Benchmark case: geometrical spreading of a 2D point source

Description

The numerical domain is simply made of a point source located at its center. The receivers are located on two axes, one following the axis of the numerical network, one following the diagonal. The numerical domain is a 2D rectilinear network, in which a point source located at the center emits the Gaussian pulse.

The Gaussian pulse used as a source signal is written as

$$\hat{f}_i^n = \exp\left(-\pi^2\left(0.5fnT_s - 1\right)^2\right), \tag{1}$$

where f is the maximal frequency, T_s the time step and n is an integer that corresponds to the time iteration. The magnitude of this 2D point source theoretically decays proportionally to the inverse square root of the distance between the source and the receiver. Therefore, the analytic formulation of the signal rewritten as

$$\hat{f}_i^n = 1/\sqrt{d_{sr}}\exp\left(-\pi^2\left(0.5f(nT_s - d_{sr}/c) - 1\right)^2\right), \tag{2}$$

where d_{sr} is distance between the source and the receiver.

Within the framework of a **grid convergence study**, the results of interest are the error made on the numerical calculation compared to the exact solution of this case. In order to observe the **convergence rate** and the **orders of accuracy**, the exact same case is calculated on a set of 5 grid sizes [1].

Name	Geometrical spreading of a 2D point source
Field	Linear Acoustics
Code	P. Chobeau, *SineCity project*, `https://github.com/pchobeau/sinecity_testcases` , BSD 3-Clause License.
Categories	
Bounded or Unbounded problems	Unbounded
Dimensionality of the case	2D
Scattering or Radiation problem	N.A. (free field propagation)
Time- or Frequency-domain problem	Time domain
Description	
PDE	Time Domain Wave Equation
Geometry	Square domain of side lengths $$L_x = L_y = L \geqslant 12 \text{ m}$$.
Spatial steps for the grid convergence study	$h = [0.01,\ 0.02,\ 0.04,\ 0.08,\ 0.16]$ m
Time steps for the grid convergence study	$T_s = [0.125, 0.25, 0.50, 1.00, 2.00] \cdot 10^{-4}$ s
Propagation medium	Air: $\rho = 1.2$ kg/m^3, $c = 340$ m/s
BCs	$Z = \infty$ at boundaries
Source	Equation (1) at the center of the domain.
Receiver	8 on axis, 8 on diagonal, 16 total, distance from the source ranging from 0.5 m to 4.0 m, every 0.5 m, see Figure 1.
Quantity to compute	Acoustic pressure
Frequencies for computation	0 Hz to 2000 Hz (the maximal frequency correspond to the cutoff frequency of the Gaussian pulse)

Geometrical details

The point source is located at the center of the domain. Two lines of 8 receivers each are placed on the axis and on the diagonal, respectively. This gives a total of 16 receivers, with a distance from the source ranging from 0.5 m to 4.0 m, every 0.5 m.

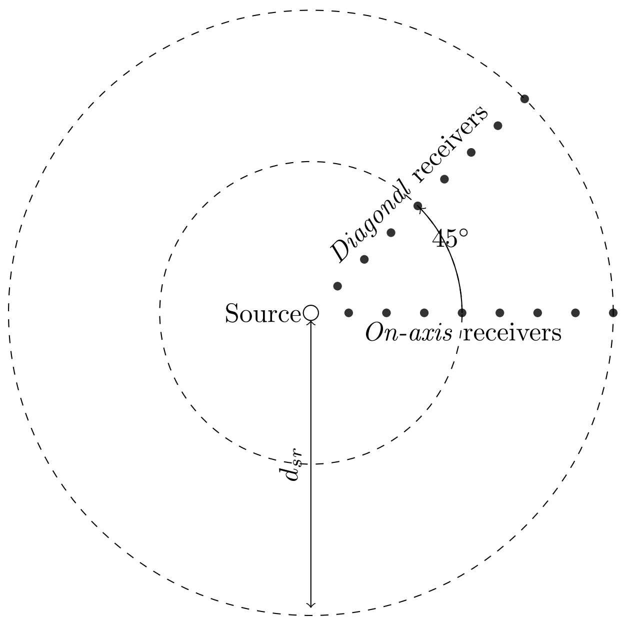

Figure 1: Geometry of the case 1: the maximal distance between the source and the receiver is $d_{sr} = 4$m, with 8 receivers per axis.

The wavefront shown in Figure 2 corresponds to the Gaussian pulse (Eq. (1)) emitted at the center of the domain.

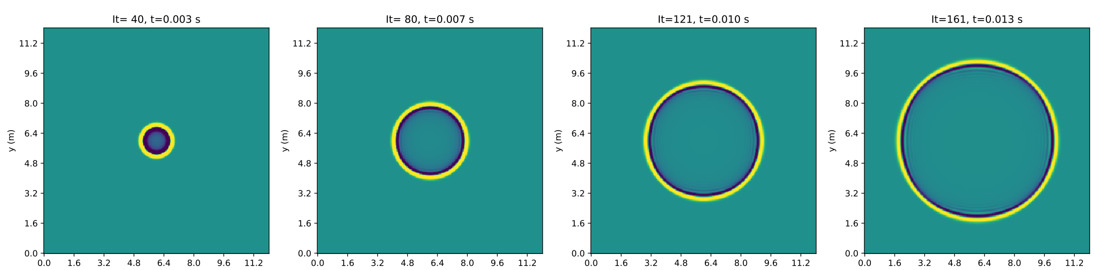

Figure 2: Wavefront emitted from a point source at the center of the domain $L_x \times L_y = 12$ m $\times$ 12 m, with a spatial step of $h = 0.02$ m . The signal is a Gaussian pulse with a cutoff frequency of 2000 Hz. Four snapshots of the pressure taken at t =3 ms, 7 ms, 10 ms and 13 ms.

References

[1] P. Chobeau and J. Picaut. A verification procedure for environmental acoustic codes. In *CFA - Le Havre*, April 2018.