Muon Capture and Muon Lifetime

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We survey a new generation of precision muon lifetime experiments. The goal of the MuCap experiment is a determination of the rate for muon capture on the free proton to 1 percent, from which the induced pseudoscalar form factor g_P of the nucleon can be derived with 7 percent precision. A measurement of the related μd capture process with similar precision would provide unique information on the axial current in the two nucleon system, relevant for fundamental neutrino reactions on deuterium. The MuLan experiment aims to measure the positive muon lifetime with 20 fold improved precision compared to present knowledge in order to determine the Fermi Coupling Constant G_F to better than 1 ppm.

1 Overview

A new generation of muon lifetime experiments is under preparation at the Paul Scherrer Institute (see Table 1). The combination of novel experimental approaches, technological advances and excellent beam quality promises a dramatic improvement in precision by typically an order of magnitude over earlier efforts. This paper surveys the scientific motivation and impact as well as the strategy and status of these experiments.

Table 1: Overview of experimental program of the MuCap and MuLan collaborations.

Project	MuCap Experiment	μd Project	MuLan Experiment
Physics	nucleon form factor, chiral symmetry	EW reactions in 2-N system, astrophysics	fundamental constant of standard model
Process	$\mu^- + p \rightarrow n + \nu_\mu$	$\mu^- + d \rightarrow n + n + \nu_\mu$	$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$
Observable	Λ_S	Λ_D	τ_{μ^+}
Precision Goal	$\leq 1\%$	$\leq 1\%$	≤ 1 ppm
Physics Goal	$g_P \leq 7\%$	axial current, L_{1A}	$G_F \leq 1$ ppm

2 Muon Capture on the Proton

2.1 Scientific Motivation

Precision measurements of muon capture by the proton provide an excellent opportunity to probe the weak axial current of the nucleon and to test our understanding of chiral symmetry breaking in QCD. The importance of such experiments is underscored by the current controversy between experiment and basic QCD predictions on the induced pseudoscalar coupling constant of the proton.

Ordinary muon capture (OMC) is a basic electroweak charged current reaction involving first-generation quarks and second-generation leptons

$$\mu^- + p \rightarrow n + \nu_\mu. \tag{1}$$

By virtue of Lorentz covariance and in the absence of second-class currents, the microscopic electroweak structure of the nucleon can be parametrized by the four form factors g_V, g_M, g_A, and g_P that determine the matrix elements of the charged vector and axial currents [4, 5]

$$\bar{u}_n(p') \left[g_V \gamma_\alpha + i \frac{g_M}{2M_n} \sigma_{\alpha\beta} q^\beta - \left(g_A \gamma_\alpha \gamma_5 + \frac{g_P}{m_\mu} \gamma_5 q_\alpha \right) \right] u_p(p). \tag{2}$$

The momentum transfer relevant for reaction (1) is $q_0^2 = (p' - p)^2 = -0.88 \, m_\mu^2$. The first three of these form factors are well determined by standard model symmetries and experimental data, leading to $g_V(q_0^2) = 0.9755(5)$, $g_M(q_0^2) = 3.582(3)$ and $g_A(q_0^2) = 1.245(3)$.

Table 2: Recent calculations of $g_P \equiv g_P(q_0^2)$ and the capture rates Λ_S and Λ_T from the singlet and triplet state of the $p\mu$ atom, respectively. Comparison of NLO and NNLO (next-to-next-to-leading order) calculations indicates good convergence of ChPT.

Reference	Year		g_P	Λ_S (s^{-1})	Λ_T (s^{-1})	Comment
[6]	1994		8.44(23)			chiral Ward identities
[7]	1997		8.21(9)			O(p^3)
[8]	2000		8.475(76)	688.4(3.8)	12.01(12)	
[9]	2001	NLO		711	14.0	small scale expansion
		NNLO		687.4	12.9	
[10]	2001	NLO		722	12.2	baryon ChPT
		NNLO		695	11.9	

