[Observables]

Ex: $|\varphi_1\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$ $\qquad$ $|\varphi_2\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle$

$\lambda_1 = 1$ $\qquad$ $\lambda_2 = -1$

A

$|\psi\rangle$ project on $|\varphi\rangle$ is P

$P = |\varphi\rangle\langle\varphi|$

$P|\psi\rangle = |\varphi\rangle\underbrace{\langle\varphi||\psi\rangle}_{\text{inner product}}$ $\longrightarrow$ rewrite $\underbrace{(\langle\varphi|\psi\rangle)}_{\text{complex \#}}|\varphi\rangle$

$A = |\varphi_1\rangle\langle\varphi_1| + (-1)|\varphi_2\rangle\langle\varphi_2|$

$|\varphi_1\rangle\langle\varphi_1| = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{2} & -\frac{i}{2} \\ \frac{i}{2} & \frac{1}{2} \end{bmatrix}$

$|\varphi_2\rangle\langle\varphi_2| = \begin{bmatrix} \frac{1}{2} & \frac{i}{2} \\ -\frac{i}{2} & \frac{1}{2} \end{bmatrix}$

$|\varphi_1\rangle\langle\varphi_1|$
$-|\varphi_2\rangle\langle\varphi_2| \longrightarrow \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = A$

$A|\varphi_1\rangle$

$= (|\varphi_1\rangle\langle\varphi_1| - |\varphi_2\rangle\langle\varphi_2|)|\varphi_1\rangle$

$= (|\varphi_1\rangle\underline{\langle\varphi_1|)\,|\varphi_1\rangle} \rightarrow$ inner product = 1

$-(|\varphi_2\rangle\underline{\langle\varphi_2|)\,|\varphi_1\rangle} \rightarrow \emptyset$

$= 1|\varphi_1\rangle - 0 = |\varphi_1\rangle$

$A|\varphi_2\rangle \longrightarrow -|\varphi_2\rangle$

In general: given $|\varphi_i\rangle$, λ_i, corresponding observable is:

$* \; A = \sum \lambda_i |\varphi_i\rangle\langle\varphi_i|$

[Schrodinger's Equation]

- Energy observable H, called the Hamiltonian of the system
  - it's eigenvectors $|\varphi_i\rangle$ are states w/ definite energy
  - the eigenvalues λ_i are the energy of the corresponding state

<u>Ex</u>: $H = \begin{bmatrix} -\frac{1}{2} & \frac{5}{2} \\ \frac{5}{2} & -\frac{1}{2} \end{bmatrix}$

- $|+\rangle$ w/ energy = 2
- $|-\rangle$ w/ energy = -3

if $|\varphi\rangle = |+\rangle$ then energy 2
if $|\varphi\rangle = |-\rangle$ then energy = -3
what about $|0\rangle$?
$\frac{1}{2}$ energy 2, $\frac{1}{2}$ energy -3

Hamiltonian of k-level quantum system

let your states be definite $|0\rangle, |1\rangle \dots |k-1\rangle$
then Hamiltonian = $\begin{bmatrix} E_0 & & & 0 \\ & E_1 & & \\ & & \ddots & \\ 0 & & & E_{k-1} \end{bmatrix}$ *each E_k corresponds to an energy state of the system

Schrodinger's Equation

$i\hbar \frac{\partial}{\partial t} |\varphi(t)\rangle = H|\varphi(t)\rangle$ more simply → given $|\varphi(0)\rangle$, H figure out $|\varphi(t)\rangle$

$i = \sqrt{-1}$ $\hbar = \frac{h}{2\pi} \rightarrow 1.055 \times 10^{-34}$ Js

if $|\varphi_j\rangle$ is some eigenvector of H w/ eigenvalue $|\lambda_j\rangle$
then $|\varphi(t)\rangle = e^{-\frac{i\lambda_j t}{\hbar}} |\varphi_j\rangle$

→ phase change! where larger λ = higher precession.

Schrödinger Equation Example

Ex: $|\varphi(0)\rangle = |0\rangle$

$H = X \longrightarrow \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

what is $|\varphi(t)\rangle$?

eigenvalues? $|+\rangle = 1$

$|-\rangle = -1$

$|\varphi(0)\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle$

$$\boxed{|\varphi(t)\rangle = \frac{1}{\sqrt{2}} e^{-\frac{it}{\hbar}} + \frac{1}{\sqrt{2}} e^{\frac{it}{\hbar}} |-\rangle}$$

state of system at time (t)

$t = \frac{\pi\hbar}{2}$

$$|\varphi(\tfrac{\pi\hbar}{2})\rangle = \frac{1}{\sqrt{2}}(i)|+\rangle + \frac{1}{\sqrt{2}}(i)|-\rangle$$

$$= -i\left[\frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle\right]$$

$$= -i|1\rangle$$

So in time $\left(\frac{\pi\hbar}{2}\right)$, where $\hbar$ is some planck constant, our quantum system evolved from state $|0\rangle$ to state $-i|1\rangle$, we were able to determine this by using Schrodinger's equation

