Notes on Ordinary Differential Equations– Summer 2025
w/Self

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Chapter 1

First Order Differential Equations

1.1 An Introduction to Differential Equations

In this chapter we will study:

- Separable
- Linear
- Homogeneous
- Bernoulli
- Exact

1.1.1 Separable

Solution Method for Separable Differential Equations

1. *Determine the equilibrium solutions.* Thsear ea ll of teh constan solutions $y = y_0$ adn are determined by solving eh equation $g(y) = 0$ for y_0.
2. *Separate the variables in a form convenient for integration.* That is, we formally write
$$\frac{1}{g(y)}dy = h(t)dt$$
and refere to this equation as teh ***differential form*** of the separable differential equation.
3. *Integrate both sides, the left-hand side with respect to y and the right-hand side with respect to t.* This yields
$$\int \frac{1}{g(y)}dy = \int h(t)dt,$$
which produces the implict solution
$$Q(y) = H(t) + c,$$
where $Q(y)$ is an antidierivative of $1/g(y)$ and $H(t)$ is an antiderivative of $h(t)$. Such antiderivatives differe by a constant c.
4. (*If possible, solve the implicit relation explicitly for y.*) $\square$

1.2 Direction Fields

1.3 Separable Differential Equations

When we have a first order differential equation of the form

$$y' = F(t, y)$$

it is ***separable*** if there exists $h(t)$ and $g(y)$ such that

$$y' = h(t)g(y)$$

Solution Method for Separable Differentiable Equations

1. *Determine the Equilibrium Solutions*

   These are all of the constant solutions $y = y_0$, determined by solving the equation $g(y) = 0$.

2. *Separate the functions in a form convenient for Integration.*

   That is, we formally write

   $$\frac{1}{g(y)}dy = h(t)dt$$

   and refer to this equation as the ***differential form*** of teh separable differential equation.

3. *Integrate both sides, the left-hand side with respect to y adn the right-hand side with respect to t.* This yields

   $$\int \frac{1}{g(y)}dy = \int h(t)dt.$$

   which produces the implicit solution

   $$Q(y) = H(t) + c.$$

   where $Q(y)$ is an antiderivatives of $1/g(y)$ and $H(t)$ is an antidierivative of $h(t)$. Such antiderivatives differe by a constant c.

4. *(if possible, solve the implicit relation explicitly for y.)*

1.4 Linear First Order Equations

Theorem 1.4.1. *Let p and f be continuous functions on an interval I. a function y is a solution of the first order linear differntial equation $y' + py = f$ on I if and only if*

$$y = \frac{1}{\mu}\int \mu f dt + \frac{c}{\mu}$$

Solution Method for First Order Linear Equations

1. Put the given linear equation in standard form: $y' + py = f$.

2. Find an inegratin factor, μ : To do this, compute an anitderivative $P = \int pdt$ and set $\mu = e^P$.

3. Multiply the equation (in standard form) by μ: This yields

   $$\mu y' + \mu' y = \mu f.$$

4. Simplify the left-hadn side: Since $(\mu y)' = \mu y' + \mu' y$, we get
$$(\mu y)' = \mu f.$$

5. Integrate both sides of the resulign equation: This yields
$$\mu y = \int \mu f dt + c.$$

6. Divide by μ to get the solution y:
$$y = \frac{1}{\mu} \int \mu f dt + \frac{c}{\mu}$$

1.5 Subsitutions

1.6 Exact Equations

1.7 Existence and Uniqueness Theorem