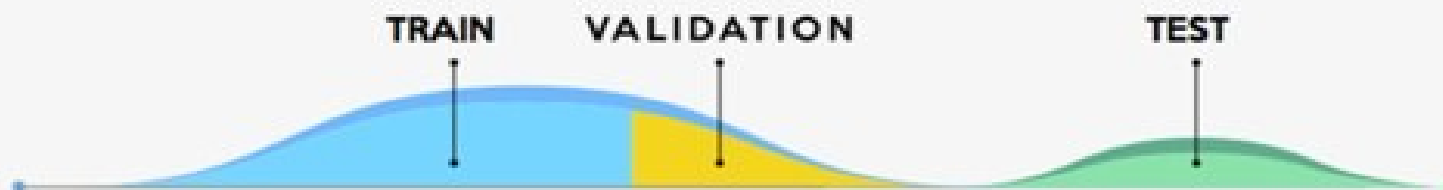
**2** For each parameter combination

Parameter (e.g., depth) <b>A</b>	4	14
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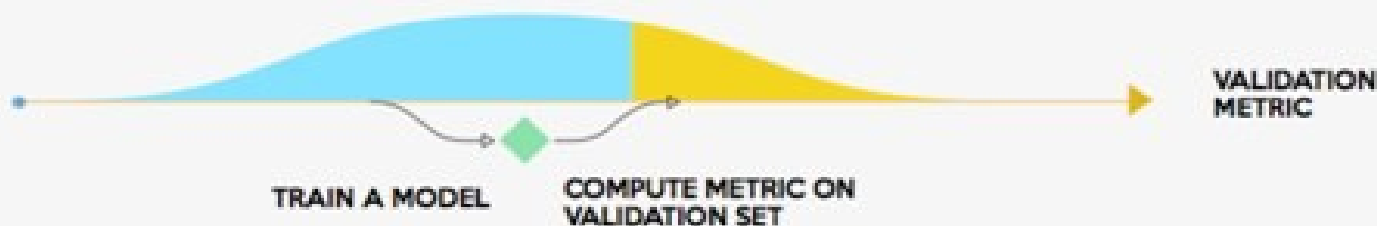
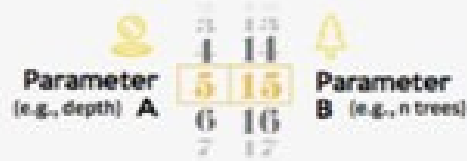


# HOLDOUT STRATEGY

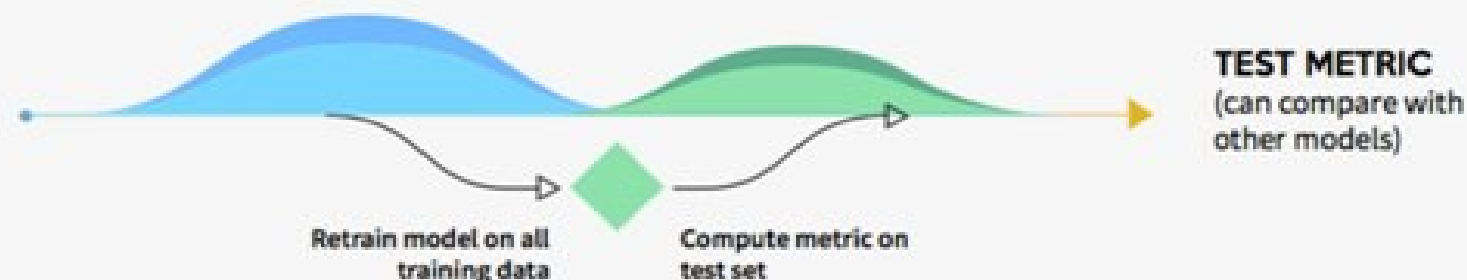
**1** Split your data into train / validation / test



**2** For each parameter combination



**3** Choose the parameter combination with the best metric



Source: [Robert Kelley](#)

# HOLDOUT STRATEGY

1

Split your data  
into train /  
validation / test



```
plot(Altura, Peso)
train <- 1:ceiling(n/2)
order.index <- order(Peso)
Peso.sort <- Peso[order.index]
Altura.sort <- Altura[order.index]
points(Altura.sort[train], Peso.sort[train], pch=16, col="red")
mean.peso <- mean(Peso.sort[train])
abline(h=mean.peso)
# El error de test es mucho mayor ya que el modelo no generaliza.
mse.train <- mse(Peso.sort[train], mean.peso); mse.train
mse.test <- mse(Peso.sort[-train], mean.peso); mse.test
```

Source: [Robert I. Kelley](#)

# HOLDOUT STRATEGY

1

Split your data  
into train /  
validation / test



# Mejor si cogemos los datos aleatoriamente.

set.seed(1) # Para obtener el mismo valor fijamos la semilla

```
train <- sample(n, ceiling(n/2))
```

```
plot(Altura, Peso)
```

```
points(Altura[train], Peso[train], pch=16, col="red")
```

#y.est=cte esa cte es la media de la variable y seleccionada en train

```
mean.peso <- mean(Peso[train])
```

```
abline(h=mean.peso)
```

```
mse.train <- mse(Peso[train], mean.peso); mse.train
```

```
mse.test <- mse(Peso[-train], mean.peso); mse.test
```

Source: [Robert I. Kelley](#)

# HOLDOUT STRATEGY

1

Split your data  
into train /  
validation / test



# Sin embargo, hay una gran variabilidad respecto a la muestra

```
plot(Altura, Peso)
for (i in c(1:5)){
  train <- sample(n,ceiling(n/2))
  mean.peso <- mean(Peso[train])
  abline(h=mean.peso)
  print(mse(Peso[-train],mean.peso))
}
```

Source: [Robert I. Kelley](#)

# HOLDOUT STRATEGY

1

Split your data  
into train /  
validation / test



# El problema se agudiza al incrementar la complejidad del modelo

```
set.seed(1)
train <- sample(n,ceiling(n/2))
plot(Altura, Peso)
points(Altura[train], Peso[train], pch=16, col="red")
Reg.2<-lm(Peso~Altura, data=Pulsaciones, subset=train)
yest.2 <- predict(Reg.2, data.frame(Altura=Altura[-train]))
mse.Reg.2<-mse(Peso[-train],yest.2); mse.Reg.2
yest.2.train <- predict(Reg.2, data.frame(Altura=Altura[train]))
mse.Reg.2.train<-mse(Peso[train],yest.2.train); mse.Reg.2.train
```

Source: [Robert I. Kelley](#)



**Generalization** is the most important feature for data driven systems:  
They must perform “well” when applied to new data (**cross-validation**).

The sample is divided in two subsets:

**train** and **test**.

- **Hold-out:**

- Variability related with the train-test splitting.
- The sample size of the test sample leads to conservative results.
- The sample size of the train sample limits the complexity of the model.
- Leave-one-out
- K-fold

1. Can we make sure that  $E_{out_1}(g)$  is close enough to  $E_{in}(g)$ ?
2. Can we make  $E_{in}(g)$  small enough?

**Generalization** is the most important feature for data driven systems:  
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- **Leave-one-out**
- K-fold

$N - 1$  points for training, and **1 point** for validation!

$$D_n = (x_1, y_1), \dots, (x_{n-1}, y_{n-1}), (\mathbf{x_n, y_n}), (x_{n+1}, y_{n+1}), \dots, (x_N, y_N)$$

Final hypothesis learned from  $D_n$  is  $g_n^-$

$$e_n = E_{\text{val}}(g_n^-) = e(g_n^-(x_n), y_n)$$

cross validation error: 
$$E_{\text{cv}} = \frac{1}{N} \sum_{n=1}^N e_n$$

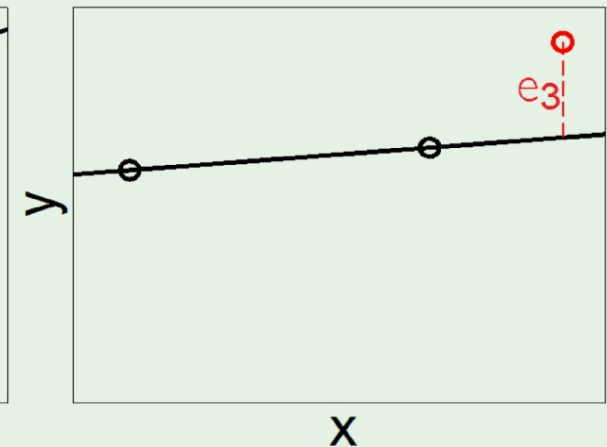
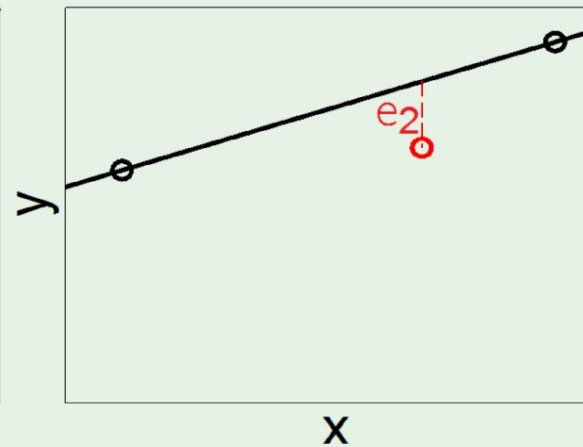
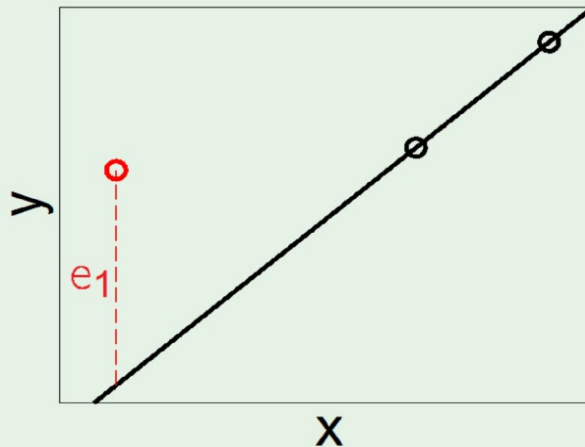
**Generalization** is the most important feature for data driven systems:  
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The sample is divided in two subsets:  
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- Hold-out:
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- **Leave-one-out**
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$N - 1$  points for training, and **1 point** for validation!

$$D_n = (x_1, y_1), \dots, (x_{n-1}, y_{n-1}), (\mathbf{x_n, y_n}), (x_{n+1}, y_{n+1}), \dots, (x_N, y_N)$$



$$E_{cv} = \frac{1}{3} (e_1 + e_2 + e_3)$$

**Generalization** is the most important feature for data driven systems: They must perform “well” when applied to new data (**cross-validation**).

The sample is divided in two subsets:

**train** and **test**.

- Hold-out:
  - Variability related with the train-test splitting.
  - The sample size of the test sample leads to conservative results.
  - The sample size of the train sample limits the complexity of the model.
- **Leave-one-out:**
  - High computational cost (**small samples**).
- K-fold

**Generalization** is the most important feature for data driven systems: They must perform “well” when applied to new data (**cross-validation**).

