Assignment 4 : Control Charts for Variables

1. The Samples of n = 6 items each are taken from a process at regular intervals. A quality characteristic is measured, $\bar{x}$ and R values are calculated for each sample. After 50 samples, we have $\sum_{i=1}^{50} \bar{x}_i = 2000,\ \sum_{i=1}^{50} R_i = 200$. Assume that the quality characteristic is normally distributed.

   (a) Compute control limits for the $\bar{x}$ and R control charts.

   $\bar{x}$ charts:

$$UCL_{\bar{x}} = \bar{\bar{x}} + A_2\,\bar{R} = 40 + 0.483 \times 4 = 41.932 \ _{\#}$$
$$CL_{\bar{x}} = \bar{\bar{x}} = 40 \ _{\#}$$
$$LCL_{\bar{x}} = \bar{\bar{x}} - A_2\,\bar{R} = 40 - 0.483 \times 4 = 38.068 \ _{\#}$$

   R charts:

$$UCL_R = D_4\,\bar{R} = 2.004 \times 4 = 8.016 \ _{\#}$$
$$CL_R = \bar{R} = 4 \ _{\#}$$
$$LCL_R = D_3\,\bar{R} = 0 \times 4 = 0 \ _{\#}$$

   (b) All points on both control charts fall between the control limits computed in part (a). What are the natural tolerance limits of the process?

$$\because\ NTL = \mu_{\bar{x}} \pm 3\,\sigma_x$$
$$= \bar{x} \pm 3 \times \frac{\bar{R}}{d_2}$$
$$= 40 \pm 3 \times \frac{4}{2.534}$$
$$\therefore\ NTL = [35.264,\ 44.736] \ _{\#}$$

   (c) If the specification limits are 41 ± 5.0 , what are your conclusions regarding the ability of the process to produce items within these specifications?

$$\because\ C_p = \frac{USL - LSL}{6\,\sigma_x}$$
$$= \frac{46 - 36}{6 \times 1.5785}$$
$$= 1.056$$
$$\therefore\ C_p > 1 \text{ , The process is capable } _{\#}$$

(d) Assuming that if an item exceeds the upper specification limit it can be reworked, and if it is below the lower specification limit it must be scrapped, what percent scrap and rework is the process producing?

Let $X \equiv$ rework, $Y \equiv$ scrap

$$P(X > USL) = P(X > 46) = 1 - \Phi\left(\frac{46 - 40}{1.5785}\right) = 1 - \Phi(3.80) = 0.00007_{\#}$$

$$P(X < LSL) = P(X < 36) = \Phi\left(\frac{36 - 40}{1.5785}\right) = \Phi(-2.53) = 0.0057_{\#}$$

2. The following $\bar{x}$ and s charts based on n = 4 have shown statistical control:

$\bar{x}$ *chart*	s *chart*
UCL = 710	UCL = 18.08
CL = 700	CL = 7.979
LCL = 690	LCL = 0

(a) Estimate the process parameters μ and s.

$$\mu = \bar{\bar{x}} = 700_{\#}$$

$$s = \frac{\bar{s}}{c_4} = \frac{7.979}{0.9213} = 8.661_{\#}$$

(b) If the specifications are at 705 ± 15, and the process output is normally distributed, estimate the fraction nonconforming.

$$\begin{aligned} &\text{Let } X \equiv \text{nonconforming} \\ P(X) &= 1 - P(690 \le X \le 720) \\ &= 1 - P\left(\frac{690 - 700}{8.661} \le Z \le \frac{720 - 700}{8.661}\right) \\ &= 1 + \Phi(-1.15) - \Phi(2.31) \\ &= 0.1355_{\#} \end{aligned}$$

(c) For the $\bar{x}$ chart, find the probability of a type I error, assuming σ is constant.

$$\begin{aligned} \alpha &= 1 - P(690 \le \bar{x} \le 710) \\ &= 1 - P\left(\frac{690 - 700}{\frac{8.661}{\sqrt{4}}} \le Z \le \frac{710 - 700}{\frac{8.661}{\sqrt{4}}}\right) \\ &= 1 + \Phi(-2.31) - \Phi(2.31) \\ &= 0.0209_{\#} \end{aligned}$$

(d) Suppose the process mean shifts to 693 and the standard deviation simultaneously shifts to 12. Find the probability of detecting this shift on the $\bar{x}$ chart on the first subsequent sample.

$$\begin{aligned}
1-\beta &= 1-P\left(690 \leq \bar{x} \leq 710 \mid \mu_x = 693,\ \sigma_x = 12\right) \\
&= 1-P\left(\frac{690-693}{\frac{12}{\sqrt{4}}} \leq Z \leq \frac{710-693}{\frac{12}{\sqrt{4}}}\right) \\
&= 1+\Phi(-0.5)-\Phi(2.83) \\
&= 0.3109_{\ \#}
\end{aligned}$$

(e) For the shift of part (d), find the average run length.

$$\begin{aligned}
&\because \text{ The process is out of control} \\
&\therefore\ ARL_1 = \frac{1}{1-\beta} = \frac{1}{0.3109} = 3.2165_{\ \#}
\end{aligned}$$