

Math 725 – Advanced Linear Algebra  
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Assignment #11 – Due 12/8/23

1. Let  $A$  be an invertible square matrix. Show that  $|\det A| = \sigma_1 \sigma_2 \cdots \sigma_n$ .
2. Let  $A$  be a nonzero  $m \times n$  matrix. Prove that  $\sigma_1 = \max\{\|Au\| : \|u\| = 1\}$ .
3. Let  $A$  and  $A'$  be two nonzero  $m \times n$  matrices with respective largest singular values  $\sigma_1$  and  $\sigma'_1$ . Prove that the largest singular value of  $A + A'$  is bounded above by  $\sigma_1 + \sigma'_1$ .
4. Suppose  $A$  is an  $m \times n$  matrix and  $B$  is  $n \times m$  matrix obtained by rotating  $A$  ninety degrees clockwise on paper (not a standard matrix operation). Do  $A$  and  $B$  have the same singular values? Prove or give a counterexample.
5. Let  $A$  be an  $m \times n$  matrix of rank  $r > 0$  with singular values  $\sigma_1, \dots, \sigma_r$ . Show that  $\|A\|_F = \sqrt{\sigma_1^2 + \dots + \sigma_r^2}$ .