

C H A P T E R 1 1

Analytic Geometry in Three Dimensions

Section 11.1	The Three-Dimensional Coordinate System	1029
Section 11.2	Vectors in Space	1037
Section 11.3	The Cross Product of Two Vectors	1042
Section 11.4	Lines and Planes in Space	1048
Review Exercises	1055
Problem Solving	1061
Practice Test	1068

CHAPTER 11

Analytic Geometry in Three Dimensions

Section 11.1 The Three-Dimensional Coordinate System

- You should be able to plot points in the three-dimensional coordinate system.
- The distance between the points x_1, y_1, z_1 and x_2, y_2, z_2 is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$
- The midpoint of the line segment joining the points x_1, y_1, z_1 and x_2, y_2, z_2 is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right).$$
- The equation of the sphere with center h, k, j and radius r is

$$(x - h)^2 + (y - k)^2 + (z - j)^2 = r^2.$$
- You should be able to find the trace of a surface in space.

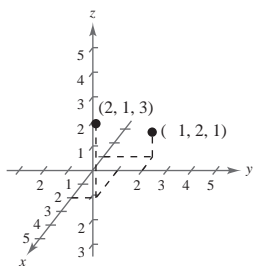
Vocabulary Check

- | | | |
|----------------------|--|------------|
| 1. three-dimensional | 2. xy -plane, xz -plane, yz -plane | 3. octants |
| 4. Distance Formula | 5. $\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}$ | 6. sphere |
| 7. surface, space | 8. trace | |

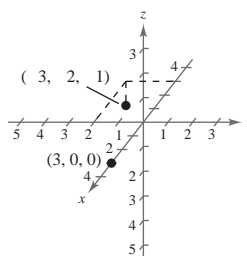
1. A $(1, 4, 3)$, B $(1, 3, 2)$, C $(3, 0, 2)$

2. A $(6, 2, 3)$, B $(2, 1, 2)$, C $(2, 3, 0)$

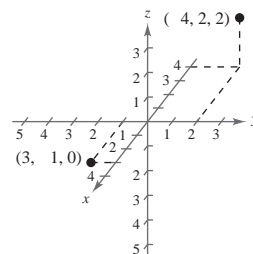
3.



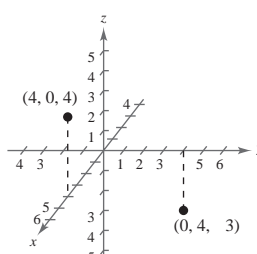
4.



5.



6.



7. $x = 3, y = 3, z = 4: 3, 3, 4$

8. $x = 6, y = 1, z = 1 \Rightarrow 6, 1, 1$

9. $y = z = 0, x = 10: 10, 0, 0$

10. $x = 0, y = 2, z = 8 \Rightarrow 0, 2, 8$

11. Octant IV

12. Octant VI

13. Octants I, II, III, IV
(above the xy -plane)

14. Octants III, IV, VII, or VIII

15. Octants II, IV, VI, VIII

16. Octants I, II, VII, or VIII

17. $d = \frac{\sqrt{5^2 + 0^2 + 2^2}}{\sqrt{25 + 4 + 36}} = \frac{\sqrt{29}}{\sqrt{65}}$
65 units

18. $d = \frac{\sqrt{7^2 + 1^2 + 0^2}}{\sqrt{36 + 16 + 52}} = \frac{\sqrt{50}}{\sqrt{104}} = \frac{5\sqrt{2}}{2\sqrt{13}}$

19. $d = \frac{\sqrt{7^2 + 3^2 + 4^2}}{\sqrt{4^2 + 2^2 + 3^2}} = \frac{\sqrt{74}}{\sqrt{29}} \approx 5.385$

20. $d = \frac{\sqrt{4^2 + 2^2 + 1^2}}{\sqrt{4^2 + 9 + 13}} = \frac{\sqrt{21}}{\sqrt{36}}$

21. $d = \frac{\sqrt{6^2 + 1^2 + 0^2}}{\sqrt{7^2 + 4^2 + 7^2}} = \frac{\sqrt{37}}{\sqrt{114}} \approx 10.677$

22. $d = \frac{\sqrt{1^2 + 2^2 + 1^2}}{\sqrt{9 + 16 + 25}} = \frac{\sqrt{6}}{\sqrt{50}} = \frac{1}{5}$

23. $d = \frac{\sqrt{1^2 + 0^2 + 0^2}}{\sqrt{1 + 9 + 100}} = \frac{1}{\sqrt{110}} \approx 10.488$

24. $d = \frac{\sqrt{2^2 + 0^2 + 4^2}}{\sqrt{4 + 100 + 9}} = \frac{\sqrt{20}}{\sqrt{113}}$

25. $d_1 = \frac{\sqrt{2^2 + 0^2 + 5^2}}{\sqrt{4 + 25 + 29}} = \frac{\sqrt{30}}{\sqrt{54}}$
 $d_2 = \frac{\sqrt{0^2 + 0^2 + 4^2}}{\sqrt{16 + 4 + 20}} = \frac{2}{\sqrt{40}} = \frac{\sqrt{10}}{2}$
 $d_3 = \frac{\sqrt{0^2 + 2^2 + 4^2}}{\sqrt{4 + 1 + 9}} = \frac{\sqrt{20}}{4} = \frac{\sqrt{5}}{2}$
 $d_1^2 + d_2^2 + d_3^2 = 29$

26. $d_1 = \frac{\sqrt{4^2 + 2^2 + 4^2}}{\sqrt{36 + 25 + 1}} = \frac{\sqrt{36}}{\sqrt{62}} = \frac{6}{\sqrt{62}}$
 $d_2 = \frac{\sqrt{4^2 + 2^2 + 4^2}}{\sqrt{16 + 1 + 1}} = \frac{\sqrt{36}}{\sqrt{18}} = \frac{2}{\sqrt{2}}$
 $d_3 = \frac{\sqrt{2^2 + 2^2 + 1^2}}{\sqrt{16 + 36 + 4}} = \frac{\sqrt{9}}{\sqrt{56}} = \frac{3}{\sqrt{56}}$
 $d_1^2 + d_2^2 + d_3^2 = 62$

$$\begin{array}{l}
 27. \ d_1 \quad \overline{2 \ 0^2 \ 2 \ 0^2 \ 1 \ 0^2} \quad \overline{4 \ 4 \ 1} \quad \overline{9} \ 3 \\
 \quad \quad \overline{2 \ 0^2 \ 4 \ 0^2 \ 4 \ 0^2} \quad \overline{4 \ 16 \ 16} \quad \overline{36} \ 6 \\
 \quad \quad \overline{2 \ 2^2 \ 4 \ 2^2 \ 4 \ 1^2} \quad \overline{36 \ 9} \quad \overline{45} \ 3 \ \overline{5} \\
 \quad \quad d_1^2 \ d_2^2 \ 9 \ 36 \ 45 \ d_3^2
 \end{array}$$

$$\begin{array}{l}
 28. \ d_1 \quad \overline{1 \ 1^2 \ 3 \ 0^2 \ 1 \ 1^2} \ 3 \\
 \quad \quad \overline{1 \ 1^2 \ 0 \ 0^2 \ 3 \ 1^2} \ 2 \\
 \quad \quad \overline{1 \ 1^2 \ 0 \ 3^2 \ 3 \ 1^2} \ \overline{13} \\
 \quad \quad d_3^2 \ 13 \ d_1^2 \ d_2^2
 \end{array}$$

$$\begin{array}{l}
 29. \ d_1 \quad \overline{5 \ 1^2 \ 1 \ 3^2 \ 2 \ 2^2} \quad \overline{16 \ 4 \ 16} \quad \overline{36} \ 6 \\
 \quad \quad \overline{5 \ 1^2 \ 1 \ 1^2 \ 2 \ 2^2} \quad \overline{36 \ 4} \quad \overline{40} \ 2 \ \overline{10} \\
 \quad \quad \overline{1 \ 1^2 \ 1 \ 3^2 \ 2 \ 2^2} \quad \overline{4 \ 16 \ 16} \quad \overline{36} \ 6 \\
 \quad \quad d_1 \ d_3 \text{ Isosceles triangle}
 \end{array}$$

$$\begin{array}{l}
 30. \ d_1 \quad \overline{7 \ 5^2 \ 1 \ 3^2 \ 3 \ 4^2} \quad \overline{4 \ 4 \ 1} \quad \overline{9} \ 3 \\
 \quad \quad \overline{3 \ 7^2 \ 5 \ 1^2 \ 3 \ 3^2} \quad \overline{16 \ 16} \quad \overline{32} \ 4 \ \overline{2} \\
 \quad \quad \overline{3 \ 5^2 \ 5 \ 3^2 \ 3 \ 4^2} \quad \overline{4 \ 4 \ 1} \quad \overline{9} \ 3 \\
 \quad \quad d_1 \ d_3 \ 3. \text{ Isosceles triangle}
 \end{array}$$

$$31. \ \frac{3}{2}, \frac{0}{2}, \frac{4}{2}, \frac{0}{2} \quad \frac{3}{2}, \ 1, 2 \qquad 32. \text{ Midpoint: } \frac{1}{2}, \frac{2}{2}, \frac{5}{2}, \frac{2}{2}, \frac{1}{2}, \frac{2}{2} \quad \frac{3}{2}, \frac{7}{2}, \frac{1}{2}$$

$$33. \text{ Midpoint: } \frac{3}{2}, \frac{3}{2}, \frac{6}{2}, \frac{4}{2}, \frac{10}{2}, \frac{4}{2} \quad 0, \ 1, 7 \qquad 34. \text{ Midpoint: } \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{3}{2}, \frac{1}{2} \quad 1, 6, \ 2$$

$$35. \text{ Midpoint: } \frac{6}{2}, \frac{4}{2}, \frac{2}{2}, \frac{2}{2}, \frac{5}{2}, \frac{6}{2} \quad 1, 0, \frac{11}{2} \qquad 36. \text{ Midpoint: } \frac{3}{2}, \frac{6}{2}, \frac{5}{2}, \frac{4}{2}, \frac{5}{2}, \frac{8}{2} \quad \frac{9}{2}, \frac{9}{2}, \frac{13}{2}$$

$$37. \text{ Midpoint: } \frac{2}{2}, \frac{7}{2}, \frac{8}{2}, \frac{4}{2}, \frac{10}{2}, \frac{2}{2} \quad \frac{5}{2}, 2, 6 \qquad 38. \text{ Midpoint: } \frac{9}{2}, \frac{9}{2}, \frac{5}{2}, \frac{2}{2}, \frac{1}{2}, \frac{4}{2} \quad 9, \ \frac{7}{2}, \ \frac{3}{2}$$

$$39. \ x \ 3^2 \ y \ 2^2 \ z \ 4^2 \ 16 \qquad 40. \ x \ 3^2 \ y \ 4^2 \ z \ 3^2 \ 4$$

$$\begin{array}{l}
 41. \ x \ 0^2 \ y \ 4^2 \ z \ 3^2 \ 3^2 \\
 \quad \quad x^2 \ y \ 4^2 \ z \ 3^2 \ 9 \\
 42. \ x \ 2^2 \ y \ 1^2 \ z \ 8^2 \ 36
 \end{array}$$

$$\begin{array}{l}
 43. \text{ Radius } \frac{\text{Diameter}}{2} \ 5 \\
 \quad \quad x \ 3^2 \ y \ 7^2 \ z \ 5^2 \ 5^2 \ 25
 \end{array}$$

$$44. \text{ Radius } \frac{\text{Diameter}}{2} \ 4: \ x \ 0^2 \ y \ 5^2 \ z \ 9^2 \ 4^2 \ 16$$

45. Center: $\frac{3}{2}, \frac{0}{2}, \frac{0}{2}, \frac{6}{2}$ $\frac{3}{2}, 0, 3$

Radius: $\sqrt{3^2 + \frac{3}{2}^2 + 0^2 + 0^2} = \sqrt{\frac{9}{4} + 9} = \sqrt{\frac{45}{4}}$

Sphere: $x - \frac{3}{2}^2 + y - 0^2 + z - 3^2 = \frac{45}{4}$

46. Center: $\frac{2}{2}, \frac{1}{2}, \frac{2}{2}, \frac{4}{2}, \frac{2}{2}, \frac{6}{2}$ $\frac{1}{2}, 1, 4$

Radius: $\sqrt{2^2 + \frac{1}{2}^2 + 2^2 + 1^2 + 2^2 + 4^2} = \sqrt{\frac{9}{4} + 9 + 4} = \sqrt{\frac{61}{4}}$

Sphere: $x - \frac{1}{2}^2 + y - 1^2 + z - 4^2 = \frac{61}{4}$

47. $x^2 - 5x + \frac{25}{4} + y^2 - z^2 - \frac{25}{4}$
 $x - \frac{5}{2}^2 + y^2 - z^2 - \frac{25}{4}$

Center: $\frac{5}{2}, 0, 0$

Radius: $\frac{5}{2}$

48. $x^2 - y^2 - 8y + 16 + z^2 - 16$
 $x^2 - y - 4^2 + z^2 - 16$

Center: $0, 4, 0$

Radius: 4

49. $x^2 - 4x + 4 + y^2 - 2y + 1 + z^2 - 6z + 9$
 $x - 2^2 + y - 1^2 + z - 3^2 = 4$

Center: $2, 1, 3$

Radius: 2

50. $x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 - 9 + 9 + 4$
 $x - 3^2 + y - 2^2 + z^2 - 4$

Center: $3, 2, 0$

Radius: 2

51. $x^2 - 4x + 4 + y^2 - z^2 - 8z + 16 + 19 + 4 + 16$
 $x - 2^2 + y^2 - z - 4^2 = 1$

Center: $2, 0, 4$

Radius: 1

52. $x^2 - y^2 - 8y + 16 + z^2 - 6z + 9 + 13 + 16 + 9$
 $x^2 - y - 4^2 + z - 3^2 = 12$

Center: $0, 4, 3$

Radius: $\sqrt{12 + 2 + 3} = \sqrt{17}$

$$\begin{array}{cccccccccccc}
 53. & & x^2 & y^2 & z^2 & 2x & \frac{2}{3}y & 8z & \frac{73}{9} & & & \\
 & x^2 & 2x & 1 & y^2 & \frac{2}{3}y & \frac{1}{9} & z^2 & 8z & 16 & \frac{73}{9} & 1 & \frac{1}{9} & 16 \\
 & & & & x & 1^2 & y & \frac{1}{3}^2 & z & 4^2 & 9 & & &
 \end{array}$$

 Center: $1, \frac{1}{3}, 4$

Radius: 3

$$\begin{array}{cccccccccccc}
 54. & x^2 & y^2 & z^2 & x & 3y & 2z & \frac{5}{2} & & & & \\
 & x^2 & x & \frac{1}{4} & y^2 & 3y & \frac{9}{4} & z^2 & 2z & 1 & \frac{5}{2} & \frac{1}{4} & \frac{9}{4} & 1 \\
 & x & \frac{1}{2}^2 & y & \frac{3}{2}^2 & z & 1^2 & 1 & & & & & &
 \end{array}$$

 Center: $\frac{1}{2}, \frac{3}{2}, 1$

Radius: 1

$$\begin{array}{cccccccc}
 55. & 9x^2 & 6x & 9y^2 & 18y & 9z^2 & 1 & \\
 & x^2 & \frac{2}{3}x & \frac{1}{9} & y^2 & 2y & 1 & z^2 & \frac{1}{9} & \frac{1}{9} & 1 \\
 & x & \frac{1}{3}^2 & y & 1^2 & z^2 & 1 & & & &
 \end{array}$$

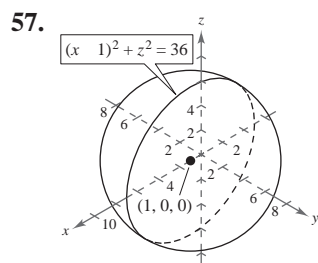
 Center: $\frac{1}{3}, 1, 0$

Radius: 1

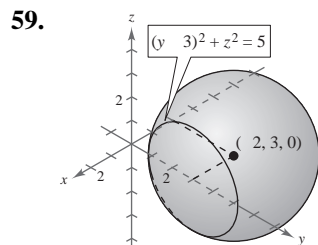
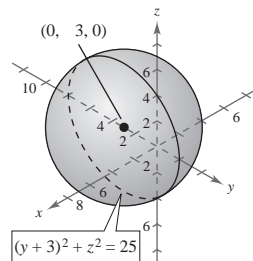
$$\begin{array}{cccccccc}
 56. & x^2 & x & \frac{1}{4} & y^2 & 8y & 16 & z^2 & 2z & 1 & \frac{33}{4} & \frac{1}{4} & 16 & 1 \\
 & x & \frac{1}{2}^2 & y & 4^2 & z & 1^2 & 9 & & & & & &
 \end{array}$$

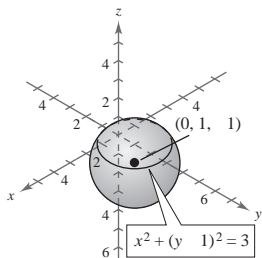
 Center: $\frac{1}{2}, 4, 1$

Radius: 3

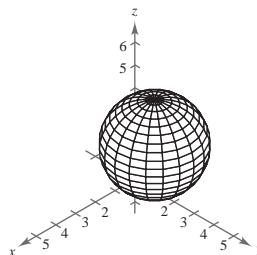


58. yz trace $x = 0$: $y^2 + z^2 = 25$ Circle

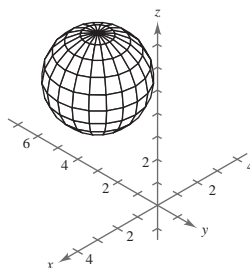


60. xy trace $z = 0 : x^2 + y^2 = 3$ Circle

61.