The induced pseudoscalar term is a direct consequence of the partial conservation of the axial vector current. Its pion pole structure plus the leading correction have been derived early on within current algebra [3]. During the last ten years g_P has been studied with increasing sophistication with baryon chiral perturbation theory (ChPT) [4, 5], i.e., within a model-independent effective theory of QCD. The theoretical results are

summarized in Table 2 and indicate remarkably robust predictions for g_P at the 2-3% level, where the main uncertainty is related to the present knowledge of the pion–nucleon coupling constant. Very recently a direct calculation confirmed that the ChPT corrections at the two-loop order are small [11]. Ref. [8] specifically addresses the sensitivity of muon capture experiments to physics beyond the standard model. A similar body of recent theoretical work has focused on the more intricate calculations of radiative muon capture in hydrogen (RMC) [4, 5].
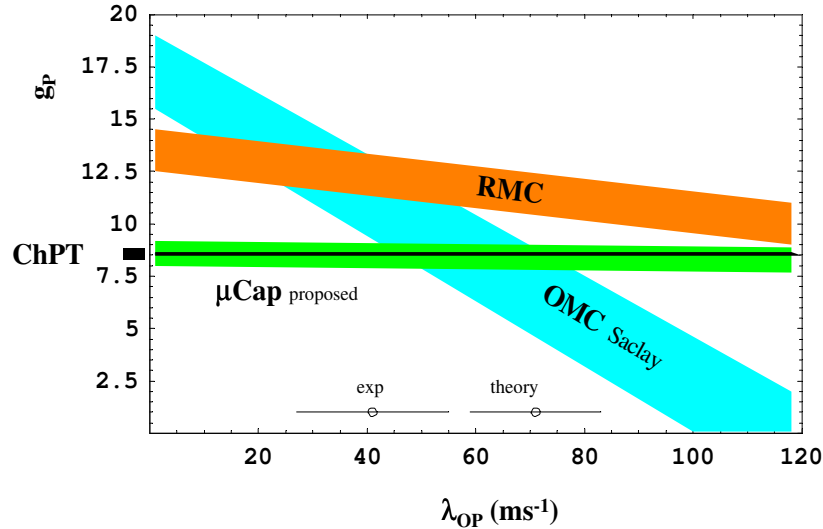

Figure 1: Current constraints on g_P as function of λ_{OP} from a recent updated analysis [5]. Experimental results from ordinary muon capture (OMC) [12], radiative muon capture (RMC) [13], and chiral perturbation theory (ChPT). The proposed MuCap experiment will be more precise and less sensitive to λ_{OP}.

Despite more than 30 years of experimental effort in this field, existing OMC data only poorly constrain g_P. In addition to the small rate Λ_S and the all neutral final state of reaction (1), a main problem lies in the kinetics of muonic hydrogen. In order to achieve acceptable muon stop rates, most experiments were performed in high density (i.e. liquid) targets, where muonic atoms quickly form $pp\mu$ molecules. The capture rates from these states differ due to the strong spin dependence of the V-A interaction, so that the uncertain transition rate λ_{OP} between the molecular ortho and para state confuses the interpretation of observed capture rates. The recent pioneering measurement of the RMC process [13] is less sensitive to λ_{OP}, but disagrees by 4.2 standard deviations from the accurate theory. Fig. 1 summarizes the present controversial situation [5]. In spite of intense theoretical scrutiny, RMC could not be reconciled with theory, whereas even the best OMC measurement cannot clarify this issue due to its dependence on muonic molecular physics. The most accurate determination of g_P in *nuclear* muon capture from a recent experiment on μ^3He capture [14] gives $g_p = 8.53 \pm 1.54$ in good agreement with theory. Note that the accuracy is limited by the theoretical extraction of g_P from the three–nucleon system. A very recent calculation argues that this uncertainty in g_P can be reduced to 6% by constraining the axial two-body current contributions from tritium beta decay [15].

