Modeling of compositional data: a multilevel approach to benthic cover Abrolhos bank.

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Framework

Compositional Data

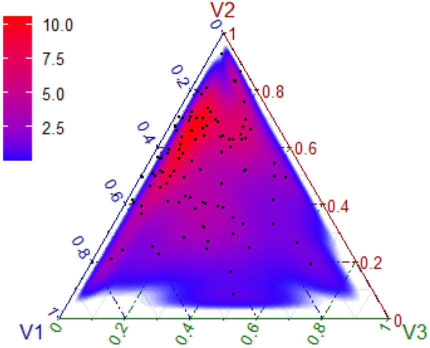
- Multivariate regression with constrained response.
- Challenge:
    - Unbalanced;
    - Lot of missing data;
    - Identificability issues

Objectives

- To model the variability effects including a hierarchical structure;
- To achieve flexibility with the proposed model, so that it can be useful in many settings.

Objective: To study the variability of the process

Case 1: $\log \phi = 1$



Case 3: $\log \phi = -1$

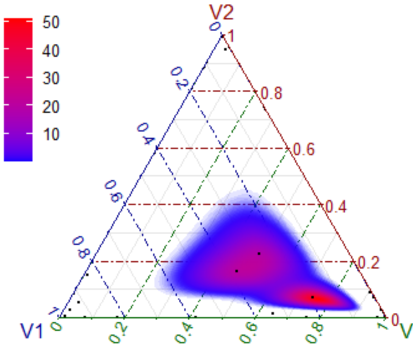


Figure: Consider a three-components (compositional data). The first simplex contains the information for high entropy (case 1), and low entropy (case 3).

Proposal

Filtered information through the decomposition

- of the Dirichlet distribution parameter into two components:
  - level and;
  - precision.
- This decomposition allows us to obtain a flexible proposal.

Notation (Maier, 2014)

Observations

- $y_{ic} \in (0,1)$: The proportion of coverage at observation i of component c.
- $\sum_{c=1}^{C} y_{ic} = 1$: Constraint

Assumptions

- $\boldsymbol{Y} \sim \mathcal{D}(\alpha)$ on $C > 2$-dimensional hyperplane or closed simplex $\mathbb{T}_C(1)$. $\alpha_c > 0$.

Model

Maier (2014) and Holger (2018)

Filtered information through the decomposition of $\boldsymbol{\alpha}$

- $\mathbf{Y}_l \sim D(\mu_l, \phi_l)$ with parameter $\alpha_{cl} = \mu_{cl}\phi_l$
- μ_{cl} : level term
- ϕ_l : precision term

Reference component: $c^\star$

- Alternative parametrization: $c^\star$ should be chosen.
- Stochastic representation for Dirichlet random vector

Sharing information equation

$$
\begin{aligned}
\beta_{cl} &= \beta_c + \epsilon_{\beta_l}, \quad \epsilon_{\beta_l} \sim \mathcal{N}(0, V_\beta) \\
\theta_l &= \theta + \epsilon_{\theta_l}, \quad \epsilon_{\theta_l} \sim \mathcal{N}(0, V_\theta)
\end{aligned}
$$

Inference procedure

Let $\boldsymbol{\Theta} = (\beta, \phi)$ be the vector of parameters

Proper independent prior distribution for the parametric vector $\boldsymbol{\Theta}$ are Normal with zero mean and precision $1/K$ for all effects of the model.

