

# Exact and Heuristic Methods to Optimize Maintenances and Flight schedules of Military Aircraft

Franco Peschiera

Supervisors:

Alain Haït, Olga Battaïa, Nicolas Dupin.

November 19th, 2020



# Outline

1. Context and state of the art
2. Exact methods
3. Valid bounds and learned constraints
4. Graph-based VND matheuristic
5. General conclusions and perspectives

# Maintenance

- ▶ Maintenance is important.

# Maintenance

- ▶ Maintenance is important.
- ▶ Maintenance is costly.

## Maintenance

- ▶ Maintenance is important.
- ▶ Maintenance is costly.
- ▶ Complexity and durability needs are growing with time.

## Maintenance

- ▶ Maintenance is important.
- ▶ Maintenance is costly.
- ▶ Complexity and durability needs are growing with time.
  - ▶ buildings that need to last 100 years.

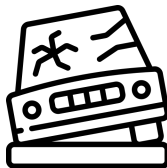
## Maintenance

- ▶ Maintenance is important.
- ▶ Maintenance is costly.
- ▶ Complexity and durability needs are growing with time.
  - ▶ buildings that need to last 100 years.
  - ▶ green energy needs lasting infrastructure.

## Maintenance planning

### Three types:

- ▶ Corrective.





## Maintenance planning

### Three types:

- ▶ Corrective.
- ▶ Preventive.



## Maintenance planning

### Three types:

- ▶ Corrective.
- ▶ Preventive.
- ▶ Predictive.



## Maintenance planning

### Three types:

- ▶ Corrective.
- ▶ **Preventive.**
- ▶ Predictive.

## Maintenance planning

### Three types:

- ▶ Corrective.
- ▶ **Preventive.**
- ▶ Predictive.

### Applications

Production, transportation.

## Maintenance planning applications

### Three elements:

- ▶ Tasks (planned over time).

## Maintenance planning applications

### Three elements:

- ▶ Tasks (planned over time).
- ▶ Resources (to do the tasks).

## Maintenance planning applications

### Three elements:

- ▶ Tasks (planned over time).
- ▶ Resources (to do the tasks).
- ▶ Recovery tasks (to care for resources).

## Maintenance planning applications

### Three elements:

- ▶ Tasks (planned over time).
- ▶ Resources (to do the tasks).
- ▶ Recovery tasks (to care for resources).

### In transportation:



## Maintenance planning applications

### Three elements:

- ▶ Tasks (planned over time).
- ▶ Resources (to do the tasks).
- ▶ Recovery tasks (to care for resources).

### In transportation:

- ▶ Tasks: flights, train schedules, routes.

## Maintenance planning applications

### Three elements:

- ▶ Tasks (planned over time).
- ▶ Resources (to do the tasks).
- ▶ Recovery tasks (to care for resources).

### In transportation:

- ▶ Tasks: flights, train schedules, routes.
- ▶ Resources: aircraft, trains, vehicles.

## Maintenance planning applications

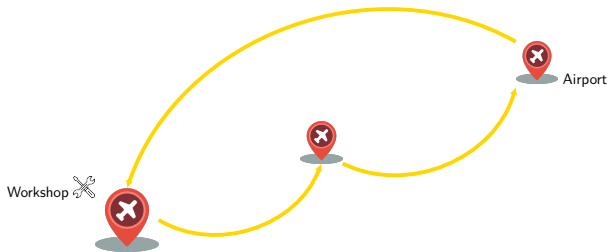
### Three elements:

- ▶ Tasks (planned over time).
- ▶ Resources (to do the tasks).
- ▶ Recovery tasks (to care for resources).

### In transportation:

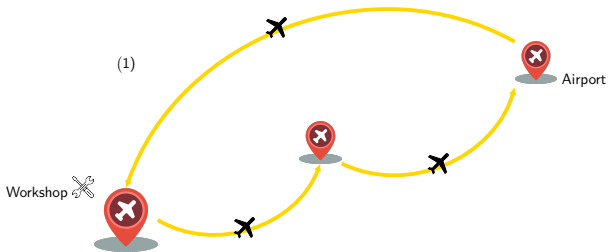
- ▶ Tasks: flights, train schedules, routes.
- ▶ Resources: aircraft, trains, vehicles.
- ▶ Recovery tasks: checks (=maintenances).

# The Flight and Maintenance Planning (FMP) problem



# The Flight and Maintenance Planning (FMP) problem

**Tail Assignment:**  
assign aircraft to flights

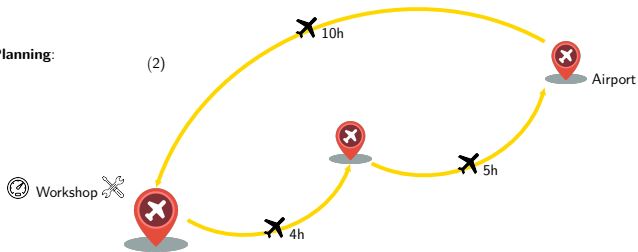


## State of the art : FMP

(1) C. Barnhart, N. L. Boland, L. W. Clarke, E. L. Johnson, G. L. Nemhauser, and R. G. Shenoi. Flight string models for aircraft fleetling and routing. *Transportation Science*, 32(3):208–220, aug 1998.

# The Flight and Maintenance Planning (FMP) problem

**Flight and Maintenance Planning:**  
assign aircraft to flights  
+ capacity in workshop  
+ hour-dependent checks



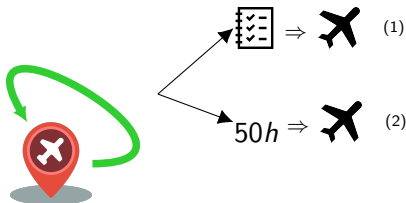
## State of the art : FMP

- (1) C. Barnhart, N. L. Boland, L. W. Clarke, E. L. Johnson, G. L. Nemhauser, and R. G. Shenoi. Flight string models for aircraft fleetling and routing. *Transportation Science*, 32(3):208–220, aug 1998.
- (2) A. Sarac, R. Batta, and C. M. Rump. A branch-and-price approach for operational aircraft maintenance routing. *European Journal of Operational Research*, 175(3):1850–1869, dec 2006.

# The Military Flight and Maintenance Planning (MFMP) problem



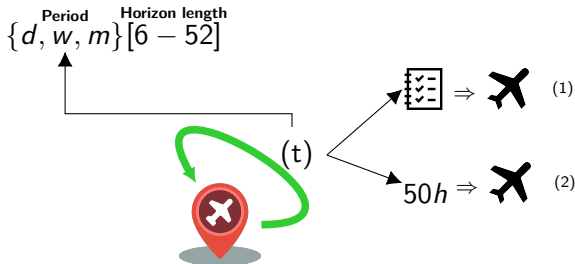
# The Military Flight and Maintenance Planning (MFMP) problem



- (1) R. A. Hahn and A. M. Newman. Scheduling United States Coast Guard helicopter deployment and maintenance at Clearwater Air Station, Florida. *Computers and Operations Research*, 35(6):1829–1843, jun 2008.
- (2) G. Kozanidis. A multiobjective model for maximizing fleet availability under the presence of flight and maintenance requirements. *Journal of Advanced Transportation*, 43(2):155–182, 2009.

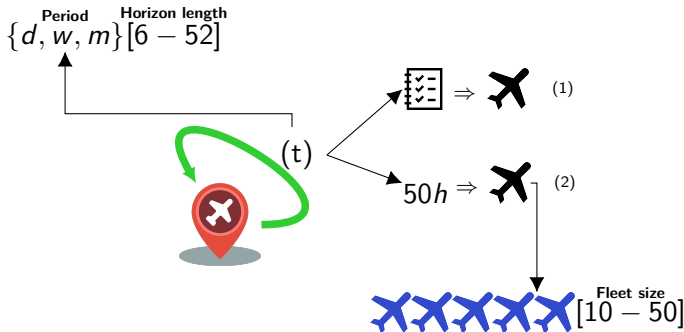


# The Military Flight and Maintenance Planning (MFMP) problem



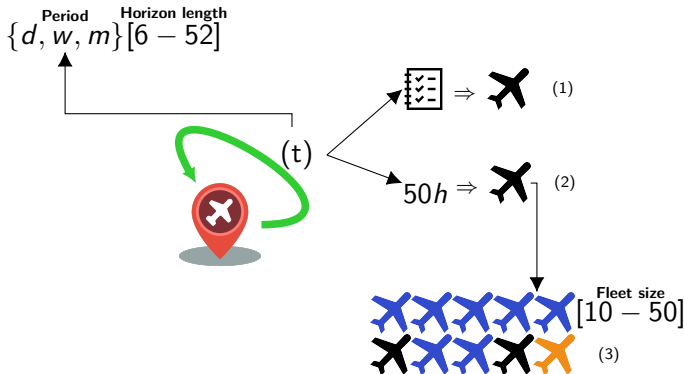
- (1) R. A. Hahn and A. M. Newman. Scheduling United States Coast Guard helicopter deployment and maintenance at Clearwater Air Station, Florida. *Computers and Operations Research*, 35(6):1829–1843, jun 2008.
- (2) G. Kozanidis. A multiobjective model for maximizing fleet availability under the presence of flight and maintenance requirements. *Journal of Advanced Transportation*, 43(2):155–182, 2009.

# The Military Flight and Maintenance Planning (MFMP) problem



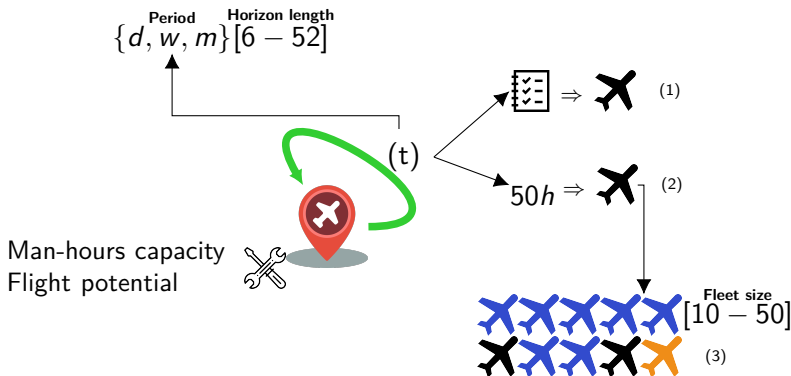
- (1) R. A. Hahn and A. M. Newman. Scheduling United States Coast Guard helicopter deployment and maintenance at Clearwater Air Station, Florida. *Computers and Operations Research*, 35(6):1829–1843, jun 2008.
- (2) G. Kozanidis. A multiobjective model for maximizing fleet availability under the presence of flight and maintenance requirements. *Journal of Advanced Transportation*, 43(2):155–182, 2009.

# The Military Flight and Maintenance Planning (MFMP) problem



- (1) R. A. Hahn and A. M. Newman. Scheduling United States Coast Guard helicopter deployment and maintenance at Clearwater Air Station, Florida. *Computers and Operations Research*, 35(6):1829–1843, jun 2008.
- (2) G. Kozanidis. A multiobjective model for maximizing fleet availability under the presence of flight and maintenance requirements. *Journal of Advanced Transportation*, 43(2):155–182, 2009.
- (3) J. Seif and A. J. Yu. An extensive operations and maintenance planning problem with an efficient solution method. *Computers and Operations Research*, 95:151–162, jul 2018.