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2.2 The MuCap experiment

The MuCap experiment [1] is based on a new method that avoids the above mentioned molecular and other key uncertainties of earlier efforts, like the neutron detector calibration if the outgoing neutron from reaction (1) is observed. The experiment is a muon lifetime measurement in ultra-pure and deuterium-depleted hydrogen gas. The measured decay lifetime $\tau_{\mu-}$ of the negative muon in hydrogen is shorter, compared with that of the positive muon $\tau_{\mu+}$, because of the additional muon capture reaction. The rate $\Lambda_S = 1/\tau_{\mu-} - 1/\tau_{\mu+}$ can be determined to 1% if both the lifetime of the positive and negative muon are measured with at least 10 ppm precision. Muons of both polarities will be stopped in an active target. Incoming muons are tracked by wire chambers and are stopped in a specially developed time projection chamber (TPC) contained in a 10-atm hydrogen pressure vessel. Two cylindrical wire chambers and a large scintillator hodoscope surround the TPC, covering an effective solid angle $\Omega/4\pi \sim 75\%$ (see Fig. 2). This system will reconstruct the trajectories of the electrons from muon decay.

Figure 2: MuCap detector showing scintillator barrel, cyclindrical wire chambers and vacuum pipe supporting the hydrogen vessel.

Several unique features allow a dramatic improvement in precision:

Unambiguous interpretation. As the target density is only 1% of LH$_2$, $pp\mu$ formation, which scales with density, is slowed. Muon capture takes place predominantly from the singlet hyperfine state of the $p\mu$ atom and is nearly independent of λ_{OP} (see fig. 1).

Clean muon stop definition. As the muon capture rate in higher-Z material can exceed Λ_S by several orders of magnitude, it is essential to eliminate muon stops and delayed diffusion to wall materials. By tracking the incident muons inside the TPC in all three dimensions, the muon stop location is determined event by event. This eliminates wall stops and allows systematic off-line studies by selective cuts on the stopping distribution.

Gas purity control. Critical background reactions leading to charged recoils can be monitored *in situ* with the TPC. This includes detection of nuclear recoils in a $\mu + Z \rightarrow Z' + \nu$ capture, where a sensitivity to impurity levels of $\leq 10^{-8}$ has been demonstrated. Also $p\mu \rightarrow d\mu$ transfer on isotopic deuterium impurities in the target gas, which subsequently leads to significant $d\mu$ diffusion, can be directly seen in the data. In addition, a recirculating purification system is under development and gas

analysis of higher-Z impurities and deuterium have been developed on the 0.01 and 1 ppm sensitivity level, respectively.

High statistics. The detector system can operate with high muon stop rates up to 30 kHz using a custom designed, dead-time free readout electronics for the TPC and the electron detectors. Pile-up effects, which have traditionally limited the acceptable rate by causing high accidental background, are reduced by identifying muon electron pairs by their common vertex.

μSR rotation. The remnant polarization for positive muons introduces a position-dependent intensity variation of the decay positrons, which can affect the lifetime measurement if the detector is not perfectly uniform. A saddle coil magnet generates a constant dipole field of ~ 80 G in the target region, which will precess the muon spin at 1 MHz. Monte Carlo studies indicate that this sinusoidal part largely decouples from the lifetime fit to the data.
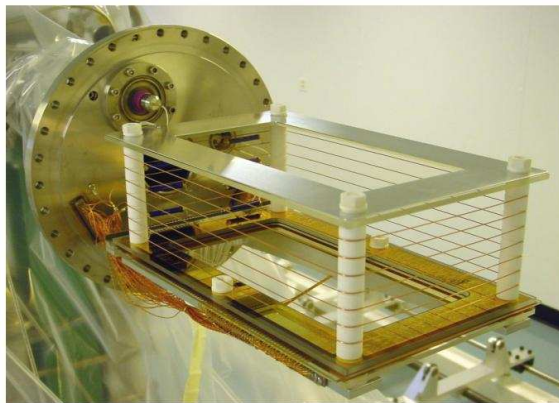

Figure 3: High-purity TPC. Drift volume height is 120 mm; area is 300×150 mm^2.

