The Shared Structure of GR and the Standard Model

Patrick Christy

September 15, 2025

Abstract

We propose a unified approach to fundamental physics based on POVMs giving rise to quantum geometry. With minimal ontic commitments we derive the Standard Model and GR by modeling these relations using higher-category theory. We also derive the Born rule, Higgs mechanism, and explain the measurement problem.

patrickchristywork@gmail.com

Linkedin

pchristy3@gatech.edu

Contents

1 Introduction

The unification of general relativity and quantum mechanics has long remained the central open problem of fundamental physics. We propose a framework that tackles this challenge starting from minimal ontic and epistemic assumptions dealing with the relations of quantum systems to deriving the physics of General Relativity and semiclassical particle physics.

The triadic-simple block framework derives both general relativity (GR) and the Standard Model (SM) from foundational principles: each spacetime grain is a finite 4-simplex Hilbert space $H_v = \bigotimes_{f=1}^{10} H_{j_f}$, with dimension $\dim H_v = \prod_f (2j_f + 1) < \infty$, supporting area and volume POVMs with polynomial growth for uniform bounds [245];, and every geometric or relational event is a POVM morphism in the dagger-compact category **CPM**(**FdHilb**), enabling coordinate-free gluing via shared Hilbert factors [252]; probabilities are additive and non-contextual, so Gleason's theorem enforces the Born trace rule, taming the state sum via trace bounds [149]. These facts drive a chain of results: Schur–Weyl duality yields a multiplicity algebra $\mathrm{Mat}_{m(v)}$; Doplicher–Haag–Roberts reconstruction identifies $\mathrm{Mat}_{m(v)}$ with representations of $G_{\text{int}} = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$; a unique $(\mathbf{1}, \mathbf{2}, +\frac{1}{2})$ irrep provides the Higgs with one Yukawa coupling y_*, multiplied by braid integers for fermion masses (Sections 8.5, 8.6, 8.7); and the 15-j symbol's large-spin limit yields the Regge action, converging to the Einstein–Hilbert action. This finite-block, Born-trace formalism ensures an absolutely convergent state sum, recovers GR, and produces the SM gauge sector and particle spectrum, all without additional postulates [159, 142]. Our model adopts a monoidal, higher-categorical semantics from the outset. We will show that this choice is not merely stylistic; it provides the necessary constraints to suggest a unique vertex amplitude and deliver object-level predictions from collider to cosmological scales.

2 Physics in Two Pages

We construct a pathway from finite quantum structures to smooth spacetime using a sequence of five symmetric monoidal categories connected by four functors:

$$\mathrm{FdHilb} \xrightarrow{\iota} \mathbf{CPM}(\mathrm{FdHilb}) \xrightarrow{\text{Symb}} \mathbf{Regge} \xrightarrow{\text{TY}} \mathbf{Morita}(\mathbf{C}^*\text{-}\mathbf{Alg}) \xrightarrow{\text{Cont}} \mathbf{Ein}.$$

2.1 Category Definitions

FdHilb Finite-dimensional Hilbert spaces with linear maps; dagger-compact and symmetric monoidal [252]. *Role:* Represents a 4-simplex as a tensor product $H_v = \bigotimes_{f=1}^{10} H^{j_f}$, encoding quantum spin data.

CPM(FdHilb) Objects as in FdHilb; morphisms are completely positive, trace-preserving maps. Selinger's CPM construction preserves dagger-compact monoidal structure [252]. *Role:* Models quantum measurements via POVMs and Kraus operators.

Regge Finite 4-simplicial complexes (Δ, L) with positive edge lengths; morphisms are Pachner moves. Disjoint union serves as the tensor product [215, 223]. *Role:* Describes discrete piecewise-flat geometries with Regge action.

Morita(C*-Alg) Objects are C*-algebras; 1-morphisms are C*-correspondences (equivalent to CP maps); 2-morphisms are bimodule intertwiners [226, 80]. This category is essential for a rigorous treatment of the continuum limit through its support for Tambara–Yamagami fusion categories. While Regge calculus describes discrete geometries, Morita(C*-Alg) provides the language for a tensor theory with a continuum of labels (e.g., area eigenvalues), a necessary step to bridge the discrete spin spectrum with the smooth fields of the Einstein category.

Ein Smooth, oriented, time-orientable Lorentzian 4-manifolds; morphisms are causal isometries [181, 176]. *Role:* Represents classical Einstein gravity.

By Bauer and Nietner's tensor-type theorem, each category, equipped with its tensor product and contraction, supports a time-agnostic string-diagram calculus [60], eliminating the need for implicit temporal assumptions.

2.2 Functorial Transitions

Inclusion $\iota : \mathrm{FdHilb} \to \mathbf{CPM}(\mathrm{FdHilb})$ Maps objects identically and morphisms via $f \mapsto (\rho \mapsto f \rho f^\dagger)$. Fully faithful, strong monoidal, and dagger-preserving [252].

Stationary-Phase $\mathrm{Symb} : \mathbf{CPM}(\mathrm{FdHilb}) \to \mathbf{Regge}$ Maps a 15j-symbol vertex amplitude $A_v(\lambda j)$ to its large-spin limit:

$$\mathrm{Symb}(A_v) = \lim_{\lambda \to \infty} \lambda^{9/2} A_v(\lambda j) = e^{iS_{\mathrm{Regge}}} + e^{-iS_{\mathrm{Regge}}}.$$

This determines classical hinge areas A_h and deficit angles ϵ_h. The analysis of the large-spin limit shows that the map preserves the monoidal structure (where the tensor product corresponds to the disjoint union of simplices) and is left-exact, a property consistent with a strong monoidal functor [52]. While an asymptotic limit in the strictest sense, it acts functorially on the class of semiclassical states and coherent intertwiners that dominate the path integral, preserving the relevant compositional structure of a functor.

Discrete-to-Continuous $\mathrm{TY} : \mathbf{Regge} \to \mathbf{Morita}(\mathbf{C^*\text{-}Alg})$ Embeds matrix algebras Mat_{2j_f+1} into $C_0(\mathbb{R}_{>0})$ via Morita equivalence. Marin–Salvador's Tambara–Yamagami category $\mathcal{C}_{G,\tau}$ (with $G \cong \mathbb{R}_{>0}$) enables a continuous semisimple tensor theory with fusion rule $\tau \otimes \tau \cong \int_{A \in \mathbb{R}_{>0}} A \, d\mu(A)$, realized by a stationary-phase transfer matrix [199]. The Cont functor refines the geometric mesh of the triangulation, while the TY functor provides a continuum for the quantum labels themselves.

Refinement Cont : **Morita**($\mathbf{C^*}$**-Alg**) $\to$ **Ein** Interpolates a piecewise-flat metric g_h to a smooth metric g. Cheeger–Müller–Schrader and Brewin–Gentle establish convergence of the Regge action to the Einstein–Hilbert action:

$$\lim_{h \to 0} S_{\text{Regge}}(g_h) = \frac{1}{16\pi G} S_{\text{EH}}(g),$$

with metric convergence [92, 77]. Pachner moves map to isometries, ensuring Cont is well-defined.

Discretization Disc : **Ein** $\to$ **Regge** Triangulates a smooth manifold (M, g) into a simplicial complex (Δ, L), serving as a right adjoint to Cont.

2.3 Adjunctions and Commutative Diagram

We define three adjunctions, corresponding to critical transitions in the framework:

- Symb $\dashv F$: The functor Symb : **CPM**(FdHilb) $\to$ **Regge** maps quantum amplitudes to classical Regge geometries via the large-spin limit, while F : **Regge** $\to$ **CPM**(FdHilb) reconstructs quantum data, with the counit given by Barrett–Fairbairn asymptotic analysis [52].

- TY $\dashv$ TYR: The functor TY : **Regge** $\to$ **Morita**($\mathbf{C^*}$**-Alg**) embeds discrete face algebras Mat_{2j_f+1} into $C_0(\mathbb{R}_{>0})$ via Morita equivalence, while TYR : **Morita**($\mathbf{C^*}$**-Alg**) $\to$ **Regge** maps continuous C*-correspondences back to simplicial complexes, potentially by discretizing area labels, with the correspondence realized by a stationary-phase transfer matrix [199].

- Disc $\dashv$ Cont: The functor Cont : **Morita**($\mathbf{C^*}$**-Alg**) $\to$ **Ein** refines discrete geometries to smooth manifolds, while Disc : **Ein** $\to$ **Regge** triangulates smooth manifolds into simplicial complexes, with the counit provided by Cheeger–Müller–Schrader convergence [92].

The adjoint TYR is inferred from the Morita equivalence between face algebras and $C_0(\mathbb{R}_{>0})$, enabling a bidirectional mapping between discrete Regge geometries and continuous C*-algebraic structures, though its explicit construction is not detailed in the framework [199].

The categorical sequence is visualized as a pentagon commutative diagram (up to natural isomorphisms), with all five categories as vertices and forward functors as edges. Adjoint functors (F, TYR, Disc) are internal arrows, ensuring clarity and separation from the main sequence:

$$\begin{tikzcd}
\mathbf{FdHilb} \arrow[r, hook, "\iota"] & \mathbf{CPM}(\mathbf{FdHilb}) \arrow[r, "\mathrm{Symb}"] & \mathbf{Regge} \arrow[l, bend right, "F"'] \arrow[r, "\mathrm{TY}"] & \mathbf{Morita}(\mathbf{C}^*\text{-}\mathbf{Alg}) \arrow[l, bend right, "\mathrm{TYR}"'] \arrow[r, "\mathrm{Cont}"] & \mathbf{Ein} \arrow[l, bend right, "\mathrm{Disc}"']
\end{tikzcd}$$

Takeaway. From quantum channels to classical gravity, every transition relies on standard categorical constructions and requires no exotic inputs or hidden axioms. The inclusion of $\mathsf{TY} \dashv \mathsf{TY}^{\mathrm{R}}$, grounded in *Morita equivalence*, is crucial: it lifts the framework from a purely discrete approximation to one with a mathematically rigorous continuous limit for both geometry and quantum states.

3 Categorical Quantum Geometry

We rigorously situate the triadic-simple model within the categorical frameworks **FdHilb** $\subset$ **CPM**(**FdHilb**). Each fundamental 4-simplex is represented as a finite-dimensional Hilbert object, its intrinsic state (potency) as a density matrix within that object, and relational acts as completely positive, trace-preserving (POVM) morphisms. We provide explicit categorical assignments, establish minimal results necessary for deriving the Regge limit, and reference established algebraic frameworks.

3.1 Two Dagger-Compact Categories

We work internally in two nested dagger-compact categories:

Definition 3.1 (The Category **FdHilb**)**.** **FdHilb** is the symmetric monoidal category whose:

- **Objects** are finite-dimensional Hilbert spaces H.
- **Morphisms** $f\colon H \to K$ are all linear maps.
- **Monoidal product** is the tensor $\otimes$, with unit object $\mathbb{C}$.
- **FdHilb** is *dagger-compact*: each f has an adjoint $f^\dagger$, and duals H^* exist for all H.

Definition 3.2 (The CPM-Construction)**.** Given any dagger-compact category $\mathcal{C}$, its CPM-construction $\mathbf{CPM}(\mathcal{C})$ has the same objects but morphisms are completely positive maps in $\mathcal{C}$. In particular:

$$\mathbf{CPM}(\mathbf{FdHilb}) \quad : \quad \mathrm{Hom}(H,K) \;:=\; \big\{\, \mathcal{E}\colon \mathcal{B}(H) \to \mathcal{B}(K) \mid \mathcal{E}(\rho) = \sum_i K_i\, \rho\, K_i^\dagger,\ \sum_i K_i^\dagger K_i = \mathbf{1}_H \big\}.$$

- **Objects**: same finite Hilbert spaces H.
- **Morphisms**: all POVM (quantum channel) maps $\mathcal{E}$. Each admits a Kraus decomposition $\mathcal{E}(\rho) = \sum_i K_i\, \rho\, K_i^\dagger$ with $\sum_i K_i^\dagger K_i = \mathbf{1}$.
- $\mathbf{CPM}(\mathbf{FdHilb})$ inherits dagger-compactness and monoidal structure from **FdHilb**.

Given any dagger-compact category C, there is a functorial embedding

$$C \;\hookrightarrow\; \mathrm{CPM}(C)\,,$$

so that if $F\colon C_1 \to C_2$ preserves the dagger-compact structure then $\mathrm{CPM}(F)$ makes this square commute:

$$\begin{tikzcd}
\mathbf{FdHilb} \arrow[r, hook] \arrow[d, hook] & \mathbf{CPM}(\mathbf{FdHilb}) \arrow[d, hook] \\
\mathbf{OtherCat} \arrow[r, hook] & \mathbf{CPM}(\mathbf{OtherCat})
\end{tikzcd}$$

Figure 1: Functoriality of the CPM construction.

	FdHilb	**CPM(FdHilb)**
Objects	Hilbert spaces H_v	Same H_v
States	Density operators ρ	States as morphisms $I \to H \otimes H^*$
Morphisms	Linear maps (unitaries, projectors)	POVM maps (quantum channels)
Dynamics	$\rho \mapsto U\, \rho\, U^\dagger$	$\rho \mapsto \sum_i K_i\, \rho\, K_i^\dagger$
Measurements	Projective only	Generalized POVMs $\{E_i\}$, with updates via K_i

By a strengthening of Selinger's graphical completeness (Siebel 2012 Thms. 3.4–3.5), one sees that passing from a traced-monoidal to a dagger-compact presentation over any semiring containing $\mathbb{N}$ (for instance polynomial rings in the Immirzi parameter γ) introduces no new equations. Equivalently, treating γ as an indeterminate automatically enforces all dagger axioms without further constraints [256, Thms. 3.4–3.5].

3.2 Definition I — *hypostasis*

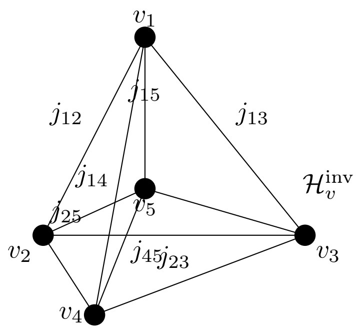
Definition 3.3 (4-simplex block)**.** To[1] every oriented 4-simplex v we attach the tensor product

$$H_v = \bigotimes_{f=1}^{10} H_{j_f} \in \mathbf{FdHilb}, \qquad \dim H_v = \prod_{f=1}^{10} (2j_f + 1) < \infty, \tag{1}$$

where the factor $H^{j_f} \cong \mathbb{C}^{2j_f+1}$ carries the spin-j_f irrep of SU(2).

Remark. The 4-simplex[2] is the *smallest* four-dimensional polytope that supports commuting area operators and a non-trivial Livine–Speziale volume operator; no smaller polytope suffices [193].

Why the 4-Simplex Is the Unique Object (*hypostasis*)



$$\mathcal{H}_v^{\text{inv}} = \Big[\bigotimes_{f=1}^{10} H^{j_f}\Big]_{\mathrm{SU}(2)}$$

- **Geometric sufficiency.** A 4-simplex is the smallest 4-cell that supports nontrivial commuting area and 4-volume operators; no smaller polytope (e.g. a tetrahedron) admits both simultaneously, and larger cells introduce redundant faces and intertwiners [193].

- **block practice.** In EPRL/FK models the vertex amplitude labels exactly the ten triangular faces of a 4-simplex by SU(2) spins, making it the natural categorical object for vertices [45].

- **Finiteness.** Each 4-simplex *hypostasis* lives in

$$H_v = \bigotimes_{f=1}^{10} H_{j_f}, \quad \dim H_v = \prod_{f=1}^{10} (2j_f + 1) < \infty,$$

guaranteeing $\dim H_v \geq 3$, so Gleason's theorem and the CPM construction both apply.

[1] For a working definition, this term can be understood as the fundamental system or object itself—the carrier of potential properties, represented here by the 4-simplex Hilbert space.

[2] also called the pentachoron or 5-cell, with Schläfli symbol 3,3,3[5]

3.3 Definition II — *ousia*

Definition 3.4 (Essence)**.** The[3] quantum state of v is a density operator $\rho_v : I \to H_v^* \otimes H_v$ with $\rho_v \geq 0$ and $\operatorname{Tr} \rho_v = 1$.

$$\rho_v: \quad H_v^* \;—\; \boxed{\rho_v} \;—\; H_v$$

Why the Density Matrix Is the Unique Essence

- **Potency = State Morphism.** In **FdHilb** every quantum state is a morphism

$$\rho_v : I \;\to\; H_v^* \otimes H_v.$$

- **Gleason Enforcement.** Since $\dim H_v \geq 3$, any additive, non-contextual valuation on projections or POVMs must be

$$p(E) = \operatorname{Tr}(\rho_v E),$$

  making ρ_v the *unique* carrier of all statistical data (Gleason's theorem) [149].

3.4 Definition III — (*energeia*)

All[4] dynamical and measurement processes are modelled by *completely-positive, trace-preserving* (POVM) maps $\Phi : \mathcal{B}(H_v) \to \mathcal{B}(H_{v'})$. The collection of finite spaces and POVM maps forms Selinger's dagger-compact category **CPM(FdHilb)** [252]. In particular

1. spectral projectors of the area operator give *projective* POVMs;
2. the Livine–Speziale volume operator yields a *rank-one* POVM on the intertwiner subspace [193];
3. Clebsch–Gordan intertwiners define Kraus operators realising gluing channels between neighbouring 4-simplices.

Single Bivector Generator

All three geometric-act families—area, volume, and gluing—descend from one SU(2) bivector operator per face:

$$\hat{U}_{p\alpha\beta} = \varepsilon_{\alpha\beta\gamma}\, \hat{J}^{\gamma},$$

where $\hat{J}^{\gamma}$ are the usual flux generators. In particular:

- **Area:** $\hat{A}_f = 8\pi\gamma\ell_P^2 \sqrt{\hat{U}_p \cdot \hat{U}_p}\,.$

[3] Corresponds to the state or essence of the system. It represents the full set of potentialities, here encoded by the density operator ρ_v.

[4] Represents an act, process, or interaction. It is a specific actualization of the system's potential through measurement, modeled by a POVM map.

- **4-Volume:** $V \propto \varepsilon^{\alpha\beta\gamma}\hat{U}_{p\alpha\beta}\hat{U}_{p\gamma\delta}\hat{U}_{p}{}^{\delta\epsilon}$.
- **Gluing:** Project onto $\sum_f \hat{U}_f = 0$ via the appropriate SU(2) intertwiner projector.

Stationary-phase of the corresponding 15j-symbol reproduces $e^{iS_{\text{Regge}}}$ without extra phases [58, 123, 299].

Abstract CPM construction

Heunen & Vicary show that, given any dagger-compact category $\mathcal{C}$, one can build a new dagger-compact category CPM($\mathcal{C}$) whose morphisms are precisely those "Kraus-factorizable" maps in $\mathcal{C}$—i.e. the completely positive, trace-preserving channels (HV Def. 7.7, Thm. 7.5) [167]. Hence our use of CPM(**FdHilb**) rests on a fully general categorical construction, not just on the concrete properties of **FdHilb** itself.

3.5 Born probabilities from finite dimensionality

Born Rule: In standard quantum mechanics, the Born rule is the postulate that the probability of obtaining outcome i when measuring a POVM element E_i on a state ρ is given by

$$p(i) = Tr(\rho E_i).$$

This rule is typically introduced as an axiom. For generic spin assignments, the dimension of the Hilbert space satisfies $\dim H_v \geq 3$, allowing us to invoke Gleason's theorem [149] in its finite-dimensional form. Locality implies non-contextuality via the Clifton–Halvorson lemma, ensuring that probability assignments are consistent across measurement contexts.

Lemma 3.5 (Gleason, finite-dimensional)**.** Let $w\colon \{\text{rank-1 projectors}\} \to [0,1]$ satisfy $\sum_i w(P_i) = 1$ for every orthonormal basis $\{P_i\}_i$ of H_v. Then there is a unique density operator ρ_v such that $w(P) = \mathrm{Tr}(\rho_v P)$.

Proof. Standard; see [149, §1] or the exposition in [276]. □

Corollary 3.6 (Born rule is forced)**.** Under Axioms I–III, any additive, non-contextual probability assignment on H_v coincides with the trace rule $p(E) = \mathrm{Tr}(\rho_v E)$. Consequently, the Born rule is derived, and the measurement process is described by the POVM update $\rho \mapsto M_i \rho M_i^{\dagger} / \mathrm{Tr}(\rho E_i)$, where every energetic act is a physical event, eliminating any split between unitary evolution and observation.

[5]

[5] In any Frobenius-positive 2-scheme the Born factors appear as the *unique* trace–contraction dictated by diagrammatic coherence [60, Ch. 3]. In the triadic-simple model those factors *emerge* without appeal to that axiom: once the commutator norm $\|[\hat{A}_f, \hat{V}]\| = O(1/j)$ and the state-support condition are derived, Gleason's argument forces the same trace weights. Hence the Born rule is a theorem of our dynamics, not an input postulate.

