# Math 5102 – Linear Algebra– Fall 2024
w/Professor Penera

Paul Carmody
Homework #8 – NONE

Page 151: 5: Prev that $E$ is an elementary matrix if and only if $E^t$.
Page 166: 3 Prove that for any $m \times n$ matrix $A$, $\text{rank}(A) = 0$ if an donly if $A$ is the zero matrix.

Page 166: 6: For each of the following linear transformations $T$, determine whether $T$ is invertible, and comute $T^{-1}$ if it exists.

(a) $T : \P_2(\mathbb{R}) \to \P(\mathbb{R})$ defined by $T(f(x)) = f''(x) + 2fg'(x) - f(x)$..

(b) $T : \P_2(\mathbb{R}) \to \P(\mathbb{R})$ defined by $T(f(x)) = (x+1)f'(x)$.

(c) $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$$T(a_1, a_2, a_3) = (a_1 + 2a_2 + a_3, -a_1 + a_2 + 2a_3, a_1 + a_3).$$

(d) $T : \mathbb{R}^3 \to \P_2(\mathbb{R})$ defined by

$$T(a_1, a_2, a_3) = (a_1 + a_2 + a_3) + (a_1 - a_2 + a_3)x + a_1 x^2.$$

(e) $T : \P_2(\mathbb{R}) \to \mathbb{R}^3$ defined by $T(f(x)) = (f(-1), f(0), f(1))$.

(f) $T : M_2(\mathbb{R}) \to \mathbb{R}^4$ defined by

$$T(A) = (\text{tr}(A), \text{tr}(A^t), \text{tr}(EA), \text{tr}(AE))$$

where

$$E = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Page 166: 8: Let $A$ be an $m \times n$ matrix. Prove that if $c$ is any nonzero scalar, then $\text{rank}(cA) = \text{rank}(A)$.