Practice Final Exam - MTH 5111 - Real Analysis 2 - Dr. Kanishka Perera - Spring 2025

Name: ______________________________

Each problem is worth 25 points. Books, notes, and formula sheets are not allowed.

$(X, \mathcal{M}, \mu)$ denotes a measure space throughout the test.

1. Show that if $p \in [1, \infty)$, $f_n \to f$ in $L^p(\mu)$, $g_n(x) \to g(x)$ for all $x \in X$, and g_n is bounded in $L^\infty(\mu)$, then $f_n\, g_n \to fg$ in $L^p(\mu)$.

2. Show that if $1 \le r < p < s \le \infty$ and $f \in L^r(\mu) \cap L^s(\mu)$, then $f \in L^p(\mu)$ and $\|f\|_p \le \max\{\|f\|_r, \|f\|_s\}$.

3. Show that if $\mu(X) = 1$ and $f, g : X \to (0, \infty)$ are measurable functions such that $fg \ge 1$, then

$$\left(\int_X f\, d\mu\right)\left(\int_X g\, d\mu\right) \ge 1.$$

4. Show that if $\varphi : \mathbb{R} \to \mathbb{R}$ is such that

$$\varphi\left(\int_0^1 f(x)\, dx\right) \le \int_0^1 \varphi(f(x))\, dx$$

for every bounded measurable function $f : \mathbb{R} \to \mathbb{R}$, then φ is convex.