Linear combinations of independent normals are normal

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See here for a PDF version of this vignette.

Prerequisites

Basic familiarity with the univariate normal distribution.

A statement of the basic property

The simple goal of this vignette is to introduce a basic property of the (univariate) normal distribution: that linear combinations of independent normal variables are also normal.

Formally, suppose Z_1 and Z_2 represent independent, normally distributed random variables. Then for any scalars a and b, the linear combination

$$X := aZ_1 + bZ_2$$

is also (univariate) normal.

Also, by basic properties of expectation and variance, $E(X) = aE(Z_1) + bE(Z_2)$ and $\text{Var}(X) = a^2\text{Var}(Z_1) + b^2\text{Var}(Z_2)$.

Example

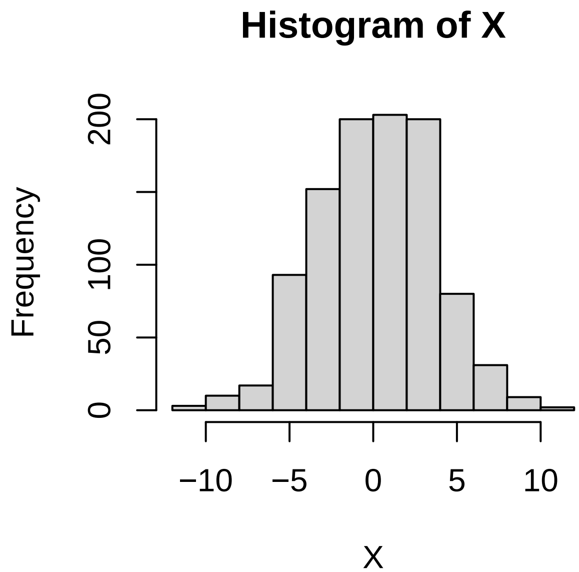
The following code provides a visual illustration of this idea with $a = 2$ and $b = 3$, but it holds for any a and b.

First we sample some values of X by randomly generating Z_1 and Z_2, and computing $X = aZ_1 + bZ_2$:

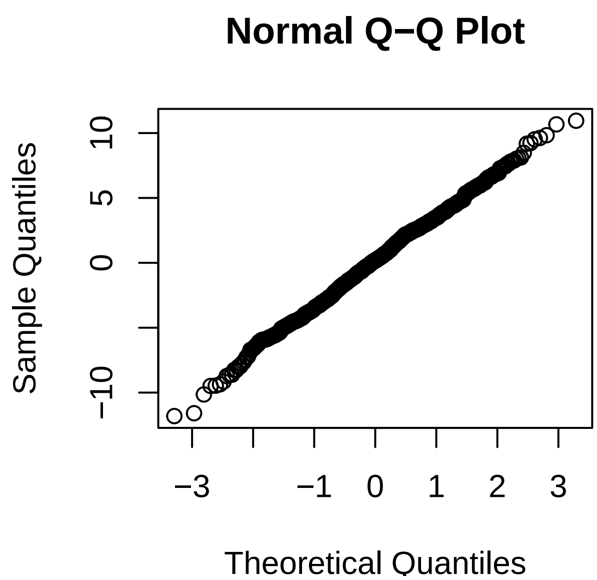
```
Z1 <- rnorm(1000)
Z2 <- rnorm(1000)
a <- 2
b <- 3
X <- a*Z1 + b*Z2
```

The property says that the samples of X look normal. A quick histogram and qqplot suggest it does. (Of course, this is not a proof that the property holds; it is just an illustration of the idea.)

```r
hist(X)
```




```r
qqnorm(X)
```



Addendum: Stable distributions

If you are curious by nature, you might now ask: is the normal distribution the only distribution that satisfies this property? The answer is "no". For example, t distributions also satisfy this property. Distributions that satisfy this property are called "stable" distributions. You can read more here.