Assignment 5 : Control Charts for Attributes

1. A process is controlled with a fraction nonconforming control chart with three-sigma limits, n = 100, UCL = 0.161, center line = 0.080, and LCL = 0.

   (a) Find the equivalent control chart for the number nonconforming.

$$
\begin{aligned}
CL &= n\bar{p} = 100 \times 0.08 = 8 \\
UCL &= n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 100 \times 0.08 + 3\sqrt{100 \times 0.08 \times 0.92} = 16.139 \\
LCL &= n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = 100 \times 0.08 - 3\sqrt{100 \times 0.08 \times 0.92} = -0.139 \\
&\therefore \text{ For the } np \text{ chart: } UCL = 16.139,\ CL = 8,\ LCL = 0 \ _{\#}
\end{aligned}
$$

   (b) Use the Poisson approximation to the binomial to find the probability of a type I error.

$$
\begin{aligned}
&X \equiv \text{the number of nonconforming} \\
&X \sim Bin(n = 100, p = 0.08) \ \approx \ X \sim Poi(\lambda = 8) \\
&\alpha = P\,(type\ I\ error) = 1 - P\,(X \leq 16.14) \\
&\quad = 1 - \sum_{i=0}^{16.14} \frac{e^{-8}\, 8^x}{x!} \\
&\quad = 1 - 0.9963 \\
&\quad = 0.0037 \ _{\#}
\end{aligned}
$$

   (c) Use the Normal approximation to find the probability of a type II error if the process fraction nonconforming shifts to 0.2.

$$
\begin{aligned}
&\beta = P\,(type\ II\ error) = P\,(0 < \hat{p} < 0.161 \mid p = 0.2) \\
&\quad = \Phi\left(\frac{0.161 - 0.2}{\sqrt{\frac{0.2 \times 0.8}{100}}}\right) - \Phi\left(\frac{0 - 0.2}{\sqrt{\frac{0.2 \times 0.8}{100}}}\right) \\
&\quad = \Phi(-0.98) - \Phi(-5.00) \\
&\quad = 0.1635 \ _{\#}
\end{aligned}
$$

   (d) What is the probability of detecting the shift in part (c) by at least the fourth sample after the shift?

$$
\begin{aligned}
P\,(detect\ the\ shift\ at\ least\ 4th\ sample) &= 1 - P\,(no\ detect\ at\ first\ 3\ sample) \\
&= 1 - \beta^3 \\
&= 1 - (0.1635)^3 \\
&= 0.9956 \ _{\#}
\end{aligned}
$$

2. A process is being controlled with a fraction nonconforming control chart. The process average has been shown to be 0.07. Three-sigma control limits are used, and the procedure calls for taking daily samples of 400 items.

   (a) Calculate the upper and lower control limits.

$$
\begin{aligned}
CL &= \bar{p} = 0.07 \\
UCL &= \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.07 + 3\sqrt{\frac{0.07 \times 0.93}{400}} = 0.108 \\
LCL &= \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.07 - 3\sqrt{\frac{0.07 \times 0.93}{400}} = 0.032
\end{aligned}
$$

$\therefore$ For the p chart: $UCL = 0.108,\ CL = 0.07,\ LCL = 0.032$ #

   (b) If the process average should suddenly shift to 0.10, what is the probability that the shift would be detected on the first subsequent sample?

$$
\begin{aligned}
1 - \beta &= P\left(detect\ on\ 1st\ sample\right) \\
&= 1 - P(0.032 < \hat{p} < 0.108 \mid p = 0.1) \\
&= 1 - \Phi\left(\frac{0.108 - 0.1}{\sqrt{\frac{0.1 \times 0.9}{400}}}\right) + \Phi\left(\frac{0.032 - 0.1}{\sqrt{\frac{0.1 \times 0.9}{400}}}\right) \\
&= 1 - \Phi(0.53) + \Phi(-4.53) \\
&= 0.298 \ _{\#}
\end{aligned}
$$

   (c) What is the probability that the shift in part (b) would be detect on the first or second sample taken after the shift?

$$
\begin{aligned}
P\left(detect\ on\ 1st\ or\ 2nd\ sample\right) &= P\left(detect\ 1st\ sample\right) + P\left(detect\ 2nd\ sample\right) \\
&= (1-\beta) + \beta(1-\beta) \\
&= 0.298 + (1 - 0.298) \times 0.298 \\
&= 0.507 \ _{\#}
\end{aligned}
$$

3. In designing a fraction nonconforming chart with center line at p = 0.20 and three-sigma control limits, what is the sample size required to yield a positive lower control limit? What is the value of n necessary to give a probability of 0.50 of detecting a shift in the process to 0.26?

- The sample size that required to yield LCL:

$$\because \text{ We want to achieve } LCL = \bar{p} - 3\sqrt{\frac{\bar{p}\,\bar{q}}{n}} > 0$$

$$\Rightarrow 0.2 - 3\sqrt{\frac{0.2 \times 0.8}{n}} > 0$$

$$\Rightarrow \left(\frac{0.2}{3}\right)^2 > \frac{0.16}{n}$$

$$\Rightarrow n > 0.16 \times \frac{9}{0.04} = 36$$

$\therefore$ **The sample size that required is greater than 36** #

- The value of n necessary to give a probability of 0.50 to detect a shift to 0.26:

$$\Rightarrow 0.26 - 0.2 = 3\sqrt{\frac{0.2 \times 0.8}{n}}$$

$$\Rightarrow \left(\frac{0.06}{3}\right)^2 = \frac{0.16}{n}$$

$$\Rightarrow n = 0.16 \times \frac{9}{0.0036} = 400$$

$\therefore$ **The value of n that necessary is 400** #