Hierarchical Dirichlet Regression Model for Benthic Cover in the Abrolhos Bank

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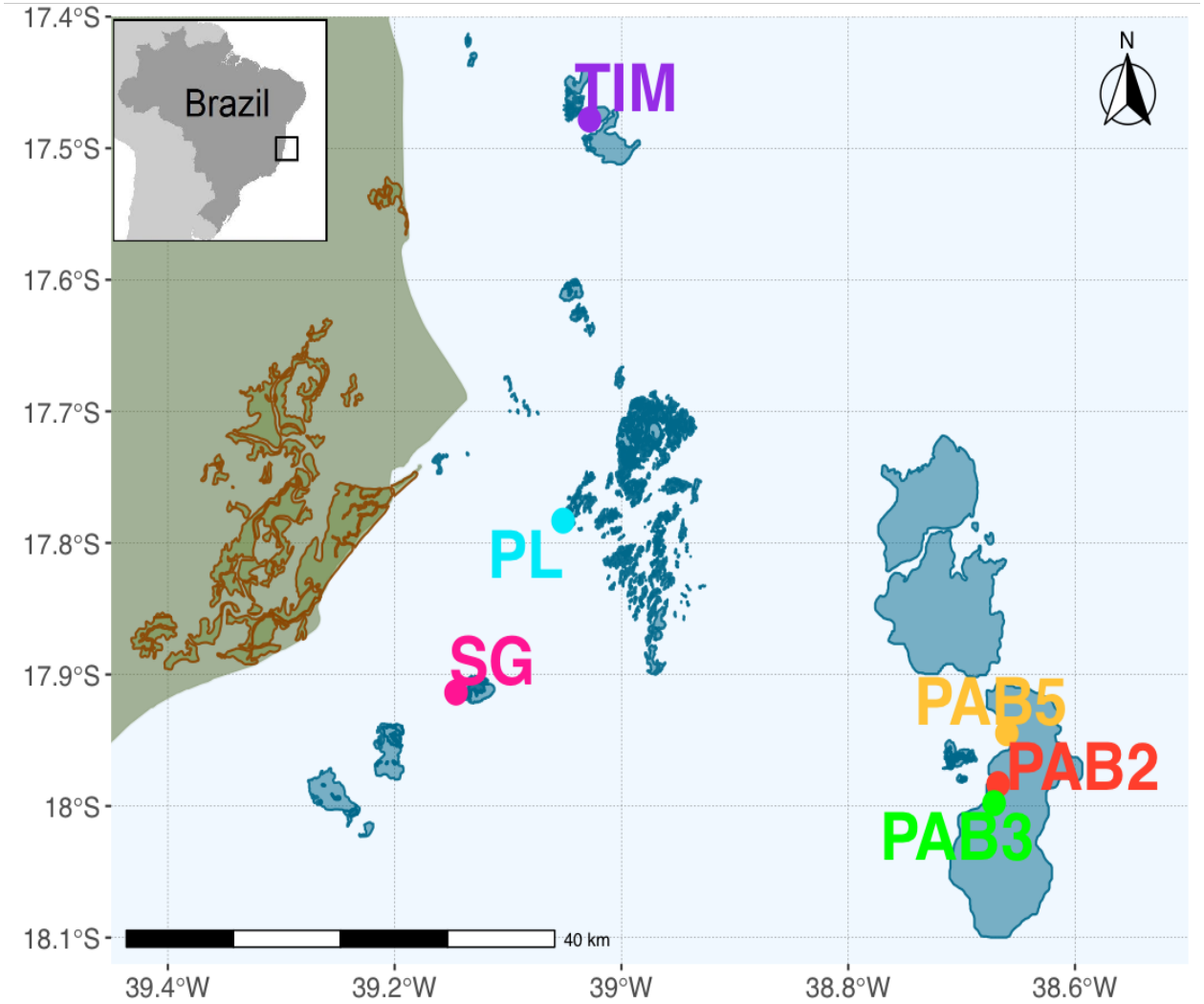
Framework

- Multivariate regression with constrained response.
- Challenge:
  - Unbalanced;
  - Lot of missing data;
  - Identificability issues