62. $x^2 + y^2 + 6y + z^2 + 8z = 16$ 21 16

$$\begin{array}{rcl} x^2 & y^2 & 6y \\ z_1 & 4 & \frac{5}{x^2} \frac{y^2}{6y} \\ z_2 & 4 & \frac{5}{x^2} \frac{y^2}{6y} \end{array}$$



63. The length of each side is 3.

Thus, $x, y, z = 3, 3, 3$.64. $x = 4, y = 4, z = 8$ 4, 4, 8

$$65. d = 165 \Rightarrow r = \frac{165}{2} = 82.5$$

$$x^2 + y^2 + z^2 = \frac{165^2}{2}$$

66. (a) $x^2 + y^2 + z^2 = 3963^2$.

(b) Assume the north and south poles are on the z -axis. Lines of longitude that run north-south are traces of planes containing the z -axis. These shapes are circles of radius 3963 miles.

(c) Latitudes are traces of planes perpendicular to the z -axis. These shapes are circles.

(d) The prime meridian is a trace of a plane containing the z -axis. It is a semi-circular arc running from pole to pole.

(e) The equator is the trace of the plane containing the x - and y -axes.

67. False. x is the directed distance from the yz -plane to P .

68. False. The trace could be a single point, or empty.

69. In the xy -plane, the z -coordinate is 0.In the xz -plane, the y -coordinate is 0.In the yz -plane, the x -coordinate is 0.

70. It is a plane.

71. The trace is a circle, or a single point.

72. The trace will be a line in the xy -plane (unless the plane is the xy -plane).

$$73. x_m = \frac{x_2 - x_1}{2} \Rightarrow x_2 = 2x_m + x_1$$

Similarly for y_2 and z_2 ,

$$x_2, y_2, z_2 = 2x_m + x_1, 2y_m + y_1, 2z_m + z_1.$$

$$74. \begin{array}{cccccc} x_2 & 2x_m & x_1 & 2.5 & 3 & 7 \\ y_2 & 2y_m & y_1 & 2.8 & 0 & 16 \\ z_2 & 2z_m & z_1 & 2.7 & 2 & 12 \\ & & & 7, 16, 12 \end{array}$$

$$75. \quad v^2 = 3v \quad \frac{9}{4} = 2 \quad \frac{9}{4}$$

$$v = \frac{3}{2} = \frac{17}{4}$$

$$v = \frac{3}{2} = \frac{17}{2}$$

$$v = \frac{3}{2} = \frac{17}{2}$$

$$76. \quad z^2 = 7z \quad \frac{49}{4} = 19 \quad \frac{49}{4}$$

$$z = \frac{7}{2} = \frac{125}{4}$$

$$z = \frac{7}{2} = \frac{5}{2}$$

$$z = \frac{7}{2} = \frac{5}{2}$$

$$77. \quad x^2 = 5x \quad \frac{25}{4} = 5 \quad \frac{25}{4}$$

$$x = \frac{5}{2} = \frac{5}{4}$$

$$x = \frac{5}{2} = \frac{5}{2}$$

$$x = \frac{5}{2} = \frac{5}{2}$$

$$78. \quad x^2 = 3x \quad \frac{9}{4} = 1 \quad \frac{9}{4}$$

$$x = \frac{3}{2} = \frac{13}{4}$$

$$x = \frac{3}{2} = \frac{13}{2}$$

$$x = \frac{3}{2} = \frac{13}{2}$$

$$79. \quad 4y^2 = 4y \quad 9 = 9$$

$$y^2 = y \quad \frac{1}{4} = \frac{9}{4} \quad \frac{1}{4}$$

$$y = \frac{1}{2} = \frac{10}{4}$$

$$y = \frac{1}{2} = \frac{10}{2}$$

$$y = \frac{1}{2} = \frac{10}{2}$$

$$80. \quad x^2 = \frac{5}{2}x \quad \frac{25}{16} = 4 \quad \frac{25}{16}$$

$$x = \frac{5}{4} = \frac{89}{16}$$

$$x = \frac{5}{4} = \frac{89}{4}$$

$$x = \frac{5}{4} = \frac{89}{4}$$

$$81. \quad \mathbf{v} = 3\mathbf{i} - 3\mathbf{j}, \text{ Quadrant IV}$$

$$\mathbf{v} = \frac{3^2}{3^2}$$

$$\frac{18}{18}$$

$$3 = \frac{18}{2}$$

$$\tan \frac{3}{3} = 1 \Rightarrow$$

$$45^\circ \text{ or } 315^\circ$$

$$82. \quad \mathbf{v} = 1, 2 \quad \text{Quadrant II}$$

$$\mathbf{v} = \frac{1^2}{2^2} = \frac{1}{5}$$

$$\tan \frac{2}{1} \Rightarrow 116.6^\circ$$

$$83. \quad \mathbf{v} = 4\mathbf{i} - 5\mathbf{j}, \text{ Quadrant I}$$

$$\mathbf{v} = \frac{16}{25} = \frac{41}{41}$$

$$\tan \frac{5}{4} \Rightarrow 51.34^\circ$$

$$84. \quad \mathbf{v} = 10, -7 \quad \text{Quadrant IV}$$

$$\mathbf{v} = \frac{100}{49} = \frac{149}{149}$$

$$\tan \frac{7}{10} \Rightarrow 325.0^\circ$$

$$85. \quad \mathbf{u} = 4, 1 \quad \mathbf{v} = 3, 5$$

$$4 = 3 \quad 1 = 5$$

$$7$$

$$86. \quad \mathbf{u} = 1, 0 \quad \mathbf{v} = 2, 6$$

$$2 = 0$$

$$2$$

$$87. \quad a_0 = 1, a_n = a_{n-1} + n^2$$

$$a_1 = 1 + 1^2 = 2$$

$$a_2 = 2 + 2^2 = 6$$

$$a_3 = 6 + 3^2 = 15$$

$$a_4 = 15 + 4^2 = 31$$

$$1 \quad 2 \quad 6 \quad 15 \quad 31$$

$$\text{First differences:} \quad 1 \quad 4 \quad 9 \quad 16$$

$$\text{Second differences:} \quad 3 \quad 5 \quad 7$$

Neither model

88. $a_0 = 0, a_n = a_{n-1} + 1$

$$a_1 = 0 + 1 = 1$$

$$a_2 = 1 + 1 = 2$$

$$a_3 = 3$$

$$a_4 = 4$$

$$\begin{array}{cccccc} & & 0 & & 1 & & 2 & & 3 & & 4 \end{array}$$

$$\text{First difference} \quad \quad \quad 1 \quad \quad \quad 1 \quad \quad \quad 1 \quad \quad \quad 1$$

$$\text{Second difference} \quad \quad \quad 0 \quad \quad \quad 0 \quad \quad \quad 0$$

Linear model

89. $a_1 = 1, a_n = a_{n-1} + 3$

$$a_2 = 1 + 3 = 4$$

$$a_3 = 4 + 3 = 7$$

$$a_4 = 7 + 3 = 10$$

$$a_5 = 10 + 3 = 13$$

$$\begin{array}{ccccccccc} & & 1 & & 4 & & 7 & & 10 & & 13 \end{array}$$

$$\text{First differences:} \quad \quad \quad 3 \quad \quad \quad 3 \quad \quad \quad 3 \quad \quad \quad 3$$

$$\text{Second differences:} \quad \quad \quad 0 \quad \quad \quad 0 \quad \quad \quad 0$$

Linear model

90. $a_1 = 4, a_n = a_{n-1} + 2n$

$$a_2 = 4 + 2 \cdot 2 = 8$$

$$a_3 = 8 + 2 \cdot 3 = 14$$

$$a_4 = 14 + 2 \cdot 4 = 22$$

$$a_5 = 22 + 2 \cdot 5 = 32$$

$$\begin{array}{ccccccccc} & & 4 & & 8 & & 14 & & 22 & & 32 \end{array}$$

$$\text{First difference} \quad \quad \quad 4 \quad \quad \quad 6 \quad \quad \quad 8 \quad \quad \quad 10$$

$$\text{Second difference} \quad \quad \quad 2 \quad \quad \quad 2 \quad \quad \quad 2$$

Quadratic model

91. $x = 5^2, y = 1^2, 49$

92. $x = 3^2, y = 6^2, 81$

93. $y = 1^2 = 4p, x = 4, p = 3$

$$y = 1^2 = 4, 3 \cdot x = 4$$

$$y = 1^2 = 12, x = 4$$

94. $x = h^2 = 4p, y = k, p = 5, h, k = 2, 5$

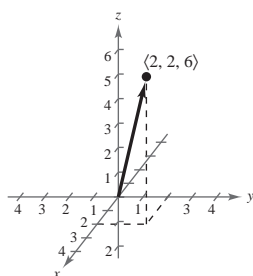
$$x = 2^2 = 4, 5 \cdot y = 5$$

$$x = 2^2 = 20, y = 5$$

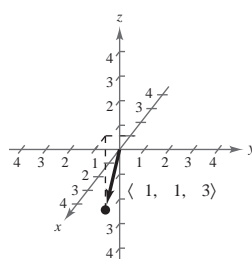
95. $a = 3, b = 2$, center: $(3, 3)$, horizontal major axis

$$\frac{x-3}{9} - \frac{y-3}{4} = 1$$

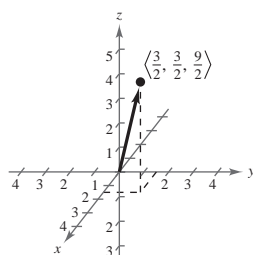
5. (a)



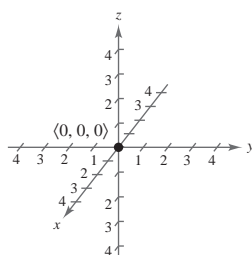
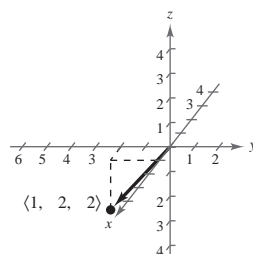
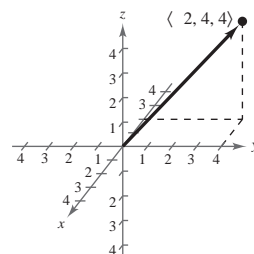
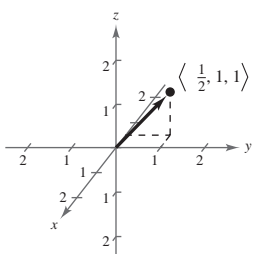
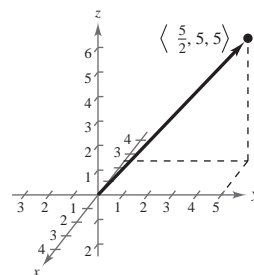
(b)



(c)



(d)

6. $\mathbf{v} = \langle 1, 2, 2 \rangle$ (a) $\mathbf{v} = \langle 1, 2, 2 \rangle$ (b) $2\mathbf{v} = \langle 2, 4, 4 \rangle$ (c) $\frac{1}{2}\mathbf{v} = \langle \frac{1}{2}, 1, 1 \rangle$ (d) $\frac{5}{2}\mathbf{v} = \langle \frac{5}{2}, 5, 5 \rangle$ 7. $\mathbf{z} = \mathbf{u} + 2\mathbf{v} = \langle 1, 3, 2 \rangle + 2\langle 1, 2, 2 \rangle = \langle 3, 7, 6 \rangle$ 8. $\mathbf{z} = 7\mathbf{u} + \frac{1}{5}\mathbf{v} = 7\langle 1, 3, 2 \rangle + \frac{1}{5}\langle 1, 2, 2 \rangle = \langle 7, 19, 13 \rangle$ 9. $2\mathbf{z} - 4\mathbf{u} = \mathbf{w} \Rightarrow \mathbf{z} = \frac{1}{2}(4\mathbf{u} + \mathbf{w}) = \frac{1}{2}(4\langle 1, 3, 2 \rangle + \langle 5, 0, 5 \rangle) = \langle \frac{1}{2}, 6, \frac{3}{2} \rangle$ 10. $\mathbf{z} = \mathbf{u} + \mathbf{v} = \langle 1, 3, 2 \rangle + \langle 1, 2, 2 \rangle = \langle 2, 5, 4 \rangle$

$$11. \mathbf{v} = \frac{7, 8, 7}{\sqrt{49 + 64 + 49}} = \frac{7, 8, 7}{\sqrt{162}} = \frac{7, 8, 7}{9\sqrt{2}}$$

$$12. \mathbf{v} = \frac{2^2, 0^2, 5^2}{\sqrt{4 + 25 + 29}}$$

$$13. \mathbf{v} = \frac{4^2, 3^2, 7^2}{\sqrt{16 + 9 + 49}} = \frac{4^2, 3^2, 7^2}{\sqrt{74}}$$

$$14. \mathbf{v} = \frac{2^2, 1^2, 6^2}{\sqrt{41}}$$

$$15. \mathbf{v} = \frac{1, 1, 0}{\sqrt{0 + 3^2 + 5^2}} = \frac{3, 1, 4}{\sqrt{34}}$$

$$16. \mathbf{v} = \frac{1, 0, 2}{\sqrt{1 + 9 + 4}} = \frac{1, 2, 0}{\sqrt{14}}$$

$$17. (a) \frac{\mathbf{u}}{\mathbf{u}} = \frac{8, 3, 1}{\sqrt{74}} = \frac{1}{\sqrt{74}} 8\mathbf{i} + 3\mathbf{j} + \mathbf{k} = \frac{\sqrt{74}}{74} 8, 3, 1$$

$$(b) -\frac{1}{\sqrt{74}} 8\mathbf{i} + 3\mathbf{j} + \mathbf{k} = -\frac{\sqrt{74}}{74} 8, 3, 1$$

$$18. (a) \frac{\mathbf{u}}{\mathbf{u}} = \frac{3, 5, 10}{\sqrt{134}} = \frac{1}{\sqrt{134}} 3\mathbf{i} + 5\mathbf{j} + 10\mathbf{k}$$

$$(b) -\frac{1}{\sqrt{134}} 3\mathbf{i} + 5\mathbf{j} + 10\mathbf{k}$$

$$19. \mathbf{u} \cdot \mathbf{v} = 4, 4, 1 \cdot 2, 5, 8 = 8 + 20 + 8 = 4$$

$$20. \mathbf{u} \cdot \mathbf{v} = 3, 4 \cdot 1, 10, 6, 1 = 28$$

$$21. \mathbf{u} \cdot \mathbf{v} = 2, 5, 3 \cdot 9, 3, 1 = 18 + 15 + 3 = 0$$

$$22. \mathbf{u} \cdot \mathbf{v} = 0, 6 \cdot 3, 4 \cdot 6, 2, 0 = 0$$

$$23. \cos \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} = \frac{8}{\sqrt{8} \sqrt{25}} \Rightarrow 124.45$$

$$24. \cos \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} = \frac{5}{\sqrt{10} \sqrt{6}} \Rightarrow 49.80$$

$$25. \cos \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} = \frac{120}{\sqrt{1700} \sqrt{73}} \Rightarrow 109.92$$

$$26. \cos \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} = \frac{100}{\sqrt{464} \sqrt{125}} \Rightarrow 65.47$$

$$27. \frac{3}{2} 8, 4, 10 \cdot 12, 6, 15 \Rightarrow \text{parallel}$$

$$28. \mathbf{u} \cdot \mathbf{v} = 2, 3, 5 \cdot 10, 0 \text{ and } \mathbf{u} = c\mathbf{v} \Rightarrow \text{neither}$$

$$29. \mathbf{u} \cdot \mathbf{v} = 3, 5, 2 \cdot 0 \Rightarrow \text{orthogonal}$$

$$30. 8\mathbf{u} = 8, 1, \frac{1}{2}, 1 \cdot 8, 4, 8 = \mathbf{v} \Rightarrow \text{parallel}$$

$$31. \mathbf{v} = 7, 5, 3, 4, 1, 1, 2, 1, 2$$

$$\mathbf{u} = 4, 7, 5, 3, 3, 1, 3, 2, 4$$

Since \mathbf{u} and \mathbf{v} are not parallel, the points are not collinear.

