Practice Test 1 - MTH 5102 - Linear Algebra - Dr. Kanishka Perera - Fall 2024

Name: ____________________

Each problem is worth 20 points. You may refer to your book/notes. Calculators and cell phones are not allowed.

1. Let V be a vector space and let W_1 and W_2 be subspaces of V. Show that if $W_1 \cup W_2$ is a subspace of V, then either $W_1 \subset W_2$ or $W_2 \subset W_1$.

2. Let V and W be vector spaces and let $S : V \to W$ and $T : V \to W$ be nonzero linear transformations. Show that if $R(S) \cap R(T) = \{0\}$, then $\{S, T\}$ is a linearly independent subset of $\mathcal{L}(V, W)$.

3. Let F be a field and define the trace of $A = (a_{ij})$ in $M_{n\times n}(F)$ by $tr(A) = \sum_{i=1}^{n} a_{ii}$. Show that the function $f : M_{n\times n}(F) \to F$ defined by $f(A) = \mathrm{tr}(A)$ is a linear functional on $M_{n\times n}(F)$.

4. Let V, W, and Z be a vector space of the same dimension and let $S : V \to W$ and $T : W \to Z$ be linear transformations. Show that if TS is an isomorphism, then S and T are isomorphisms.

5. Let V and W be vector spaces and let $T : V \to W$ be a linear transformation. Show that $N(T^t) = \{g \in W^* : g(w) = 0 \text{ for all } w \in R(T)\}$.