Assignment 2

ESO208

1.

$$4x_1 + 2x_2 = 10$$
$$2x_1 + 4x_2 + x_3 = 11.5$$
$$x_2 + 5x_3 = 5$$

```
L matrix is:

4.000000        0.000000        0.000000
2.000000        3.000000        0.000000
0.000000        1.000000        4.666667


U matrix is:

1.000000        0.500000        0.000000
0.000000        1.000000        0.333333
0.000000        0.000000        1.000000


Solution vactor is:

1.517857
1.964286
0.607143
```

2. Solve $\begin{bmatrix} 9.3746 & 3.0416 & -2.4371 \\ 3.0416 & 6.1832 & 1.2163 \\ -2.4371 & 1.2163 & 8.4429 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9.2333 \\ 8.2049 \\ 3.9339 \end{bmatrix}$

```
A(upper triangualar matrix):

9.374600        3.041600        -2.437100
0.000000        5.196349        2.007020
0.000000        -0.000000       7.068908


Solution vector x:
0.900177
0.761367
0.624216
```

3. $\boldsymbol{A} = \begin{bmatrix} 9 & 3 & -2 \\ 3 & 6 & 1 \\ -2 & 1 & 9 \end{bmatrix}$

a) Reduce it to an upper triangular matrix using Gauss Elimination Procedure.

b) Synthesize a lower triangular matrix $\boldsymbol{L}$ and an upper triangular matrix $\boldsymbol{U}$ from the steps of (a) above such that $\boldsymbol{A} = \boldsymbol{LU}$.

c) Compute $\boldsymbol{A}^{-1}$ using the $\boldsymbol{LU}$ decomposition obtained in (b).

(a)

```
A(upper triangualar matrix):

9.000000        3.000000        -2.000000
0.000000        5.000000        1.666667
0.000000        0.000000        8.000000
```

(b)

```
L matrix is:

1.000000        0.000000        0.000000
0.333333        1.000000        0.000000
-0.222222       0.333333        1.000000


U matrix is:

9.000000        3.000000        -2.000000
0.000000        5.000000        1.666667
0.000000        0.000000        8.000000
```

[c]

```
inverse of the matrix A is (By Gauss jordan method)

0.147222        -0.080556       0.041667
-0.080556       0.213889        -0.041667
0.041667        -0.041667       0.125000
```

4.

```
Solution using thomas algorithm:

-1.933333
-0.866667
-0.533333
0.733333
```

5.

```
inverse of the matrix A is (By Gauss jordan method)

25.000000       -41.000000      10.000000       -6.000000
-41.000000      68.000000       -17.000000      10.000000
10.000000       -17.000000      5.000000        -3.000000
-6.000000       10.000000       -3.000000       2.000000
```

```
Solution vector x:
1.000000
-1.000000
1.000000
-1.000000
```

Problem 2.

1. Consider the following matrix:

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 \\ 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

a) Find the eigenvalue of maximum absolute magnitude and the corresponding eigenvector using the Power method with an accuracy of 0.001% of relative approximate error on the eigenvalue.

(a)

```
Direct Power method

Eigenvalue
5.302686

Eigenvector
0.302787
-1.000000
0.999928
-0.302742


Iterations
28
```

(b)

d) Obtain all the eigenvalues using *QR* algorithm and compare with those obtained in (a), (b) and (c) above.

```
QR decomposition

Eigenvalues
5.302727
3.618067
1.696465
1.382585


iterations
16
```

(2)

2. Consider the following Matrix:

$$\begin{bmatrix} 7 & -2 & 1 \\ -2 & 10 & -2 \\ 1 & -2 & 7 \end{bmatrix}$$

(a)

```
Direct Power method

Eigenvalue
12.000092

Eigenvector
0.500023
-1.000000
0.500000


Iterations
17
```

(b)

```
QR decomposition

Eigenvalues
11.999979
6.000010
6.000000


iterations
11
```

[3]

3. Consider the following matrix:

$$\begin{bmatrix} 3 & 4 & 1 \\ 3 & 5 & 1 \\ 2 & 2 & 1 \end{bmatrix}$$

a) Find the eigenvalue of maximum absolute magnitude and the corresponding eigenvector using the Power method.

d) Obtain the eigenvalues using *QR* algorithm.

{a}

```
Direct Power method

Eigenvalue
8.156859

Eigenvector
0.877404
1.000000
0.524645


Iterations
6
```

(b)

```
QR decomposition

Eigenvalues
8.156848
0.657015
0.186596


iterations
6
```