$$32. \mathbf{v} = 4, 2, 8, 7, 1, 4, 2, 1, 3$$

$$\mathbf{u} = 0, 4, 6, 8, 7, 1, 4, 2, 6$$

Since $\mathbf{u} = 2\mathbf{v}$, the points are collinear.

$$33. \mathbf{v} = 1, 1, 2, 3, 5, 2, 2, 1, 3$$

$$\mathbf{u} = 3, 1, 4, 2, 1, 5, 4, 2, 6$$

Since $\mathbf{u} = 2\mathbf{v}$, the points are collinear.

$$34. \mathbf{v} = 1, 0, 5, 4, 6, 4, 1, 1, 2$$

$$\mathbf{u} = 2, 1, 6, 5, 7, 6, 1, 1, 1$$

Since \mathbf{u} and \mathbf{v} are not parallel, the points are not collinear.

$$35. \mathbf{v} = 2, 4, 7 \quad q_1 = 1, q_2 = 5, q_3 = 0 \Rightarrow$$

$$\begin{aligned} 2 &= q_1 = 1 \Rightarrow q_1 = 3 \\ 4 &= q_2 = 5 \Rightarrow q_2 = 1 \Rightarrow \\ 7 &= q_3 = 7 \end{aligned}$$

Terminal point is $(3, 1, 7)$.

$$36. (4, 1, 1) - (x, 6, y) = (4, z, 3) \Rightarrow x, y, z = 10, 5, 2$$

$$37. \mathbf{v} = 4, \frac{3}{2}, \frac{1}{4} \quad q_1 = 2, q_2 = 1, q_3 = \frac{3}{2}$$

$$4 = q_1 = 2 \Rightarrow q_1 = 6$$

$$\frac{3}{2} = q_2 = 1 \Rightarrow q_2 = \frac{5}{2}$$

$$\frac{1}{4} = q_3 = \frac{3}{2} \Rightarrow q_3 = \frac{7}{4}$$

Terminal point: $(6, \frac{5}{2}, \frac{7}{4})$

$$38. (\frac{5}{2}, \frac{1}{2}, 4) - (x, 3, y) = (2, z, \frac{1}{2}) \Rightarrow x, y, z = (\frac{11}{2}, \frac{3}{2}, \frac{7}{2})$$

$$\begin{aligned} 39. c\mathbf{u} + c\mathbf{i} + 2c\mathbf{j} + 3c\mathbf{k} \\ c\mathbf{u} + \frac{c^2}{4c^2} + \frac{4c^2}{9c^2} + \frac{9c^2}{c^2} = c + \frac{1}{4} + 3 \Rightarrow \\ c = \frac{3}{\frac{1}{4}} = \frac{3}{\frac{1}{4}} = 12 \end{aligned}$$

$$\begin{aligned} 40. c\mathbf{u} + c\mathbf{u} + c\mathbf{u} + \frac{4}{4} + \frac{4}{4} + \frac{16}{16} = c + \frac{24}{24} = 12 \\ \Rightarrow c = \frac{12}{\frac{24}{24}} = \frac{6}{6} = 6 \Rightarrow c = 6 \end{aligned}$$

$$41. \mathbf{v} = q_1, q_2, q_3$$

Since \mathbf{v} lies in the yz -plane, $q_1 = 0$. Since \mathbf{v} makes an angle of 45° with $q_2 = q_3$. Finally, $|\mathbf{v}| = 4$ implies that $q_2^2 + q_3^2 = 16$. Thus, $q_2 = q_3 = \frac{2}{\sqrt{2}} = \sqrt{2}$ and $\mathbf{v} = (0, \sqrt{2}, \sqrt{2})$, or $q_2 = -\sqrt{2}$ and $q_3 = -\sqrt{2}$ and $\mathbf{v} = (0, -\sqrt{2}, -\sqrt{2})$.

$$42. \mathbf{v} \text{ lies in } xz\text{-plane} \Rightarrow y = 0.$$

$$\mathbf{v} = 10 \sin 60^\circ, 0, \cos 60^\circ = 5\sqrt{3}, 0, 5, \text{ or}$$

$$\mathbf{v} = 10 \sin 60^\circ, 0, \cos 60^\circ = 5\sqrt{3}, 0, 5$$

$$43. \overrightarrow{AB} = 0, 70, 115 \cdot F_1 = C_1 0, 70, 115$$

$$\overrightarrow{AC} = 60, 0, 115 \cdot F_2 = C_2 60, 0, 115$$

$$\overrightarrow{AD} = 45, -65, 115 \cdot F_3 = C_3 45, -65, 115$$

$$F_1 = F_2 = F_3 = 0, 0, 500. \text{ Thus}$$

$$60C_2 = 45C_3 = 0$$

$$70C_1 = 65C_3 = 0$$

$$115C_1 = 115C_2 = 115C_3 = 500$$

$$\text{Solving this system yields } C_1 = \frac{104}{69}, C_2 = \frac{28}{23}, C_3 = \frac{112}{69}.$$

Thus,

$$\mathbf{F}_1 = 202.919 \text{ N}$$

$$\mathbf{F}_2 = 157.909 \text{ N}$$

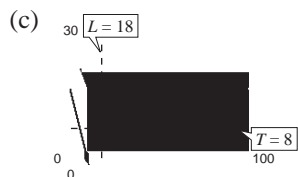
$$\mathbf{F}_3 = 226.521 \text{ N}$$

$$44. (a) \sin^{-1} \frac{18}{L}$$

$$T = \frac{8}{\cos \sin^{-1} \frac{18}{L}} = \frac{8}{\cos \sin^{-1} \frac{18}{L}} = \frac{8}{\frac{18^2}{L^2 - 18^2}} = \frac{8L}{L^2 - 18^2}$$

Domain: $L > 18$

(b) L	20	25	30	35	40	45	50
T	18.4	11.5	10	9.3	9.0	8.7	8.6



Vertical asymptote: $L = 18$

Horizontal asymptote: $T = 8$

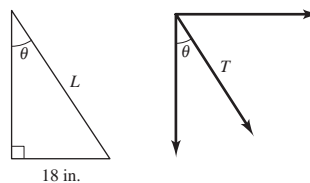
The minimum tension in each cable is 8 pounds and the minimum cable length is 18 inches.

$$(d) 10 = \frac{8}{\cos \sin^{-1} \frac{18}{L}} \Rightarrow \cos \sin^{-1} \frac{18}{L} = \frac{8}{10} = \frac{4}{5}$$

$$\sin^{-1} \frac{18}{L} = \cos^{-1} \frac{4}{5}$$

$$\frac{18}{L} = \sin \cos^{-1} \frac{4}{5} = \frac{3}{5}$$

$$L = \frac{90}{3} = 30 \text{ inches}$$



45. True. $\cos^{-1} 0 \Rightarrow 90^\circ$ 46. True

47. If $\mathbf{u} \cdot \mathbf{v} < 0$, then $\cos \theta < 0$ and the angle between \mathbf{u} and \mathbf{v} is obtuse, $180^\circ > \theta > 90^\circ$.

48. Let $\mathbf{v} = v_1, v_2, v_3$ and $\mathbf{u} = u_1, u_2, u_3$.

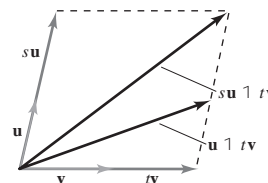
Then $t\mathbf{v} = tv_1, tv_2, tv_3$

$$\mathbf{u} + t\mathbf{v} = u_1 + tv_1, u_2 + tv_2, u_3 + tv_3$$

$$\text{and } s\mathbf{u} + t\mathbf{v} = su_1 + tv_1, su_2 + tv_2, su_3 + tv_3$$

The endpoints of these three vectors are collinear, as indicated in the figure.

So, the figure is a line.



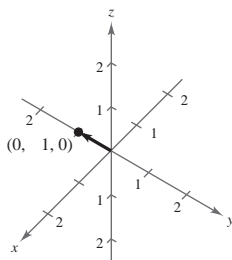
49. (a) $x = t, y = 3t - 2$

(b) $x = t - 1, y = 3t - 1, z = 2 - 3t - 1$

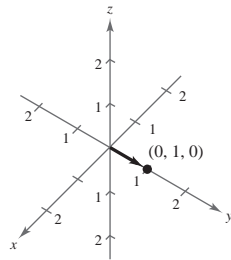
50. (a) $x = t, y = \frac{2}{t}$

(b) $x = t - 1, y = \frac{2}{t - 1}$

$$\begin{array}{rcccl}
 & \mathbf{i} & \mathbf{j} & \mathbf{k} & \\
 3. \mathbf{i} & \mathbf{k} & 1 & 0 & 0 \\
 & & 0 & 0 & 1
 \end{array} \quad \mathbf{j}$$



$$\begin{array}{rcccl}
 & \mathbf{i} & \mathbf{j} & \mathbf{k} & \\
 4. \mathbf{k} & \mathbf{i} & 0 & 0 & 1 \\
 & & 1 & 0 & 0
 \end{array} \quad \mathbf{j}$$



$$\begin{array}{rcccl}
 & \mathbf{i} & \mathbf{j} & \mathbf{k} & \\
 5. \mathbf{u} & \mathbf{v} & 3 & 2 & 5 \\
 & & 0 & 1 & 1
 \end{array} \quad 3, 3, 3$$

$$\begin{array}{rcccl}
 \mathbf{u} & \mathbf{v} & \mathbf{u} & 3, 3, 3 & 3, 2, 5 \\
 \mathbf{u} & \mathbf{v} & \mathbf{v} & 3, 3, 3 & 0, 1, 1
 \end{array} \quad 0$$

$$\begin{array}{rcccl}
 & \mathbf{i} & \mathbf{j} & \mathbf{k} & \\
 6. \mathbf{u} & \mathbf{v} & 6 & 8 & 3 \\
 & & 4 & 1 & 4
 \end{array} \quad 29, 36, 38$$

$$\begin{array}{rcccl}
 & \mathbf{i} & \mathbf{j} & \mathbf{k} & \\
 7. \mathbf{u} & \mathbf{v} & 10 & 0 & 6 \\
 & & 7 & 0 & 0
 \end{array} \quad 0, 42, 0$$

$$\begin{array}{rcccl}
 \mathbf{u} & \mathbf{v} & \mathbf{u} & 0, 42, 0 & 10, 0, 6 \\
 \mathbf{u} & \mathbf{v} & \mathbf{v} & 0, 42, 0 & 7, 0, 0
 \end{array} \quad 0$$

$$\begin{array}{rcccl}
 & \mathbf{i} & \mathbf{j} & \mathbf{k} & \\
 8. \mathbf{u} & \mathbf{v} & 5 & 5 & 11 \\
 & & 2 & 2 & 3
 \end{array} \quad 7, 37, 20$$

$$\begin{array}{rcccl}
 & \mathbf{i} & \mathbf{j} & \mathbf{k} & \\
 9. \mathbf{u} & \mathbf{v} & 6 & 2 & 1 \\
 & & 1 & 3 & 2
 \end{array} \quad 7, 13, 16$$

$$7\mathbf{i} - 13\mathbf{j} + 16\mathbf{k}$$

$$\begin{array}{rcccl}
 & \mathbf{i} & \mathbf{j} & \mathbf{k} & \\
 10. \mathbf{u} & \mathbf{v} & 1 & \frac{3}{2} & \frac{5}{2} \\
 & & \frac{1}{2} & \frac{3}{4} & \frac{1}{4}
 \end{array} \quad \frac{3}{2}, \frac{3}{2}, \frac{3}{2} \quad \frac{3}{2}\mathbf{i} - \frac{3}{2}\mathbf{j} + \frac{3}{2}\mathbf{k}$$

$$\begin{array}{rcccl}
 & \mathbf{i} & \mathbf{j} & \mathbf{k} & \\
 11. \mathbf{u} & \mathbf{v} & 0 & 0 & 6 \\
 & & 1 & 3 & 1
 \end{array} \quad 18, -6, 0$$

$$18\mathbf{i} - 6\mathbf{j}$$

$$\begin{array}{rcccl}
 & \mathbf{i} & \mathbf{j} & \mathbf{k} & \\
 12. \mathbf{u} & \mathbf{v} & \frac{2}{3} & 0 & 0 \\
 & & 0 & \frac{1}{3} & 3
 \end{array} \quad 2\mathbf{j} - \frac{2}{9}\mathbf{k}$$

$$\begin{array}{rcccl}
 & \mathbf{i} & \mathbf{j} & \mathbf{k} & \\
 13. \mathbf{u} & \mathbf{v} & 1 & 0 & 1 \\
 & & 0 & 1 & 2
 \end{array} \quad 1, -2, 1$$

$$\mathbf{i} - 2\mathbf{j} + \mathbf{k}$$

$$\begin{array}{rcccl}
 & \mathbf{i} & \mathbf{j} & \mathbf{k} & \\
 14. \mathbf{u} & \mathbf{v} & 1 & 0 & 2 \\
 & & 0 & 1 & 1
 \end{array} \quad 0, 2, 1 \quad 0\mathbf{i} + 2\mathbf{j} + 1\mathbf{k}$$

$$\begin{array}{l}
 \mathbf{15.} \quad \mathbf{u} \cdot \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \mathbf{i} - 3\mathbf{j} + 3\mathbf{k} \\
 \|\mathbf{u} \cdot \mathbf{v}\| = \sqrt{19} \\
 \text{Unit vector} = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u} \cdot \mathbf{v}\|} = \frac{1}{\sqrt{19}} \mathbf{i} - 3\mathbf{j} + 3\mathbf{k} \\
 \frac{\sqrt{19}}{19} \langle 1, -3, 3 \rangle
 \end{array}$$

$$\begin{array}{l}
 \mathbf{16.} \quad \mathbf{u} \cdot \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 0 \\ 1 & 0 & 3 \end{vmatrix} = 6\mathbf{i} - 3\mathbf{j} + 2\mathbf{k} \\
 \|\mathbf{u} \cdot \mathbf{v}\| = \sqrt{36 + 9 + 4} = 7 \\
 \text{Unit vector} = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u} \cdot \mathbf{v}\|} = \frac{6}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}
 \end{array}$$