Objective: To study the variability by site



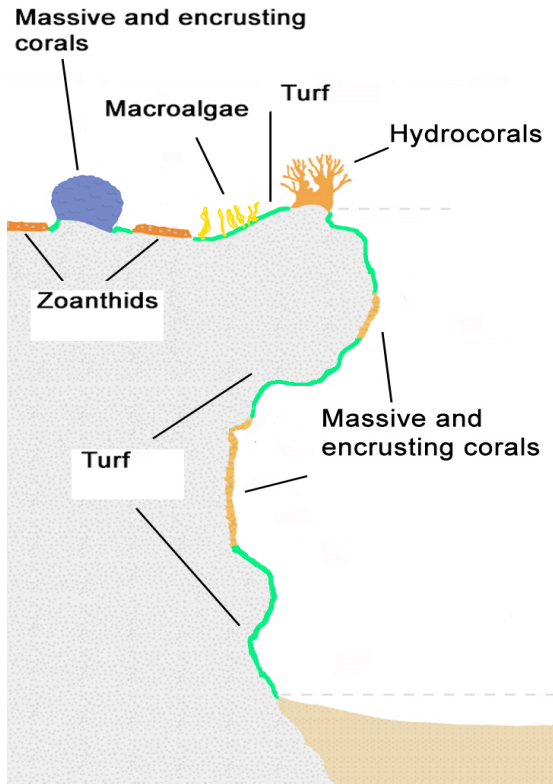


Figure:

Model

Maier (2014) and Holger (2018)

Filtered information through the decomposition of $\boldsymbol{\alpha}$

- $\mathbf{Y}_l \sim D(\mu_l, \phi_l)$ with parameter $\alpha_{cl} = \mu_{cl}\phi_l$
- μ_{cl} : level term
- ϕ_l : precision term

Reference component: $c^\star$

- Alternative parametrization: $c^\star$ should be chosen.
- Stochastic representation for Dirichlet random vector

Sharing information equation

$$\begin{aligned} \beta_{cl} &= \beta_c + \epsilon_{\beta_l}, \quad \epsilon_{\beta_l} \sim \mathcal{N}(0, V_\beta) \\ \theta_l &= \theta + \epsilon_{\theta_l}, \quad \epsilon_{\theta_l} \sim \mathcal{N}(0, V_\theta) \end{aligned}$$

Inference procedure

Let $\boldsymbol{\Theta} = (\beta, \phi)$ be the vector of parameters

Proper independent prior distribution for the parametric vector $\boldsymbol{\Theta}$ are Normal with zero mean and precision $1/K$ for all effects of the model.

The joint posterior distribution does not have a known closed form

$$\pi(\boldsymbol{\Theta} \mid \mathbf{y}) \propto L(\boldsymbol{\Theta} \mid \mathbf{y}) \prod_{l}^{L} \pi(\phi_l) \prod_{c}^{C} \pi(\beta_{cl}) \tag{1}$$

Sampling from the posterior distribution

by Markov chain Monte Carlo (MCMC) via the Stan software.

Results and Conclusions

The results validate the original hypotheses

Sites near the coast (inshore) are more variable than the offshore sites.

Main conclusions

- The proposed model quantifies the heteroscedasticity through precision effects via hierarchical structures by site;
- The method is flexible;
- The reference component has been chosen using objective criteria;
- The proposal allows to obtain adequate predictions.
- This work contributes to the United Nations's Sustainable Development Goal 14 - "Life Under Water".

References

- Gelman, Andrew, and Jennifer Hill. 2006. Data Analysis Using Regression and Multilevel/Hierarchical Models. Analytical Methods for Social Research. Cambridge University Press. https://doi.org/10.1017/CBO9780511790942.
- Holger, and Sennhenn-Reulen. 2018. "Bayesian Regression for a Dirichlet Distributed Response Using Stan." ArXiv.org. https://arxiv.org/abs/1808.06399.
- Maier, Marco J. 2014. "DirichletReg: Dirichlet Regression for Compositional Data in R." Research Report Series/Department of Statistics and Mathematics 125. Vienna: WU Vienna University of Economics and Business. http://epub.wu.ac.at/4077/.
- Wang, K., G. Tian, and M Tang. 2011. Dirichlet and Related Distributions: Theory, Methods and Applications. Wiley Series in Probability; Statistics.

Thank you

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