

Green Sea Turtle Population

MATH 3006 Class Project

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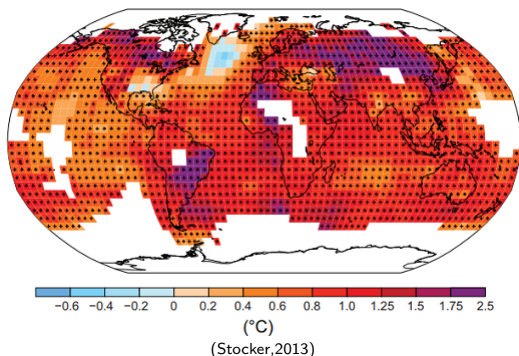
University of Cincinnati

April 28, 2018

A consequence of global warming

"Quantitative genetic analyses and behavioral data suggest that populations with temperature-dependent sex determination may be unable to evolve rapidly enough to counteract the negative fitness consequences of rapid global temperature change." (Janzen, 1994)

Observed change in surface temperature 1901–2012



Evidence of Janzen's claim



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This population of green sea turtles is nearly all female, signalling a major problem

by DANIELLA SILVA

- The green sea turtle (*Chelonia Mydas*) is an endangered species (Seminoff, 2004).
- Global warming predicts a temperature increase in the coming decades (Stocker, 2013).
- Recent data shows 116 female green sea turtles to every 1 male green sea turtle (Jensen et al., 2018).

(Image credit: <https://www.nbcnews.com/science/environment/population-green-sea-turtles-nearly-all-female-signalling-major-problem-n837341>)

Nesting near the equator

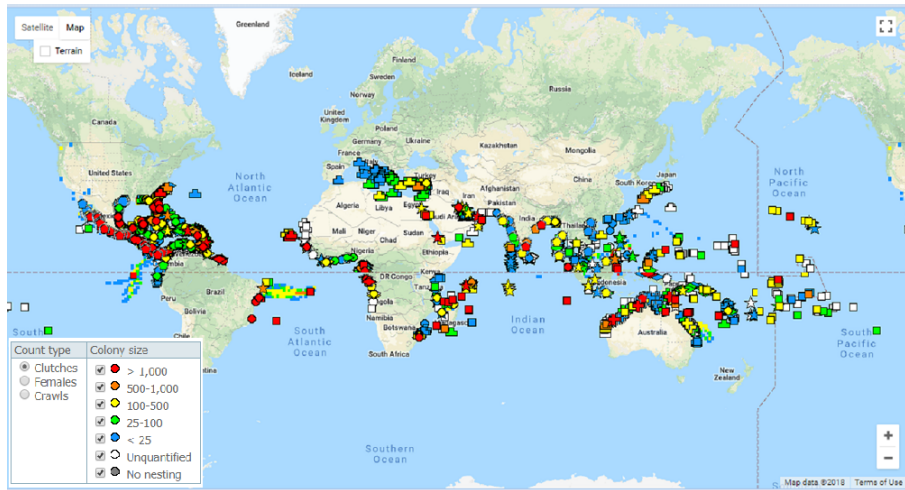


Figure: Worldwide green turtle nesting sites, aka. clutches (Halpin, 2009).

Northern Great Barrier Reef (nGBR) subpopulation

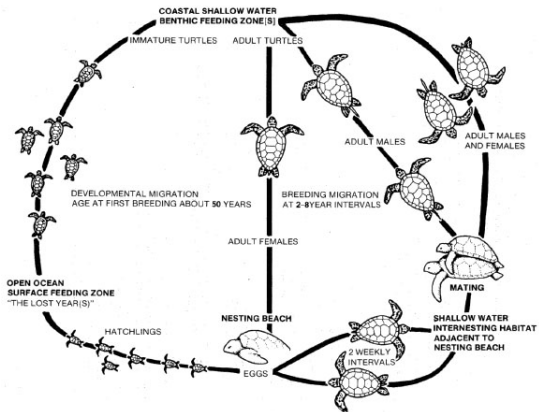
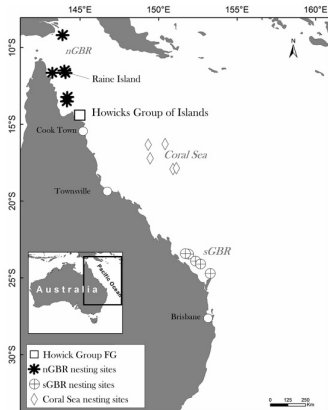


