Math 5102 – Linear Algebra– Fall 2024

w/Professor Penera

Paul Carmody

Homework #6 – NONE

Page 107: 3, 4, 5, 15, 16

Page 107:3. Which of the following pairs of vector spaces are isomorphic? Justify your answers.

(a) F^3 and $P_3(F)$.

(b) F^4 and $P_3(F)$.

(c) $M_{2\times 2}(\mathbb{R})$ and $P_3(\mathbb{R})$.

(d) $V = \{A \in M_{2\times 2}(\mathbb{R}) : \text{tr}(A) = 0\}$ and $\mathbb{R}^4$.

Page 107:4. Let A and B be $n \times n$ matrices. Prove taht AB is invertible and $(AB)^{-1} = B^{-1}A^{-1}$.

Page 107:5. Let A be invertible. Prove that A^t is invertible and $(A^t)^{-1} = (A^{-1})^t$.

Page 107:15. Let V and W be n-dimensional vector spaces, and let $T : V \to W$ be a linear transformation. Suppose that β is a basis for V. Prove that T is an isomorphism if and only if $T(\beta)$ is a basis for W.

Page 107:16. Let B be an $n \times n$ invertyible matrix. Define $\Phi : M_{n\times n}(F) \to M_{n\times n} \to (F)$ by $\Phi(A) = B^{-1}AB$. Prove that Φ is an isomorphism.

Page 116: 4, 11

Page 116:4) Let T be teh linear operator $\mathbb{R}^2$ defined by

$$T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2a + b \\ a - 3b \end{pmatrix}$$

Let β be the standard ordered basis for $\mathbb{R}^2$, and let

$$\beta' = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$$

Use Theorem 2.23 and the fact that

$$\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$

to find $[T]_{\beta'}$

Page 116:11) Let V be a finite-dimensional vector space with ordered bases α, β and γ.

(a) Prove that if Q and R are the changed of coordinate matrices that change α-coordinates in β-coordinates and β-coordiantes into γ-coordinates, respectiely, then RQ si the change of coordinate matrix that changes α-coordinates to γ-coordinates.

(b) Prove that if Q chjanges α-coordinates into β-coordinates, then Q^{-1} changes β-coordinates into α-coordinates.