# Benchmark case: ground reflection

# Computational details

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|---|---|
| **Computational technique** | the finite difference time domain (FDTD) method [4] and the transmission line matrix (TLM) method [1, 3] |
| **Computed results** | see the commented figures in the next Section. |
| **Programming language** | Python 2.7.14 - additional packages: numpy, scipy, matplotlib, os, site. |
| **Programming details** | All details are available at `https://github.com/pchobeau/sinecity_testcases/tree/master/num_methods` , BSD 3-Clause License. |
| **Code accessibility** | `https://github.com/pchobeau/sinecity_testcases`, BSD 3-Clause License. |
| **Processing details** | e.g. for an FDTD calculation, it starts from `case3_ground.py` in which the main parameters are set. The initialization of the domain (geometry, boundaries, source and receiver locations) are defined in `init_fdtd_ground.py`. The update scheme calculation is performed in `upd_fdtd.py`. The results are processed in `errors_calc_ground.py`. |
| **Computational complexity** | N.A. |
| **Notes** | Both time and space discretization are tested. This case can be further extended to impedance boundary condition. |
| **References** | [1–4] |
| **Contributing institute** | Laboratoire d'Acoustique de L'Université du Maine (LAUM), Le Mans Acoustique (LMAc), UMRAE. |

# Results

The attenuation as a function of frequency for the TLM, the FDTD and the analytic solutions are shown in Figure 1 at 4 receiver locations. The results from the numerical methods seem to be in agreement with the analytic solution.

The absolute error$(x_i, y_i, t_n) = \left|\hat{p}_i^{\mathrm{n}} - p^{\mathrm{exact}}_{(x,y,t)}\right|$ is the absolute value of the difference between the numerical result and the analytic formulation (**??**).

Figure 2 shows the errors using the two-norm and the max-norm for each numerical method and for all the 112 receivers. The error for each grid has been averaged over the whole frequency range of interest, *i.e.* from $f=$ 25 Hz to $f$=1000 Hz. Therefore, the convergence rate of second order is verified for the whole frequency range.

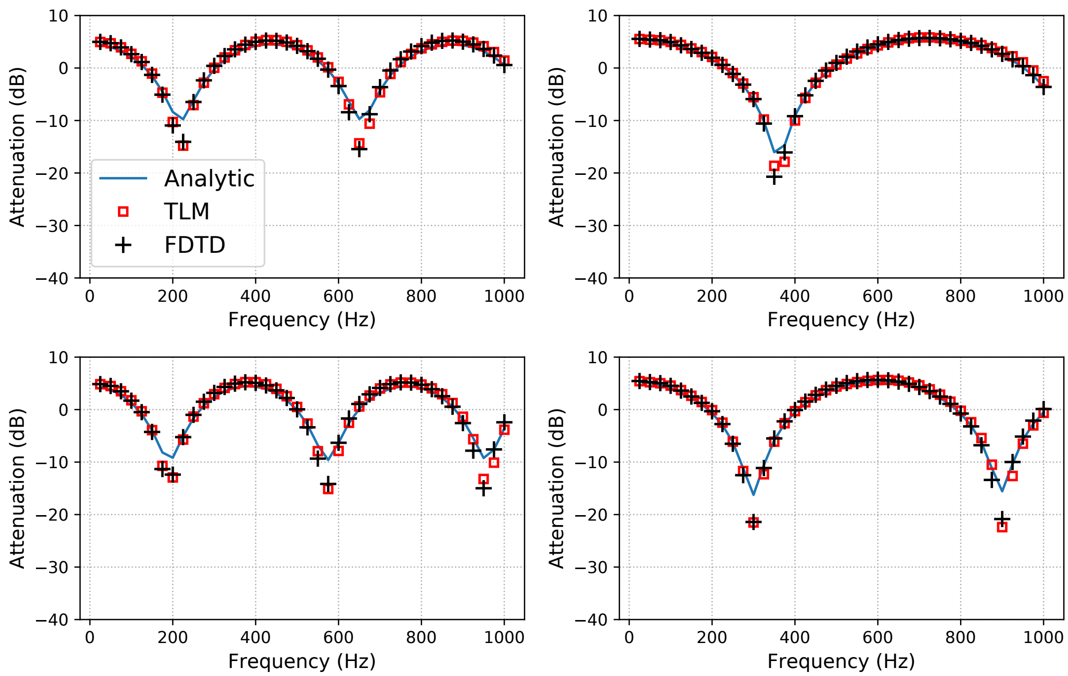


Figure 1: Attenuation relative to free-field propagation for 4 receiver positions: from left to right, top to bottom, the coordinates expressed in meter $(x_1 = 1.28, y_1 = 0.96)$ ; $(x_2 = 2.88, y_2 = 0.96)$ ; $(x_3 = 4.80, y_3 = 0.96)$ ; $(x_4 = 0.96, y_4 = 2.88)$.