Figure: Left: Map of the region of interest (Jansen et al., 2018).
Right: Life cycle of green sea turtle (coraldigest.com)

The life of a turtle

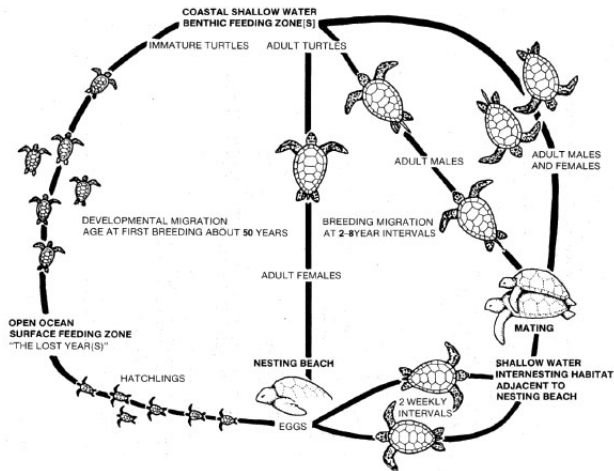


Figure: 6 stages in the life cycle (coraldigest.com)

Our Approach

Develop a two sex population model using a modified Leslie matrix.

Determine the relationship between temperature and the proportion of the population which is female. Call this function $\chi(T)$.

Determine the relationship between air temperature (AT), sea surface temperature (SST), and nesting temperature (T). Call this function $T(SST, AT)$.

Apply an annual temperature increase function $T(t)$ to $T(SST, AT)$ and apply to $\chi(T)$ so we have $\chi(t)$ to use in modified Leslie matrix.

Compare to previous models.

Predict future female proportion in the region.

Model Terminology

The following terms have important meaning:

- **Survival Rate** is the proportion of the turtles which survive each year.
- **Progression Rate** is the proportion of turtles which survive and advance to the next life stage.
- **Remaining Rate** is the proportion of turtles which survive but remain in the same life stage for another year.
- **Duration** is the length of time it takes for a turtle to grow to the next size.

Class divisions

The stages are divided by size. Each stage has a growth rate, and thus a duration for each stage.

Stage	Name	τ (years)	Size (cm CCL*)	% Mature
1	Egg-Hatchling	1	< 5	0
2	Pelagic Juvenile	4-5	5-40	0
3	Benthic Juvenile	11	40-62	0
4	Subadult	19	62-91	0
5	Maturing Adult	5	91-97	<50
6	Adult	18-19	> 97	>50

Curved Carpace Length (CCL) is width of the underside of the shell.

(Chaloupka, 2004).

A turtle which has a 5cm CCL is 1 year old.

Note: 97cm indicates the size when turtles become more reproductive.

Parameter Table

Symbol	Meaning
χ	Proportion of hatched female
$1 - \chi$	Proportion of hatched male
σ_i^m	Survival rate of male
σ_i^f	Survival rate of female
p_i^m	Remaining rate of male
p_i^f	Remaining rate of female
q_i^m	Progression rate of male
q_i^f	Progression rate of female
f_i	Reproductive rate
τ_i^m	Male duration (years)
τ_i^f	Female duration (years)
T	Temperature (Celsius)
t	time (year, ie. 2018)

note: i denote i^{th} stage, and $i = 1, 2, 3, 4, 5, 6$.

Assumptions

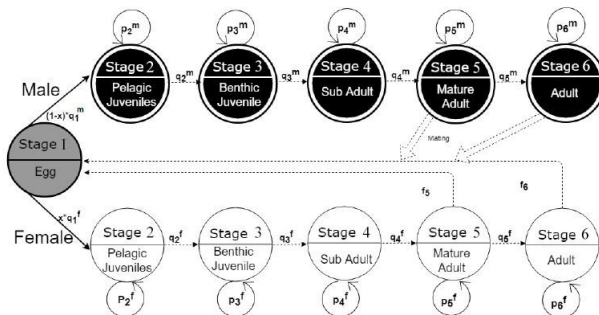
- ① The model consists of six stages for both male and female turtles. These stages are based on both size and the probability of being sexually mature. (Chaloupka, 2002)
- ② Migratory behavior is ignored (Crouse et al., 2008).
- ③ The female proportion can only be affected by temperature.
- ④ The proportion of female offspring (χ) changes each year.
- ⑤ The parameters other than χ , T and t remain constant from generation to generation (Crouse et al., 2008; Chaloupka & Limpus, 2005).
- ⑥ The life of turtle will end after the adult stage (stage 6).
- ⑦ The temperature (T) does not affect the survival rate (σ).

