Math 5411 – Mathematical Statistics I– Fall 2024
w/Nezamoddini-Kachouie

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Homework #6 – September 30, 2024

1. Derive the CDF for X Uniform($a = min, b = max$). Start with the PDF of X and use the definition of CDF. It means find the $P(X \leq x) = F(X = x)$ for $x < a, a < x < b$, and $x > b$.

$$\begin{aligned}
P(X = x) &= h, x \in [a, b] \\
F(X = x) = P(X \leq x) &= \int_{-\infty}^{x} P(t)\, dt \\
&= \int_{-\infty}^{a} P(t) dt + \int_{a}^{x} P(t)\, dt \\
&= \int_{-\infty}^{a} 0\, dt + \int_{a}^{x} h\, dt \\
&= h(x - a)
\end{aligned}$$

which is zero for $x \leq a$, a line with slope h for x from a to b, and 1 for $x \geq b$.

2. Show $P(x_1 \leq x \leq x_2) = F(x_2) - F(x_1)$

$$\begin{aligned}
F(X = x) = P(X \leq x) &= \int_{-\infty}^{x} P(t) dt \\
F(X = x_2) = P(x \leq x_2) &= \int_{-\infty}^{x_2} P(t) dt
\end{aligned}$$

Since $x_1 \leq x \leq x_2$

$$\begin{aligned}
F(x_2) = P(x \leq x_2) &= \int_{-\infty}^{x_1} P(t) dt + \int_{x_1}^{x_2} P(t) dt \\
&= P(x \leq x_2) = F(x_1) + P(x_1 \leq x \leq x_2)
\end{aligned}$$

$$\therefore P(x_1 \leq x \leq x_2) = F(x_2) - F(x_1)$$

3. Find CDF for X Normal(μ, σ).

$$\begin{aligned}
p(x) &= \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \\
c(t) &= \int_{-\infty}^{t} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx \\
\text{Let } u &= \frac{x - \mu}{\sigma\sqrt{2}},\ du = \frac{1}{\sigma\sqrt{2}} dx \\
c(t) &= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{t} e^{-u^2}\, du
\end{aligned}$$