Benchmark case: Laplace operator eigenfunction

Computational details

Computational technique	Finite Differences applied to the Helmholtz equation (FDH) [1]
Computed results	see the commented figures in the next Section.
Programming language	Python 2.7.14 - additional packages: numpy, scipy, matplotlib, os, site.
Programming details	all details are provided at `https://github.com/pchobeau/sinecity_testcases/tree/master/num_methods/fd_wave_eq/fd_core`, BSD 3-Clause License.
Code accessibility	`https://github.com/pchobeau/sinecity_testcases`, BSD 3-Clause License.
Processing details	It starts from the main folder in which `case0_laplace.py` sets the main parameters. The initialization of the domain (geometry, boundaries, source and receiver location) is done in `init_fd_modes.py`. The update calculation is done in `upd_fd.py`. Finally, the results are processed in `errors_fd_verif.py`, where the spectrums and errors are returned.
Computational complexity	N.A.
Notes	Limited to Laplacian test in the frequency domain.
References	[1, 2]
Contributing institute	Laboratoire d'Acoustique de L'Université du Maine (LAUM), Le Mans Acoustique (LMAc), UMRAE.

Results

The absolute error$(x_i, y_i, t_n) = \left|\hat{p}_i^{\mathrm{n}} - p_{(x,y,t)}^{\mathrm{exact}}\right|$ is the absolute value of the difference between the numerical result and the analytic formulation. The norms of the error are calculated over all grid points for each spatial step. Figure 1 shows the two-norm and the max-norm of the errors. The second order convergence rate is observed for max-norm, whereas the two-norm shows a 2.5 convergence order. This 2.5 convergence remains to be explained, as the expected rate is around 2. As expected, the observed orders of accuracy start fluctuating around the formal order of convergence when the grids get coarser.

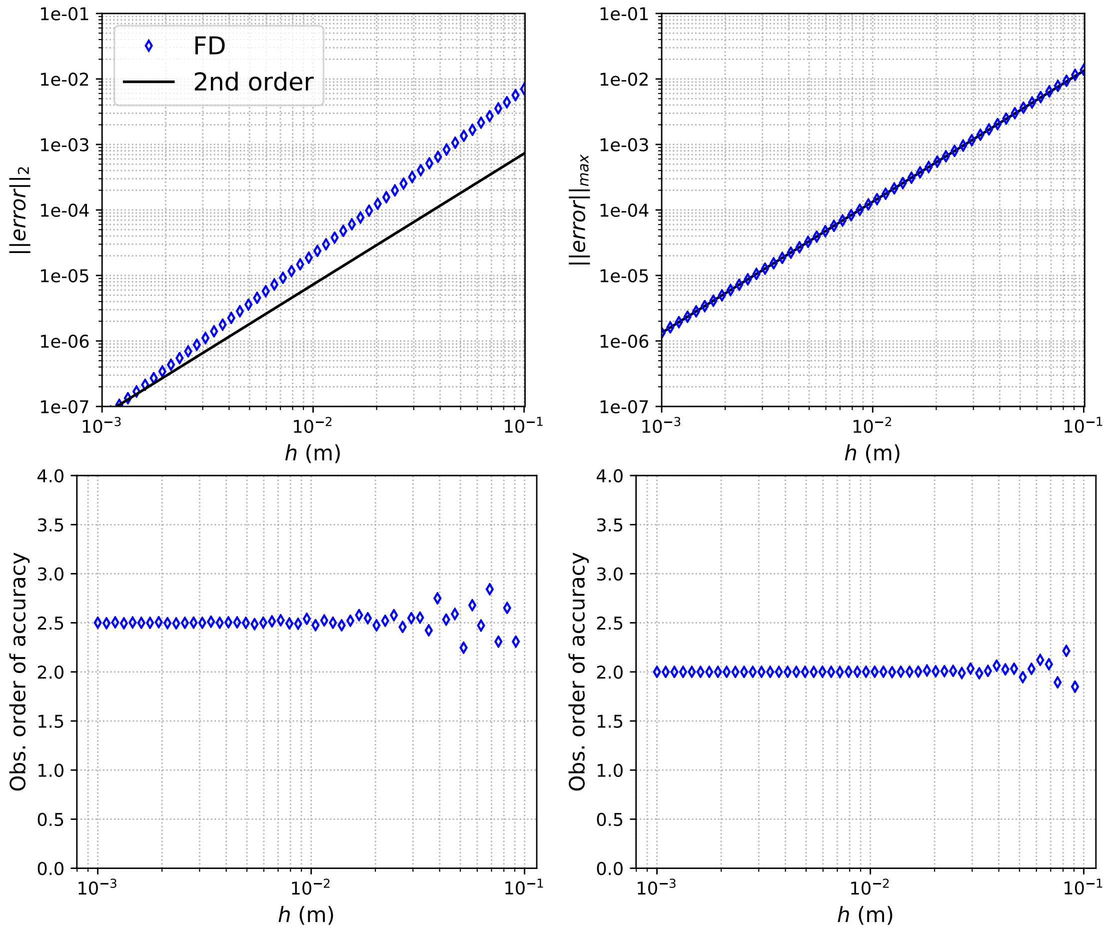

Figure 1: Two-norm and max-norm of the absolute error (top) and the corresponding observed orders of accuracy (bottom) for case 1, using the FDH method.

References

[1] G. Hegedüs and M. Kuczmann. Calculation of the Numerical Solution of Two-dimensional Helmholtz Equation. *Acta Technica Jaurinensis*, 3(1):75–86, 2010.

[2] G. Sutmann. Compact finite difference schemes of sixth order for the helmholtz equation. *Journal of Computational and Applied Mathematics*, 203:15–31, 2007.