Assumptions

- 8 The mating process is ignored.
- 9 The Sea Surface Temperature (SST) in the nGBR region is constant and assumed 21°C
- 10 When the size is greater than 60cm CCL, females grow faster than males (Chaloupka et al.,2004). This leads to the following assumptions about the duration of each stage:

Stage	Name	Duration of male τ_i^m (years)	Duration of female τ_i^f (years)
1	Egg-Hatchling	1	1
2	Pelagic Juvenile	4	4
3	Benthic Juvenile	11	11
4	Subadult	20	18
5	Maturing Adult	6	4
6	Adult	18	22

Flowchart



Symbol	Meaning
x	Proportion female
p_i	Remaining rate
q_i	Progression rate
f_i	Reproductive rate

Figure: The population is divided by sex along the top and bottom paths. The rates of remaining, progression and reproduction are all represented by arrows. Flowchart created using <https://www.draw.io/>.

Population vector

Stage	Symbol	Meaning
1	A_t	Eggs in year t
2	B_t	Pelagic Juveniles in year t
3	C_t	Benthic Juveniles in year t
4	D_t	Subadults in year t
5	E_t	Mature Adults in year t
6	F_t	Adults in year t

Note: t in year name, ie. 2018

Modified Leslie matrix

If we combine two genders into one matrix we get the form:

$$\begin{bmatrix} A_{t+1} \\ B_{t+1}^m \\ C_{t+1}^m \\ D_{t+1}^m \\ E_{t+1}^m \\ F_{t+1}^m \\ B_{t+1}^f \\ C_{t+1}^f \\ D_{t+1}^f \\ E_{t+1}^f \\ F_{t+1}^f \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & f_5^f & f_6^f \\ (1-\chi)q_1^m & p_2^m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & q_2^m & p_3^m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & q_3^m & p_4^m & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & q_4^m & p_5^m & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & q_5^m & p_6^m & 0 & 0 & 0 & 0 & 0 \\ \chi * q_1^f & 0 & 0 & 0 & 0 & 0 & p_2^f & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & q_2^f & p_3^f & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & q_3^f & p_4^f & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & q_4^f & p_5^f & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & q_5^f & p_6^f \end{bmatrix} \begin{bmatrix} A_t \\ B_t^m \\ C_t^m \\ D_t^m \\ E_t^m \\ F_t^m \\ B_t^f \\ C_t^f \\ D_t^f \\ E_t^f \\ F_t^f \end{bmatrix}$$

Equation

We call upon earlier work on another species of turtles to derive the parameters p_i and q_i .

$$p_i = \left(\frac{1 - \sigma_i^{\tau_i - 1}}{1 - \sigma_i^{\tau_i}} \right) \sigma_i$$

$$q_i = \frac{\sigma_i^{\tau_i} (1 - \sigma_i)}{1 - \sigma_i^{\tau_i}}$$

(Crouse et al., 2008)

Symbol	Meaning
σ_i	Survival rate %
p_i	Remaining rate %
q_i	Progression rate %

Let's denote $\sigma_i^{\tau_i}$ for the probability for the turtles in stage i surviving τ years. Then, we can have the total percentage of Remaining rate of stage i :

$$p_i = \left(\frac{1 + \sigma_i + \sigma_i^2 + \dots + \sigma_i^{\tau_i-2}}{1 + \sigma_i + \sigma_i^2 + \dots + \sigma_i^{\tau_i-1}} \right) \sigma_i$$

by geometric series

$$1 + \sigma + \sigma^2 + \dots + \sigma^{\tau-1} \Rightarrow \frac{(1 - \sigma^{\tau})}{(1 - \sigma)}$$

we can rewrite p_i as

$$p_i = \left(\frac{1 - \sigma_i^{\tau_i-1}}{1 - \sigma_i^{\tau_i}} \right) \sigma_i$$

Next, we assume that the proportion of the population that grows into the next stage class and survives q so we have:

$$q_i = \left(\frac{\sigma_i^{\tau_i-1}}{1 + \sigma_i + \sigma_i^2 + \dots + \sigma_i^{\tau_i-1}} \right) \sigma_i$$

by geometric series, the equation can be rewritten as as

$$q_i = \frac{\sigma_i^{\tau_i}(1 - \sigma_i)}{1 - \sigma_i^{\tau_i}}$$

The proof above is from research into the loggerhead sea turtles, we use the method to get the remaining rate p , and the progression rate q (Crouse et al., 2008).

