Quantum Error Correcting Codes

1st look at classical repetition codes

- simplest example is send 3-bit repetition code
  - * logic - encode all bits into desired bit → either 000 or 111 if Alice is sending Bob the bits, Bob just needs to take the majority bit.

    the pr[correct transmission] = $(1-p)^3 + 3(1-p)^2 p \longrightarrow 1 - (3p^2 - 2p^3)$

Quantum analogue?

Alice transmits qubit to Bob

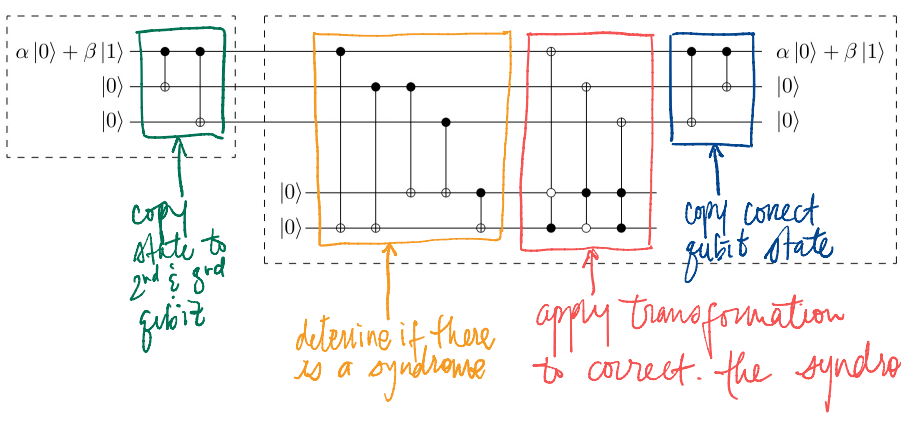
repetition code = $\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|000\rangle + \beta|111\rangle$ (not great)

* what does a bit flip look like quantumly?

$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ unitary operation

the use of 2 ancilla qubits $|00\rangle$ helps identify the "syndrome" if $|00\rangle$ is returned from the circuit unchanged, no bit flip happened.

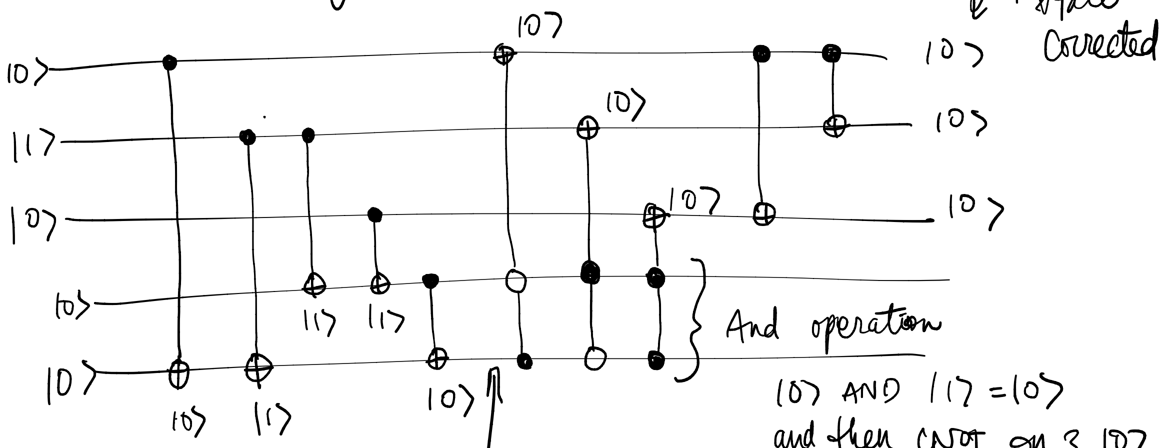
Full diagram of quantum error correcting code



* note: this circuit can only recover 1 bit flip w/ certainty, for more than 1 bit flip you lose some certainty.

Quantum Error Correction

|01x⟩ → syndrome = 10   WORKED EXAMPLE



|0⟩ AND |1⟩ = |0⟩
and then CNOT on 3 |0⟩

$|\psi_1\rangle = |10\rangle$ matches syndrome

GENERAL QUANTUM ERRORS - CSS Codes

Some notⁿ → $\mathbb{Z}_2$ → group of integers mod n, so $\mathbb{Z}_2 \Rightarrow \{0,1\}$

CSS codes - named after 3 people who found it.

- demonstrates how some classical error correction codes can be turned into quantum codes.

*I'm going to watch some Eigenchris videos to get familiar w/ classical error correcting code methods. *