Math 5411 – Mathematical Statistics I– Fall 2024
w/Nezamoddini-Kachouie

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Homework #4 – September 16, 2024

§42, 49, 60, and 80 from chapter 1.

Exercise 42 How many ways can 11 boys on a soccer team be grouped in 4 forwards, 3 midfielders, 3 defenders, and 1 goalie?

$$\binom{11}{4,3,3,1} = \frac{11!}{4!\,4!\,3!\,1!} = \frac{39916800}{24 \cdot 24 \cdot 6} = \frac{39916800}{345} = 11550$$

Exercise 49 A fair coin is tossed three times

1. What is the probability of two or more heads given that there were at least one head?

   Let A be the event of "at least one head" and B be the event of "two or more heads".

$$\begin{aligned}
|\,\Omega\,| &= 2^3 = 8 \text{ and } A^c = \{\,(t,t,t)\,\} \\
P(A) &= 7/8 \\
B^c &= \{\,(t,t,t),(h,t,t),(t,h,t),(t,t,h)\,\} \\
P(B) &= 1/2 \\
B \cap A &= \{\,(h,t,t),(t,h,t),(t,t,h)\,\} \\
P(B \cap A) &= 3/8 \\
P(B|A) = P(B \cap A)/P(A) &= \frac{3/8}{4/8} = 3/4
\end{aligned}$$

2. What is the probabilty give that there was at least on tail?

   Let C be the event "at least one tail".

$$\begin{aligned}
P(C) &= 1/2 \\
B \cap C &= \{\,(h,h,t),(t,h,h),(h,t,h)\,\} \\
P(B \cap C) &= 3/8 \\
P(B|C) = P(B \cap C)/P(C) &= \frac{3/8}{4/8} = 3/4
\end{aligned}$$

Exercise 60 A factory runs three shifts. In a given day, 1% of the items produced by the first are defective, 2% of the second shifts items are defective, and 5% of the third shift's items are defective. If the shifts all have the same productivity, what percentage of the items produced in a day are defective? If an item is defective, what is the probability that it was produced on the third shift?

Let N be the amount of items produced per shift. The total number of defectives items T will be $T = 0.01N + 0.02N + 0.05N = 0.08N$ will be $0.08N$ out of $3N$ items or 2.667%. Let A be the probability that it is a defective item and C be the probability that it came from the third shift. $P(C) = 1/3$ and $P(A) = 0.02667$. $P(C|A) = P(C \cap A)/P(A) = 0.02667/0.05 = 0.5334$.

Exercise 80 If a parent has genotype Aa, he transmits either A or a to an offspring (each with a $\frac{1}{2}$ chance). The gene he transmits to one offspring is independent of the one he transmits to another. Consider a parent with three children and the following events: $A = \{$children 1 and 2 have the same gene$\}$, $B = \{$ children 1 and 3 have the same gene$\}$, $C = \{$children 2 and 3 have the same gene$\}$. Show that these events are pairwise independent but not mutually independent.

We have three events but only two outcomes. If 1 & 2 (A) have the same gene AND 2 & 3 (B) have the same gene then it logically falls through transitivity that 1 & 3 (C) must have the same gene. Thus, in terms of propability $P\,(\,C\,|\,(A\,|\,B)\,) = 1$.