Benchmark case: acoustic modes of a 2D rectangular domain

Computational details

Computational technique	Finite Differences applied to the Helmholtz equation (FDH) [4], the finite difference time domain (FDTD) method [3] and the transmission line matrix (TLM) method [1, 2].
Computed results	
Programming language	Python 2.7.14 - additional packages: numpy, scipy, matplotlib, os, site.
Programming details	all details are available at `https://github.com/pchobeau/sinecity_testcases` , BSD 3-Clause License.
Code accessibility	BSD 3-Clause License
Processing details	e.g. for a FDTD calculation, it starts from the main folder with `case2_modes.py` in which the main parameters are set. The initialization of the domain (geometry, boundaries, source and receiver locations) are described in `init_fdtd_modes.py`. The update calculation is carried out in `upd_fdtd.py`. Finally, the results are processed in `errors_calc2_modes.py`.
Computational complexity	N.A.
Notes	This case can be used with both time domain method - using initial condition, and frequency domain method - assuming the harmonic behavior of the exact solution, *i.e.* $\omega_{i,j} n\delta t = 0$. It takes into account perfectly reflecting boundary conditions only.
References	[1–5]
Contributing institute	Laboratoire d'Acoustique de L'Université du Maine (LAUM), Le Mans Acoustique (LMAc), UMRAE.

Results

Figure 1 shows the norms of the errors for the three numerical methods. All are second order convergent as expected from the local truncation error - see *e.g.* [3, Sec. II.F.].

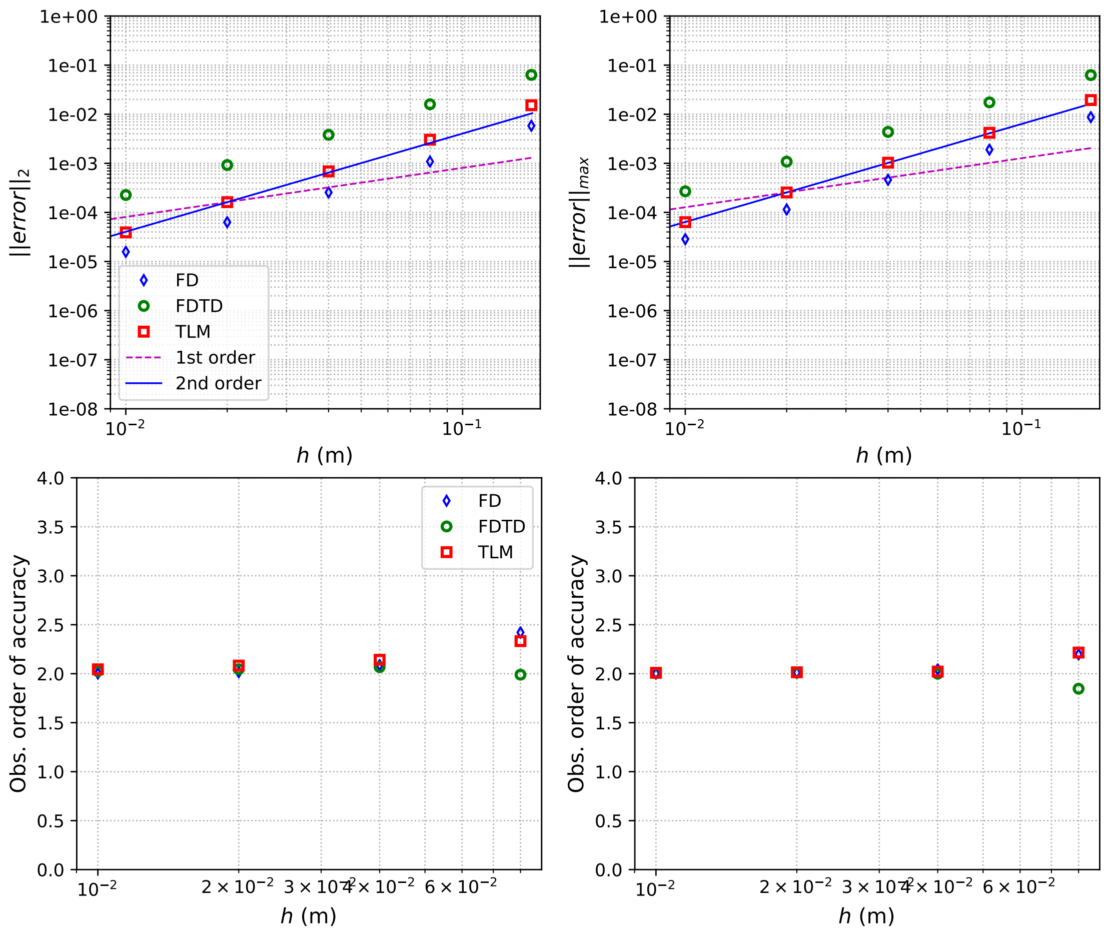

Figure 1: Two-norm and the max-norm of the absolute error (top) and the corresponding observed orders of accuracy (bottom) for case 3, using the FDH, the FDTD and TLM methods.

References

[1] P. Aumond, G. Guillaume, B. Gauvreau, C. Lac, V. Masson, and M. Berengier. Application of the Transmission Line Matrix method for outdoor sound propagation modelling - Part 2: Experimental validation using meteorological data derived from the meso-scale model Meso-NH. *Applied Acoustics*, 76:107–112, 2014.

[2] G. Guillaume, P. Aumond, B. Gauvreau, and G. Dutilleux. Application of the transmission line matrix method for outdoor sound propagation modelling - Part 1: Model presentation and evaluation. *Applied Acoustics*, 76:113–118, 2014.

[3] B. Hamilton and S. Bilbao. FDTD Methods for 3-D Room Acoustics Simulation With High-Order Accuracy in Space and Time. *IEEE/ACM Transactions on Audio, Speech and Language Processing (TASLP)*, 2017.

[4] G. Hegedüs and M. Kuczmann. Calculation of the Numerical Solution of Two-dimensional Helmholtz Equation. *Acta Technica Jaurinensis*, 3(1):75–86, 2010.

[5] G. Sutmann. Compact finite difference schemes of sixth order for the helmholtz equation. *Journal of Computational and Applied Mathematics*, 203:15–31, 2007.