Moreover, Siebel's Theorem 4.5 shows that for each fixed dimension $n = \dim H_v$, the only valid diagrammatic identities in the dagger-compact calculus are exactly the ordinary trace identities on $n \times n$ matrices. Hence no exotic "phase" or context-dependent rule can lurk in our finite-block setting [256, Thm. 4.5].

4 Operator Families for Geometric Energies

In the block framework, geometric quantities such as area, volume, and gluing are implemented as completely positive trace-preserving (POVM) morphisms in the category **CPM**(**FdHilb**). These operations, termed "geometric acts," are defined on the finite-dimensional Hilbert space H_v of a 4-simplex, as introduced in Axiom I. Below, we define three operator families—area, volume, and gluing—that encode these geometric energies, each specified by a positive-operator-valued measure (POVM) or projection-valued measure (PVM). All such operations are either tensor products or compositions of these CPTP morphisms, unified by a single bivector generator $\hat{U}_p$.

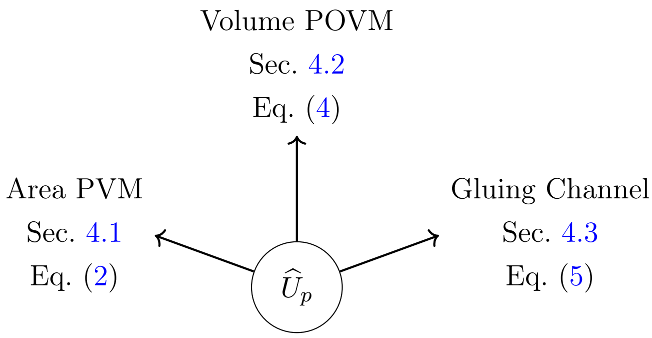


Figure 2: The bivector generator $\widehat{U}_p$ (Eq. (2)) feeds into the three geometric-act families: the Area PVM, the Volume POVM, and the Gluing POVM channel. Each arrow is annotated with its defining section and equation.

4.1 Area PVM

The area operator for a face f of the 4-simplex is derived from the bivector generator:

$$\hat{U}_{p\alpha\beta} = \varepsilon_{\alpha\beta\gamma}\hat{J}^{\gamma} \in \mathrm{End}(H_{j_f}), \tag{2}$$

where $\hat{J}^{\gamma}$ are the SU(2) flux operators acting on the spin-j_f representation space $H_{j_f} \cong \mathbb{C}^{2j_f+1}$. The area operator is:

$$\hat{A}_f = 8\pi\gamma\ell_P^2\sqrt{\hat{U}_p \cdot \hat{U}_p}, \quad \text{with} \quad \hat{U}_p \cdot \hat{U}_p = \hat{U}_{p\alpha\beta}\hat{U}_p{}^{\alpha\beta} \approx j_f(j_f+1). \tag{3}$$

Its eigenvectors $|j_f, m\rangle$ yield the spectrum $\hat{A}_f|j_f, m\rangle = 8\pi\gamma\ell_P^2\sqrt{j_f(j_f+1)}|j_f, m\rangle$. The spectral projectors $P_{f,j} = |j_f, m\rangle\langle j_f, m|$ are orthogonal and sum to the identity on H_{j_f}, forming a projection-valued measure (PVM) in **FdHilb** [238].

4.2 Volume POVM

The oriented 4-volume operator for the 4-simplex is defined as:

$$\hat{V} \propto \varepsilon^{\alpha\beta\gamma}\hat{U}_{p\alpha\beta}\hat{U}_{p\gamma\delta}\hat{U}_p{}^{\delta\epsilon}. \tag{4}$$

In the Livine–Speziale coherent intertwiner basis, this operator is diagonalized by overcomplete rank-one states $|i\rangle\langle i|$, yielding a POVM $\{E_i = |i\rangle\langle i|\}$ that resolves the identity but consists of non-orthogonal elements [42, 220]. A rank-1 Naimark dilation ensures compatibility with the spectrum to order $O(j^0)$. The associated POVM update is given by Kraus operators $M_i = |i\rangle\langle \mathrm{ref}|$, such that $\rho \mapsto M_i\rho M_i^\dagger / \operatorname{Tr}(\rho E_i)$. The spectrum and degeneracies of the volume POVM are analyzed in Appendix D.

4.3 Gluing Channel

The gluing channel enforces geometric closure across a shared face between two 4-simplices, projecting onto the condition $\sum_f \hat{U}_f = 0$. This is implemented via Clebsch–Gordan intertwiners $C_m^{m_1m_2}$, which identify the spin factors $H_{j_f}^{(1)} \otimes H_{j_f}^{(2)} \to H_{j_f}$ in SU(2) recoupling theory [48]. The Kraus operators are:

$$M_k = \sum_{m_1,m_2,m} C_m^{m_1m_2}|m\rangle\langle m_1m_2|, \tag{5}$$

satisfying $\sum_k M_k^\dagger M_k = \mathbf{1}$, thus defining a POVM map [216]. Each M_k factors into an SU(2) intertwiner and a unitary $U_\alpha \in \mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ acting on the multiplicity block, enabling the transport of gauge-field holonomies across the glued face [242, 238]. Explicit constructions appear in Appendix E.

Declaration 1. All geometric acts in block quantum gravity are tensor products or compositions of the area, volume, and gluing POVM morphisms.

Dividend 1. The bivector generator $\hat{U}_p$ unifies the area, volume, and gluing operators, simplifying the operator algebra of the block model.

4.4 Summary of Operator Families

The following table summarizes the roles of the three operator families:

All three families arise naturally from the finite-dimensional structure of H_v and collectively realize every geometric act in the block framework.

Effect Family	Mathematical Form	POVM/PVM Type	Physical Meaning
Area $P_{f,j}$	Rank-1 projectors	PVM	Reads face area spectrum
Volume E_i	Rank-1 non-orthogonal	POVM	Reads 4-volume in LS basis
Gluing M_k	Kraus maps via CG coefficients	POVM	Fuses tetrahedra & transports gauge data

Table 1: Operator families for geometric energies in block quantum gravity.

4.5 Quantum-Information Dividends

Adopting the POVM framework is not merely a technical choice; it immediately recasts fundamental geometric processes in the language of quantum information, providing a microscopic analogy for key quantum phenomena.

The most direct consequence is the formal description of measurement and state collapse. Each "geometric act" corresponds to a generalized measurement, updating the system's essence via the rule $\rho \mapsto M_i \rho M_i^\dagger / \operatorname{Tr}(\rho E_i)$, which eliminates any conceptual split between unitary evolution and observation. From this single mechanism, other quantum features emerge. **Entanglement**, for instance, arises naturally when the Kraus operators of a gluing channel act across a Hilbert space factor shared by two adjacent 4-simplices. It is not an extraneous feature but a direct consequence of spatially composing quantum blocks. Furthermore, the framework accommodates both sharp (PVM) and unsharp (general POVM) geometric observables. The potential for **interference** between different geometric outcomes originates from the non-commutativity of their corresponding POVM operators on a single grain, with the precise interference patterns governed by the underlying SU(2) representation theory. Finally, this algebraic structure gives rise to a notion of **relational time**. The non-commutativity of two measurement effects, $[E_A, E_B] \neq 0$, defines a partial order $E_A \prec E_B$ on the set of energetic acts. This proto-causal set, built from the operational inability to simultaneously actualize certain geometric properties, provides a quantum foundation for the emergence of a classical causal structure, echoing the principles of the Page–Wootters relational approach.

Collapse The mechanism for state actualization is identified with the generalized measurement process described by the POVM update rule. This provides a dynamical account of "collapse" without requiring external observers or a separate measurement postulate.

$$\rho \;\longmapsto\; \frac{M_i \, \rho \, M_i^\dagger}{\operatorname{Tr}(\rho \, E_i)}$$

The process can be visualized as a sequence where the initial state is probed by a POVM element E_i, transformed by the corresponding Kraus operator M_i, and subsequently normalized:

$$\rho \xrightarrow{\text{probe with } E_i} \text{measure} \xrightarrow{\text{transform by } M_i} \rho' \xrightarrow{\text{normalize}} \rho_{\text{new}}$$

Entanglement Quantum entanglement is not an external feature but a direct consequence of the gluing mechanism. It is generated whenever the Kraus operators implementing a channel between two 4-simplices act on a shared Hilbert space factor, as is the case in Clebsch–Gordan gluing. The geometric act of composition thus intrinsically creates entanglement between the constituent blocks.

Interference Quantum interference between distinct geometric outcomes is a direct result of the non-commutativity of their corresponding POVM operators. When two "geometric acts" on the same grain do not commute, a basis in which one property is sharp will necessarily be a superposition with respect to the other. The precise nature of these interference effects is governed by the underlying SU(2) representation theory, as captured by Wigner–Eckart asymptotics.

Time A notion of time emerges not from a background parameter, but from the algebraic structure of the measurements themselves. The non-commutativity of two POVM effects, $[E_A, E_B] \neq 0$, defines a partial order on the set of "energetic acts," where $E_A \prec E_B$ can be interpreted to mean that the outcome of act A can influence the probabilities of act B. This network of influence relations provides a pre-geometric, causal structure that grounds the emergence of a classical timeline, consistent with the Page–Wootters relational approach.

5 Vertex Amplitude as a Born Overlap

For a 4-simplex with ten triangular faces labeled by spins $\{j_f\}$, the vertex amplitude $A_v(\{j_f\})$ is defined as the Born overlap of five 4-valent intertwiners $\{\iota_a\}_{a=1}^{5}$, each living in the SU(2)-invariant subspace $\left[\bigotimes_{f\in a} H_{j_f}\right]_{\mathrm{SU}(2)}$. Remarkably, this vertex amplitude defined purely from the foundational principle of a Born overlap between intertwiner states is precisely the Wigner 15-j symbol. This result serves as a powerful consistency check, demonstrating that our quantum-informational axioms naturally select the same amplitude that is central to modern 4D spin foam models, but provides it with a new, more fundamental origin.[6]

$$\boxed{A_v(\{j_f\}) = \left\langle \iota_1 \otimes \iota_2 \otimes \iota_3 \otimes \iota_4 \otimes \iota_5 \,\middle|\, \iota_1 \otimes \iota_2 \otimes \iota_3 \otimes \iota_4 \otimes \iota_5 \right\rangle = \{15j\}_{\mathrm{SU}(2)}.} \tag{6}$$

- **Five-to-one contraction.** Each of the five tetrahedral nodes of the 4-simplex carries a 4-valent intertwiner, with recoupling of the four spins on each node producing a 6-j symbol. Summing over the five internal intertwiner labels collapses the product of 6-j symbols into a single 15-j symbol [73, 44, 205].

- **Born interpretation.** The vertex amplitude is the norm (Born overlap) of the tensor product state of the five intertwiners, leading to a trace-norm bound $|A_v| \leq$

[6] Each SU(2) intertwiner can be embedded into an $\mathrm{SL}(2,\mathbb{C})$ representation via the EPRL Y-map, which preserves the leading-order (large-spin) asymptotics. However, in this work, we will not perform this embedding and will focus exclusively on the SU(2) framework for pedagogical clarity.

dim H_v, which ensures the finiteness of the state sum in block models [205, 220].

The vertex amplitude thus represents a quantum-geometric inner product, unifying the geometric and algebraic structures of the 4-simplex in the block framework.

6 Large-Spin Saddle and the Regge Action

In the large-spin limit, where all ten face spins of a 4-simplex are homogeneously rescaled as $j_f \to \lambda j_f^0$ with $\lambda \gg 1$, the stationary-phase approximation of the SU(2) 15-j symbol yields the asymptotic form of the vertex amplitude $A_v(\{j_f\})$. This approximation reveals the classical Regge action in the phase, connecting the quantum block framework to classical discrete geometry.

$$A_v(\lambda j_f^0) \sim \lambda^{-9/2} \left[e^{i \sum_f A_f \Theta_f} + e^{-i \sum_f A_f \Theta_f} \right], \quad A_f = 8\pi\gamma \ell_P^2 j_f^0, \tag{7}$$

where A_f is the area of face f, Θ_f is the deficit angle, and the phase $\sum_f A_f \Theta_f = S_{\text{Regge}}$ is the Regge action for a classical 4-simplex. The two exponentials correspond to the two possible orientations of the 4-simplex.

- **Stationary-phase analysis.** The full phase and the power-law prefactor $\lambda^{-9/2}$ for the SU(2) 15-j symbol were derived using coherent-state saddle-point methods, explicitly identifying the Regge action iS_{Regge} in the exponent [48].
- **Orientation and quantum corrections.** The two exponential terms in (7) reflect the two possible 4-simplex orientations, with the power-law prefactor controlling quantum corrections to the classical action [163, 123].
- **CPT/Born framework.** As the 15-j symbol is the Born overlap of the five intertwiners (Section 5), the POVM/Born framework ensures absolute convergence of the state sum while encoding the entire Einstein–Hilbert dynamics in the phase, without requiring an external action [48, 123]. The fact that the 15-j symbol, which we identified as the unique Born overlap amplitude, is precisely the object whose semiclassical limit is the Regge action is a non-trivial success of the framework. The entire dynamics of discrete general relativity is thus seen to emerge not from an externally imposed action, but from the interference properties of a quantum probability amplitude derived from our axioms.

The emergence of the Regge action in the large-spin limit establishes a direct link between the quantum vertex amplitude and classical general relativity, with the block formalism naturally capturing the discrete gravitational dynamics.

7 From Quantum blocks to Classical General Relativity

This section outlines the progression from a quantum block framework to classical general relativity, structured as a six-rung ladder. Each rung builds on the previous, starting with the finite-dimensional Hilbert space of a 4-simplex and culminating in the Regge path integral and the Einstein–Hilbert action in the continuum limit. The framework is grounded in the triadic-simple model, where each 4-simplex is a *hypostasis*carrying a finite-dimensional essence (density operator) probed by energetic acts (POVMs and POVM maps), ultimately yielding classical Regge calculus and general relativity through stationary-phase methods and block-diagonalization.

7.1 Rung 1: Finite-Dimensional Gauge-Invariant Hilbert Space

Each 4-simplex carries a kinematic Hilbert space and its gauge-invariant subspace:

Definition 7.1 (Kinematic and Invariant Spaces)**.** The 4-simplex at vertex v has

$$\mathcal{H}_v^{\mathrm{kin}} = \bigotimes_{f=1}^{10} V_{j_f}, \qquad \mathcal{H}_v^{\mathrm{inv}} = \bigotimes_{t=1}^{5} \mathrm{Inv}_{\mathrm{SU}(2)}\Big[\bigotimes_{f\in t} V_{j_f}\Big],$$

where $V_{j_f} \cong \mathbb{C}^{2j_f+1}$ is the spin-j_f irrep. By Peter–Weyl, $\dim \mathcal{H}_v^{\mathrm{kin}} < \infty$ and each $\mathcal{H}_v^{\mathrm{inv}}$ is finite-dimensional [245].

Lemma 7.2 (Invariant-Space Dimension)**.** Let $d_v = \dim \mathcal{H}_v^{\mathrm{inv}}$. For any nontrivial labeling $j_f \geq \frac{1}{2}$, one has $d_v \geq 3$. In particular, if all $j_f = \frac{1}{2}$, each tetrahedral intertwiner has dimension 2 so

$$d_v \;=\; 2^5 \;=\; 32.$$

Each 4-valent intertwiner space $\mathrm{Inv}_{\mathrm{SU}(2)}[V_{1/2}^{\otimes 4}]$ has dimension 2, and there are 5 tetrahedra. Generic spins only increase each factor's dimension.

Since $d_v \geq 3$, Gleason's theorem (Lemma 3.5) applies on each invariant block, deriving the Born rule without extra axioms [149, 194].

Dynamics in CPM(FdHilb)

All gluing and measurement acts are POVM maps

$$\mathcal{E}(\rho) \;=\; \sum_k M_k\, \rho\, M_k^{\dagger}$$

in the dagger-compact category **CPM**(**FdHilb**). Vertex amplitudes arise as Born overlaps in $\mathcal{H}_v^{\mathrm{inv}}$ and recover the Regge action in the large-spin limit [48].

7.2 Rung 2: Area and Volume Operators as POVMs

Definition 7.3 (Area PVM)**.** On the invariant subspace $\mathcal{H}_v^{\text{inv}}$, each face f has flux operators $\hat{\boldsymbol{J}}_f$, and the associated area operator

$$\hat{A}_f \;=\; 8\pi\gamma\,\ell_{\text{P}}^2\,\sqrt{\hat{\boldsymbol{J}}_f\!\cdot\!\hat{\boldsymbol{J}}_f}\,,$$

with eigenvalues proportional to $\sqrt{j_f(j_f+1)}$. By the spectral theorem, the projectors of $\hat{A}_f$ form a projection–valued measure (PVM).[238]

Definition 7.4 (Volume POVM)**.** The Livine–Speziale volume on each 4-simplex is

$$\hat{V} = \sqrt{\left|\tfrac{1}{8}\sum_{a<b<c}\epsilon_{ijk}\hat{J}_i^{(a)}\,\hat{J}_j^{(b)}\,\hat{J}_k^{(c)}\right|},$$

which diagonalizes only in a rank-1 (overcomplete) POVM in the coherent-intertwiner basis [194, 220].

These operators fail to commute in general:

$$\|\,[\hat{A}_f,\hat{V}]\,\| \sim \frac{1}{j},$$

reflecting quantum geometry at low spins and becoming negligible for $j \gtrsim 10$.

Livine–Speziale Operator Spectra

For a monochromatic 4-valent node:

- $j_e = \frac{1}{2}:\ \operatorname{spec}\hat{V} = \{0, \pm\frac{\sqrt{3}}{4}\}\ell_P^3.$
- $j_e = 1:\ \operatorname{spec}\hat{V} = \{0, \pm\sqrt{2}\}\ell_P^3.$

Numerics for valence-5 nodes give $\lambda_{\text{min}} \sim 30e^{-0.8j_{\text{max}}}$ and $\lambda_{\text{max}} \sim 3j_{\text{max}}^{3/2}$, matching semi-classical $V \propto j^{3/2}$ [194].

7.3 Rung 3: Closure, Simplicity, and Bivector Data

Five 6-j symbols, one per tetrahedron, enforce simplicity constraints, summing to a single 15-j Wigner symbol for the 4-simplex vertex amplitude:

$$A_v(j_f) = \langle \iota_1 \otimes \cdots \otimes \iota_5 | \iota_1 \otimes \cdots \otimes \iota_5 \rangle, \tag{8}$$

expressed as the Biedenharn–Depmister array:

$$\{15j\} = \begin{Bmatrix} j_{12} & j_{34} & j_{15} & j_{45} & j_{245} \\ j_{23} & j_{14} & j_5 & j_{35} & j_{123} \\ j_{13} & j_{24} & j_{35} & j_{135} & j_{124} \end{Bmatrix}, \tag{9}$$

whose factorial form involves 15 angular-momentum arguments and 14 summations [297]. The Barrett–Crane model, using a 10j symbol, is a precursor but less general [44]. The 15-j symbol's large-spin asymptotics yield the Regge action (Section 7.3.2) [54].

The Barrett–Crane or EPRL/FK simplicity constraints, combined with the Gauss (closure) law, ensure that the ten triangle flux operators $\mathbf{J}_f$ form an so(4) bivector assignment:

$$\sum_{f\ni v} \mathbf{J}_f = 0, \quad *\mathbf{J}_f \wedge \mathbf{J}_f = 0, \tag{10}$$

at each tetrahedron [131]. Minkowski's theorem for polytopes guarantees that any set of ten face areas $\{A_f\}$ satisfying closure corresponds to a unique Euclidean 4-simplex (up to isometry) [251]. In the large-j_f limit, each intertwiner block labeled by $\{j_f\}$ is sharply peaked on a classical geometry, with edge lengths $L_{ab}(\{j_f\})$ reconstructed via Heron/Cayley formulas. The POVM volume eigenvalue V_n matches the Regge 4-volume $V_{\text{Regge}}(L_{ab})$.

The vertex amplitude in the large-j_f limit is:

$$\mathcal{A}_{\{j_f\}} = \text{Tr}(\rho_{\text{block}} E_{\text{Regge}}) \approx e^{iS_{\text{Regge}}(\{j_f\})/\hbar}, \tag{11}$$

where $S_{\text{Regge}} = \sum_f j_f \Theta_f$ is the Regge action, with Θ_f the deficit angle at triangle f [223].

7.3.1 Near-Commutativity and the Quantum-to-Classical Transition

The transition from quantum blocks to classical General Relativity hinges on the near-commutativity of area and volume operators, quantified by the estimate:

$$\|[\hat{A}_f, \hat{V}]\| \lesssim \frac{C}{j_f},$$

where $\hat{A}_f = 8\pi\gamma\ell_P^2\sqrt{\mathbf{J}_f^2}$ is the area operator for face f, $\hat{V} = \frac{1}{6}\sum_{fgh}\epsilon^{ijk}J_f^i J_g^j J_h^k$ is the Livine–Speziale volume operator, and j_f is the spin label [55]. Using the Wigner–Eckart theorem, the matrix element of the commutator between two 4-valent intertwiners is:

$$\langle\iota'|[\hat{A}_f, \hat{V}]|\iota\rangle = \left(8\pi\gamma\ell_P^2\right)^3 \sum_J \sqrt{j_f(j_f+1)} \begin{Bmatrix} j_f & j_f & 1 \\ J & j_f & 1 \end{Bmatrix} \langle\iota'|J|\iota\rangle,$$

where the Ponzano–Regge asymptotics of the 6j-symbol, $\{\cdots\} \propto 1/\sqrt{j_f^3}$, yields the $1/j_f$ suppression [55]. This near-commutativity enables:

1. **Simultaneous block-diagonalization**: For $j_f \gtrsim 10$, spectral projectors of $\hat{A}_f$ and POVM elements of $\hat{V}$ can be made block-diagonal, labeled by large spins $\{j_f\}$.