# The Military Flight and Maintenance Planning (MFMP) problem



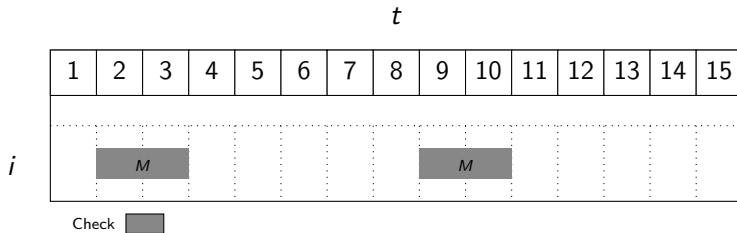
- (1) R. A. Hahn and A. M. Newman. Scheduling United States Coast Guard helicopter deployment and maintenance at Clearwater Air Station, Florida. *Computers and Operations Research*, 35(6):1829–1843, jun 2008.
- (2) G. Kozanidis. A multiobjective model for maximizing fleet availability under the presence of flight and maintenance requirements. *Journal of Advanced Transportation*, 43(2):155–182, 2009.
- (3) J. Seif and A. J. Yu. An extensive operations and maintenance planning problem with an efficient solution method. *Computers and Operations Research*, 95:151–162, jul 2018.

## Maintenance example

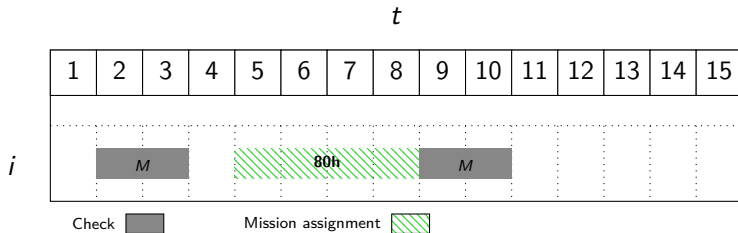
$t$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$i$															

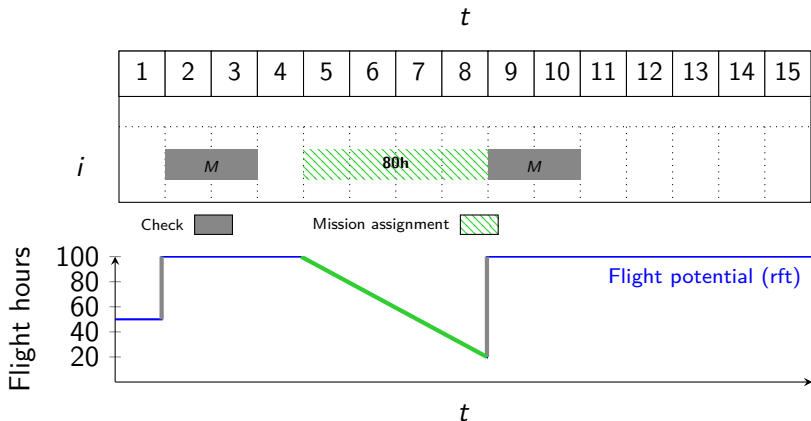
## Maintenance example



## Maintenance example

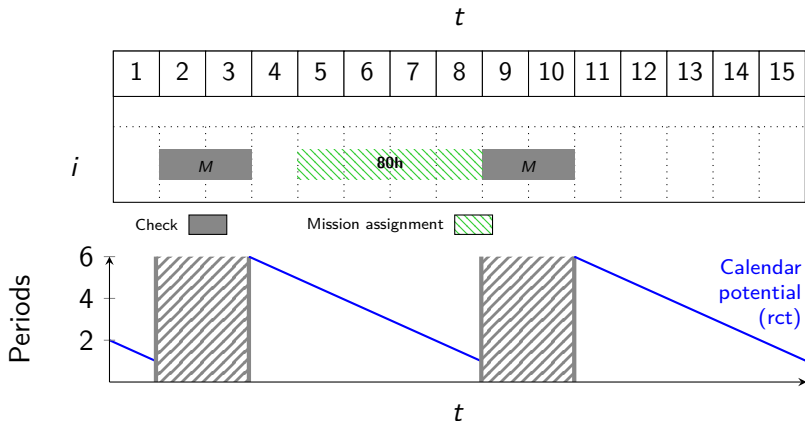


## Maintenance example





## Maintenance example

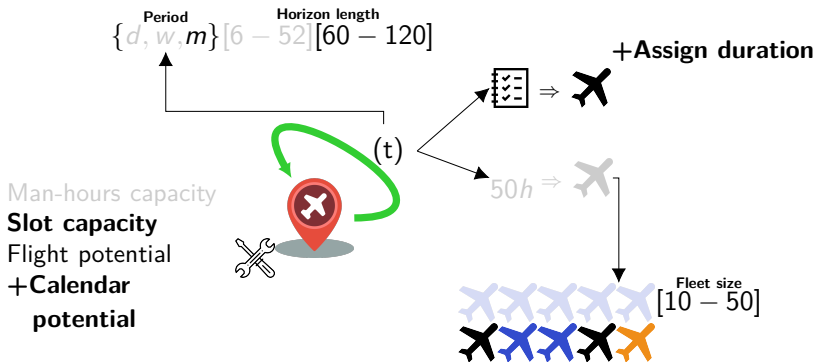


# The long term Military Flight and Maintenance Planning problem

The French Air Force variant of the MFMP for the Mirage 2000 series.

## The long term Military Flight and Maintenance Planning problem

The French Air Force variant of the MFMP for the Mirage 2000 series.



## Comparison of several MFMP problems

Table with most relevant characteristics of variants of the MFMP in the literature. C= Constraint; O=Objective.

Reference	Maintenance					Missions					Fleet	
	CP	FD	MS	RC	MT	DA	HD	HF	MD	HT	AV	SU
Kozanidis				C			C				O(c)	O(d)
Hahn et al.		C				C	C		O	C		C(a)
Winata		C				C	C		O			
Cho		C				C						C(a)
Verhoeff et al.				C			C			C	C(b)	O(d)
Li et al.		C				C						
Shah et al.		C					C				O(a)	C(a)
Gavranis et al.				C			C					O(e),O(f)
Marlow and Dell				C			C			O		O(b)
Seif and Yu			C	C	C		C	C				O(e)
This thesis	C	C				C		C	C		O(a),O(b)	O(c)

## Comparison of several MFMP problems

Table with most relevant characteristics of variants of the MFMP in the literature. C= Constraint; O=Objective.

Reference	Maintenance					Missions					Fleet	
	CP	FD	MS	RC	MT	DA	HD	HF	MD	HT	AV	SU
Kozanidis				C			C				O(c)	O(d)
Hahn et al.		C				C	C		O	C		C(a)
Winata		C				C	C		O			
Cho		C				C						C(a)
Verhoeff et al.				C			C			C	C(b)	O(d)
Li et al.		C				C						
Shah et al.		C					C				O(a)	C(a)
Gavranis et al.				C			C					O(e),O(f)
Marlow and Dell				C			C			O		O(b)
This thesis	C	C				C		C	C		O(a),O(b)	O(c)

## Objectives

- Formulate the French Air Force variant of the MFMP and study its complexity.

## Objectives

- ▶ Formulate the French Air Force variant of the MFMP and study its complexity.
- ▶ Build exact methods to solve the French Air Force MFMP problem.

## Objectives

- ▶ Formulate the French Air Force variant of the MFMP and study its complexity.
- ▶ Build exact methods to solve the French Air Force MFMP problem.
- ▶ Expand solution methods to cope with large scale real-life MFMP problems.



## Objectives

- ▶ Formulate the French Air Force variant of the MFMP and study its complexity.
- ▶ Build exact methods to solve the French Air Force MFMP problem.
- ▶ Expand solution methods to cope with large scale real-life MFMP problems.
- ▶ Deliver a decision making prototype for test in real-life data sets.

# Outline

1. Context and state of the art
2. **Exact methods**
3. Valid bounds and learned constraints
4. Graph-based VND matheuristic
5. General conclusions and perspectives

# The long term MFMP problem

## Basic problem

# The long term MFMP problem

## Basic problem

►  $j \in \mathcal{J}$  missions.

j	$MT_j^{min}$	$Start_j$	$End_j$	$H_j$	$R_j$
0	2	1	4	24	1
1	2	5	7	34	3
2	3	8	11	18	3
3	3	12	15	30	3
4	2	16	18	35	3
5	2	19	20	25	1

# The long term MFMP problem

## Basic problem

- ▶  $j \in \mathcal{J}$  missions.
- ▶  $i \in \mathcal{I}$  **aircraft**.

	$Rct_i^{Init}$	$Rft_i^{Init}$
i		
0	7	120
1	13	220
2	7	140
3	8	140
4	6	160

## The long term MFMP problem

### Basic problem

- ▶  $j \in \mathcal{J}$  missions.
- ▶  $i \in \mathcal{I}$  aircraft.
- ▶ **Maintenances.**

Frequency (in time, in flight hours).

Duration.

Capacity.

## The long term MFMP problem

### Basic problem

- ▶  $j \in \mathcal{J}$  missions.
- ▶  $i \in \mathcal{I}$  aircraft.
- ▶ Maintenances.

### More constraints

Fleet-status, mission-aircraft compatibility.

# The long term MFMP problem

## Basic problem

- ▶  $j \in \mathcal{J}$  missions.
- ▶  $i \in \mathcal{I}$  aircraft.
- ▶ Maintenances.

## More constraints

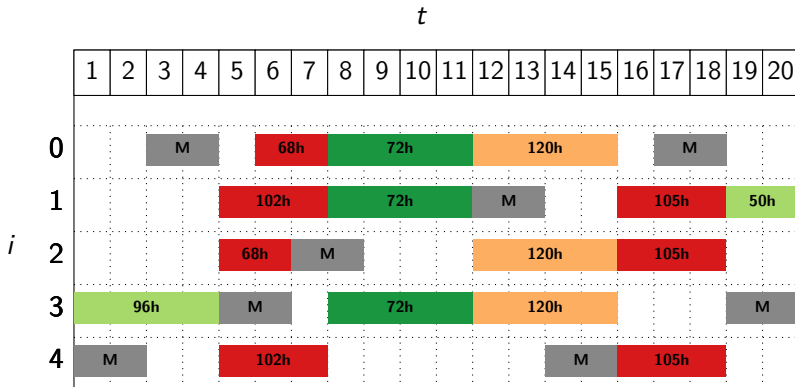
Fleet-status, mission-aircraft compatibility.

## Objectives

Maximize the availability, minimize the number of checks, minimize maintenance capacity.



## An example of an MFMP solution



## Complexity analysis

Take an instance  $I$  from the MFMP problem.

## Complexity analysis

Take an instance  $I$  from the MFMP problem.

- ▶ Take out maintenances needs.

## Complexity analysis

Take an instance  $I$  from the MFMP problem.

- ▶ Take out maintenances needs.
- ▶ No partial assignments of missions.

## Complexity analysis

Take an instance  $I$  from the MFMP problem.

- ▶ Take out maintenances needs.
- ▶ No partial assignments of missions.
- ▶ Only one aircraft needed per each mission.

## Complexity analysis

Take an instance  $I$  from the MFMP problem.

- ▶ Take out maintenances needs.
- ▶ No partial assignments of missions.
- ▶ Only one aircraft needed per each mission.
- ▶ Take out objective function: feasibility.