The sample is divided in two subsets:

**train** and **test**.

- Hold-out:
  - Variability related with the train-test splitting.
  - The sample size of the test sample leads to conservative results.
  - The sample size of the train sample limits the complexity of the model.
- **Leave-one-out**:
  - High computational cost (**small samples**).
- K-fold

### # Leave-One-Out Cross-Validation

```
yest.3<-rep(NA, length(train)) # La actualización es ineficiente
```

```
train <- 1:n
```

```
for (i in train){
```

```
  Reg.i<-lm(Peso~Altura, data=Pulsaciones, subset=train[-i])
```

```
  yest.3[i]<-predict(Reg.i,data.frame(Altura=Altura[i]))
```

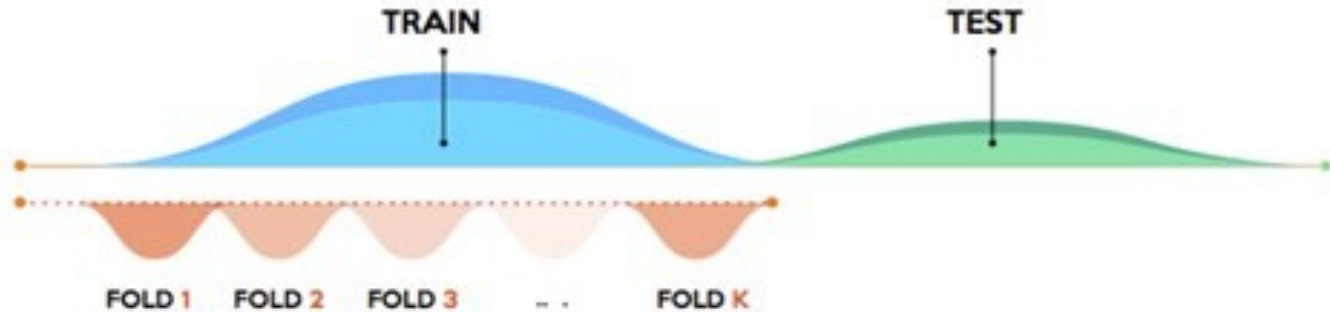
```
}
```

```
mse.Reg.3<-mse(Peso,yest.3); mse.Reg.3
```

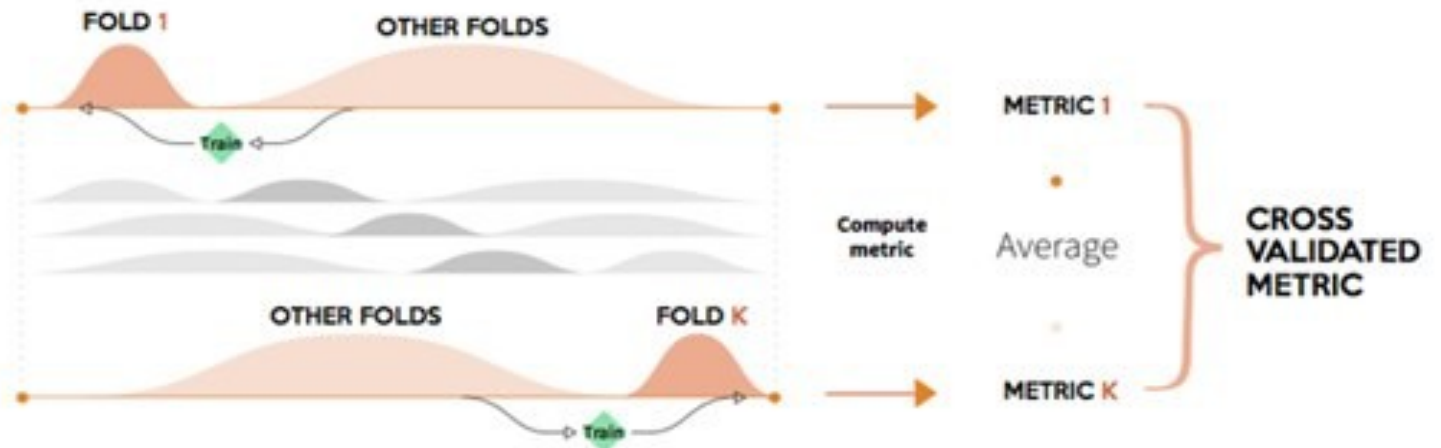


# K-FOLD STRATEGY

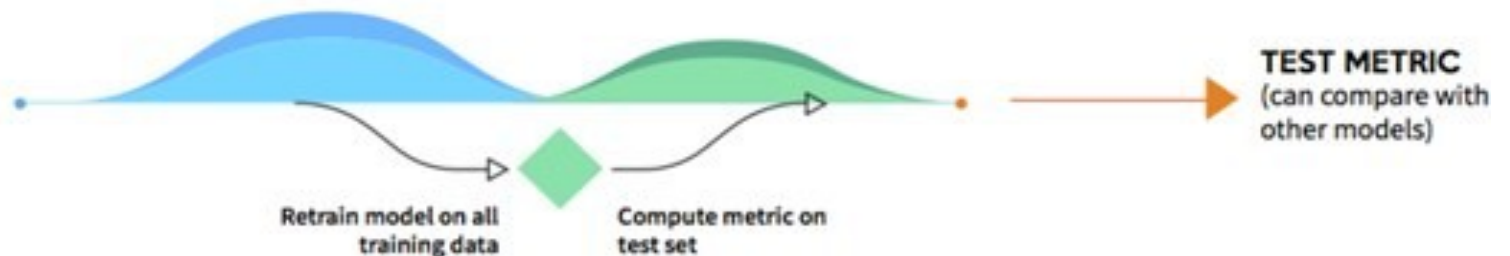
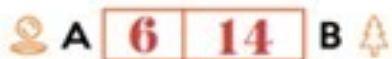
**1** Set aside the test set and split the train set into k folds



**2** For each parameter combination

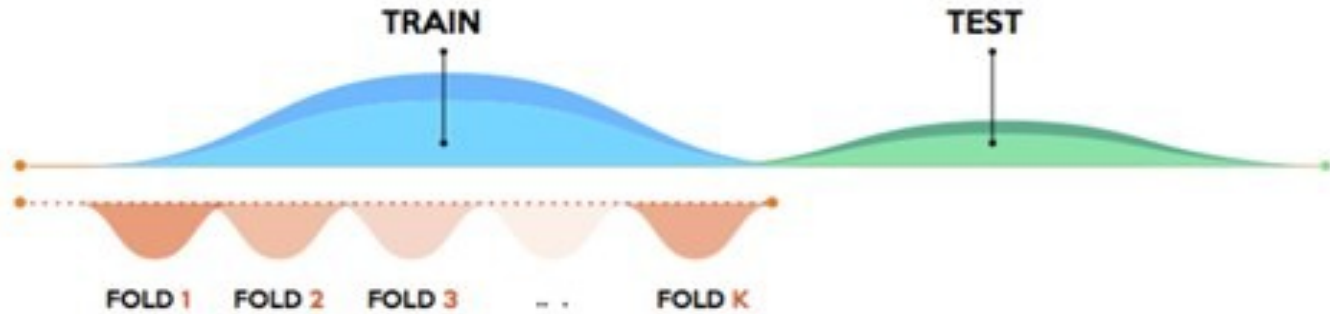


**3** Choose the parameter combination with the best metrics

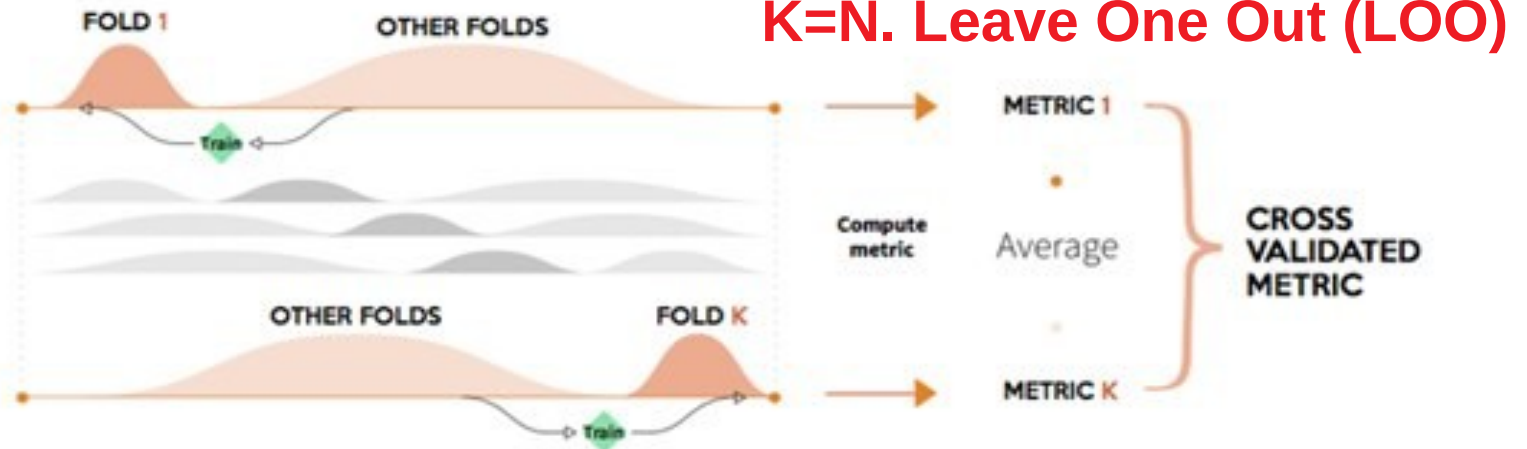


# K-FOLD STRATEGY

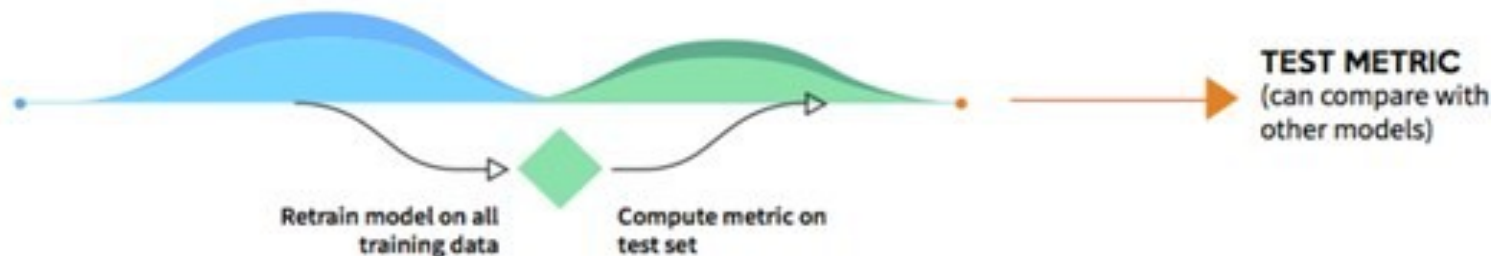
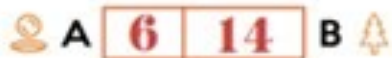
**1** Set aside the test set and split the train set into k folds



**2** For each parameter combination



**3** Choose the parameter combination with the best metrics



**Generalization** is the most important feature for data driven systems: They must perform “well” when applied to new data (**cross-validation**).

The sample is divided in two subsets:  
**train** and **test**.

- Hold-out:
  - Variability related with the train-test splitting.
  - The sample size of the test sample leads to conservative results.
  - The sample size of the train sample limits the complexity of the model.
- Leave-one-out:
  - High computational cost (**small samples**).
- **K-fold**:
  - Symilar results than leave-one-out with low number of folds.
  - Statistical analysis of the validation measures.