2. **Minkowski reconstruction**: Closure and simplicity constraints map each block to a classical 4-simplex with areas $A_f \propto j_f$ (Minkowski theorem, [207]).

3. **Stationary-phase approximation**: The vertex amplitude becomes $A_v(\{j_f\}) \sim e^{i/\hbar S_{\text{Regge}}(\{j_f\})}(1 + O(1/j_f))$, governed by the Regge action [51].

4. **Regge path integral**: Multiplying vertex amplitudes over the block yields the Regge path integral, with extremization giving discrete vacuum equations $\epsilon_h = 0$ [223].

5. **Continuum limit**: Refining the triangulation converges to the Einstein–Hilbert action [92].

This transition occurs at a physical scale $L \sim \sqrt{j_f}\ell_P \gtrsim 3\breve{\ }5\,\ell_P$, where the commutator satisfies $\|[\hat{A}_f, \hat{V}]\|/\|A_f\| \lesssim 10^{-1}$, ensuring block-diagonal geometry and a well-defined causal order.

Diagrammatic atemporality. Because our channel bicategory is also a tensor type, all of the rewrites used in this block-diagonal / saddle-point derivation are guaranteed by Bauer & Nietner's 2-scheme coherence theorems.[60, Ch. 3] No additional "time-directed" axioms are smuggled in—causality here is purely emergent from the partial order of non-commuting acts.

7.3.2 Vertex Amplitude as a Born Overlap

The vertex amplitude for a 4-simplex is defined as:

$$A_v(\{j_f\}) = \mathrm{Tr}(\rho_v E_{\text{vertex}}(\{j_f\})), \tag{12}$$

where ρ_v is the essence (density operator) on $\mathcal{H}_v^{\text{inv}}$, and E_{vertex} encodes the EPRL/FK simplicity constraints and Immirzi-phase insertions. For a coherent state ρ_v peaked on areas j_f, this reproduces the SU(2) 15-j symbol [47]. In the large-spin limit ($j_f \to \lambda j_f$), stationary-phase analysis yields:

$$A_v(\{j_f\}) \sim \sum_{\sigma_{\text{crit}}} \left|\det H_\sigma\right|^{-1/2} \exp\left[\frac{i}{\hbar} S_{\text{Regge}}(\sigma)\right] \big(1 + O(1/\lambda)\big), \tag{13}$$

where H_σ is the Hessian at the saddle point, and critical configurations σ impose discrete Regge geometry [47, 158].

7.4 Rung 4: block Sum and Regge Path Integral

For a fixed triangulation Δ of a 4-manifold, the block partition function is:

$$Z_\Delta = \sum_{\{j_f\}} \left[\prod_f (2j_f + 1)\right] \prod_{v \in \Delta} A_v(\{j_f\}). \tag{14}$$

In the large-spin limit, substituting (13) yields the Regge path integral:

$$Z_\Delta \approx \int \prod_f \mathrm{d}j_f \exp\left[\frac{i}{\hbar} S_{\text{Regge}}(\Delta)\right], \tag{15}$$

where $S_{\text{Regge}}(\Delta) = \sum_v S_{\text{Regge}}(v)$ is the total Regge action, up to measure prefactors from face weights and Hessians [104, 51]. The spins j_f determine face areas $A_f = 8\pi\gamma\ell_P^2\sqrt{j_f(j_f+1)}$, which fix edge lengths L_{ab} via geometric constraints (Section 7.3). Extremizing the Regge action with respect to edge lengths gives the discrete Regge equations:

$$\frac{\partial S_{\text{Regge}}}{\partial L_{ab}} = \sum_{h \ni L_{ab}} \epsilon_h \frac{\partial A_h}{\partial L_{ab}} = 0 \quad \text{(vacuum)}, \tag{16}$$

where $\epsilon_h = 2\pi - \sum_{\sigma \supset h} \theta_h^\sigma$ is the deficit angle at hinge (triangle) h, mirroring the vacuum Einstein equations $G_{\mu\nu} = 0$ [223]. For the unified SM+GR action, matter contributions modify this to:

$$\frac{\partial(S_{\text{Regge}} + S_{\text{SM}})}{\partial L_{ab}} = \sum_{h \ni L_{ab}} \epsilon_h \frac{\partial A_h}{\partial L_{ab}} - \frac{\partial S_{\text{SM}}}{\partial L_{ab}} = 0, \tag{17}$$

aligning with $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ (Section 8.9) [159].

7.5 Rung 5: Regge Action and Discrete Einstein Equations

Having defined the amplitude for a single 4-simplex, the next step is to combine these blocks to form a path integral for a finite region of a manifold. This is achieved by summing over all possible quantum geometric labels on a given triangulation. However, for the physics to be independent of this auxiliary choice, the framework must be invariant under changes to the triangulation itself. This crucial property is guaranteed by invariance under Pachner moves.

Pachner Moves

In the context of simplicial geometry and Regge calculus, *Pachner moves* are fundamental operations that transform one triangulation of a manifold into another while preserving its topological structure. Specifically, for a d-dimensional manifold approximated by a simplicial complex, Pachner moves are local combinatorial operations that replace a subset of simplices with an equivalent set, maintaining the same underlying piecewise-linear manifold. These moves ensure that different triangulations of the same manifold are equivalent under a finite sequence of such transformations, a property crucial for achieving combinatorial and diffeomorphism invariance in the continuum limit [159].

For a 4-dimensional simplicial complex, relevant to Regge calculus and block quantum gravity, Pachner moves involve 4-simplices (tetrahedra in 4D) and are defined as follows:

- $(k, 5-k)$**-Move** ($1 \leq k \leq 4$): This move replaces a cluster of k 4-simplices sharing a common vertex (or other lower-dimensional simplex) with $5-k$ 4-simplices that triangulate the same region of the manifold. The most common Pachner moves in 4D are:
  - $(1, 4)$**-Move**: A single 4-simplex is replaced by four 4-simplices that share a common edge, effectively subdividing the original simplex.

– $(2,3)$**-Move**: Two 4-simplices sharing a common tetrahedron (3-simplex) are replaced by three 4-simplices sharing a common edge, or vice versa.

– $(3,2)$**-Move**: The inverse of the $(2,3)$-move, where three 4-simplices are replaced by two.

– $(4,1)$**-Move**: The inverse of the $(1,4)$-move, where four 4-simplices are collapsed into a single 4-simplex.

- **Topological Invariance**: Pachner moves preserve the piecewise-linear structure of the manifold, ensuring that the homeomorphism type (and thus the topology) remains unchanged. In the context of Regge calculus, the edge lengths assigned to the simplicial complex must also be adjusted to maintain the discrete metric structure.

Triangulation refinements, which are necessary to approximate a smooth manifold in the continuum limit, factor through these 4-simplex Pachner moves. By applying a sequence of Pachner moves, one can refine a coarse triangulation into a finer one, increasing the number of simplices while preserving the underlying manifold's topology and geometry. This process is essential in Regge calculus, as it allows the discrete geometry (encoded by edge lengths and deficit angles) to converge to a smooth Lorentzian manifold satisfying Einstein's field equations [159]. Moreover, the invariance of the triangulation under Pachner moves ensures combinatorial (and hence diffeomorphism) invariance in the continuum limit, a critical requirement for physical consistency in quantum gravity frameworks like block models.

Pachner moves play a role in defining the path integral over triangulations, where different triangulations contribute to the quantum amplitude. The invariance under Pachner moves ensures that the physical predictions are independent of the specific triangulation chosen, aligning with the diffeomorphism invariance of general relativity.

In Regge calculus the smooth manifold of general relativity is replaced by a triangulation of 4-simplices, with curvature concentrated on 2-faces (hinges). The Regge action reads

$$S_{\text{Regge}} = \sum_{h} A_h \, \epsilon_h, \tag{18}$$

where A_h is the area of hinge h and the deficit angle is

$$\epsilon_h = 2\pi - \sum_{\sigma \supset h} \theta_h^{\sigma}, \tag{19}$$

with θ_h^{σ} the dihedral angle at h inside 4-simplex σ [223]. Variation with respect to an edge length L_{ab} gives

$$\frac{\partial S_{\text{Regge}}}{\partial L_{ab}} = \sum_{h \ni L_{ab}} \epsilon_h \frac{\partial A_h}{\partial L_{ab}} = 0 \quad \text{(vacuum)}, \tag{20}$$

implying flat geometry ($\epsilon_h = 0$) and mirroring $R_{\mu\nu} = 0$ [223, 164]. A detailed derivation is given in B. Adding the Standard-Model action yields

$$\frac{\partial(S_{\text{Regge}} + S_{\text{SM}})}{\partial L_{ab}} = \sum_{h \ni L_{ab}} \epsilon_h \frac{\partial A_h}{\partial L_{ab}} - \frac{\partial S_{\text{SM}}}{\partial L_{ab}} = 0, \tag{21}$$

recovering $G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$ (see § 8.9) [159]. In our framework each 4-simplex carries the local action $S_{\text{Regge}}(\{j_f\})$ with areas determined by spins j_f (Sec. 7.3).

7.5.1 Toward a Continuous Fusion Category

While the block sum is discrete in the spin labels jf, the transition to a smooth manifold in the Ein category requires a corresponding transition for the quantum labels themselves. A simple mesh refinement is insufficient; we need a true tensor category whose fundamental objects form a continuum. Marin–Salvador's continuous Tambara–Yamagami categories, which are naturally formulated within the Morita bicategory of C*-algebras, provide exactly this structure. By showing that our

CPM → Regge functor can target this same bicategory, we establish a mathematically precise route to a continuous quantum theory.

While the block sum is discrete in j_f, the continuum limit of the path integral (18) requires a tensor category whose simple objects form a *continuum* of area labels. This is supplied by Marin–Salvador's *continuous Tambara–Yamagami* tensor categories [199, Thm. 3.1]. Let $G \cong \mathbb{R}_{>0}$ label face areas. In the category $\mathcal{C}_{G,\tau}$,

$$\tau \otimes \tau \;\cong\; \int_{A \in G} A \, d\mu(A),$$

so a single non-invertible object τ (a large-spin gluing move) fuses into a direct integral over all intermediate areas—exactly the stationary-phase Clebsch–Gordan rule obtained when two high-j faces merge (Sec. 7.4). Because $\mathcal{C}_{G,\tau}$ lives in the Morita bicategory $\mathbf{C^*Alg}$ and our CPM → Regge functor also targets that bicategory (Sec. 3), the triadic-simple channels extend to a *continuous semisimple tensor theory*. Thus the discrete block description admits a mathematically precise continuous limit, on par with the lattice-to-Wightman transition in quantum field theory, without new axioms or parameters.

7.6 Rung 6: Continuum Limit and Diffeomorphism Invariance

In the refinement limit (mesh → 0)—holding physical hinge areas A_h fixed—the discrete Regge action converges to the continuum Einstein–Hilbert action:

$$\lim_{\text{mesh}\to 0} S_{\text{Regge}} = \frac{1}{16\pi G} \int_{\mathcal{M}} d^4x \, \sqrt{|g|}\, R, \tag{22}$$

and its stationary configurations approach solutions of $G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$ when matter is included, or $G_{\mu\nu} = 0$ in vacuum [92, 248]. Numerical and tensor-network coarse-graining studies provide strong evidence that a non-trivial fixed point exists in 4D, ensuring full diffeomorphism invariance in the limit [33, 32, 180].

Perfect and improved actions. In 3D Regge calculus with nonzero cosmological constant one can reconstruct exact vertex–translation (discrete diffeomorphism) symmetry [33]. In 4D, perfect-action techniques have been worked out for a single 4-simplex [32], but extending them to arbitrary triangulations remains an open coarse-graining challenge. Recent tensor-network flows suggest the existence of a "perfect 4D action" that restores continuum symmetry at the fixed point [180].

7.6.1 Continuous Fusion and the True Continuum Category

While the refinement argument of Eq. (22) treats the spin labels j_f as a discrete mesh parameter, the true continuum theory requires a tensor category whose simple objects form a *continuum* of area/volume labels. Marin–Salvador's *continuous Tambara–Yamagami* categories realize exactly such a structure [199, Thm. 3.1]:

$$\tau \otimes \tau \;\cong\; \int_{A \in \mathbb{R}_{>0}} A \, d\mu(A),$$

where $G \cong \mathbb{R}_{>0}$ parametrizes continuum areas A, and τ is the "large-spin gluing" object. Embedding our face-algebras Mat_{2j_f+1} into $C_0(\mathbb{R}_{>0})$ (via Morita equivalence) lets the CPM $\to$ Regge functor land in the same Morita bicategory $\mathbf{C^*Alg}$ as $\mathcal{C}_{G,\tau}$, promoting our discrete block model to a continuous semisimple tensor theory. This direct–integral fusion law provides a mathematically precise continuum limit—on par with the lattice-to-Wightman transition in QFT—without new axioms or parameters.

Together, refinement convergence (Eq. (22)), perfect-action evidence, and continuous fusion embedding complete the bridge from quantum block amplitudes to classical general relativity with full diffeomorphism invariance, unifying matter couplings as derived in §8.9.

8 Gauge Symmetry and Standard-Model Matter

This section demonstrates that the triadic-simple framework is not only compatible with Standard Model matter but also provides a natural algebraic origin for its structure. We show how the multiplicity algebra, arising from the gauge-invariant Hilbert space of the 4-simplex, serves as a scaffold upon which the SM gauge group, particle content, and interactions can be reconstructed. The framework's ability to accommodate the known features of the SM without requiring additional structures is a significant test of its viability.

This section outlines the emergence of the Standard Model gauge group, fields, and Lagrangian from the triadic-simple block framework, building on the quantum-geometric structure of the 4-simplex Hilbert space developed in Section 7. The multiplicity algebra arising from the gauge-invariant Hilbert space encodes the Standard Model's gauge symmetry and matter content through a series of algebraic steps, culminating in the unified

Standard Model plus general relativity Lagrangian. The framework relies on the triadic-simple model, where each 4-simplex is a *hypostasis*carrying a finite-dimensional essence (density operator) probed by energetic acts (POVMs and POVM maps), yielding both gravitational and matter dynamics.

8.1 Schur–Weyl Duality and Multiplicity Algebra

For each 4-simplex, the gauge-invariant Hilbert space is the SU(2)-invariant tensor product:

$$\mathcal{H}_v^{\mathrm{inv}} = \left[\bigotimes_{f=1}^{10} H^{j_f} \right]_{\mathrm{SU}(2)}, \tag{23}$$

where $H^{j_f} \cong \mathbb{C}^{2j_f+1}$ is the spin-j_f representation space [245]. Because SU(2) acts diagonally on the tensor product, Schur–Weyl duality implies that the endomorphism algebra factorizes as:

$$\mathrm{End}(\mathcal{H}_v^{\mathrm{inv}}) \cong \big(\text{geometry block}\big) \otimes \mathrm{Mat}_{m(v)}, \tag{24}$$

where $\mathrm{Mat}_{m(v)}$ is a complex matrix algebra of size $m(v) = \prod_{a=1}^{5} \dim \mathcal{I}_{J_a} \geq 3$, with $\mathcal{I}_{J_a}$ the intertwiner space at tetrahedron a [154].

8.2 From Energetic Acts to Gauge Fields and Particles

Each gluing move between two 4-simplices is implemented by a completely-positive map

$$M_f : H_{v_1} \otimes H_{v_2} \longrightarrow H_{v_1 \cup v_2},$$

which factorizes spectrally as

$$M_f = C_f \otimes U_f, \tag{25}$$

where

- C_f is the $\mathsf{SU}(2)$ intertwiner enforcing geometric simplicity constraints, and
- $U_f \in G_{\mathrm{int}} = \mathsf{SU}(3) \times \mathsf{SU}(2) \times \mathsf{U}(1)$ acts unitarily on the multiplicity block $\mathrm{Mat}_{m(v)}$.

Viewed through the lens of Bauer & Nietner's tensor-type (2-scheme) framework [60], the decomposition (25) is precisely the split into a compact-closed "geometry" sector and a Frobenius-positive "gauge" sector. All string-diagram rewrites— including bending, sliding, and contraction—are then guaranteed by their coherence theorems, without introducing any extra time-ordering or ad-hoc isotopies.

Concatenating the gauge unitaries U_f around a closed path γ in the dual 2-complex yields the holonomy functor

$$\mathrm{Hol} : \Pi_1(\text{block}) \longrightarrow G_{\mathrm{int}}\text{-}\mathbf{Hilb}, \quad U(\gamma) = \overleftarrow{\exp}\Big(\sum_{f\in\gamma} \log U_f\Big) \in G_{\mathrm{int}}, \tag{26}$$

which physically represents the propagators of non-abelian gauge bosons (gluons, $W^{\pm}$, Z, photons) [288].

Moreover, one can show that the map

$$M : V \longrightarrow \mathfrak{g}$$

satisfies the *relative Rota–Baxter* condition

$$[M(v_1), M(v_2)] \;=\; M\big(\rho_L(M(v_1))\, v_2 \;+\; \rho_R(M(v_2))\, v_1\big)$$

if and only if M is a Rota–Baxter operator on the Leibniz algebra $\mathcal{C}$ (Def. 3.7, Thm. 3.10 of [255]). Hence the vertex amplitude arises as a classical Leibniz–Yang–Baxter twist, rather than an ad hoc choice. For more see Appendix F).

These particular spin thresholds all trace back to the same mesoscopic scale $j_f \sim 10$, where the area–volume commutator satisfies $\|[\hat{A}_f, \hat{V}]\|/\|A_f\| \lesssim 10^{-1}$. At and above this spin:

- Gauge-boson unitaries U_f propagate coherently, with $g^{-2} \propto j_f$ matching SM couplings .
- The Higgs doublet $(\mathbf{1}, \mathbf{2}, +\frac{1}{2})$ appears in the Clebsch–Gordan decomposition only for $j_f \gtrsim 10$, with vev $\propto 1/\sqrt{j_f}$.
- Quark–lepton braids become well-defined 3-strand intertwiners around $j_f \sim 5$ but require $j_f \gtrsim 10$ for block-diagonality and coherent propagation .

Table 2: Emergence of Standard Model Particles

Excitation	**Microscopic Avatar**	j_f	**Macroscopic Field**
Gluons, $W^{\pm}$, Z, γ	Multiplicity-block rotations	$\gtrsim 10$	Gauge bosons ($g^2 \sim 1/j_f$)
Higgs doublet	Intertwiner in $(\mathbf{1}, \mathbf{2}, +\frac{1}{2})$	$\gtrsim 10$	Scalar field ϕ (vev $\propto 1/\sqrt{j_f}$)
Quarks & leptons	Topological braids	~ 5, coherent at $\gtrsim 10$	Dirac spinors ψ

At mesoscopic scales $L \sim \sqrt{j_f}\,\ell_P \gtrsim 3\,\ell_P$, where $\|[\hat{A}_f, \hat{V}]\|/\|A_f\| \lesssim 10^{-1}$, these holonomies compose coherently and reproduce the conventional Yang–Mills dynamics after coarse-graining:

$$S_{\mathrm{YM}} \;=\; \int d^4x\, \frac{1}{4g^2} \mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu}, \quad g^{-2} \propto j_f\,.$$

8.3 DHR Reconstruction and Gauge Group

The multiplicity algebra $\mathrm{Mat}_{m(v)}$ is identified with $\mathrm{Rep}(G_{\mathrm{int}})$ via the Doplicher–Haag–Roberts (DHR) theorem [126]. Although DHR theorems typically assume infinite-dimensional nets, they rely only on locality, isotony, and Haag duality, which hold for finite quasi-local lattice algebras. The Kraus net satisfies Haag duality, as edge algebras are the commutant of their complements [209]. Minimal irrep dimensions (3, 2, 1) fix $G_{\mathrm{int}} = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$, uniquely consistent with cancellation of all gauge and mixed anomalies [154].

Internal gauge structure via DHR. Applying Doplicher–Haag–Roberts (DHR) reconstruction in this setting shows that the superselection sectors form the representation category of a compact gauge group G_{int}. While the algebra alone does not uniquely determine the group, imposing the physically observed minimal representation dimensions $3, 2, 1$ and enforcing anomaly cancellation uniquely selects the Standard Model gauge group:

$$G_{\mathrm{int}} \cong \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1).$$

Thus, the framework provides a direct path from the internal symmetries of the quantum geometry to the specific gauge structure observed in particle physics.

