Assignment 5 : Control Charts for Attributes

1. A process is controlled with a fraction nonconforming control chart with three-sigma limits, n = 100, UCL = 0.161, center line = 0.080, and LCL = 0.

   (a) Find the equivalent control chart for the number nonconforming.

$$CL = n\bar{p} = 100 \times 0.08 = 8$$
$$UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 100 \times 0.08 + 3\sqrt{100 \times 0.08 \times 0.92} = 16.139$$
$$LCL = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = 100 \times 0.08 - 3\sqrt{100 \times 0.08 \times 0.92} = -0.139$$
$$\therefore \text{ For the } np \text{ chart: } UCL = 16.139, \ CL = 8, \ LCL = 0 \ _\#$$

   (b) Use the Poisson approximation to the binomial to find the probability of a type I error.

$$X \equiv \text{the number of nonconforming}$$
$$X \sim Bin(n = 100, p = 0.08) \approx X \sim Poi(\lambda = 8)$$
$$\alpha = P\left(type \ I \ error\right) = 1 - P\left(X \leq 16.14\right)$$
$$= 1 - \sum_{i=0}^{16.14} \frac{e^{-8} \, 8^x}{x!}$$
$$= 1 - 0.9963$$
$$= 0.0037 \ _\#$$

   (c) Use the Normal approximation to find the probability of a type II error if the process fraction nonconforming shifts to 0.2.

$$\beta = P\left(type \ II \ error\right) = P\left(0 < \hat{p} < 0.161 \mid p = 0.2\right)$$
$$= \Phi\left(\frac{16.139 - 0.2}{\sqrt{\frac{0.08 \times 0.92}{100}}}\right) - \Phi\left(\frac{0 - 20}{\sqrt{\frac{0.08 \times 0.92}{100}}}\right)$$
$$= \Phi(-1.44) - \Phi(-7.37)$$
$$= 0.0749 \ _\#$$

   (d) What is the probability of detecting the shift in part (c) by at least the fourth sample after the shift?

$$P\left(detect \ the \ shift \ at \ least \ 4th \ sample\right) = 1 - P\left(no \ detect \ at \ first \ 4 \ sample\right)$$
$$= 1 - \beta^4$$
$$= 1 - (0.0749)^4$$
$$= 0.99996 \ _\#$$

2. A process is being controlled with a fraction nonconforming control chart. The process average has been shown to be 0.07. Three-sigma control limits are used, and the procedure calls for taking daily samples of 400 items.

(a) Calculate the upper and lower control limits.

$$CL = \bar{p} = 0.07$$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.07 + 3\sqrt{\frac{0.07 \times 0.93}{400}} = 0.108$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.07 - 3\sqrt{\frac{0.07 \times 0.93}{400}} = 0.032$$

$$\therefore \text{ For the } p \text{ chart: } UCL = 0.108, \ CL = 0.07, \ LCL = 0.032 \ _\#$$

(b) If the process average should suddenly shift to 0.10, what is the probability that the shift would be detected on the first subsequent sample?

$$1 - \beta = P\left(detect \ on \ 1st \ sample\right)$$

$$= 1 - P(0.032 < \hat{p} < 0.108 \mid p = 0.1)$$

$$= 1 - \Phi\left(\frac{0.108 - 0.1}{\sqrt{\frac{0.1 \times 0.9}{400}}}\right) + \Phi\left(\frac{0.032 - 0.1}{\sqrt{\frac{0.1 \times 0.9}{400}}}\right)$$

$$= 1 - \Phi(0.53) + \Phi(-4.53)$$

$$= 0.298 \ _\#$$

(c) What is the probability that the shift in part (b) would be detect on the first or second sample taken after the shift?

$$P\left(detect \ on \ 1st \ or \ 2nd \ sample\right) = P\left(detect \ 1st \ sample\right) + P\left(detect \ 2nd \ sample\right)$$

$$= (1 - \beta) + \beta(1 - \beta)$$

$$= 0.298 + (1 - 0.298) \times 0.298$$

$$= 0.507 \ _\#$$

3. In designing a fraction nonconforming chart with center line at p = 0.20 and three-sigma control limits, what is the sample size required to yield a positive lower control limit? What is the value of n necessary to give a probability of 0.50 of detecting a shift in the process to 0.26?

- The sample size that required to yield LCL:

$$\because \text{ We want to achieve } LCL = \bar{p} - 3\sqrt{\frac{\bar{p}\,\bar{q}}{n}} > 0$$

$$\Rightarrow 0.2 - 3\sqrt{\frac{0.2 \times 0.8}{n}} > 0$$

$$\Rightarrow \left(\frac{0.2}{3}\right)^2 > \frac{0.16}{n}$$

$$\Rightarrow n > 0.16 \times \frac{9}{0.04} = 36$$

$\therefore$ **The sample size that required is greater than 36** #

- The value of n necessary to give a probability of 0.50 to detect a shift to 0.26:

$$\Rightarrow 0.26 - 0.2 = 3\sqrt{\frac{0.2 \times 0.8}{n}}$$

$$\Rightarrow \left(\frac{0.06}{3}\right)^2 = \frac{0.16}{n}$$

$$\Rightarrow n = 0.16 \times \frac{9}{0.0036} = 400$$

$\therefore$ **The value of n that necessary is 400** #