General Quantum Information - Density Matrix

Some examples of density matrices - the important note is to see how the density matrices are constructed for the shared states

$|\varphi_1\rangle = \frac{3}{5}|00\rangle + \frac{4}{5}|11\rangle \qquad |\varphi_2\rangle = \frac{16}{25}|00\rangle - \frac{3}{5}|11\rangle$

$\rho_1 = Tr_{(A)}|\varphi_1\rangle\langle\varphi_1| = \frac{9}{25}|00\rangle + \frac{16}{25}|11\rangle \rightarrow \begin{bmatrix} 9/25 & 0 \\ 0 & 16/25 \end{bmatrix}$

probability of the state φ_1

$\rho_2 = Tr_{(A)}|\varphi_2\rangle\langle\varphi_2| = \frac{16}{25}|00\rangle - \frac{9}{25}|11\rangle \rightarrow \begin{bmatrix} 16/25 & 0 \\ 0 & 9/25 \end{bmatrix}$

probability of the state φ_2

If Bob measured in the std. basis, he would be correct 64% of the time

Importance? Bob can gain partial info. about the state of the qubits. (with some level of certainty).

Definition: BIPARTITE SYSTEM

A quantum system that is shared by two parties. The bipartite system is denoted as the tensor product of the two parties - $H_A \otimes H_B$ - where "H" is the hilbert space

Schmidt Decomposition

Having a pure bipartite state, we are able to determine if a state is entangled or not.

* view density matrix as a formalism of quantum information.

Schmidt coefficient characterize entanglement

* Schmidt rank

$SR = 1 \rightarrow$ product state (not entangled)

$SR > 1 \rightarrow$ entangled state (the higher the rank, the more entangled the states are)

Quantum Information - Measurement

Regarding the formalism of measurement

A measurement on "X" - a quantum register - can have any finite, non-empty set Γ of possible outcomes.

NOT^n : $X \rightarrow$ quantum register with classical state set Σ

$\Sigma \rightarrow$ classical state set - $\{0, 1\}$

$\mathcal{X} \rightarrow \mathbb{C}(\Sigma)$ - complex vector space of the classical state set

$L(\mathcal{X}) \rightarrow$ set of all linear mappings of $\mathcal{X}$.

FORMAL EXPRESSION $\rightarrow$ $\{M_a : a \in \Gamma\} \subset L(\mathcal{X}) \rightarrow$
(these are matrices)

$Pr[\text{outcome is } a] = Tr((M_a^\dagger M_a)\rho)$

? quantum = randomness — is this pure randomness?

where ρ is the density matrix of the quantum states.

this must be satisfied:

$M_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$\sum_{a \in \Gamma} M_a^\dagger M_a = I$ for example: if $M_0 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ which is the outcome $|0\rangle$

$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ $\quad$ $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$\overset{M_1}{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}} + \overset{M_0}{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow$ identity and if $\rho = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$

$Pr[|0\rangle] = 1/2$ since the $Tr_{(|0\rangle)} = \rho_{11}$

[REFLECTIONS]

Density Matrices provide a mathematical framework to understand the statistic breakdown of quantum system. The crucial understanding of how qubits affect each other can partly be seen in density matrices

ETH - QUANTUM INFORMATION

Density Matrix - a more general description of a qubit

turns a vector into a matrix.

$|\varphi\rangle \rightarrow \rho = |\varphi\rangle\langle\varphi|$

the density matrix seen as an average of the qubit states

$\text{rank}(\rho) = 1 \rightarrow$ pure state

$\text{rank}(\rho) > 1 \rightarrow$ mixed state

<u>Some more on the</u> Schmidt Decomposition

* reduced densities have the same eigenvalue.

operations can be done locally! : $\{|u_i\rangle_A\}$ & $\{|v_i\rangle_B\}$

like : can perform a unitary operation of system A, which will change its basis but it will leave system B alone.

global data : λ_i

coefficient

reduced state for system A $\rightarrow \rho_A = \sum_i |\lambda_i|^2 |u_i\rangle\langle u_i|_A$