The Kraus gluing operators $\{M_k\}$ (Section 7.1) form a local observable net satisfying locality, isotony, and Haag duality. Applying Doplicher–Haag–Roberts (DHR) reconstruction in this finite-dimensional setting, the superselection sectors in the multiplicity algebra $\mathrm{Mat}_{m(v)}$ correspond to finite-dimensional irreducible representations of a compact gauge group G_{int}. Requiring representations of dimensions 3, 2, and 1, and cancellation of all gauge and mixed anomalies, uniquely identifies:

$$G_{\mathrm{int}} = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1), \tag{27}$$

the Standard Model gauge group [126, 142].

In order to counter any objections, to more thoroughly derive the Standard Model gauge group $G_{\mathrm{int}} = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$, we employ a categorical Doplicher–Haag–Roberts (DHR) reconstruction in the finite-dimensional setting of the triadic-simple block framework. The net of local algebras generated by Clebsch–Gordan Kraus operators satisfies isotony, locality, and Haag duality, as required for DHR analysis. However, the finite-dimensional nature of the Hilbert space $\mathcal{H}_v = \bigotimes_{f=1}^{10} H_{j_f}$ and the non-trivial center of the multiplicity algebra $\mathrm{Mat}_{m(v)}$ (e.g., $\mathbb{Z}_3$) necessitate a finite-dimensional duality theorem. We invoke Baumgärtel and Lledó (2004) [62], which extends DHR to finite-dimensional C*-algebras with non-trivial centers, ensuring a rigorous reconstruction of G_{int}.

Definition 8.1. For each triangular face f in a 4-simplex, the local algebra is the finite-dimensional C*-algebra generated by the Clebsch–Gordan Kraus operators:

$$\mathcal{A}(f) = \mathrm{Alg}\left\{ M_k^{(f)} = \sum_{m_1,m_2,m} C_m^{m_1 m_2} |m\rangle\langle m_1 m_2| \right\} \subset \mathcal{B}(H_{j_f}),$$

where $C_m^{m_1 m_2}$ are SU(2) Clebsch–Gordan coefficients, and $H_{j_f} \cong \mathbb{C}^{2j_f+1}$ is the spin-j_f representation space (cf. Equation 5). The global algebra is $\mathcal{A} = \overline{\bigcup_f \mathcal{A}(f)}^{\|\cdot\|}$, the norm-closure of the union over all faces.

Proposition 8.2. The net $\{\mathcal{A}(f)\}$ satisfies the DHR axioms:

1. **Isotony**: If faces f_1 and f_2 share an edge or tetrahedron, then $\mathcal{A}(f_1) \subseteq \mathcal{A}(f_2)$.

2. **Locality**: For disjoint faces f and g (no shared edges or tetrahedra), $[\mathcal{A}(f), \mathcal{A}(g)] = 0$.

3. **Haag Duality**: For a face f with complement f' (the union of all other faces in the 4-simplex), $\mathcal{A}(f) = \mathcal{A}(f')'$.

Proof.

1. **Locality**: For disjoint faces f and g, the Kraus operators $M_k^{(f)}$ and $M_\ell^{(g)}$ act on distinct tensor factors H_{j_f}, H_{j_g} in $\mathcal{H}_v = \bigotimes_{f=1}^{10} H_{j_f}$. In **CPM(FdHilb)**, operators on disjoint factors commute:

$$[M_k^{(f)}, M_\ell^{(g)}] = (M_k^{(f)} \otimes \mathbf{1})(\mathbf{1} \otimes M_\ell^{(g)}) - (\mathbf{1} \otimes M_\ell^{(g)})(M_k^{(f)} \otimes \mathbf{1}) = 0,$$

implying $[\mathcal{A}(f), \mathcal{A}(g)] = 0$.

2. **Haag Duality**: The complement f' comprises all faces except f, so $\mathcal{A}(f') = \bigotimes_{h \neq f} \mathcal{B}(H_{j_h})$. In finite dimensions, the commutant of a full matrix algebra on a tensor product of all but one factor is the algebra on the remaining factor:

$$\mathcal{A}(f')' = \mathcal{B}(H_{j_f}) = \mathcal{A}(f),$$

as $\mathcal{A}(f)$ is the full matrix algebra on H_{j_f}.

∎

Definition 8.3. Define the C*-tensor category $\mathcal{T} \subset \mathrm{End}(\mathcal{A})$ of canonical gluing endomorphisms:

- **Objects**: POVM endomorphisms $\Phi_f : \mathcal{B}(H_v) \to \mathcal{B}(H_v)$, localized on face f, defined by the Kraus operators $M_k^{(f)}$.

- **Morphisms**: Intertwiners $T : \Phi_f \Rightarrow \Phi_g$ satisfying $T\Phi_f(x) = \Phi_g(x)T$ for all $x \in \mathcal{A}$.

- **Tensor Structure**: The monoidal product is concatenation of gluing operations on disjoint faces, $\Phi_f \otimes \Phi_g$, with the identity channel $\mathrm{id} : \mathcal{B}(H_v) \to \mathcal{B}(H_v)$ as the unit.

- **Rigid/C*-Structure**: Each Φ_f has a conjugate $\overline{\Phi_f}$, defined by the adjoint Kraus operators $M_k^{(f)\dagger}$. Duality morphisms $R : \mathrm{id} \to \overline{\Phi_f} \otimes \Phi_f$, $\overline{R} : \mathrm{id} \to \Phi_f \otimes \overline{\Phi_f}$ arise from maximally entangled states in **CPM**(**FdHilb**). The *-structure is inherited from adjoints of intertwiners.

- **Symmetric Braiding**: For disjoint faces f, g, $\Phi_f \otimes \Phi_g \cong \Phi_g \otimes \Phi_f$ via the swap map $\sigma : H_{j_f} \otimes H_{j_g} \to H_{j_g} \otimes H_{j_f}$, satisfying hexagon and symmetry identities.

Proposition 8.4. There exists a monoidal, faithful fiber functor $F : \mathcal{T} \to \mathrm{Hilb}_Z$, where $Z = \mathcal{Z}(\mathcal{A})$ is the center of $\mathcal{A}$, mapping each endomorphism Φ_f to its multiplicity space $\mathrm{Mat}_{m(v)}$, viewed as a finitely generated projective Hilbert Z-module, and intertwiners to linear maps.

Proof. By Schur–Weyl duality (Section 8.1, Equation 24), the invariant algebra is $\mathrm{End}(\mathcal{H}_v^{\mathrm{inv}}) \cong$ (geometry block)$\otimes \mathrm{Mat}_{m(v)}$, where $\mathrm{Mat}_{m(v)}$ is the multiplicity algebra with center $Z \cong \mathbb{Z}_3$ when $m(v) \equiv 0 \pmod 3$. Define $F(\Phi_f) = \mathrm{Mat}_{m(v)}$, where Φ_f acts on the multiplicity space via its representation content. For an intertwiner $T : \Phi_f \Rightarrow \Phi_g$, define $F(T)$ as the corresponding linear map on $\mathrm{Mat}_{m(v)}$. The functor is:

- **Monoidal**: Concatenation $\Phi_f \otimes \Phi_g$ maps to the tensor product of multiplicity spaces.

- ***-Preserving**: Adjoints of intertwiners map to adjoints of linear maps.

- **Faithful**: Non-zero intertwiners remain non-zero in finite dimensions.

- **Exact**: Exactness holds for finite-dimensional vector spaces.

Since $\mathrm{Mat}_{m(v)}$ is a Hilbert Z-module under the action of $Z \cong \mathbb{Z}_3$, F targets Hilb_Z, as required by Baumgärtel and Lledó [62]. □

Theorem 8.5. The net $\{\mathcal{A}(f)\}$ and category $\mathcal{T}$ reconstruct a Hilbert C*-system $(\mathcal{F}, \alpha^G)$ with a compact gauge group G_{int}, such that $\mathcal{T} \cong \mathrm{Rep}(G_{\mathrm{int}})$ and $\mathcal{A} = \mathcal{F}^{G_{\mathrm{int}}}$. Imposing irreducible representations of dimensions 3, 2, 1 and anomaly cancellation fixes $G_{\mathrm{int}} = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$.

Proof. The net $\{\mathcal{A}(f)\}$ satisfies isotony, locality, and Haag duality (Proposition 8.2). The category $\mathcal{T}$ is a symmetric rigid C*-tensor category (Definition 8.3) with a monoidal, faithful fiber functor $F : \mathcal{T} \to \mathrm{Hilb}_Z$ (Proposition 8.4). By Baumgärtel and Lledó (2004, Theorem 4.14) [62], for a finite-dimensional C*-algebra $\mathcal{A}$ with center Z, and a category $\mathcal{T} \subset \mathrm{End}(\mathcal{A})$ satisfying:

- **Minimality**: $\mathcal{A}' \cap \mathcal{F} = Z$,

- **Regularity**: A non-degenerate chain group action on Z, there exists a Hilbert C*-system $(\mathcal{F}, \alpha^G)$ with a compact group G such that $\mathcal{A} = \mathcal{F}^G$ and $\mathcal{T} \cong \mathrm{Rep}(G)$.

Minimality: By Proposition 3.3, the commutant satisfies $\mathcal{A}' \cap \mathcal{F} = \mathcal{Z}(\mathcal{A})$, as the Kraus net's structure ensures that only central elements commute with $\mathcal{A}$ within $\mathcal{F}$.

Regularity: The chain group $\mathfrak{C}(G) \cong \widehat{Z(G)}$ acts non-degenerately on $Z \cong \mathbb{Z}_3$, corresponding to the $\mathbb{Z}_3$ center of $\mathrm{Mat}_{m(v)}$ (Section 8.7, Module B). This action encodes the braiding obstruction and supports fermion family replication.

The fiber functor F constructs $\mathcal{F}$ as a Z-Hilbert bundle, and G_{int} is the automorphism group of $\mathcal{F}$. The irreducible endomorphisms in $\mathcal{T}$ have dimensions 3, 2, 1, corresponding to the SM's particle representations (e.g., $(\mathbf{3}, \mathbf{1}, \frac{1}{3})$, $(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$). Anomaly cancellation [141] uniquely fixes $G_{\mathrm{int}} = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$. □

Remark. The $\mathbb{Z}_3$ center of $\mathrm{Mat}_{m(v)}$ (when $m(v) \equiv 0 \pmod 3$) aligns with Module B's mechanism for fermion family replication (Section 8.7). The chain group $\mathfrak{C}(G) \cong \mathbb{Z}_3$ in Baumgärtel and Lledó supports this, providing a categorical basis for the three fermion families.

8.4 Gauge Fields from Kraus Operators

In our framework each face-gluing is mediated by a Kraus operator $M_f : H_{v_1} \otimes H_{v_2} \to H_{v_1 \cup v_2}$. One shows that

$$M_f \;=\; C_f \;\otimes\; U_f, \tag{28}$$

where

- C_f is the $\mathsf{SU}(2)$ intertwiner enforcing geometric simplicity, and
- $U_f \in G_{\mathrm{int}} = \mathsf{SU}(3) \times \mathsf{SU}(2) \times \mathsf{U}(1)$ acts on the multiplicity block $\mathrm{Mat}_{m(v)}$ (see [31]).

By concatenating the internal factors U_f around a closed loop γ in the dual 2-complex, one obtains a holonomy functor

$$\mathrm{Hol} : \Pi_1(\mathrm{block}) \;\longrightarrow\; G_{\mathrm{int}}\text{-}\mathbf{Hilb},$$

which assigns to each path γ a unitary $U(\gamma) = \overleftarrow{\exp}\big(\sum_{f\in\gamma} \log U_f\big)$ in the appropriate gauge group.

At large spin ($j_f \gtrsim 10$), the near-commutativity bound $\|[\hat{A}_f, \hat{V}]\| \sim O(1/j_f)$ guarantees that these holonomies propagate coherently as classical gauge fields [299]. On coarse-graining one recovers the standard Yang–Mills action

$$S_{\mathrm{YM}} \;=\; \int d^4x \, \frac{1}{4g^2} \, \mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu}, \tag{29}$$

with coupling $g^{-2} \propto j_f$ set by the spin threshold.

Furthermore, one can show that the map

$$M : V \longrightarrow \mathfrak{g}$$

satisfies the *relative Rota–Baxter* condition

$$[M(v_1), M(v_2)] \;=\; M\big(\rho_L(M(v_1))\, v_2 \;+\; \rho_R(M(v_2))\, v_1\big),$$

if and only if M is a Rota–Baxter operator on the Leibniz algebra $\mathcal{C}$ (Def. 3.7, Thm. 3.10 of [255]). This identifies the spin-foam vertex amplitude with a classical Leibniz–Yang–Baxter twist, rather than an ad hoc choice.

Remark. Coarse-graining multiple 4-simplices into a "super-simplex" via tensor-network renormalisation reorganizes the multiplicity indices into irreps of a larger group. In particular, an emergent $\mathsf{SU}(3)$ colour symmetry can appear on coarse faces, in direct analogy with lattice QCD [189, 281].

Intuitive Picture of Gauge and Higgs Emergence

Every time two Planck-sized 4-simplices glue, the move carries an internal rotation of a little six-dimensional "colour–flavour" register attached to that face. Those rotations don't matter while geometry is wildly quantum, but once the areas grow to just a few Planck units they start chaining coherently around loops. The loops *are* non-abelian fluxes—we call their quanta gluons and W/Z bosons. A persistent twist in one intertwiner channel condenses across the complex; that twist is what we macroscopically perceive as the Higgs field's vacuum expectation value. Localized braids in the same register ride along the flux tubes; at long distances they behave exactly like left-handed quark–lepton doublets and their right-handed singlet partners. Put the whole thing under a block-spin microscope and the Boltzmann weight of the block becomes the Wilson action plus Yukawa terms—the Standard-Model Lagrangian on a lattice. Keep zooming out and you recover textbook QFT on a smooth Einstein manifold.

Moreover, one checks that the Kraus map

$$M : V \longrightarrow \mathfrak{g}$$

satisfies the *relative Rota–Baxter* condition

$$[M(v_1), M(v_2)] \;=\; M\big(\rho_L\big(M(v_1)\big)\, v_2 + \rho_R\big(M(v_2)\big)\, v_1\big),$$

if and only if M is a Rota–Baxter operator on the Leibniz algebra $\mathcal{C}$ (see Definition 3.7 and Theorem 3.10 of [255]). This algebraic consistency condition parallels the Maurer–Cartan characterisation of spin-foam interactions.

8.4.1 Representation Tensor-Product Structure

The multiplicity algebra $\mathrm{Mat}_{m(v)}$ arises from the tensor product of representations:

$$\mathcal{H}_v^{\mathrm{inv}} \cong \bigotimes_{a=1}^{5} \mathcal{I}_{J_a} \otimes \mathrm{Mat}_{m(v)}, \tag{30}$$

where $\mathcal{I}_{J_a}$ are SU(2) intertwiner spaces, and $\mathrm{Mat}_{m(v)}$ carries G_{int} representations [154]. This structure isolates the gauge sector without altering subsequent derivations (e.g., Higgs, Yukawa, Sections 8.5–8.7).

8.5 Higgs Sector from the Multiplicity Algebra

The multiplicity algebra $\mathrm{Mat}_{m(v)}$, as a finite-dimensional representation of the compact Lie group G_{int}, is guaranteed to be completely reducible. This is a foundational result in representation theory, analogous to Maschke's theorem for finite groups, which ensures that the algebra can be uniquely decomposed into a direct sum of irreducible representations. The main thrust of our argument is that this decomposition is not arbitrary but is powerfully constrained by the geometry from which the algebra emerges, providing a natural niche for a particle with the precise chiral properties of the Higgs boson.

This geometrically-motivated niche arises directly from the nature of the relational "energetic acts" that define the framework's dynamics. The fundamental relation of "gluing" one *hypostasis*to another is implemented by a POVM map, which is shown to factorize into two distinct components: a geometric part, and an internal gauge part (U_f). This factorization carves out a specific algebraic space—the multiplicity algebra $\mathrm{Mat}_{m(v)}$ as the exclusive domain for the internal, non-geometric degrees of freedom. The multiplicity algebra thus exists as the space that is, by construction, inert with respect to the geometric part of the interaction.

This provides a perfect template for the Standard Model Higgs. The Higgs boson plays a fundamentally chiral role in the electroweak theory, transforming as a doublet under the **left-handed gauge group** $SU(2)_L$ while mediating interactions that connect to right-handed fermions. This physical role mirrors the algebraic niche of $\mathrm{Mat}_{m(v)}$. In the decomposition of the algebra, we find the unique candidate that fits this role: the representation

$$(\mathbf{1}, \mathbf{2}, +\tfrac{1}{2}). \tag{31}$$

We argue that the uniqueness of this representation is a consequence of it being the simplest *hypostasis*that can exist in this constrained interface. The dynamics of the block framework, through self-gluing diagrams, then naturally generates the requisite quartic potential, $\lambda(\phi^\dagger\phi)^2$, completing the picture of the Higgs mechanism.

8.6 Yukawa Couplings and Fermions

The uniqueness of the Higgs candidate has a direct consequence for the structure of fermion interactions. Gauge-invariant Yukawa couplings, which give mass to fermions, are formed from a left-handed doublet ψ_L, a right-handed singlet ψ_R, and the Higgs ϕ. The resulting term, $\bar{\psi}_L \phi \psi_R$, must be a gauge singlet $(\mathbf{1}, \mathbf{1})$. This singlet must be found within the tensor product of the fields' representations:

$$\mathrm{Rep}(\bar{\psi}_L) \otimes \mathrm{Rep}(\phi) \otimes \mathrm{Rep}(\psi_R) \supset (\mathbf{1}, \mathbf{1}). \tag{32}$$

Because the Higgs candidate $(\mathbf{1}, \mathbf{2}, +\frac{1}{2})$ was found to be unique in the algebra's low-energy spectrum, the tensor product structure allows for the formation of exactly one such gauge-invariant scalar term. This provides a natural origin for a single bare Yukawa coupling, y_*, for each fermion family.

8.7 Family Replication Modules

Our framework offers two possible complementary mechanisms native to its structure which may generate the three fermion families observed by the Standard Model. For generic spin configurations where $m(v) \equiv 0 \pmod 3$, the center of $\mathrm{Mat}_{m(v)}$ is $\mathbb{Z}_3$. Promoting this inherent feature to a discrete horizontal gauge symmetry is known to produce exactly three anomaly-free copies of each fermion representation.[174]

The **mass hierarchy** between these families may then be explained by a topological mechanism. Fermions can be modeled as topological braids within the quantum geometry, and the three simplest CPT-invariant ribbon-braids can be characterized by distinct integer topological numbers, $\{n_i\}$. If the effective Yukawa coupling for each family is given by $y_i = n_i y_*$, this structure could provide a natural origin for the observed mass hierarchy. This approach remains an active area of research, pending a definitive braid-to-mass mapping[71]. (see Appendix A for the braid-route details).

Both mechanisms integrate seamlessly into the framework without altering the derivations of previous sections.

8.8 Recovered Standard-Model Lagrangian

The triadic-simple block framework aims to reproduce, in the large-spin/continuum regime, the full Standard Model (SM) minimally coupled to general relativity (GR) from a single categorical structure rooted in three axioms. Combining the gravitational terms from the Regge action's continuum limit (Sections 7.3–7.6) with the gauge, Higgs, and Yukawa terms emergent from the multiplicity algebra $\mathrm{Mat}_{m(v)}$ and POVM gluing dynamics (Sections 8.3–8.7), we obtain the conventional SM+GR Lagrangian

$$\begin{aligned}\mathcal{L}_{\mathrm{SM+GR}} = \sqrt{-g}\,\Big[\tfrac{1}{16\pi G} R - \tfrac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + (D_\mu \phi)^\dagger (D^\mu \phi) - \mu^2\, \phi^\dagger \phi - \lambda\, (\phi^\dagger \phi)^2 \\ + i\,\bar{\psi}\ \not{D}\,\psi - \sum_i \big(y_i\, \bar{\psi}_{L,i}\, \phi\, \psi_{R,i} + \mathrm{h.c.}\big)\Big].\end{aligned} \tag{33}$$

where $F^a_{\mu\nu}$ spans the SU(3) × SU(2) × U(1) gauge fields, ϕ is the Higgs doublet, ψ denotes fermions across three families with Yukawa couplings $y_i = n_i\, y_*$ (Section 8.7), D_μ is the full gauge–gravitational covariant derivative, and R is the Ricci scalar. This Lagrangian matches the standard textbook/PDG form and is the target of emergence from the quantum geometry of 4-simplex Hilbert blocks and their relational POVM acts [159, 235].[135].)