## Complexity analysis

Take an instance  $I$  from the MFMP problem.

- ▶ Take out maintenances needs.
- ▶ No partial assignments of missions.
- ▶ Only one aircraft needed per each mission.
- ▶ Take out objective function: feasibility.

Becomes the NP-Complete Shift Satisfaction Personnel Task Scheduling Problem  $\star$ .

$\star$  E. M. Arkin and E. B. Silverberg. Scheduling jobs with fixed start and end times. Discrete Applied Mathematics, 1987.

## Solution approaches

**Mixed Integer Programming model**

**Simulated Annealing: initial solution**



## Solution approaches

**Mixed Integer Programming model**

Simulated Annealing: initial solution

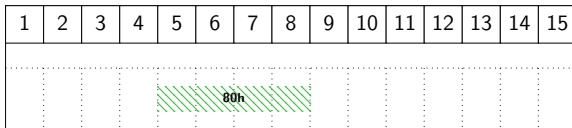




# Mixed Integer Programming model

## Binary variables

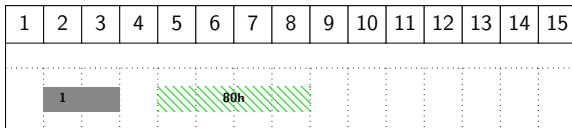
- $a_{ijt}^s$  : assignment of mission  $j$  to aircraft  $i$  starts in period  $t$ .  
 $a_{ijt}$  : mission  $j$  assigned in period  $t$  to aircraft  $i$ .



# Mixed Integer Programming model

## Binary variables

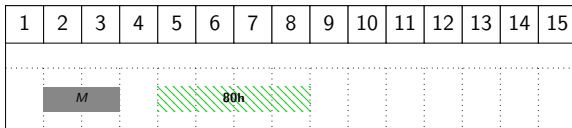
- $a_{ijt}^s$  : assignment of mission  $j$  to aircraft  $i$  starts in period  $t$ .  
 $a_{ijt}$  : mission  $j$  assigned in period  $t$  to aircraft  $i$ .  
 $m_{it}$  : aircraft  $i$  starts a check in period  $t$ .



# Mixed Integer Programming model

## Binary variables

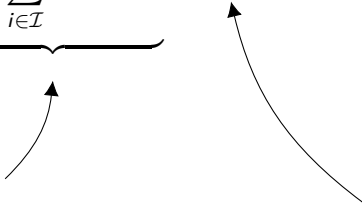
- $a_{ijt}^s$  : assignment of mission  $j$  to aircraft  $i$  starts in period  $t$ .  
 $a_{ijt}$  : mission  $j$  assigned in period  $t$  to aircraft  $i$ .  
 $m_{it}$  : aircraft  $i$  starts a check in period  $t$ .



# Mixed Integer Programming model

## Binary variables

- $a_{ijt}^s$  : assignment of mission  $j$  to aircraft  $i$  starts in period  $t$ .  
 $a_{ijt}$  : mission  $j$  assigned in period  $t$  to aircraft  $i$ .  
 $m_{it}$  : aircraft  $i$  starts a check in period  $t$ .

$$\underbrace{\sum_{t' \in \mathcal{T}_t^s} \sum_{i \in \mathcal{I}} m_{it'}}_{\text{Number of aircraft in maintenance in period } t} + N_t \leq C^{max} \quad t \in \mathcal{T}$$


Number of aircraft  
in maintenance in  
period  $t$

Maintenance capacity

# Mixed Integer Programming model

## Binary variables

- $a_{ijt}^s$  : assignment of mission  $j$  to aircraft  $i$  starts in period  $t$ .  
 $a_{ijt}$  : mission  $j$  assigned in period  $t$  to aircraft  $i$ .  
 $m_{it}$  : aircraft  $i$  starts a check in period  $t$ .

$$\sum_{t' \in \mathcal{T}_t^s} \sum_{i \in \mathcal{I}} m_{it'} + N_t \leq C^{\max} \quad t \in \mathcal{T}$$

$$\underbrace{\sum_{i \in \mathcal{I}_j} a_{ijt}}_{\text{Number of aircraft assigned to mission } j \text{ in period } t} \geq R_j \quad j \in \mathcal{J}, t \in \mathcal{T}_j$$

Number of aircraft  
assigned to mission  $j$   
in period  $t$

Mission needs of mis-  
sion  $j$



# Mixed Integer Programming model

## Binary variables

- $a_{ijt}^s$  : assignment of mission  $j$  to aircraft  $i$  starts in period  $t$ .  
 $a_{ijt}$  : mission  $j$  assigned in period  $t$  to aircraft  $i$ .  
 $m_{it}$  : aircraft  $i$  starts a check in period  $t$ .

$$\sum_{t' \in \mathcal{T}_t^s} \sum_{i \in \mathcal{I}} m_{it'} + N_t \leq C^{\max} \quad t \in \mathcal{T}$$

$$\sum_{i \in \mathcal{I}_j} a_{ijt} \geq R_j \quad j \in \mathcal{J}, t \in \mathcal{T}_j$$

$$\underbrace{\sum_{t' \in \mathcal{T}_t^s} m_{it'} + \sum_{j \in \mathcal{J}_t \cap \mathcal{O}_i} a_{ijt}}_{\text{Assignments of aircraft } i \text{ in period } t} \leq 1 \quad t \in \mathcal{T}, i \in \mathcal{I}$$

Assignments of aircraft  $i$  in period  $t$

## Mixed Integer Programming model: Flight hours rule

### Continuous variables

$u_{it}$  : flight hours by aircraft  $i$  during period  $t$ .

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0h	0h	0h	0h	20h	20h	20h	20h	0h	0h	0h	0h	0h	0h	0h

# Mixed Integer Programming model: Flight hours rule

## Continuous variables

$u_{it}$  : flight hours by aircraft  $i$  during period  $t$ .

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0h	0h	0h	0h	20h	20h	20h	20h	0h	0h	0h	0h	0h	0h	0h

$$u_{it} \geq \sum_{j \in \mathcal{J}_t \cap \mathcal{O}_i} a_{ijt} H_j \quad t = 1, \dots, \mathcal{T}, i \in \mathcal{I}$$

$$u_{it} \geq U^{\min} (1 - \sum_{t' \in \mathcal{T}_t^s} m_{it'}) \quad t = 1, \dots, \mathcal{T}, i \in \mathcal{I}$$

$$u_{it} \in [0, \max_j \{H_j\}] \quad t = 1, \dots, \mathcal{T}, i \in \mathcal{I}$$

## Mixed Integer Programming model: Flight hours rule

### Continuous variables

$u_{it}$  : flight hours by aircraft  $i$  during period  $t$ .

$rft_{it}$  : remaining flight hours for aircraft  $i$  during period  $t$ .

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
50h	100h	100h	100h	80h	60h	40h	20h	20h	20h	20h	20h	20h	20h	20h

## Mixed Integer Programming model: Flight hours rule

### Continuous variables

$u_{it}$  : flight hours by aircraft  $i$  during period  $t$ .

$rft_{it}$  : remaining flight hours for aircraft  $i$  during period  $t$ .

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
50h	100h	100h	100h	80h	60h	40h	20h	20h	20h	20h	20h	20h	20h	20h

$$rft_{i0} = Rft_i^{Init} \quad i \in \mathcal{I}$$

$$rft_{it} \leq rft_{i(t-1)} + H^M m_{it} - u_{it} \quad t = 1, \dots, \mathcal{T}, i \in \mathcal{I}$$

$$rft_{it} \geq H^M m_{it'} \quad t \in \mathcal{T}, t' \in \mathcal{T}_t^s, i \in \mathcal{I}$$

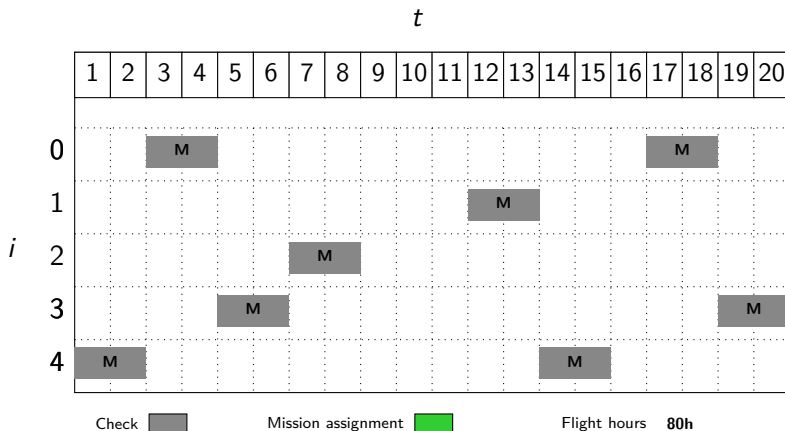
$$rft_{it} \in [0, H^M] \quad t \in \mathcal{T}, i \in \mathcal{I}$$