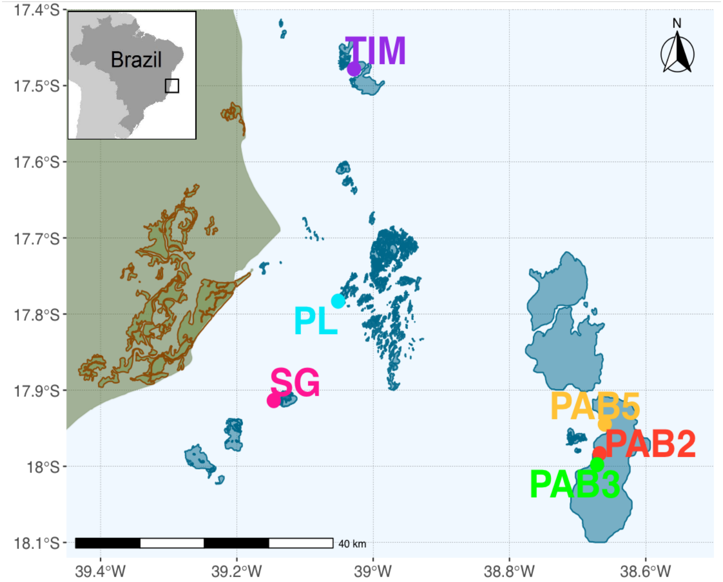
The joint posterior distribution does not have a known closed form

$$\pi(\boldsymbol{\Theta} \mid \mathbf{y}) \propto L(\boldsymbol{\Theta} \mid \mathbf{y}) \prod_{l}^{L} \pi(\phi_l) \prod_{c}^{C} \pi(\beta_{cl}) \tag{1}$$

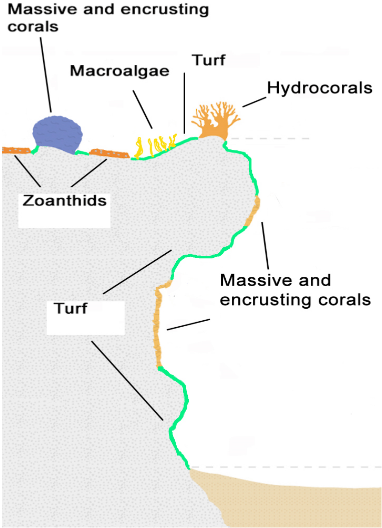
Sampling from the posterior distribution

by Markov chain Monte Carlo (MCMC) via the Stan software.

Application: The Area

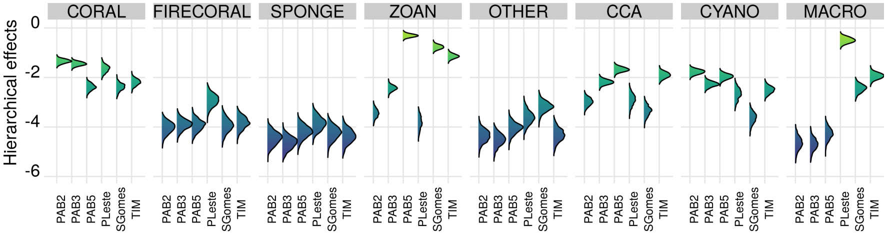


Composition of benthic communities

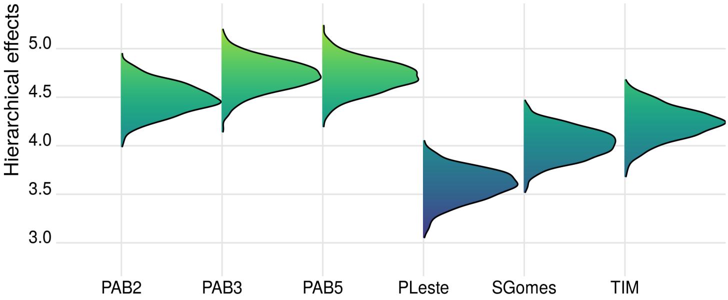


(Teixeira, Chiroque-Solano, et all. 2021)

Results: Posterior density of the β effect for each of the nine components by site and habitat levels.



Results: Posterior density of the θ effect for each of the nine components by site and habitat levels.



The results validate the original hypotheses

Sites near the coast (inshore) are more variable than the offshore sites.

Conclusions and Future Work

Main conclusions

- The proposed model quantifies the heteroscedasticity through precision effects via hierarchical structures by site;
- The method is flexible;
- The reference component has been chosen using objective criteria;
- The proposal allows to obtain adequate predictions.
- This work contributes to the United Nations's Sustainable Development Goal 14 - "Life Under Water".

References

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