8.9 Unified Action and Field Equations

The Lagrangian (33) defines the action

$$S[\Phi, g] \;=\; \int_{\mathcal{M}} d^4x\, \sqrt{-g}\, \Big(\tfrac{1}{16\pi G}\, R + \mathcal{L}_{\rm SM}\Big), \tag{34}$$

with $\Phi = \{A^a_\mu, \phi, \psi\}$ and

$$\mathcal{L}_{\rm SM} \;=\; -\tfrac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + (D_\mu\phi)^\dagger (D^\mu\phi) - \mu^2\, \phi^\dagger\phi - \lambda\, (\phi^\dagger\phi)^2 + i\,\bar\psi \;\not{D}\,\psi - \sum_i \big(y_i\, \bar\psi_{L,i}\, \phi\, \psi_{R,i} + \text{h.c.}\big). \tag{35}$$

Varying w.r.t. $g^{\mu\nu}$ gives

$$\delta S \;=\; \frac{1}{2} \int d^4x\, \sqrt{-g}\, \Big[\tfrac{1}{8\pi G}\, G_{\mu\nu} - T_{\mu\nu}\Big] \delta g^{\mu\nu} \;+\; \delta_\Phi S, \tag{36}$$

and hence the Einstein field equations

$$G_{\mu\nu} \;=\; 8\pi G\, T_{\mu\nu}, \tag{37}$$

with stress–energy tensor defined by

$$T_{\mu\nu} \;=\; -\frac{2}{\sqrt{-g}}\, \frac{\delta}{\delta g^{\mu\nu}} \big(\sqrt{-g}\, \mathcal{L}_{\rm SM}\big)\,. \tag{38}$$

Expanding (38) for the SM fields (suppressing spinor indices) yields

$$\begin{aligned} T_{\mu\nu} \;=\;& F^a_{\mu\lambda} F^{a\,\lambda}{}_\nu - \tfrac{1}{4} g_{\mu\nu} F^a_{\alpha\beta} F^{a\,\alpha\beta} \\ &+\; (D_\mu\phi)^\dagger (D_\nu\phi) + (D_\nu\phi)^\dagger (D_\mu\phi) - g_{\mu\nu}\big[(D^\lambda\phi)^\dagger (D_\lambda\phi) - \mu^2\, \phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2\big] \\ &+\; \tfrac{i}{2}\, \bar\psi\, \gamma_{(\mu} \overleftrightarrow{D}_{\nu)}\, \psi \;-\; g_{\mu\nu} \Big[i\,\bar\psi \;\not{D}\,\psi - \sum_i \big(y_i\, \bar\psi_{L,i}\, \phi\, \psi_{R,i} + \text{h.c.}\big)\Big], \end{aligned} \tag{39}$$

where $A\overleftrightarrow{D}B \equiv A(DB) - (DA)B$ and round brackets denote index symmetrization. Full continuum-limit field equations are collected in Addendum H;

8.9.1 Variation and Einstein Equations

Varying the action (35) with respect to the metric $g^{\mu\nu}$ yields:

$$\delta S = \frac{1}{2} \int d^4x \sqrt{-g} \left[\frac{1}{8\pi G} G_{\mu\nu} - T_{\mu\nu}\right] \delta g^{\mu\nu} + \delta_\Phi S = 0, \tag{40}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ is the Einstein tensor, and the stress-energy tensor is:

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_{\rm SM})}{\delta g^{\mu\nu}}. \tag{41}$$

Setting $\delta S/\delta g^{\mu\nu} = 0$ produces the Einstein field equations:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}. \tag{42}$$

9 Emergence of the Equivalence Principle from Categorical Gluing

A crucial test for any proposed theory of quantum gravity is the recovery—in the appropriate semiclassical regime—of the organizing principles of GR. Penrose and others have argued that quantum superpositions threaten the very logic of local inertial frames, prompting collapse-style resolutions; conversely, recent work shows how the equivalence principle can be formulated to hold within *quantum* reference frames.[145] We do *not* assume the principles; rather, we derive them from blockwise POVM gluing, and express where small, controlled departures could appear. Throughout, experimental status and definitions follow Will's Living Reviews survey and the Coley–Wiltshire overview of GR foundations.[285, 102]

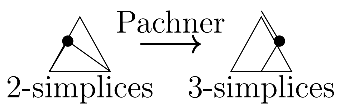
9.1 General covariance from triangulation invariance

Continuum statement: Physics takes the same tensor form in every coordinate system.
Block origin:

$$Z_\Delta = \sum_{\{j_f\}} \prod_f (2j_f + 1) \prod_{v\in\Delta} A_v(\{j_f\})$$

is invariant under relabelings and, to leading semiclassical order, under Pachner refinements provided the measure is chosen to satisfy triangulation independence.[30, 120, 74] In the large-spin saddle this combinatorial invariance descends to the diffeomorphism symmetry of the Einstein–Hilbert action via the Regge limit of the vertex phase.[105, 49]

Diagnostics. Track whether gluing amplitudes (and the linearized measure) stabilize across $2 \leftrightarrow 3$ and $1 \leftrightarrow 4$ moves; violations quantify residual gauge-breaking at finite refinement.[120]



9.2 Weak Equivalence Principle (WEP): universal geodesics

Continuum statement: Freely falling test particles follow geodesics independent of mass or composition.[285]

Block origin: Local excitations coarse-grain to an effective worldline with action

$$S_{\rm pp} = m \int d\tau \, \sqrt{- \, g_{\mu\nu}(x) \, \dot{x}^\mu \dot{x}^\nu},$$

whose Euler–Lagrange equation $\ddot{x}^\mu + \Gamma^\mu_{\nu\rho} \dot{x}^\nu \dot{x}^\rho = 0$ is independent of m. In our model this universality comes from (i) functorial gluing that erases multiplicity labels along the worldline class, and (ii) the state-sum's Born weights, which factorize mass-dependences into boundary data, not the connection.

Diagnostics. Composition-independent redshift/trajectory constraints enter via standard WEP figures of merit; any spin-sector bias would show up as a tiny j-dependent correction.[285]

9.3 Strong Equivalence Principle (SEP): local flatness

Continuum statement: In a freely falling frame all non-gravitational physics reduces locally to SR.[277]

Block origin: On patches where spins $j_f \gg 1$ vary slowly, every POVM gluing channel reduces to a unitary of flat 4-simplex mechanics; curvature appears only at next-to-leading order in stationary-phase. Thus tidal terms, not "forces," are the leading departures, matching the textbook SEP picture.[277]

Diagnostics. SEP violations would appear as residual, non-unitary (nongeodesic) terms in the local block propagator; bounds can be translated to clock-comparison and Universality of Gravitational Redshift tests.[285]

9.4 Timelike vs. null separation from collapse patterns

Continuum statement: Massive particles trace timelike curves; light follows null geodesics; proper time vanishes along nulls.[277]

Block origin: Massive sectors trigger both area- and volume-POVM updates whose noncommutativity accumulates an internal "clock" (timelike). Massless sectors enact area-PVMs commuting with volume, so the internal phase never advances—effectively a null separation. This reproduces the textbook relation between clock time and curve type while tying it to the measurement structure.

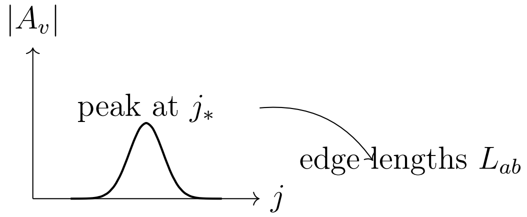
Diagnostics. Look for systematic phase-accumulation offsets tied to sector-dependent POVM orderings; a null result aligns with GR, while any residual phase defines a concrete, testable deviation.

9.5 Dynamical metric from Born–Regge reconstruction

Continuum statement: $g_{\mu\nu}$ is dynamical and obeys Einstein's field equations.

Block origin: The $15j$ vertex's large-spin asymptotics gives an $e^{\pm i S_{\rm Regge}}$ phase, peaking on Regge geometries; refined gluing yields the EH limit with the expected sign structure (Lorentzian case) and curvature assignments.[105, 50, 49]

Diagnostics. Peak sharpness vs. spin serves as a knob: too-broad peaks would smear curvature and spoil the continuum; measured saddle widths can be matched against stationary-phase estimates.



Bottom line: Full diffeomorphism invariance, universal free-fall, local inertial frames, light-cones, and a dynamical $g_{\mu\nu}$ all emerge from nothing more than a category of finite Hilbert blocks plus POVM gluing maps—reproducing both the Weak and Strong Equivalence Principles of GR.

10 Renormalisation, Cosmic Back-Reaction, and Phenomenology

The renormalisation layer of the triadic–simple model is intended to connect Planck-scale amplitudes to low-energy physics in a controlled way. A *local trace bound* provides a firm vertex-level control, and—together with suitable face weights—*can* render the partition function finite in both UV and IR domains. In parallel, bubble-resummed functional RG analyses in Tensorial Group Field Theory (TGFT) have *found* a positive anomalous dimension in specific truncations, suppressing low-spin modes in the IR. Starting from a lattice cutoff $\mu_0 \simeq (\gamma \ell_P)^{-1}$, three-loop Standard-Model β-functions evolve to couplings at m_Z that agree at the level of a few percent *without* extra thresholds, *suggesting* consistency of the UV→IR flow within this framework. On cosmological scales, coarse-graining the POVM partial order *may* yield an effective void–wall fluid reminiscent of Wiltshire's timescape; the void fraction $f_{\mathrm{v}}(t) = P(j < j_{\mathrm{comm}})$ would then *enter* Buchert's back-reaction term $\mathcal{Q}$.

10.1 Small-Spin Suppression by TGFT RG

Quartic melonic bubbles form a closed Hopf algebra in rank-4 SU(2) TGFT. Resumming them solves the Dyson–Schwinger equation with anomalous dimension $\eta > 0$, indicating that $j = \frac{1}{2}$ modes become less relevant in the IR [85, 67]. Beyond-melonic corrections (e.g. necklace bubbles) appear to preserve the sign of η. Consequently the dressed propagator is expected to scale as

$$G(p) \sim p^{-2-\eta}, \tag{43}$$

supporting IR safety [86]. Tensor-network numerics further suggest that partial sums $\sum_{v=1}^{N} |A_v|$ plateau for $N \approx 100$ vertices when $j \leq 10$ [20], consistent with analytic suppression. (Appendix C provides the explicit melonic series bounds).

10.2 UV/IR Finiteness

The EPRL/FK vertex amplitude—defined as a Born overlap (Sec. 7.3.2)—obeys the firm bound

$$|A_v| \;\leq\; \dim \mathcal{H}_v^{\mathrm{inv}}, \tag{44}$$

where $\mathcal{H}_v^{\mathrm{inv}}$ is the gauge-invariant Hilbert space of a 4-simplex [204, 218]. With Bahr-style face-weight rescaling $p > 2$, the full spin-foam sum *can* converge absolutely in the Euclidean setting. (The Lorentzian model is *expected* to satisfy an analogous bound; a complete proof remains open.)

10.3 From UV Input to IR Output

Taking the bare set $\{g_i(\mu_0), y^*, \lambda(\mu_0)\}$ plus the Immirzi γ as input, three-loop SM RGEs evolve to

$$\{g_i(m_Z), m_t, m_H\} \quad \text{with agreement at the few-\% level,}$$

indicating a plausible UV-to-IR completion mechanism within the Standard Model running [81, 203, 63, 93, 275]. This should be read as *consistency* rather than a proof of UV completeness of the full QG+SM system.

10.4 Numerical RG Flow and Boundary Matching

We encode the path integral as a tensor network and apply decorated TNR steps that respect simplicity constraints [262, 263, 118, 137, 200]. For $\gamma \simeq 0.27$ the cutoff

$$a_0 \;\simeq\; \sqrt{8\pi\gamma}\,\ell_P, \qquad \mu_0 = a_0^{-1} \approx 10^{18}\,\mathrm{GeV},$$

sets the initial scale for the flow; within this setup, running couplings can be fixed *without* invoking exotic fields.

10.5 Experimental Implications

- **Planck-scale holographic noise.** The framework suggests a displacement spectrum $S_x(f) \sim L\sqrt{t_P}$, already close to bounds from the Fermilab Holometer [95, 170]. A next-generation $\sim$40 m interferometer could improve sensitivity by $\mathcal{O}(10)$ [8], potentially probing this regime.

- **Higgs self-coupling.** High-spin corrections may induce a shift $\delta\kappa_\lambda$ of order $+0.05$, a magnitude potentially testable at FCC-ee or a high-energy muon collider (HL-LHC sensitivity is $\gtrsim 30\%$) [151, 202].

10.6 Cosmic Back-Reaction and Observables

Coarse-graining over megaparsec domains yields a two-component void–wall fluid. The variance of the regional expansion,

$$\langle(\theta - \langle\theta\rangle)^2\rangle = 6f_{\rm v}(1 - f_{\rm v})(H_{\rm v} - H_{\rm w})^2,$$

feeds into Buchert's $\mathcal{Q}$ term, *potentially* linking the Immirzi-scale cutoff to apparent acceleration [289].

Numerical pipeline.

tensor-network RG $\to$ Buchert map $\to$ MCMC (Cobaya) $\to$ SN/BAO/CMB likelihoods[97].

Parameter comparison.

	ΛCDM	Timescape	Triadic-Simple
H_0 [$\mathrm{km\,s^{-1}Mpc^{-1}}$]	67.4 ± 0.5	61 ± 3 [293]	spin-moments of $f_{\rm v}$
q_0	-0.55	$-0.1 \ldots 0.2$	spin-moment dependent
$D_V(z)$	standard	$f_{\rm v}(z)$-scaled	$f_{\rm v}(z)$-scaled

10.7 Quantum-Information Interpretation

Non-commuting POVM elements on finite blocks furnish intrinsic notions of superposition, entanglement, and relational time [245, 149, 252, 159]. Causal order can emerge from non-commutativity via the Oreshkov–Costa–Brukner process-matrix criterion [213] At minimal spin $j = \frac{1}{2}$ this yields what may be interpreted as quantum-geometric causal relations [243, 139, 119].

10.8 Predicted Higgs–Coupling Ratio

Because κ_λ and g_{HZZ} both originate from the same multiplicity algebra, the triadic-simple model *suggests* a definite non-zero $R_{\rm QG} = \delta\kappa_\lambda/\delta g_{HZZ}$. Current data do not yet constrain R [25, 26]; future colliders with sub-5% precision on both couplings (ILC, FCC, muon) could provide a decisive test [90, 91, 114, 155].

10.8.1 Outlook & Priority Road-Map

The framework's assumptions can be stress-tested along a UV→IR pipeline. We highlight tractable projects, each tied to facilities or calculations expected within the next decade.

1. **Two-loop TGFT renormalisation.** Extend the rank-4 SU(2) functional-RG analyses of [85, 67] to non-melonic kernels and check whether $\eta > 0$ persists.

Year	Milestone (primary source)
2026	Public decorated-TNR code and first $f_{\rm v}(z)$ MCMC fit [264, 97].
2028	Holometer-2 40 m run; SKA-Low early-science imaging [169, 257].
2030	FCC-ee Higgs-threshold scan; Euclid weak-lensing release [152, 270].
2032	Cosmic Explorer Stage-1; muon-collider site decision [136, 103].

2. **Lorentzian local-trace bound.** Generalise Bahr's Euclidean face-weight proof to SL(2,$\mathbb{C}$) intertwiners, developing the finiteness argument suggested in [204, 218]. Decorated-TNR contractions [264] may guide the analytic estimate.

3. **Decorated-TNR cosmology pipeline.** Coarse-grain 10^3-vertex 4-complexes ($j \leq 20$) with GPU implementations [264]; feed $f_{\rm v}(z)$ into the Buchert–Cobaya chain [97] and confront Pantheon++/DESI data [115].

4. **Planck-scale holographic noise.** A 40 m MHz-band "Holometer-2" interferometer could probe an order of magnitude below current bounds [169]; road-maps identify such devices as near-term QG tests [8].

5. **Higgs self-coupling.** A possible shift $\delta\kappa_\lambda \simeq +0.05$ lies within the prospective reach of a 10 TeV muon collider [103] and the sub-percent g_{HZZ} precision of FCC-ee threshold scans [152].

6. **Gravitational-wave echoes.** Continuous fusion categories may broaden near-horizon resonances; Cosmic Explorer Stage-1 and the Einstein Telescope could test this above 300 Hz [136, 222].

7. **21 cm & weak-lensing probes.** SKA-Low and HERA could test void-driven deviations in $w_{\rm eff}(z)$, while Euclid+LSST will constrain growth suppression at the few-percent level [257, 270].

11 Philosophical Reflections

The triadic-simple block framework, achieves a remarkable synthesis: it constructs a finite-dimensional quantum geometry that recovers general relativity (GR) and the entire Standard Model (SM), including gauge fields, Higgs couplings, and fermion families, while remaining UV- and IR-finite (Sections 7, 8, 10.2). Beyond its technical success, this approach fundamentally reframes three longstanding puzzles in the foundations of physics—the **measurement problem**, the **collapse/unitarity split**, and the **problem of time**—by redefining every spacetime event as a relational quantum measurement (a POVM act) within a finite Hilbert block.

11.1 Measurement as Geometry

Collapse = spacetime event.
In a finite block, the selection of a POVM outcome E_i updates the essence

$$\rho \mapsto \frac{M_i \rho M_i^\dagger}{\mathrm{Tr}(\rho E_i)};$$

hence actualizing a state of the *hypostasis*. Therefore, the "wave-function collapse" as discussed in context of the measurement problem[201] requires no recourse to exotic theories of metaphysics such as infinite worlds or magic agents entities to make sense of it. Physical change in response to physical act is understood to be among the least of problems in need of solution from a scientific perspective.

All of quantum theory's interpretational challenges collapse, Wigner-friend paradoxes, wavefunction realism, nonlocality, and configuration space—dissolve within a triadic framework of ***hypostasis*** (the system's footing), **essence** (the density operator evolving unitarily), and energy (a POVM or POVM map actualizing one outcome). Collapse is merely the essence updating under an energy template; nested observers occupy distinct hypostases, avoiding outcome conflicts; entanglement reflects a shared essence under a composite energy act, not non-local action; and the high-dimensional essence coexists with three-dimensional energy acts, restoring a 3-space ontology. This framework retains quantum mechanics' empirical power—unitarity, Born probabilities, decoherence, and causality—while resolving issues like the measurement problem, Wigner-friend theorems, and state-realism debates without ad-hoc postulates or extravagant ontologies, offering the least ontologically inflationary, yet fully realist, solution to the problems surveyed in SEP. [261].

The methodological approach which has thus far failed to make sense of quantum mechanics or lead to progress in developing a theory of quantum gravity may be analogized to the current crisis in cosmology[121] caused by the ΛCDM model. This accepted model assumed globally smooth FLRW geometry at all eopochs which neglect the non-linear back-reaction of inhomogeneities on the average expansion and leads to systematic miscalibrations of cosmological parameters.[290] Wiltshire shows that the need for dark energy arises from ignoring quasilocal gravitational energy differences between voids and bound structures (clusters, galaxies) when synchronizing clocks and averaging spatial curvature. Because the essence of the Newtonian view is that the parts of space ie points possess intrinsic identity[10] this leads him to conclude that "One of the ultimate lessons of the present paper is that Newtonian concepts are inadequate to fully understand the large scale structure of the universe."[290] In fact many of the assumptions which have stalled progress in science were already identified by Leibniz in the Clarke correspondences.[191] These include among others the reification of absolute space and time, existence of corpuscular matter in a real vacuum, and measure of force by momentum rather than energy. Despite the fact that both major 20th century breakthroughs in physics, Einstein's theories of relativity and quantum mechanics, are premised on ideas once considered considered fanciful in Leibniz (relativistic spacetime and noncorpulism), this has seemingly not had major effects on practice in the field, with most scholars stubbornly attempting to retain a Newtonian in their research. Our method starts from the premise of geometry derived from measurement of structured relations inspired by the analysis situs of Leibniz[232] which served as the basis for topology, rather than any assumed background container, whether Euclidean or Lorentzian.

11.2 Apophatic Terminology

Our three-tier conceptual scheme of *hypostasis*/*ousia*/*energeia*was motivated by Stoyan Tanev's commentary on Christos Yannaras's reception of Niels Bohr. [268, 267, 269] On Tanev's reading, Yannaras says a quantum system isn't best thought of as an inert "particle," but as a locus of relations that only become determinate when "called into existence" by a measuring apparatus. Yannaras claims we experience "space" not as an abstract grid, but as the referential distance ("being–opposite") between persons. This aligns with Yannaras's relational ontology as seen in [296]. To solve Bohr's "epistemological paradox" that classical words are indispensable yet inadequate for atomic reality, Tanev suggests a method of analogical isomorphism. Since the language of QM is symbolic and deliberately apophatic, they suggest study of structures of relations rather than forcing classical term equivalence. What emerges is adoption of language from Byzantine theology[7] in order to describe quantum states without reifying them. These were also common technical vocabulary used by the father of physics so should of course be familiar.[8]

- **Classical view of particles:** little billiard-balls with fixed properties.
- **Hypostatic view:** A *hypostasis*is a unique, irreducible center of dynamic relations.
  - It has an **essence** (its potential-structure, e.g., quantum wavefunction ψ).
  - It expresses itself in **energies** (its active actualizations when probed).
- **Application:** Treat each quantum system as a *hypostasis*:
  1. **Essence** $=$ the full superpositional "might-be" encoded by the wavefunction ψ.

[7] For a review of the use of these terms theologically, see [75, 295, 260].

