Math XXXX – Independent Study: Manifolds– Summer 2025

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Basic Category Theory – Tom Leinster– August, 2025

Chapter 1

Categories, Functors, and Natural Transformations

1.1 Categories

1.1.1 Exercises

1.1.12 Find three examples of categories not mentioned above.

1.1.13 Show that a map in a category can have at most one inverse. That is, given a map $f : A \to B$ there is at most one map $g : B \to A$ such that $gf = \mathbb{I}_A$ and $fg = \mathbb{I}_B$.

1.1.14 Let $\mathscr{A}$ and $\mathscr{B}$ be categories. Construct 1.1.11 defined by the product category $\mathscr{A} \times \mathscr{B}$, except that the definitions of composition and identities in $\mathscr{A} \times \mathscr{B}$ are not given. There is only one sensible way to define the: write it down.

1.1.15 There is a category call **Toph** whose objects are topological spaces and whose maps $X \to Y$ are homotopy classes of continuous maps X to Y. What do we need to know about homotopy in order to prove that **Toph** is a category? What does it mean in pure topological terms for two objects of **Toph** to be isomorphic?