# Math 5102 – Linear Algebra– Fall 2024
## w/Professor Penera

Paul Carmody
Homework #6 – NONE

Page 107: 3, 4, 5, 15, 16

Page 107:3. Which of the following pairs of vector spaces are isomorphic? Justify your answers.

(a) $F^3$ and $P_3(F)$.

(b) $F^4$ and $P_3(F)$.

(c) $M_{2\times 2}(\mathbb{R})$ and $P_3(\mathbb{R})$.

(d) $V = \{A \in M_{2\times 2}(\mathbb{R}) : \text{tr}(A) = 0\}$ and $\mathbb{R}^4$.

Page 107:4. Let $A$ and $B$ be $n \times n$ matrices. Prove taht $AB$ is invertible and $(AB)^{-1} = B^{-1}A^{-1}$.

Page 107:5. Let $A$ be invertible. Prove that $A^t$ is invertible and $(A^t)^{-1} = (A^{-1})^t$.

Page 107:15. Let $V$ and $W$ be $n$-dimensional vector spaces, and let $T : V \to W$ be a linear transformation. Suppose that $\beta$ is a basis for $V$. Prove that $T$ is an isomorphism if and only if $T(\beta)$ is a basis for $W$.

Page 107:16. Let $B$ be an $n \times n$ invertyible matrix. Define $\Phi : M_{n\times n}(F) \to M_{n\times n} \to (F)$ by $\Phi(A) = B^{-1}AB$. Prove that $\Phi$ is an isomorphism.

Page 116: 4, 11

Page 116:4) Let $T$ be teh linear operator $\mathbb{R}^2$ defined by

$$T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2a+b \\ a-3b \end{pmatrix}$$

Let $\beta$ be the standard ordered basis for $\mathbb{R}^2$, and let

$$\beta' = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$$

Use Theorem 2.23 and the fact that

$$\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$

to find $[T]_{\beta'}$

Page 116:11) Let $V$ be a finite-dimensional vector space with ordered bases $\alpha, \beta$ and $\gamma$.

(a) Prove that if $Q$ and $R$ are the changed of coordinate matrices that change $\alpha$-coordinates in $\beta$-coordinates and $\beta$-coordiantes into $\gamma$-coordinates, respectiely, then $RQ$ si the change of coordinate matrix that changes $\alpha$-coordinates to $\gamma$-coordinates.

(b) Prove that if $Q$ chjanges $\alpha$-coordinates into $\beta$-coordinates, then $Q^{-1}$ changes $\beta$-coordinates into $\alpha$-coordinates.