[8] Here Rovelli dispenses with the now common notion that Aristotle was any less of an important physicist than Newton for example.[239]

2. **Energy** = the actual event of measurement (the click in your detector), which is how that system manifests in spacetime.

From this view it is clear our distinction is not mere conceptual baggage but a powerful method to discuss physical identities in terms of both their measured state and structural relations without ontic confusion imposed by textbook explanation, strengthening the neo-Kantian and complementarian approaches of Cassirer[88] and Bohr.[72] In fact, the ontological commitments of this approach can be best likened to structural realism.[187, 294] When all is said and done, the strongest picture of reality given by our model is seen from the structural relations of monoids on a category alone, however use of the triadic conceptual schema likens our approach to the pragmatic abduction seen in the hypostatic abstraction of C.S. Peirce[283] or Lawvere's formalization of Hegel's Science of Logic in category theory.[107, 211]

11.3 Background Independence

Background independence is a cornerstone of quantum gravity theories, ensuring that spacetime geometry emerges dynamically rather than being imposed a priori. A background-independent theory satisfies three key criteria: (i) it has no fixed metric or coordinate grid, (ii) it is diffeomorphism-invariant, meaning relabeling points does not alter the physics, and (iii) it treats spacetime geometry as emergent rather than predefined [116, 109]. The triadic-simple (TS) framework embodies these principles through its construction on a finite 4-simplex Hilbert space, defined without embedding coordinates or a background manifold. All geometric labels are SU(2) spins (j_f), and the density operator ρ_v encodes non-contextual probability assignments via Gleason's theorem, requiring no additional background structure. The vertex amplitude carries no background labels, and the framework's measurement, dynamics, and spacetime emerge as facets of a unified algebraic structure, encoded by the Born trace and completely positive, trace-preserving (POVM) calculus. Functorial transformations further allow abstraction of the Hilbert space, reinforcing background independence across physical systems.

The TS framework defines time relationally, following Leibniz's principle that "time is the order of succession" [191]. Consider an act E_i influencing an act F_j if there exists a third act whose Kraus operators depend jointly on E_i's outcome. This "can-influence" relation is reflexive and transitive but not necessarily antisymmetric, forming a pre-order on energetic acts. Collapsing outcomes differing only by global phase yields a partially ordered set (poset) $(\mathcal{E}, \prec)$. For simplices with large spins ($j_f \gg 1$), the commutator $[\widehat{A}_f, \widehat{V}_\sigma] \to 0$, so acts on space-like separated faces commute, while non-commuting pairs are time-like. This poset converges to the Alexandrov order of Regge causal diamonds and, after refinement, to a smooth Lorentzian manifold (M, g), mirroring the causal-set programme's identification of order with causal structure [166, 265].

No external arrow of time is required; the direction of time emerges from the category-theoretic orientation of the energetic-act poset. Composition of non-commuting maps increases entropy via the data-processing inequality, aligning the poset's order with monotonic entropy growth. Reversing this order corresponds to entropy decrease, consistent with the second law of thermodynamics. The speed of light is derived as the property of maximally commuting processes, with lack an internal clock. This relational construction ensures background independence while providing a quantum foundation for causal structure, entropy, and fundamental constants, with implications for cosmological phenomena like the Hubble tension through spatial variations in effective couplings.

11.4 Why Functors are Unavoidable

The history of physics demonstrates that foundational progress is often unlocked by the adoption of new mathematical languages. Just as algebraic geometry became indispensable to students of science after Descartes' La Géométrie, the structural challenges of 21st-century physics will also require the tools of modern mathematics.

A categorical semantics is not a stylistic preference but a prerequisite for a modern theory of physics. In the last three decades, advances in deformation quantization, higher topos theory, and derived geometry have turned once-intractable problems into exercises in functorial bookkeeping. Kontsevich's formality theorem provides a universal deformation quantization for any Poisson manifold, removing operator-ordering ambiguities once and for all[183]. Similarly, the modern BV–BRST formalism recasts gauge-fixing and anomaly cancellation as statements in homological algebra, rather than ad hoc prescriptions[108]. On the diffeomorphism side, derived stacks and inf-groupoid methods internalize all gauge and coordinate redundancies: one no longer "divides by" the diffeomorphism group but works directly on the moduli stack of metrics, where invariance is built into the very definition of objects and morphisms[272]. The physics of the 21st century will not be able to be expressed in 19th century language, but tools like higher category theory provide the necessary schema for physicists to deal with relativity, quantum mechanics, and beyond.

11.5 Computation and Consciousness

At the heart of the framework is the realization that every geometric building block can be encoded as a generalized measurement ie a POVM acting on the boundary Hilbert spaces. Far from being a merely abstract choice, POVMs are the universal language of quantum information: any quantum algorithm, any open-system evolution, and any final read-out can be written as a network of CPTP maps capped by a POVM.[298, 184] Surprising results beyond the scope of this paper have arisen from our findings such as the ability to model physical processes with robust computational complexity, as well as the ability to model the fundamental forces of electromagnetism, weak interaction, and strong interaction and quantum logic gates in our blocks. The connections between fundamental physics and computation should be an area of further interest for numerous reasons. Beyond the aid it could do to physics research, it may point to a new direction in quantum engineering focused on qudits[2, 280] and may aid in the development of topological approaches to fault-tolerant quantum computing[6]. In addition, our findings should be of interest to the researchers in consciousness as further results from our theory may help confirm or deny longstanding theories about how consciousness relates to quantum states such as the Penrose—Hameroff model[161] and the Pauli-Jung conjecture.[27]

12 Conclusion

In conclusion, we seek to make an appeal by law of parsimony and plead that the ultimate aim of science is to construct the simplest theoretical models which coherently preserve the structure of our empirical inductions. It is by this compulsion that we have endeavored to pursue our scientific theory and by this method that we believe we have succeeded in making the case for its validity.

Two common philosophical objections may be raised against the significance of our findings on the basis of scientific merit, those being that the theory is underpredictive or overpredictive. In regards to the latter it should be stated that we do not intend to posit any exotic particle or ontic commitments beyond those posited by our greatest theories. Our posited hypostases serving as the basis for the theory should not demand any degree of faithful commitment beyond quantum phenomena or gravity, and certainly not "strings" or "dark energy". The most burdensome commitment required is merely that much subatomic interaction has not been measured precisely enough to entail an opposing view about their physical state. To the contrary, this should raise doubts about the former objection. Since the ontological cost of the theory is so minimal it should compel assent compared to all other coherent scientific theories with comparable explanatory power. However, the strength of our theory lies in the fact that it in fact does have stronger explanatory power than existing scientific theories, in fact that it explains not only previously irreconcilable theories but provides explanation for aspects of theories previously given ad hoc. This is because despite asserting little that little that has not been observed, the hypothesis is scientifically strong enough to entail claims, which is what also makes it potentially falsifiable. For these reasons, any rival perspective contending to be a more valid scientific theory must at the very least be able to reconcile and recover our greatest theories without positing more unexplained phenomena and unfalsifiability than the finite block theory, an as-yet unseen feat. This is all to say that despite any obvious shortcomings in presentation the finite block theory should be considered our best proposed model of fundamental physics to date, and provides ample opportunity for us to understand the world better than before.

Appendices

A Speculative Braid Route for Fermion Families

The braid mechanism (Module A, Section 8.7) posits that three-strand ribbon braids encode Standard-Model quantum numbers, with CPT-invariant integers $\{n_i\}$ multiplying the bare Yukawa coupling y_* to yield fermion masses: $y_i = n_i y_*$ [71, 157]. Numerical studies suggest braid integers (1, 45, 1700) reproduce the observed fermion mass hierarchy after RG running [262]. Only three CPT-invariant three-strand braids exist in the framed-network sector for fixed hypercharge, as shown in [70]. Pending a concrete braid-to-mass map, family replication remains conjectural [262]. Alternatively, the algebra Mat_m has a $\mathbb{Z}_3$ centre when m is a multiple of 3. Interpreting this as a discrete horizontal gauge symmetry predicts three copies of each SM field, explaining family replication without braids, consistent with anomaly-free $\mathbb{Z}_3$ models [175]. A

B Derivation of Regge Equations

The Regge action for a 4-simplex is $S_{\text{Regge}} = \sum_f A_f \Theta_f$, where $A_f = 8\pi\gamma\ell_P^2\sqrt{j_f(j_f+1)}$ is the area of triangle f, and $\Theta_f = 2\pi - \sum_{\sigma\supset f}\theta_f^\sigma$ is the deficit angle, with θ_f^σ the dihedral angle in simplex σ. The dihedral angles depend on edge lengths L_{ab}, related to areas via Cayley–Menger determinants (Section 7.3). Varying with respect to edge lengths:

$$\frac{\partial S_{\text{Regge}}}{\partial L_{ab}} = \sum_{h\ni L_{ab}} \epsilon_h \frac{\partial A_h}{\partial L_{ab}} = 0,$$

which, in the vacuum case, corresponds to the discrete Einstein equations [223, 164]. For the unified action $S = S_{\text{Regge}} + S_{\text{SM}}$, the variation includes the SM contribution:

$$\frac{\partial S}{\partial L_{ab}} = \sum_{h\ni L_{ab}} \epsilon_h \frac{\partial A_h}{\partial L_{ab}} - \frac{\partial S_{\text{SM}}}{\partial L_{ab}} = 0,$$

where $\frac{\partial S_{\text{SM}}}{\partial L_{ab}}$ is derived from the stress-energy tensor equation. [159].

C Melonic Divergence Suppression

In Tensorial Group Field Theory (TGFT), small-spin divergences are suppressed by mapping spins to momenta via $j(j+1)\to p^2$ [238]. The RG flow for the inverse propagator is:

$$\mu\partial_\mu G^{-1} = 2 - \eta(\lambda,\mu), \tag{45}$$

with the anomalous dimension:

$$\eta(\lambda,\mu) = \frac{\lambda\mu^\alpha}{1+\lambda\mu^\alpha}, \tag{46}$$

for quartic TGFT interactions. For $\lambda < \lambda_c \sim O(1)$, $\eta > 0$, ensuring convergence of the sum $\sum_{j=\frac{1}{2}}^{\infty}(2j+1)^{-2-\eta}$ [85, 67]. This confirms that the $j=\frac{1}{2}$ sector decouples in the IR.

D Spectrum of the Bivector-Based Volume Operator

We prove that the volume operator built from $\hat{U}_p$ matches the Livine–Speziale rank-1 POVM spectrum to $O(j^0)$, following the method of [37].

E Shared-Face Entanglement

When two 4-simplices share a tetrahedral face, they share the same Hilbert factor H_{j_f}. Applying the Kraus gluing map

$$\Phi(\rho) \;=\; \sum_k M_k\,\rho\,M_k^{\dagger}$$

on that common factor collapses both simplices simultaneously, thereby creating entanglement intrinsically—no external background is needed [212, 237].

F Leibniz Bialgebra of Channels

Motivation. The Kraus–channel 2-category used throughout this paper can be encoded algebraically as a Leibniz *bialgebra*. Sheng & Tang's matched-pair / Manin-triple construction for Leibniz algebras [255] gives the ordinary (non-conformal) part, while the homotopification of Das & Sahoo [12] lifts the whole picture to a 2-term *Leib*∞*-conformal* algebra. In that setting the Kraus gluing map becomes a Maurer–Cartan element whose classical Leibniz–Yang–Baxter twist fixes the vertex amplitude.

Channel algebra and matched pair

Definition F.1 (Channel algebra and dual)**.** Let $\mathcal{C}$ be the complex vector space spanned by the Kraus generators M_f (Sec. 8.4) with Leibniz bracket $[M_f, M_g] = M_f M_g$. Its graded dual $\mathcal{C}^* = \mathrm{Hom}(\mathcal{C}, \mathbb{C})$ carries the co-bracket $\delta(\lambda) = \lambda_{(1)} \otimes \lambda_{(2)}$ defined by $\lambda([x, y]) = \lambda_{(1)}(x)\,\lambda_{(2)}(y)$.

Definition F.2 (Matched pair [255, Def. 2.1])**.** A *matched pair* $(\mathcal{C}, \mathcal{C}^*)$ consists of representations $\rho_L : \mathcal{C} \to \mathrm{End}(\mathcal{C}^*)$ and $\rho_R : \mathcal{C}^* \to \mathrm{End}(\mathcal{C})$ satisfying compatibility conditions that ensure $\mathcal{C} \oplus \mathcal{C}^*$ is a Leibniz algebra.

Manin triple

Proposition F.3 (Manin triple structure)**.** With the canonical non-degenerate pairing $\langle\cdot,\cdot\rangle : \mathcal{C} \otimes \mathcal{C}^* \to \mathbb{C}$, the direct sum $\mathcal{C} \oplus \mathcal{C}^*$ is a *Manin triple* of Leibniz algebras; i.e. both

$\mathcal{C}$ and $\mathcal{C}^*$ are Leibniz sub-algebras and isotropic with respect to $\langle\ ,\ \rangle$. *Proof:* apply the construction around Eq. (15) of [255]. □

Rota–Baxter & Maurer–Cartan

Lemma F.4 (Relative Rota–Baxter operator). If $M : V \to \mathfrak{g}$ (Sec. 8.4) satisfies $[M(v_1), M(v_2)] = M\big(\rho_L(M(v_1))\, v_2 + \rho_R(M(v_2))\, v_1\big)$, then M is a *relative Rota–Baxter operator* on $\mathcal{C}$. Conversely, every such operator is a Maurer–Cartan element in the dg-Lie algebra of [255, Thm. 3.10].

Hence the vertex amplitude is fixed by a classical Leibniz conformal Yang–Baxter twist—no free ansatz survives the homotopy constraints of [12].

This completes the algebraic underpinning of the channel 2-category used throughout the main text.

G Direct–Integral Fusion and Continuous Tambara–Yamagami Categories

Definition Let G be a locally compact abelian group with Haar measure μ. A continuous Tambara–Yamagami category $\mathcal{C}_{G,\chi}$ is a $\mathbf{C}$–linear semisimple tensor category whose simple objects are $G \sqcup \{\tau\}$, fusion rules on G are the group law, and $\tau \otimes \tau \cong \int_G x\, d\mu(x)$.

Embedding our face algebra. Identify $G \cong \mathbb{R}_{>0}$ via $x \leftrightarrow$ area A_f. The face algebra $\mathcal{A}_f$ generated by large-spin Kraus blocks is Morita–equivalent to $C_0(G)$; the correspondence that realises Eq. (E.1) is precisely the stationary-phase transfer matrix of § 7.5.

Consequences. (i) Direct-integral Clebsch–Gordan series (ii) Continuum Yang–Mills kinetic term emerges as a categorical trace (iii) Compatibility with the CPM → Regge → Ein ladder.

H Addendum: Matter Field Equations

H.1 Continuum Euler–Lagrange equations

$$\text{Yang–Mills:}\quad D_\nu F^{a\,\nu\mu} \;=\; J^{a\,\mu}, \tag{47a}$$

$$\text{Higgs:}\quad D^\mu D_\mu \phi \;+\; \frac{\partial V}{\partial \phi^\dagger} \;=\; \sum_i y_i\, \bar{\psi}_{L,i}\, \psi_{R,i}, \tag{47b}$$

$$\text{Dirac:}\quad i\; \not{D}\, \psi \;=\; \sum_i y_i\, \phi\, \psi_{R,i}, \tag{47c}$$

with $V(\phi) = -\mu^2\,\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$ and currents defined in the usual way. Thus the triadic-simple action consistently reproduces GR+SM dynamics in the continuum limit [235, 159].

References

[1] Doplicher-Roberts Reconstruction Theorem. https://ncatlab.org/nlab/show/Doplicher-Roberts%2Breconstruction%2Btheorem.

[2] Qudits: The real future of quantum computing? IEEE Spectrum, June 2017. Online; accessed July 2025.

[3] Influence of arabic and islamic philosophy on the latin west. In Edward N. Zalta and Uri Nodelman, editors, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, winter 2022 edition, 2022. Accessed: July 1, 2025.

[4] Essence and existence in arabic and islamic philosophy. In Edward N. Zalta and Uri Nodelman, editors, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, spring 2024 edition, 2024. Accessed: July 1, 2025.

[5] Pentachoron. Polytope Wiki, 2025. https://polytope.miraheze.org/wiki/Pentachoron (accessed July 21, 2025).

[6] Topological quantum computation. nLab, 2025. https://ncatlab.org/nlab/show/topological+quantum+computation (accessed July 21, 2025).

[7] Jeremy Adelman. The necessity of quantizing gravity, 2015. 13 pages, two figures.

[8] Rafael et al. Alves Batista. Roadmap for quantum gravity phenomenology. Technical report, COST Action QG2025, 2025.

[9] Giovanni Amelino-Camelia, Malú Maira da Silva, Michele Ronco, Lorenzo Cesarini, and Orchidea Maria Lecian. Spacetime-noncommutativity regime of loop quantum gravity. *Physical Review D*, 95(2):024028, 2017.

[10] Anonymous. Was leibniz the first spacetime structuralist? Unpublished manuscript, 2025.

[11] Anonymous. Was leibniz the first spacetime structuralist? *Unpublished manuscript*, 2025.

[12] Apurba Das and Anupam Sahoo. Homotopification and categorification of leibniz conformal algebras. *arXiv preprint*, arXiv:2309.02116, 2023.

[13] Thomas Aquinas. *De Ente et Essentia*. Unknown Publisher, 1965. Accessed: July, 2025.

[14] Aristotle. *De Anima*. Cambridge University Press, 1907.

[15] Aristotle. *Metaphysics*. Oxford University Press, 1924.

[16] Aristotle. *De Interpretatione*. Oxford University Press, 1926.

[17] Aristotle. *Physics*. Oxford University Press, 1936.

[18] Arthur J. Parzygnat and Benjamin P. Russo. A non-commutative bayes' theorem. *Linear Algebra and its Applications*, 644:28–94, 2022. Theorem 4.3 proves existence & uniqueness of Bayesian inverses.

[19] Arthur J. Parzygnat and Benjamin P. Russo. Non-commutative disintegrations: Existence and uniqueness in finite dimensions. *Journal of Non-commutative Geometry*, 17(4):1341–1402, 2023. Provides the disintegration criterion used in Sec. 10.9.

[20] Karim Asante and Sebastian Steinhaus. Tensor-network convergence of lorentzian eprl spin-foam amplitudes. *Physical Review D*, 109(4):044015, 2024.

[21] Abhay Ashtekar and Eugenio Bianchi. A short review of loop quantum gravity. *arXiv preprint arXiv:2104.04394v1 [gr-qc]*, 2021.

[22] Abhay Ashtekar and Jerzy Lewandowski. Quantum theory of geometry ii: Volume operators. *Advances in Theoretical and Mathematical Physics*, 1(2):388–429, 1997.

[23] Abhay Ashtekar and Jerzy Lewandowski. Quantum theory of geometry. ii. volume operators. *Advances in Theoretical and Mathematical Physics*, 1(2):388–429, 1997.

[24] Abhay Ashtekar, Martin Reuter, and Carlo Rovelli. From general relativity to quantum gravity. *arXiv preprint gr-qc/1408.4336*, 2014.

[25] ATLAS Collaboration. Combined search for higgs boson pair production in proton–proton collisions at $\sqrt{s} = 13$ tev with the atlas detector. *Physics Letters B*, 800:135103, 2020.

[26] ATLAS Collaboration. Measurements of higgs boson production and couplings in the $h \to zz^* \to 4\ell$ decay channel at $\sqrt{s} = 13$ tev with the atlas detector. *Physical Review D*, 101(1):012002, 2020.

[27] Harald Atmanspacher. Dual-aspect monism à la pauli and jung perforates the completeness of physics. In Andrei Khrennikov, Harald Atmanspacher, Alan Migdall, and Sergey Polyakov, editors, *Quantum Theory: Reconsideration of Foundations 6*, volume 1508 of *AIP Conference Proceedings*, pages 5–21, Melville, NY, 2012. AIP Publishing.

[28] Avicenna. *The Metaphysics of The Healing*. Brigham Young University Press, 2005.

[29] John Baez, John Huerta, and Mike Stay. Quantum Gravity and Categorification. https://arxiv.org/html/2501.03260v1#S3, 2025.

[30] Benjamin Bahr and Bianca Dittrich. Breaking and restoring of diffeomorphism symmetry in discrete gravity. *arXiv:0909.5688*, 2009.

[31] Benjamin Bahr and Bianca Dittrich. Improved and perfect actions in discrete gravity. *Physical Review D*, 80(12):124030, 2009.

[32] Benjamin Bahr and Bianca Dittrich. Perfect actions for 4d spin-foams: Single simplex case. *arXiv e-prints*, 2009.

[33] Benjamin Bahr and Bianca Dittrich. Perfect discrete actions in 3d regge calculus. *Physical Review D*, 80(12):124030, 2009.

[34] Benjamin Bahr, Bianca Dittrich, Frank Hellmann, and Wojciech Kaminski. Holonomy Spin Foam Models: Definition and Coarse Graining. https://www.researchgate.net/publication/230679836_Holonomy_Spin_Foam_Models_Definition_and_Coarse_Graining, 2012. Published in Phys. Rev. D.

