ESO207A: Data Structures and Algorithms

Practice Set *5: Shortest Paths* –

Problem 1. Give a linear-time algorithm for the following problem. Given an undirected graph $G = (V, E)$ with unit edge weights and a distinct source vertex s, find the *number* of shortest paths from s to v, for every vertex $v \in V - \{s\}$.

Problem 2. Given a graph with possible negative weight edges, add a large enough positive constant to each edge weight so that it becomes positive, and now run Dijkstra's algorithm to compute shortest paths. Give a counterexample and also explain briefly.

Problem 3. Given a directed graph $G = (V, E)$ with (possibly negative) edge weights along with a specific node $s \in V$ and a tree $T = (V, E')$, give a (linear-time) algorithm that checks whether T is the shortest path tree for G with starting point s.

Problem 4. Given a directed graph $G = (V, E)$ with (possibly negative) edge weights and an additional guarantee that between any two vertices there exists a shortest path of length k (no of hops), give an $O(k|E|)$ time algorithm to compute the single source shortest path problem.

Problem 5. You are given a directed graph with positively weighted edges. Give an efficient algorithm that returns the length of the shortest cycle in the graph. (Can you do this in time $O(|V|^3)$?)

Problem 6. You are given an undirected graph $G = (V, E)$ with positive edge weights and an edge $e \in E$. Find the length of the shortest cycle containing the edge e.