Benchmark case: plane wave scattering by a circular obstacle

Computational details

Computational technique	the finite difference time domain (FDTD) method [4] and the transmission line matrix (TLM) method [1, 3]
Computed results	see the commented figures in the next Section.
Programming language	Python 2.7.14 - additional packages: numpy, scipy, matplotlib, os, site.
Programming details	All details are provided at `https://github.com/pchobeau/sinecity_testcases/tree/master/num_methods` , BSD 3-Clause License.
Code accessibility	`https://github.com/pchobeau/sinecity_testcases`, BSD 3-Clause License.
Processing details	e.g. for an FDTD calculation, it starts from `case4_scattering.py`, initialization of the domain, source and receiver `init_fdtd_scattering.py`, update calculation in `upd_fdtd.py`. Results processing done in `errors_calc2_scat.py`
Computational complexity	N.A.
Notes	Both time and space discretization are tested. This case could be extended to impedance circular obstacles. This verification is only performed for a single frequency. However, it could be extended to a frequency range once the normalization procedure of the numerical signals would have been clarified.
References	[1–4]
Contributing institute	Laboratoire d'Acoustique de L'Université du Maine (LAUM), Le Mans Acoustique (LMAc), UMRAE.

Results

The pressure scattered around the obstacle recorded by the circle of receiver located at $r = 0.5$ m from the center is shown in Figure 1 for $f = 449$ Hz. The results obtained from the two numerical methods (TLM and FDTD) are in agreement with the analytic solution.
The absolute error$(x_i, y_i, t_n) = \left|\hat{p}_i^{\mathrm{n}} - p_{(x,y,t)}^{\mathrm{exact}}\right|$ is the absolute value of the difference between the numerical result and the analytic formulation.
Figure 2 shows the errors in two norms for the frequency $f = 449$ Hz. In this case, each frequency requires a distinct fit in magnitude proportional to the analytic solution magnitude. Therefore, the error should be recalculated for each frequency. Although the global trend of the error approaches the second order of accuracy, it can be seen that the observed orders of accuracy is more fluctuating than for the previous cases. This can be explained by the normalization procedure of the magnitude for the numerical method that is applied for one chosen direction of incidence. A refined method for fitting the polar diagram might give smoother orders of accuracy.

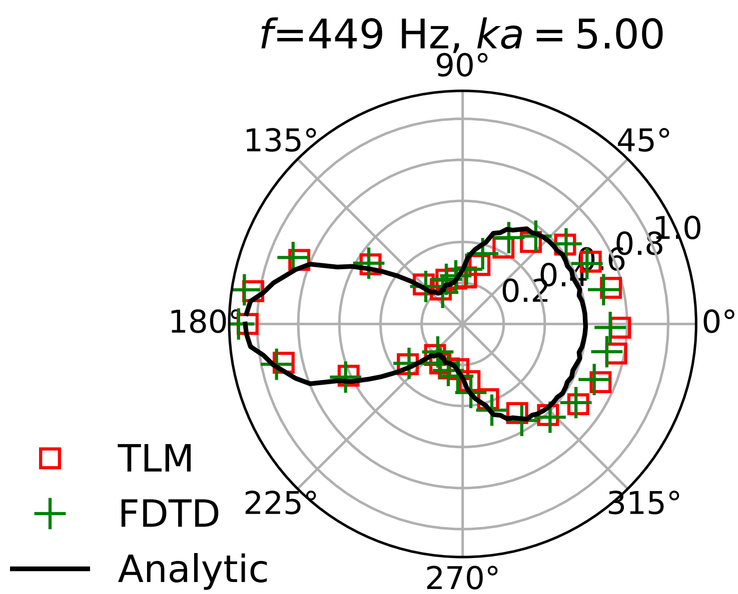

Figure 1: Polar diagrams of the scattered field for $f = 449$ Hz calculated with the TLM, the FDTD and the analytic solution at the distance $r = 0.5$ m from the center of the scatterer.

References

[1] P. Aumond, G. Guillaume, B. Gauvreau, C. Lac, V. Masson, and M. Berengier. Application of the Transmission Line Matrix method for outdoor sound propagation modelling - Part 2: Experimental validation using meteorological data derived from the meso-scale model Meso-NH. *Applied Acoustics*, 76:107–112, 2014.

