Benchmark case: Laplace operator eigenfunction

Description

This test case corresponds to the sample eigenfunction of the Laplace operator [2] that is written as

$$\hat{P}_i = \sin(\pi i h/L_x)\sin(\pi j h/L_y), \tag{1}$$

where L_x and L_y are the lengths for the domain. Here $L_x = L_y = L = 1$ m. This gives the source function

$$\hat{F}_i = \left(\frac{3\pi^2}{L^2} + k^2\right)\sin(\pi i h/L)\sin(\pi j h/L). \tag{2}$$

Within the framework of a **grid convergence study**, the results of interest are the error made on the numerical calculation compared to the exact solution of this case. In order to observe the **convergence rate** and the **orders of accuracy**, the exact same case is calculated on a set of 5 grid sizes [1].

Name	Laplace operator eigenfunction
Field	Linear Acoustics
Code	P. Chobeau, *SineCity project*, `https://github.com/pchobeau/sinecity_testcases` , BSD 3-Clause License.
Categories	
Bounded or Unbounded problems	Unbounded
Dimensionality of the case	2D
Scattering or Radiation problem	N.A. (free field Laplacian test)
Time- or Frequency-domain problem	Frequency domain
Description	
PDE	Helmholtz Equation
Geometry	Square domain of side lengths $L_x = L_y = L = 1$ m, see Figure 1
Spatial steps for the grid convergence study	$h = [0.01,\ 0.02,\ 0.04,\ 0.08,\ 0.16]$ m
Propagation medium	Air: $\rho = 1.2$ kg/m^3, $c = 340$ m/s
BCs	$Z = \infty$ at boundaries
Source	$\hat{F}_i = \left(\frac{3\pi^2}{L^2} + k^2\right)\sin(\pi i h/L)\sin(\pi j h/L)$, at all points of the domain.
Receiver	All points of the domain (rectilinear grids).
Quantity to compute	Acoustic pressure
Frequencies for computation	N.A.

Geometrical details

The numerical domain consists of 2D square plate of 1 m side lengths. This plate is meshed using one of the 5 grid sizes and the source signal is implemented using Equation (2). The pressure field is compared to the exact solution (Eq. (1)) for the 5 grids.

The pressure field inside the numerical domain of the finite difference Helmholtz equation is depicted in Figure 1. It shows a maximum pressure at the center of domain and zero pressure at the boundaries. This pressure field has been normalized for the dynamic to be constricted within the range 0 to 1 Pa.

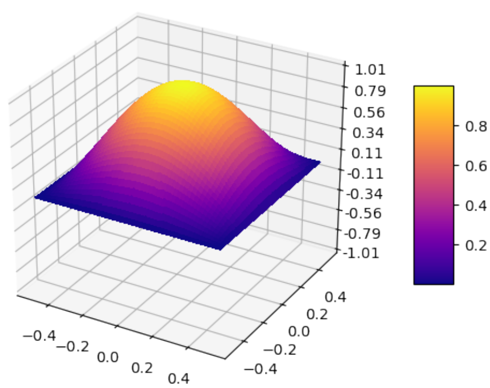

Figure 1: Real part of the normalized pressure obtained using the Finite Difference Helmholtz equation.

References

[1] P. Chobeau and J. Picaut. A verification procedure for environmental acoustic codes. In *CFA - Le Havre*, April 2018.

[2] G. Sutmann. Compact finite difference schemes of sixth order for the helmholtz equation. *Journal of Computational and Applied Mathematics*, 203:15–31, 2007.