Math 5111 – Real Analysis II– Sprint 2025
w/Professor Liu

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Homework (not graded) – May 2025

Pg 30, 3

Prove that if f is a real function on a measurable space X such that $\{x : f(x) \geq r\}$ is measurable for for every rational r, then f is measurable.

Pg 30, 4

Let $\{a_n\}$ and $\{b_n\}$ be sequencesin $[-\infty, \infty]$, and prove the following assertions:

(a) $\limsup\limits_{n\to\infty} (-a_n) = -\liminf\limits_{n\to\infty} a_n$.

(b) $\limsup\limits_{n\to\infty} (a_n + b_n) \leq \limsup\limits_{n\to\infty} a_n + \limsup\limits_{n\to\infty} b_n$ provided none of the sums is $\infty - \infty$.

(c) if $a_n \leq b_n$, for all n, then

$$\liminf_{n\to\infty} a_n \leq \liminf_{n\to\infty} b_n$$

Show by example that strick inequalty can hold in (b).

Pg 30, 5

(a) Suppose $f : X \to [-\infty, \infty]$ and $g; X \to [-\infty, \infty]$ are measurable. Prove that the sets

$$\{x : f(x) < g(x)\}, \{x : f(x) = g(x)|$$

are measurable.