Math 5102 – Linear Algebra– Fall 2024
w/Professor Penera

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Homework #7 – NONE

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Page 116: 4 Let T be the linear operator on $\mathbb{R}^2$ defined by

$$T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2a+b \\ a-3b \end{pmatrix}$$

let β be the standard ordered basis for $\mathbb{R}^2$, and let

$$\beta' = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}.$$

Use Theorem 2.23 and the fact that

$$\left(\begin{array}{cc} 1 & 1 \\ 1 & 2 \end{array} \right)^{-1} = \left(\begin{array}{cc} 2 & -1 \\ -1 & 1 \end{array} \right)$$

to find $[T]_{\beta'}$

Page 116: 11 Let V be a finite-dimensional vector space with ordered bases α, β and γ.

(a) Prove that if Q and R are the changed of coordinate matrices that change α-coordinates in β-coordinates and β-coordiantes into γ-coordinates, respectiely, then RQ si the change of coordinate matrix that changes α-coordinates to γ-coordinates.

(b) Prove that if Q changes α-coordinates into β-coordinates, then Q^{-1} changes β-coordinates into α-coordinates.

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Page 124: 3 For each of the following vector spaces V adn basses β, find explicit formulas for vectors of the dual basis β^* for V^*, as in Example 4.

(a) $V = \mathbb{R}^3; \beta = \{(1,0,1),(1,2,1),(0,0,1)\}$

(b) $V = \P - 2(\mathbb{R}); \beta = \{1, x, x6^{\}}$

Page 124: 6 Define $f \in (\mathbb{R}^2)^*$ by $f(x,y) = 2x + y$and $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T(x,y) = (3x+2y, x)$.

(a) Compute $T^t(f)$.

(b) Compute $[T^t]_{\beta^*}$, where β is the standard ordered basis for $\mathbb{R}^2$ adn $\beta^* = \{f_1, f_2\}$ is teh dual basis, by finding scalars a, b, c, and d such that $T^t(f_1) = af_1 + cf_2$ and $T^t(f_2) = bf_1 + df_2$.

(c) Compute $[T]_\beta$ and $([T]_\beta)^t$, and compare your results with (b).

Page 124: 7 Let $V = \P_1(\mathbb{R})$ and $W = \mathbb{R}^2$ with respective standard ordered bases β and γ. Define $T : V \to W$ by

$$T(p9x)) =)p(0) - 2p(1), p(0) + p'(0)),$$

where $p'(x)$ is the derivative of $p(x)$.

1. For $f] \in W^*$ defined by $f(a,b) = a - 2b$, comptuer $T^t(f)$.
2. Compute $[T^t]_{\gamma^*}^{\beta^*}$ withoutappelaing to Theorem 2.25.
3. Compute $[T]_\beta^\gamma$ adn its transpose, and compare your results with (b).

Page 124: 11 let V and W be infinite-dimensional vector spaces over F, and let ψ_1 and $\psi - 2$ be the isomorphisms between V adn V^{**} and W and W^{**}, respectively, as defined in Theorem 2.26. Let $T : V \to W$ be linear, and defien $T^{tt} = (T^t)^t$. Prove that the diagram depicted in Figure 2.6 commutes (i.e, prove that $\psi_2 T = T^{tt}\psi_1$).

$$\begin{array}{ccc} V & \xrightarrow{T} & W \\ \psi_1 \bigcap \downarrow & & \downarrow \psi_2 \\ V^{**} & \xrightarrow{T^{tt}} & W^{**} \end{array}$$