Graphs, Games, and Decisions:

How Does a Winner *Slither*?

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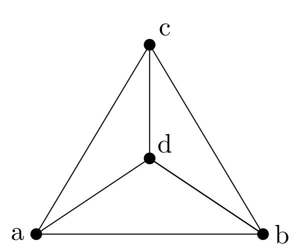
1 Introduction

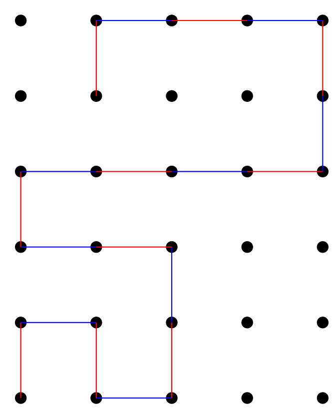
1.1 Graph

Definition. A graph, $G(V, E)$, is a set V of vertices and a subset E of pairs of vertices, called edges. Take an example of a graph where the set of vertices is $V = \{a, b, c, d\}$ and the set of edge is $E = \{(a, b), (b, c), (c, a), (a, d), (b, d), (c, d)\}$. This can be graphically represented as follows:



1.2 A Game on Graph: *Slither*

The game of Slither first appeared in June 1972 issue of *Scientific American* in "Mathematical Games" series of Martin Gardner. The game was conceived by David L. Silverman in his book *Your Move*. The earliest version of the game, which we name "classic *Slither*," goes like this: Two players are given a planar lattice of dimension 5×6. Each player take turns drawing an orthogonal unit segment such that the new segment is connected to the preceding path, on either side. The player that is forced to enclose a boundary loses.



Let us assume that the game began at the left-bottom corner with red as player 1. The game above is a player 1 win game. Although the game is still not over, the reader can see why. Was it possible for player 2 to win this game at all?

Conjecture 1. The classic *Slither* is a player 1 win game if the lattice has an even number of vertices.

Proof?

⋮

Conjecture 2. The classic *Slither* is a player 2 win game if the lattice has an odd number of vertices.

Proof?

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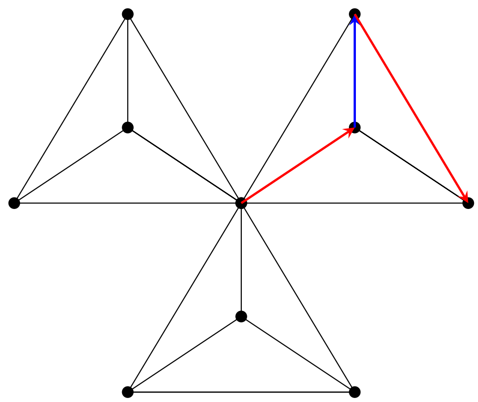
For the next version of the game, We are given an undirected host graph. The players successively make their moves such that they trace a directed path. The player unable to make further move loses. Notice that the winning goal here is different from the earlier game. We call this a 2-Player progressive game.



In the game above, given that the red started the game from the central vertex, the game is a player 1 win. Was it possible for player 2 to win this game at all?

Theorem 1. [Hoffmann] The 2-player progressive *Slither* is a player 1 win game if player 1 starts at a universal vertex of the maximal matching.

Proof.

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The paper will continue to present other modifications to the game of *Slither*, such as the number of players, classes of host graphs, probabilistic moves, and so on. Even though a simple game, ideas in *Slither* are applicable to matchings, combinatorial game theory, algorithmic game theory, and so on.