Appears in the International Joint Conference on Artificial Intelligence (IJCAI), 1995

## A Study of Cross-Validation and Bootstrap for Accuracy Estimation and Model Selection

Ron Kohavi


Computer Science Department

Stanford University

Stanford, CA. 94305

Over 3000 scitations


## Reassessing Statistical Downscaling Techniques for Their Robust Application under Climate Change Conditions

J. M. Gutiérrez ; D. San-Martin; S. Brands; R. Manzananas; S. Herrera

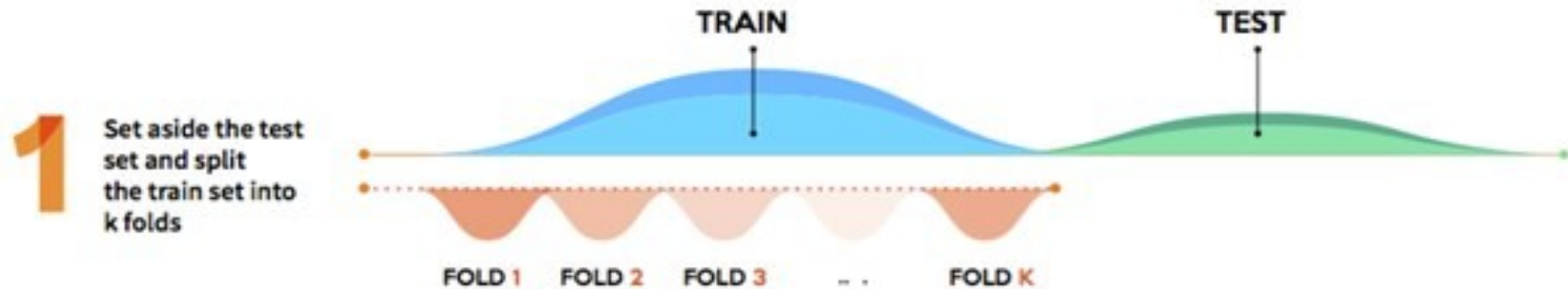
*J. Climate* (2013) 26 (1): 171–188.

Cross  
Valida

<https://doi.org/10.1175/JCLI-D-11-00687.1>

Article history 

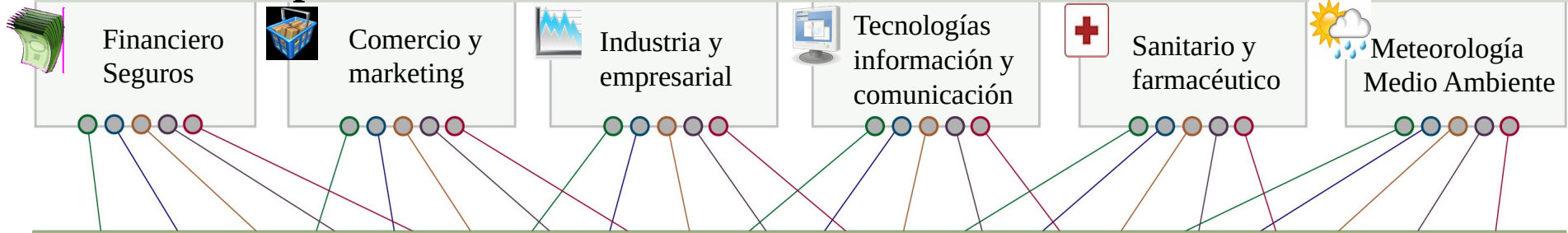
# K-FOLD STRATEGY



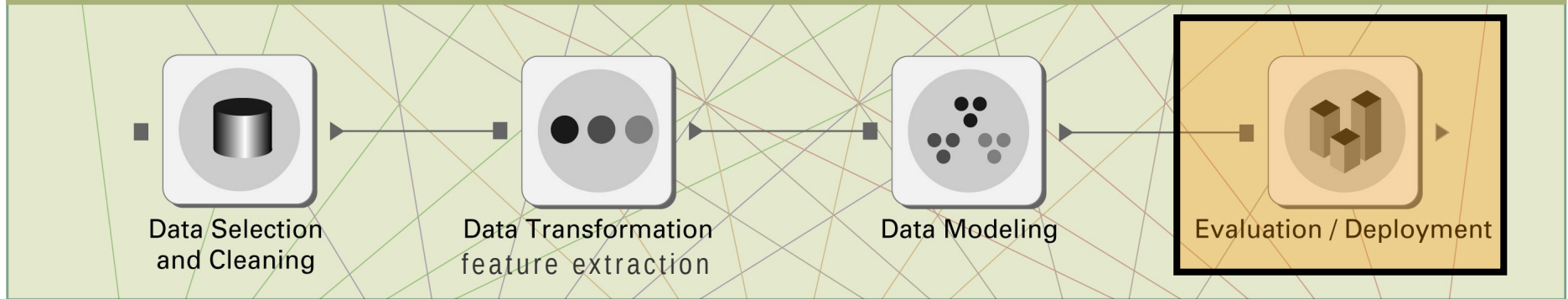
## # 10-Fold Cross-Validation

```
idx.aleatorios <- sample(1:n,n,replace=F)
K <- 10
tam <- ceiling(n/K)
yest4 <- rep(NA, length(train)) # La actualización es ineficiente
for (i in 0:(K-1)){
  idx.test <- idx.aleatorios[(i*tam+1):((i+1)*tam)]
  idx.test <- idx.test[!is.na(idx.test)]
  lm4 <- lm(Peso~Altura, subset=-idx.test)
  yest4[idx.test] <- predict(lm4, data.frame(Altura=Altura[idx.test]))
}
mse4 <- mse(Peso,yest4); mse4
```

## Sectores de aplicación

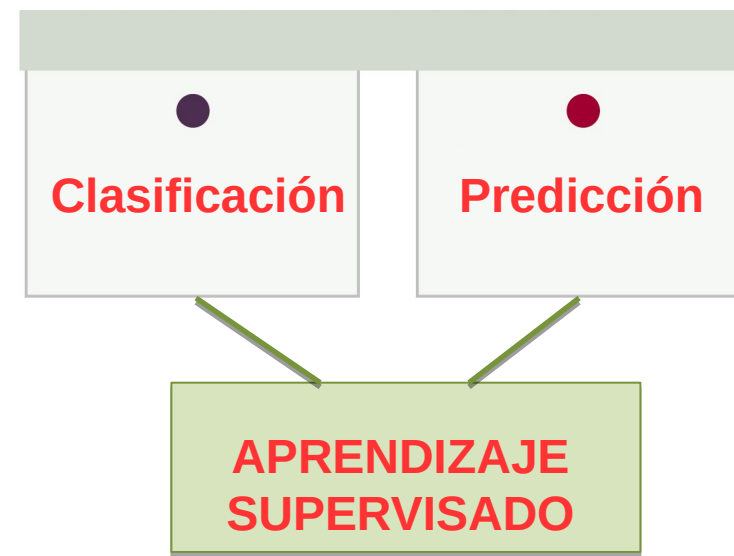


## Proceso de Minería de Datos

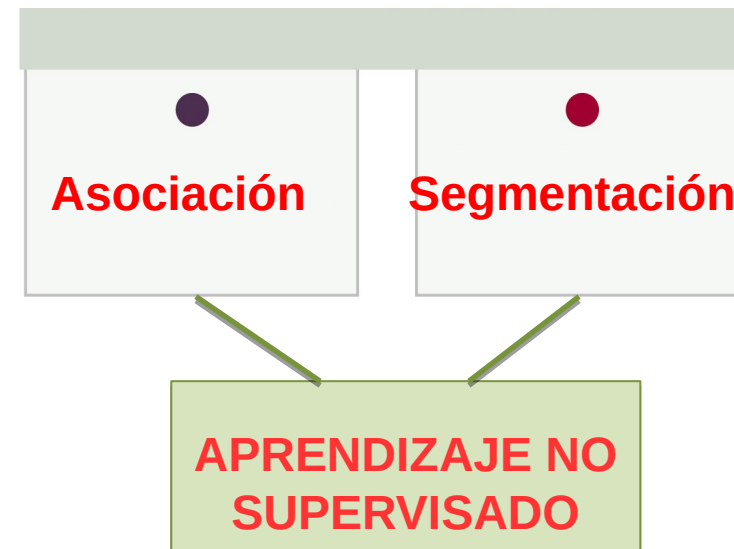


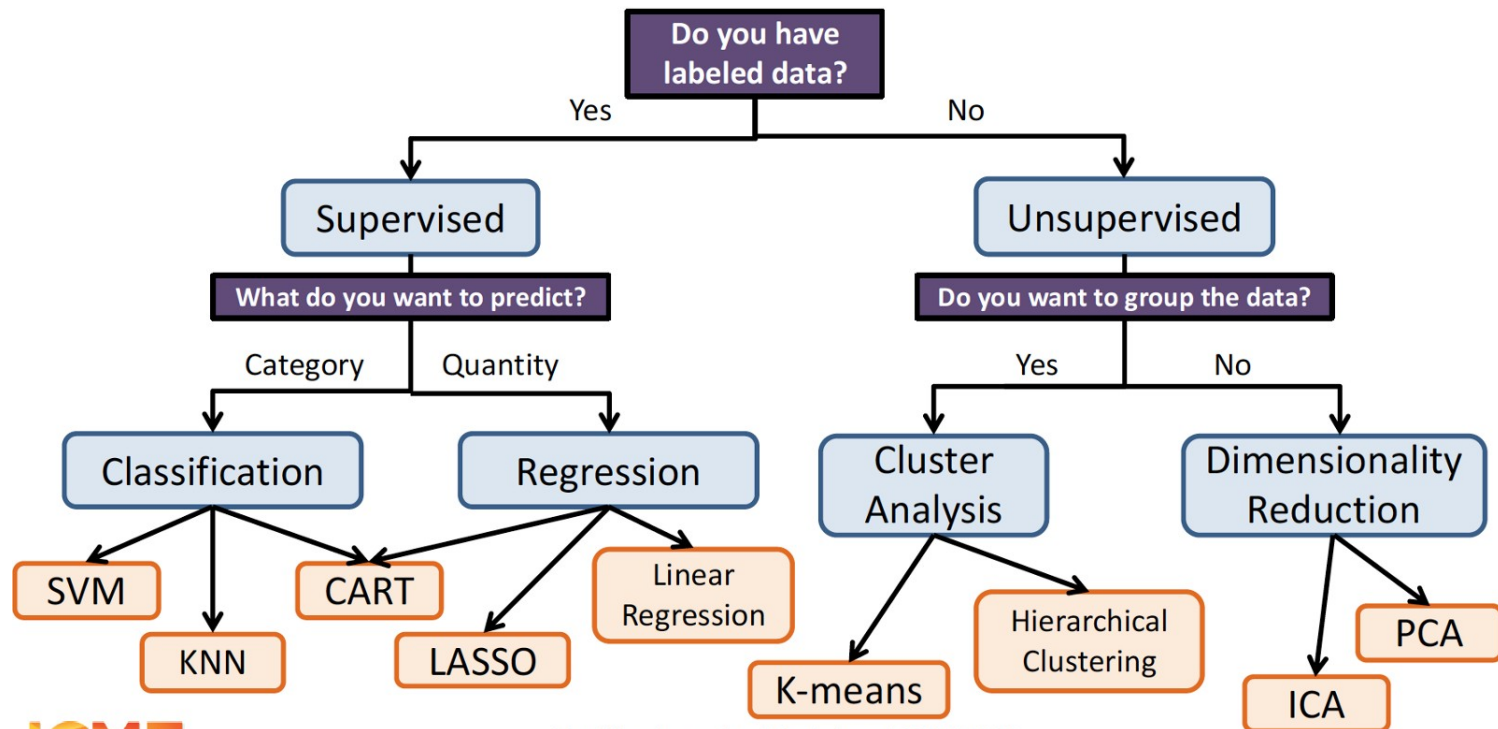
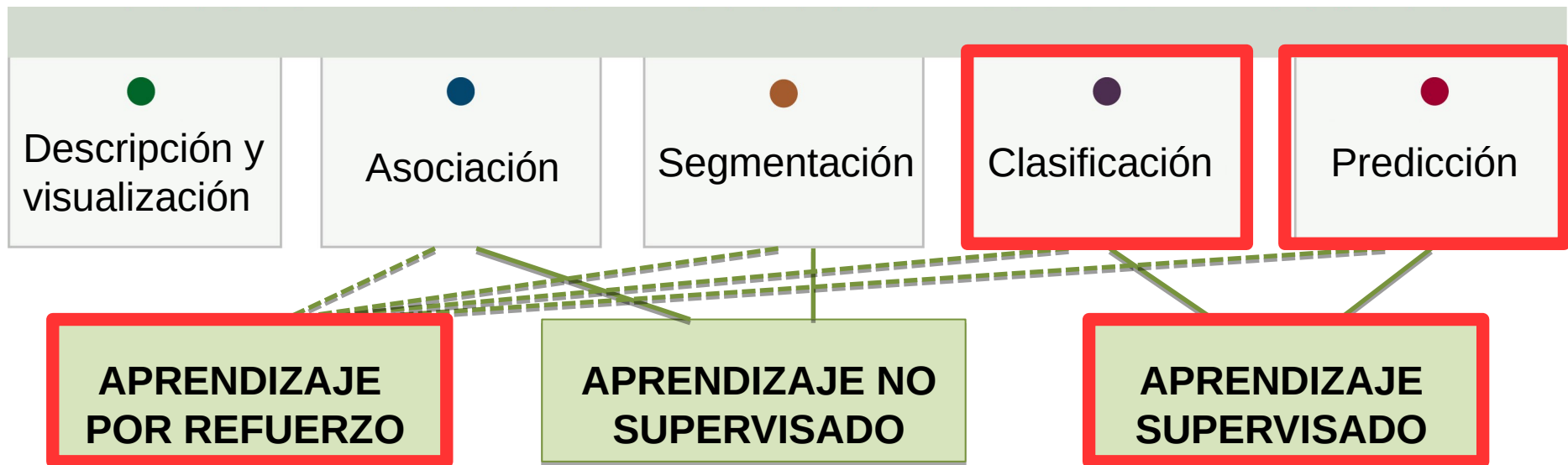


