Recent progress in the generation, manipulation and storage of distant entangled quantum states has opened up an avenue to the construction of a small-scale quantum network over metropolitan scale distances in the near future. This motivates the exploration of different potential methods for the efficient generation of entangled links between nodes. Here we have examined three different protocols using both semi-analytical and Monte Carlo methods.

# I. PROTOCOLS

The central challenge for generating entanglement over long distances is the communication overhead required to determine whether entanglement generation succeeded.

## A. Multiplexed Barrett and Kok

The first scheme that we consider is a version of the Barrett and Kok protocol. In this scheme, entanglement is generated at both nodes between the spin state of the NV and the modal occupation of single photon (typically encoded in the photonic time degree of freedom for NVs). Interfering these single photons on a beamsplitter erases which-path information, allowing entanglement to be generated between the two NV spins, conditional on detecting one output photon in each of the time bins.

In the standard version of this scheme, the clock rate is limited by the classical communication time required to establish whether the protocol succeeded. For 50 km, this is 250 $\mu$s, limiting the attempt rate of the scheme to 4 kHz.

## B. Multiplexed Extreme Photon Loss Protocol

## C. Kwiat scheme

# II. MODELLING

## A. Parameters

In our models we have considered two different regimes of parameters, as shown in Table I. The 'ambitious' regime parameters represent our optimistic expectations for the performance

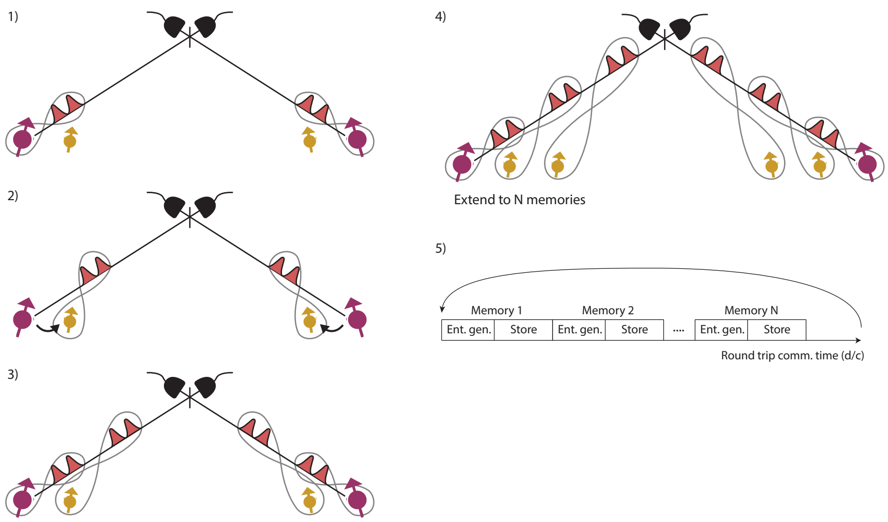


FIG. 1.

of quantum nodes based on Nitrogen Vacancy (NV) solid-state spins in the medium term (5-10 years), while the 'near-term' parameters reflect our expectations for the achievable performance in the shorter term. Both regimes rely on the availability of a cavity system to enhance the ZPL emission of the NV, and on the availability of frequency conversion to convert emitted photons to telecom wavelengths. These technologies are viewed as prerequisites to a scalable quantum network.

TABLE I. Variables used in these simulations

| Variable | Description | Ambitious value | Near-term value |
|---|---|---|---|
| $d$ | separation between nodes in km | | |
| $n$ | number of memories (including NV) | 4 | 2 |
| $\eta$ | speed of light in fibre | 0.2 km/$\mu$s | 0.2 km/$\mu$s |
| $\eta$ | fibre attenuation at telecom wavelengths | 0.2 dB/km | 0.2 dB/km |
| $p_{\mathrm{fc}}$ | frequency conversion efficiency | 0.3 | 0.1 |
| $p_{\mathrm{out}}$ | NV outcoupling efficiency | 0.3 | 0.1 |
| $t_{\mathrm{eg}}$ | spin-photon entanglement generation time | 1 $\mu$s | 2 $\mu$s |
| $t_{\mathrm{cg}}$ | NV-carbon gate time | 10 $\mu$s | 100 $\mu$s |
| $p_m$ | SPDC source photon-pair emission probability | 0.1 | 0.01 |

### B. Observations

1) Even for the highly ambitious values, storing in memory takes a long time, comparable to the time required for information to propagate from one node to the other. This limits the number of memories that can be usefully utilised, as can be seen in Fig. 2, in which the dashed red line denotes the distance below which the time required to store states in the memories is longer than the communication time between the nodes. For distances less than this, there is no point in using this number of memories. This is particularly damaging to the Kwiat scheme, since this scheme relies on being able to make memory attempts to generate entanglement during one communication cycle.

