Math 5411 – Mathematical Statistics I– Fall 2024
w/Nezamoddini-Kachouie

Paul Carmody
Homework #1 – August 27, 2024

1. Prove the property C and D in page 5 of the textbook.

Property C. If $A \subset B$, then $P(A) \leq P(B)$.

Since $A \subset B$ let $C = B \backslash A$.

$$\text{Let } C = B \backslash A$$
$$B = A \sqcup C$$
$$P(B) = P(A) + P(C)$$
$$P(B) \geq P(A)$$

Property D Addition Law $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Notice that

$$P(A) = P\left((A \backslash B) \sqcup (A \cap B)\right) = P(A \backslash B) + P(A \cap B)$$
$$P(B) = P\left((B \backslash A) \sqcup (A \cap B)\right) = P(B \backslash A) + P(A \cap B)$$
$$\text{and}$$
$$A \cup B = (A \backslash B) \sqcup (A \cap B) \sqcup (B \backslash A)$$

Then

$$P(A \cup B) = P(A \backslash B) + P(A \cap B) + P(B \backslash A)$$
$$P(A \cup B) + P(A \cap B) = \left(P(A \backslash B) + P(A \cap B)\right) + \left(P(B \backslash A) + P(A \cap B)\right)$$
$$= P(A) + P(B)$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

2. Read the textbook up to page 11 and solve modified Example F as: How many people must you ask to have a 50:50 chance of finding someone who share your birth month?

Using a similar argument. Asking n people with the event is that one of those birthdays is the same month as my own. The total number of outcomes is 12^n and the total number of failures, $A^c = 11^n$. Thus, $P(A^c) = \frac{11^n}{12^n}$ and $P(A) = 1 - \frac{11^n}{12^n}$. Therefore, for $P(A) = 0.5$ we have

$$\frac{11^n}{12^n} = 0.5$$
$$n = \frac{\ln 0.5}{\ln \frac{11}{12}} \approx 8$$