ESO207A: Data Structures and Algorithms	
Homework *4b: Basic Graph Algorithms*	HW Due Date: Nov 2, 2018

Instructions.

1. Start each problem on a new sheet. For each problem, write your name, Roll No., the problem number, the date and the names of any students with whom you collaborated. Remember that you must write the answer and the algorithm in your own words.
2. For questions in which algorithms are asked for, first summarize the problem you are solving and your results (including time/space complexity as appropriate). The body of the write-up should provide the following:
   (a) A clear description of the algorithm in English and/or pseudo-code, where, helpful.
   (b) At least one worked example or diagram to show more precisely how your algorithm works.
   (c) A proof/argument of the correctness of the algorithm.
   (d) An analysis of the running time of the algorithm.

   Remember, your goal is to communicate. *Full marks will be given only to correct solutions which are described clearly.* Convoluted and unclear descriptions will receive *low marks*.

Problem 1. *Topological Sorting.* In class, a basic algorithm was presented to compute a topological sorting of a given *DAG* G.

repeat
    Find a source vertex and delete it from the graph G.
until the graph is empty.

A source vertex is any vertex of the graph that has no incoming edges. Prove the following.

1. Prove that this algorithm produces a valid linearization of the input DAG.
2. Show that this algorithm can be implemented in linear time.

Problem 2. Design a (simple) linear-time algorithm that given an undirected graph G and a specific edge e, determines whether G has a cycle containing the edge e.

Problem 3. *Modeling using Graphs.* There is a city *Graphotopia* where all the streets are one-way. The mayor claims that there is a way to drive legally from any intersection in town to any other intersection. The opposition is not convinced and you have to test the veracity of the mayor's claims.

1. Model the problem graph theoretically and show how can it be solved in linear time.

2. Suppose the above claim is false. The mayor now changes her claim. She claims that if one starts at town hall via the one-way streets, then no matter where you reach, you can always drive legally back to the town hall. Model this claim graph theoretically and give an algorithm to solve it in linear time.

Problem 4. You are given a curriculum of courses for a certain department in a certain , all of which are compulsory. The pre-requisite graph G is a directed graph that has a node for each course and there is an edge from course u to course v if course u is a pre-requisite for course v. Design an algorithm that takes as input G and finds the minimum number of semesters necessary to complete the curriculum (assuming that a student can take any number of courses in a semester).