Assignment 3 : Inferences About Process Quality

1. The inside diameters of bearings used in an aircraft landing gear assembly are known to have a standard deviation of σ = 0.002 cm. A random sample of 15 bearings has an average inside diameter of 8.2535 cm.

   (a) Test the hypothesis that the mean inside bearing diameter is 8.25 cm. Use a two-sided alternative and α = 0.05.

$$
\begin{aligned}
&H_0 : \mu = 8.25 \\
&H_1 : \mu \neq 8.25 \\
&Z_0 = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{8.2535 - 8.25}{\frac{0.002}{\sqrt{15}}} = 6.7777 \\
&Z_{\frac{\alpha}{2}} = Z_{0.025} = 1.645 \\
&\because 1.645 < 6.7777 \\
&\therefore \textbf{reject } H_0
\end{aligned}
$$

   (b) Find the P-value for this test.

$$P - value = 1 - \Phi(6.7777) = 1 - 0.9998 = 0.0002$$

   (c) Construct a 95% two-sided confidence interval on the mean bearing diameter.

$$
\begin{aligned}
&\bar{x} - Z_{\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + Z_{\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}} \\
&8.2535 - 1.645 \times \frac{0.002}{\sqrt{15}} \leq \mu \leq 8.2535 + 1.645 \times \frac{0.002}{\sqrt{15}} \\
&8.2527 \leq \mu \leq 8.2543 \\
&\therefore \text{the 95\% C.I. is } [8.2527,\ 8.2543]
\end{aligned}
$$

2. The service life of a battery used in a cardiac pacemaker is assumed to be normally distributed. A random sample of ten batteries is subjected to an accelerated life test by running them continuously at an elevated temperature until failure, and the following lifetimes (in hours) are obtained: 25.5, 26.1, 26.8, 23.2, 24.2, 28.4, 25.0, 27.8, 27.3, and 25.7.

   (a) The manufacturer wants to be certain that the mean battery life exceeds 25hr. What conclusions can be drawn from these data. (α = 0.05).

$$n = 10,\ \bar{x} = 26,\ s = 1.6248$$
$$H_0 : \mu = 25$$
$$H_1 : \mu > 25$$

$$t_0 = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{26 - 25}{\frac{1.6248}{\sqrt{10}}} = 1.9463$$
$$t_{\frac{\alpha}{2},\ n-1} = t_{0.025,\ 9} = 2.262$$
$$\because 2.262 > 1.9463$$
$\therefore$ **fail to reject** H_0

(b) Construct a 90% two-sided confidence interval on mean life in the accelerated test.

$$\bar{x} - t_{\frac{\alpha}{2},\ n-1} \times \frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{\frac{\alpha}{2},\ n-1} \times \frac{s}{\sqrt{n}}$$
$$26 - 1.833 \times \frac{1.6248}{\sqrt{10}} \leq \mu \leq 26 + 1.833 \times \frac{1.6248}{\sqrt{10}}$$
$$25.0582 \leq \mu \leq 26.9418$$
$\therefore$ the 90% C.I. is $[25.0582,\ 26.9418]$

(c) Construct a 95% lower confidence interval on mean battery life. Why would the manufacturer be interested in a one-sided confidence interval?

$$\bar{x} - t_{\frac{\alpha}{2},\ n-1} \times \frac{s}{\sqrt{n}} \leq \mu$$
$$26 - 2.262 \times \frac{1.6248}{\sqrt{10}} \leq \mu$$
$$24.8378 \leq \mu$$
$\therefore$ the 95% lower C.I. is $[24.8378,\ \infty]$

that means only if $\mu < 24.8378$, then they need to inspect their process.