

Rooks problem

GitHub repo: <https://github.com/pchs20/rooks-problem>

Parameters

$n \in \mathbb{N}$	Number of columns and rows of the chessboard.
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Sets

$R = \{1, \dots, n\}$	Set of rows (or ranks) of the chessboard.
$C = \{1, \dots, n\}$	Set of columns (or files) of the chessboard.

Variables

$rook_{r,c} \in \{0, 1\}$	Equals 1 if a rook is placed at row r and column c . Equals 0 otherwise.	$\forall r \in R, \forall c \in C$
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Constraints

At most one rook per row

Limit the number of rooks per row to at most one.

$$\sum_{c \in C} (rook_{r,c}) \leq 1, \forall r \in R$$

At most one rook per column

Limit the number of rooks per column to at most one.

$$\sum_{r \in R} (rook_{r,c}) \leq 1, \forall c \in C$$

No rook on first position (additional)

Do not allow a rook on the first position of the chessboard.

Note that this is not necessary to solve the problem. It is an additional constraint to accomplish a non-trivial solution for the problem.

$$rook_{1,1} = 0$$

Objective

Have the maximum number of rooks on the board.

$$\max \sum_{r \in R} \sum_{c \in C} (rook_{r,c})$$