

Math 5411 – Mathematical Statistics I– Fall 2024

w/Nezamoddini-Kachouie

Paul Carmody
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§1.8 Page 31 exercises 49 & 50

49. A fair coin is tossed three times.

a. What is the probability of two or more heads given that there was at least one head?

The sample space is $\Omega = \{(H, H, H), (H, H, T), (H, T, H), (T, H, H), (H, T, T), (T, T, H), (T, H, T), (T, T, T)\}$ and they are equally likely.

Let $A = \{\text{at least one head}\} = \{(H, H, H), (H, H, T), (H, T, H), (T, H, H), (H, T, T), (T, T, H), (T, H, T)\}$ and $B = \{\text{two or more heads}\} = \{(H, H, H), (H, H, T), (H, T, H), (T, H, H)\}$. $|A| = 7$ and $P(A) = 7/8$. $|B| = 4$ and $P(B) = 1/2$.

The probability of two or more heads given that there was at least one is $P(B|A) = P(A \cap B)/P(A)$. $A \cap B = B$ and $P(A \cap B) = P(B)$ thus $P(B|A) = P(A \cap B)/P(A) = P(B)/P(A) = 4/7$.

b. What is the probability given that there was at least one tail?

Let $C = \{\text{at least one tail}\} = \{(H, H, T), (H, T, H), (T, H, H), (H, T, T), (T, T, H), (T, H, T), (T, T, T)\}$ and $|C| = 7$ thus $P(C) = 7/8$

$B \cap C = \{(H, H, T), (H, T, H), (T, H, H)\}$ and $|B \cap C| = 3$ thus $P(B \cap C) = 3/8$.

$P(B|C) = P(B \cap C)/P(C) = \frac{3/8}{7/8} = 3/7$

59. Two dice are rolled and the sum of the face values is six. What is the probability that at least one of the dice came up a three?

$$\begin{aligned}\Omega = \{ & 2, 3, 4, 5, 6, 7, \\ & 3, 4, 5, 6, 7, 8, \\ & 4, 5, 6, 7, 8, 9, \\ & 5, 6, 7, 8, 9, 10, \\ & 6, 7, 8, 9, 10, 11, \\ & 7, 8, 9, 10, 11, 12 \} \\ |\Omega| = & 36\end{aligned}$$

Let $A = \{\text{sum is 6}\}$. $|A| = 5$ and $P(A) = 5/36$

$B = \{\text{at least one of the dice came up three}\}$. Let $B_1 = \{\text{the first die is a three}\}$ and $B_2 = \{\text{the second die is a three}\}$. $B = (B_1 \cup B_2) \setminus P(B_1 \cap B_2)$ thus $|B| = |B_1| + |B_2| - |B_1 \cap B_2| = 6 + 6 - 1 = 11$ and $P(B) = 11/36$.

$A \cap B = \{(3, 3)\}$, $|A \cap B| = 1$, $P(A \cap B) = 1/36$ thus $P(A|B) = P(A \cap B)/P(B) = 1/11$.