Math 5411 – Mathematical Statistics I– Fall 2024
w/Nezamoddini-Kachouie

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§1.8 Page 31 excercises 49 & 50

49. A fair coin is tossed three times.

a. What is the probability of two or more heads given that there was at least one head?

The sample space is $\Omega = \{(H, H, H), (H, H, T), (H, T, H), (T, H, H), (H, T, T), (T, T, H), (T, H, T), (T, T, T)\}$ and they are equally likely.
Let $A = \{$at least one head $\} = \{(H, H, H), (H, H, T), (H, T, H), (T, H, H), (H, T, T), (T, T, H), (T, H, T)\}$ and $B = \{$ two or more heads $\} = \{(H, H, H), (H, H, T), (H, T, H), (T, H, H)\}$. $|A| = 7$ and $P(A) = 7/8$. $|B| = 4$ and $P(B) = 1/2$.
The probability of two or more heads given that there was at least one is $P(B|A) = P(A \cap B)/P(A)$. $A \cap B = B$ and $P(A \cap B) = P(B)$ thus $P(B|A) = P(A \cap B)/P(A) = P(B)/P(A) = 4/7$.

b. What it the probability given that there was at least one tail?

Let $C = \{$ at least one tail $\} = \{(H, H, T), (H, T, H), (T, H, H), (H, T, T), (T, T, H), (T, H, T), (T, T, T)\}$ and $|C| = 7$ thus $P(C) = 7/8$
$B \cap C = \{(H, H, T), (H, T, H), (T, H, H)\}$ and $|B \cap C| = 3$ thus $P(B \cap C) = 3/8$.
$P(B|C) = P(B \cap C)/P(C) = \frac{3/8}{7/8} = 3/7$

59. Two dice are rolled and the sum of the face values is six. What is the probability that at least one of the dice came up a three?

$$\Omega = \{\, 2, 3, 4, 5, 6, 7,$$
$$3, 4, 5, 6, 7, 8,$$
$$4, 5, 6, 7, 8, 9,$$
$$5, 6, 7, 8, 9, 10,$$
$$6, 7, 8, 9, 10.11,$$
$$7, 8, 9, 10, 11, 12\,\}$$
$$|\,\Omega\,| = 36$$

Let $A = \{$ sum is 6 $\}$. $|A| = 5$ and $P(A) = 5/36$
$B = \{$ at least one of the dice came up three $\}$. Let $B_1 = \{$ the first die is a three $\}$ and $B_2 = \{$ the second die is a three $\}$. $B = (B_1 \cup B_2) \backslash P(B_1 \cap B_2)$ thus $|B| = |B_1| + |B_2| - |B_1 - B_2| = 6 + 6 - 1 = 11$ and $P(B) = 11/36$.
$A \cap B = \{(3, 3)\}, |A \cap B| = 1, P(A \cap B) = 1/36$ thus $P(A|B) = P(A \cap B)/P(B) = 1/11$.