[2] M. Bruneau. *Fundamentals of Acoustics*. Wiley-Blackwell, Jan 2006.

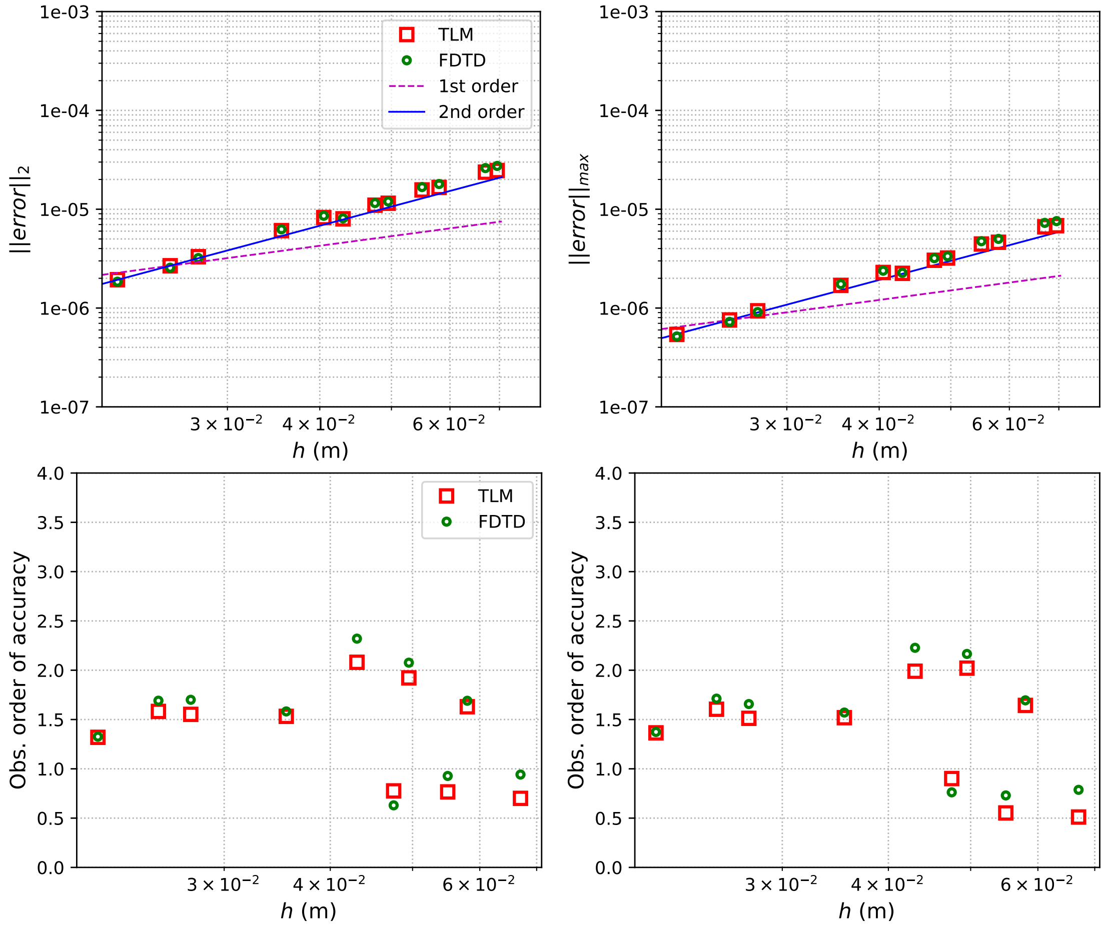

Figure 2: Two-norm and max-norm of the absolute error (top) and their observed orders of accuracy (bottom) for case 5, with $f = 449$ Hz.

[3] G. Guillaume, P. Aumond, B. Gauvreau, and G. Dutilleux. Application of the transmission line matrix method for outdoor sound propagation modelling - Part 1: Model presentation and evaluation. *Applied Acoustics*, 76:113–118, 2014.

[4] B. Hamilton and S. Bilbao. FDTD Methods for 3-D Room Acoustics Simulation With High-Order Accuracy in Space and Time. *IEEE/ACM Transactions on Audio, Speech and Language Processing (TASLP)*, 2017.