Math 5411 – Mathematical Statistics I– Fall 2024
w/Nezamoddini-Kachouie

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Example 1: Deal two cards from a well-shuffled eck. let the random variable X be the number of aces dealt and let the randome variable Y bt the number of face cards dealt. Find $f(x, y)$ and calculate the probability that the hand will contain more aces than face cards.

There are four aces, $|X| = 4$ and 12 face cards $|Y| = 12$. The entire sample set of valid hands is

$$\mathcal{A} = \{(x, y) \mid (0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (2, 0)\}$$

$$f_{X,Y}(0,0) = \frac{\binom{36}{2}}{\binom{52}{2}}, \frac{\text{number of numbered hands}}{\text{number of cards}}$$
$$f_{X,Y}(0,1) = \frac{\binom{36}{1}\binom{12}{1}}{\binom{52}{2}}, \frac{\text{one numbered, one face}}{\text{number of cards}}$$
$$f_{X,Y}(0,2) = \frac{\binom{12}{2}}{\binom{52}{2}}, \frac{\text{two face}}{\text{number of numbered hands}}$$
$$f_{X,Y}(1,0) = \frac{\binom{4}{1}\binom{36}{1}}{\binom{52}{2}}, \frac{\text{one ace, one numbered}}{\text{number of numbered hands}}$$
$$f_{X,Y}(1,1) = \frac{\binom{4}{1}\binom{12}{1}}{\binom{52}{2}}, \frac{\text{one ace, one face}}{\text{number of numbered hands}}$$
$$f_{X,Y}(2,0) = \frac{\binom{4}{2}}{\binom{52}{2}}, \frac{\text{two aces}}{\text{number of numbered hands}}$$

more aces than face cards is

$$f_{X,Y}(2,0) + f_{X,Y}(1,0) = \frac{\binom{4}{2}}{\binom{52}{2}} + \frac{\binom{4}{}\binom{36}{1}}{\binom{52}{2}} = \frac{4 * 3 + 4 \cdot 36}{(52 \cdot 51)/2} = 0.118$$

Example 3 Jordan and Greta agree to meet at the library between 2:00 PM and 3:00 PM. Their arrival times are independent and uniformly distributed, between 2:00 and 3:00. If they wait 20 minutes for the other, find the probability that they meet.

This is a bivariant uniform distribution. Let X be the arrival time (minute) for Jordan and Y be the arrival time (minute) for Greta. Thus

$$f_x(x) = \frac{1}{60}, \; 0 \le x \le 60$$
$$f_Y(y) = \frac{1}{60}, \; 0 \le y \le 60$$
$$\mathcal{A} = \{(x, y) \mid 0 \le x \le 60, \; 0 \le y \le 60\}$$
$$f_{X,Y}(x, y) = \frac{1}{60 * 60} = \frac{1}{3600}, \text{ probability of meeting at the exact same minute}$$
$$f_{X,Y}(|X - Y| \le 20) = \frac{3600 - (60 - 20)^2}{3600}, \frac{\text{area of coincidence}}{\text{area of all possible outcomes}}$$
$$= 0.5555$$

Note the "area of non-coincidence" are two isosceles right trianges of side 40, making a square.