- Target Variable:
  - $Y$  (*discrete/factor* or *continuous*)
- Predictive Model  $\rightarrow Y = f(X_1, X_2, \dots, X_N)$

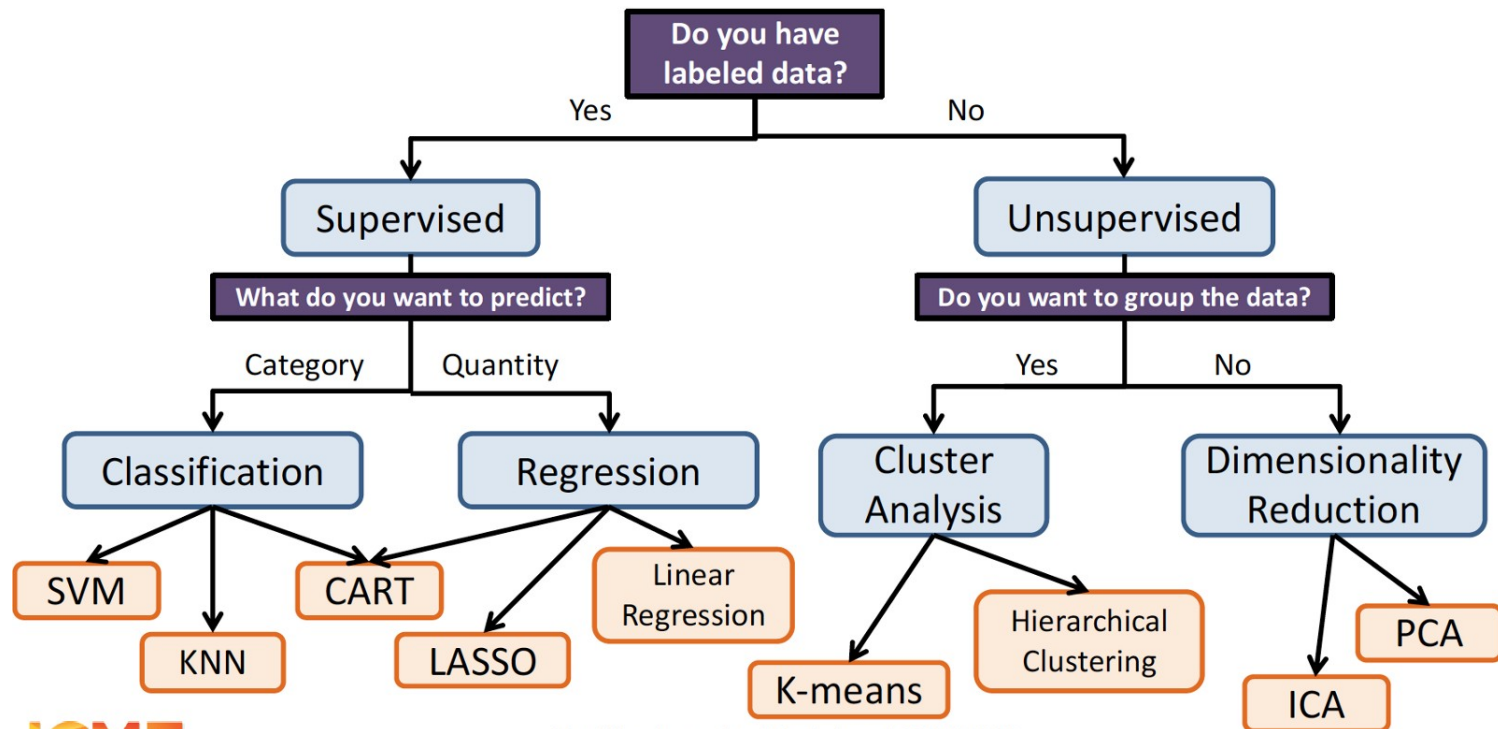
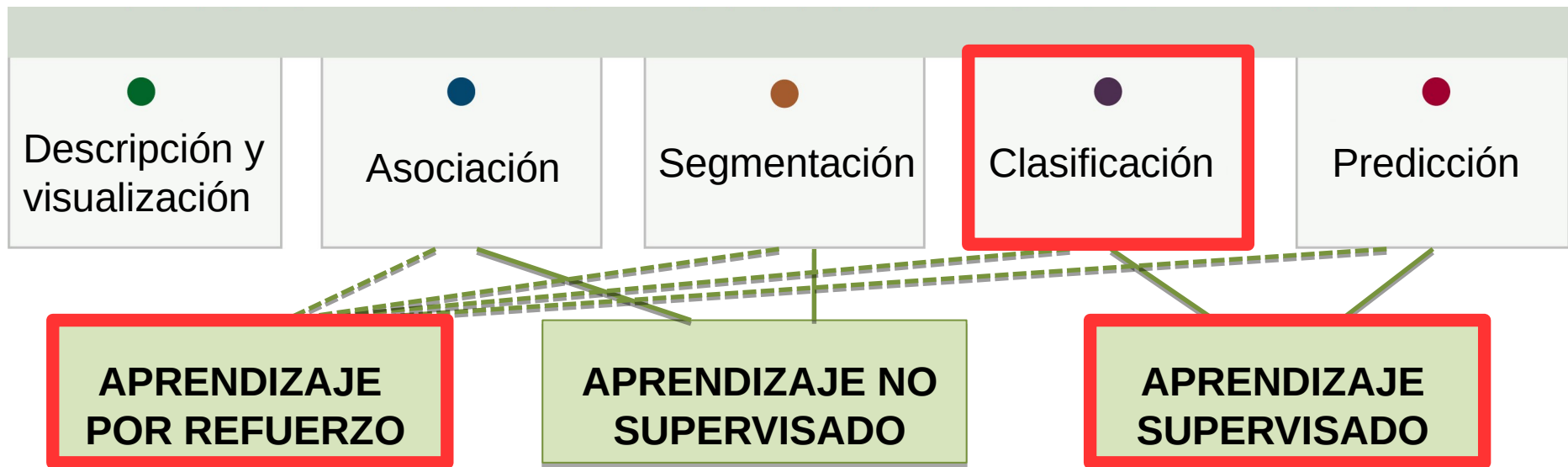


- There is no target variable:
  - *Association* or *segmentation*
- Predictive Model  $\rightarrow$  Algorithmic









# Confusion Matrix

	Predicted label class 1	Predicted label class 2
True label class 1	<b>correct</b> true positive for class 1	<b>wrong</b> false positive for class 2
True label class 2	<b>wrong</b> false positive for class 1	<b>correct</b> true positive for class 2

accuracy =  $\frac{\text{orange} + \text{blue}}{\text{orange} + \text{yellow} + \text{blue} + \text{green}}$

Source: <https://towardsdatascience.com/handling-imbalanced-datasets-in-machine-learning-7a0e84220f28>

# Confusion Matrix

	Predicted label class 1	Predicted label class 2
True label class 1	<b>correct</b> true positive for class 1	<b>wrong</b> false positive for class 2
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**Fatal Genetic Defect**  
*10 out of every 100000 babies*

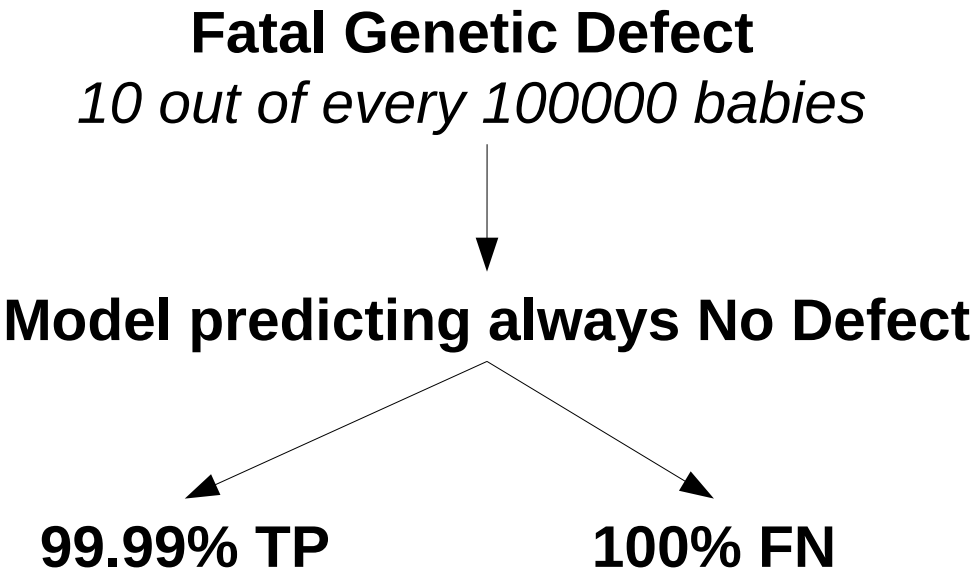
accuracy = 
$$\frac{\text{orange} + \text{blue}}{\text{orange} + \text{yellow} + \text{blue} + \text{green}}$$

Source: <https://towardsdatascience.com/handling-imbalanced-datasets-in-machine-learning-7a0e84220f28>

