Benchmark case: ground reflection

Description

This case corresponds to the propagation of an impulse above a reflecting ground.

The analytic solution for this case can be found in [1]. The total pressure field p_{total} that takes into account both the direct and reflected waves that respectively follow the paths R_1 and R_2 can be written as

$$p_{\text{total}} = \frac{e^{\jmath k R_1}}{R_1} + Q\frac{e^{\jmath k R_2}}{R_2}, \tag{1}$$

where k is the wave number and Q is given by

$$Q = R_p + (1 - R_p)F_{\text{w}}, \tag{2}$$

where R_p is the reflection coefficient written as

$$R_p = \frac{\cos(\theta) - 1/Z}{\cos(\theta) + 1/Z}, \tag{3}$$

and F_{w} can be written as

$$F_{\text{w}} = 1 + \jmath\sqrt{\pi} w e^{-w^2} \operatorname{erfc}(-\jmath w). \tag{4}$$

and

$$w = \frac{1+\jmath}{2}\sqrt{kR_2}\left(\cos(\theta) + \frac{1}{Z}\right), \tag{5}$$

where Z is the surface impedance of the ground and θ the angle between the axis normal to the ground and the reflected path.

Within the framework of a **grid convergence study**, the results of interest are the error made on the numerical calculation compared to the exact solution of this case. In order to observe the **convergence rate** and the **orders of accuracy**, the exact same case is calculated on a set of 5 grid sizes [2].

Name	Ground reflection
Field	Linear Acoustics
Code	P. Chobeau, *SineCity project*, `https://github.com/pchobeau/sinecity_testcases` , BSD 3-Clause License.
Categories	
Bounded or Unbounded problems	Half-bounded - perfectly reflecting ground
Dimensionality of the case	2D
Scattering or Radiation problem	N.A.
Time- or Frequency-domain problem	Time or Frequency domain
Description	
PDE	Time Domain Wave Equation or Helmholtz Equation
Geometry	Semi infinite domain of length $L_x = 6$ m, $L_y = 3$ m if absorbing layers, see Figure 1
Spatial steps for the grid convergence study	$h = [0.01,\ 0.02,\ 0.04,\ 0.08,\ 0.16]$ m
Time steps for the grid convergence study	T_s is set at the Courant limit for each grid.
Propagation medium	Air: $\rho = 1.2$ kg/m^3, $c = 340$ m/s
BCs	$Z = \infty$ at boundaries
Source	a Gaussian pulse: $$\hat{f}_i^n = \exp\left(-\pi^2\left(0.5fnT_s - 1\right)^2\right),$$ at the source point.
Receiver	Total of 112 receivers placed on a grid (coarsest spatial step), see Figure 1 (right).
Quantity to compute	Acoustic pressure
Frequencies for computation	N.A.

Geometrical details

The geometry of the numerical domain is depicted in Figure 1. The source height is $h_s = 1.50$ m. The receivers are located on the grids with an horizontal spacing of 0.5 m - *i.e.* between $d_{sr} = 0.5$ m and $d_{sr} = 7.0$ m and a vertical spacing of 0.2 m - *i.e.* between $h_r = 0.2$ m and $h_r = 1.4$ m. The spatial steps h and the time steps T_s used for the grids are identical to case 2.

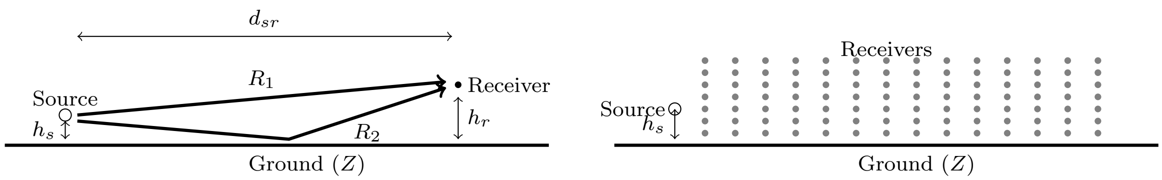

Figure 1: Geometry of the case 4 (left) with $h_s = 0.64$ m, $d_{sr} = [0.32 : 0.32 : 5.12]$ m and $h_r = [0.32 : 0.32 : 2.24]$ m ; depiction of the receiver grid (right) with a total of 112 receivers.

References

[1] M. C. Bérengier, B. Gauvreau, Ph. Blanc-Benon, and D. Juvé. Outdoor sound propagation: A short review on analytical and numerical approaches. *Acta Acustica united with Acustica*, 89(6):980–991, 2003.

[2] P. Chobeau and J. Picaut. A verification procedure for environmental acoustic codes. In *CFA - Le Havre*, April 2018.