

50 min. No books, no notes, no electronic devices. You must show your work to receive credit.

1.[18] For the function $f(x) = xe^{-x}$

(a) Find the critical points of f .

(b) Find $f''(x)$.

(c) Find the open intervals on which f is concave up and the open intervals on which f is concave down.

(d) Identify any inflection points for the graph of f . Give the actual points.

(e) Use the second derivative test to identify any local extrema for the graph of f .

2.[5] Use l'Hôpital's rule to compute $\lim_{x \rightarrow 1} \frac{\ln x}{\sin(x-1)}$. Indicate which case of $l'H$ you're using.

3.[6] A rectangular field is to be bounded by a fence on three sides and a straight stream on the fourth side. Find the dimensions of the field with maximum area that can be enclosed using 800 ft. of fence.

4.[7] Find the general antiderivative (indefinite integral): $\int \left(2 \sin x - \frac{1}{1+x^2} + e^{3x} \right) dx$

5.[5] Let f be integrable on $[1, 3]$. If $\int_1^3 f(x) \, dx = -8$ and $\int_1^2 f(x) \, dx = 1$, what's $\int_2^3 f(x) \, dx$?

6.[6] Use the Fundamental Theorem of Calculus to find the *average value* of $f(x) = x^2 - 1$ on $[-1, 1]$.

7.[13] Use the Fundamental Theorem of Calculus to evaluate the following definite integrals. Simplify your final answers.

(a) $\int_1^4 \left(\frac{1}{x} + \sqrt{x} \right) dx.$

(b) $\int_0^{\ln 2} e^{-x} \, dx.$