## Solution approaches

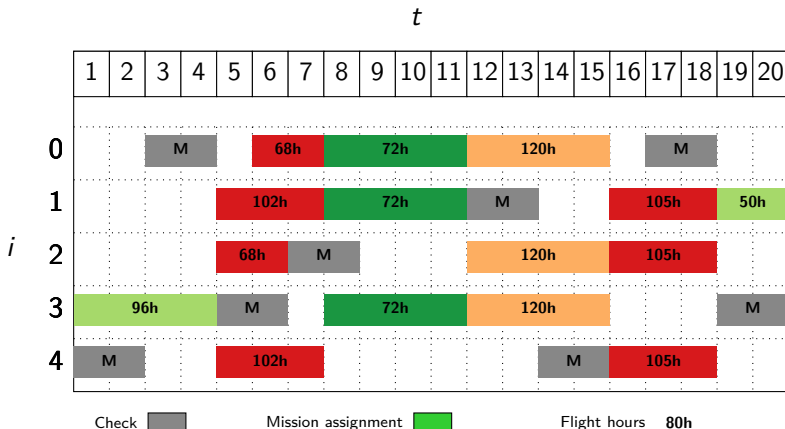
Mixed Integer Programming model

**Simulated Annealing: initial solution**

## Simulated Annealing: initial solution

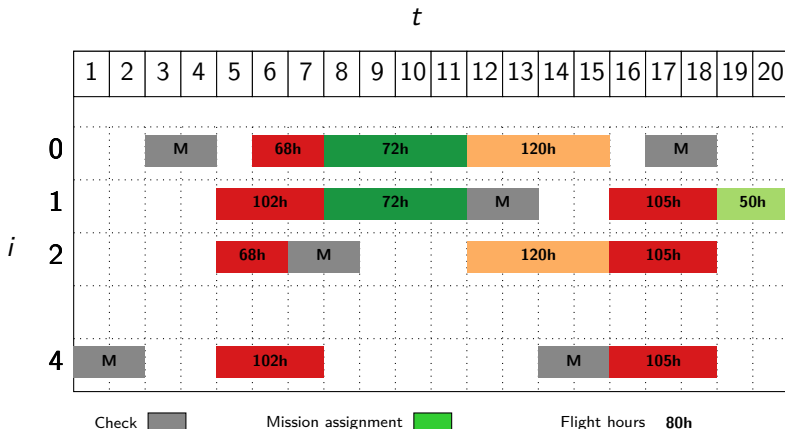


## Simulated Annealing: initial solution

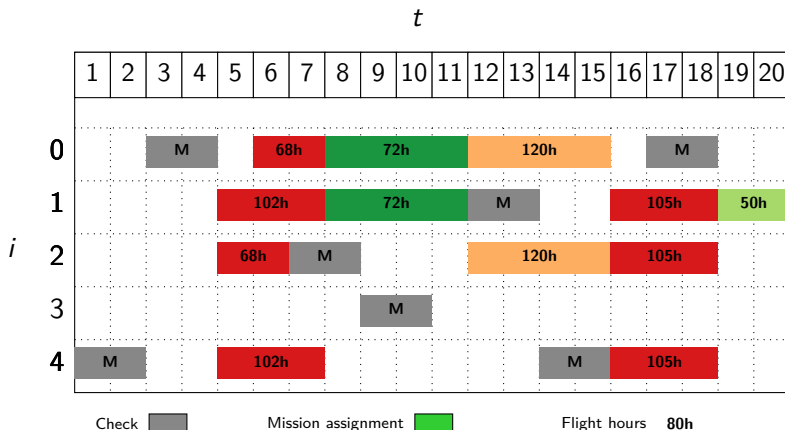




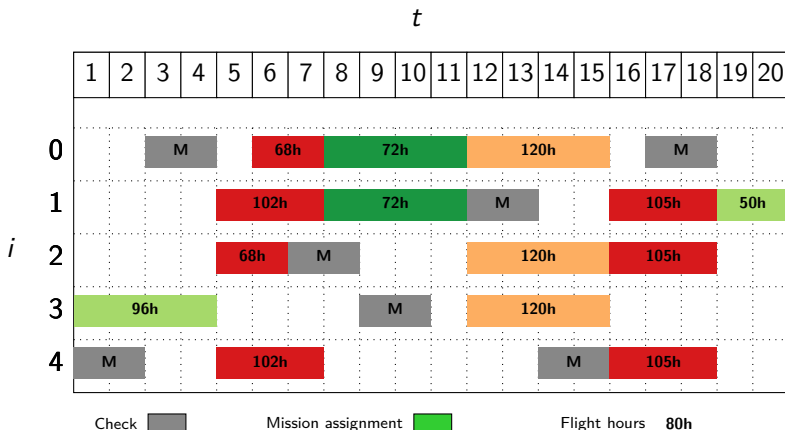
## Simulated Annealing: initial solution



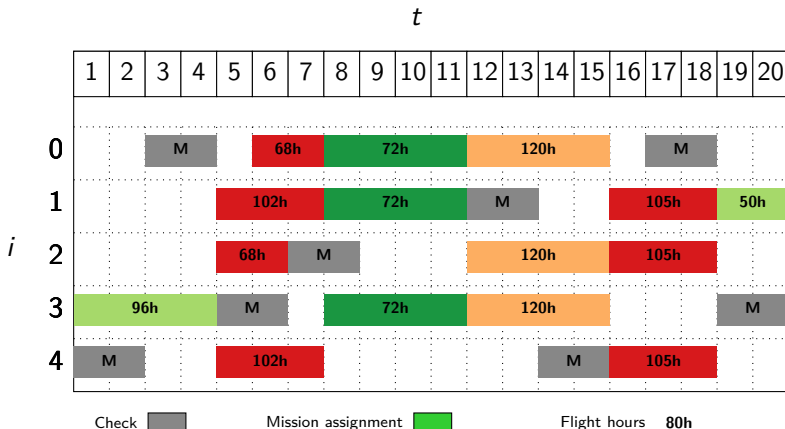
# Simulated Annealing: initial solution



## Simulated Annealing: initial solution



## Simulated Annealing: initial solution



## Experiments

**Instance simulator.**

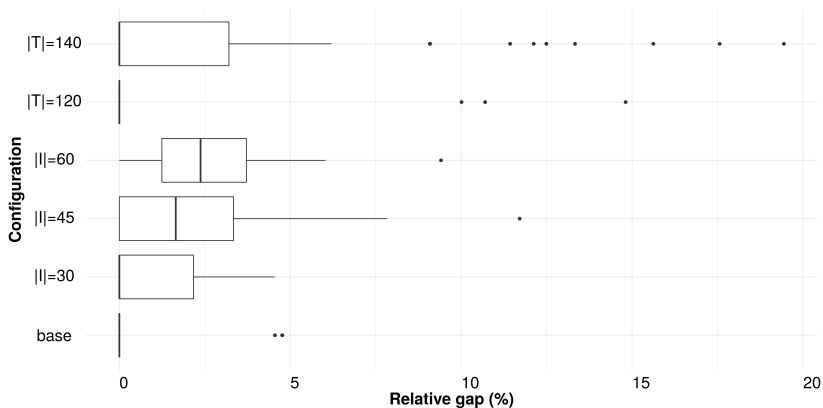
## Experiments

### Instance simulator.

- ▶ Several parameters and configurations.
  - ▶ Fleet sizes ( $|\mathcal{I}|$ ): 15 (base), 30, 45, 60.
  - ▶ Planning horizons ( $|\mathcal{T}|$ ): 90 (base), 120, 140.
- ▶ Number of instances per dataset: 50.
- ▶ Resolution: CPLEX with a time limit of 1 hour.

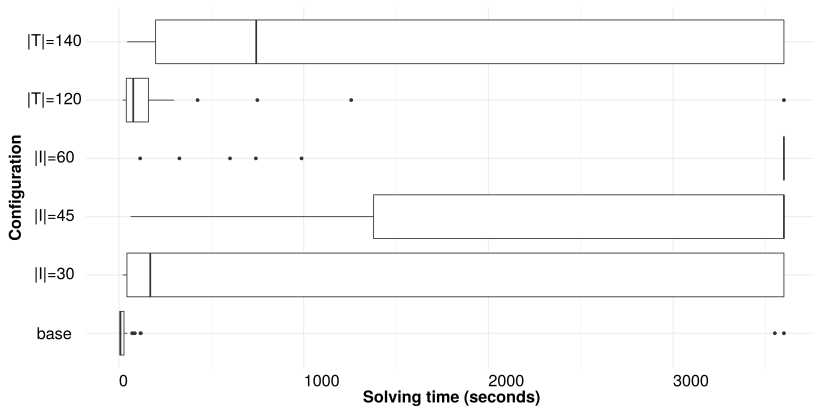
## Results

- Small gaps overall.



## Results

- ▶ Small gaps overall.
- ▶ Solution times highly sensible on the size of the instance.





## Preliminary conclusions

- ▶ **The long term MFMP problem is presented** along with a complexity analysis and a configurable instance generator.

## Preliminary conclusions

- ▶ **The long term MFMP problem is presented** along with a complexity analysis and a configurable instance generator.
- ▶ **Two solving techniques are tested:** a MIP model and a Simulated Annealing approach.

## Preliminary conclusions

- ▶ **The long term MFMP problem is presented** along with a complexity analysis and a configurable instance generator.
- ▶ **Two solving techniques are tested:** a MIP model and a Simulated Annealing approach.
- ▶ **Improvements are needed** to reduce the model's sensibility to the size of instances.

## Preliminary conclusions

- ▶ **The long term MFMP problem is presented** along with a complexity analysis and a configurable instance generator.
- ▶ **Two solving techniques are tested:** a MIP model and a Simulated Annealing approach.
- ▶ **Improvements are needed** to reduce the model's sensibility to the size of instances.

**Submission:** Franco Peschiera, Olga Battaïa, Alain Haït, Nicolas Dupin. Long term planning of military aircraft flight and maintenance operations. Annals of Operations Research.

# Outline

1. Context and state of the art
2. Exact methods
3. **Valid bounds and learned constraints**
4. Graph-based VND matheuristic
5. General conclusions and perspectives

## Contents

Predicting learned-cuts

Applying learned-cuts to the MFMP

# Contents

## Predicting learned-cuts

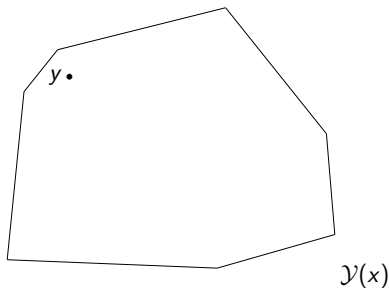
## Applying learned-cuts to the MFMP

## Predicting learned-cuts



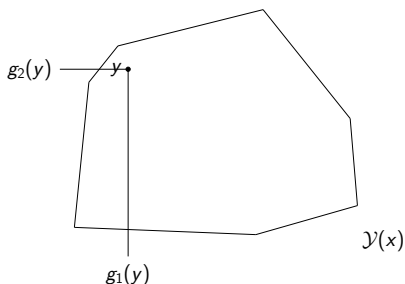
## Predicting learned-cuts

**Optimization problem**  $y^*(x) \equiv \arg \min_{y \in \mathcal{Y}(x)} C(x, y)$



## Predicting learned-cuts

**Optimization problem**  $y^*(x) \equiv \arg \min_{y \in \mathcal{Y}(x)} C(x, y)$   
**Solution features**  $G(y) = \{g_n(y) \mid \forall n \in \mathcal{N}\}$

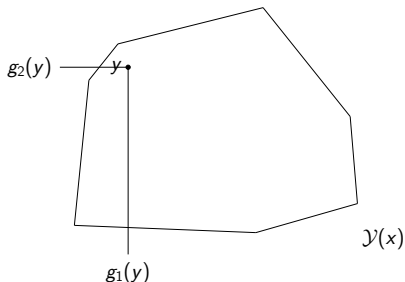


## Predicting learned-cuts

**Optimization problem**  $y^*(x) \equiv \arg \min_{y \in \mathcal{Y}(x)} C(x, y)$

**Solution features**  $G(y) = \{g_n(y) \mid \forall n \in \mathcal{N}\}$

**Input features**  $H(x) = \{h_m(x) \mid \forall m \in \mathcal{M}\}$



## Predicting learned-cuts

**Optimization problem**

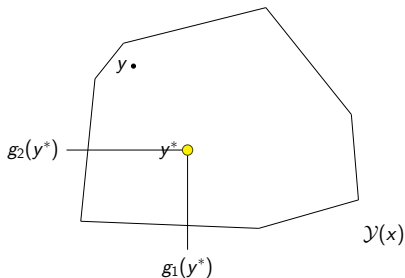
$$y^*(x) \equiv \arg \min_{y \in \mathcal{Y}(x)} C(x, y)$$

**Solution features**

$$G(y) = \{g_n(y) \mid \forall n \in \mathcal{N}\}$$

**Input features**

$$H(x) = \{h_m(x) \mid \forall m \in \mathcal{M}\}$$



## Predicting learned-cuts

**Optimization problem**

$$y^*(x) \equiv \arg \min_{y \in \mathcal{Y}(x)} C(x, y)$$

**Solution features**

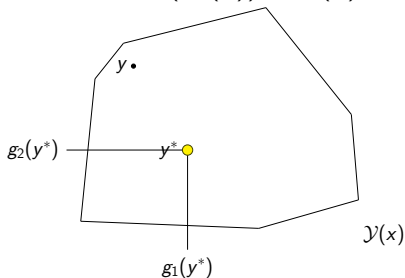
$$G(y) = \{g_n(y) \mid \forall n \in \mathcal{N}\}$$

**Input features**

$$H(x) = \{h_m(x) \mid \forall m \in \mathcal{M}\}$$

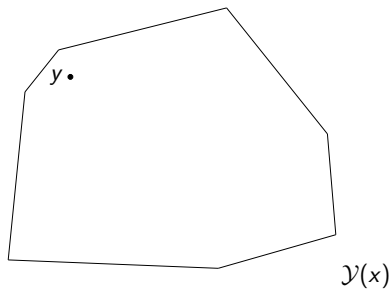
**Training for optimal**

$$G(y^*(x)) \approx \hat{G}(x) = f(H(x)) \star$$



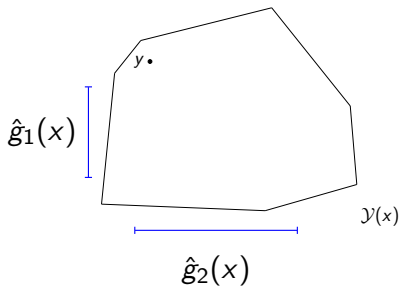
★ E. Larsen, S. Lachapelle, Y. Bengio, E. Frejinger, S. Lacoste-Julien, and A. Lodi. Predicting Tactical Solutions to Operational Planning Problems under Imperfect Information. arXiv, 2018.