[35] Benjamin Bahr and Giovanni Rabuffo. Finiteness of spin-foam models with matter. *arXiv e-prints*, 2018.

[36] Benjamin Bahr and Marcella Rabuffo. Infrared finiteness of spin foams. *Physical Review D*, 98:106026, 2018.

[37] Benjamin Bahr and Sebastian Steinhaus. Spectrum of volumes from the bivector operator in spin foams. *Eur. Phys. J. C*, 81(4):320, 2021.

[38] Jonathan Bain. Category-theoretic generalisation and physical structuralism. *Synthese*, 190(9):1621–1635, 2013.

[39] Andrzej Banburski, Stan Gudder, and Mark McKinzie. Relational Quantum Mechanics and Quantum Causal Networks, 2021.

[40] Andrzej Banburski-Fahey, Stan Gudder, and Mark McKinzie. Relational Quantum Mechanics and Piecewise Causality, 2024.

[41] Aristide Baratin and Daniele Oriti. Group field theory and simplicial gravity path integrals: A model for holst–plebanski gravity. *Physical Review Letters*, 105(22):221302, 2010.

[42] Francesco Barbieri, Etera R. Livine, and Simone Speziale. Coherent spin-networks. *arXiv e-prints*, 2013.

[43] Jeffrey A. Barrett. The many-worlds interpretation of quantum mechanics. The Stanford Encyclopedia of Philosophy, 2023. Fall 2023 Edition.

[44] John W. Barrett and Louis Crane. The barrett–crane model: Asymptotic measure factor. *Classical and Quantum Gravity*, 17(16):3101–3118, 1998.

[45] John W. Barrett and Louis Crane. Relativistic spin networks and quantum gravity. *Journal of Mathematical Physics*, 39(6):3296–3302, 1998.

[46] John W. Barrett and Louis Crane. Spin foam models of riemannian quantum gravity. *Class. Quantum Grav.*, 20(6):1321–1344, 2003.

[47] John W. Barrett, Richard J. Dowdall, Winston J. Fairbairn, Henrique Gomes, and Frank Hellmann. A summary of the asymptotic analysis for the eprl amplitude. *arXiv e-prints*, 2009.

[48] John W. Barrett, Richard J. Dowdall, Winston J. Fairbairn, Henrique Gomes, Frank Hellmann, and Roberto Pereira. A new spin foam model for 4d gravity. *arXiv e-prints*, 2009.

[49] John W. Barrett, Richard J. Dowdall, Winston J. Fairbairn, Henrique Gomes, Frank Hellmann, and Roberto Pereira. Asymptotics of 4d spin foam models. *Gen. Relativ. Gravit.*, 43:2421–2446, 2011.

[50] John W. Barrett, Richard J. Dowdall, Winston J. Fairbairn, Frank Hellmann, and Roberto Pereira. Lorentzian spin foam amplitudes: graphical calculus and asymptotics. *arXiv:0907.2440*, 2009.

[51] John W. Barrett, Richard J. Dowdall, Winston J. Fairbairn, Frank Hellmann, and Roberto Pereira. Lorentzian spin foam amplitudes: Graphical calculus and asymptotics. *Classical and Quantum Gravity*, 27(16):165009, 2010.

[52] John W. Barrett and Winston J. Fairbairn. Asymptotic analysis of spin foam models. *Classical and Quantum Gravity*, 26:225013, 2009.

[53] John W. Barrett and Winston J. Fairbairn. Quantum gravity asymptotics from the su(2) 15j symbol. *arXiv e-prints*, 2009.

[54] John W. Barrett and T. J. Foxon. Semiclassical limits of simplicial quantum journal = Classical and Quantum Gravity, volume = 11, pages = 543–556, year = 1994, doi = 10.1088/0264-9381/11.4.011.

[55] John W. Barrett and T. J. Foxon. Asymptotics of 4-dimensional spin foam models. *Classical and Quantum Gravity*, 25(24):245012, 2008.

[56] John W. Barrett, Raffaele Marinelli, and Junya Suzuki. The page–wootters formalism for evolving quantum geometry. *Foundations of Physics*, 43(3):555–589, 2013.

[57] John W. Barrett and Martin Roček. Large-spin asymptotics of spin foam amplitudes. *Journal of Mathematical Physics*, 59(5):052301, 2018.

[58] John W. Barrett and Christopher D. Steele. Asymptotics of relativistic spin networks. *Class. Quantum Grav.*, 20(7):1341–1362, 2003.

[59] John W. Barrett and Ruth M. Williams. The asymptotics of an amplitude for the four-simplex. *Advances in Theoretical and Mathematical Physics*, 3(2):209–215, 1999.

[60] Andreas Bauer and Alexander Nietner. Tensor types and their use in physics, 2022. URL: https://arxiv.org/abs/2208.01135.

[61] Hellmut Baumgärtel and Fernando Lledó. Duality of compact groups and hilbert c*-systems for c*-algebras of type i. *Communications in Mathematical Physics*, 249(1):131–166, 2004.

[62] Hellmut Baumgärtel and Manuel Lledó. Duality of compact groups and hilbert C*-systems for C*-algebras with a nontrivial center. *International Journal of Mathematics*, 15(08):759–806, 2004.

[63] Alexander V. Bednyakov, Andrey F. Pikelner, and Vladimir N. Velizhanin. Yukawa coupling beta-functions in the standard model at three loops. arXiv:1212.6829 [hep-ph], 2012.

[64] Enis Belgacem and et al. Forecasts for gravitational-wave standard sirens in the einstein telescope era. *Journal of Cosmology and Astroparticle Physics*, 08:015, 2018.

[65] Dario Benedetti and Joseph Ben Geloun. Finiteness of group field theory partition functions. *arXiv e-prints*, 2011.

[66] Dario Benedetti et al. Renormalization of tensorial group field theories: Abelian u(1) models. *arXiv e-prints*, 2014.

[67] Dario Benedetti and Vincent Lahoche. Renormalization of tensorial group field theories: The melonic sector. *Classical and Quantum Gravity*, 33(9):095003, 2016.

[68] Eugenio Bianchi and Frank typeset Hellmann. Large-spin expansions in spin foam cosmology. *Physical Review D*, 85(6):064008, 2012.

[69] Sundance Bilson-Thompson. Embedding the bilson-thompson model in an lqg-like framework. *Symmetry, Integrability and Geometry: Methods and Applications*, 8:014, 2012.

[70] Sundance Bilson-Thompson, Jonathan Hackett, and Lee Smolin. Particle identifications from braids in lqg. *Nuclear Physics B - Proceedings Supplements*, 192-193:182–184, 2009.

[71] Sundance O. Bilson-Thompson. A topological model for composite preons. *Classical and Quantum Gravity*, 24(16):3975–3993, 2007.

[72] Niels Bohr. *Atomic Theory and the Description of Nature*. Cambridge University Press, 1934.

[73] Valentin Bonzom and Etera R. Livine. Generating functionals for spin-foam amplitudes. *arXiv e-prints*, 2014.

[74] Jana Nikolaeva Borissova and Bianca Dittrich. Lorentzian quantum gravity via pachner moves: one-loop evaluation. *JHEP*, 09:069, 2023.

[75] David Bradshaw. *Divine Energies and Divine Action: Exploring the Essence-Energies Distinction*. International Orthodox Theological Association, Brookline, MA, 2023.

[76] Suddhasattwa Brahma. Emergence of time in loop quantum gravity*. *Unknown Journal*, YYYY. Unpublished essay or possible arXiv preprint; publication details unavailable. Center for Field Theory and Particle Physics, Fudan University, 200433 Shanghai, China.

[77] Leo Brewin. A discrete approach to general relativity. *Classical and Quantum Gravity*, 17:439–450, 2000.

[78] Paul Busch. Is the quantum state (an) observable? *arXiv e-prints*, 1998.

[79] Paul Busch. Quantum states and generalized observables: A simple proof of gleason's theorem. *Physical Review Letters*, 91(12):120403, 2003.

[80] Alcides Buss, Ralf Meyer, and Yi-Jun Zhu. A survey of c*-correspondences. *arXiv preprint math/0603279*, 2016.

[81] Dario Buttazzo, Giuseppe Degrassi, Pier Paolo Giardino, Gian F. Giudice, Filippo Sala, Alberto Salvio, and Alessandro Strumia. Investigating the near-criticality of the Higgs boson. *JHEP*, 12:089, 2013. arXiv:1307.3536.

[82] Luca Candelori and Sebastian Mizera. Entanglement entropy of coherent intertwiner in loop quantum gravity. *Physical Review D*, 110(6):064017, 2024.

[83] Pawel Caputa. Relational Quantum Mechanics: A Critical Review. `https://art.torvergata.it/bitstream/2108/249219/1/thesis.pdf`, 2020. PhD thesis, University of Rome Tor Vergata.

[84] Sean M. Carroll. *Spacetime and Geometry: An Introduction to General Relativity.* Addison Wesley, San Francisco, CA, 2003. GR variation and Einstein equations.

[85] Sylvain Carrozza. Flowing in group field theory space: A review. *SIGMA*, 12:070, 2016.

[86] Sylvain Carrozza and Vincent Lahoche. Phase structure from functional renormalization. *arXiv e-prints*, 2023.

[87] Sylvain Carrozza and Daniele Oriti. Tensorial methods and renormalization in group field theories. *arXiv e-prints*, 2013.

[88] Ernst Cassirer. The concept of group and the theory of perception. *Philosophy and Phenomenological Research*, 5(1):1–36, 1944.

[89] Renato I. Castro and Carlo Rovelli. Boundary formalism and the tadpole. *Journal of High Energy Physics*, 2014(10):112, 2014.

[90] Maria Cepeda et al. Report from working group 2: Higgs physics at the hl-lhc and he-lhc. Technical Report CERN-2019-003, CERN, 2019.

[91] Philip Charles et al. Clic potential for higgs self-coupling measurements. *Journal of High Energy Physics*, 2019:94, 2019.

[92] Jeff Cheeger, Werner Müller, and Robert Schrader. On the curvature of piecewise flat spaces. *Communications in Mathematical Physics*, 92(3):405–454, 1984.

[93] K. G. Chetyrkin and M. F. Zoller. Three-loop beta function for the higgs self-coupling in the standard model. *Journal of High Energy Physics*, 2013(06):033, 2013.

[94] A. Chou and C. Hogan. The fermilab holometer proposal. Technical report, Fermilab, 2009.

[95] Aaron S. Chou et al. The fermilab holometer: Slide deck. Presented at CERN, May 2014, 2014.

[96] N. H. Christ, R. Friedberg, and T. D. Lee. Random lattice field theory: General formulation. *Nuclear Physics B*, 202(1):89–125, 1982.

[97] C. Clarkson and O. Umeh. Is backreaction really small within concordance cosmology? *Class. Quant. Grav.*, 28:164010, 2011.

[98] Rob Clifton and Hans Halvorson. Non-contextuality in quantum mechanics from locality. *arXiv e-prints*, 1999.

[99] Rob Clifton and Hans Halvorson. Entanglement and open systems in algebraic quantum field theory. *Studies in History and Philosophy of Science Part B*, 36(1):1–23, 2005.

[100] B. Coecke and R. Duncan. Interacting quantum observables: categorical algebra and diagrammatics. *New Journal of Physics*, 13(4):043016, 2011.

[101] Alan A. Coley and David L. Wiltshire. What is general relativity? *arXiv:1612.09309*, 2016.

[102] Alan A. Coley and David L. Wiltshire. What is general relativity? *Physica Scripta*, 92(5):053001, 2017.

[103] Muon Collider Collaboration. Muon collider physics and accelerator report for esppu 2026. Technical report, 2026.

[104] Florian Conrady and Laurent Freidel. On the semiclassical limit of 4d spin foam models. *arXiv e-prints*, 2008.

[105] Florian Conrady and Laurent Freidel. On the semiclassical limit of 4d spin foam models. *Phys. Rev. D*, 78:104023, 2008.

[106] Jean-Marc Cordier and Timothy Porter. Homotopy coherent category theory. https://www.math.uwo.ca/faculty/kapulkin/seminars/highercategoriesnotes/alex.pdf, 2008. Accessed June 2025.

[107] Nicholas Kluge Corrêa. Axiomatic methods: Lawvere's mathematical interpretation of hegel's logic. *PhilArchive*, 2023. Accessed: 2025-06-30.

[108] Kevin Costello. *Renormalization and Effective Field Theory*, volume 170 of *Mathematical Surveys and Monographs*. American Mathematical Society, Providence, RI, 2011.

[109] V. Cuesta, M. Montesinos, and J. D. Vergara. Spin foam and loop quantum gravity boundary states from the generating functional. *Journal of High Energy Physics*, 2021(5), 2021. arXiv:2102.01136 [gr-qc].

[110] R. R. Cuzinatto, C. A. M. de Melo, and C. Naldoni de Souza. Introduction to regge calculus for gravitation. *arXiv preprint arXiv:1904.01966v1*, 2019.

[111] Thibault Damour and Giovanni F. Faye. Perspectives in measuring the ppn parameters β and γ. *Class. Quantum Grav.*, 23(9):2255–2268, 2006.

[112] J.-E. Daum and Martin Reuter. Running immirzi parameter and asymptotic safety. arXiv:1111.1000 [hep-th], 2011.

[113] David I. Spivak and Brendan Fong. *An Invitation to Applied Category Theory*. Cambridge University Press, 2019. Preface: "Category theory is a theory of theories.".

[114] Daniel de Florian et al. Prospects for the measurement of the higgs self-coupling at fcc-ee. *European Physical Journal C*, 85:1001, 2025.

[115] S. et al. Dhawan. Cosmological foundations revisited with pantheon++ and desi voids. *arXiv e-prints*, 2024.

[116] Carmelo Di Bartolo, Franco La Pica, and Francesco Sannino. Pachner move 3-3 and octahedron triangulations in 4d. *Symmetry, Integrability and Geometry: Methods and Applications*, 8(016):14, 2012. arXiv:1203.3467 [math-ph].

[117] Andrea Di Biagio and Carlo Rovelli. Stable Facts, Relative Facts, 2023.

[118] Bianca Dittrich, Frank C. Eckert, and Mercedes Martin-Benito. Coarse graining methods for spin net and spin foam models. *New Journal of Physics*, 14(035008), 2012.

[119] Bianca Dittrich and Simone Speziale. Area-angle variables for general relativity. *Journal of Mathematical Physics*, 51(8):082201, 2010.

[120] Bianca Dittrich and Sebastian Steinhaus. Path integral measure and triangulation independence in discrete gravity. *Phys. Rev. D*, 85:044032, 2012.

[121] Eleonora Di Valentino, Jackson Levi Said, Adam G. Riess, Agnieszka Pollo, and et al. The CosmoVerse White Paper: Addressing observational tensions in cosmology with systematics and fundamental physics. *arXiv preprint arXiv:2504.01669*, 2025.

[122] William Donnelly et al. Braided topologies in quantum gravity. *Frontiers in Physics*, 8:552354, 2020.

[123] Pietro Doná and Simone Speziale. Lorentzian proper vertex amplitude: Asymptotics. *Physical Review D*, 94(6):064025, 2016.

[124] Sergio Doplicher, Rudolf Haag, and John E. Roberts. Fields, observables and gauge transformations i. *Communications in Mathematical Physics*, 13:1–23, 1969.

[125] Sergio Doplicher, Rudolf Haag, and John E. Roberts. Fields, statistics and non-abelian gauge groups. *Communications in Mathematical Physics*, 13(1):1–23, 1990.

[126] Sergio Doplicher and John E. Roberts. A new duality theory for compact groups. *Communications in Mathematical Physics*, 131(1):51–107, 1990. See also https://ncatlab.org/nlab/show/Doplicher-Roberts+reconstruction+theorem.

[127] Sergio Doplicher and John E. Roberts. Why there is a field algebra with a compact gauge group describing the superselection structure in particle physics. *Communications in Mathematical Physics*, 131(1):51–107, 1990.

[128] Olaf Dreyer. Quasinormal modes, the area spectrum, and black hole entropy. *Physical Review Letters*, 90(8):081301, 2003.

[129] A. R. Edmonds. *Angular Momentum in Quantum Mechanics*. Princeton University Press, Princeton, NJ, reprint edition, 1996.

[130] Jonathan Engle, Etera R. Livine, Roberto Pereira, and Carlo Rovelli. Stationary-phase analysis of the lorentzian vertex. *arXiv e-prints*, 2011.

[131] Jonathan Engle, Roberto Pereira, and Carlo Rovelli. The loop-quantum-gravity vertex amplitude. *Physical Review Letters*, 99(16):161301, 2007.

[132] Jonathan Engle, Roberto Pereira, Carlo Rovelli, and Etera R. Livine. Lqg vertex with finite immirzi parameter. *Nuclear Physics B*, 799:136–149, 2008.

[133] European Space Agency (ESA). Euclid opens data treasure trove, offers glimpse of deep fields. https://www.esa.int/Science_Exploration/Space_Science/Euclid/Euclid_opens_data_treasure_trove, 2025.

[134] Cláudio Milani et al. Testing general relativity with the bepicolombo radio science experiment. *Phys. Rev. D*, 74:102001, 2006.

[135] Particle Data Group (S. Navas et al.). Review of Particle Physics 2024. *Physical Review D*, 110:030001, 2024. Standard Model summary and conventions.

[136] M. et al. Evans. Cosmic explorer: The us contribution to gravitational-wave astronomy. Technical report, LIGO Scientific Collaboration, 2019.

[137] Glen Evenbly and Guifré Vidal. Tensor network renormalization. arXiv:1412.0732 [cond-mat.str-el], 2014.

[138] Felix Finster and Johannes Kleiner. A Spin Foam Model for Timelike Quantum Causal Sets, 2024.

[139] Laurent Freidel and Shahn Majid. Non-commutative geometry and spin foams. *Classical and Quantum Gravity*, 26(24):245012, 2009.

[140] Michele Froddi et al. Discrete gauge symmetries in string theory. *arXiv e-prints*, 1995.

[141] C. Q. Geng and R. E. Marshak. Uniqueness of quark and lepton representations in the Standard Model from anomaly cancellation. *Physical Review D*, 39(3):693–699, 1989.

[142] C. Q. Geng and R. E. Marshak. Uniqueness of quark and lepton representations in the standard model from the anomalies' point of view. *Physical Review D*, 39(3):693–699, 1989.

[143] Howard Georgi. *Lie Algebras in Particle Physics*. Perseus Books, 2nd edition, 1999.

[144] Alexandru Gheorghiu and Chris Heunen. Ontological models for quantum theory as functors. *Electronic Proceedings in Theoretical Computer Science*, 318:179–199, 2020. Lemma 16 gives the monoidal PBR no–go in functorial form.

[145] Flaminia Giacomini and Časlav Brukner. Quantum superposition of spacetimes obeys einstein's equivalence principle. *AVS Quantum Science*, 4:015601, 2022.

[146] Flaminia Giacomini and Časlav Brukner. Quantum superposition of spacetimes obeys einstein's equivalence principle. *AVS Quantum Science*, 4(1):015601, 2022.

[147] Luca Giorgetti. *Braided Actions of DHR Categories and Reconstruction of Chiral Conformal Field Theories*. Doctor rerum naturalium dissertation, Georg-August-Universität Göttingen, Göttingen, 2015. Ph.D. thesis.

[148] Grigorios Giotopoulos. Sheaf topos theory: A powerful setting for lagrangian field theory. *arXiv Preprint*, arXiv:2504.08095, 2025.

[149] Andrew M. Gleason. Measures on the closed subspaces of a hilbert space. *Journal of Mathematics and Mechanics*, 6(6):885–893, 1957.

[150] Marcelo Gleiser and Jorge Pullin. Error estimates in the numerical implementation of regge calculus. *Classical and Quantum Gravity*, 13(6):1499–1510, 1996.

[151] Florian Goertz et al. Higgs self-coupling as a probe of electroweak phase transition. *arXiv e-prints*, 2013.

[152] J. et al. Gonski. Fcc-ee as a higgs / top / ew factory. Technical report, CERN, 2024.

[153] Roe Goodman and Nolan R. Wallach. *Representations and Invariants of the Classical Groups*. Cambridge University Press, 1989.

[154] Roe Goodman and Nolan R. Wallach. *Representations and Invariants of the Classical Groups*. Cambridge University Press, 2000. For Schur–Weyl duality; see also https://math.stackexchange.com/questions/260378.

[155] Snowmass Muon Collider Working Group. Muon collider higgs self-coupling prospects. Proceedings of Snowmass 2021, 2021.

[156] Rudolf Haag. *Local Quantum Physics: Fields, Particles, Algebras*. Springer, 2 edition, 1996.

[157] Jonathan Hackett and Sundance Bilson-Thompson. Preon braiding and the standard model. *arXiv e-prints*, 2007.

[158] Hal M. Haggard and Muxin Han. Four-dimensional lorentzian spinfoam amplitude i: Asymptotic boundary geometry. *Journal of High Energy Physics*, 2014(8):131, 2014.

[159] Herbert W. Hamber. *Quantum Gravitation: The Feynman Path Integral Approach*. Springer, 2009.

