Math 5411 – Mathematical Statistics I– Fall 2024
w/Nezamoddini-Kachouie

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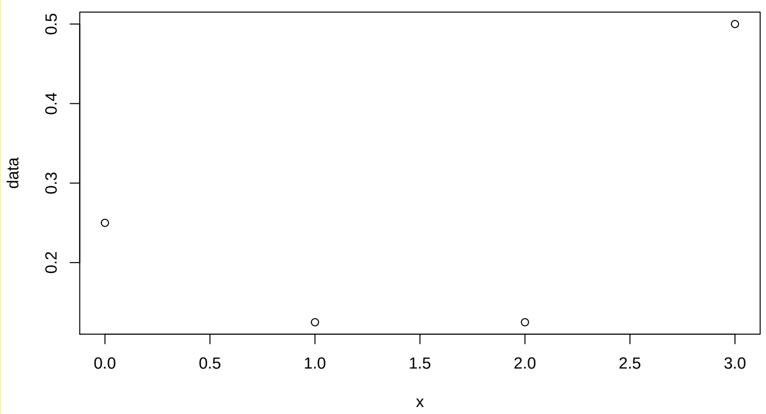
Homework #5 – September 23, 2024

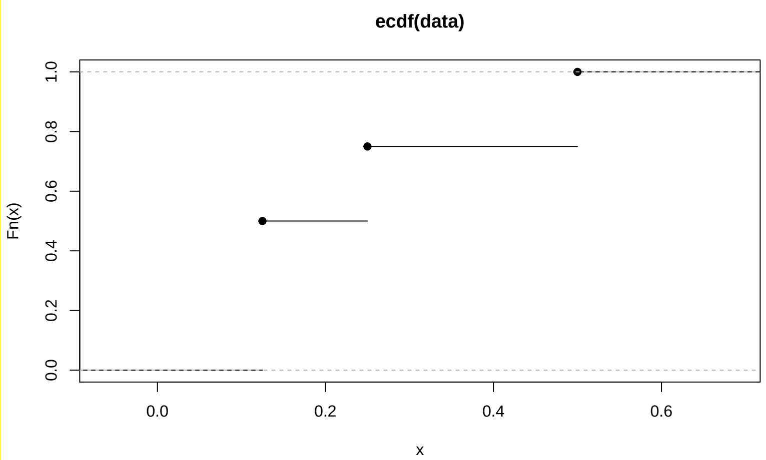
Questions 1, 3, 7, 10, 14, 27, 31 from Chapter 2 starting on Page 64.

1. Suppose that X is a discrete random variable with $P(x = 0) = .25P(X = 1) = .125, P(x = 2) = .125$, and $P(x = 3) = .5$. Graph the frequencey function and the cumulative distribution function of X.

The frequency function is



The cummulative distribution function is



3. The following table shows the cumulative distribution function of a discrete random variable. Find the frequency function.

k	$F(k)$
0	0
1	.1
2	.3
3	.7
4	.8
5	1.0

The cummulative distribution function, $F(X = x_i) = \sum_{n=0}^{i} P(x_i), i = 0, \ldots, 5$. Thus $P(X = k) = F(k) - F(k-1)$

k	$P(X = k)$
0	0
1	.1
2	.2
3	.4
4	.1
5	.2

We can verify this by checking that $\sum_{k=0}^{5} P(X = k) = 1$.

7. Find the cdf of a Bernouilli random variable.

Let p be the probabily of success. Then, $F(0) = 1 - p, F(1) = p + (1 - p) = 1$

10. Appending three extra bits to a 4-bit word in a particular way (a Hamming code) allows detection and correction of up to one error in any of the bits. If each bit has probability of .05 of being changed during communication, and the bits are changed independently of each other, what is the probability that the word is correctly received (that is, 0 or 1 bit is in error)? How does this probability compare to the probabiliy that the word will be transmitted correctly with no check bits, in which case all four bits would have to be transmitted correctly for the word to be correct?

7 bits means that the size of the sample space is $2^7 = 128$. Allowing 0 or 1 bits to be in error

with $p = 0.5$ and 7 bits or $\frac{\binom{7}{2} \cdot 0.05}{2^7} = \frac{21 * 0.05}{128} = 0.0082$.. That is a 99.18% accuracy.

No check bits. The probability of each correct bit is $p = .95$ with four bits,

the probability that all are correct is $= (.95)^4 = .8145$ or 81.45%.

14. Two boys play basketball in the following way. They take turns shooting and stop when a basket is made. Player A goes first and has a probability p_1 of making a basket on any throw. Player B, who shoots second, has a probability p_2 of making a second basket. The outcomes of the successive trials are assumed to be independent.

   (a) Find the frequency function for the total number of attempts.

k	$F(k)$	
0	p_1	
1	$(1 - F(0))p_2$	$= (1 - p_1)p_2$
2	$(1 - F(1))p_1$	$= (1 - (1 - p_1)p_2)p_1$
3	$(1 - F(2))p_2$	$= (1 - (1 - (1 - p_1)p_2)p_1)p_2$

   (b) What is the probability that player A wins?

27. Suppose that a rare disease has an incidence of 1 in 1000. Assuming that members of the population are affected independently, find the probability of k cases in a population of 100,000 for $k = 0, 1, 2$.

31. Phone calls are received at a certain residence as a Poisson process with parameter $\lambda = 2$ per hour.

   (a) If Diane takes a 10-min. shower, what is the probability that the phone rings during that time?

   This is a Poisson distribution with a modified $\lambda = 2/6$ for the ten minute interval in quesitons

   which comes to $P(X = 1) = \frac{\lambda^k}{k!} e^{-\lambda} = \frac{(1/3)^1}{1!} e^{-1/3} = e^{-1/3}/3 = 0.24$

(b) How long can her shower be if she wishes the probability of receiving no phone calls to be at most .5?

$P(X=0) = 0.5 = \frac{\lambda_{p=0.5}^k}{k!} e^{-\lambda_{p=0.5}} = e^{-\lambda_{p=0.5}}$ or $\lambda_{p=0.5} = -\ln 0.5 = .69$ or $\lambda_{p=0.5} = 0.69 = 2x$

where $x = .346$ hours or 20 minutes.