Practice Test 2 - MTH 5102 - Linear Algebra - Dr. Kanishka Perera - Fall 2024

Name: ______________________________

Each problem is worth 25 points. You may refer to your book/notes. Calculators and cell phones are not allowed.

1. Let $T : M_{2\times 2}(\mathbb{R}) \to \mathbb{R}^4$ be defined by

$$T(A) = (\operatorname{tr}(A), \operatorname{tr}(A^t), \operatorname{tr}(EA), \operatorname{tr}(A)),$$

where

$$E = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Determine whether T is invertible, and compute T^{-1} if it exists.

2. Let $A, B \in M_{n\times n}(F)$ be such that $AB + BA = 0$. Show that if n is odd and F is not a field of characteristic 2, then A or B is not invertible.

3. Let $T : P_3(\mathbb{R}) \to P_3(\mathbb{R})$ be defined by

$$T(f(x)) = xf'(x) + f''(x) - f(2).$$

Find the eigenvalues of T and an ordered basis β for $P_3(\mathbb{R})$ such that $[T]_\beta$ is a diagonal matrix.

4. Let T be an invertible linear operator on a finite dimensional vector space V. Show that if T is diagonalizable, then T^{-1} is diagonalizable.