# Confusion Matrix

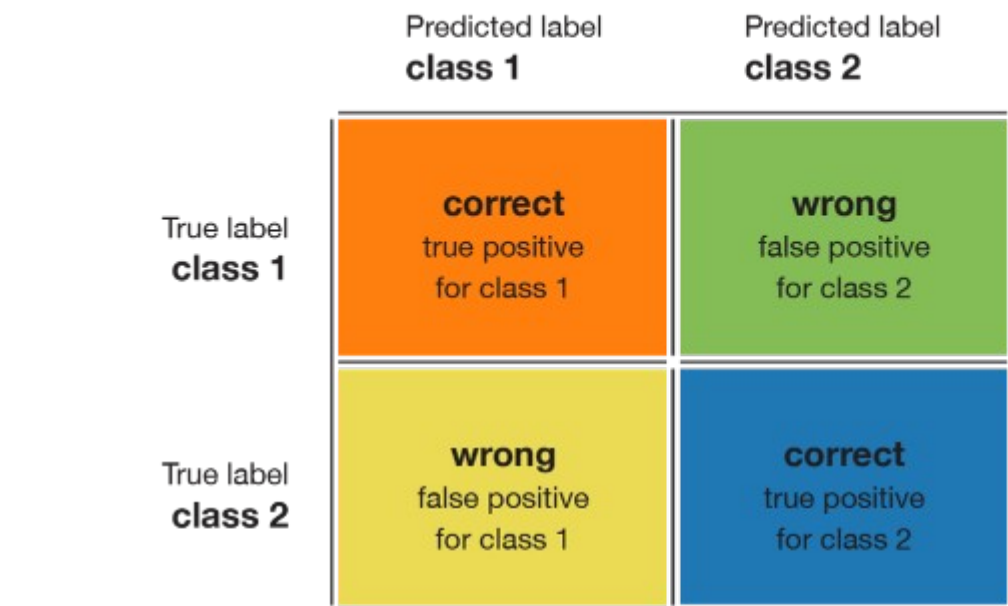
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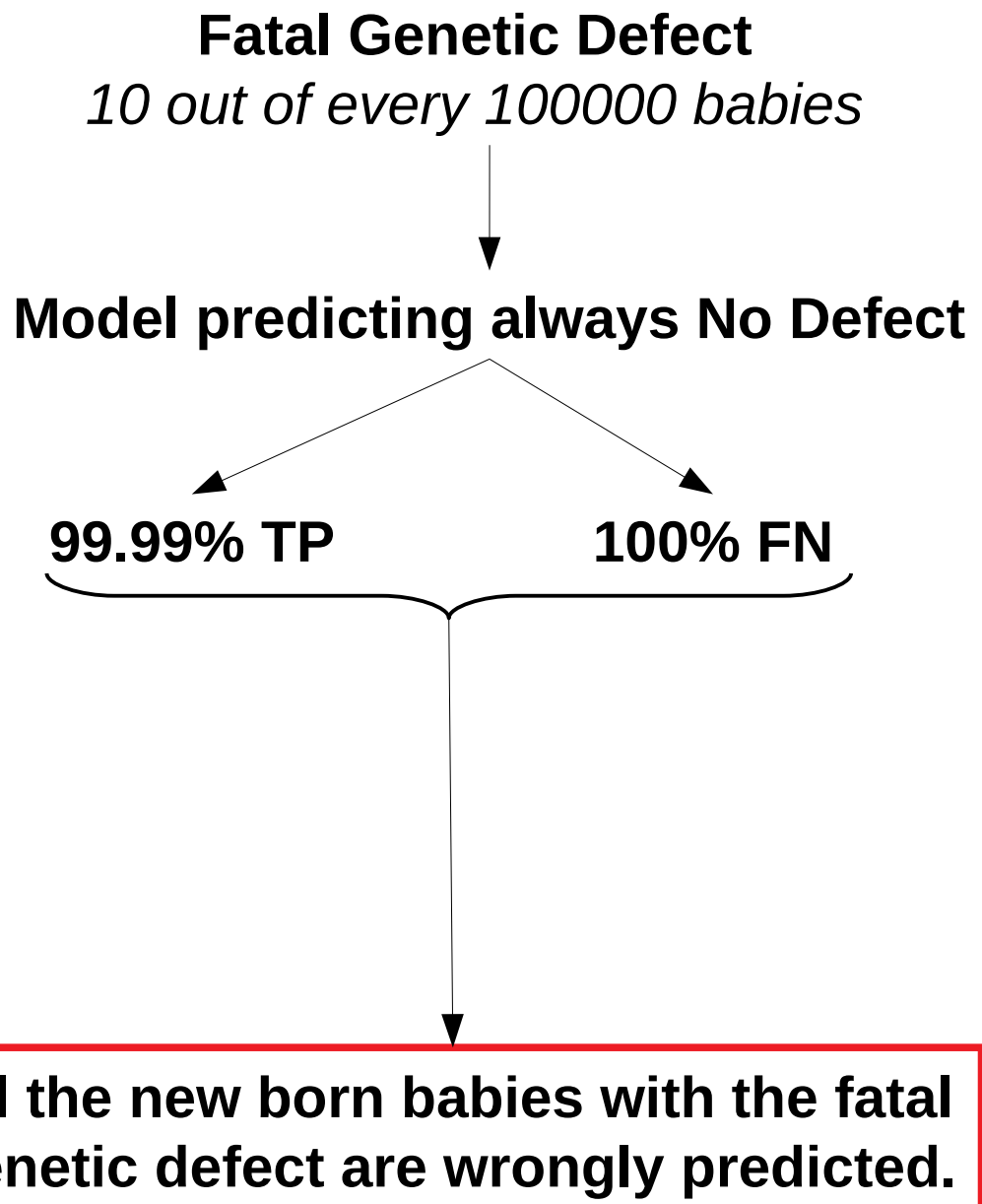


Source: <https://towardsdatascience.com/handling-imbalanced-datasets-in-machine-learning-7a0e84220f28>

# Confusion Matrix



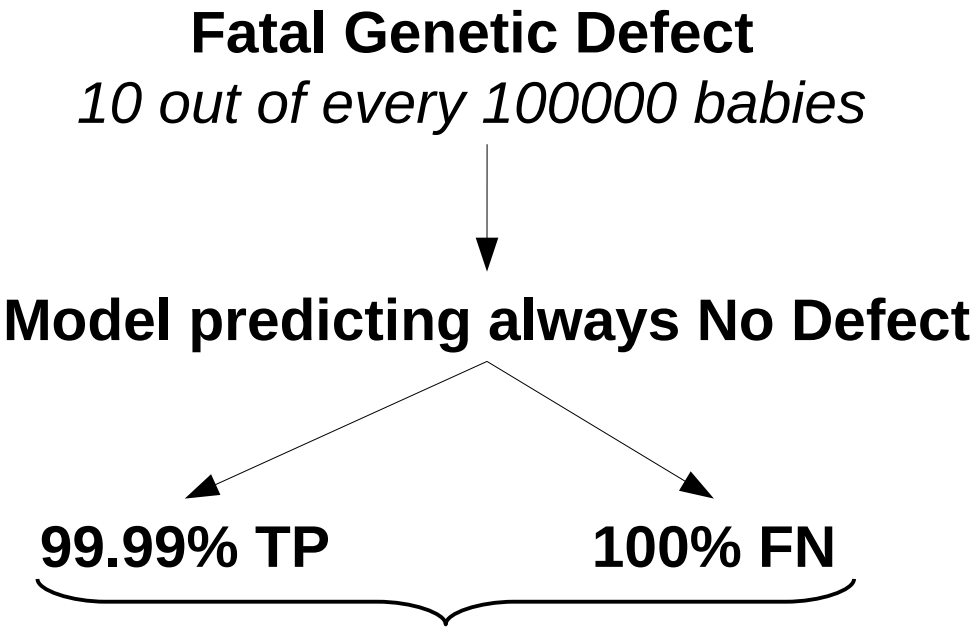
accuracy =  $\frac{\text{orange} + \text{blue}}{\text{orange} + \text{yellow} + \text{blue} + \text{green}}$



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accuracy =  $\frac{\text{orange} + \text{blue}}{\text{orange} + \text{yellow} + \text{blue} + \text{green}}$

class 1  
precision =  $\frac{\text{orange}}{\text{orange} + \text{yellow}}$

class 2  
precision =  $\frac{\text{blue}}{\text{blue} + \text{green}}$

class 1  
recall =  $\frac{\text{orange}}{\text{orange} + \text{green}}$

class 2  
recall =  $\frac{\text{blue}}{\text{blue} + \text{yellow}}$

**Precision:** *define how trustable is the result*

**Recall:** *expresses how well the model is able to detect that class*

Source: <https://towardsdatascience.com/handling-imbalanced-datasets-in-machine-learning-7a0e84220f28>

# Confusion Matrix

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**Fatal Genetic Defect**  
*10 out of every 100000 babies*

**Model predicting always No Defect**

**99.99% TP**

**100% FN**

accuracy =  $\frac{\text{orange} + \text{blue}}{\text{orange} + \text{yellow} + \text{blue} + \text{green}}$

class 1  
precision =  $\frac{\text{orange}}{\text{orange} + \text{yellow}}$

class 2  
precision =  $\frac{\text{blue}}{\text{blue} + \text{green}}$

class 1  
recall =  $\frac{\text{orange}}{\text{orange} + \text{green}}$

class 2  
recall =  $\frac{\text{blue}}{\text{blue} + \text{yellow}}$

**HR/HP:** class is perfectly handled by the model

**LR/HP:** model can't detect the class well but is highly trustable when it does

**HR/LP:** class is well detected but the model include points of other classes in it

**LR/LP:** class is poorly handled by the model

Source: <https://towardsdatascience.com/handling-imbalanced-datasets-in-machine-learning-7a0e84220f28>



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accuracy =  $\frac{\text{orange} + \text{blue}}{\text{orange} + \text{yellow} + \text{blue} + \text{green}}$

class 1  
precision =  $\frac{\text{orange}}{\text{orange} + \text{yellow}}$

class 2  
precision =  $\frac{\text{blue}}{\text{blue} + \text{green}}$

class 1  
recall =  $\frac{\text{orange}}{\text{orange} + \text{green}}$

class 2  
recall =  $\frac{\text{blue}}{\text{blue} + \text{yellow}}$

**F1-Score:**

$$\frac{2 * \text{precision} * \text{recall}}{(\text{precision} + \text{recall})}$$

