A verification procedure for environmental acoustic codes

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Outline

Introduction

Verification and Validation (V&V): checking that a code **returns the correct value for a given application.**

Criteria for code assessment in order of increasing rigor [1]:

1. **expert judgment**: e.g. spectrum overlapping (it fits!),
2. **error quantification**: comparison with a reference,
3. **consistency/convergence**: behavior of the solution,
4. **order of accuracy**: rate of convergence.

[1] P. Knupp, K. Salari, Verification of Computer Codes in Computational Science and Engineering, Chapman and Hall/CRC, Boca Raton, FL, 2003.

Definitions

- **Verification:** process of assessing software correctness and numerical accuracy of the solution to a given mathematical model [2].
- **Validation:** process of assessing the physical accuracy of a mathematical model based on comparisons between computational results and experimental data [2].

[2] W. L. Oberkampf and C. J. Roy. *Verification and Validation in Scientific Computing*, Cambridge University Press, 2010.

Errors

Absolute errors:

$$\text{error}(x, y, t) = \left| \hat{p}^{\text{n}}_{i,j} - p^{\text{exact}}_{(x,y,t)} \right| \tag{1}$$

- Verification = simplified cases,
- use of analytical solutions $\rightarrow p^{\text{exact}}_{(x,y,z,t)}$ is known.

Norms (continuous) - 2D:

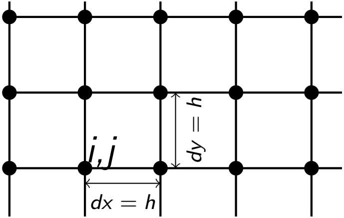
$$||\text{error}||_q = \left(\int_{x,y} |\text{error}(x, y, t = t_n)|^q dxdy \right)^{1/q} \tag{2}$$

Norms

Norms (discretized):

$$||\mathrm{error}||_q = \left(\sum_{i,j} |\mathrm{error}_{i,j}^n|^q h^2\right)^{1/q}, \tag{3}$$

$$||\mathrm{ERROR}||_q = \left(\sum_{i,j} |\mathrm{ERROR}_{i,j}^{f_n}|^q h^2\right)^{1/q}. \tag{4}$$

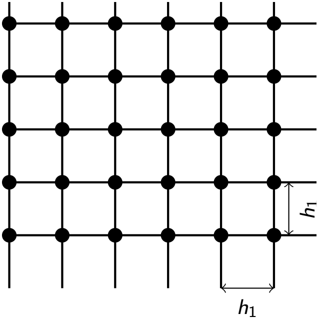


2D rectilinear grid

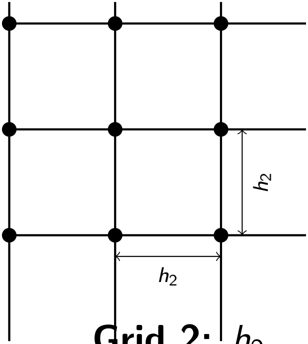
Order of accuracy

Observed order of accuracy:

$$p_{\text{obs}} = \frac{\log_{10}(||\text{error}||_q(h_1)/||\text{error}||_q(h_2))}{\log_{10}(h_1/h_2)} \tag{5}$$



Grid 1: h_1



Grid 2: h_2

A verification procedure

The 5 steps of the procedure:

1. use of an **initial condition** or a well **known signal**,
2. calculation of the **absolute error**,
3. **repetition of step 2** for at least five spatial steps,
4. calculation of the spatial **norms**,
5. **convergence rate** and **observed order of accuracy**.

Test cases

'... all numerical wave models have advantages and disadvantages concerning the computational efficiency and the **accuracy** *of the results. And* **this strongly depends on the case study** *chosen... '* [3]

[3] M. Vorländer, *Computer simulations in room acoustics: Concepts and uncertainties*, The Journal of the Acoustical Society of America, 2012.

5 test cases:

- Laplace eigenfunction,
- geometrical spreading,
- acoustic modes,
- ground reflection,
- scattering by a circle.

3 numerical methods (2D):

- Finite Difference on Helmholtz (**FDH**),
- Finite Difference Time Domain (**FDTD**),
- Transmission Line Matrix (**TLM**).

