

Math 5411 – Mathematical Statistics I– Fall 2024  
w/Nezamoddini-Kachouie

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Homework #1 – August 27, 2024

1. Prove the property C and D in page 5 of the textbook.

**Property C.** If  $A \subset B$ , then  $P(A) \leq P(B)$ .

Since  $A \subset B$  let  $C = B \setminus A$ .

$$\text{Let } C = B \setminus A$$

$$B = A \sqcup C$$

$$P(B) = P(A) + P(C)$$

$$P(B) \geq P(A)$$

**Property D** Addition Law  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ .

Notice that

$$P(A) = P((A \setminus B) \sqcup (A \cap B)) = P(A \setminus B) + P(A \cap B)$$

$$P(B) = P((B \setminus A) \sqcup (A \cap B)) = P(B \setminus A) + P(A \cap B)$$

and

$$A \cup B = (A \setminus B) \sqcup (A \cap B) \sqcup (B \setminus A)$$

Then

$$P(A \cup B) = P(A \setminus B) + P(A \cap B) + P(B \setminus A)$$

$$P(A \cup B) + P(A \cap B) = (P(A \setminus B) + P(A \cap B)) + (P(B \setminus A) + P(A \cap B))$$

$$= P(A) + P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

2. Read the textbook up to page 11 and solve modified Example F as: How many people must you ask to have a 50:50 chance of finding someone who share your birth month?

Using a similar argument. Asking  $n$  people and the event is that one of those birthdays is the same. The total number of outcomes is  $12^n$  and the total number of failures,  $A^c = 11^n$ . Thus,  $P(A^c) = \frac{11^n}{12^n}$  and  $P(A) = 1 - \frac{11^n}{12^n}$ . Therefore, for  $P(A) = 0.5$  we have

$$\begin{aligned}\frac{11^n}{12^n} &= 0.5 \\ n &= \frac{\ln 0.5}{\ln \frac{11}{12}} \approx 8\end{aligned}$$