## Applying learned-cuts



## Applying learned-cuts

We predict the optimal features  $\hat{G}(x) = \hat{g}_n(x) \forall n \in \mathcal{N}$



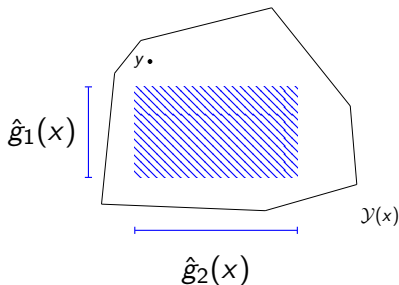
## Applying learned-cuts

**We predict the optimal features**

$$\hat{G}(x) = \hat{g}_n(x) \quad \forall n \in \mathcal{N}$$

**We predict the optimal “zone”**

$$\mathcal{Y}'(x) = \{y \in \mathcal{Y} \mid \hat{G}(x) = G(y)\}$$





## Applying learned-cuts

**We predict the optimal features**

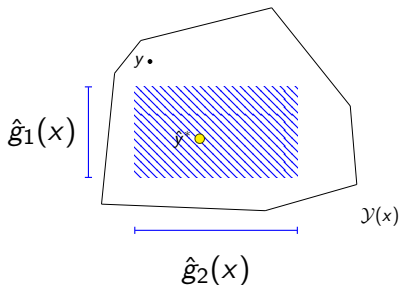
**We predict the optimal “zone”**

**We solve the predicted model**

$$\hat{G}(x) = \hat{g}_n(x) \quad \forall n \in \mathcal{N}$$

$$\mathcal{Y}'(x) = \{y \in \mathcal{Y} \mid \hat{G}(x) = G(y)\}$$

$$\hat{y}^*(x) \equiv \arg \min_{y \in \mathcal{Y}'(x)} C(x, y)$$



## Applying learned-cuts

**We predict the optimal features**

**We predict the optimal “zone”**

**We solve the predicted model**

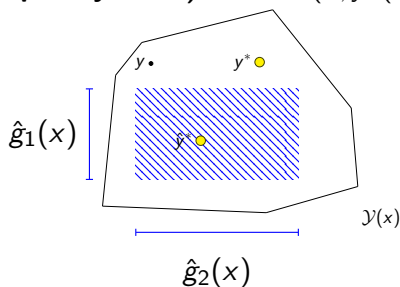
**With some (hopefully small) loss**

$$\hat{G}(x) = \hat{g}_n(x) \quad \forall n \in \mathcal{N}$$

$$\mathcal{Y}'(x) = \{y \in \mathcal{Y} \mid \hat{G}(x) = G(y)\}$$

$$\hat{y}^*(x) \equiv \arg \min_{y \in \mathcal{Y}'(x)} C(x, y)$$

$$C(x, \hat{y}^*(x)) \approx C(x, y^*(x))$$



Context and SoA  
oooooooo

Exact methods  
oooooooooooo

ML-based cuts  
ooo●oooooooooooo

Graph-based VND  
oooooooooooooooo

Conclusions  
ooooo

# Motivation

## Motivation

1. **Performance:** a smaller model is easier to solve.

## Motivation

1. **Performance:** a smaller model is easier to solve.
2. **User feedback:** direct feedback about the solution without needing to solve any model.

## Contents

Predicting learned-cuts

**Applying learned-cuts to the MFMP**

## New formulation

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$i$															

## New formulation

$a'_{ijtt'}$  : aircraft  $i$  is in mission  $j$  between  $t$  and  $t'$ .

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$i$															

$$a'_{ij58} = 1$$

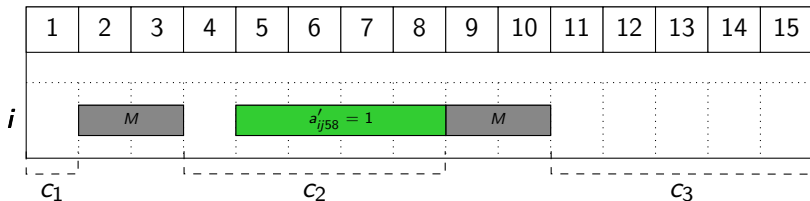




## New formulation

$a'_{ijtt'}$  : aircraft  $i$  is in mission  $j$  between  $t$  and  $t'$ .

$m'_{ip}$  : aircraft  $i$  uses check pattern  $p$ .



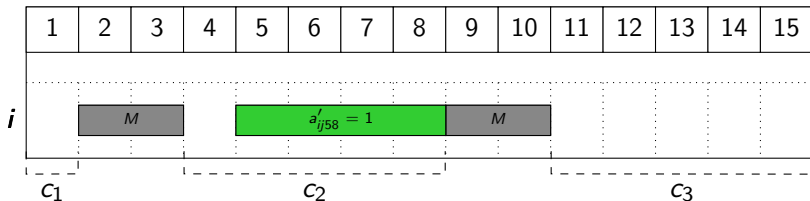
## New formulation

$a'_{ijtt'}$  : aircraft  $i$  is in mission  $j$  between  $t$  and  $t'$ .

$m'_{ip}$  : aircraft  $i$  uses check pattern  $p$ .

$$\sum_{(j,t,t') \in \mathcal{JTT}_{ic}} a'_{ijtt'} H'_{jtt'} + U'_{tc}$$

$$i \in \mathcal{I}, p \in \mathcal{P}, c \in \mathcal{C}_p$$



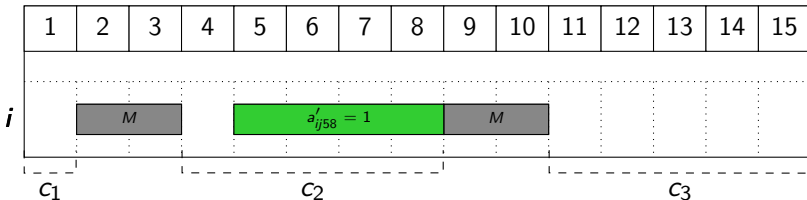
## New formulation

$a'_{ijtt'}$  : aircraft  $i$  is in mission  $j$  between  $t$  and  $t'$ .

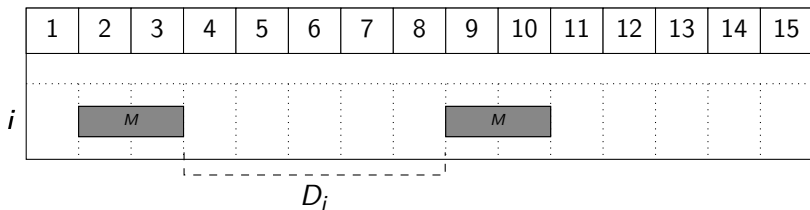
$m'_{ip}$  : aircraft  $i$  uses check pattern  $p$ .

$$\sum_{(j,t,t') \in \mathcal{JTT}_{ic}} a'_{ijtt'} H'_{jtt'} + U'_{tc} \leq H^M + \text{big}M(1 - m'_{ip})$$

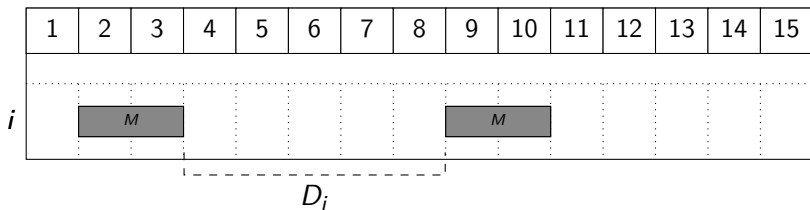
$$i \in \mathcal{I}, p \in \mathcal{P}, c \in \mathcal{C}_p$$



## Solution features: $G(y)$

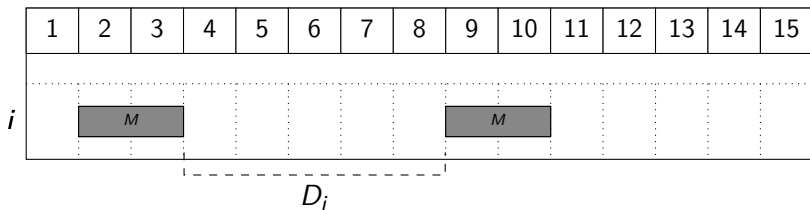


## Solution features: $G(y)$



For each aircraft  $i$ :  $D_i(y) \in [E^{min}, E^{max}] \forall y \in \mathcal{Y}(x)$ .

## Solution features: $G(y)$



For each aircraft  $i$ :  $D_i(y) \in [E^{min}, E^{max}] \forall y \in \mathcal{Y}(x)$ .

## Average distance between maintenances

$$g_1(y) = \mu_D = \frac{\sum_{i \in \mathcal{I}} D_i(y)}{I}.$$

## Input features: $H(x)$

### Mission related

Max, average, variance of flight hours per period. Median period.



## Input features: $H(x)$

### Mission related

Max, average, variance of flight hours per period. Median period.

### Fleet related

Sum of initial flight potential.

## Input features: $H(x)$

### Mission related

Max, average, variance of flight hours per period. Median period.

### Fleet related

Sum of initial flight potential.

### Compatibility related

Sum of all special mission flight hours.

## Forecasting technique

### Quantile regressions

Upper bound and lower bound at 10% and 90%.

$$\mu_D \rightarrow [\hat{\mu}_D^{lb}, \hat{\mu}_D^{ub}]$$

## Forecasting technique

### Quantile regressions

Upper bound and lower bound at 10% and 90%.

$$\mu_D \rightarrow [\hat{\mu}_D^{lb}, \hat{\mu}_D^{ub}]$$

### Training / test set

of 5000 small instances solved to optimal and divided into 70/30.