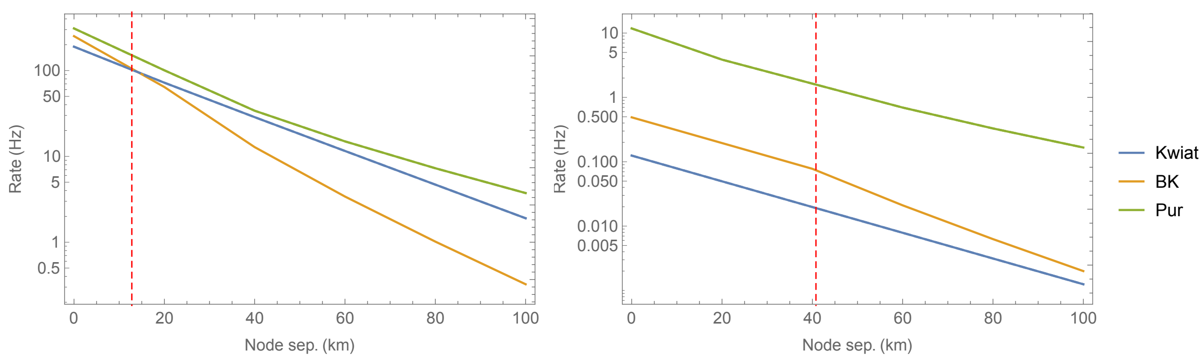


FIG. 2.

2) This is apparent when fixing the distance between the nodes and changing the number of available memories. For both the ambitious and the near-term values, the Kwiat scheme derives no benefit from additional memories (in essence, the chance of a successful event in one communication round is very low for both regimes (numbers), and so having additional memories with which to store successful events is of no benefit). For the near-term values, there is no benefit for BK or Purification, since, as the red dashed line shows, beyond 2 memories, all time is taken up with storing states. In contrast, for the ambitious values, there is a benefit to increasing the number of memories.

3) Rate is only half of the story. Changing the parameters also impacts the fidelity of the state produced. In the previous plots, there was no constraint on the minimum acceptable fidelity, and so the rate was maximised. However, in any useful protocol this will not be the case. For a given set of parameters, it is possible to constrain the maximum number of

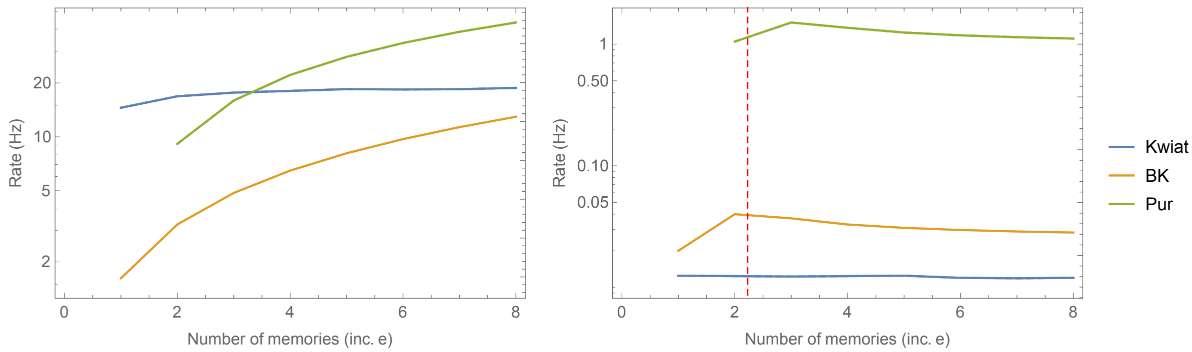


FIG. 3.

entanglement generation attempts that a given stored state will be subjected to. This will increase the fidelity of the state, at the expense of the entanglement generation rate.

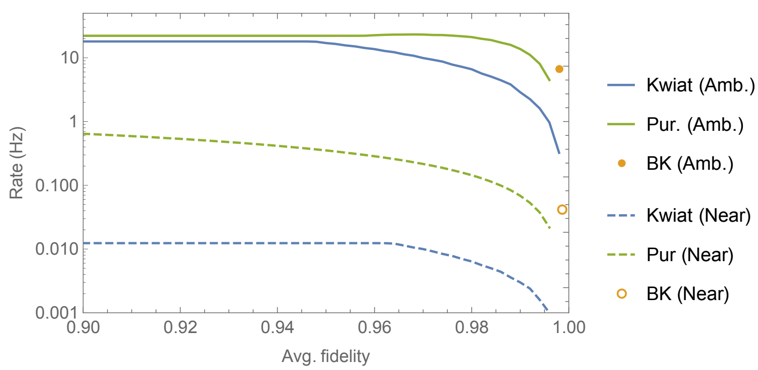


FIG. 4.