Math 5050 – Special Topics: Manifolds– Spring 2025
w/Professor Berchenko-Kogan

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Section 2: 1, 3, 4

2.1. Vector fields

Let X be the vector field $x\partial/\partial x + y\partial/\partial y$ and $f(x,y,z)$ the function $x^2 + y^2 + z^2$ on $\mathbb{R}^3$. Complute Xf.

2.3. Vector space structure on derivations at a point

Let D and D' be derivatiosn at p in $\mathbb{R}^n$, and $c \in \mathbb{R}$. Prove that

(a) the sum $D + D'$ is a derivation at p.

(b) the scalar multiple cD is a derivation at p.

2.4. Product of derivations

Let A be an algebra over a field K. If D_1 and D_2 are derivations of A, show that $D_1 \circ D_2$ is not necessarily a derivation (it is if D_1 or $D_2 = 0$), but $D_1 \circ D_2 - D_2 \circ D_1$ is always a derivation of A.