## Applying learned-cuts

### Pattern filtering:

$$D_{ip} \in [\hat{\mu}_D^{lb} - tol, \hat{\mu}_D^{ub} + tol] \rightarrow p \in \mathcal{P}_i$$

## Applying learned-cuts

### Pattern filtering:

$$D_{ip} \in [\hat{\mu}_D^{lb} - tol, \hat{\mu}_D^{ub} + tol] \rightarrow p \in \mathcal{P}_i$$

### Pattern recycling: with probability $\alpha$

$$D_{ip} \notin [\hat{\mu}_D^{lb} - tol, \hat{\mu}_D^{ub} + tol] \wedge P(\alpha) \rightarrow p \in \mathcal{P}_i$$

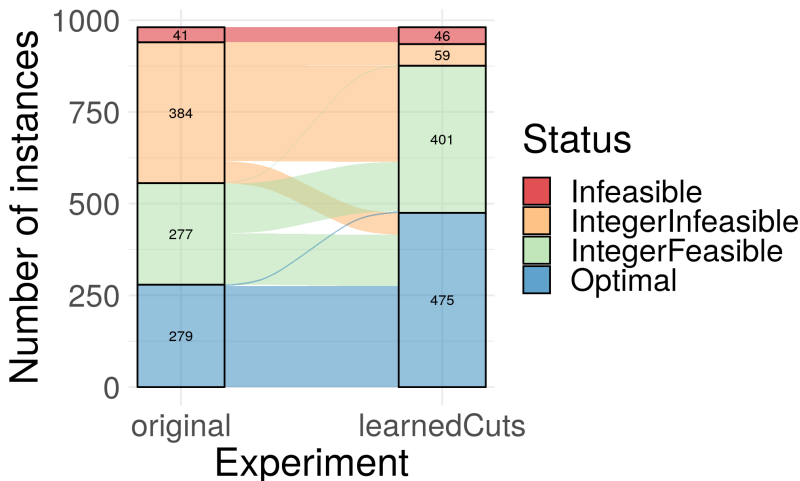
## Experiments

- ▶ Number of instances: medium (1000), large (1000) and very large (1000).
- ▶ We seeded instance generation for better comparison.
- ▶ CPLEX running 1 thread and limited to 1 hour.

Largest instances have 60 aircraft, 90 periods.

## Results: performance

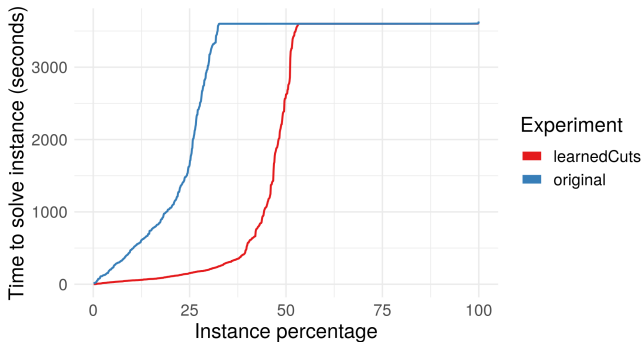
- More solutions.





## Results: performance

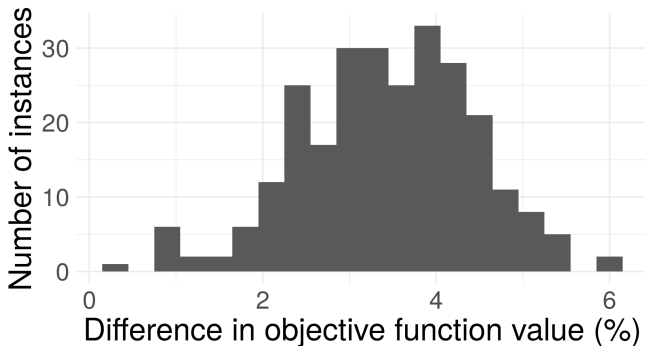
- ▶ More solutions.
- ▶ Faster solutions.



## Results: optimality

We compare instances where the two models returned “optimal”.

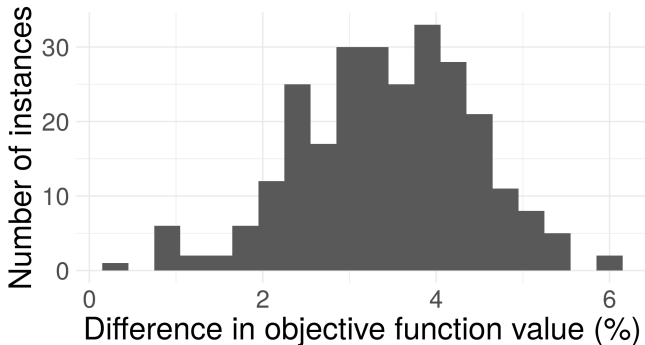
- ▶ A 82.1% average reduction in solution time for these instances.



## Results: optimality

We compare instances where the two models returned “optimal”.

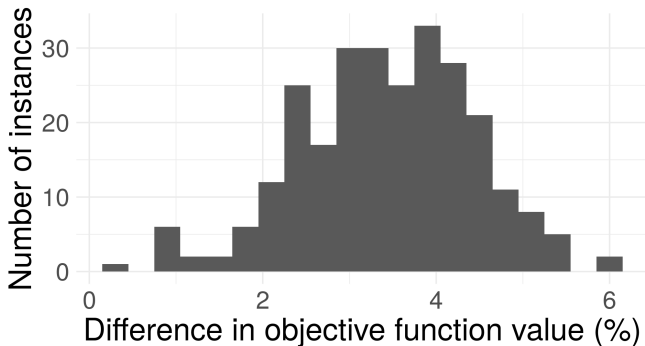
- ▶ A 82.1% average reduction in solution time for these instances.
- ▶ Less than 7% loss of optimality for  $\geq 95\%$  of these instances. Most below 4%.



## Results: optimality

We compare instances where the two models returned “optimal”.

- ▶ A 82.1% average reduction in solution time for these instances.
- ▶ Less than 7% loss of optimality for  $\geq 95\%$  of these instances. Most below 4%.
- ▶ Better predictions can reduce this loss even further.



Context and SoA  
oooooooo

Exact methods  
oooooooooooo

ML-based cuts  
oooooooooooooooo●

Graph-based VND  
oooooooooooooooo

Conclusions  
ooooo

## Preliminary conclusions

## Preliminary conclusions

- ▶ **A new “check pattern” model for the MFMP** provides performance advantages over previous models.

## Preliminary conclusions

- ▶ **A new “check pattern” model for the MFMP** provides performance advantages over previous models.
- ▶ **A supervised learning for optimization scheme** provides good heuristic solutions.

## Preliminary conclusions

- ▶ **A new “check pattern” model for the MFMP** provides performance advantages over previous models.
- ▶ **A supervised learning for optimization scheme** provides good heuristic solutions.
- ▶ **Its application on the MFMP problem** shows the benefits of such an approach.



## Preliminary conclusions

- ▶ **A new “check pattern” model for the MFMP** provides performance advantages over previous models.
- ▶ **A supervised learning for optimization scheme** provides good heuristic solutions.
- ▶ **Its application on the MFMP problem** shows the benefits of such an approach.
- ▶ **The performance gains are more visible** when the test instance size is close to the trained instance size.

## Preliminary conclusions

- ▶ **A new “check pattern” model for the MFMP** provides performance advantages over previous models.
- ▶ **A supervised learning for optimization scheme** provides good heuristic solutions.
- ▶ **Its application on the MFMP problem** shows the benefits of such an approach.
- ▶ **The performance gains are more visible** when the test instance size is close to the trained instance size.

## Preliminary conclusions

- ▶ **A new “check pattern” model for the MFMP** provides performance advantages over previous models.
- ▶ **A supervised learning for optimization scheme** provides good heuristic solutions.
- ▶ **Its application on the MFMP problem** shows the benefits of such an approach.
- ▶ **The performance gains are more visible** when the test instance size is close to the trained instance size.

**Publication:** Peschiera, F., Dell, R., Royset, J. et al. A novel solution approach with ML-based pseudo-cuts for the Flight and Maintenance Planning problem. OR Spectrum, 2020.

# Outline

1. Context and state of the art
2. Exact methods
3. Valid bounds and learned constraints
4. **Graph-based VND matheuristic**
5. General conclusions and perspectives

## Contents

**Graph representation of a solution space**

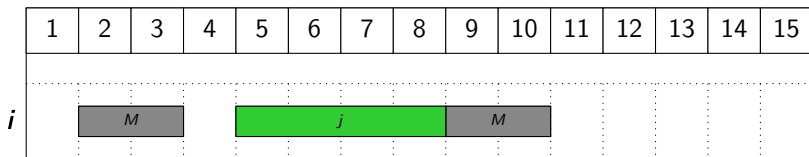
**Applying said graphs within a Variable Neighborhood  
Descent**

# Contents

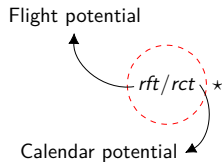
## Graph representation of a solution space

## Applying said graphs within a Variable Neighborhood Descent

# Patterns



## Pattern representation

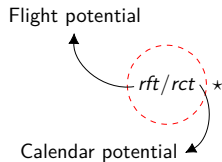


	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$i$															

\* Q. Deng, B. F. Santos, and R. Curran. A practical dynamic programming based methodology for aircraft maintenance check scheduling optimization. European Journal of Operational Research, 2020.



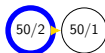
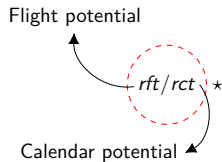
## Pattern representation



50/2

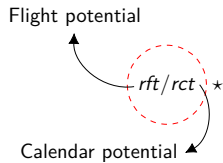
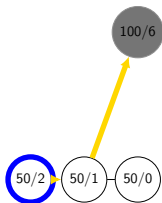
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$i$															

## Pattern representation



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$i$															

## Pattern representation

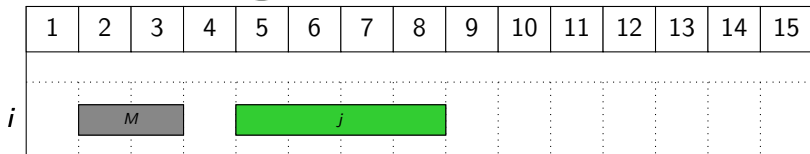
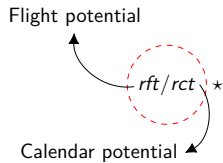
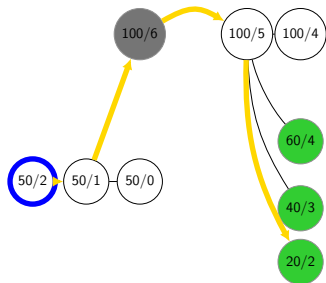


	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$i$															

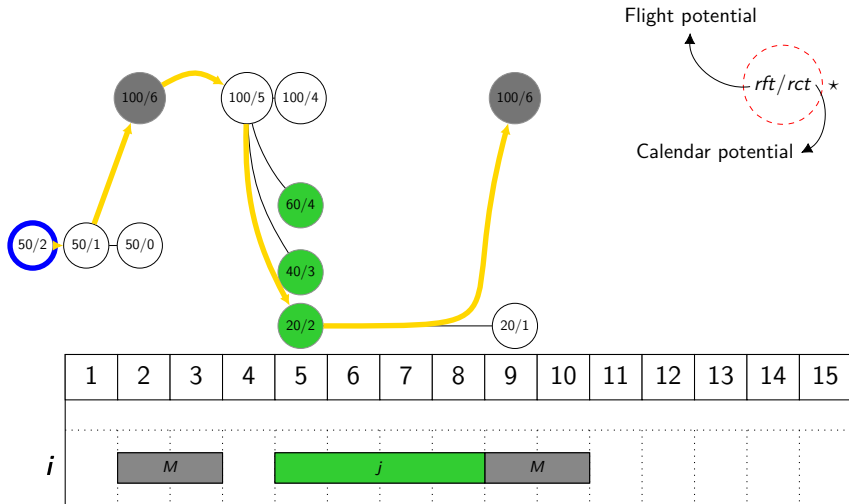
A diagram showing a grid structure. The top row contains indices 1 through 15. Below it is a grid with two rows. The first row of the grid is labeled  $i$  on the left. The second row of the grid contains a shaded gray rectangle labeled  $M$  spanning columns 2 and 3. Dotted lines indicate the grid structure.