$$\begin{array}{l}
 \mathbf{17.} \quad \mathbf{u} \cdot \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & 5 \\ \frac{1}{2} & \frac{3}{4} & \frac{1}{10} \end{vmatrix} = \frac{71}{20}\mathbf{i} - \frac{11}{5}\mathbf{j} + \frac{5}{4}\mathbf{k} \\
 \text{Consider the parallel vector } \mathbf{w} = \langle 71, -44, 25 \rangle \\
 \|\mathbf{w}\| = \sqrt{71^2 + 44^2 + 25^2} = \sqrt{7602} \\
 \text{Unit vector} = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u} \cdot \mathbf{v}\|} = \frac{1}{\sqrt{7602}} \langle 71, -44, 25 \rangle \\
 \frac{\sqrt{7602}}{7602} \langle 71, -44, 25 \rangle
 \end{array}$$

$$\begin{array}{l}
 \mathbf{18.} \quad \mathbf{u} \cdot \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & 14 & 5 \\ 14 & 28 & 15 \end{vmatrix} = 70\mathbf{i} - 175\mathbf{j} + 392\mathbf{k} \\
 \|\mathbf{u} \cdot \mathbf{v}\| = \sqrt{70^2 + 175^2 + 392^2} = \sqrt{189,189} = 429 \\
 \text{Unit vector} = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u} \cdot \mathbf{v}\|} = \frac{1}{429} \langle 70, -175, 392 \rangle \\
 \frac{1}{429} \langle 70, -175, 392 \rangle \\
 \frac{1}{429} \langle 10, -25, 56 \rangle \\
 \frac{\sqrt{429}}{1287} \langle 10, -25, 56 \rangle
 \end{array}$$

$$\begin{array}{l}
 \mathbf{19.} \quad \mathbf{u} \cdot \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 2\mathbf{i} - 2\mathbf{j} \\
 \|\mathbf{u} \cdot \mathbf{v}\| = 2\sqrt{2} \\
 \text{Unit vector} = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u} \cdot \mathbf{v}\|} = \frac{1}{2\sqrt{2}} \langle 2, -2, 0 \rangle \\
 \frac{1}{2\sqrt{2}} \langle 1, -1, 0 \rangle \\
 \frac{\sqrt{2}}{2} \langle 1, -1, 0 \rangle
 \end{array}$$

$$\begin{array}{l}
 \mathbf{20.} \quad \mathbf{u} \cdot \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 2 \\ 2 & 1 & 2 \end{vmatrix} = 6\mathbf{i} - 6\mathbf{j} + 3\mathbf{k} \\
 \|\mathbf{u} \cdot \mathbf{v}\| = \sqrt{36 + 36 + 9} = 9 \\
 \text{Unit vector} = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u} \cdot \mathbf{v}\|} = \frac{1}{9} \langle 6, -6, 3 \rangle \\
 \frac{2}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}
 \end{array}$$

$$\begin{array}{l}
 \mathbf{21.} \quad \mathbf{u} \cdot \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{vmatrix} = \mathbf{j} \\
 \text{Area} = \|\mathbf{u} \cdot \mathbf{v}\| = 1 \text{ square unit}
 \end{array}$$

$$\begin{array}{l}
 \mathbf{22.} \quad \mathbf{u} \cdot \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 2 \\ 1 & 0 & 1 \end{vmatrix} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k} \\
 \text{Area} = \|\mathbf{u} \cdot \mathbf{v}\| = \sqrt{4 + 1 + 4} = 3 \text{ square units}
 \end{array}$$

$$\begin{array}{l}
 \mathbf{23.} \quad \mathbf{u} \cdot \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 4 & 6 \\ 2 & 1 & 5 \end{vmatrix} = 26\mathbf{i} - 3\mathbf{j} + 11\mathbf{k} \\
 \text{Area} = \|\mathbf{u} \cdot \mathbf{v}\| = \sqrt{26^2 + 3^2 + 11^2} = \sqrt{806} \text{ square units}
 \end{array}$$

$$\begin{array}{l}
 \mathbf{24.} \quad \mathbf{u} \cdot \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & 2 \\ 1 & 2 & 4 \end{vmatrix} = 8\mathbf{i} - 10\mathbf{j} + 7\mathbf{k} \\
 \text{Area} = \|\mathbf{u} \cdot \mathbf{v}\| = \sqrt{8^2 + 10^2 + 7^2} = \sqrt{213} \text{ square units}
 \end{array}$$

$$\begin{array}{rcl}
 & \mathbf{i} & \mathbf{j} & \mathbf{k} \\
 25. \mathbf{u} \times \mathbf{v} & \begin{vmatrix} 2 & 2 & 3 \\ 0 & 2 & 3 \end{vmatrix} & & 12, 6, 4 \\
 \text{Area} & \frac{1}{2} \sqrt{12^2 + 6^2 + 4^2} & & \\
 & 14 \text{ square units} & &
 \end{array}$$

$$\begin{array}{rcl}
 & \mathbf{i} & \mathbf{j} & \mathbf{k} \\
 26. \mathbf{u} \times \mathbf{v} & \begin{vmatrix} 4 & 3 & 2 \\ 5 & 0 & 1 \end{vmatrix} & & 3, 6, 15 \\
 \text{Area} & \frac{1}{2} \sqrt{3^2 + 6^2 + 15^2} & & \\
 & 270 \text{ square units} & &
 \end{array}$$

27. (a) $\vec{AB} = \langle 3, 2, 1 \rangle$, $\vec{AC} = \langle 1, 2, 4 \rangle$, $\vec{AD} = \langle 1, 2, 2 \rangle$ is parallel to $\vec{BC} = \langle 0, 1, 5 \rangle$, $\vec{BD} = \langle 3, 6, 8 \rangle$, $\vec{CD} = \langle 1, 2, 2 \rangle$.
 $\vec{AD} = \langle 3, 4, 4 \rangle$ is parallel to $\vec{BC} = \langle 3, 4, 4 \rangle$.

(b) $\vec{AB} = \langle 1, 2, 2 \rangle$, $\vec{AD} = \langle 3, 4, 4 \rangle$, $\vec{AC} = \langle 16, 2, 10 \rangle$
 Area $\frac{1}{2} \sqrt{16^2 + 2^2 + 10^2} = \frac{1}{2} \sqrt{360} = 6\sqrt{10}$ square units

(c) $\vec{AB} \cdot \vec{AD} = 1 \cdot 3 + 2 \cdot 4 + 2 \cdot 4 = 15 \neq 0 \Rightarrow$ not a rectangle

28. (a) $\vec{AB} = \langle 1, 2, 3 \rangle$, $\vec{CD} = \langle 1, 2, 3 \rangle$

Opposites are parallel and same length. Thus ABCD form a parallelogram.

(b) $\vec{AB} = \langle 1, 2, 3 \rangle$, $\vec{AC} = \langle 5, 4, 1 \rangle$, $\vec{BC} = \langle 10, 14, 6 \rangle$
 Area $\frac{1}{2} \sqrt{10^2 + 14^2 + 6^2} = \frac{1}{2} \sqrt{360} = 6\sqrt{10}$ square units

(c) $\vec{AB} \cdot \vec{AC} = 1 \cdot 5 + 2 \cdot 8 + 3 \cdot 1 = 16 \neq 0 \Rightarrow$ not a rectangle.

29. $\mathbf{u} = \langle 1, 2, 3 \rangle$, $\mathbf{v} = \langle 3, 0, 0 \rangle$

$$\begin{array}{rcl}
 & \mathbf{i} & \mathbf{j} & \mathbf{k} \\
 \mathbf{u} \times \mathbf{v} & \begin{vmatrix} 1 & 2 & 3 \\ 3 & 0 & 0 \end{vmatrix} & & 0, 9, 6 \\
 \text{Area} & \frac{1}{2} \sqrt{0^2 + 9^2 + 6^2} & & \frac{1}{2} \sqrt{117} = \frac{3}{2} \sqrt{13} \text{ square units}
 \end{array}$$

30. $\mathbf{u} = \langle 2, 1, 0 \rangle$, $\mathbf{v} = \langle 4, 2, 3 \rangle$

$$\begin{array}{rcl}
 & \mathbf{i} & \mathbf{j} & \mathbf{k} \\
 \mathbf{u} \times \mathbf{v} & \begin{vmatrix} 2 & 1 & 0 \\ 4 & 2 & 3 \end{vmatrix} & & 3, 6, 3 \\
 \text{Area} & \frac{1}{2} \sqrt{3^2 + 6^2 + 3^2} & & \frac{1}{2} \sqrt{81} = \frac{9}{2} \text{ square units}
 \end{array}$$

31. $\mathbf{u} = \langle 2, 2, 2 \rangle$, $\mathbf{v} = \langle 3, 2, 0 \rangle$

$$\begin{array}{rcl}
 & \mathbf{i} & \mathbf{j} & \mathbf{k} \\
 \mathbf{u} \times \mathbf{v} & \begin{vmatrix} 2 & 2 & 2 \\ 3 & 2 & 0 \end{vmatrix} & & 4, 4, 10 \\
 \text{Area} & \frac{1}{2} \sqrt{4^2 + 4^2 + 10^2} & & \frac{1}{2} \sqrt{120} = \sqrt{30} \text{ square units}
 \end{array}$$

32. $\mathbf{u} = \langle 2, 2, 4 \rangle$, $\mathbf{v} = \langle 0, 2, 0 \rangle$

$$\begin{array}{rcl}
 & \mathbf{i} & \mathbf{j} & \mathbf{k} \\
 \mathbf{u} \times \mathbf{v} & \begin{vmatrix} 2 & 2 & 4 \\ 0 & 2 & 0 \end{vmatrix} & & 8, 8, 0 \\
 \text{Area} & \frac{1}{2} \sqrt{8^2 + 8^2} & & \frac{1}{2} \sqrt{128} = 4\sqrt{2} \text{ square units}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{ccc} 2 & 3 & 3 \\ 4 & 4 & 0 \\ 0 & 0 & 4 \end{array} \\
 \hline
 2 \ 16 \quad 3 \ 16 \quad 3 \ 0 \quad 16
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{ccc} 2 & 3 & 1 \\ 1 & 1 & 0 \\ 4 & 3 & 1 \end{array} \\
 \hline
 2 \ 1 \quad 3 \ 1 \quad 1 \ 7 \quad 2
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{ccc} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{array} \\
 \hline
 \text{Volume} \quad \mathbf{u} \quad \mathbf{v} \quad \mathbf{w} \quad 2 \text{ cubic units}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{ccc} 0 & 2 & 2 \\ 0 & 0 & 2 \\ 3 & 0 & 2 \end{array} \\
 \hline
 0 \ 2 \ 6 \quad 2 \ 0 \quad 12 \\
 \text{Volume} \quad \mathbf{u} \quad \mathbf{v} \quad \mathbf{w} \quad 12 \text{ cubic units}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{ccc} 4 & 0 & 0 \\ 0 & 2 & 3 \\ 0 & 5 & 3 \end{array} \\
 \hline
 \mathbf{u} \quad \mathbf{v} \quad \mathbf{w} \quad 4 \ 21 \quad 84 \\
 \text{Volume} \quad 84 \quad 84 \text{ cubic units}
 \end{array}$$

$$43. \mathbf{V} = \frac{1}{2} \cos 40^\circ \mathbf{j} + \sin 40^\circ \mathbf{k}$$

$$\mathbf{F} = p\mathbf{k}$$

$$\begin{array}{r}
 \begin{array}{ccc} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ (a) \mathbf{V} \cdot \mathbf{F} & 0 & \frac{1}{2} \cos 40^\circ \\ & 0 & 0 \end{array} \\
 \hline
 \frac{1}{2} p \cos 40^\circ \mathbf{i}
 \end{array}$$

$$T = \mathbf{V} \cdot \mathbf{F} = \frac{p}{2} \cos 40^\circ$$

$$\begin{array}{r}
 \begin{array}{ccc} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{array} \\
 \hline
 34. \mathbf{u} \quad \mathbf{v} \quad \mathbf{w} \quad 6
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{ccc} 1 & 4 & 7 \\ 2 & 0 & 4 \\ 0 & 3 & 6 \end{array} \\
 \hline
 36. \mathbf{u} \quad \mathbf{v} \quad \mathbf{w} \quad 1 \ 0 \quad 12 \quad 4 \ 12 \quad 0 \ 7 \ 6 \quad 6
 \end{array}$$

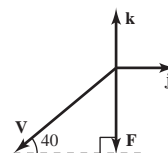
$$\begin{array}{r}
 \begin{array}{ccc} 1 & 1 & 3 \\ 0 & 3 & 3 \\ 3 & 0 & 3 \end{array} \\
 \hline
 38. \mathbf{u} \quad \mathbf{v} \quad \mathbf{w} \quad 1 \ 9 \quad 1 \ 9 \quad 3 \ 9 \quad 9 \\
 \text{Volume} \quad \mathbf{u} \quad \mathbf{v} \quad \mathbf{w} \quad 9 \ 9 \text{ cubic units}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{ccc} 1 & 2 & 1 \\ 1 & 2 & 2 \\ 2 & 0 & 1 \end{array} \\
 \hline
 40. \mathbf{u} \quad \mathbf{v} \quad \mathbf{w} \quad 1 \ 2 \quad 2 \ 1 \quad 4 \ 1(0 \ 4 \ 16 \\
 \text{Volume} \quad \mathbf{u} \quad \mathbf{v} \quad \mathbf{w} \quad 16 \text{ cubic units}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{ccc} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & 1 \end{array} \\
 \hline
 42. \overrightarrow{AB} = 1, 1, 0, \overrightarrow{AC} = 1, 0, 2, \overrightarrow{AD} = 0, 1, 1 \\
 \mathbf{u} \quad \mathbf{v} \quad \mathbf{w} \quad 1 \ 2 \quad 1 \ 1 \quad 3 \\
 \text{Volume} \quad 3 \text{ cubic units}
 \end{array}$$

(b)

p	T
15	5.75
20	7.66
25	9.58
30	11.49
35	13.41
40	15.32
45	17.24



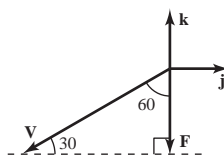
44. $\mathbf{V} = 0.16 \cos 30^\circ \mathbf{j} + \sin 30^\circ \mathbf{k}$

$\mathbf{F} = 2000\mathbf{k}$

$$\mathbf{V} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.16 \cos 30^\circ & 0.16 \sin 30^\circ \\ 0 & 0 & 2000 \end{vmatrix}$$

$$= 2000(0.16 \cos 30^\circ) \mathbf{i} - 160 \sqrt{3} \mathbf{j}$$

$T = \mathbf{V} \times \mathbf{F} = 160 \sqrt{3} \text{ ft-lb}$



45. True. The cross product is not defined for two-dimensional vectors.

 46. False. $\mathbf{u} \times \mathbf{v} = -\mathbf{v} \times \mathbf{u}$

47. If the magnitudes of two vectors are doubled, the magnitude of the cross product will be four times as large.

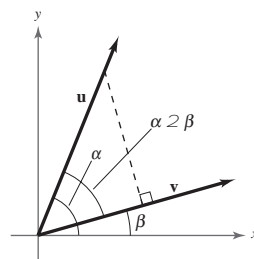
$$48. \mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos \alpha & \sin \alpha & 0 \\ \cos \beta & \sin \beta & 0 \end{vmatrix} = (\sin \alpha \cos \beta - \sin \beta \cos \alpha) \mathbf{k}$$

 Area of triangle formed by the unit vectors \mathbf{u} and \mathbf{v} is

$$\frac{1}{2} \text{ base} \times \text{height} = \frac{1}{2} (1) \sin \alpha$$

 The area is also given by $\frac{1}{2} |\mathbf{u} \times \mathbf{v}| = \frac{1}{2} |\sin \alpha \cos \beta - \sin \beta \cos \alpha|$

 Notice that $\sin \alpha \cos \beta - \sin \beta \cos \alpha$ is negative.

