Math 5411 – Mathematical Statistics I– Fall 2024
w/Nezamoddini-Kachouie

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Extra Credit – December 8, 2024

	Brent	WTT
μ	$\bar{X_b} = 72.6$	$\bar{X_W} = 69.89$
σ^2	$S_B^2 = 2.5$	$S_W^2 = 30$
N	$n_B = 32$	$n_W = 40$
N_E	$n_{B,E} = 26$	$n_{W,E} = 31$

$$
\begin{aligned}
\hat{p_B} &= \frac{x_B}{n_B} = 26/32 = 0.813, \\
\hat{p_W} &= \frac{x_W}{n_W} = 31/40 = 0.775 \\
\hat{p} &= \frac{x_B + x_W}{n_B + n_W} = \frac{26+31}{32+40} = 0.792 \\
\hat{q} &= 1 - \hat{p} = 0.208 \\
z &= \frac{(\hat{p_B} - \hat{p_W}) - (p_B - p_W)}{\sqrt{\frac{\hat{p}\hat{q}}{n_B} + \frac{\hat{p}\hat{q}}{n_W}}} = \frac{(0.813 - 0.775) - 0}{\sqrt{\frac{(0.792)(0.208)}{26} + \frac{(0.792)(0.208)}{31}}} = \frac{0.038}{\sqrt{\frac{0.647}{26} + \frac{0.647}{31}}} = 0.177 \\
z_{0.98} &= 2.053 \\
z_{0.85} &= 1.036
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{\text{diff}} &= \sqrt{\frac{S_B^2}{n_B} \frac{S_W^2}{n_W}} = 0.214 \\
CI &= (\hat{p_B} - \hat{p_W}) \pm z_{1-\alpha}\sigma_{\text{diff}} \\
CI_{98} &= (0.813 - 0.775) \pm 2.053(0.214) \\
&= 0.038 \pm 0.439 \\
CI_{85} &= (0.813 - 0.775) \pm 1.036(0.214) \\
&= 0.038 \pm 0.228
\end{aligned}
$$

Judging by the z-score we cannot must reject the null hypothesis for 98% and, naturally, appears to work for 85%.