

Homework 3, Due 02/27/2025

February 21, 2025

Question 1 (20 points)

Consider the function $f(x) = \cos(x)$ with the domain of $[-1, 1]$:

- (a) Approximate $f(x)$ with the 4th order polynomial $P_4(x)$ obtained from Taylor Expansion. Estimate the error bound for $|f(x) - P_4(x)|$ and compare the actual largest error with the error bound
- (b) Construct the 3rd order polynomial $P_3(x)$ that is the closest to $P_4(x)$, compute the error bound for $|P_4(x) - P_3(x)|$
- (c) Based on (a) and (b), compute the error bound for $|f(x) - P_3(x)|$. Compare the real error with this error bound.

Question 2 (20 points)

Consider the 1D Poisson's equation $-u_{xx} = 1 + x$ with boundary condition $u(0) = 0$ and $u(1) = 0$:

- (a) Find the solution analytically over the domain $[0, 1]$
- (b) Solve the equation numerically by centered difference discretization, with $h = 0.1, 0.01$, and 0.001
- (c) Plot the error distributions over space, as well as 2-norms of the error vs. h . Analyze the order of accuracy of your discretization.