Assignment 3 : Inferences About Process Quality

1. The inside diameters of bearings used in an aircraft landing gear assembly are known to have a standard deviation of σ = 0.002 cm. A random sample of 15 bearings has an average inside diameter of 8.2535 cm.

   (a) Test the hypothesis that the mean inside bearing diameter is 8.25 cm. Use a two-sided alternative and α = 0.05.

$$H_0 : \mu = 8.25$$
$$H_1 : \mu \neq 8.25$$
$$Z_0 = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{8.2535 - 8.25}{\frac{0.002}{\sqrt{15}}} = 6.7777$$
$$Z_{\frac{\alpha}{2}} = Z_{0.025} = 1.645$$
$$\because 1.645 < 6.7777$$
$$\therefore \textbf{reject} \ \ H_0$$

   (b) Find the P-value for this test.

$$P - value = 1 - \Phi(6.7777) = 1 - 0.9998 = 0.0002 \approx 0$$

   (c) Construct a 95% two-sided confidence interval on the mean bearing diameter.

$$\bar{x} - Z_{\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + Z_{\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}}$$
$$8.2535 - 1.645 \times \frac{0.002}{\sqrt{15}} \leq \mu \leq 8.2535 + 1.645 \times \frac{0.002}{\sqrt{15}}$$
$$8.2527 \leq \mu \leq 8.2543$$
$$\therefore \text{the 95\% C.I. is } \ [8.2527, \ 8.2543]$$

2. The service life of a battery used in a cardiac pacemaker is assumed to be normally distributed. A random sample of ten batteries is subjected to an accelerated life test by running them continuously at an elevated temperature until failure, and the following lifetimes (in hours) are obtained: 25.5, 26.1, 26.8, 23.2, 24.2, 28.4, 25.0, 27.8, 27.3, and 25.7.

   (a) The manufacturer wants to be certain that the mean battery life exceeds 25hr. What conclusions can be drawn from these data. ($\alpha = 0.05$).

$$n = 10,\ \bar{x} = 26,\ s = 1.6248$$
$$H_0 : \mu = 25$$
$$H_1 : \mu > 25$$

$$t_0 = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{26 - 25}{\frac{1.6248}{\sqrt{10}}} = 1.9463$$
$$t_{\alpha,\ n-1} = t_{0.05,\ 9} = 1.833$$
$$\because 1.833 < 1.9463$$
$$\therefore \textbf{reject}\ \ H_0$$

   (b) Construct a 90% two-sided confidence interval on mean life in the accelerated test.

$$\bar{x} - t_{\frac{\alpha}{2},\ n-1} \times \frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{\frac{\alpha}{2},\ n-1} \times \frac{s}{\sqrt{n}}$$
$$26 - 1.833 \times \frac{1.6248}{\sqrt{10}} \leq \mu \leq 26 + 1.833 \times \frac{1.6248}{\sqrt{10}}$$
$$25.0582 \leq \mu \leq 26.9418$$
$$\therefore \text{the 90\% C.I. is } [25.0582,\ 26.9418]$$

   (c) Construct a 95% lower confidence interval on mean battery life. Why would the manufacturer be interested in a one-sided confidence interval?

$$\bar{x} - t_{\alpha,\ n-1} \times \frac{s}{\sqrt{n}} \leq \mu$$
$$26 - 1.833 \times \frac{1.6248}{\sqrt{10}} \leq \mu$$
$$25.0582 \leq \mu$$
$$\therefore \text{the 95\% lower C.I. is } [25.0582,\ \infty]$$

that means only if $\mu < 25.0582$, then they need to inspect their process.