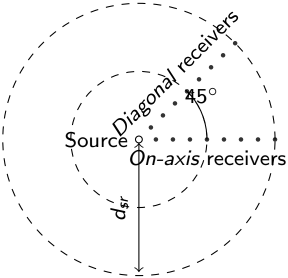
Outline

2 Geometrical spreading

Geometry of the simulations

Simulations setup:

- Spatial steps: $h = [0.01, 0.02, 0.04, 0.08, 0.16]$ m,
- Time steps: $T_s = [1.25e-05, 2.50e-05, 5.00e-05, 1.00e-04, 2.00e-04]$ s,
- Simulation length: $T_{\mathrm{sim}} = 0.06$ s,
- Source signal: Gaussian pulse with $f_{\mathrm{max}} = 2000$ Hz.



Geometry of the case 1: the maximal distance between the source and the receiver is $d_{sr} = 4$ m, with 8 receivers per axis.

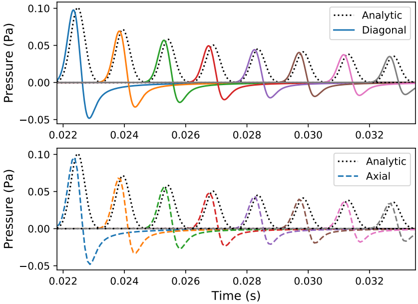
Formulation

Gaussian source term:

$$\hat{f}_{i,j}^{n} = \exp\left(-\pi^2 \left(0.5 f n T_s - 1\right)^2\right), \qquad (6)$$

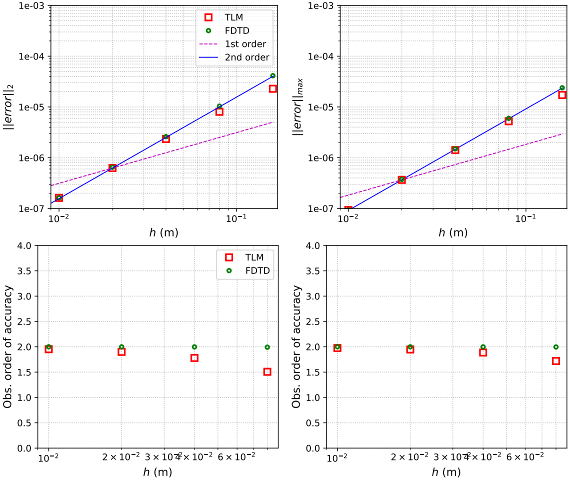
Pressure at receivers:

$$\hat{p}_{i,j}^{n} = 1/\sqrt{d_{sr}} \exp\left(-\pi^2 \left(0.5 f \left(n T_s - d_{sr}/c\right) - 1\right)^2\right). \qquad (7)$$



Time signals for the FDTD compared to analytic.

Results



Two-norm and max-norm of the absolute error (top) and the corresponding observed orders of accuracy (bottom) for case 2, using the FDTD and the TLM methods.

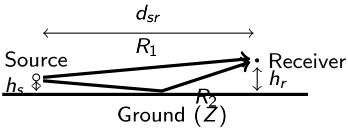
Outline

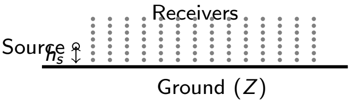
3 Ground reflection

Geometry

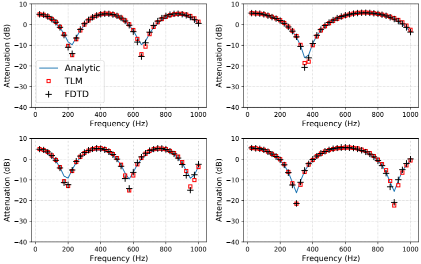
Simulations setup:

- Ground: perfectly reflecting,
- Spatial steps: $h = [0.01, 0.02, 0.04, 0.08, 0.16]$ m,
- Time steps: $T_s = [1.25e-05, 2.50e-05, 5.00e-05, 1.00e-04, 2.00e-04]$ s,
- Simulation length: $T_{\mathrm{sim}} = 0.04$ s,
- Source signal: Gaussian pulse with $f_{\max} = 2000$ Hz.