Male Parameter Table					
	(τ_i^m)	(σ_i^m)	(p_i^m)	(q_i^m)	(f_i^m)
Stage 1	1	0.4394	0	0.4394	0
Stage 2	4	0.6445	0.5703	0.0741	0
Stage 3	11	0.8804	0.8413	0.0391	0
Stage 4	20	0.8474	0.8416	0.0058	0
Stage 5	6	0.9482	0.8104	0.1378	0
Stage 6	18	0.9482	0.9159	/	0
(Chaloupka, 2005)					

Female Parameter Table					
	(τ_i^f)	(σ_i^f)	(p_i^f)	(q_i^f)	(f_i^f)
Stage 1	1	0.4394	0	0.4394	0
Stage 2	4	0.6445	0.5704	0.0741	0
Stage 3	11	0.8804	0.8413	0.0391	0
Stage 4	18	0.8474	0.8392	0.0082	0
Stage 5	4	0.9482	0.7297	0.2185	36.3
Stage 6	22	0.9482	0.9248	/	72.6
(Chaloupka, 2005)					

Modified Leslie matrix

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 36.3 & 72.6 \\ (1 - \chi) * 0.4394 & 0.5703 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0741 & 0.8413 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0391 & 0.8416 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0058 & 0.8104 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1378 & 0.9159 & 0 & 0 & 0 & 0 & 0 & 0 \\ \chi * 0.4394 & 0 & 0 & 0 & 0 & 0 & 0.5704 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0741 & 0.8413 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0391 & 0.8392 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0082 & 0.7297 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2185 & 0.9248 & 0 \end{bmatrix}$$

Next we turn to the relationship between temperature and the female proportion of the population.

Temperature dependent sex function

$$\chi(T) = \frac{1}{1 + e^{\beta(T-T_0)}}$$

Let χ be the proportion of female turtle hatchlings at sand temperature T . We expect the following behavior based on temperature dependent sex determination.

$$T \rightarrow T_{HotDeath}, \chi(T) \rightarrow 1.$$

$$T \rightarrow T_{50/50}, \chi(T) \rightarrow 0.5.$$

$$T \rightarrow T_{ColdDeath}, \chi(T) \rightarrow 0.0.$$

β is established as -1.3 and $T_0 = 29.18$ Celsius (Hays, 2017). Therefore,

$$\chi(T) = \frac{1}{1 + e^{-1.3(T-29.18)}}$$

Sand Temperature function

The temperature in the proportion female function is the sand temperature. The sand temperature of the nest is dependent on the air temperature (AT) and sea surface temperature (SST) (Fuentes et.al, 2009).

Moulter Cay is the near Raine Island and serves as a fair comparison The Temperature increase for the offshore nesting sites in the area.

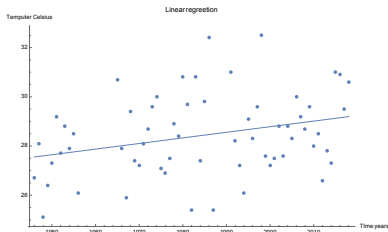
Moulter Cay Sand temperature (Fuentes et. al., 2009) =

$$T(SST, AT) = -0.236SST + 1.022AT + 6.915$$

Temperature Increase from global warming

We assume the sea surface temperature (SST) is constant 21°C . Air Temperature (AT) is what is subject change from global warming. To model the increased air temperature we used data from the area and fit a linear regression model to determine the following for t in year (ie, 2017).