 Thus, $\sin \alpha \cos \beta - \sin \beta \cos \alpha = -\sin(\alpha - \beta)$


49. $\cos 480^\circ = \cos 120^\circ = -\frac{1}{2}$

50. $\tan 300^\circ = -\sqrt{3}$

51. $\sin 690^\circ = \sin 330^\circ = -\frac{1}{2}$

52. $\cos 930^\circ = \cos 210^\circ = -\frac{\sqrt{3}}{2}$

53. $\sin \frac{19\pi}{6} = \sin \frac{7\pi}{6} = -\frac{1}{2}$

54. $\cos \frac{17\pi}{6} = \cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$

55. $\tan \frac{15\pi}{4} = \tan \frac{7\pi}{4} = -1$

56. $\tan \frac{10\pi}{3} = \tan \frac{4\pi}{3} = \sqrt{3}$

57. $z = 6x - 4y$

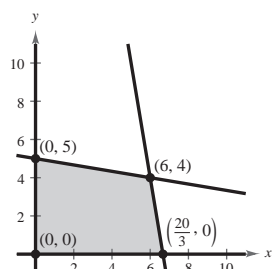
At $(0, 5)$: $z = 6(0) - 4(5) = -20$

At $(0, 0)$: $z = 6(0) - 4(0) = 0$

At $(\frac{20}{3}, 0)$: $z = 6(\frac{20}{3}) - 4(0) = 40$

At $(6, 4)$: $z = 6(6) - 4(4) = 52$

 The maximum value of z , $z = 52$, is found at $(6, 4)$.

 The minimum value of z , $z = -20$, is found at $(0, 5)$.


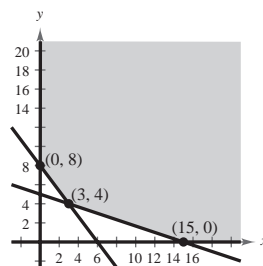
58. $0, 8 : z = 6(0) + 7(8) = 56$

$3, 4 : z = 6(3) + 7(4) = 46$

$15, 0 : z = 6(15) + 7(0) = 90$

 Minimum: 46 at $(3, 4)$.

Maximum: Unbounded



4. $x = x_1 + at + 5 - 4t, y = y_1 + bt = 0 + 0t, z = z_1 + ct = 10 + 3t$

(a) Parametric equations: $x = 5 - 4t, y = 0, z = 10 + 3t$

(b) Symmetric equations: $\frac{x-5}{-4} = \frac{z-10}{3}, y = 0$

5. $x = x_1 + at = 2 + 2t, y = y_1 + bt = 3 + 3t, z = z_1 + ct = 5 + t$

(a) Parametric equations: $x = 2 + 2t, y = 3 + 3t, z = 5 + t$

(b) Symmetric equations: $\frac{x-2}{2} = \frac{y-3}{3} = \frac{z-5}{1}$

6. (a) $\mathbf{v} = \langle 3, -2, 1 \rangle$

$x = 1 + 3t, y = -2t, z = 1 + t$

(b) Symmetric equations: $\frac{x-1}{3} = \frac{y}{-2} = \frac{z-1}{1}$

7. (a) $\mathbf{v} = \langle 1, 2, 4 \rangle, \mathbf{w} = \langle 0, 3, 2 \rangle, \mathbf{u} = \langle 1, 4, 5 \rangle$

Point: $(2, 0, 2)$

$x = 2 + t, y = 4t, z = 2 + 5t$

(b) $\frac{x-2}{1} = \frac{y}{4} = \frac{z-2}{5}$

8. (a) $\mathbf{v} = \langle 8, 5, 12 \rangle$

Point: $(2, 3, 0)$

Parametric equations: $x = 2 + 8t, y = 3 + 5t, z = 12t$

(b) Symmetric equations: $\frac{x-2}{8} = \frac{y-3}{5} = \frac{z}{12}$

9. (a) $\mathbf{v} = \langle 1, 3, 2 \rangle, \mathbf{w} = \langle 8, 16, 15 \rangle, \mathbf{u} = \langle 4, 10, 1 \rangle$

Point: $(3, 8, 15)$

$x = 3 + 4t, y = 8 + 10t, z = 15 + t$

(b) $\frac{x-3}{4} = \frac{y-8}{10} = \frac{z-15}{1}$

10. $\langle 2, 3, -1 \rangle, \langle 1, -5, 3 \rangle$

(a) Let $P = (2, 3, -1)$, $Q = (1, -5, 3)$

$\vec{PQ} = \langle 1 - 2, -5 - 3, 3 - (-1) \rangle = \langle -1, -8, 4 \rangle$

Direction numbers: $a = -1, b = -8, c = 4$

Choose P as the initial point:

$x = 2 - t, y = 3 - 8t, z = -1 + 4t$

(b) $\frac{x-2}{-1} = \frac{y-3}{-8} = \frac{z+1}{4}$

11. 3, 1, 2, 1, 1, 5

(a) $\mathbf{v} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \\ -1 \end{pmatrix}$
 Parametric: $x = 3 + 0t, y = 1 - 2t, z = 2 - t$

(b) Since $b = 0$, there are no symmetric equations.

12. 2, 1, 5, 2, 1, 3

(a) Let $P = (2, 1, 5), Q = (2, 1, 3)$
 $\vec{PQ} = \begin{pmatrix} 2 - 2 \\ 1 - 1 \\ 3 - 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix}$

Direction numbers: $a = 0, b = 2, c = 8$

Choose P as the initial point:

$x = 2 + 0t, y = 1 + 2t, z = 5 - 8t$

(b) Since the direction number $a = 0$, no set of symmetric equations are possible.

13. $\frac{1}{2}, 2, \frac{1}{2}, 1, \frac{1}{2}, 0$

(a) $\mathbf{v} = \begin{pmatrix} 1 \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} -1 \\ \frac{1}{2} \\ 0 \end{pmatrix}$

Direction numbers: 3, 5, 1

Parametric: $x = \frac{1}{2} - 3t, y = 2 + 5t, z = \frac{1}{2} + t$

(b) Symmetric: $\frac{2x - \frac{1}{2}}{6} = \frac{y - 2}{5} = \frac{2z - 1}{2}$

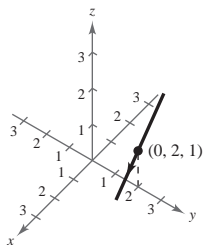
14. (a) $\mathbf{v} = \begin{pmatrix} 3 \\ \frac{3}{2} \\ 5 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \\ \frac{9}{2} \end{pmatrix} = \begin{pmatrix} -1 \\ -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$, or $\begin{pmatrix} 1 \\ \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$

Point: $(3, 5, 4)$

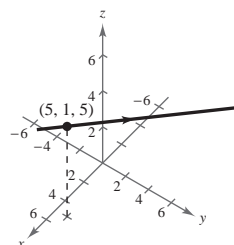
Parametric equations: $x = 3 - t, y = 5 - \frac{1}{2}t, z = 4 + \frac{1}{2}t$

(b) Symmetric equations: $\frac{x - 3}{-1} = \frac{y - 5}{-\frac{1}{2}} = \frac{z - 4}{\frac{1}{2}}$

15.



16.



17. $a x_1 + b y_1 + c z_1 = 0$

$1 x_2 + 0 y_2 + 0 z_2 = 0$

$x_3 = 2$

19. $2 x_1 + 1 y_1 + 2 z_1 = 3$

$2 x_2 + y_2 + 2 z_2 = 10$

18. $a x_0 + b y_0 + c z_0 = 0$

$0 x_1 + 1 y_1 + 0 z_1 = 3$

$z_2 = 3$

20. $0 x_1 + 3 y_1 + 5 z_1 = 0$

$3 y_2 + 5 z_2 = 0$

$$21. \mathbf{n} \quad 1, 2, 1 \Rightarrow \begin{matrix} 1x & 2 & 2y & 0 & 1z & 0 & 0 \\ & x & 2y & z & 2 & 0 \end{matrix}$$

$$22. \mathbf{n} \quad 1, 1, 2$$

$$\begin{matrix} 1x & 0 & 1y & 0 & 2z & 6 & 0 \\ x & y & 2z & 12 & 0 \end{matrix}$$

$$23. \mathbf{u} \quad 1, 0, 2 \quad \mathbf{v} \quad 0, 3, 0 \quad \mathbf{w} \quad 1, 2, 3$$

$$\begin{matrix} & \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \mathbf{n} & \mathbf{u} & \mathbf{v} & \mathbf{w} & & \\ & 1 & 2 & 3 & 3, 9, 7 \\ & 2 & 3 & 3 & \end{matrix}$$

$$\begin{matrix} 3x & 0 & 9y & 0 & 7z & 0 & 0 \\ & 3x & 9y & 7z & 0 \\ & 3x & 9y & 7z & 0 \end{matrix}$$

$$24. \mathbf{u} \quad 2, 6, 2, \mathbf{v} \quad 3, 3, 0$$

$$\begin{matrix} & \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \mathbf{u} & \mathbf{v} & & & \\ & 2 & 6 & 2 & 6, 6, 24 \\ & 3 & 3 & 0 \end{matrix}$$

$$\mathbf{n} \quad 1, 1, 4$$

$$\text{Plane: } \begin{matrix} 1x & 4 & 1y & 1 & 4z & 3 & 0 \\ & x & y & 4z & 7 & 0 \end{matrix}$$

$$25. \mathbf{u} \quad 3, 2, 4 \quad \mathbf{v} \quad 3, 2, 2 \quad \mathbf{w} \quad 1, 1, 4$$

$$\begin{matrix} & \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \mathbf{n} & \mathbf{u} & \mathbf{v} & \mathbf{w} & & \\ & 1 & 1 & 4 & 18, 6, 3 \\ & 1 & 4 & 2 \end{matrix}$$

$$\begin{matrix} 18x & 2 & 6y & 3 & 3z & 2 & 0 \\ & 18x & 6y & 3z & 24 & 0 \\ & 6x & 2y & z & 8 & 0 \end{matrix}$$

$$26. \mathbf{u} \quad 4, 0, 2, \mathbf{v} \quad 1, 2, 5$$

$$\begin{matrix} & \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \mathbf{u} & \mathbf{v} & & & \\ & 4 & 0 & 2 & 4, 22, 8 \\ & 1 & 2 & 5 \end{matrix}$$

$$\mathbf{n} \quad 2, 11, 4$$

$$\text{Plane: } \begin{matrix} 2x & 1 & 11y & 1 & 4z & 2 & 0 \\ & 2x & 11y & 4z & 5 & 0 \end{matrix}$$

$$27. \mathbf{n} \quad \mathbf{j}: 0x + 2y + 5z = 3$$

$$y + 5z = 0$$

$$28. \quad 1, 2, 1 \quad 2, 1, 1 \quad 3, 1, 2 \quad \text{and} \quad 2, 3, 1 \quad \text{are parallel to plane.}$$

$$\begin{matrix} & \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \mathbf{n} & & & & \\ & 3 & 1 & 2 & 7, 1, 11 \\ & 2 & 3 & 1 \end{matrix}$$

$$\begin{matrix} 7x & 2 & 1y & 2 & 11z & 1 & 0 \\ 7x & y & 11z & 5 & 0 \end{matrix}$$

$$29. \mathbf{n}_1 \quad 5, 3, 1, \mathbf{n}_2 \quad 1, 4, 7$$

$$\mathbf{n}_1 \cdot \mathbf{n}_2 = 5 + 12 + 7 = 0; \text{ orthogonal}$$

$$30. \mathbf{n}_1 \quad 3, 1, 4, \mathbf{n}_2 \quad 9, 3, 12$$

$$3\mathbf{n}_1 = 9, 3, 12 \quad \mathbf{n}_2 \Rightarrow \text{parallel planes}$$

$$31. \mathbf{n}_1 \quad 2, 0, 1, \mathbf{n}_2 \quad 4, 1, 8$$

$$\mathbf{n}_1 \cdot \mathbf{n}_2 = 8 + 8 = 0; \text{ orthogonal}$$

$$32. \mathbf{n}_1 \quad 1, 5, 1$$

$$\mathbf{n}_2 \quad 5, 25, 5 \quad 5\mathbf{n}_1 \Rightarrow \text{parallel}$$

33. (a) $\mathbf{n}_1 = 3, 4, 5$, $\mathbf{n}_2 = 1, 1, 1$; normal vectors to planes

$$\cos \frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{\|\mathbf{n}_1\| \|\mathbf{n}_2\|} = \frac{6}{\sqrt{50} \sqrt{3}} = \frac{6}{150} \Rightarrow 60.67$$

(b) $3x - 4y + 5z = 6$ Equation 1

$x - y + z = 2$ Equation 2

3 times Equation 2 added to Equation 1 gives

$$7y - 8z = 0$$

$$y = \frac{8}{7}z.$$

Substituting back into Equation 2, $x - 2 + y + z = 2 \Rightarrow x = 2 - \frac{8}{7}z + z = 2 - \frac{1}{7}z.$

Letting $t = z/7$, we obtain $x = 2 - t, y = 8t, z = 7t.$

34. (a) $\mathbf{n}_1 = 1, 3, 1$, $\mathbf{n}_2 = 2, 0, 5$

$$\cos \frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{\|\mathbf{n}_1\| \|\mathbf{n}_2\|} = \frac{7}{\sqrt{11} \sqrt{29}} = \frac{7}{319} \Rightarrow 66.93$$

(b) $2x - 5z = 3 \Rightarrow x = \frac{1}{2}(5z + 3)$

Then $3y - x + z = 2 \Rightarrow \frac{1}{2}(5z + 3) - 3y + z = 2 \Rightarrow \frac{3}{2}z - 3y = \frac{1}{2} \Rightarrow y = \frac{1}{2}z - \frac{1}{6}$

Let $z = t$. Parametric equations: $x = \frac{5}{2}t + \frac{3}{2}, y = \frac{1}{2}t - \frac{1}{6}, z = t$

or equivalently, let $z = 2t$ and you obtain $x = 5t + \frac{3}{2}, y = t - \frac{1}{6}, z = 2t.$

35. (a) $\mathbf{n}_1 = 1, 1, 1$, $\mathbf{n}_2 = 2, 5, 1$; normal vectors to planes

$$\cos \frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{\|\mathbf{n}_1\| \|\mathbf{n}_2\|} = \frac{2}{\sqrt{3} \sqrt{30}} = \frac{2}{90} \Rightarrow 77.83$$

(b) $x - y + z = 0$ Equation 1

$2x - 5y + z = 1$ Equation 2

2 times Equation 1 added to Equation 2 gives

$$7y - z = 1$$

$$y = \frac{z + 1}{7}.$$

Substituting back into Equation 1, $x - z + y + z = 0 \Rightarrow x = -y = -\frac{z + 1}{7} = -\frac{z}{7} - \frac{1}{7}.$

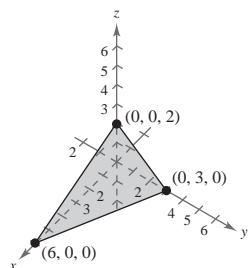
Letting $z = t$, $x = -\frac{6t + 1}{7}, y = \frac{t + 1}{7}.$

Equivalently, let $y = t, z = 7t - 1$ and $x = 6t - 1.$

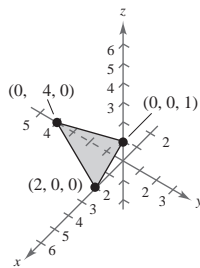
36. The planes are parallel because $\mathbf{n}_1 = 2, 4, 2$ is a multiple of $\mathbf{n}_2 = 3, 6, 3$.

The planes do not intersect.

