Math 5102 – Linear Algebra– Fall 2024
w/Professor Penera

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Homework #3 – NONE

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4. Do the polynomials $x^3 - 2x^2 + 1, 4x^2 - x + 3$, and $3x - 2$ generate $P_3(\mathbb{R})$?

5. Is $\{(1, 4, -6), (1, 5, 8), (2, 1, 1), (0, 1, 0)\}$ a linearly independent subset of $\mathbb{R}^3$? Justify your answer.

7. The vectors $u_1 = (2, -3, 1), u_2 = (1, 4, -2), u_3 = (-8, 12, -4), u_4 = (1, 37, -17)$, and $u_5 = (-3, -5, 8)$ generate $\mathbb{R}^3$. Find a subset of the set $\{u_1, u_2, u_3, u_4, u_5\}$ that is a basis for $\mathbb{R}^3$.

13. The set of solutions to the system of linear equations

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ 2x_2 - 3x_2 + x_3 &= 0 \end{aligned}$$

is a subspace of $\mathbb{R}^3$. Find abasis for this subspace.

14. Find bases for the following subspaces of F^5.

$$W_1 = \{(a_1, a_2, a_3, a_4, a_5\} \in F^5 \,:\, a_1 - a_3 - a_4 = 0\}$$

and

$$W_2 = \{(a_1, a_2, a_3, a_4, a_5) \in F^5 \,:\, a_2 = a_3 = a_4 \text{ and } a_1 + a_5 = 0\}$$

What are the dimensions of W_1 and W_2

15. The set of all $n \times n$ matrices having trace equal to zero is a subspace W of $M_{n\times n}(F)$(see Example 4 of Sections 1.3). find a basis for W. What is the dimension of W?

16. The set of all upper triangular $n \times n$ matrices is a subspace W of $M_{n\times n}(F)$(See Exercise 12 of Section 1.3). Find a basis for W. What is the dimension of W?

26. Let V, W, and Z be as in Exercise 21 of Section 1.2. If V and W are vector spaces over F of dimensions m and n, determine the dimension of Z.