Math 5301 – Numerical Analysis– Spring 2025
w/Professor Du

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Homework #1 – Januar 24, 2025

Question 1 (20 points)

Using Newton's Divided Difference Table, construct a quadratic polynomial to interpolate the function $f(x) = \sin x$ at $x + 0 = 0, x_1 = \pi/4$ and $x_2 = \pi/2$.

(a) Write the polynomial in the form $P_2(x) = ax^2 + bx + c$, include the divided difference table you use.

i	x_i	$f[x_i]$	$f[x_{i-1}, x_i]$
0	0	0	
1	$\pi/4$	$\frac{0.707}{0.785} = 0.9003$	
2	$\pi/2$	$\frac{0.2635}{-0.785}$	-0.3357

$$P_2(x) = 0.9003x - 0.3357x^2$$

(b) Estimate the error bound for the interpolation.

(c) Estimate (graphically) the largest real error by comparing the plots of $y = f(x)$ and $y = P_2(x)$. Attach computer generated plots.

(d) Compare the real error with the error bound computed in step (b) and comment on the comparison.

Question 2 (20 points)

Suppose we do piecewise interpolation over equally-spaced nodes with $[1, 4]$ for $f(x) = 1/x$. We would like to keep the largest error under 10^{-3}.

(a) How many nodes are required for piecewise linear interpolation?

(b) How many nodes are required for piecewise quadratic interpolation?

(c) Use Matlab to confirm your calculation in (a).