$$AT[t] = 0.0227692x - 16.7534$$



the data is from <http://www.bom.gov.au/other/copyright.shtml>

Then we got our female proportion function

$$\chi(t) = \frac{1}{1 + 9.51172 \times 10^{24} e^{(-0.0302512t)}}$$

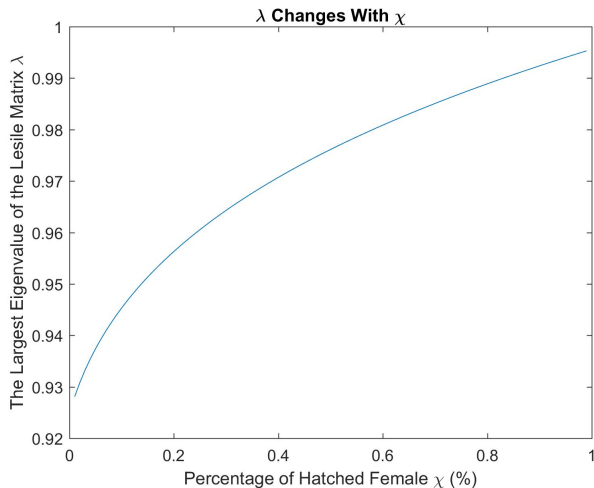
	0	0	0	0	0	0	0	0	0	36.3	72.6
$(1 - \chi(t)) * 0.4394$	0.5703	0	0	0	0	0	0	0	0	0	0
	0.0741	0.8413	0	0	0	0	0	0	0	0	0
	0	0.0391	0.8416	0	0	0	0	0	0	0	0
	0	0	0.0058	0.8104	0	0	0	0	0	0	0
	0	0	0	0.1378	0.9159	0	0	0	0	0	0
$\chi(t) * 0.4394$	0	0	0	0	0	0.5704	0	0	0	0	0
	0	0	0	0	0	0.0741	0.8413	0	0	0	0
	0	0	0	0	0	0	0.0391	0.8392	0	0	0
	0	0	0	0	0	0	0	0.0082	0.7297	0	0
	0	0	0	0	0	0	0	0	0.2185	0.9248	0

Data Fitting

Temperature data with proportion female for hatching from Taiwan (Rowena, 2013).

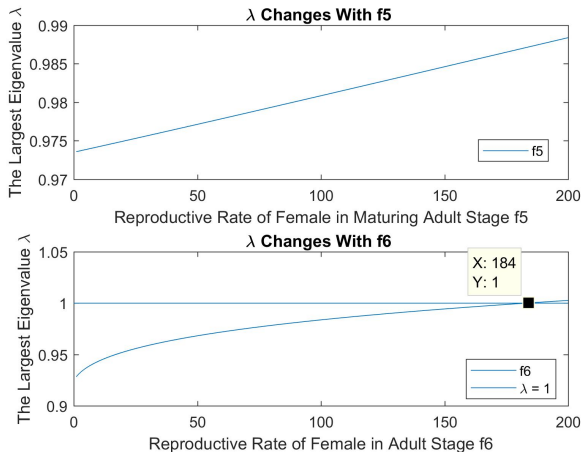
Year	2010	2011
Number of eggs hatched	142	102
Number of females	87	98
Average temperature	26.7875	30.275
Percentage of female in data	0.612676	0.960784
Our prediction of female percentage	0.781875	0.979835