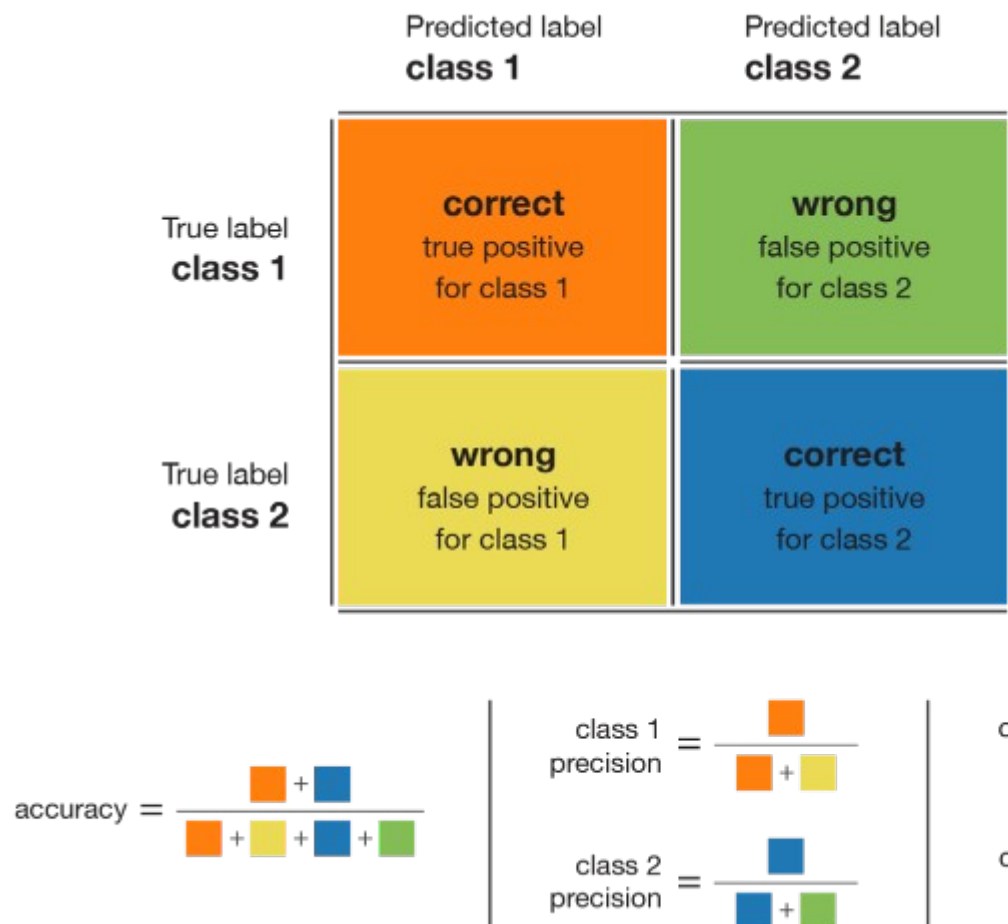
**Precision:** *define how trustable is the result*

**Recall:** *expresses how well the model is able to detect that class*

**F1:** *combines precision and recall of a class in one metric*

Source: <https://towardsdatascience.com/handling-imbalanced-datasets-in-machine-learning-7a0e84220f28>

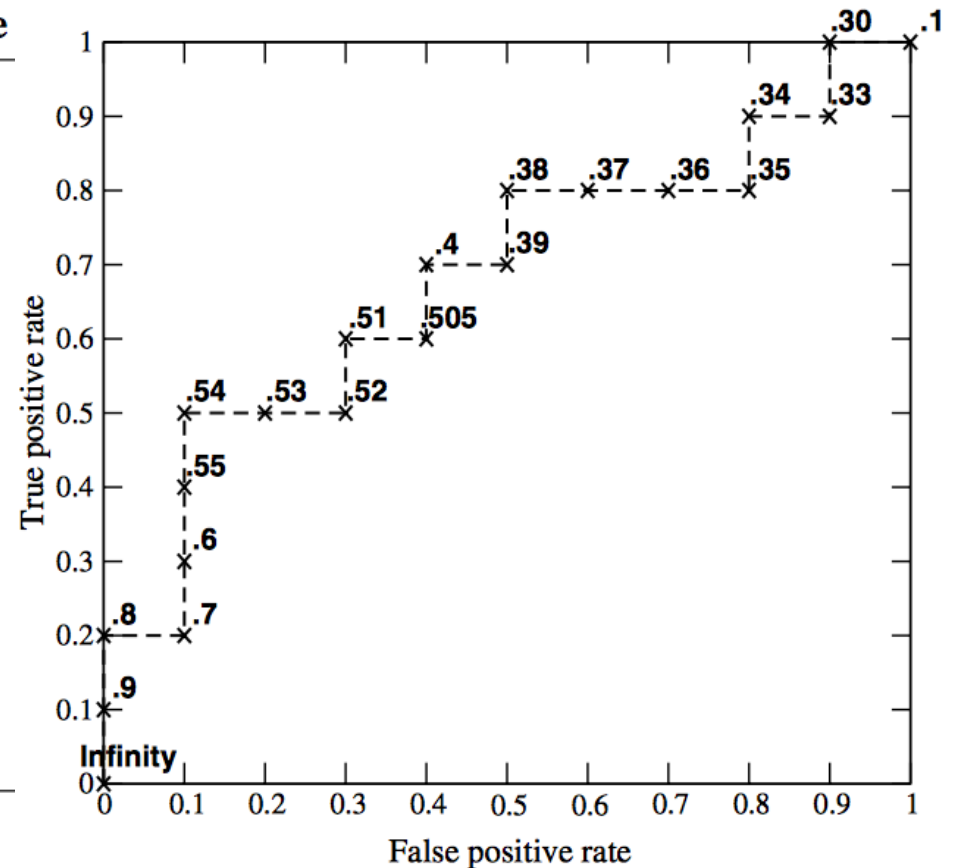
# Confusion Matrix



Summarizes the performance of the system over all possible probability thresholds.

```
library(pROC)
obs<-c(rep(0,50),rep(1,50));
prd<-obs+2*(runif(100)-0.5);
prd[which(prd<0)]<-0; prd[which(prd>1)]<-1;
plot(roc(obs,prd), print.auc=TRUE)
hist(prd)
```

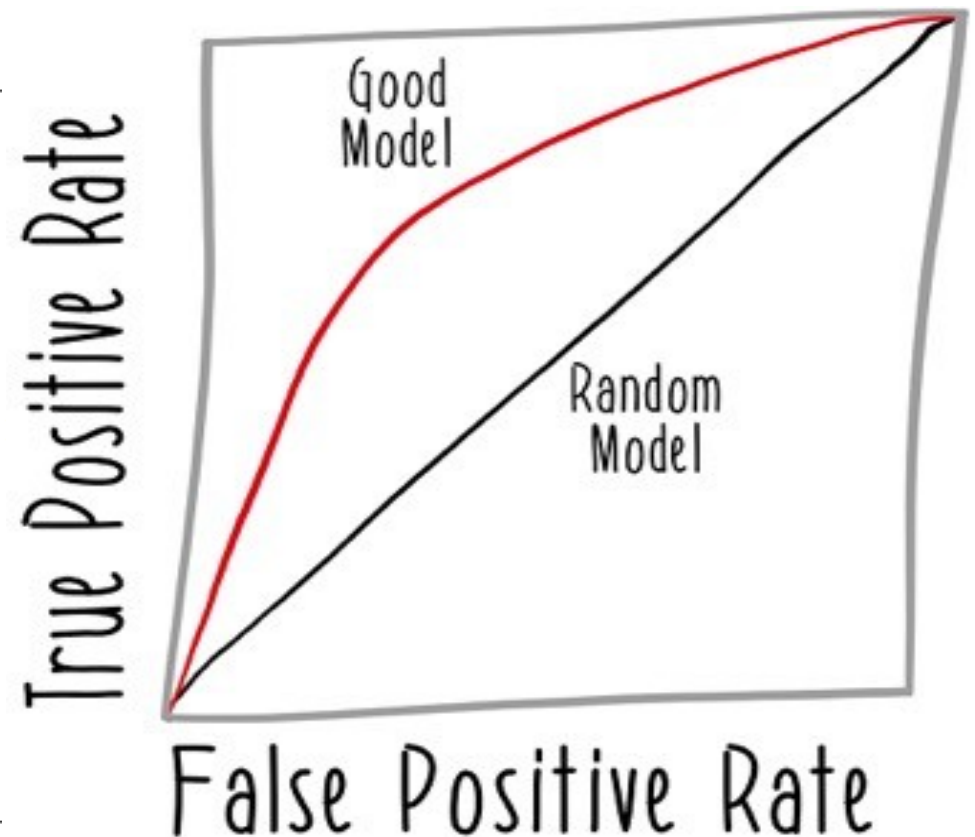
Inst#	Class	Score	Inst#	Class	Score
1	<b>p</b>	.9	11	<b>p</b>	.4
2	<b>p</b>	.8	12	<b>n</b>	.39
3	<b>n</b>	.7	13	<b>p</b>	.38
4	<b>p</b>	.6	14	<b>n</b>	.37
5	<b>p</b>	.55	15	<b>n</b>	.36
6	<b>p</b>	.54	16	<b>n</b>	.35
7	<b>n</b>	.53	17	<b>p</b>	.34
8	<b>n</b>	.52	18	<b>n</b>	.33
9	<b>p</b>	.51	19	<b>p</b>	.30
10	<b>n</b>	.505	20	<b>n</b>	.1



Summarizes the performance of the system over all possible probability thresholds.

```
library(pROC)
obs<-c(rep(0,50),rep(1,50));
prd<-obs+2*(runif(100)-0.5);
prd[which(prd<0)]<-0; prd[which(prd>1)]<-1;
plot(roc(obs,prd), print.auc=TRUE)
hist(prd)
```

Inst#	Class	Score	Inst#	Class	Score
1	p	.9	11	p	.4
2	p	.8	12	n	.39
3	n	.7	13	p	.38
4	p	.6	14	n	.37
5	p	.55	15	n	.36
6	p	.54	16	n	.35
7	n	.53	17	p	.34
8	n	.52	18	n	.33
9	p	.51	19	p	.30
10	n	.505	20	n	.1



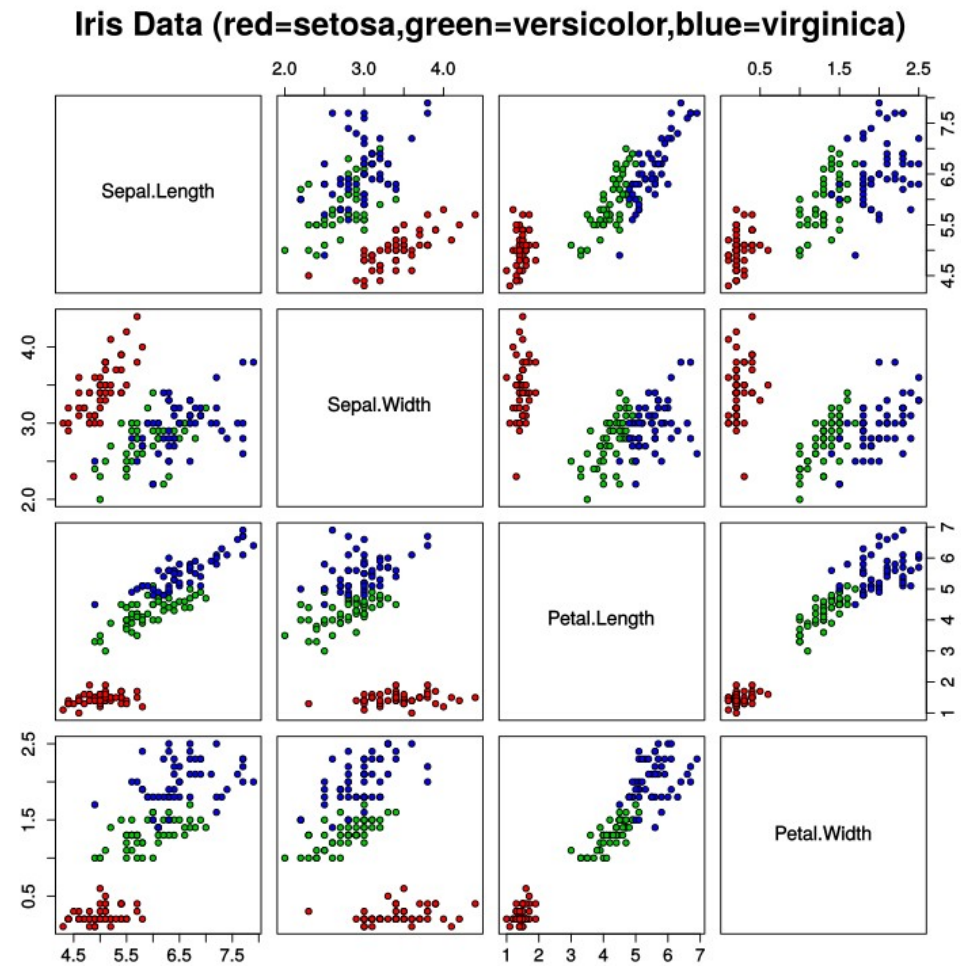
<https://www.kdnuggets.com/2018/01/machine-learning-model-metrics.html>

```

data(iris)
fitControl <- trainControl(method="none",
                           number=1,
                           repeats=1,
                           verboseIter=TRUE)

modelFit <- train(Species ~ ., data=iris, method="knn", trControl=fitControl)
pred <- predict(modelFit, newdata = iris[,-5])
acc<-confusionMatrix(iris$Species,pred)
print(acc)

