Practice Test 1 - MTH 5111 - Real Analysis 2 - Dr. Kanishka Perera - Spring 2025

Name: ______________________________

Each problem is worth 25 points. Books, notes, and formula sheets are not allowed.

$(X, \mathcal{M}, \mu)$ denotes a measure space throughout the test.

1. Show that if the function $f : X \to [-\infty, \infty]$ has the property that $\{x \in X : f(x) > r\}$ is measurable for all rational numbers r, then f is measurable.

2. Show that if $f : X \to [0, \infty]$ is measurable and $E \in \mathcal{M}$, then

$$\int_X f \, d\mu = \int_E f \, d\mu + \int_{X \setminus E} f \, d\mu.$$

3. Show that if $f : X \to [0, \infty]$ is measurable, then

$$\lim_{n \to \infty} \int_X n \log\left(1 + \frac{f}{n}\right) d\mu = \int_X f \, d\mu.$$

(Hint: Use the monotone convergence theorem.)

4. Let $\mathcal{F}$ be a collection of measurable functions $f : X \to [0, \infty]$ and let

$$c = \inf_{f \in \mathcal{F}} \int_X f \, d\mu.$$

Assume that every sequence (f_n) in $\mathcal{F}$ has a subsequence (f_{n_k}) such that $f_{n_k}(x) \to f(x)$ for all $x \in X$ for some function $f \in \mathcal{F}$. Show that there exists a function $f \in \mathcal{F}$ such that

$$\int_X f \, d\mu = c.$$

(Hint: Use Fatou's lemma.)