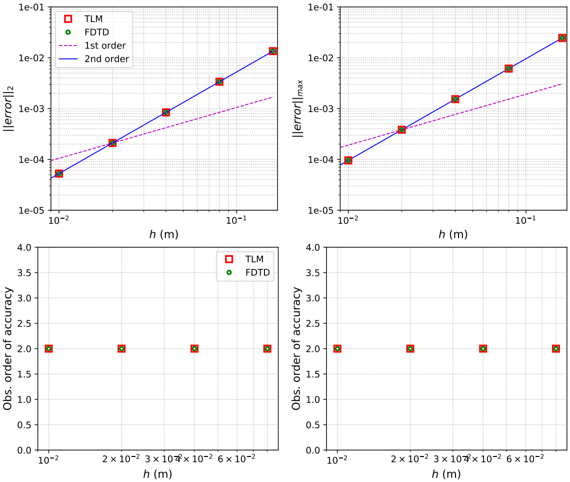


Geometry of the simulation with a total of 112 receivers.



Attenuation relative to free-field propagation for 4 receiver positions.

Results



Two-norm and max-norm of the absolute error (top) and the corresponding observed orders of accuracy (bottom) for case 4, using the FDTD and the TLM methods.

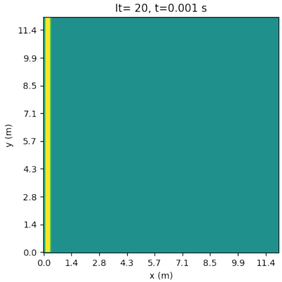
Outline

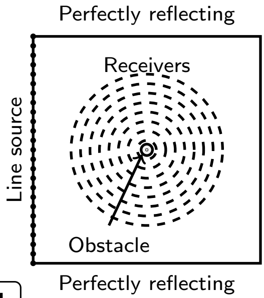
4. Scattering by a circular obstacle

Geometry

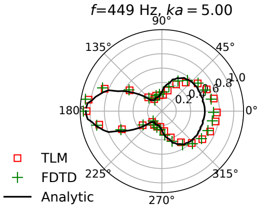
Simulations setup:

- Lengths: $L_x = 12$ m and $L_y = 12$ m.
- Spatial steps: $h = [0.0213, \ldots, 0.0695]$ m,
- Simulation length: $T_{\mathrm{sim}} = 0.08$ s,
- Source signal: Gaussian pulse with $f_{\max} = 2000$ Hz, line source.



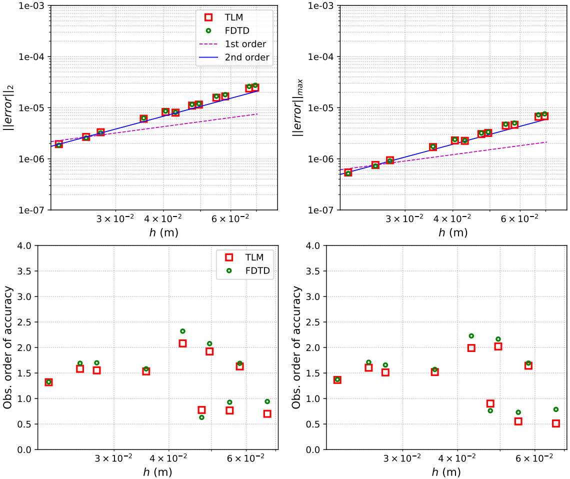


Geometry of the numerical domain.



Polar diagrams of the scattered fields for $f = 449$ Hz.

Results



Two-norm and max-norm of the absolute error (top) and their observed orders of accuracy (bottom) for case 5, with $f = 449$ Hz.

Outline

5 Conclusion

Conclusion

Numerical methods have been **verified** for several **test cases** related to outdoor sound propagation using:

- **grid convergence study**,
- **convergence rate**,
- **order of accuracy**.

All codes available at:
`https://github.com/pchobeau/sinecity_testcases`
Test cases pushed on the benchmarks EAA webpage:
`https://eaa-bench.mec.tuwien.ac.at/main/`.

Future work

- verification procedure **as a function of frequency**,
- increase of the complexity: **3D** and **additional cases** (meteo, impedance models, noise barriers...),
- **validation** procedure using measurement results ('real world'),
- development of a 'user friendly' **online tool** (e.g. use of java script for online calculations).

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Thanks for your attention.

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