Several engineering runs with prototype TPCs have been performed at PSI to establish the feasibility of the new method and to optimize physics and detector parameters [16, 17]. Fig. 3 shows the new TPC during assembly in the hydrogen pressure vessel. The design was optimized for the extremely stringent experimental purity requirements. Only UHV-proof materials were used, bakeable to 130° C. Chamber glass frames with metallic coatings were developed onto which the gold coated tungsten wires are soldered. The glass matches the small thermal expansion of tungsten. The drift voltage of 30 kV produces a homogeneous electrical field of 2 kV/cm. In 10 bar hydrogen, this leads to drift velocities of ~ 5 mm/μs. In the MWPC located at the bottom of the TPC, anode wires and cathode wires are read out giving two-dimensional coordinates. Operation at ~ 6.5 kV high voltage results in a typical gas amplification of 10^4.

The final hydrogen chamber system has been constructed and is presently being conditioned. The main part of the electron detector was commissioned in fall of 2002 and was used for a high statistics μ^+ lifetime measurement in a realistic set-up. This included the precession magnet, the full data acquisition chain and a selection of target materials to study μSR effects. For 2003, two runs are planned: a commissioning/integration run of the TPC and first physics data taking in fall. In 2004, we foresee a high statistics MuCap run with a continuous beam or with the advanced Muon-On-Request beam [18], which is developed in the context of the MuLan experiment.

3 Muon Capture on the Deuteron

Muon capture on the deuteron,

$$\mu^- + d \to n + n + \nu_\mu, \tag{3}$$

results from the same hadronic current as shown in Eq. 2 and its rate Λ_D from the doublet state of the $d\mu$ atoms shows a similar sensitivity to g_P as Λ_S. There are additional interesting features. The final 3-body state covers a broad range of momentum transfer to the 2-N system (see Fig. 4). The properties of the 2-N system enter, in particular the deuteron wave function and the neutron scattering length a_{nn} in the final state. Furthermore, two-body currents contribute, making process (3) uniquely suited to study the axial meson exchange currents (MEC) in the 2-N system.
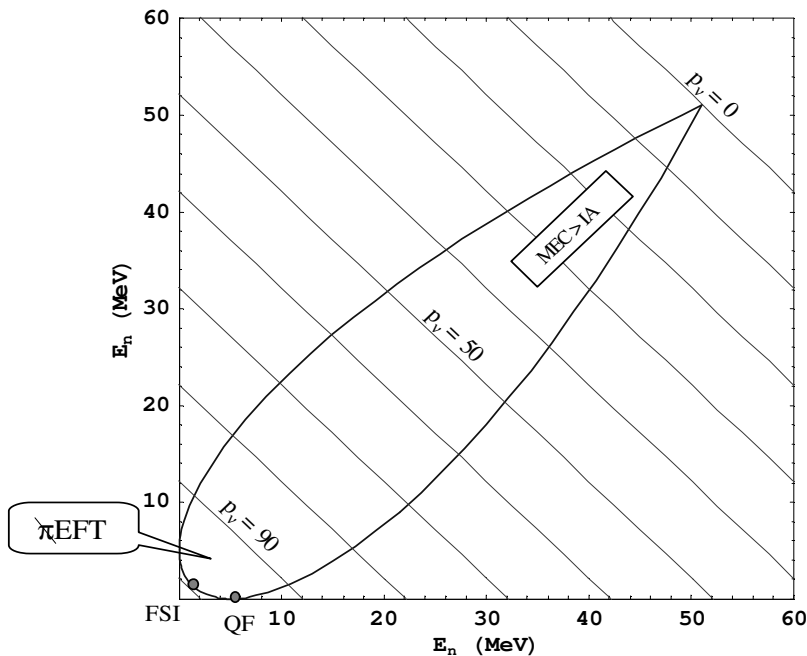

Figure 4: The μd capture Dalitz plot as function of neutron energy. Diagonal lines indicate constant neutrino momentum p_ν (MeV/c). Interesting kinematic regions include: final state interaction (FSI) ; quasifree (QF) ; $p_\nu \geq 90$ MeV/c, where pionless EFT applies; small p_ν, where MECs dominate over impulse approximation.

