Museum guards problem

GitHub repo: https://github.com/pchs20/museum-guards-problem

Statement: https://www.lix.polytechnique.fr/~liberti/swarchex.pdf (section 5.1)

We understand a **museum** as a building with rooms that are connected with doors.

A museum director must decide how many guards should be employed to control a new wing. Budget cuts have forced him to station guards at each door, guarding two rooms at once. **We must have at least one guard controlling each room**.

The museum will be modeled as a **graph**, where each node represents a room and each door an edge between the rooms that interconnect. The objective is to find the minimum subset of edges (i.e. doors) from the graph that need to be guarded.

Sets

$r \in R$	Rooms of the museum.	
$\{r1, r2\} \in D$	Doors between two rooms of the museum.	$r1, r2 \in R$

So the museum will consist on a non-directed and non-weighted graph $M = (R, D)$.

Variables

$guard_{\{r1,r2\}} \in \{0, 1\}$	Equals 1 if a guard is placed at the door $\{r1, r2\}$. Equals 0 otherwise.	$\forall \{r1, r2\} \in D$

Constraints

At least one guard on each room

Force that each room is guarded for at least one guard. This means that at least in one of the doors of each room, a guard should be placed.

$$\sum_{s \in R : \{r, s\} \in D} (guard_{\{r, s\}}) \geq 1, \forall r \in R$$

Objective

Have the minimum number of doors guarded.

$$\min \sum_{\{r1, r2\} \in D} (guard_{\{r1, r2\}})$$