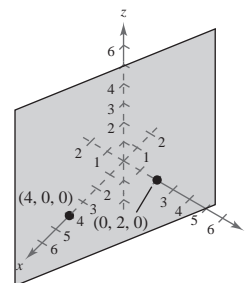
37. $x + 2y + 3z = 6$



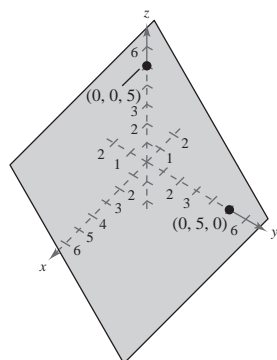
38. $2x + y + 4z = 4$



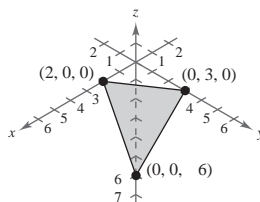
39. $x + 2y + 4z = 4$



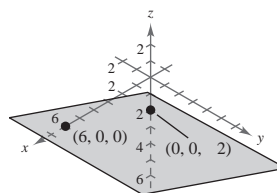
40. $y + z = 5$



41. $3x + 2y + z = 6$



42. $x + 3z = 6$



43. $D = \frac{\overrightarrow{PQ} \cdot \mathbf{n}}{\|\mathbf{n}\|}$
 $P = 1, 0, 0$ on plane, $Q = 0, 0, 0$,
 $\mathbf{n} = 8, -4, 1$, $\overrightarrow{PQ} = 1, 0, 0$
 $D = \frac{1 \cdot 8 + 0 \cdot (-4) + 0 \cdot 1}{\sqrt{64 + 16 + 1}} = \frac{8}{\sqrt{81}} = \frac{8}{9}$

44. $P = 4, 0, 0$ on plane, $Q = 3, 2, 1$, $\mathbf{n} = 1, -1, 2$
 $\overrightarrow{PQ} = 1, 2, 1$
 $D = \frac{\overrightarrow{PQ} \cdot \mathbf{n}}{\|\mathbf{n}\|} = \frac{1 \cdot 1 + 2 \cdot (-1) + 1 \cdot 2}{\sqrt{1 + 1 + 4}} = \frac{0}{\sqrt{6}} = 0$

45. $D = \frac{\overrightarrow{PQ} \cdot \mathbf{n}}{\|\mathbf{n}\|}$
 $P = 2, 0, 0$ on plane, $Q = 4, -2, -2$,
 $\mathbf{n} = 2, -1, 1$, $\overrightarrow{PQ} = 2, -2, -2$
 $D = \frac{2 \cdot 2 + (-2) \cdot (-1) + (-2) \cdot 1}{\sqrt{4 + 1 + 1}} = \frac{4 - 2 - 2}{\sqrt{6}} = \frac{0}{\sqrt{6}} = 0$

46. $P = 6, 0, 0$ on plane, $Q = 1, 2, 5$,
 $\overrightarrow{PQ} = 7, 2, 5$, $\mathbf{n} = 2, 3, 1$
 $D = \frac{\overrightarrow{PQ} \cdot \mathbf{n}}{\|\mathbf{n}\|} = \frac{7 \cdot 2 + 2 \cdot 3 + 5 \cdot 1}{\sqrt{4 + 9 + 1}} = \frac{14 + 6 + 5}{\sqrt{14}} = \frac{25}{\sqrt{14}}$

47. (a) $z = 0.81x + 0.36y + 0.2$

Year	x	y	z (Actual)	z (Model)
1999	6.2	7.3	7.8	7.85
2000	6.1	7.1	7.7	7.70
2001	5.9	7.0	7.4	7.50
2002	5.7	7.0	7.3	7.34
2003	5.6	6.9	7.2	7.22

- (b) The approximations are very similar to the actual values of z .
- (c) If the consumption of the two types of milk increases (or decreases), so does the consumption of the third type of milk.

48. The plane containing $P(6, 0, 0)$, $S(0, 0, 0)$, $T(1, -1, 8)$ has normal vector

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6, 0, 0 & 1, -1, 8 & 0, 0, 0 \end{vmatrix} = \begin{vmatrix} 6 & 0 & 0 \\ 1 & -1 & 8 \end{vmatrix} = 0, 48, -6$$

or $\mathbf{n}_1 = 0, 8, 1$.

The plane containing $P(6, 0, 0)$, $Q(6, 6, 0)$, and $R(7, 7, 8)$ has normal vector

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0, -6, 0 & 1, 1, 8 & 48, 0, 6 \end{vmatrix} = \begin{vmatrix} 0 & -6 & 0 \\ 1 & 1 & 8 \end{vmatrix} = 48, 0, 6$$

or $\mathbf{n}_2 = 8, 0, 1$.

The angle between two adjacent sides is given by

$$\cos \theta = \frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{\|\mathbf{n}_1\| \|\mathbf{n}_2\|} = \frac{8}{\sqrt{65} \sqrt{65}} = \frac{8}{65} \Rightarrow \theta \approx 89.12^\circ$$

49. False. They might be skew lines, such as:

$$L_1: x = t, y = 0, z = 0 \text{ (} x\text{-axis)}$$

$$\text{and } L_2: x = 0, y = t, z = 1$$

50. True

51. The lines are parallel:

$$\frac{3}{2} \begin{pmatrix} 10 \\ 18 \\ 20 \end{pmatrix} = \begin{pmatrix} 15 \\ 27 \\ 30 \end{pmatrix}$$

52. (a) Sphere: $x^2 + y^2 + z^2 = 4$

(b) Two planes parallel to given plane. Let $Q(x, y, z)$ be a point on one of these planes, and pick $P(0, 0, 10)$ on the given plane. By the distance formula,

$$\begin{aligned} \frac{\|\overrightarrow{PQ}\|^2}{\|\mathbf{n}\|^2} &= \frac{(x-0)^2 + (y-0)^2 + (z-10)^2}{4^2 + 3^2 + 1^2} \\ &= \frac{x^2 + y^2 + z^2 - 20z + 100}{26} \end{aligned}$$

(Two planes parallel to given plane)

53. $x^2 + y^2 = 10^2 = 100$

54. $\frac{3}{4} \Rightarrow \tan \theta = \frac{3}{4} \Rightarrow \theta = \arctan \frac{3}{4}$ (line)

55. $r = 3 \cos \theta$

$$r^2 = 3r \cos \theta$$

$$x^2 + y^2 = 3x$$

56. $r = \frac{1}{2 \cos \theta} \Rightarrow 2r = r \cos \theta = 1 \Rightarrow 2 \sqrt{x^2 + y^2} = x + 1$

$$\Rightarrow 2 \sqrt{x^2 + y^2} = x + 1 \Rightarrow 4(x^2 + y^2) = x^2 + 2x + 1 \Rightarrow 3x^2 + 4y^2 - 2x = 1$$

57. $r^2 = 49$

$$r = 7$$

11. Midpoint: $\frac{10}{2}, \frac{8}{2}, \frac{6}{2}, \frac{2}{2}, \frac{12}{2}, \frac{6}{2}$ 1, 2, 9

12. Midpoint: $\frac{5}{2}, \frac{7}{2}, \frac{3}{2}, \frac{9}{2}, \frac{1}{2}, \frac{5}{2}$ 6, 6, 2

13. $x^2 + y^2 + z^2 = 1$

14. $x^2 + y^2 + z^2 = 16$

15. Radius: 6

$x^2 + y^2 + z^2 = 36$

16. Radius $\frac{15}{2}$

$x^2 + y^2 + z^2 = \frac{225}{4}$

17. $x^2 - 4x + 4 + y^2 - 6y + 9 + z^2 - 4z + 4 = 9$
 $x^2 - 2^2 + y^2 - 3^2 + z^2 - 2^2 = 9$

Center: 2, 3, 0

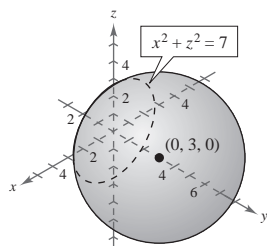
Radius: 3

18. $x^2 - 10x + 25 + y^2 - 6y + 9 + z^2 - 4z + 4 = 34$
 $x^2 - 5^2 + y^2 - 3^2 + z^2 - 2^2 = 4$

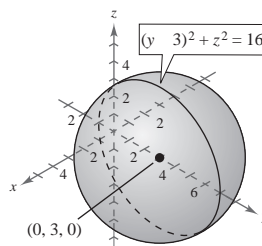
Center: 5, 3, 2

Radius: 2

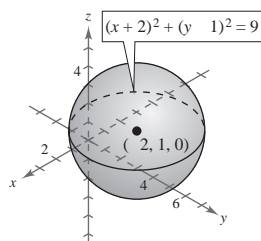
19. (a) xz -trace $y = 0$: $x^2 + z^2 = 7$, circle



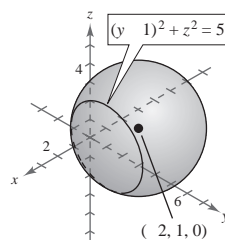
(b) yz -trace $x = 0$: $(y - 3)^2 + z^2 = 16$, circle



20. (a) xy -trace $z = 0$: $(x + 2)^2 + (y - 1)^2 = 9$ circle



(b) yz -trace $x = 0$: $(y - 1)^2 + z^2 = 5$ circle



21. Initial point: 2, 1, 4

Terminal point: 3, 3, 0

(a) $\mathbf{v} = \langle 3 - 2, 3 - 1, 0 - 4 \rangle = \langle 1, 2, -4 \rangle$

(b) $|\mathbf{v}| = \sqrt{1^2 + 2^2 + 4^2} = \sqrt{21}$

(c) $\mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|} = \left\langle \frac{1}{\sqrt{21}}, \frac{2}{\sqrt{21}}, \frac{-4}{\sqrt{21}} \right\rangle$

22. (a) $\overrightarrow{PQ} = \langle 3 - 1, 2 - 3, 2 - 5 \rangle = \langle 2, -1, -3 \rangle$

(b) $|\overrightarrow{PQ}| = \sqrt{2^2 + (-1)^2 + (-3)^2} = \sqrt{14}$

(c) Unit vector: $\frac{1}{\sqrt{14}} \langle 2, -1, -3 \rangle = \left\langle \frac{2}{\sqrt{14}}, \frac{-1}{\sqrt{14}}, \frac{-3}{\sqrt{14}} \right\rangle$

23. Initial point: 7, 4, 3

Terminal point: 3, 2, 10

(a) $\mathbf{v} = \frac{3-7}{10^2}, \frac{2-4}{6^2}, \frac{10-3}{7^2} = \frac{-4}{100}, \frac{-2}{36}, \frac{7}{49} = -\frac{1}{25}, -\frac{1}{18}, \frac{1}{7}$

(b) $\mathbf{v} = \frac{3-7}{10^2}, \frac{2-4}{6^2}, \frac{10-3}{7^2} = \frac{-4}{100}, \frac{-2}{36}, \frac{7}{49} = -\frac{1}{25}, -\frac{1}{18}, \frac{1}{7}$

(c) $\mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{-\frac{1}{25}, -\frac{1}{18}, \frac{1}{7}}{\sqrt{\frac{1}{625} + \frac{1}{324} + \frac{1}{49}}} = \frac{-\frac{1}{25}, -\frac{1}{18}, \frac{1}{7}}{\frac{1}{185}} = -\frac{185}{25}, -\frac{185}{18}, \frac{185}{7}$

24. (a) $\overrightarrow{PQ} = \langle 5-0, 8-3, 6-1 \rangle = \langle 5, 5, 5 \rangle$

(b) $|\overrightarrow{PQ}| = \sqrt{5^2 + 5^2 + 5^2} = \sqrt{75} = 5\sqrt{3}$

(c) Unit vector: $\frac{1}{5\sqrt{3}} \langle 5, 5, 5 \rangle = \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$

25. $\mathbf{u} \cdot \mathbf{v} = 1 \cdot 0 + 4 \cdot 6 + 3 \cdot 5 = 9$

26. $\mathbf{u} \cdot \mathbf{v} = 8 \cdot 2 + 4 \cdot 5 + 2 \cdot 2 = 0$

27. $\mathbf{u} \cdot \mathbf{v} = 2 \cdot 1 + 1 \cdot 0 + 1 \cdot 1 = 3$

28. $\mathbf{u} \cdot \mathbf{v} = 2 \cdot 1 + 1 \cdot 3 + 2 \cdot 2 = 5$

29. Since $\mathbf{u} \cdot \mathbf{v} = 0$, the angle is 90° .

30. $\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}| |\mathbf{v}|} = \frac{12 \cdot 5 + 2 \cdot 2}{\sqrt{11^2 + 45^2} \sqrt{15^2 + 45^2}} = \frac{15}{11 \cdot 45} \Rightarrow \theta = 47.61^\circ$

31. Since $\frac{2}{3} \langle 39, 12, 21 \rangle = \langle 26, 8, 14 \rangle$, the vectors are parallel.

32. $\mathbf{u} \cdot \mathbf{v} = 8 \cdot 5 + 8 \cdot 2 + 4 \cdot \frac{1}{2} = 16 + 20 + 4 = 40$

Orthogonal

33. First two points: $\mathbf{u} = \langle 3, 4, 1 \rangle$

Last two points: $\mathbf{v} = \langle 0, 2, 6 \rangle$

Since $\mathbf{u} \cdot \mathbf{v} = 0$, the points are not collinear.

34. First two points: $\langle 1, 5, 4 \rangle$

Last two points: $\langle 2, 10, 8 \rangle$

Since $\langle 2, 10, 8 \rangle = 2 \langle 1, 5, 4 \rangle$, the 3 points are collinear.

35. Let \mathbf{a} , \mathbf{b} , and \mathbf{c} be the three force vectors determined by $A \langle 0, 10, 10 \rangle$, $B \langle 4, 6, 10 \rangle$ and $C \langle 4, 6, 10 \rangle$.

$\mathbf{a} = \frac{1}{\sqrt{2}} \langle 0, 10, 10 \rangle = \langle 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

$\mathbf{b} = \frac{1}{\sqrt{152}} \langle 4, 6, 10 \rangle = \langle \frac{2}{38}, \frac{3}{38}, \frac{5}{38} \rangle$

$\mathbf{c} = \frac{1}{\sqrt{152}} \langle 4, 6, 10 \rangle = \langle \frac{2}{38}, \frac{3}{38}, \frac{5}{38} \rangle$

Must have $\mathbf{a} + \mathbf{b} + \mathbf{c} = 300\mathbf{k}$. Thus:

$$\frac{2}{38} \mathbf{b} + \frac{2}{38} \mathbf{c} = 0$$

$$\frac{1}{2} \mathbf{a} + \frac{3}{38} \mathbf{b} + \frac{3}{38} \mathbf{c} = 0$$

$$\frac{1}{2} \mathbf{a} + \frac{5}{38} \mathbf{b} + \frac{5}{38} \mathbf{c} = 300.$$

CONTINUED

35. **CONTINUED**

From the first equation $\mathbf{b} = \mathbf{c}$. From the second equation, $-\frac{1}{2}\mathbf{a} = -\frac{6}{38}\mathbf{b}$.

From the third equation, $-\frac{1}{2}\mathbf{a} = 300 - \frac{10}{38}\mathbf{b}$. Thus,

$$-\frac{6}{38}\mathbf{b} = 300 - \frac{10}{38}\mathbf{b} \Rightarrow \frac{16}{38}\mathbf{b} = 300 \text{ and } \mathbf{b} = \mathbf{c} = \frac{75 \cdot 38}{4} = 115.58.$$

$$\text{Finally, } \mathbf{a} = 2 \left(\frac{6}{38} - \frac{75 \cdot 38}{4} \right) = \frac{225}{2} = 112.5.$$

36. Let \mathbf{a} , \mathbf{b} , \mathbf{c} be the three force vectors determined by $A(0, 10, 10)$, $B(4, 6, 10)$ and $C(4, 6, 10)$.

$$\mathbf{a} = \frac{1}{\sqrt{2}}(0, 10, 10) = \frac{1}{\sqrt{2}}(0, 1, 1)$$

$$\mathbf{b} = \frac{1}{\sqrt{152}}(4, 6, 10) = \frac{1}{\sqrt{152}}\left(\frac{2}{38}, \frac{3}{38}, \frac{5}{38}\right)$$

$$\mathbf{c} = \frac{1}{\sqrt{152}}(4, 6, 10) = \frac{1}{\sqrt{152}}\left(\frac{2}{38}, \frac{3}{38}, \frac{5}{38}\right)$$

We must have $\mathbf{a} + \mathbf{b} + \mathbf{c} = 200\mathbf{k}$. Thus,

$$-\frac{2}{38}\mathbf{b} = -\frac{2}{38}\mathbf{c} = 0$$

$$-\frac{1}{2}\mathbf{a} = -\frac{3}{38}\mathbf{b} = -\frac{3}{38}\mathbf{c} = 0$$

$$-\frac{1}{2}\mathbf{a} = -\frac{5}{38}\mathbf{b} = -\frac{5}{38}\mathbf{c} = 200$$

Solving this system, $\mathbf{a} = 106.1$, $\mathbf{b} = \mathbf{c} = 77.1$.

