Benchmark case: plane wave scattering by a circular obstacle

Description

This case corresponds to the scattering of a wavefront by a circular obstacle.

The monochromatic incident pressure p_i, propagating in the vicinity of a circular scatterer with a radius a, can be written in a cylindrical coordinate system (r, φ, z) as the sum of cylindrical waves as follows [1]

$$p_i = P_0 \sum_{m=0}^{\infty} (2 - \delta_{m0}) i^m \; J_m \; (kr) \cos(im\varphi) e^{i\omega t}, \tag{1}$$

where k is the wave number, ω the angular frequency, P_0 the amplitude of the incident wave, δ_{m0} the Dirac function and J_m the Bessel function of the first kind for real order m. The scattered wave is expressed as a divergent cylindrical wave as

$$p_s = P_0 \sum_{n=0}^{\infty} A_n H_n^{(1)} \; (kr) \cos(in\varphi) e^{i\omega t}, \tag{2}$$

where $H_n^{(1)}$ is the Hankel function of the first kind for real order n. The coefficients A_n are derived from the boundary condition, which can be written at $r = a$ as

$$\frac{i}{k\rho c} \frac{\partial}{\partial r} (p_i + p_s) = \frac{-1}{\beta} (p_i + p_s), \text{ for } r = a, \tag{3}$$

where β is the specific acoustic admittance [**?**] of the cylinder *i.e.* $\beta = \rho c / \gamma$, ρ is the mass density and c the sound speed in the air. The coefficients A_n are given by:

$$A_n = -\frac{(2 - \delta_{n0}) i^n [i J_n'(ka) + (\rho c/\beta) J_n(ka)]}{i H_n^{(1)'}(ka) + (\rho c/\beta) H_n^{(1)}(ka)}. \tag{4}$$

Within the framework of a **grid convergence study**, the results of interest are the error made on the numerical calculation compared to the exact solution of this case. In order to observe the **convergence rate** and the **orders of accuracy**, the exact same case is calculated on a set of 5 grid sizes [2].

Name	Plane wave scattering by a circular obstacle
Field	Linear Acoustics
Code	P. Chobeau, *SineCity project*, `https://github.com/pchobeau/sinecity_testcases` , BSD 3-Clause License.
Categories	
Bounded or Unbounded problems	Unbounded
Dimensionality of the case	2D
Scattering or Radiation problem	N.A.
Time- or Frequency-domain problem	Time or Frequency domain
Description	
PDE	Time Domain Wave Equation or Helmholtz Equation
Geometry	Semi infinite domain of length $L_x = 6$ m, $L_y = 6$ m, if absorbing layers, see Figure 1
Spatial steps for the grid convergence study	h =[0.0213, 0.0251, 0.0274, 0.0355, 0.0405, 0.0430, 0.0475, 0.0495, 0.0550, 0.0580, 0.0670, 0.0695] m
Time steps for the grid convergence study	T_s is set at the Courant limit for each grid.
Propagation medium	Air: $\rho = 1.2$ kg/m^3, $c = 340$ m/s
BCs	$Z = \infty$ at boundaries and rigid circle of radius a=0.3 m at the center.
Source	Line source emitting correlated Gaussian pulses: $$\hat{f}_i^n = \exp\left(-\pi^2 \left(0.5fnT_s - 1\right)^2\right),$$ at the source point.
Receiver	8 circles centered on the obstacle, with a radius ranging from 0.5 m to 4.0 m distance, every 0.5 m, see Figure 1.
Quantity to compute	Acoustic pressure
Frequencies for computation	N.A.

Geometrical details

As shown in Figure 1, the pulse emitted by the line source propagates from the left to the right of the domain, and is scattered by the circular obstacle located at the center of the domain.

The scatterer is located at the center of the computational domain and its radius is $a = 0.30$ m. The receivers are located on the 8 circles centered on the obstacle, with a radius ranging from 0.5 m to 4.0 m distance, every 0.5 m. All boundaries are considered as perfectly reflecting. The simulations duration is

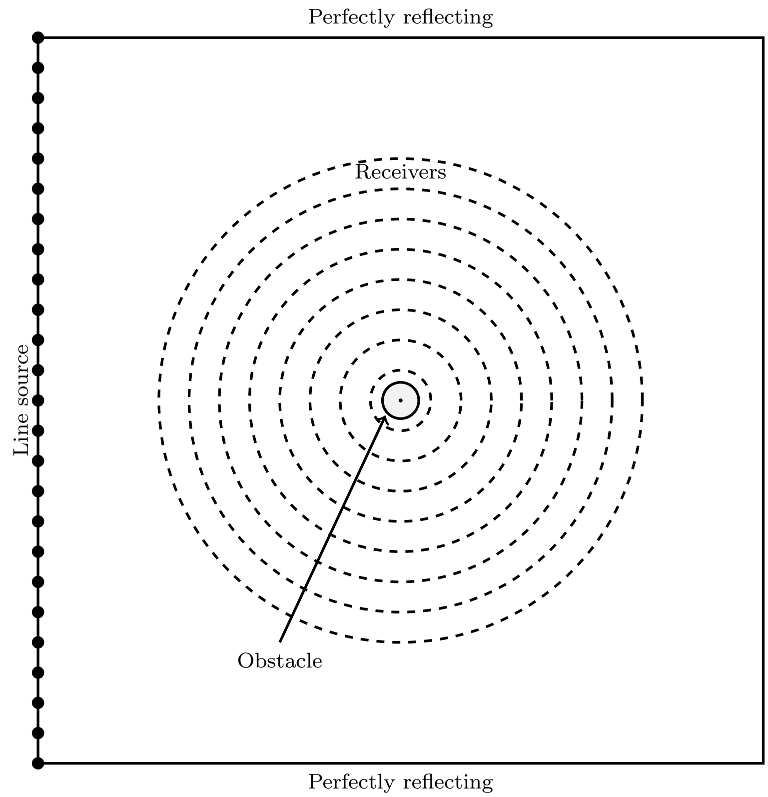

Figure 1: Geometry of the numerical domain for the study of a pulse scattered by a circular obstacle.

set to not exceed 0.06 s, which is short enough to avoid any unwanted reflection at each receiver location.

The spatial steps for the 12 grids are h =[0.0213, 0.0251, 0.0274, 0.0355, 0.0405, 0.0430, 0.0475, 0.0495, 0.0550, 0.0580, 0.0670, 0.0695] m, and the corresponding time steps T_s are set at the Courant limit for each grid.

References

[1] M. Bruneau. *Fundamentals of Acoustics*. Wiley-Blackwell, Jan 2006.

[2] P. Chobeau and J. Picaut. A verification procedure for environmental acoustic codes. In *CFA - Le Havre*, April 2018.