```



# Confusion Matrix

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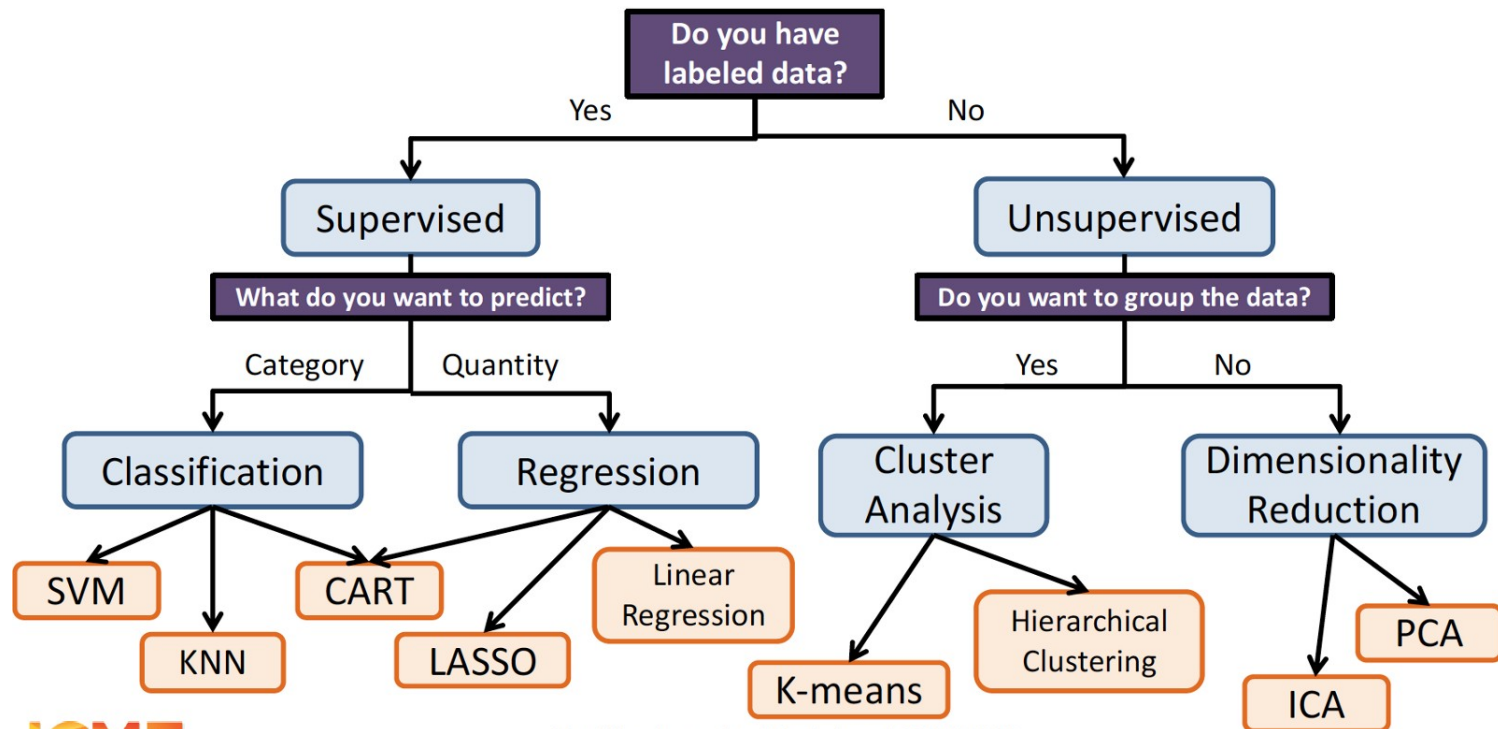
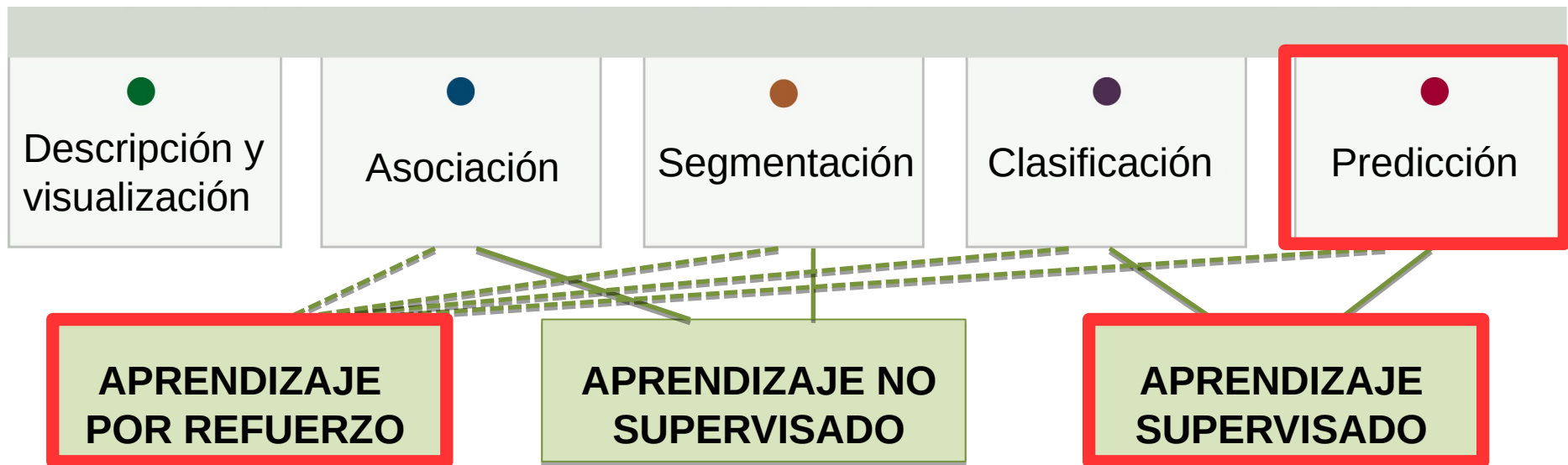
**Fatal Genetic Defect**  
*10 out of every 100000 babies*

$$\text{accuracy} = \frac{\text{orange} + \text{blue}}{\text{orange} + \text{yellow} + \text{blue} + \text{green}}$$

Dealing with unbalanced data in machine learning  
[https://shiring.github.io/machine\\_learning/2017/04/02/unbalanced](https://shiring.github.io/machine_learning/2017/04/02/unbalanced)

Source: <https://towardsdatascience.com/handling-imbalanced-datasets-in-machine-learning-7a0e84220f28>







## Model accuracy (training and validation).

Some models are trained using an **empirical error (cost) function**, which measures **model accuracy** as the difference between the predicted and the actual value. In this case, this is a natural **validation measure**.

- This cost function could be anything:
  - Sum of absolute errors:  $J = \sum |y - u|$ .
  - Sum of square errors:  $J = \sum (y - u)^2$ .
  - As long as the minimum occurs when the distributions are the same, in theory it would work.
- One good idea is that  $u$  represents the parameters of the distribution of  $y$ .
  - Rationale: often natural processes are fuzzy, and any input might have a range of outputs.
  - This approach also gives a smooth measure of how accurate we are.
  - The maximum likelihood principle says that:  $\theta_{\text{ML}} = \arg \max_{\theta} p(y; u)$
  - Thus we want to minimize:  $J = -p(y; u)$
  - For  $i$  samples:  $J = -\prod_i p(y_i; u)$
  - Taking log both sides:  $J' = -\sum_i \log p(y_i; u)$ .
  - This is called cross-entropy.
- Applying the idea for:  $y \sim \text{Gaussian}(\text{center} = u)$ :
  - $p(y; u) = e^{-(y-u)^2}$ .
  - $J = -\sum \log e^{-(y-u)^2} = \sum (y - u)^2$
  - This motivates sum of squares as a good choice.

Correlation (Pearson, Spearman)

## Model performance: Validation diagnostics and metrics.

There are several domain-dependent diagnostics (computed separately for prediction 'p' and observation 'o') and metrics/errors for validating model performance.

**Distributional consistency:** evaluates the model capability to reproduce the distribution of the observed data.

- **Bias** =  $\text{mean } p - \text{mean } o$
- **Variance ratio** =  $\text{var } p / \text{var } o$
- **Distributional similarity:** ks-score, Von Misses, pdf-score, etc.

The **quantile-quantile plot** is a typical tool to evaluate, in a graphical way, the distributional similarity of the order statistics (e.g. **percentiles**).

**Different diagnostics for different fields.**

**Accuracy:** assess the correspondence of the simulated and observed sequences. Two typical scores are usually used: Root Mean Square Error (**RMSE**) and the (Pearson/Spearman/Kendall) **Correlation**.

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (y_i - \hat{y}_i)^2}$$

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}}$$

**Distributional consistency:** evaluates the model capability to reproduce the distribution of the observed data. The most popular are the **bias** (mean difference) or the **ratio of variances/standard deviation**. In addition, there are hypothesis tests to evaluate in a global way the similarity of the observed and simulated series (e.g. **Kolmogorov-Smirnov**, **Perkins**, **Von Misses**, etc).

The **quantile-quantile plot** is a typical tool to evaluate, in a graphical way, the distributional similarity of the order statistics (e.g. **percentiles**).

```
## Example with R:  
?qqplot  
require(graphics)  
y<-rt(200,df=5)  
qqnorm(y)  
qqline(y,col=2)  
qqplot(y,rt(300,df=5))
```

**Accuracy:** assess the correspondence of the simulated and observed sequences. Two typical scores are usually used: Root Mean Square Error (**RMSE**) and the (Pearson/Spearman/Kendall) **Correlation**.

