Benchmark case: acoustic modes of a 2D rectangular domain

Description

The present case aims at testing the acoustic modes calculation in a 2D rectangular domain. The exact solution [2, p.73-74] of such a setup at the iteration n is written as

$$p_{\text{exact}}(i,j,n) = \cos\left(\frac{n_i \pi i}{L_i}\right) \cos\left(\frac{n_j \pi j}{L_j}\right) \cos(\omega_{i,j} n \delta t), \tag{1}$$

where

$$\omega_{i,j} = \pi c \sqrt{\left(\frac{n_i}{L_i}\right)^2 + \left(\frac{n_j}{L_j}\right)^2}, \tag{2}$$

where i, j are the discrete node location on the grid, n_i and n_j are the modes numbers. The exact solution (1) is used as an initial condition and imposed at each point of the domain at the first time step.

Within the framework of a **grid convergence study**, the results of interest are the error made on the numerical calculation compared to the exact solution of this case. In order to observe the **convergence rate** and the **orders of accuracy**, the exact same case is calculated on a set of 5 grid sizes [1].

Name	Acoustic modes of a 2D rectangular domain
Field	Linear Acoustics
Code	P. Chobeau, *SineCity project*, `https://github.com/pchobeau/sinecity_testcases` , BSD 3-Clause License.
Categories	
Bounded or Unbounded problems	Bounded - perfectly reflecting boundaries
Dimensionality of the case	2D
Scattering or Radiation problem	N.A.
Time- or Frequency-domain problem	Time **and** Frequency domain
Description	
PDE	Time Domain Wave Equation or Helmholtz Equation
Geometry	rectangular domain is of side lengths $L_x = 2.36$ m $\times$ $L_y = 1.28$ m, see Figure 1
Spatial steps for the grid convergence study	$h = [0.01,\ 0.02,\ 0.04,\ 0.08,\ 0.16]$ m
Time steps for the grid convergence study	$T_s = [0.125, 0.25, 0.50, 1.00, 2.00] \cdot 10^{-4}$ s
Propagation medium	Air: $\rho = 1.2$ kg/m^3, $c = 340$ m/s
BCs	Perfectly reflecting: $\gamma = 1/Z \to \infty$ at boundaries
Source	Initial condition (1) imposed at all points of the domain.
Receiver	All points of the domain (rectilinear grids).
Quantity to compute	Acoustic pressure
Frequencies for computation	N.A.

Geometrical details

The two modes studied in this test are shown in Figure 1 for an FDTD simulation. From this pressure rendering, the FDTD simulations seem to be in good agreement with the analytic solutions.

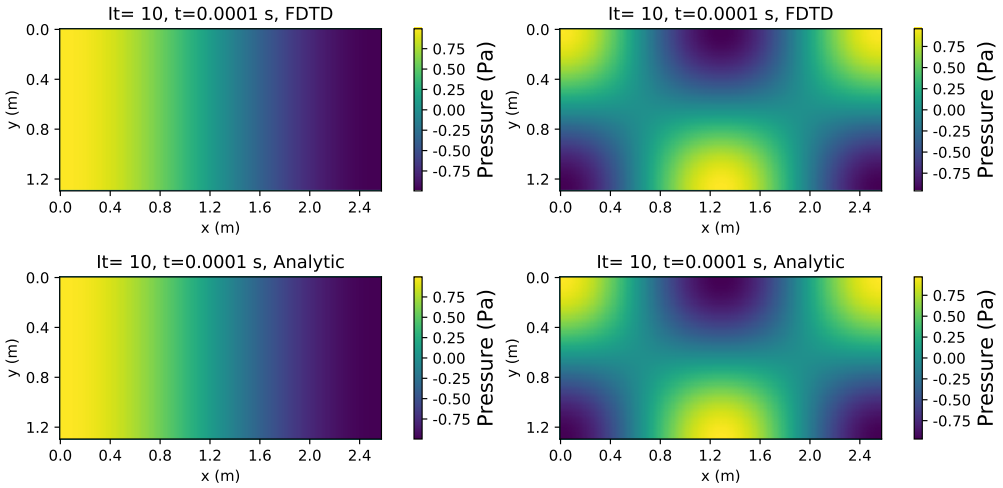

Figure 1: Snapshots of the pressure returned by the FDTD update inside the 2D rectangular domain using the finest grid ($h = 0.01$ m and $T_s = 0.125 \cdot 10^{-4}$ s) for the modes: (a) $n_x = 1,\ n_y = 0$; (b) $n_x = 2,\ n_y = 1$.

References

[1] P. Chobeau and J. Picaut. A verification procedure for environmental acoustic codes. In *CFA - Le Havre*, April 2018.

[2] Heinrich Kuttruff. *Room Acoustics*. Spon Press, 2009.