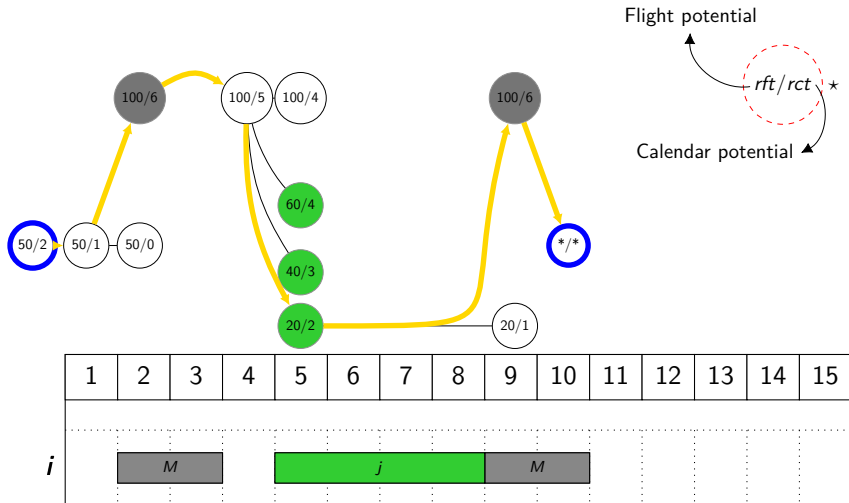
## Pattern representation



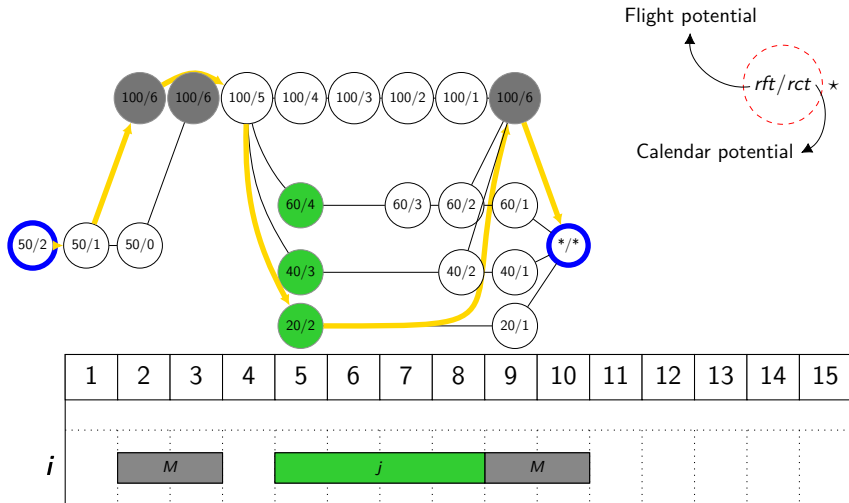
## Pattern representation



## Pattern representation



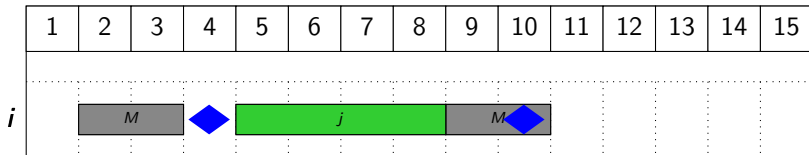
## Pattern representation



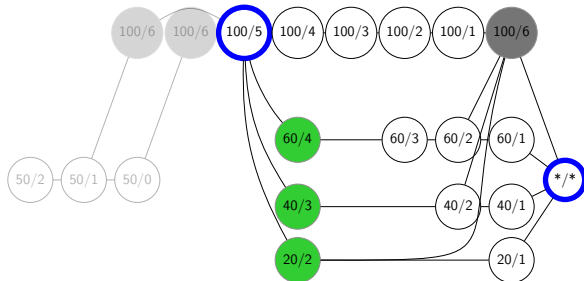
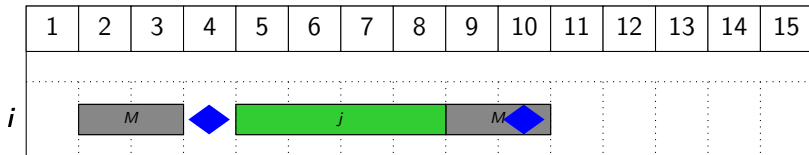




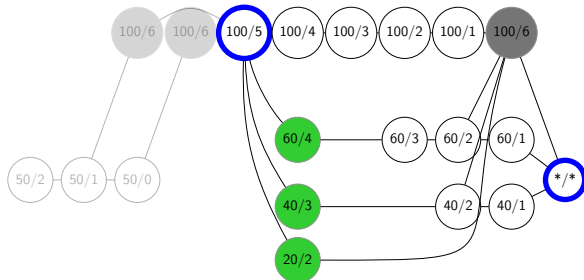
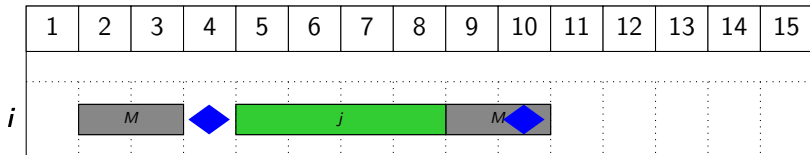
## Pattern extraction



## Pattern extraction

*rft/rct*

## Pattern extraction

*rft/rct*

## Pattern extraction

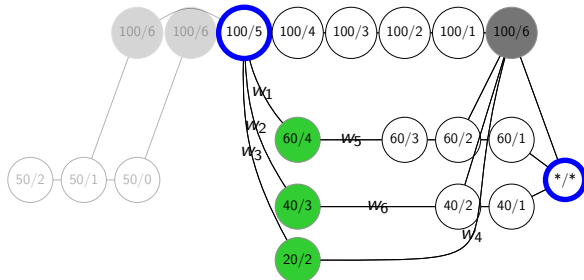
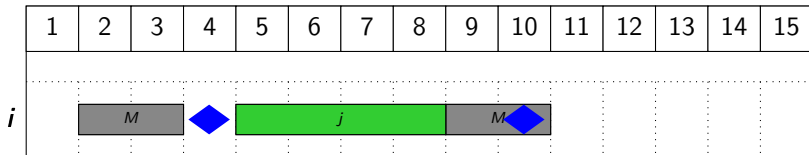
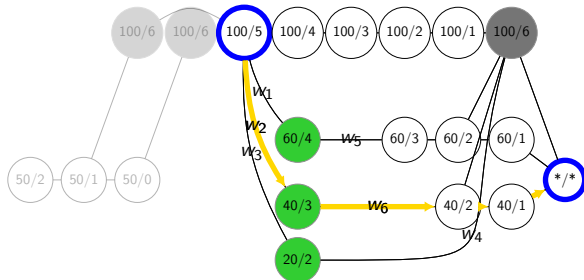


Diagram illustrating a 1D lattice with 15 sites. The sites are numbered 1 to 15. Site 1 is occupied by a fermion  $i$ . Site 2 is occupied by a boson  $M$ . Site 4 is occupied by a fermion  $j$ . Site 10 is occupied by a fermion. Sites 3, 5, 6, 7, 8, 9, 11, 12, 13, 14, and 15 are empty.

 $rft/rct$

## Contents

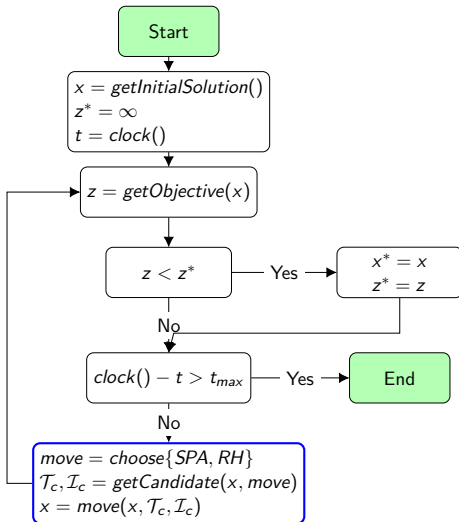
Graph representation of a solution space

**Applying said graphs within a Variable Neighborhood  
Descent**

## Solution approach: Variable Neighborhood Descent



## Solution approach: Variable Neighborhood Descent



## Neighborhood 1: Shortest Path Algorithm (SPA)

## Neighborhood 1: Shortest Path Algorithm (SPA)

$$SPA(x, \mathcal{I}_c, \mathcal{T}_c)$$

## Neighborhood 1: Shortest Path Algorithm (SPA)

$$SPA(x, \mathcal{I}_c, \mathcal{T}_c)$$

$$A_c = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ \boxed{0} & \boxed{-1} & \boxed{-1} & \boxed{-1} & \boxed{-1} & \boxed{0} \\ 0 & 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

## Neighborhood 1: Shortest Path Algorithm (SPA)

$$SPA(x, \mathcal{I}_c, \mathcal{T}_c)$$

$$A_c = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A_{c+1} = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

SPA

## Neighborhood 2: Rolling Horizon (RH)

## Neighborhood 2: Rolling Horizon (RH)

$$RH(x, \mathcal{I}_c, \mathcal{T}_c)$$

## Neighborhood 2: Rolling Horizon (RH)

$$RH(x, \mathcal{I}_c, \mathcal{T}_c)$$

$$A_c = \begin{pmatrix} 1 & \boxed{1 \quad 1 \quad 1} & 0 & 0 \\ -1 & \boxed{0 \quad 0 \quad 0} & 0 & -1 \\ 0 & \boxed{0 \quad 0 \quad 0} & 2 & 2 \\ 0 & \boxed{0 \quad 0 \quad 0} & 0 & 0 \end{pmatrix}$$



## Neighborhood 2: Rolling Horizon (RH)

$$RH(x, \mathcal{I}_c, \mathcal{T}_c)$$

$$A_c = \begin{pmatrix} 1 & \boxed{1 \ 1 \ 1} & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$
  

$$A_{c+1} = \begin{pmatrix} 1 & \boxed{0 \ 0 \ 0} & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 1 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

RH

## Initial solution

►  $SPA(\emptyset, \mathcal{I}, \mathcal{T})$

## Initial solution

►  $SPA(\emptyset, \mathcal{I}, \mathcal{T})$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

## Initial solution

►  $SPA(\emptyset, \mathcal{I}, \mathcal{T})$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \boxed{0} & \boxed{1} & \boxed{1} & \boxed{1} & \boxed{2} & \boxed{-1} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

## Initial solution

►  $SPA(\emptyset, \mathcal{I}, \mathcal{T})$

$$\begin{pmatrix} \boxed{1} & \boxed{1} & \boxed{1} & \boxed{1} & 0 & 0 \\ 0 & 1 & 1 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

## Initial solution

►  $SPA(\emptyset, \mathcal{I}, \mathcal{T})$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 2 & -1 \\ \boxed{0 & 0 & 0 & 0 & 2 & 2} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

## Initial solution

►  $SPA(\emptyset, \mathcal{I}, \mathcal{T})$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 2 & 2 \\ -1 & -1 & 0 & 0 & 0 & 2 \end{pmatrix}$$