In recent years the structure of the two-nucleon system and its response to electro-weak probes have received considerable attention. Progress has been made in complementary nuclear physics approaches, the calculation within the framework of phenomenological Lagrangian models [19], the development of pionless effective theory [20] of nucleon-nucleon interaction and the hybrid MEEFT approach [21], which incoporates both effective field theory (EFT) and Lagrangian models. These sophisticated calculations were partially fueled by astrophysics interest, the importance of solar fusion $p+p \to d+e^+ +\nu$ as well as charged and neutral current $\nu + d$ scattering for the solar neutrino problem

and the analysis of the SNO experiment. In particular, it has been shown in model-independent EFT [22], that up to NNLO all neutrino deuteron break-up channels as well as pp fusion are related by one isovector axial two-body current, parametrized by the counterterm L_{1A}. This stimulated intense effort [23] to calibrate all of these reactions by a single accurate measurement. Alas, precision measurements of electroweak reactions in the 2-N system are extremely difficult, so that the most precise constraint of L_{1A} is derived from the 3-N system [19, 23].

It was suggested by Kammel and Chen [17, 25], that a determination of Λ_D with $\sim 1\%$ precision could provide the most accurate measurement of an electroweak reaction in the two-nucleon system and, in particular, determine L_{1A}. Such an experiment is under consideration by our collaboration. In this context, one has to address at least two questions: (a) Is reaction (3) soft enough so that it can be compared to solar neutrino reactions? (b) Is a measurement at the required precision feasible?

A recent exploratory calculation [24] within the framework of baryon ChPT constraining the axial MECs by tritium decay gives an affirmative answer to (a). Λ_D can be calculated with 1% precision, while the contribution from the small p_ν region, where ChPT is questionable, is negligible. The L_{1A} term contributes about 4% to the rate. On the other hand, the applicability of pionless EFT is limited to $p_\nu \geq 90$ MeV/c and it would be preferable to measure the reduced rate for this region [25].
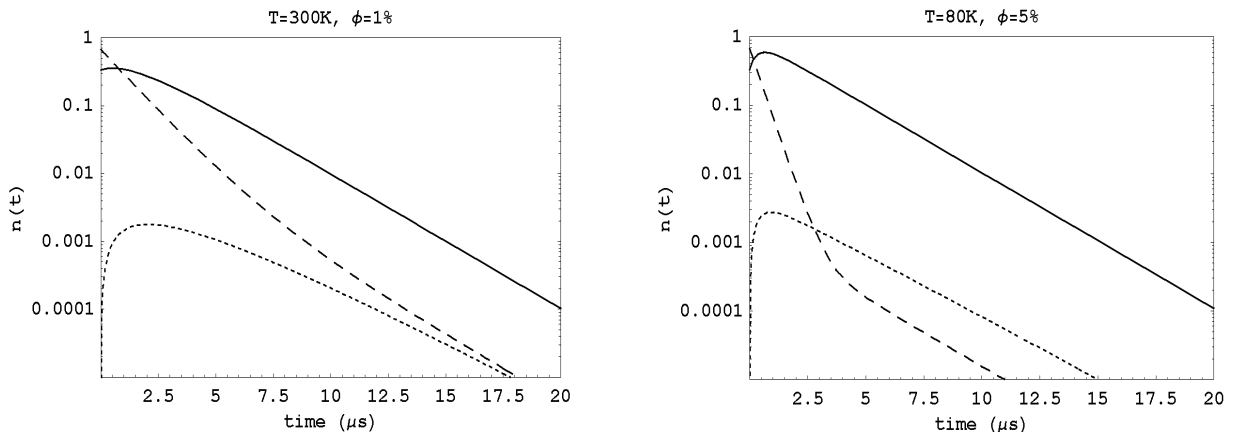

Figure 5: Time distributions of relevant states for different deuterium densities ϕ relative to LD$_2$ and temperatures. $d\mu(\uparrow\downarrow)$ (solid), $d\mu(\uparrow\uparrow)$ (dashed), $^3He\mu$ (dotted).

