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Discrete Applied Mathematics
ELSEVIER

Re: Submission of the article entitled “New lower bounds for Schur and weak Schur numbers”

Dear Editors,

In 1916 Schur formulated the sum-free property of sets of integers. A set is sum-free if no element z can be constructed by adding two, non-necessarily different, elements x and y of this set, i.e. $z \neq x+y$. He studied consecutive integers $\{1,2,\dots,p\}$ partitioned into n sets and proved that there is a maximum number $S(n)$ starting from which a n sum-free partition does not exist. In other words, for any $p > S(n)$ a split of n consecutive integers into n sum-free sets is not possible. The number $S(n)$ is traditionally referred to as “a Schur number”.

In the 1950s Sierpiński included the prohibition of summing up two identical numbers in the definition of the sum-free property (the weak sum-free property). The weak sum-free partitions possess the similar property of a maximum number $WS(n)$ starting from which a p weak sum-free partition does not exist (i.e. none of partitions of $\{1,2,\dots,p\}$, $p > WS(n)$, is weakly sum-free). The number $WS(n)$ is traditionally referred to as “a weak Schur number”. The weak sum-free partitions are consequently larger than those respecting the original Schur constraint: $WS(n) \geq S(n)$.

The problems of Schur and weak Schur numbers are approached from both sides. On the one hand, mathematicians try to bound $S(n)$ – or $WS(n)$ – from above. On the other hand, mathematicians and computer scientists aim at raising the lower bound of $S(n)$ – or $WS(n)$ – by exhibiting an n-partition which is [weakly] sum-free.

In 1972 Abbott and Hanson improved a Schur number lower bound compared to the one given by Schur himself. Their idea was to solve a certain homogeneous linear equation with non-zero integer coefficients. Recently Rowley has observed that there is a kind of template hidden behind the original construction. He exploited this fact and raised lower bounds on Schur numbers. He also produced $\mathrm{WS}(n)$ for $n=6, 7, \ldots, 10$.

We submit to your journal our study in which we focused on a template-based approach. We established a new template for Schur numbers and generalized it for weak Schur numbers. Thanks to the templates developed, we obtained inequalities for $\mathrm{S}(n)$ and $\mathrm{WS}(n)$ that allowed us to increase lower bounds for both these numbers: $\mathrm{S}(9)$, $\mathrm{S}(10)$, and $\mathrm{WS}(6)$, $\mathrm{WS}(9)$, $\mathrm{WS}(10)$.

We believe that our approach opens up numerous trails to pursue the quest of $\mathrm{S}(n)$ or $\mathrm{WS}(n)$. For this reason we submit it to your journal in the hope that it will be accepted.

Sincerely yours,

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