ESO207: Data Structures and Algorithms

Programming Assignment *1* *Due: August 26 midnight*

Instructions: The precise input-output format will be specified by Programming TAs for SPOJ.

Problem 1. FFT

1. Write the following program. *Input:* Given a polynomial $A(x)$ that is specified by providing its degree $n-1$, and its n coefficients $a_0, a_1, \ldots, a_{n-1}$ in order. Let a be the n-dimensional vector of its coefficients. *Output* $DFT(a,n)$. Recall that $DFT(a,n)$ is defined as
$$DFT(a,n) = \begin{bmatrix} A(w_n^0) \\ A(w_n^1) \\ \vdots \\ A(w_n^{n-1}) \end{bmatrix} .$$
2. Write the following program. Given an n-dimensional vector of complex numbers $y = [y_0, y_1, \ldots, y_{n-1}]$ output $DFT_n^{-1}(y)$. See note 2 below.

Notes.

1. Note that the input coefficients for $A(x)$ can be complex numbers.
2. Let F_n denote the $n \times n$ DFT matrix. That is,
$$F_n = \begin{bmatrix} 1 & 1 & 1 & \ldots & 1 \\ 1 & \omega_n & \omega_n^2 & \ldots & \omega_n^{n-1} \\ 1 & \omega_n^2 & \omega_n^4 & \ldots & \omega_n^{2(n-1)} \\ & \vdots & \vdots & \vdots & \\ 1 & \omega_n^{n-1} & \omega_n^{2(n-1)} & \ldots & \omega_n^{(n-1)^2} \end{bmatrix}$$
Recall by taking inner-product of any two columns that $F_n^* F_n = nI$. Hence, $F_n^{-1} = \frac{1}{n} F_n^*$ and therefore,
$$(DFT)_n^{-1}(y) = (1/n) F_n^*(y) .$$
We obtain two ways of computing DFT_n^{-1}. From definition of F_n^*, we have that F_n^* is the same as that of F_n^* with ω_n replaced by $\overline{\omega_n} = \omega_n^{-1} = e^{-2\pi i/n}$. So, in the computation of $F_n y$, if we replace the role of ω_n by ω_n^{-1} appropriately throughout, and divide by n, we should obtain $DFT^{-1}(y)$. The second method comes by observing the rows of F_n^* and relating them to rows of F_n.
$$F_n^* = \begin{bmatrix} 1 & 1 & 1 & \ldots & 1 \\ 1 & \overline{\omega_n} & \overline{\omega_n}^2 & \ldots & \overline{\omega_n}^{n-1} \\ 1 & \overline{\omega_n}^2 & \overline{\omega_n}^4 & \ldots & \overline{\omega_n}^{2(n-1)} \\ & \vdots & \vdots & \vdots & \\ 1 & \overline{\omega_n}^{n-1} & \overline{\omega_n}^{2(n-1)} & \ldots & \overline{\omega_n}^{(n-1)^2} \end{bmatrix}$$