As regards (b) current experimental results are of 6-10% precision and only marginally agree with each other [26]. However, the MuCap strategy should allow significant improvement. Moreover - if necessary - the reduced rate for the kinematic region $p_\nu \geq 90$ MeV/c can be determined as the difference between a precise measurement of the total rate Λ_D (via the lifetime method) and a measurement of the Dalitz plot of neutrons with $E_n \geq 10$ MeV (with neutron detectors). The latter measurement needs only limited precision of 5-10%, as the high-energy part contains only a small fraction of the total intensity. The main challenge is, in a different guise as for $\mu + p$ capture, the muon induced kinetics. The hyperfine quenching of the upper $d\mu(\uparrow\uparrow)$ quartet to the $d\mu(\uparrow\downarrow)$ doublet state is slow. In addition, $^3He\mu$ atoms are formed after $dd\mu$ formation and fusion, where muon capture is larger than in deuterium. The $d\mu$ system has been intensively studied

as the prototype for resonant muon-catalyzed fusion [27]. For a clean interpretation the target conditions should be chosen such that the $d\mu(\uparrow\downarrow)$ state dominates and the population of states can be verified *in-situ* by the observation of muon-catalyzed fusion reactions. A preliminary optimization (Fig. 5) indicates promising conditions at $\phi = 5\%$ and T=80 K.

4 Muon Lifetime

4.1 Scientific Motivation

The Fermi Coupling Constant G_F is a fundamental constant of nature. In particular, together with $\alpha = 1/137.03599976(50)$ (0.0037 ppm) and $M_Z = 91.1876(21)\text{GeV}$ (23 ppm), G_F defines the gauge couplings of the electroweak sector of the standard model. The most precise determination of G_F comes from the measurement of the muon lifetime τ_μ [28, 29]

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}(1 + \Delta q). \tag{4}$$

Here Δq encapsulates the higher order QED and QCD corrections calculated in the Fermi theory. The remaining electroweak corrections are contained in the quantity [30] Δr defined by

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}(1 + \Delta r) \tag{5}$$

where g and M_W are the $SU(2)_L$ coupling constant and the W boson mass, respectively. Interesting quantum loop effects are absorbed in this quantity, including a remarkable sensitivity to the top quark mass and to the Higgs mass as well as potential new physics [29]. At the moment, G_F is not the limiting factor in exploiting these loop contributions, as the precision of other electroweak observables still has to be improved considerably. Recent 2-loop QED calculations [28, 31] led to a revised value and error of $G_F = 1.16637(1) \times 10^{-5}\,\text{GeV}^{-2}$(9 ppm). The hitherto dominant theoretical uncertainty in Eq. 4 was reduced to a negligible level. Thus an extraction of G_F from experiment down to the level of 0.5 ppm is becoming feasible before encountering theoretical limitations. This fact together with the spectacular precision achieved for other electroweak parameters, most notably the Z mass, has stimulated a new generation of μ^+ lifetime experiments at PSI [2, 32] and Rutherford Appleton Laboratory [33].

The extraction of G_F from Eq. 4 involves the following experimental contributions

$$\frac{\delta G_F}{G_F} = \sqrt{(\tfrac{5}{2}\tfrac{\delta m_\mu}{m_\mu})^2 + (\tfrac{1}{2}\tfrac{\delta\tau_\mu}{\tau_\mu})^2 \left[+(4\tfrac{m_{\nu_\mu}^2}{m_\mu^2})^2\right]} = \sqrt{0.38^2 + 9^2 \left[+10^2\right]}\,ppm \tag{6}$$

The last contribution is based on the current upper bound of $m_{\nu_\mu} \leq 170\,\text{keV}$ from direct experiments. However, given the present empirical information on neutrino masses from ν-oscillation and cosmic microwave background measurements, such a large m_{ν_μ} appears unrealistic. Thus the main uncertainty in G_F is due to τ_μ.

