Practice Test 2 - MTH 5111 - Real Analysis 2 - Dr. Kanishka Perera - Spring 2025

Name: ______________________________

Each problem is worth 25 points. Books, notes, and formula sheets are not allowed.

$(X, \mathcal{M}, \mu)$ denotes a measure space throughout the test.

1. Show that if $f \in L^1(X)$ and $E_1 \subset E_2 \subset \cdots$ is a sequence of measurable sets such that $\bigcup_n E_n = X$, then
$$\lim_{n\to\infty} \int_{E_n} f \, d\mu = \int_X f \, d\mu.$$

2. Show that if $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are lower semicontinuous functions, then $f + g$ is lower semicontinuous.

3. Show that if $f \in L^1(X)$, then for each $\varepsilon > 0$, $\exists \delta > 0$ such that $\int_E |f| \, d\mu < \varepsilon$ whenever $\mu(E) < \delta$.

4. Show that if X is a locally compact Hausdorff space and λ and μ are outer regular Borel measures on X such that $\lambda(V) = \mu(V)$ for all open sets $V \subset X$, then $\lambda = \mu$.