Math 5102 – Linear Algebra– Fall 2024
w/Professor Penera

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Homework #4 – NONE

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3. $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(a_1, a_2, a_3) = (a_1 - a_2, 2a_3)$.

5. $T : P_2(\mathbb{R}) \to P_3(\mathbb{R})$ defined by $T(f(x)) = xf(x) + f'(x)$.

13. Let V and W be vector spaces, let $T : V \to W$ be linear, and let $\{w_1, w_2, \dots, w_k\}$ be a linearly independent subset of $R(T)$. Prove that $S = \{v_1, v_2, \dots, v_k\}$ is chosen so that $T(v_i) = w_i$ for $i = 1, 2, \dots, k$, then S is linearly independent.

17. Let V and W be finite-dimensional vector spaces and $T : v \to W$ be linear.
    (a) Prove that if $\dim(V) < \dim(W)$, then T cannot be onto.
    (b) Prove that if $\dim(V) > \dim(W)$, then T cannot be one-to-one.

20. Let V and W be vector spaces with subspaces V_1 adn W_1, respectively. If $T : V \to W$ is linear, prove that $T(V_1)$ is subspace of W and that $\{x \in V : T(x) \in W_1\}$ is a subspace of V.