Eigenvalue

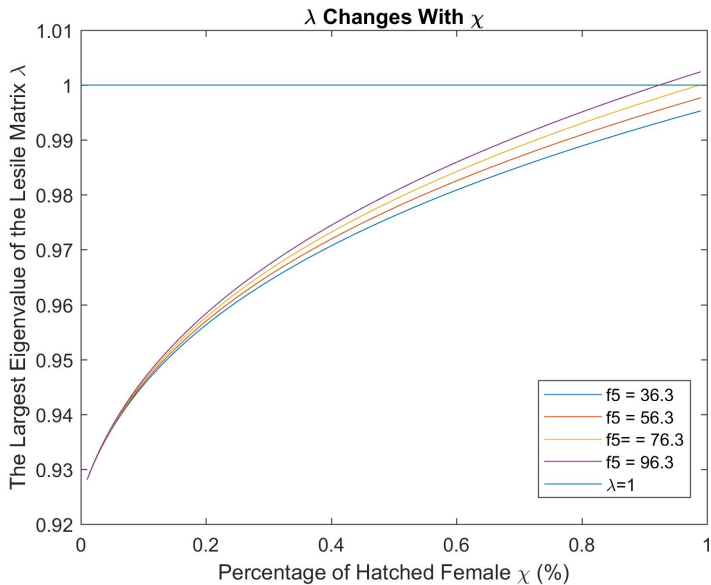


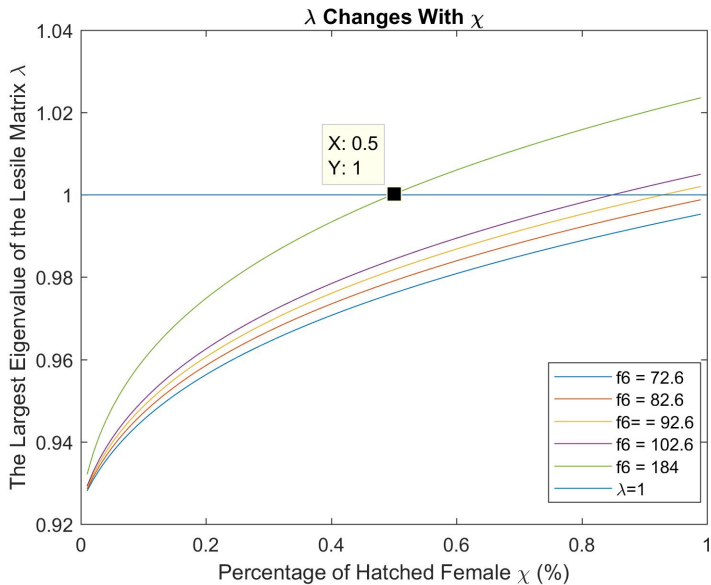
$\lambda < 1$ for all possible χ

Parameter Effects

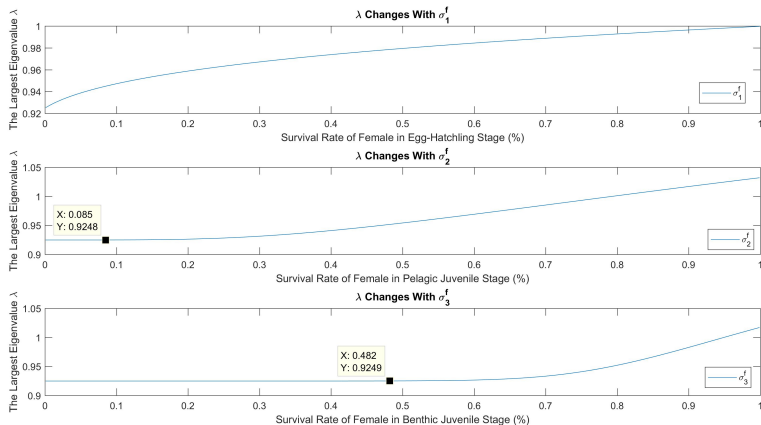


note: $\chi = 0.5$



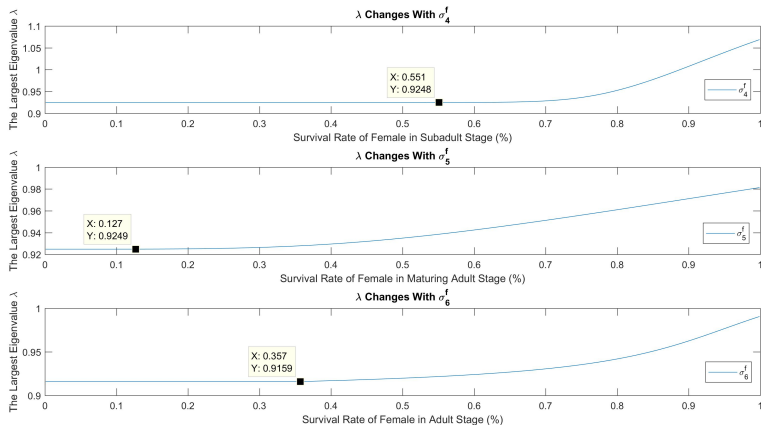


Parameter Effects

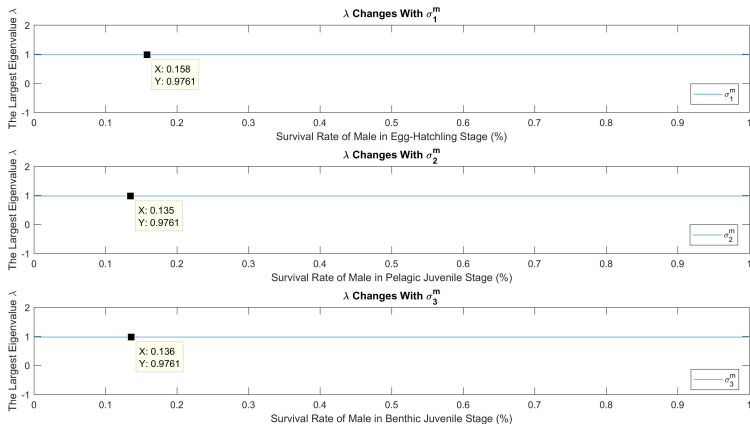


note: $\chi = 0.5$ ($\sigma_2^f = 0.6445$, $\sigma_3^f = 0.8804$)

