Math 5102 – Linear Algebra– Fall 2024
w/Professor Penera

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Homework #9 – NONE

Page 180: 6 Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be defined by $T(a, b, c) = (a + b, 2a - c)$. Determine $T^{-1}(1, 11)$.

$$a + b = 1$$
$$2a - c = 11$$
$$b = 1 - a \text{ and } c = 2a - 11$$
$$T^{-1}(a, b) = (a, 1 - a, 2a - 11)$$
$$T(a, b, c) = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 0 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$
$$T^{-1}(a, b) = \begin{pmatrix} u & v \\ w & x \\ y & z \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$
$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 0 & -1 \end{pmatrix} \begin{pmatrix} u & v \\ w & x \\ y & z \end{pmatrix} = \begin{pmatrix} u + w & v + x \\ 2u - y & 2v - z \end{pmatrix}$$
$$A^{-1} = \begin{pmatrix} u & v \\ 1 - u & -v \\ 2u & 2v - 1 \end{pmatrix}$$

Page 180:8 Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(a, b, c) = (a + b, b - 2c, a + 2c)$. For each vector v in $\mathbb{R}^3$, determine whether $v \in R(T)$.

(a) $v = (1, 3, -2)$

(b) $v = (2, 1, 1)$