Diet problem

GitHub repo: https://github.com/pchs20/truck-allocation

Original statement: https://colab.ampl.com/notebooks/diet-lecture.html

Sets

$Days$	Set of days of the diet.
$Meals = \{'Breakfast', 'Lunch', 'Dinner'\}$	Set of meal types that the diet includes per each day.
$Dishes$	Set of dishes names that can be used in the diet.

Parameters

Parameters about the diet

$cal^{min} \in \mathbb{R}^+$ $cal^{max} \in \mathbb{R}^+$	Minimum and maximum number of **calories** per day[kcal].
$p^{min} \in \mathbb{R}^+$ $p^{max} \in \mathbb{R}^+$	Minimum and maximum amount of **protein** that a day can contain [g].
$c^{min} \in \mathbb{R}^+$ $c^{max} \in \mathbb{R}^+$	Minimum and maximum amount of **carbs** that a day can contain [g].
$f^{min} \in \mathbb{R}^+$ $f^{max} \in \mathbb{R}^+$	Minimum and maximum amount of **fat** that a day can contain [g].
$ve \in \{0, 1\}$	Equals 1 if the diet is **vegetarian** and 0 otherwise.
$vg \in \{0, 1\}$	Equals 1 if the diet is **vegan** and 0 otherwise.

Parameters about the dishes

$suitable_{dish,\,meal} \in \{0,\,1\}$	Equals 1 if the dish is suitable for the meal and 0 otherwise.	$\forall dish \in Dishes$ $\forall meal \in Meals$
$cal_{dish} \in \mathbb{R}^+$	Calories of each dish [kcal].	$\forall dish \in Dishes$
$p_{dish} \in \mathbb{R}^+$	Protein that each dish contains [g].	$\forall dish \in Dishes$
$c_{dish} \in \mathbb{R}^+$	Carbs that each dish contains [g].	$\forall dish \in Dishes$
$f_{dish} \in \mathbb{R}^+$	Fat that each dish contains [g].	$\forall dish \in Dishes$
$ve_{dish} \in \{0,\,1\}$	Equals 1 if the dish is vegetarian and 0 otherwise.	$\forall dish \in Dishes$
$vg_{dish} \in \{0,\,1\}$	Equals 1 if the dish is vegan and 0 otherwise.	$\forall dish \in Dishes$
$cost \in \mathbb{R}^+$	Cost per serving of the dish [€].	$\forall dish \in Dishes$

Variables

$x_{dish,\,meal,\,day} \in \{0,\,1\}$	Equals 1 if a dish is used in a meal of a day and 0 otherwise.	$\forall dish \in Dishes$ $\forall meal \in Meals$ $\forall day \in Days$

Constraints

Each meal has between one and three dishes

$$1 \leq \sum_{dish \in Dishes} x_{dish,meal,day} \leq 3,\ \forall meal \in Meals,\ \forall day \in Days$$

Note: This constraint is implemented with two constraints.

Dishes must belong to their meal types

Dishes can only be selected for the meal type they are suitable for.

$$x_{dish,\,meal,\,day} \leq suitable_{dish,meal},\ \forall dish \in Dishes,\ \forall meal \in Meals,\ \forall day \in Days$$

Note: This constraint could be modified to consider that a certain dish can be included in different types of meal (e.g. avocado toast could be considered suitable for both breakfast and dinner).

Respect vegetarian diet

If the diet is vegetarian, all dishes must be vegetarian. If the diet is not vegetarian, dishes can be either vegetarian or not.

$$x_{dish,meal,day} \cdot \left(1 - ve_{dish}\right) \leq (1 - ve),\ \forall dish \in Dishes,\ \forall meal \in Meals,\ \forall day \in Days$$

- If a dish is not selected ($x_{dish,meal,day}$ equals 0) or the dish is vegetarian (ve_{dish} equals 1), the left side equals 0, so we do not force anything.
- However, if a dish is selected and not vegetarian, we force the right side to be 1, therefore, that the diet is not vegetarian (ve equals 0).
- Equivalently, if the diet is vegetarian (so the right side equals 0), we force to not select the dish or that it is vegetarian.

Note: We assume that all vegan dishes are also introduced as vegetarian. This means: $vg_{dish} \rightarrow ve_{dish},\ \forall dish \in Dishes$.

Respect vegan diet

Same as the previous constraint but for a vegan diet.

$$x_{dish,meal,day} \cdot \left(1 - vg_{dish}\right) \leq (1 - vg),\ \forall dish \in Dishes,\ \forall meal \in Meals,\ \forall day \in Days$$

Respect calories

Calorie count per day should be between the allowed bounds.

$$cal^{min} \leq \sum_{meal \in Meals} \sum_{dish \in Dishes} x_{dish,meal,day} \cdot cal_{dish} \leq cal^{max}, \forall day \in Days$$

Note: This constraint is implemented with two constraints.

Respect protein

Protein amount per day must be between the allowed bounds.

$$p^{min} \leq \sum_{meal \in Meals} \sum_{dish \in Dishes} x_{dish,meal,day} \cdot p_{dish} \leq p^{max}, \forall day \in Days$$

Note: This constraint is implemented with two constraints.

Respect carbs

Carbs amount per day must be between the allowed bounds.

$$c^{min} \leq \sum_{meal \in Meals} \sum_{dish \in Dishes} x_{dish,meal,day} \cdot c_{dish} \leq c^{max}, \forall day \in Days$$

Note: This constraint is implemented with two constraints.

Respect fat

Fat amount per day must be between the allowed bounds.

$$f^{min} \leq \sum_{meal \in Meals} \sum_{dish \in Dishes} x_{dish,meal,day} \cdot f_{dish} \leq f^{max}, \forall day \in Days$$

Note: This constraint is implemented with two constraints.

(Future) Dish not repeated more than X times in the whole diet

(Future) Dish not repeated more than X times in a week/last 2 days/…

Objective

Minimize the total cost of the diet.

$$\min \left(\sum_{day \in Days} \sum_{meal \in Meals} \sum_{dish \in Dishes} x_{dish,\, meal,day} \cdot cost_{dish} \right)$$