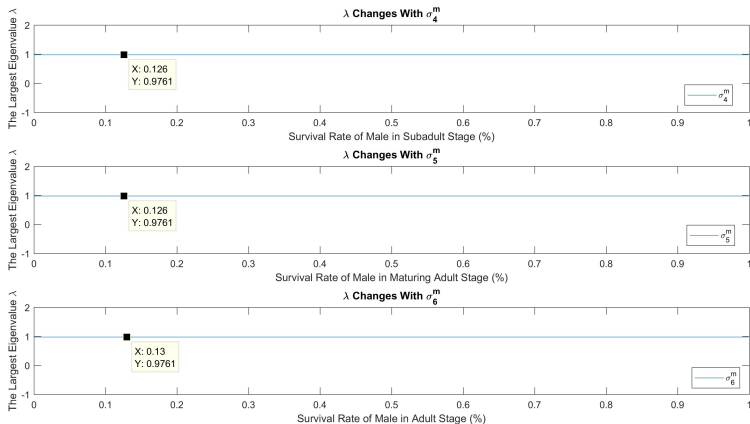
Parameter Effects



note: $\chi = 0.5$ ($\sigma_4^f = 0.8474$, $\sigma_5^f = 0.9482$, $\sigma_6^f = 0.9482$)

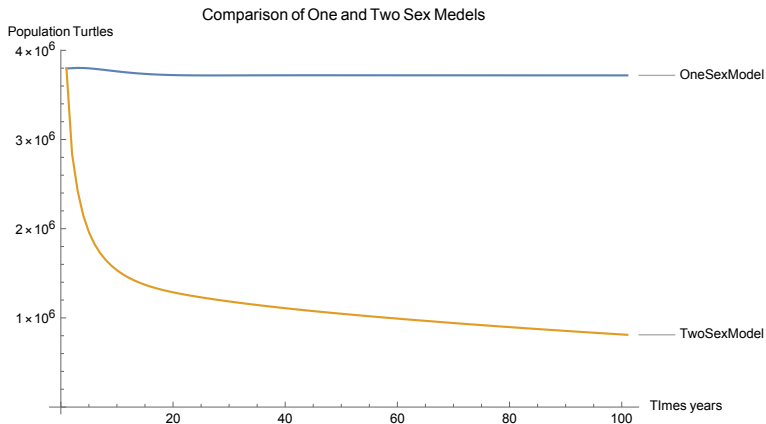


note: $\chi = 0.5$



note: $\chi = 0.5$

Two sex model vs. one sex model



note: The initial year is 2018

The reasons we used a two-sex model to determine the population of the green sea turtle.

- Determine how temperature can affect the total population.
- Most researchers use size to determine the age of the turtle, in fact the growth rates differ for males and females.
- Both males and females have multiple mating partners during a breeding season.

Based on our model

- Our model shows the decreasing trend on green sea turtle's population.
- The increasing of temperature can affect the total population.
- Increasing the survival rate in stage 4 σ_4 is more easy to change the decreasing trend.
 - Have more laws about fishing.
 - Set some special region for the green sea turtle.
- Increasing the reproductive rate(f_5 and f_6) can help to save the green sea turtle's population.
 - Develop artificial reproduction.
 - Clean up the beach.

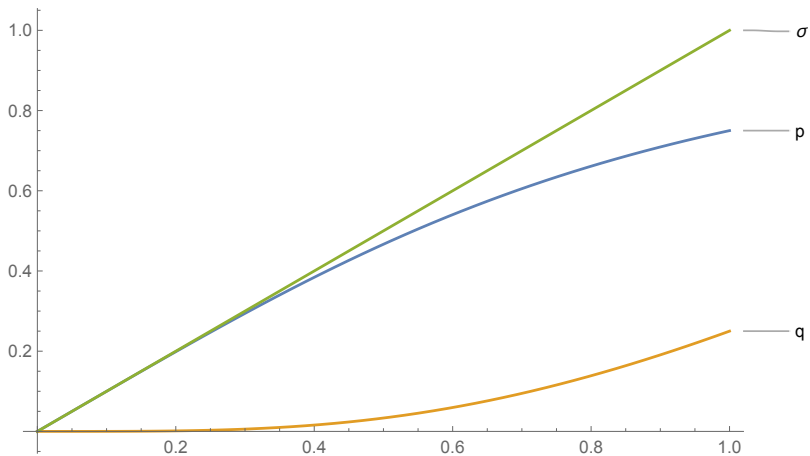
what we did before

let P_0 be the initial p_i value that we got in our matrix [?]