# References

[1] P. Aumond, G. Guillaume, B. Gauvreau, C. Lac, V. Masson, and M. Berengier. Application of the Transmission Line Matrix method for outdoor sound propagation modelling - Part 2: Experimental validation using meteorological data derived from the meso-scale model Meso-NH. *Applied Acoustics*, 76:107–112, 2014.

[2] M. C. Bérengier, B. Gauvreau, Ph. Blanc-Benon, and D. JuvÃ©. Outdoor sound propagation: A short review on analytical and numerical approaches. *Acta Acustica united with Acustica*, 89(6):980–991, 2003.

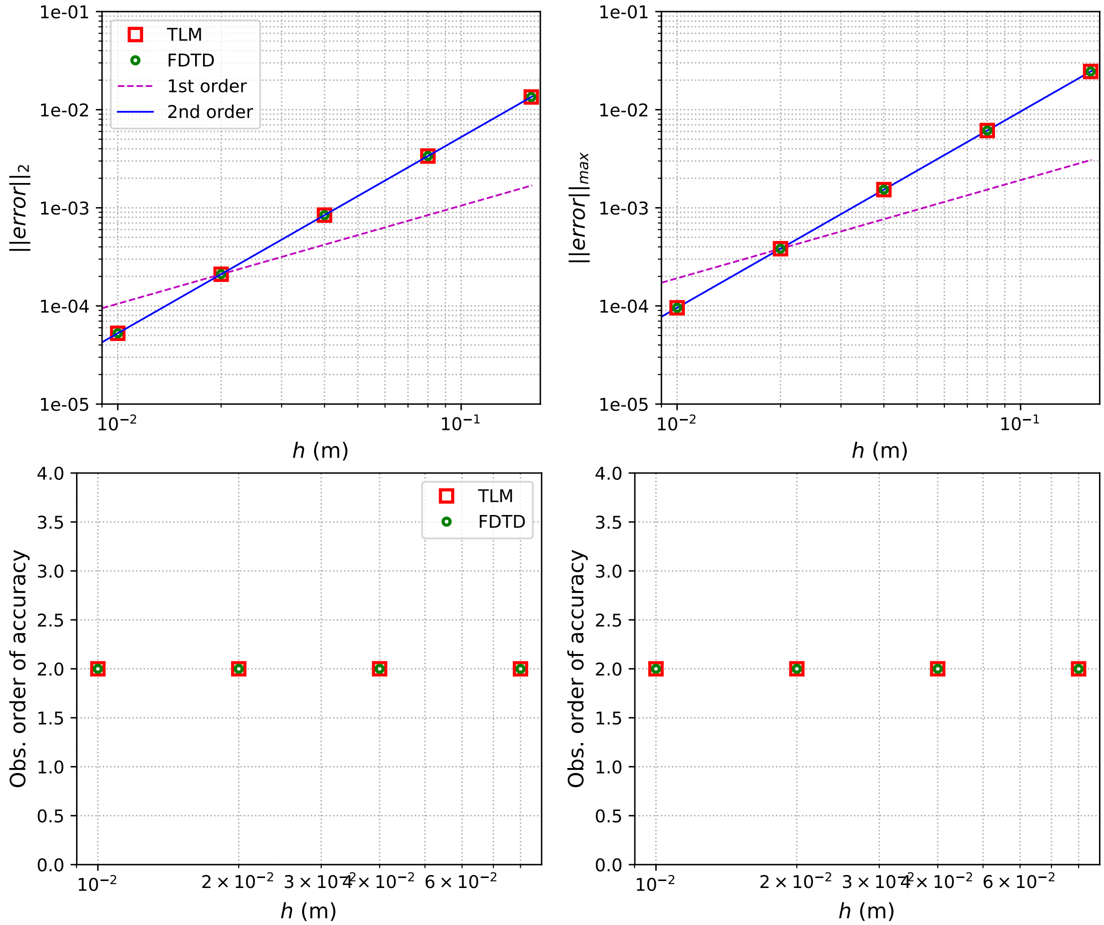


Figure 2: Two-norm and max-norm of the absolute error (top) and the corresponding observed orders of accuracy (bottom) for case 4, using the FDTD and the TLM methods.


[3] G. Guillaume, P. Aumond, B. Gauvreau, and G. Dutilleux. Application of the transmission line matrix method for outdoor sound propagation modelling - Part 1: Model presentation and evaluation. *Applied Acoustics*, 76:113–118, 2014.

[4] B. Hamilton and S. Bilbao. FDTD Methods for 3-D Room Acoustics Simulation With High-Order Accuracy in Space and Time. *IEEE/ACM Transactions on Audio, Speech and Language Processing (TASLP)*, 2017.