Thus, the tensions are 106.1, 77.1 and 77.1 pounds.

$$37. \mathbf{u} = \frac{1}{\sqrt{2}}(2, 8, 2) = \frac{1}{\sqrt{2}}(1, 4, 1)$$

$$38. \mathbf{u} = \frac{1}{\sqrt{152}}(10, 15, 5) = \frac{1}{\sqrt{152}}(2, 3, 1)$$

$$39. \mathbf{u} = \frac{1}{\sqrt{7602}}(3, 2, 5) = \frac{1}{\sqrt{7602}}(3, 2, 5)$$

$$\mathbf{u} = \frac{1}{\sqrt{7602}}(3, 2, 5)$$

$$\text{Unit vector: } \frac{1}{\sqrt{7602}}(3, 2, 5)$$

$$40. \mathbf{u} = \frac{1}{\sqrt{16}}(0, 4, 12) = \frac{1}{4}(0, 1, 3)$$

41. First two points: 3, 2, 3

Last two points: 3, 2, 3

First and third points: 2, 2, 0

$$\begin{array}{ccc} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & 3 \\ 2 & 2 & 0 \end{array} \quad \begin{array}{c} 6, \quad 6, 10 \\ \hline 36 \quad 36 \quad 100 \\ \hline 172 \end{array}$$

Area $\frac{1}{2} \sqrt{172} = 2\sqrt{43}$ square units

43. The parallelogram is determined by the three vectors with initial point 0, 0, 0.

$\mathbf{u} = 3, 0, 0$, $\mathbf{v} = 2, 0, 5$, $\mathbf{w} = 0, 5, 1$

$$\begin{array}{ccc} & 3 & 0 & 0 \\ \mathbf{u} & \mathbf{v} & \mathbf{w} & \\ 2 & 0 & 5 & 75 \\ 0 & 5 & 1 & \end{array}$$

Volume $\frac{1}{6} \sqrt{75} = \frac{5\sqrt{3}}{2}$ cubic units

42. $\mathbf{u} = 1, 0, 1$, $\mathbf{v} = 1, 0, 1$ opposite sides parallel and equal length.

Adjacent sides: $\mathbf{u} = 1, 0, 1$, $\mathbf{w} = 0, 2, 0$

$$\begin{array}{ccc} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \mathbf{u} & \mathbf{w} & \\ 1 & 0 & 1 \\ 0 & 2 & 0 \end{array} \quad \begin{array}{c} 2, 0, 2 \\ \hline 4 \quad 4 \quad 2 \end{array}$$

Area $\frac{1}{2} \sqrt{4 + 4 + 2} = \sqrt{3}$ square units

44. $\mathbf{u} = 2, 0, 0$, $\mathbf{v} = 0, 4, 0$, $\mathbf{w} = 0, 0, 6$

$$\begin{array}{ccc} & 2 & 0 & 0 \\ \mathbf{u} & \mathbf{v} & \mathbf{w} & \\ 0 & 4 & 0 & 48 \\ 0 & 0 & 6 & \end{array}$$

Volume $\frac{1}{6} \sqrt{48} = 4$ cubic units

45. $\mathbf{v} = 3, 1, 6$, $\mathbf{u} = 4, 3, 6$, point: 1, 3, 5

(a) Parametric equations: $x = 1 + 4t$, $y = 3 + 3t$, $z = 5 + 6t$

(b) Symmetric equations: $\frac{x-1}{4} = \frac{y-3}{3} = \frac{z-5}{6}$

46. (a) $\mathbf{v} = 5, 20, 3$

$$x = 5t, y = 20t, z = 3t$$

$$(b) \frac{x}{5} = \frac{y}{20} = \frac{z}{3}$$

47. Use $2\mathbf{v} = 4, 5, 2$, point: 0, 0, 0.

(a) Parametric equations: $x = 4t$, $y = 5t$, $z = 2t$

(b) Symmetric equations: $\frac{x}{4} = \frac{y}{5} = \frac{z}{2}$

48. (a) $\mathbf{v} = 1, 1, 1$

$$x = 3t, y = 2t, z = 1t$$

$$(b) \frac{x-3}{1} = \frac{y-2}{1} = \frac{z-1}{1} \text{ or}$$

$$x - 3 = y - 2 = z - 1$$

49. $\mathbf{u} = 5, 0, 2$, $\mathbf{v} = 2, 3, 8$

$$\begin{array}{ccc} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \mathbf{u} & \mathbf{v} & \\ 5 & 0 & 2 \\ 2 & 3 & 8 \end{array} \quad \begin{array}{c} 6, \quad 36, 15 \\ \hline 2 \quad 3 \quad 8 \end{array}$$

$\mathbf{n} = 2, 12, 5$

$$\begin{array}{ccccccc} ax & x_0 & by & y_0 & cz & z_0 & 0 \\ 2x & 0 & 12y & 0 & 5z & 0 & 0 \\ & & 2x & 12y & 5z & 0 & \end{array}$$

50. $\mathbf{u} = 5, 5, 2$, $\mathbf{v} = 3, 5, 2$

$$\begin{array}{ccc} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \mathbf{n} & \mathbf{u} & \mathbf{v} \\ 5 & 5 & 2 \\ 3 & 5 & 2 \end{array} \quad \begin{array}{c} 0, \quad 16, 40 \\ \hline 3 \quad 5 \quad 2 \end{array}$$

$$\begin{array}{ccccccc} \text{Plane: } 0x & 1 & 16y & 3 & 40z & 4 & 0 \\ & & 2y & 3 & 5z & 4 & 0 \\ & & & 2y & 5z & 14 & 0 \end{array}$$

51. $\mathbf{n} = \mathbf{k}$, normal vector

$$\begin{array}{ccccccc} \text{Plane: } 0x & 5 & 0y & 3 & 1z & 2 & 0 \\ & & & & & z & 2 & 0 \end{array}$$

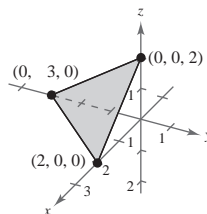
52. $\mathbf{n} = 1, 1, 2$, point: $(0, 0, 6)$

$$1x + 0y + 2z = 6 \quad 0$$

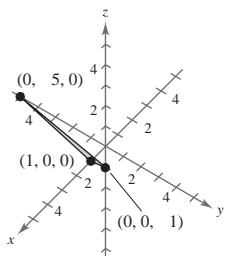
$$x + y + 2z = 12 \quad 0$$

$$x + y + 2z = 12 \quad 0$$

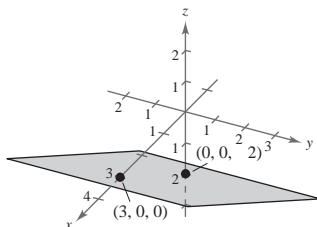
53. $3x + 2y + 3z = 6$



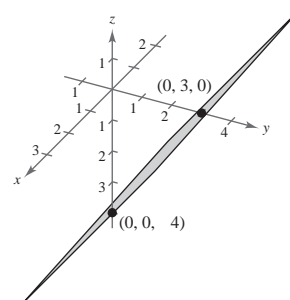
54. $5x + y + 5z = 5$



55. $2x + 3z = 6$



56. $4y + 3z = 12$



57. $\mathbf{n} = 2, -20, 6$, $P = (0, 0, 1)$ in plane, $Q = (2, 3, 10)$, $\vec{PQ} = 2, 3, 9$

$$D = \frac{\vec{PQ} \cdot \mathbf{n}}{\|\mathbf{n}\|} = \frac{2}{440} - \frac{1}{110} - \frac{110}{110} = 0.0953$$

58. $D = \frac{\vec{PQ} \cdot \mathbf{n}}{\|\mathbf{n}\|}$

$$Q = (1, 2, 3), P = (2, 0, 0) \text{ on plane. } \vec{PQ} = 1, 2, 3, \mathbf{n} = 2, -1, 1$$

$$D = \frac{1, 2, 3 \cdot 2, -1, 1}{\sqrt{6}} = \frac{2 - 1 + 3}{\sqrt{6}} = \frac{4}{\sqrt{6}}$$

59. $\mathbf{n} = 1, -10, 3$, $P = (2, 0, 0)$ in plane, $Q = (0, 0, 0)$, $\vec{PQ} = 2, 0, 0$

$$D = \frac{\vec{PQ} \cdot \mathbf{n}}{\|\mathbf{n}\|} = \frac{2}{1 + 100 + 9} = \frac{2}{110} = \frac{2}{55} \approx 0.191$$

60. $D = \frac{\vec{PQ} \cdot \mathbf{n}}{\|\mathbf{n}\|}$

$$Q = (0, 0, 0), P = (0, 0, 12) \text{ on plane. } \vec{PQ} = 0, 0, -12, \mathbf{n} = 2, 3, 1$$

$$D = \frac{0, 0, -12 \cdot 2, 3, 1}{\sqrt{14}} = \frac{-12}{\sqrt{14}} = -\frac{6\sqrt{14}}{7}$$

61. False. $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$

62. True. See page 831.

63. $\mathbf{u} \cdot \mathbf{u} = 3^2 + 2^2 + 1^2 = 14$

$$9 + 4 + 1 = 14$$

$$14$$

$$\mathbf{u}^2$$

5. The largest angle in a triangle is always opposite the longest side of the triangle. First, determine the lengths of the three sides. Then, once the largest angle has been identified, use the fact that $\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|}$, where \mathbf{u} and \mathbf{v} are defined to be the vectors that form θ . If $\mathbf{u} \cdot \mathbf{v} = 0$, the angle is a right angle. If $\mathbf{u} \cdot \mathbf{v} > 0$, the angle is acute. If $\mathbf{u} \cdot \mathbf{v} < 0$, the angle is obtuse.

(a) A: 1, 2, 0

B: 0, 0, 0

C: 2, 1, 0

$$d AB = \sqrt{5}, d AC = \sqrt{10}, d BC = \sqrt{5}$$

Angle B is largest.

$$\overrightarrow{BA} = \langle 1, 2, 0 \rangle, \overrightarrow{BC} = \langle 2, 1, 0 \rangle$$

$$\overrightarrow{BA} \cdot \overrightarrow{BC} = 0 \Rightarrow \text{The triangle is a right triangle.}$$

(b) A: 3, 0, 0

B: 0, 0, 0

C: 1, 2, 3

$$d AB = 3, d AC = \sqrt{29}, d BC = \sqrt{14}$$

Angle B is largest.

$$\overrightarrow{BA} = \langle 3, 0, 0 \rangle, \overrightarrow{BC} = \langle 1, 2, 3 \rangle$$

$$\overrightarrow{BA} \cdot \overrightarrow{BC} = 3 < 0 \Rightarrow \text{The triangle is an obtuse triangle.}$$

(c) A: 2, 3, 4

B: 0, 1, 2

C: 1, 2, 0

$$d AB = \sqrt{24}, d AC = \sqrt{50}, d BC = \sqrt{6}$$

Angle B is largest.

$$\overrightarrow{BA} = \langle 2, 4, 2 \rangle, \overrightarrow{BC} = \langle 1, 1, 2 \rangle$$

$$\overrightarrow{BA} \cdot \overrightarrow{BC} = 10 > 0 \Rightarrow \text{The triangle is an acute triangle.}$$

(d) A: 2, 7, 3

B: 1, 5, 8

C: 4, 6, 1

$$d AB = \sqrt{178}, d AC = \sqrt{189}, d BC = \sqrt{107}$$

Angle B is largest.

$$\overrightarrow{BA} = \langle 3, 12, 5 \rangle, \overrightarrow{BC} = \langle 5, 1, 9 \rangle$$

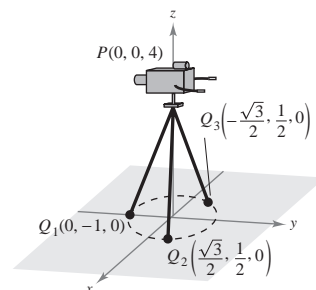
$$\overrightarrow{BA} \cdot \overrightarrow{BC} = 48 > 0 \Rightarrow \text{The triangle is an acute triangle.}$$

$$\begin{aligned}\vec{PQ}_1 &= 0, 0, 1, 0, 0, 4, 0, 1, 4 \\ \vec{PQ}_2 &= \frac{\sqrt{3}}{2}, 0, \frac{1}{2}, 0, 0, 4, -\frac{\sqrt{3}}{2}, \frac{1}{2}, 4 \\ \vec{PQ}_3 &= \frac{\sqrt{3}}{2}, 0, \frac{1}{2}, 0, 0, 4, \frac{\sqrt{3}}{2}, \frac{1}{2}, 4\end{aligned}$$

$$\text{Note that } \vec{PQ}_1 \cdot \vec{PQ}_2 = \vec{PQ}_2 \cdot \vec{PQ}_3 = \vec{PQ}_3 \cdot \vec{PQ}_1 = 17.$$

Unit vectors:

$$\begin{aligned}\mathbf{u}_1 &= \frac{1}{\sqrt{17}} 0, 1, 4 \\ \mathbf{u}_2 &= \frac{1}{\sqrt{17}} \frac{\sqrt{3}}{2}, \frac{1}{2}, 4 \\ \mathbf{u}_3 &= \frac{1}{\sqrt{17}} \frac{\sqrt{3}}{2}, \frac{1}{2}, 4\end{aligned}$$



The unit vectors $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$ give the directions of the force in each leg. Since the legs are the same length, the total weight is distributed equally among the legs. So,

$$\begin{aligned}\mathbf{F}_1 &= \frac{120}{3} \mathbf{u}_1 = \frac{40}{\sqrt{17}} 0, 1, 4 \\ \mathbf{F}_2 &= \frac{120}{3} \mathbf{u}_2 = \frac{40}{\sqrt{17}} \frac{\sqrt{3}}{2}, \frac{1}{2}, 4 \\ \mathbf{F}_3 &= \frac{120}{3} \mathbf{u}_3 = \frac{40}{\sqrt{17}} \frac{\sqrt{3}}{2}, \frac{1}{2}, 4\end{aligned}$$

7. Let A lie on the y -axis and the wall on the x -axis. Then, $A = 0, 10, 0$, $B = 8, 0, 6$, $C = 10, 0, 6$ and

$$\vec{AB} = 8, -10, 6, \vec{AC} = 10, -10, 6.$$

$$|\vec{AB}| = \sqrt{8^2 + 10^2 + 6^2} = 10\sqrt{2}$$

$$|\vec{AC}| = \sqrt{10^2 + 10^2 + 6^2} = 2\sqrt{59}$$

$$\mathbf{F}_1 = 420 \frac{\vec{AB}}{|\vec{AB}|} = \frac{420}{10\sqrt{2}} 8, -10, 6 = \frac{84}{\sqrt{2}} 4, -5, 3$$

$$\mathbf{F}_2 = 650 \frac{\vec{AC}}{|\vec{AC}|} = \frac{650}{2\sqrt{59}} 10, -10, 6 = \frac{650}{\sqrt{59}} 5, -5, 3$$