4.2 The MuLan experiment

The MuLan collaboration [2] plans to reduce the uncertainty of the present experimental value of 2197.03± 0.04 ns (18 ppm) to 1 ppm. Both the statistics and systematics must be dramatically improved, relying on the following experimental concept.
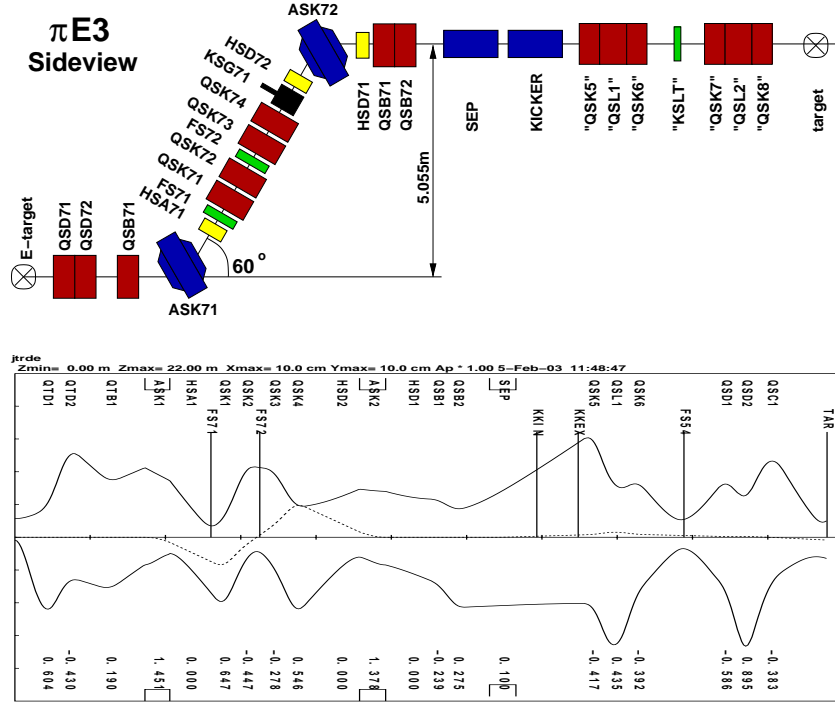

Figure 6: New $\pi E3$ beamline and calculated beam envelopes used in chopper tests.

Statistics of 10^{12} events. This statistics cannot be achieved with the conventional "one-muon-at-a-time" method. Instead, muons from the continuous PSI beam will be electrostatically chopped [18] at ~ 50 kHz in the secondary surface muon beam line of the high intensity $\pi E3$ area. This allows to accumulate pulses with several muons and then have beam free interval of $\sim 20\,\mu s$ for measurement.

Pile-up suppression. Decay positrons are registered in a soccer ball shaped arrangement of 170 tile elements consisting of 3 mm thick fast plastic scintillator pairs, which covers $\sim 75\%$ of 4π. The average inner diameter of the ball is ~ 80 cm. The analog signals from the photomultipliers coupled to the scintillators will be digitized by 8-bit 500 MHz waveform digitizers. High segmentation, excellent pulse pair resolution and energy information are crucial to identify and suppress pile-up of two electrons hitting a tile simultaneously, which would systematically distort the observed time spectrum.

Detector stability. Target and detector material are minimized and low thresholds are used, so that the majority of Michel electrons hitting a tile are detected and the effect of time-dependent gain/threshold changes is reduced.

μSR effect. As discussed for MuCap, residual μ^+ polarization and subsequent precession and depolarization can lead to systematic distortions of the decay spectra. This effect is suppressed in several steps: by the highly symmetric detector geometry, the choice of depolarizing target material and the use of a transverse magnetic field to reduce the overall ensemble polarization per pulse and to monitor the muon spin asymmetry.

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The effect of muonium formation to the lifetime has been found to be negligible [34].

A recent milestone for MuLan was the development of a new kickable tune of the $\pi E3$ channel, based on extensive phase space simulations and measurements. The tune minimizes the angular divergence in the kicker plane, while simultaneously optimizing the phase space acceptance to obtain the required muon flux. The measured extinction factor of $\leq 10^{-3}$ at a beam intensity of $\sim$15 MHz lies comfortably within the proposed design specifications. The new chopper is presently being built at TRIUMF and PSI. Two MuLan runs are scheduled for 2004, the first for commissioning the chopper and beamline and the second for commissioning the newly built detector, electronics and high rate data acquisition. The new chopper supports different time structures, providing an exciting new facility for MuLan, MuCap and other fundamental muon experiments.

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