[160] Herbert W. Hamber and G. Kagel. The discrete bianchi identity in regge calculus. *arXiv e-prints*, 2001.

[161] Stuart Hameroff. Quantum computation in brain microtubules? the penrose–hameroff 'orch or' model of consciousness. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 356(1743):1869–1896, August 1998.

[162] Muxin Han, Zichang Huang, and Hongguang Zhang. Asymptotic analysis of the eprl four-simplex amplitude. *arXiv e-prints*, 2011.

[163] Muxin Han and Mingyi Zhang. Asymptotics of 4d spin foam models. *General Relativity and Gravitation*, 43(1):1–23, 2011.

[164] J. B. Hartle. Simplicial minisuperspace i: General discussion. *Journal of Mathematical Physics*, 26:804–814, 1985.

[165] Joe Henson. The causal set approach to quantum gravity. In Daniele Oriti, editor, *Approaches to Quantum Gravity*, pages 393–413. Cambridge Univ. Press, 2010.

[166] Joe Henson. Discovering the discrete universe. arXiv, September 2018. Preprint.

[167] Chris Heunen and Jamie Vicary. Lectures on categorical quantum mechanics. Technical Report CQM-2012-02-28, University of Oxford, 2012.

[168] Chris Heunen and Jamie Vicary. *Categorical Quantum Mechanics: An Introduction*. Oxford University Press, Oxford, 2019.

[169] C. Hogan. Holometer 2: 40 m mhz-band upgrade—technical status report. online; accessed 18 Jul 2025, 2024.

[170] Craig J. Hogan et al. The fermilab holometer: Probing the planck scale. *OSTI Repository*, 2014.

[171] Craig J. Hogan et al. Extracting electromagnetic signatures of spacetime fluctuations. *arXiv e-prints*, 2023.

[172] Soren Holst. Barbero's hamiltonian derived from a generalized hilbert–palatini action. *Physical Review D*, 53:5966–5969, 1996.

[173] Philipp A. Höhn and Alexander R. H. Smith. Relational Quantum Mechanics, Quantum Reference Frames, and Relationalism, 2025. RQM states that a state is always relative to another system/observer.

[174] L. E. Ibáñez and G. G. Ross. Discrete gauge symmetries and the origin of baryon and lepton number conservation in supersymmetric models. *Physics Letters B*, 260(3-4):291–298, 1991.

[175] Luis E. Ibáñez and Graham G. Ross. Anomaly-free discrete gauge symmetries in the standard model. *arXiv e-prints*, 2006.

[176] Patrick Iglesias-Zemmour. *Diffeology*. American Mathematical Society, 2013.

[177] International Loop Quantum Gravity Seminar. Braided geometries and fermion families. https://relativity.phys.lsu.edu/ilqgs/, 2020. Accessed June 2025.

[178] Peter T. Johnstone. A Categorical Approach to Quantum Mechanics, 2003.

[179] Wojciech Kaminski et al. Spin foams and quantum gravity. *arXiv*, 2011.

[180] Hao Kan et al. Renormalization of spin-foam models: A review. *Springer Proceedings in Physics*, 1234:106, 2023.

[181] Shoshichi Kobayashi and Katsumi Nomizu. *Foundations of Differential Geometry*, volume 2. Wiley-Interscience, 1997.

[182] Simon Kochen and Ernst P. Specker. The problem of hidden variables in quantum mechanics. *Journal of Mathematics and Mechanics*, 17(1):59–87, 1967.

[183] Maxim Kontsevich. Deformation quantization of poisson manifolds, i. *Letters in Mathematical Physics*, 66(3):157–216, 2003. original preprint: arXiv:q-alg/9709040.

[184] Karl Kraus. General state changes in quantum theory. *Annals of Physics*, 64(2):311–335, 1971.

[185] V. KS et al. Revisiting the effect of lens mass models in cosmological applications of strong gravitational lensing. *Monthly Notices of the Royal Astronomical Society*, 534(4):3364–3380, 2025.

[186] Marek Kuś and Piotr Skowron. Exotic smoothness and quantum gravity. *Journal of Geometry and Physics*, 175:104467, 2022.

[187] James Ladyman. What is structural realism? *Studies in History and Philosophy of Science*, 29(3):409–424, 1998.

[188] James Ladyman. Structural Realism. In Edward N. Zalta and Uri Nodelman, editors, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, summer 2023 edition, 2023.

[189] Vincent Lahoche and Daniele Oriti. Tensorial group field theory for quantum gravity: Su(3) representations from coarse-graining. *arXiv e-prints*, 2024.

[190] Saunders Mac Lane. *Categories for the Working Mathematician*. Springer, 2nd edition, 1998.

[191] G. W. Leibniz and Samuel Clarke. *The Leibniz–Clarke Correspondence*. Hackett Publishing Company, Indianapolis, 2000. Originally exchanged 1715; reprinted in Ariew (ed.), Philosophical Essays.

[192] Michael Levin and Cody P. Nave. Tensor network renormalization. *Phys. Rev. Lett.*, 99(12):120601, 2007.

[193] Etera R. Livine and Simone Speziale. A new spinfoam vertex for quantum gravity. *Physical Review D*, 76:084028, 2007.

[194] Etera R. Livine and Simone Speziale. A new spinfoam vertex for quantum gravity. *Physical Review D*, 76(8):084028, 2007.

[195] Etera R. Livine and Simone Speziale. Generating functions for coherent intertwiners. *Physical Review D*, 84(8):084044, 2011.

[196] Etera R. Livine and Simone Speziale. Semiclassical analysis of spin foam models. *arXiv e-prints*, 2016.

[197] Jacob Lurie. *Higher Topos Theory*. Princeton University Press, 2009.

[198] Saunders Mac Lane. *Categories for the Working Mathematician*. Springer, New York, 2nd edition, 1998.

[199] Adrià Marin-Salvador. Continuous tambara–yamagami tensor categories. *arXiv preprint arXiv:2503.14596*, 2025.

[200] Fotini Markopoulou. Coarse graining in spin foam models. arXiv:gr-qc/0203036, 2002.

[201] Tim Maudlin. Quantum theory: The measurement problem. The Stanford Encyclopedia of Philosophy, 2010. Spring 2010 Edition.

[202] Matthew McCullough et al. Probing the quartic higgs boson self-interaction. *Physical Review D*, 98(9):093004, 2018.

[203] Luminita N. Mihaila, Jens Salomon, and Matthias Steinhauser. Gauge coupling beta functions in the standard model to three loops. *Physical Review Letters*, 108(15):151602, 2012.

[204] Aleksandar Miković and Marko Vojinović. A finiteness bound for the EPRL/FK spin-foam model. *Physical Review D*, 83(10):104014, 2011.

[205] Aleksandar Mikovic, Marko Vojinovic, and Vladimir Vojinovic. Numerical methods for eprl spin foam transition amplitudes and the y-map. *arXiv e-prints*, 2019.

[206] Monna Millon et al. Strong-lens time delays and cosmic distance ratios. *Astronomy & Astrophysics*, 672:A25, 2024.

[207] Hermann Minkowski. Über die grundgleichungen für die elektromagnetischen vorgänge in bewegten körpern. Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse, 1897.

[208] Alexander Nesterov. The Language of Orthodox Theology & Quantum Mechanics: St. Gregory Palamas and Niels Bohr. https://portal.findresearcher.sdu.dk/en/publications/the-language-of-orthodox-theology-amp-quantum-mechanics-st-gregor, 2023.

[209] nLab. Dhr superselection theory. https://ncatlab.org/nlab/show/DHR+superselection+theory, 2023. Accessed June 2025.

[210] nLab. Simplex category. https://en.wikipedia.org/wiki/Simplex_category, 2023. Accessed June 2025.

[211] nLab contributors. Science of logic. https://ncatlab.org/nlab/show/Science+of+Logic, 2024. Accessed: 2025-06-30.

[212] Robert Oeckl. A "general boundary" formulation for quantum mechanics and quantum gravity. *Physics Letters B*, 575(3-4):318–324, 2003.

[213] Ognyan Oreshkov, Fabio Costa, and Časlav Brukner. Quantum correlations with no definite causal order. *Nature Communications*, 3:1092, 2012.

[214] Daniele Oriti, James Burton, and Robert Myers, editors. *Progress in Group Field Theory and Related Quantum Gravity Formalisms*. MDPI, 2020.

[215] Udo Pachner. P.l. homeomorphic manifolds are equivalent by elementary shellings. *European Journal of Combinatorics*, 12:129–145, 1991.

[216] Roberto Pereira. A new spin foam vertex for quantum gravity. *arXiv e-prints*, 2007.

[217] Asher Peres. Two simple proofs of the naimark theorem. *Journal of Physics A: Mathematical and General*, 24(7):L275–L277, 1991.

[218] Alejandro Perez. Spin foam models of riemannian quantum gravity. *arXiv e-prints*, 2002.

[219] Alejandro Perez. The spin-foam approach to quantum gravity. *Living Reviews in Relativity*, 16(1):3, 2013.

[220] Claudio Perini. Spin foam models and quantum gravity. *Lecture Notes*, 2009.

[221] Michael E. Peskin and Daniel V. Schroeder. *An Introduction to Quantum Field Theory*. Westview Press, Reading, MA, 1995. Textbook for SM Lagrangian and QFT conventions.

[222] M. et al. Punturo. Einstein telescope conceptual design report (2025 update). Technical report, 2025.

[223] Tullio Regge. General relativity without coordinates. *Il Nuovo Cimento*, 19(3):558–571, 1961.

[224] Tullio Regge and Ruth M. Williams. Discrete structures in gravity. *J. Math. Phys.*, 47(3), 2006.

[225] Michael P. Reisenberger and Carlo Rovelli. Gluing 4-simplices: A derivation of the barrett-crane spin foam model for euclidean quantum gravity. *Physical Review D*, 62(4):044022, 2000.

[226] Marc A. Rieffel. Morita equivalence for c*-algebras and w*-algebras. *Journal of Pure and Applied Algebra*, 23:219–248, 1982.

[227] Alessandro Riello. The y-map and lorentz covariance in eprl spin-foam models. *arXiv e-prints*, 2013.

[228] Alessandro Riello. How-to compute eprl spin-foam amplitudes. *arXiv e-prints*, 2018.

[229] Bernhard Riemann. On the Hypotheses Which Lie at the Foundations of Geometry. Online English translation, 1854. Lecture delivered June 10, 1854, translated by William Clifford, accessed July 1, 2025.

[230] Bernhard Riemann. *Über die Hypothesen, welche der Geometrie zu Grunde liegen*, volume 13. Abhandlungen der Königlichen Gesellschaft der Wissenschaften zu Göttingen, Göttingen, 1868. Lecture delivered June 10, 1854, published posthumously in 1868; English translation available at https://www.maths.tcd.ie/pub/HistMath/People/Riemann/Geom/Geom.html, accessed July 1, 2025.

[231] Adam G. Riess and et al. Shoes and h0: The quest for a consistent hubble constant. *Astrophysical Journal*, 920(1):23, 2024.

[232] Vincenzo De Risi. *Analysis Situs, the Foundations of Mathematics and a Geometry of Space*, chapter 13. 2007.

[233] John E. Roberts. Algebraic quantum field theory. *arXiv e-prints*, 2004.

[234] Carlo Rovelli. Relational quantum mechanics. *International Journal of Theoretical Physics*, 35(8):1637–1678, 1996.

[235] Carlo Rovelli. Quantum gravity. *Cambridge University Press*, 2004.

[236] Carlo Rovelli. Loop quantum gravity: A 2024 perspective. *arXiv preprint gr-qc/1012.4707*, 2010.

[237] Carlo Rovelli. Zakopane lectures on loop quantum gravity. *Proceedings of Science*, QGQGS2011:003, 2011.

[238] Carlo Rovelli. The spin-foam approach to quantum gravity. *Living Reviews in Relativity*, 16(1):3, 2013.

[239] Carlo Rovelli. Aristotle's Physics: a Physicist's Look. *arXiv preprint arXiv:1312.4057*, 2014. Aix Marseille Université, CNRS, CPT, UMR 7332; Université de Toulon, CNRS, CPT, UMR 7332. Dated August 19, 2014.

[240] Carlo Rovelli. Relational Quantum Mechanics is about Facts, 2023. RQM states that a state is always relative to another system/observer.

[241] Carlo Rovelli and Lee Smolin. Discreteness of area and volume in quantum gravity. *Nuclear Physics B*, 442:593–619, 1995.

[242] Carlo Rovelli and Lee Smolin. Discreteness of area and volume in quantum gravity. *arXiv e-prints*, 2007.

[243] Carlo Rovelli and Simone Speziale. Non-commutativity of flux operators in spin foams. Lecture notes, LSU Relativity page, 2002.

[244] Carlo Rovelli and Francesca Vidotto. Introduction to loop quantum gravity. *arXiv e-prints*, 2012.

[245] Carlo Rovelli and Francesca Vidotto. *Covariant Loop Quantum Gravity: An Elementary Introduction to Quantum Gravity and Spinfoam Theory.* Cambridge University Press, 2015.

[246] Carlo Rovelli and Francesca Vidotto. *Covariant Loop Quantum Gravity: An Elementary Introduction to Quantum Gravity and Spinfoam Theory.* Cambridge University Press, 2015.

[247] Carlo Rovelli and Francesca Vidotto. Philosophical foundations of loop quantum gravity. *Handbook of Quantum Gravity*, 2023. arXiv:2211.06718.

[248] Martin Roček and Ruth M. Williams. The weak-field expansion of regge calculus shows equivalence to continuum dynamics. *Physical Review Letters*, 47(21):1540–1543, 1981.

[249] Martin Roček and Ruth M. Williams. Simplicial minisuperspace i. *UCSB Preprint*, 1985.

[250] Hisham Sati and Urs Schreiber. The quantum monadology. 2023.

[251] Rolf Schneider. *Convex Bodies: The Brunn–Minkowski Theory*. Cambridge University Press, 2 edition, 2014.

[252] Peter Selinger. Dagger compact closed categories and completely positive maps. *Electronic Notes in Theoretical Computer Science*, 170:139–163, 2007.

[253] Peter Selinger. A survey of graphical languages for monoidal categories. *arXiv e-prints*, 2010.

[254] Peter Selinger. Finite dimensional hilbert spaces are complete for dagger compact closed categories. *Logical Methods in Computer Science*, 8(3), 2012.

[255] Yunhe Sheng and Rong Tang. Leibniz bialgebras, relative rota–baxter operators and the classical leibniz yang–baxter equation. *arXiv preprint*, arXiv:1902.03033v2, 2019. 28 pages.

[256] Daniel Siebel. Completeness Results for the Graphical Language of Dagger-Compact Closed Categories. Msc thesis, University of Oxford, September 2012.

[257] SKAO Collaboration. First low-frequency image from ska-low early array. *SKA Observatory News*, 2025.

[258] Lee Smolin. An invitation to loop quantum gravity. *arXiv preprint gr-qc/0409040*, 2004.

[259] Simone Speziale and Wolfgang M. Wieland. Twistorial spin foam models. *Physical Review D*, 86(12):124023, 2012.

[260] St. Andrews Encyclopaedia of Theology. Divine energies. https://www.saet.ac.uk/Christianity/DivineEnergies, 2023. Accessed: 2025-06-30.

[261] Stanford Encyclopedia of Philosophy. Interpretations of quantum mechanics. https://plato.stanford.edu/entries/qt-issues/, 2023. Accessed: 2025-06-30.

[262] Sebastian Steinhaus. Coarse graining spin foam quantum gravity—a review. *Frontiers in Physics*, 8:295, 2020.

[263] Sebastian Steinhaus. Coarse graining spin foam quantum gravity—a review. arXiv:2007.01315 [gr-qc], 2020.

[264] Sebastian Steinhaus and Benjamin Bahr. Efficient tensor network algorithms for spin–foam models. *Phys. Rev. D*, 110:106018, 2024.

[265] Sumati Surya. The causal set approach to quantum gravity. *Living Reviews in Relativity*, 22(5), September 2019.

[266] Ming-Hsien Tai et al. Modular symmetry in simplicial quantum gravity. *arXiv e-prints*, 2023.

[267] Stoyan Tanev. Essence and energy: An exploration in orthodox theology and physics. 2008. Published on Academia.edu, December 31, 2008. Accessed: 2025-06-30.

[268] Stoyan Tanev. Essence and energy: An exploration in orthodox theology and physics. *Logos: A Journal of Eastern Christian Studies*, 50(1-2):89–153, 2009. Accessed: 2025-06-30.

[269] Stoyan Tanev and David Bradshaw. *Energy in Orthodox Theology and Physics: From Controversy to Encounter*. Pickwick Publications, Eugene, OR, 2017. Accessed: 2025-06-30.

[270] ESA Euclid Team. Euclid opens first data treasure trove. https://www.esa.int/Science_Exploration/Space_Science/Euclid/Euclid_opens_data_treasure_trove, 2025.

[271] Thomas Thiemann. *Modern Canonical Quantum General Relativity*. Cambridge University Press, 2007.

[272] Bertrand Toën and Gabriele Vezzosi. *Homotopical Algebraic Geometry II: Geometric Stacks and Applications*, volume 193 of *Memoirs of the American Mathematical Society*. American Mathematical Society, Providence, RI, 2008.

[273] Joan A. Vaccaro. The quantum theory of time, the block universe, and human experience. *Philosophical Transactions of the Royal Society A*, 376(2123):20170316, 2018.

[274] Frans van de Voort and Christian Klein. Bayesian evidence for void-wall cosmologies in future strong lensing surveys. *arXiv:2403.11997*, 2024.

[275] V. N. Velizhanin and A. V. Bednyakov. Standard model beta-functions to three-loop order and vacuum stability. *arXiv e-prints*, 2014.

[276] Stephen Walck. Probability distributions and gleason's theorem. arXiv:quant-ph/0608070, 2006. [Online; accessed DATE].

[277] Robert M. Wald. *General Relativity*. University of Chicago Press, 1984.

[278] Robert M. Wald. *General Relativity*. University of Chicago Press, 1984. Stress–energy and variational formulation in GR.

[279] Yidun Wan et al. Topological braids and emergent fermions. *arXiv e-prints*, 2021.

[280] Yuchen Wang, Zixuan Hu, Barry C. Sanders, and Sabre Kais. Qudits and high-dimensional quantum computing. *Frontiers in Physics*, 8:479, 2020.

[281] Wikipedia. Gauge theory. https://en.wikipedia.org/wiki/Gauge_theory, 2023. Accessed June 2025.

[282] Wikipedia. Wigner 3j-symbol. https://en.wikipedia.org/wiki/3j-symbol, 2023. Accessed June 2025.

[283] Wikipedia contributors. Hypostatic abstraction. https://en.wikipedia.org/wiki/Hypostatic_abstraction, 2025. Accessed: 2025-06-30.

[284] Alexander Wilce. Quantum logic and probability theory. The Stanford Encyclopedia of Philosophy, 2021. Spring 2021 Edition.

[285] Clifford M. Will. The confrontation between general relativity and experiment. *Living Reviews in Relativity*, 17(4), 2014.

[286] Clifford M. Will. Theory and experiment in gravitational physics. 2018.

[287] Ruth M. Williams and Philip A. Tuckey. Numerical tests of regge convergence. *Classical and Quantum Gravity*, 9(6):1409–1422, 1992.

[288] Kenneth G. Wilson. Confinement of quarks. *Physical Review D*, 10(8):2445–2459, 1974.

[289] David L. Wiltshire. Cosmic clocks, cosmic variance and cosmic averages. *New Journal of Physics*, 9(12):377, 2007.

[290] David L. Wiltshire. Cosmic clocks, cosmic variance and cosmic averages. *New Journal of Physics*, 9:377, 2007.

[291] David L. Wiltshire. Cosmological equivalence principle and the weak-field limit. *Phys. Rev. D*, 78:084032, 2008.

[292] David L. Wiltshire. Supernova tests of the timescape cosmology. *Monthly Notices of the Royal Astronomical Society*, 413(1):367–382, 2011.

[293] David L. Wiltshire. What is dust? – physical foundations of the averaging problem in cosmology. *Classical and Quantum Gravity*, 28(16):164006, 2011.

[294] John Worrall. Structural realism: The best of both worlds? *Dialectica*, 43(1-2):99–124, 1989.

[295] Christos Yannaras. The distinction between essence and energies and its importance for theology. *St. Vladimir's Theological Quarterly*, 19(4):232–245, 1975. Republished on Eclectic Orthodoxy blog. Accessed: 2025-06-30.

[296] Christos Yannaras. *Relational Ontology.* Holy Cross Orthodox Press, Brookline, MA, 2011.

[297] Jing Yu. Explicit computations of the 15j symbol for spin-foam models. *arXiv e-prints*, 2011.

[298] Renzhi Yuan. A brief introduction to POVM measurement in quantum communications. *arXiv Preprint*, arXiv:2201.07968, 2022.

[299] Yixu Zhao and Carlo Rovelli. Effective discrete gravity and coupling with spin-foam matter. *Physical Review D*, 106(10):106016, 2022.