Aaronson HW 3

[SEPARABLE & ENTANGLED STATES]

Determine whether each of the following two qubit states are separable or entangled

$$|\psi\rangle = \frac{|00\rangle + i|01\rangle + i|10\rangle - |11\rangle}{2}$$

$$= \frac{1}{\sqrt{2^2}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ i \end{bmatrix} \rightarrow (|0\rangle + i|1\rangle)(|0\rangle + i|1\rangle)$$

$$= |00\rangle + i|01\rangle + i|10\rangle + i^2|11\rangle$$

$$= |00\rangle + i|01\rangle + i|10\rangle - |11\rangle$$

$|\psi\rangle$ = separable

$$|\psi\rangle = \frac{3}{5}|01\rangle - \frac{4}{5}|10\rangle \qquad \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \rightarrow |01\rangle$$

cannot get combination of these 2 states they are distinct

$$= \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \rightarrow |10\rangle$$

↓ density matrix

$$\rho = \sum p_i |\psi_i\rangle\langle\psi_i|$$

$$\begin{bmatrix} \sum 1 & \sum 1 & \sum 1 & \sum 1 \\ \sum 1 & \sum 1 & \sum 1 & \sum 1 \end{bmatrix}$$

$|00\rangle + i|01\rangle + i|10\rangle - |11\rangle$

$$= |0\rangle\langle 0| + i|0\rangle\langle 1| + i|1\rangle\langle 0| - |1\rangle\langle 1| \Big/ 2$$

$\begin{bmatrix}1\\0\end{bmatrix} \times \begin{bmatrix}1 & 0\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$ each outer product occupies a space of the matrix

$$i|0\rangle\langle 1| = i\begin{bmatrix}1\\0\end{bmatrix} \times \begin{bmatrix}0 & 1\end{bmatrix} = i\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix} = \begin{bmatrix}0 & i\\0 & 0\end{bmatrix}$$

$$i|1\rangle\langle 0| = i\begin{bmatrix}0\\1\end{bmatrix} \times \begin{bmatrix}1 & 0\end{bmatrix} = i\begin{bmatrix}0 & 0\\1 & 0\end{bmatrix} = \begin{bmatrix}0 & 0\\i & 0\end{bmatrix}$$

$$-|1\rangle\langle 1| = \begin{bmatrix}0\\1\end{bmatrix} \times \begin{bmatrix}0 & 1\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & -1\end{bmatrix}$$

density matrix $= \begin{bmatrix}1 & i\\i & -1\end{bmatrix} \Big/ 2$

$$\frac{3}{5}|0\rangle\langle 1| = \frac{3}{5}\begin{bmatrix}1\\0\end{bmatrix} \times \begin{bmatrix}0 & 1\end{bmatrix} = \frac{3}{5}\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix} = \begin{bmatrix}0 & 3/5\\0 & 0\end{bmatrix}$$

$$-\frac{4}{5}|1\rangle\langle 0| = -\frac{4}{5}\begin{bmatrix}0\\1\end{bmatrix} \times \begin{bmatrix}1 & 0\end{bmatrix} = -\frac{4}{5}\begin{bmatrix}0 & 0\\1 & 0\end{bmatrix} = \begin{bmatrix}0 & 0\\-4/5 & 0\end{bmatrix}$$

$\begin{bmatrix}0 & 3/5\\-4/5 & 0\end{bmatrix}$ ← $|\Phi\rangle$ density matrix