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## Example with R:

?qqplot

require(graphics)

y<-rt(200,df=5)

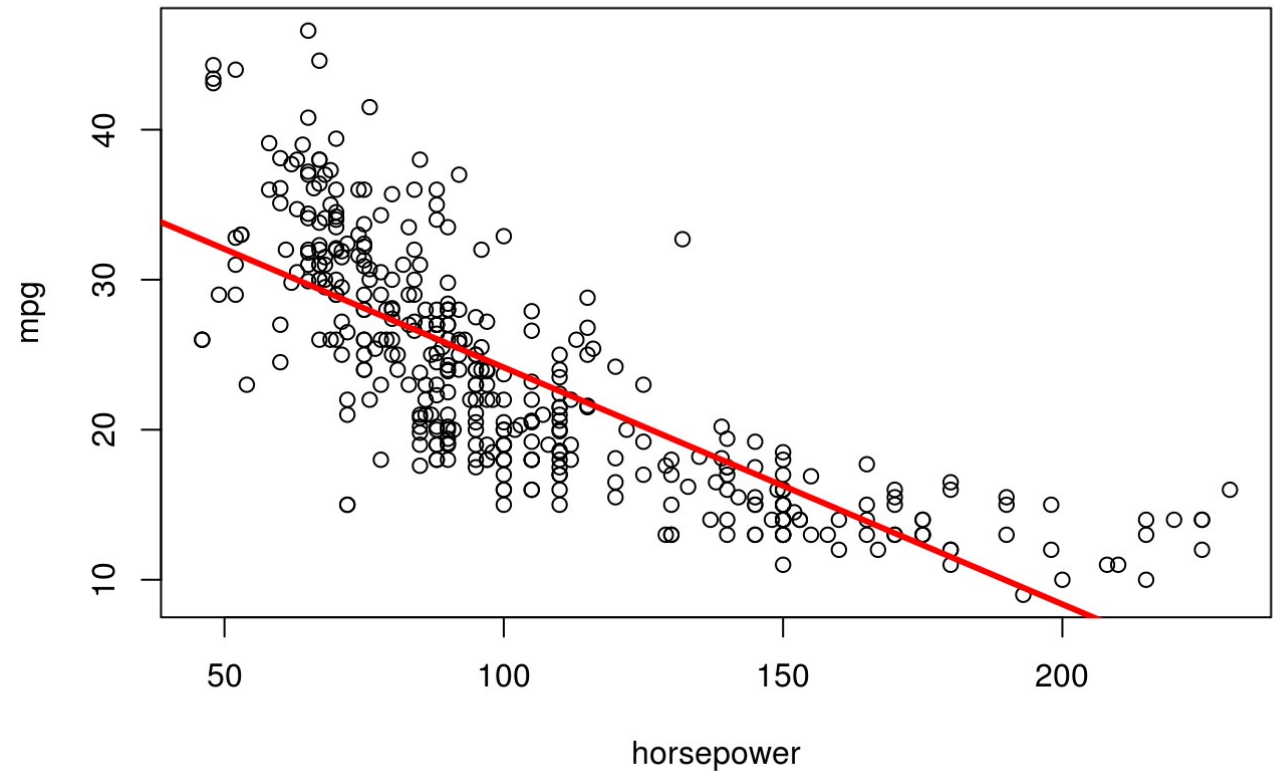
qqnorm(y)

qqline(y,col=2)

qqplot(y,rt(300,df=5))

How to create and use our own functions, including validation measures in R?

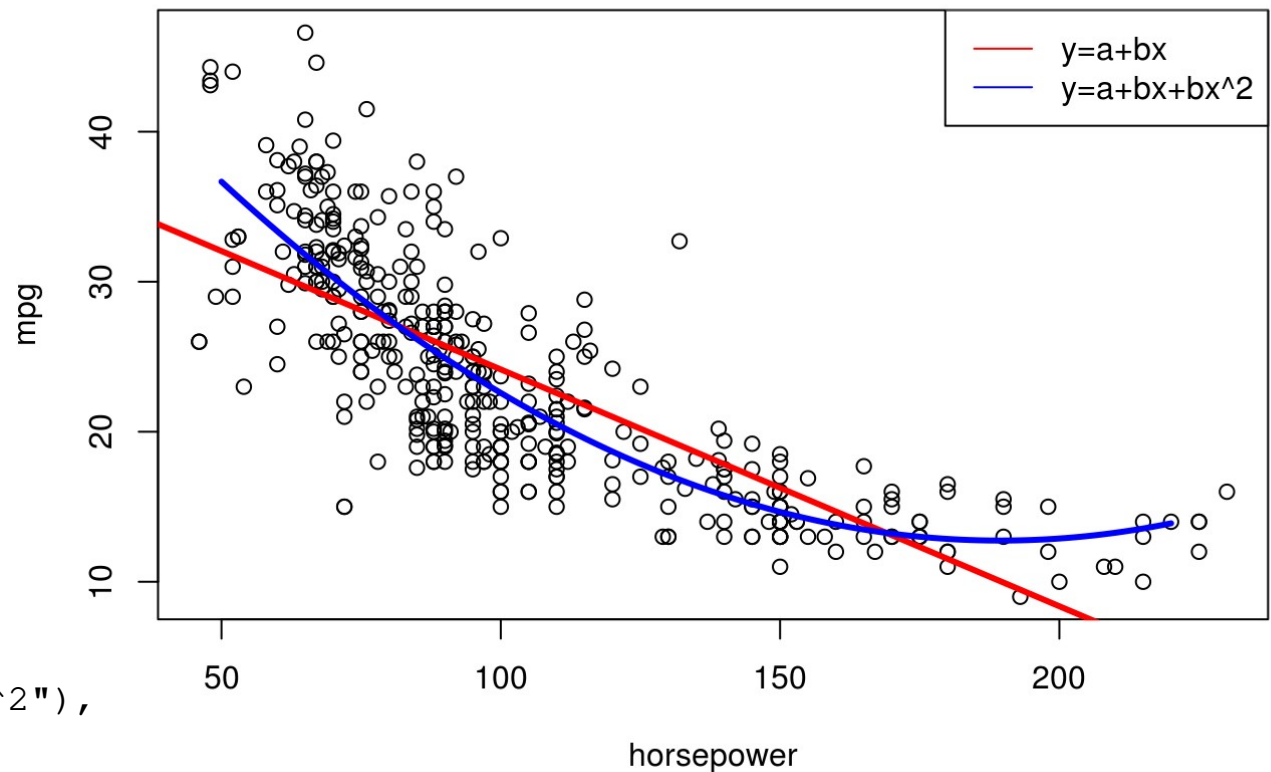
```
# install.packages('ISLR')
library(ISLR)
attach(Auto)
summary(Auto)
n <- length(mpg)
plot(horsepower, mpg)
lm1 <- lm(mpg~horsepower)
abline(lm1, col="red", lwd=3)
summary(lm1)
```

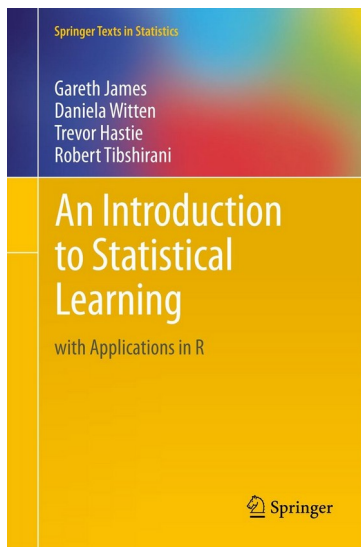


```

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attach(Auto)
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n <- length(mpg)
plot(horsepower, mpg)
lm1 <- lm(mpg~horsepower)
abline(lm1, col="red", lwd=3)
summary(lm1)
plot(horsepower, mpg)
abline(lm1, col="red", lwd=3)
lm2 <- lm(mpg~poly(horsepower,2))
xs <- seq(50,220,length=100)
ys <- predict(lm2,
  data.frame(horsepower=xs))
lines(xs,ys, type="l",
  lwd=3, col="blue")
legend("topright",
  legend=c("y=a+bx", "y=a+bx+bx^2"),
  lty=1 , col=c("red", "blue"))
summary(lm2)

```





# An Introduction to Statistical Learning: With Applications in R

James, G., Witten, D., Hastie, T., Tibshirani, R.

Springer (2013)

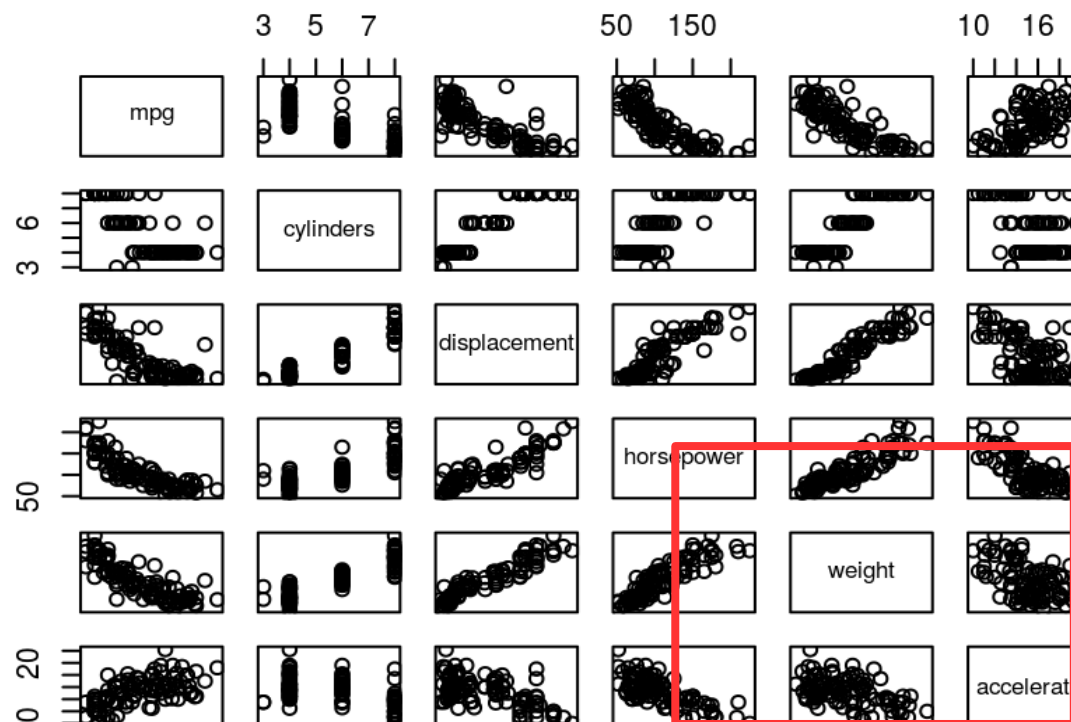
<http://www-bcf.usc.edu/~gareth/ISL>

```
install.packages("ISLR")
library("ISLR")
library(help = "ISLR")
```

```
> data(Auto)
> str(Auto)
```

```
'data.frame':    392 obs. of  9 variables:
 $ mpg          : num  18 15 18 16 17 ...
 $ cylinders     : num   8  8  8  8  8  8  8 ...
 $ displacement : num  307 350 318 304 ...
 $ horsepower    : num  130 165 150 150 ...
 $ weight        : num  3504 3693 3436 ...
 $ acceleration  : num   12 11.5 11 12 ...
 $ year          : num   70  70  70  70  70 ...
 $ origin        : num    1  1  1  1  1  1  1 ...
 $ name          : Factor w/ 304 levels ...
```

```
> pairs(Auto)
```





**CARET** (CIAssification and REgresion Training) is a wrapper of a number of standard machine learning packages which performs model tuning (optimization of the model parameters) and cross-validation strategies.  
<http://topepo.github.io/caret/index.html>

```
> modelLookup(model = "lm")
  model parameter      label forReg forClass probModel
  lm intercept intercept  TRUE    FALSE    FALSE
```

```
trainControl(method , number, ...)
  method: "none", "cv", "LOOCV"
  number: For "cv" (2 => hold-out, 10 => 10-fold)
```

```
> ctrl <- trainControl(method = "LOOCV")
> mod <- train(weight ~ horsepower,
              data = Auto,
              method = "lm",
              trControl = ctrl)
# metric="RMSE",
# preProc = c("center", "scale")
```

```
> mod
```

```
Linear Regression | 392 samples | 1 predictor | No pre-  
processing  
Resampling: Leave-One-Out Cross-Validation  
Summary of sample sizes: 391, 391, 391, 391, 391, 391, ...  
Resampling results:
```

RMSE	Rsquared	MAE
429.5254	0.7436498	347.5039

```
> str(model$control$index$Fold001)  
int [1:391] 2 3 4 5 6 7 8 9 10 11 ...
```

```
> plot(mod$pred$obs, type="l");  
  lines(1:392,mod$pred$pred,col="red")
```