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \frac{4}{\sqrt{2}} \frac{84}{\sqrt{2}} + \frac{5}{\sqrt{59}} \frac{650}{\sqrt{59}}, \frac{5}{\sqrt{2}} \frac{84}{\sqrt{2}} - \frac{5}{\sqrt{59}} \frac{650}{\sqrt{59}}, \frac{3}{\sqrt{2}} \frac{84}{\sqrt{2}} + \frac{3}{\sqrt{59}} \frac{650}{\sqrt{59}}$$

$$= 185.526, -720.099, 432.059$$

$$|\mathbf{F}| = 860.0 \text{ lb}$$

8. Note that $\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}| |\mathbf{v}|}$. Therefore,

$$|\mathbf{u} \times \mathbf{v}| \sin \theta = |\mathbf{u} \times \mathbf{v}| \sqrt{1 - \cos^2 \theta} = |\mathbf{u} \times \mathbf{v}| \sqrt{1 - \frac{(\mathbf{u} \cdot \mathbf{v})^2}{|\mathbf{u}|^2 |\mathbf{v}|^2}} = \frac{|\mathbf{u} \times \mathbf{v}|^2}{|\mathbf{u}| |\mathbf{v}|}$$

$$= \frac{u_1^2 + u_2^2 + u_3^2 + v_1^2 + v_2^2 + v_3^2 - u_1 v_1 - u_2 v_2 - u_3 v_3}{u_1 v_1 + u_2 v_2 + u_3 v_3}$$

$$|\mathbf{u} \times \mathbf{v}| \sin \theta = |\mathbf{u} \times \mathbf{v}|$$

Since \mathbf{u} and \mathbf{v} are orthogonal, $\theta = 90^\circ$ and $\sin \theta = 1$. So, $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}| |\mathbf{v}|$, \mathbf{u}, \mathbf{v} orthogonal.

$$9. \mathbf{u} = a_1 \mathbf{i} + b_1 \mathbf{j} + c_1 \mathbf{k}, \mathbf{v} = a_2 \mathbf{i} + b_2 \mathbf{j} + c_2 \mathbf{k}, \mathbf{w} = a_3 \mathbf{i} + b_3 \mathbf{j} + c_3 \mathbf{k}$$

$$\mathbf{v} \times \mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = (b_2 c_3 - b_3 c_2) \mathbf{i} + (a_2 c_3 - a_3 c_2) \mathbf{j} + (a_2 b_3 - a_3 b_2) \mathbf{k}$$

$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & b_1 & c_1 \\ b_2 c_3 - b_3 c_2 & a_2 c_3 - a_3 c_2 & a_2 b_3 - a_3 b_2 \end{vmatrix}$$

$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} b_1 & a_2 b_3 & a_3 b_2 & c_1 & a_3 c_2 & a_2 c_3 & \mathbf{i} & a_1 & a_2 b_3 & a_3 b_2 & c_1 & b_2 c_3 & b_3 c_2 & \mathbf{j} \\ a_1 & a_3 c_2 & a_2 c_3 & b_1 & b_2 c_3 & b_3 c_2 & \mathbf{k} \\ a_2 & a_1 a_3 & b_1 b_3 & c_1 c_3 & a_3 & a_1 a_2 & b_1 b_2 & c_1 c_2 & \mathbf{i} \\ b_2 & a_1 a_3 & b_1 b_3 & c_1 c_3 & b_3 & a_1 a_2 & b_1 b_2 & c_1 c_2 & \mathbf{j} \\ c_2 & a_1 a_3 & b_1 b_3 & c_1 c_3 & c_3 & a_1 a_2 & b_1 b_2 & c_1 c_2 & \mathbf{k} \end{vmatrix}$$

$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = \mathbf{u} \times (\mathbf{v} \times \mathbf{w})$$

$$10. \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} = \begin{vmatrix} u_1 & v_2 & v_3 \\ u_2 & w_2 & w_3 \\ u_3 & w_1 & w_2 \end{vmatrix} = \begin{vmatrix} u_1 & v_1 & v_3 \\ u_2 & w_1 & w_3 \\ u_3 & w_1 & w_2 \end{vmatrix}$$

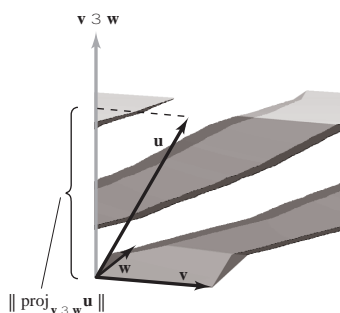
$$= \begin{vmatrix} u_1 & v_2 & v_3 \\ u_2 & w_2 & w_3 \\ u_3 & w_1 & w_2 \end{vmatrix} \mathbf{i} + \begin{vmatrix} u_1 & v_1 & v_3 \\ u_2 & w_1 & w_3 \\ u_3 & w_1 & w_2 \end{vmatrix} \mathbf{j} + \begin{vmatrix} u_1 & v_1 & v_2 \\ u_2 & w_1 & w_2 \\ u_3 & w_1 & w_2 \end{vmatrix} \mathbf{k}$$

$$= u_1 \mathbf{i} + \frac{v_2}{w_2} \mathbf{j} + \frac{v_3}{w_3} \mathbf{k} = u_2 \mathbf{j} + \frac{v_1}{w_1} \mathbf{j} + \frac{v_3}{w_3} \mathbf{j} = u_3 \mathbf{k} + \frac{v_1}{w_1} \mathbf{k} + \frac{v_2}{w_2} \mathbf{k}$$

$$= u_1 \mathbf{i} + u_2 \mathbf{j} + u_3 \mathbf{k} = \frac{v_2}{w_2} \mathbf{i} + \frac{v_3}{w_3} \mathbf{i} + \frac{v_1}{w_1} \mathbf{j} + \frac{v_3}{w_3} \mathbf{j} + \frac{v_1}{w_1} \mathbf{k} + \frac{v_2}{w_2} \mathbf{k}$$

$$\mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} = \mathbf{u} \times \mathbf{v} \times \mathbf{w}$$

11.

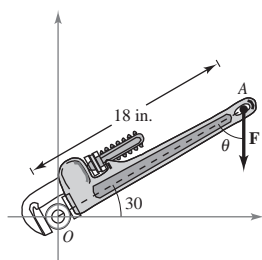

 $\mathbf{v} \times \mathbf{w}$ area of base and $\text{proj}_{\mathbf{v} \times \mathbf{w}} \mathbf{u}$ height of parallelepiped

Therefore, the volume is

$$V = \text{height} \times \text{area of base} = \text{proj}_{\mathbf{v} \times \mathbf{w}} \mathbf{u} \times \mathbf{v} \times \mathbf{w}$$

$$= \frac{\mathbf{u} \times \mathbf{v} \times \mathbf{w}}{\mathbf{v} \times \mathbf{w}} \times \mathbf{v} \times \mathbf{w} = \mathbf{u} \times \mathbf{v} \times \mathbf{w}.$$

12.



(a) $O = 0, 0, 0$

$A = 18 \cos 30^\circ, 18 \sin 30^\circ, 0 = 9\sqrt{3}, 9, 0$

$\overrightarrow{OA} = 9\sqrt{3}\mathbf{i} + 9\mathbf{j}, 0$

$\mathbf{M} = \overrightarrow{OA} \times \mathbf{F} = \overrightarrow{OA} \times \mathbf{F} \sin$

$$= \sqrt{9^2 + 3^2} \times 9 \times 0^2 \times 60 \sin$$

$$1080 \sin$$

1440

0
0

180

CONTINUED

12. ⚙ CONTINUED ⚙

(b) $\vec{M} = 1080 \sin 45^\circ = 1080 \frac{\sqrt{2}}{2} = 540\sqrt{2} = 763.7 \text{ in-lb}$

(c) $\vec{M} = 1080 \sin \theta$ has its maximum value at 90° . In order to generate the maximum torque, the force should be applied in a direction perpendicular to the wrench handle.

13. (a) In inches: $\vec{AB} = 15\mathbf{j} - 12\mathbf{k}$

In feet: $\vec{AB} = \frac{5}{4}\mathbf{j} - \mathbf{k}$

$\mathbf{F} = 200 \cos \theta \mathbf{j} + \sin \theta \mathbf{k}$

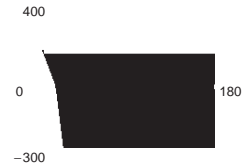
(b) $\vec{AB} \cdot \mathbf{F} = 0 + \frac{5}{4} \sin \theta - 1 \cos \theta = 250 \sin \theta - 200 \cos \theta$

$\vec{AB} \cdot \mathbf{F} = 250 \sin \theta - 200 \cos \theta = 250 \sin \theta - 200 \cos \theta$

(c) When $\theta = 30^\circ$: $\vec{AB} \cdot \mathbf{F} = 250 \sin 30^\circ - 200 \cos 30^\circ = 125 - 173.2 = -48.2$

(d) From the graph we see that the maximum value occurs when $\theta = 51.34^\circ$.

(e) From the graph we see that the zero occurs when $\theta = 141.34^\circ$, the angle making \vec{AB} parallel to \mathbf{F} .



14. The area of the triangle is one-half of the area of any of the 3 parallelograms having the following adjacent sides:

\mathbf{b} and \mathbf{c} , \mathbf{b} and \mathbf{a} , \mathbf{c} and \mathbf{a}

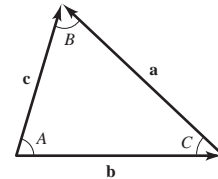
So,

$$\text{Area} = \frac{1}{2} \begin{vmatrix} \mathbf{b} & \mathbf{c} \\ \mathbf{a} & \mathbf{c} \end{vmatrix} = \frac{1}{2} \begin{vmatrix} \mathbf{a} & \mathbf{c} \\ \mathbf{a} & \mathbf{b} \end{vmatrix} = \frac{1}{2} \begin{vmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{c} & \mathbf{a} \end{vmatrix}$$

$$\mathbf{b} \cdot \mathbf{c} \sin A = \mathbf{a} \cdot \mathbf{c} \sin B = \mathbf{a} \cdot \mathbf{b} \sin C$$

Divide by $\mathbf{a} \cdot \mathbf{b} \cdot \mathbf{c}$:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



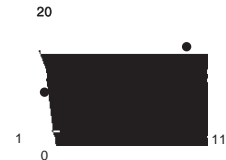
15. First insect: $x = 6 - t, y = 8 - t, z = 3 - t$

Second insect: $x = 1 - t, y = 2 - t, z = 2 - t$

(a) When $t = 0$ the first insect is located at $(6, 8, 3)$ and the second insect is located at $(1, 2, 0)$.

$d = \sqrt{(6-1)^2 + (8-2)^2 + (3-0)^2} = \sqrt{25 + 36 + 9} = \sqrt{70} \text{ inches}$

(b) $d = \sqrt{(6-t-1+t)^2 + (8-t-2+t)^2 + (3-t-2+t)^2} = \sqrt{5^2 + 6^2 + t^2} = \sqrt{61 + t^2}$



t	0	1	2	3	4	5	6	7	8	9	10
d	$\sqrt{70}$	$\sqrt{45}$	$\sqrt{30}$	5	$\sqrt{30}$	$\sqrt{45}$	$\sqrt{70}$	$\sqrt{105}$	$\sqrt{150}$	$\sqrt{205}$	$\sqrt{270}$

⚙ CONTINUED ⚙

15. CONTINUED

- (c) The distance between the two insects appears to lessen in the first 3 seconds, but then begins to increase with time.
- (d) When $t = 3$, the insects get within 5 inches of each other.

16. (a) $x = 2 + 4t, y = 3, z = 1 - t \Rightarrow$ direction vector $\mathbf{u} = 4, 0, -1$

Let P be the point on the line with $t = 0$:

$$\begin{aligned}
 P &= (2, 3, 1), Q = (1, 5, 2) \\
 \overrightarrow{PQ} &= \langle 1, 2, 1 \rangle \\
 \overrightarrow{PQ} \cdot \mathbf{u} &= \langle 1, 2, 1 \rangle \cdot \langle 4, 0, -1 \rangle = 4 - 1 = 3 \\
 \|\overrightarrow{PQ}\| &= \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6} \\
 D &= \frac{|\overrightarrow{PQ} \cdot \mathbf{u}|}{\|\mathbf{u}\|} = \frac{3}{\sqrt{16 + 0 + 1}} = \frac{3}{\sqrt{17}} \approx 0.736
 \end{aligned}$$

- (b) $x = 2t, y = 3 - t, z = 2 - 2t \Rightarrow$ direction vector $\mathbf{u} = 2, -1, -2$

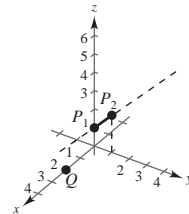
Let P be the point on the line with $t = 0$:

$$\begin{aligned}
 P &= (0, 3, 2), Q = (1, 2, 4) \\
 \overrightarrow{PQ} &= \langle 1, -1, 2 \rangle \\
 \overrightarrow{PQ} \cdot \mathbf{u} &= \langle 1, -1, 2 \rangle \cdot \langle 2, -1, -2 \rangle = 2 + 1 - 4 = -1 \\
 \|\overrightarrow{PQ}\| &= \sqrt{1^2 + (-1)^2 + 2^2} = \sqrt{6} \\
 D &= \frac{|\overrightarrow{PQ} \cdot \mathbf{u}|}{\|\mathbf{u}\|} = \frac{1}{\sqrt{4 + 1 + 4}} = \frac{1}{\sqrt{9}} = \frac{1}{3} \approx 0.333
 \end{aligned}$$

17. (a) $\mathbf{u} = 0, 1, 1$ direction vector of line determined by P_1 and P_2

$$\begin{aligned}
 D &= \frac{|\overrightarrow{P_1Q} \cdot \mathbf{u}|}{\|\mathbf{u}\|} = \frac{|(2, 0, 1) \cdot (0, 1, 1)|}{\sqrt{0^2 + 1^2 + 1^2}} \\
 &= \frac{|1|}{\sqrt{2}} = \frac{1}{\sqrt{2}}
 \end{aligned}$$

- (b) The shortest distance to the line segment is $P_1Q = \sqrt{2^2 + 0^2 + 1^2} = \sqrt{5}$.



18. (a) $x = t - 3, y = \frac{1}{2}t - 1, z = 2t - 1 \Rightarrow$ direction vector $\mathbf{u} = 1, \frac{1}{2}, 2$

Let $Q = 4, 3, s$.

Let P be the point on the line corresponding to $t = 0$: $P = 3, 1, -1$

$$\overrightarrow{PQ} = 4 - 3, 3 - 1, s - (-1) = 1, 2, s + 1$$

$$\overrightarrow{PQ} \cdot \mathbf{u} = 1 \cdot 1 + 2 \cdot \frac{1}{2} + (s + 1) \cdot 2 = 1 + 1 + 2s + 2 = 4 + 2s$$

$$|\mathbf{u}| = \sqrt{1^2 + \left(\frac{1}{2}\right)^2 + 2^2} = \sqrt{1 + \frac{1}{4} + 4} = \sqrt{\frac{21}{4}} = \frac{\sqrt{21}}{2}$$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{u}|}{|\mathbf{u}|} = \frac{|4 + 2s|}{\frac{\sqrt{21}}{2}} = \frac{2|4 + 2s|}{\sqrt{21}} = \frac{4|2 + s|}{\sqrt{21}}$$

(b)

$$D = \frac{4|2 + s|}{\sqrt{21}}$$

Minimum distance is $D = 2.23607$ at $s = -1$.

(c) $D = \frac{4|2 + s|}{\sqrt{21}}$

As s approaches very large (very positive) or very small (very negative) values, the expression under the radical is dominated by the term $105s^2$:

$$105s^2 - 210s + 2310 \approx 105s^2, \text{ for large } |s|$$

So, at large values of $|s|$,

$$D \approx \frac{\sqrt{105s^2}}{\frac{\sqrt{21}}{2}} = \frac{\sqrt{105}}{\frac{\sqrt{21}}{2}} s = 2\sqrt{5}s$$

Asymptotes: $D = 2\sqrt{5}s$

