$$R_x(\frac{\pi}{2}) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix} == x$$

$$R_x(\frac{-\pi}{2}) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} == -x$$

$$\hat{x}|\mathrm{x}\rangle = \frac{1}{\sqrt{2}}(1-i)|\mathrm{x}\rangle$$

$$\hat{x}|\text{-x}\rangle = \frac{1}{\sqrt{2}}(1+i)|\text{-x}\rangle$$

$$\hat{-x}|\mathrm{x}\rangle = \frac{1}{\sqrt{2}}(1+i)|\mathrm{x}\rangle$$

$$\hat{-x}|\text{-x}\rangle = \frac{1}{\sqrt{2}}(1-i)|\text{-x}\rangle$$

X-readout, incoming state: $\alpha|\mathbf{x}\rangle + \beta|\text{-}\mathbf{x}\rangle$

After the y-rotation on the electron spin, the system is in state:

$$|\psi_s\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes (\alpha|\mathrm{x}\rangle + \beta|\text{-x}\rangle) \tag{1}$$

We perform either a $\hat{\pm x}$- or $\hat{\mp x}$-rotation to the carbon spin, followed by an x-rotation on the electron spin:

$$|\psi_s\rangle = \frac{1}{2\sqrt{2}}(|0\rangle - i|1\rangle) \otimes (\alpha(1 \mp i)|\mathrm{x}\rangle + \beta(1 \pm i)|\text{-x}\rangle) \tag{2}$$

$$+ \frac{1}{2\sqrt{2}}(-i|0\rangle + |1\rangle) \otimes (\alpha(1 \pm i)|\mathrm{x}\rangle + \beta(1 \mp i)|\text{-x}\rangle) \tag{3}$$

Working this out, we get dependent on the direction of the *Ren*-gate the following final states:

$$\hat{\pm x} \rightarrow \alpha|0\rangle \otimes |\mathrm{x}\rangle + \beta|1\rangle \otimes |\text{-x}\rangle \tag{4}$$

$$\hat{\mp x} \rightarrow \beta|0\rangle \otimes |\text{-x}\rangle + \alpha|1\rangle \otimes |\mathrm{x}\rangle \tag{5}$$

Which shows that for the $\hat{\pm x}$-gate, the electron RO directly corresponds to the RO of the carbon spin in the x-direction, however, for the $\hat{\mp x}$-gate, the RO should get a minus-sign.