## Initial solution

►  $SPA(\emptyset, \mathcal{I}, \mathcal{T})$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 2 & 2 \\ -1 & -1 & 0 & 0 & 0 & 2 \end{pmatrix}$$

►  $RH(\emptyset, \mathcal{I}, \mathcal{T})$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$



## Initial solution

►  $SPA(\emptyset, \mathcal{I}, \mathcal{T})$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 2 & 2 \\ -1 & -1 & 0 & 0 & 0 & 2 \end{pmatrix}$$

►  $RH(\emptyset, \mathcal{I}, \mathcal{T})$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 2 & 2 \\ -1 & -1 & 0 & 0 & 0 & 2 \end{pmatrix}$$

## Initial solution

►  $SPA(\emptyset, \mathcal{I}, \mathcal{T})$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 2 & 2 \\ \boxed{-1 & -1 & 0 & 0 & 0 & 2} \end{pmatrix}$$

►  $RH(\emptyset, \mathcal{I}, \mathcal{T})$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 2 & 2 \\ \boxed{-1 & -1 & 0 & 0 & 0 & 2} \end{pmatrix}$$

► Simulated Annealing()

## Experiments

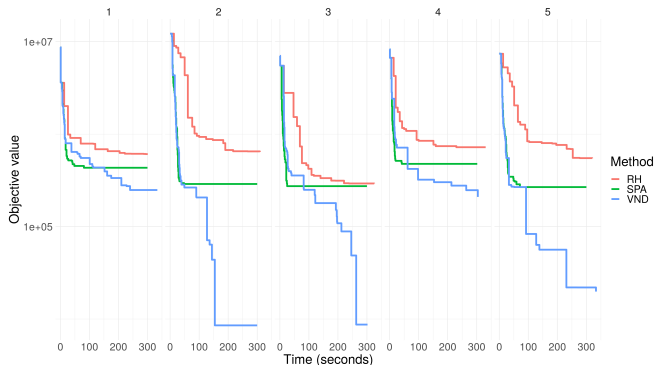
- ▶ Instance sizes: large ( $|I|=60$ ), very large ( $|I|=100$ ) and **very** large ( $|I|=255$ ).
- ▶ 1 graph per cluster of aircraft and node aggregation with respect to remaining flight time.
- ▶ CPLEX running 1 thread for up to 20 minutes.

All instances have 90 periods.

## Results: comparing neighborhoods

- ▶ SPA: fast but reaches local minima.
- ▶ RH: slow but avoids local minima.
- ▶ VND=SPA+RH: fast and avoids local minima.

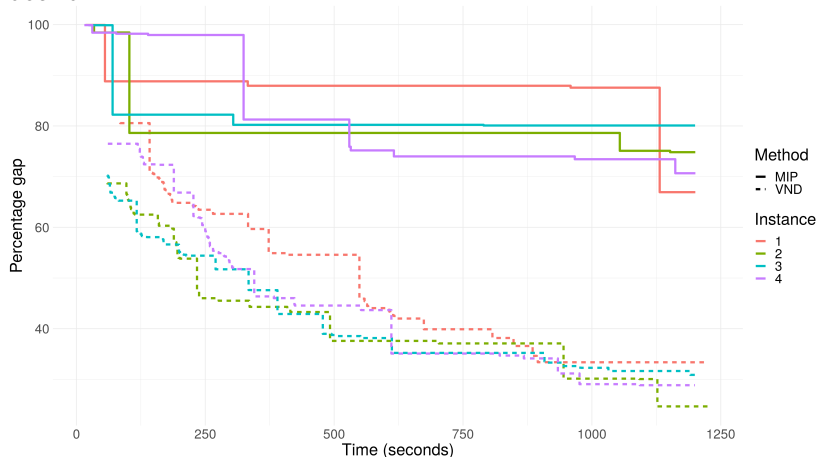
Example of one instance ( $|I|=60$ ) solved 5 times with different random seeds, for 5 minutes.



## Results: large instances

- ▶ VND outperforms MIP for very large instances (255 aircraft).

Example comparing the percentage gap to the best known lower bound.



Context and SoA  
oooooooo

Exact methods  
oooooooooooo

ML-based cuts  
oooooooooooooooooooo

Graph-based VND  
oooooooooooooooo●

Conclusions  
ooooo

## Preliminary conclusions

## Preliminary conclusions

- ▶ **A Graph representation is built** to efficiently explore the solution space of an aircraft and solve the MFMP.

## Preliminary conclusions

- ▶ **A Graph representation is built** to efficiently explore the solution space of an aircraft and solve the MFMP.
- ▶ **Complementary neighborhoods** are combined to avoid falling quickly into local minima.



## Preliminary conclusions

- ▶ **A Graph representation is built** to efficiently explore the solution space of an aircraft and solve the MFMP.
- ▶ **Complementary neighborhoods** are combined to avoid falling quickly into local minima.
- ▶ **Efficiently solve large instances** by reducing gaps in very large instances from 67.1% to 23.1%.

## Preliminary conclusions

- ▶ **A Graph representation is built** to efficiently explore the solution space of an aircraft and solve the MFMP.
- ▶ **Complementary neighborhoods** are combined to avoid falling quickly into local minima.
- ▶ **Efficiently solve large instances** by reducing gaps in very large instances from 67.1% to 23.1%.

## Preliminary conclusions

- ▶ **A Graph representation is built** to efficiently explore the solution space of an aircraft and solve the MFMP.
- ▶ **Complementary neighborhoods** are combined to avoid falling quickly into local minima.
- ▶ **Efficiently solve large instances** by reducing gaps in very large instances from 67.1% to 23.1%.

**To be submitted:** Franco Peschiera, Alain Haït, Nicolas Dupin, Olga Battaïa. Novel Graph-based matheuristic to solve the Flight and Maintenance Planning problem.

# Outline

1. Context and state of the art
2. Exact methods
3. Valid bounds and learned constraints
4. Graph-based VND matheuristic
5. **General conclusions and perspectives**

## General conclusions

- ▶ **The long term MFMP problem is formalized**, analyzed and compared with similar problems.

## General conclusions

- ▶ **The long term MFMP problem is formalized**, analyzed and compared with similar problems.
- ▶ **Exact methods** provide good quality solutions (less than 5% gap) in medium-sized instances.

## General conclusions

- ▶ **The long term MFMP problem is formalized**, analyzed and compared with similar problems.
- ▶ **Exact methods** provide good quality solutions (less than 5% gap) in medium-sized instances.
- ▶ **ML-based methods** reduce the solution times (by 35% on average) of exact methods with small optimality loss.

## General conclusions

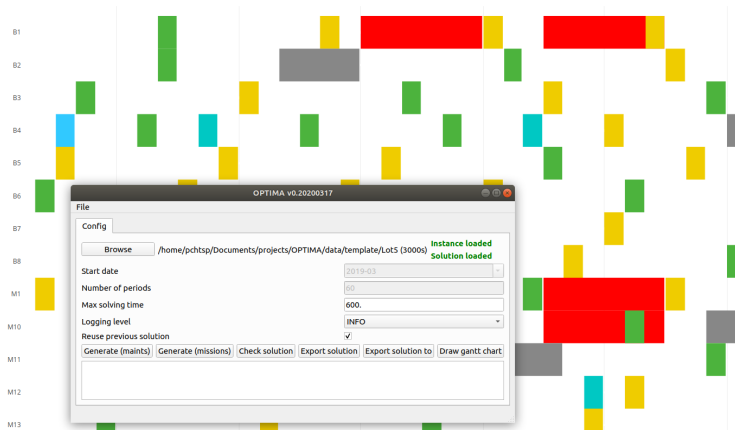
- ▶ **The long term MFMP problem is formalized**, analyzed and compared with similar problems.
- ▶ **Exact methods** provide good quality solutions (less than 5% gap) in medium-sized instances.
- ▶ **ML-based methods** reduce the solution times (by 35% on average) of exact methods with small optimality loss.
- ▶ **Graph-based methods** solve very large scale instances efficiently (60% better solutions than MIP on average).



## General conclusions

- ▶ **The long term MFMP problem is formalized**, analyzed and compared with similar problems.
- ▶ **Exact methods** provide good quality solutions (less than 5% gap) in medium-sized instances.
- ▶ **ML-based methods** reduce the solution times (by 35% on average) of exact methods with small optimality loss.
- ▶ **Graph-based methods** solve very large scale instances efficiently (60% better solutions than MIP on average).
- ▶ **A functional desktop application** was developed, deployed and validated successfully by Dassault Aviation on real-life datasets.

# Desktop application



## Perspectives

- ▶ **Extend the problem** to include some additional secondary constraints.

## Perspectives

- ▶ **Extend the problem** to include some additional secondary constraints.
- ▶ **Integrate** novel techniques (ML and graph) inside known frameworks (Column Generation, GRASP).

## Perspectives

- ▶ **Extend the problem** to include some additional secondary constraints.
- ▶ **Integrate** novel techniques (ML and graph) inside known frameworks (Column Generation, GRASP).
- ▶ **Generalize ML methodology** to obtain a probability distribution for patterns, automatize feature extraction.

## Perspectives

- ▶ **Extend the problem** to include some additional secondary constraints.
- ▶ **Integrate** novel techniques (ML and graph) inside known frameworks (Column Generation, GRASP).
- ▶ **Generalize ML methodology** to obtain a probability distribution for patterns, automatize feature extraction.
- ▶ **Combine ML and VND**, e.g., by applying learned-cuts during path-sampling to extract promising patterns.

# Contributions

## Journal articles

- ▶ F. Peschiera, R. Dell, J. Royset, A. Haït, N. Dupin, and O. Battaïa. A novel solution approach with ML-based pseudo-cuts for the Flight and Maintenance Planning problem. *OR Spectrum*, pages 1–30, jun 2020. ISSN 0171-6468. doi: 10.1007/s00291-020-00591-z.
- ▶ F. Peschiera, A. Haït, N. Dupin, and O. Battaïa. Long term planning of military aircraft flight and maintenance operations. Technical report, ISAE-SUPAERO, Université de Toulouse, France, 2020 (submitted).
- ▶ F. Peschiera, N. Dupin, A. Haït, and O. Battaïa. Novel graph-based matheuristic to solve the flight and maintenance planning problem. *Forthcoming (to be submitted)*.

## Conferences

- ▶ F. Peschiera, A. Haït, N. Dupin, and O. Battaïa. Maintenance planning on french military aircraft operations. In *Congrès annuel de la société Française de Recherche Opérationnelle et d'Aide à la Décision (ROADEF)*, pages 1–2, Lorient, FR, 2018.
- ▶ F. Peschiera, O. Battaïa, A. Haït, and N. Dupin. Bi-objective mip formulation for the optimization of maintenance planning on french military aircraft operations. 2018.
- ▶ F. Peschiera, A. Haït, N. Dupin, and O. Battaïa. A novel mip formulation for the optimization problem of maintenance planning of military aircraft. In *XIX Latin-Iberoamerican Conference on Operations Research*, Lima, PE, 2018.
- ▶ F. Peschiera, N. Dupin, O. Battaïa, and A. Haït. An alternative mip formulation for the military flight and maintenance planning problem. In *Congrès annuel de la société Française de Recherche Opérationnelle et d'Aide à la Décision (ROADEF)*, Montpellier, FR, 2020.

Thank you for your attention.