parameter	P_0	when $\lambda = 1$	σ_0	σ
p_2^f	0.5704	0.813	0.644 5	1.19735
p_3^f	0.8413	0.93	0.8804	1.0516
p_4^f	0.8392	0.93	0.8474	0.9719
p_5^f	0.7297	0.883	0.9482	1.5385
p_6^f	0.9248	0.971	0.9482	1.04059

let Q_0 be the initial q_i value that we got in our matrix

parameter	Q_0	$\lambda = 1 \Rightarrow q_i =$	σ_0	σ
q_1^f	0.4394	0.999	0.4394	0.999
q_2^f	0.0741	0.09	0.6445	0.6878
q_3^f	0.0391	0.172	0.8804	1.1249
q_4^f	0.0082	0.019	0.8474	0.9055
q_5^f	0.2185	0.557	0.9482	1.42



Future development

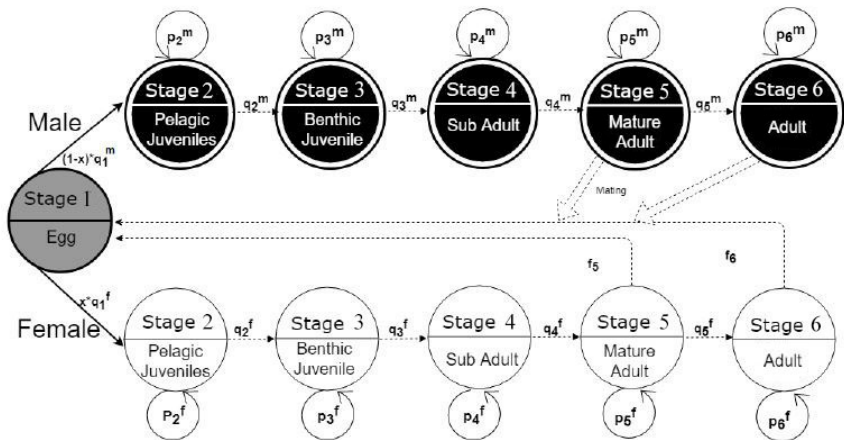
Stage	Natural Risk	Human Risk
1	Beach predators ie. Coyotes	Development loss of habitat
2,3	Ocean predators ie. Tiger Shark	Fishing bycatch Troll fishing nets
4,5,6	Ocean predators	Fishing bycatch
(Stocker, 2013).		

We have created a model which can be used to investigate the population dynamics of the green sea turtle, but we ignored some factors. Therefore, our model do not consider some extreme condition. The ideas for solving those problem

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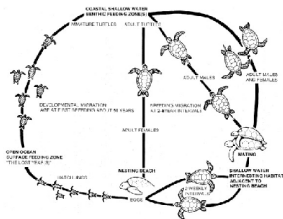
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- <http://environmentalprotectionofasia.com/ztc/p/biology/lifehistory.htm> (2018)
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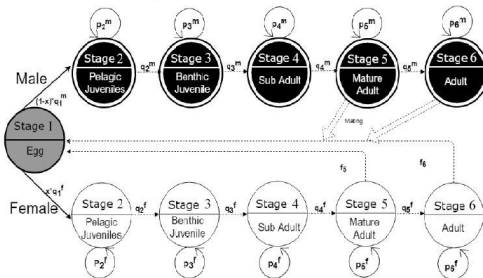


Stage	Symbol	Meaning
1	A_t	Eggs in year t
2	B_t	Pelagic Juveniles in year t
3	C_t	Benthic Juveniles in year t
4	D_t	Subadults in year t
5	E_t	Mature Adults in year t
6	F_t	Adults in year t

Symbol	Meaning
χ	Proportion of hatched female
$1 - \chi$	Proportion of hatched male
σ_i^m	Survival rate of male
σ_i^f	Survival rate of female
p_i^m	Remaining rate of male
p_i^f	Remaining rate of female
q_i^m	Progression rate of male
q_i^f	Progression rate of female
f_i	Reproductive rate
τ_i^m	Male duration (years)
τ_i^f	Female duration (years)
T	Temperature (Celsius)
t	time (year, ie. 2018)



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1	A_t	Eggs in year t
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p_i^m	Remaining rate of male
p_i^f	Remaining rate of female
q_i^m	Progression rate of male
q_i^f	Progression rate of female
f_i	Reproductive rate
τ_i^m	Male duration (years)
τ_i^f	Female duration (years)
T	Temperature (Celsius)
t	time (year, ie. 2018)