Hierarchical Dirichlet Regression Model for Benthic Cover in the Abrolhos Bank

Pamela M. Chiroque-Solano

Sciences Department at University of Lisboa, CE3C, Portugal
Institute of Biology and SAGE-COPPE, Federal University of Rio de Janeiro, Brazil.

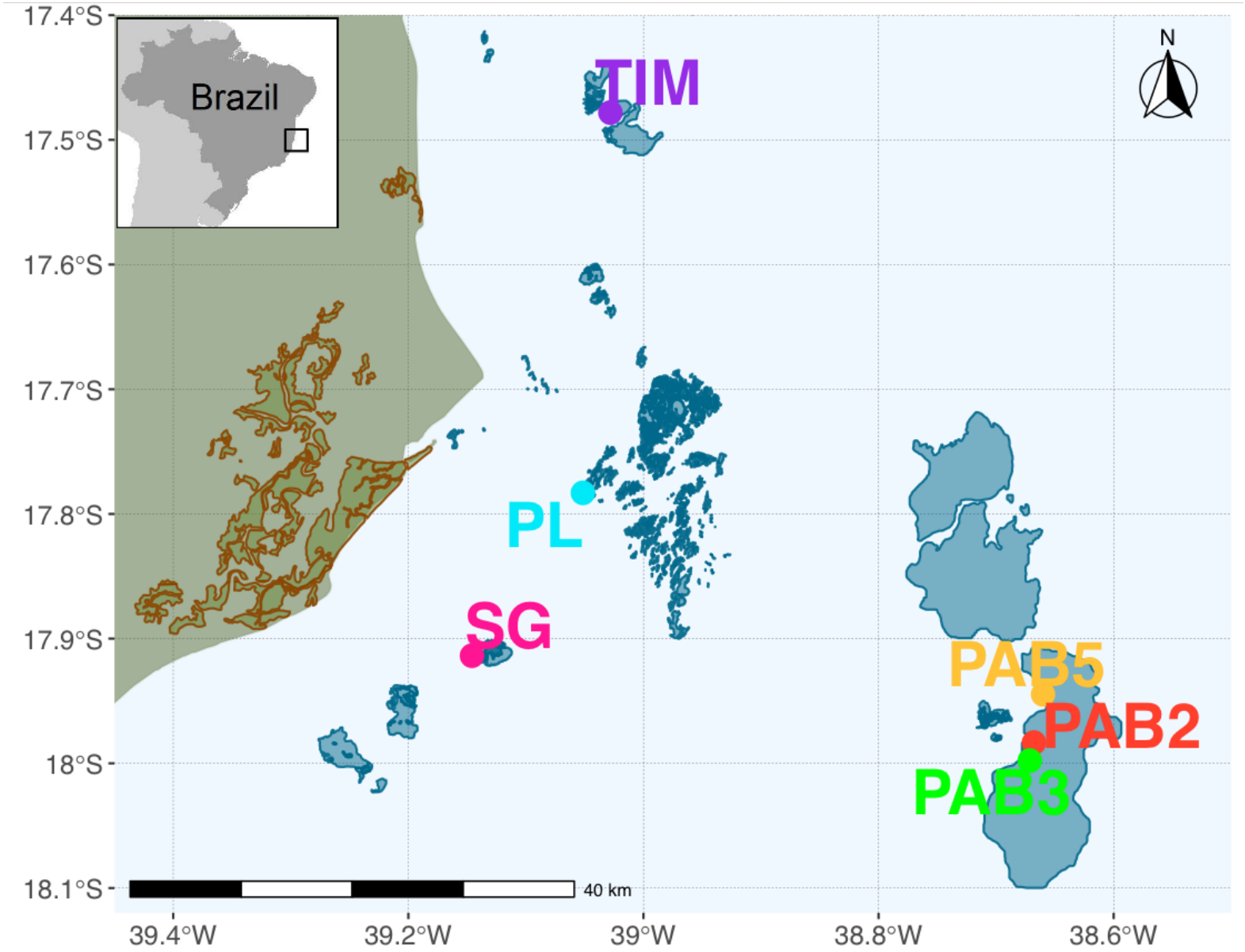
This work was done in collaboration with Mariana S. Sá and Larissa M. Martins.

11th of August, 2021

Framework

- Multivariate regression with constrained response.
- Challenge:
  - Unbalanced;
  - Lot of missing data;
  - Identificability issues
- Problem: To understand the variability by localization

Objective


Brazil
TIM
PL
SG
PAB5
PAB2
PAB3
N
17.4°S
17.5°S
17.6°S
17.7°S
17.8°S
17.9°S
18°S
18.1°S
40 km
39.4°W
39.2°W
39°W
38.8°W
38.6°W

Model

Maier (2014) and Holger (2018)

Alternative parametrization

- $\mathbf{Y}_l \sim D(\mu_l, \phi_l)$ with parameter $\alpha_{cl} = \mu_{cl}\phi_l$
- μ_{cl} : level term
- ϕ_l : precision term

Reference component: $c^\star$

$c^\star$ should be chosen

Sharing information equation

$$\begin{aligned} \beta_{cl} &= \beta_c + \epsilon_{\beta_l}, \quad \epsilon_{\beta_l} \sim \mathcal{N}(0, V_\beta) \\ \theta_l &= \theta + \epsilon_{\theta_l}, \quad \epsilon_{\theta_l} \sim \mathcal{N}(0, V_\theta) \end{aligned}$$

Inference procedure

Let $\boldsymbol{\Theta} = (\beta, \phi)$ be the vector of parameters

Proper independent prior distribution for the parametric vector $\boldsymbol{\Theta}$ are Normal with zero mean and precision $1/K$ for all effects of the model.

The joint posterior distribution does not have a known closed form

$$\pi(\boldsymbol{\Theta} \mid \mathbf{y}) \propto L(\boldsymbol{\Theta} \mid \mathbf{y}) \prod_{l}^{L} \pi(\phi_l) \prod_{c}^{C} \pi(\beta_{cl}) \tag{1}$$

Sampling from the posterior distribution

by Markov chain Monte Carlo (MCMC) via the Stan software.

Results and Conclusions

The results validate the original hypotheses

Sites near the coast (inshore) are more variable than the offshore sites.

Main conclusions

- The proposed model quantifies the heteroscedasticity through precision effects via hierarchical structures by site;
- The method is flexible;
- The reference component has been chosen using objective criteria;
- The proposal allows to obtain adequate predictions.
- This work contributes to the United Nations's Sustainable Development Goal 14 - "Life Under Water".

References

- Gelman, Andrew, and Jennifer Hill. 2006. Data Analysis Using Regression and Multilevel/Hierarchical Models. Analytical Methods for Social Research. Cambridge University Press. https://doi.org/10.1017/CBO9780511790942.
- Holger, and Sennhenn-Reulen. 2018. "Bayesian Regression for a Dirichlet Distributed Response Using Stan." ArXiv.org. https://arxiv.org/abs/1808.06399.
- Maier, Marco J. 2014. "DirichletReg: Dirichlet Regression for Compositional Data in R." Research Report Series/Department of Statistics and Mathematics 125. Vienna: WU Vienna University of Economics and Business. http://epub.wu.ac.at/4077/.
- Wang, K., G. Tian, and M Tang. 2011. Dirichlet and Related Distributions: Theory, Methods and Applications. Wiley Series in Probability; Statistics.

Thank you

pamela@dme.ufrj.br