Hamiltonian → matrix operator correlating to energy states (observable)

$$\begin{bmatrix} e_1 & & & 0 \\ & e_2 & & \\ & & e_3 & \\ 0 & & & \ddots \; e_n \end{bmatrix}$$

[Particle in a Box]

Solving Schrodinger's Equation: (for H) hamiltonian

2nd derivative

$$H = \frac{-\hbar^2}{2m} \cdot \frac{\partial^2}{\partial x^2} |\varphi\rangle$$ (where φ is some quantum state)

Eigenstate $\varphi(x) = e^{ikx}$

$$S.E = \frac{-\hbar^2}{2m} \cdot \frac{\partial^2}{\partial x^2} \cdot e^{ikx} \rightarrow \text{do } 2^{nd} \text{ derivative}$$

$$= \frac{-\hbar^2}{2m} \cdot (ik)^2 e^{ikx}$$

$$= \frac{\hbar^2 k^2}{2m} \cdot e^{ikx} \rightarrow \left[E_k e^{ikx}\right] \quad \text{where } E_k = \frac{\hbar^2 k^2}{2m}$$

* e^{ikx} & e^{-ikx} same energy

$$\left[\psi_E(x) = Ae^{ikx} + Be^{-ikx}\right]$$

$$= C \sin kx + D \cos kx$$

$$\psi_E(x) = C \sin kx + D \cos kx \qquad E_k = \frac{\hbar^2 k^2}{2m}$$

$$\psi_E(\emptyset) = \emptyset \quad = \quad 0 = D \cdot 1 \Rightarrow D = \emptyset$$

$$\psi_E(L) = \emptyset \rightarrow C \sin kL = \emptyset \qquad kL = n\pi \quad n \text{ is an integer}$$

$$k_n = \frac{n\pi}{L}$$

$$E_n = \frac{\hbar^2 k_n^2}{2m} \rightarrow \frac{\hbar^2 \left(\frac{n\pi}{L}\right)^2}{2m}$$

[Adiabatic Quantum Computation]

Unstructured search:

thm: any quantum algorithm must take at least $\sqrt{n}$ time

& CANNOT look inside "U_f"

In adiabatic QC, you can see inside "U_f" - speed up?

READ ARTICLES

[BQP]

P - polynomial time

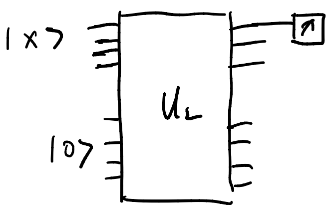
BPP - probabilistic polynomial time

Input - x
Output - yes/no } primality Testing
SAT problem

let $L = \{ x : \text{answer yes} \}$ (x: input)

$L_{primality} = \{ x : x \text{ is prime} \}$

$L \in BQP$ if there is a sequence of quantum circuits



$\begin{cases} 1 \text{ w.p } 2/3 & \text{if } x \in L \\ 0 \text{ w.p } 2/3 & \text{if } x \notin L \end{cases}$

& # of gates in $U_L = O(poly(n))$

$\boxed{BPP \subseteq BQP}$ is NP contained in BQP ← quantum algo. complexity

↳ $\subseteq$ PSPACE (polynomial space)

P = PSPACE ?? Big question

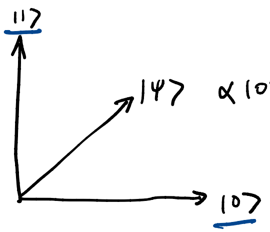
Fourier checking & PH Read article

[Spin as a Qubit]

how do you design physical qubit?

1. Initialize
2. Manipulate - gate (unitary trans) using Hamiltonian
3. Measure

for ex:



$\alpha|0\rangle + \beta|1\rangle$

$|0\rangle$ & $|1\rangle$ eigenstates

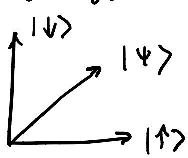
Unitary transformation as Hamiltonian on for some time (t)

then unitary transformation $U = e^{iHt/\hbar}$

Defining "spin":

intrinsic angular momentum

$|\uparrow\rangle$, $|\downarrow\rangle$



$|\psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$

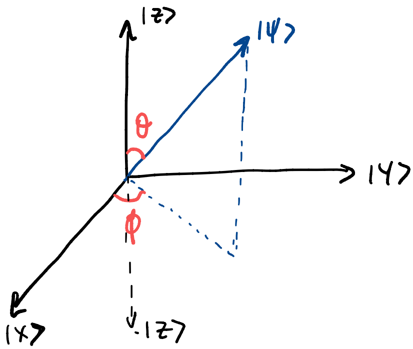
BLOCH SPHERE

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

$0 \le \theta \le \pi$

$0 \le \phi \le 2\pi$

2 real parameters θ, ϕ are only needed to represent state $|\psi\rangle$ in 3 dimensions



$$|z\rangle = |0\rangle$$
$$-|z\rangle = |1\rangle$$

$$|x\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \rightarrow |+\rangle$$
$$-|x\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \rightarrow |-\rangle$$
$$|y\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$$
$$-|y\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle$$

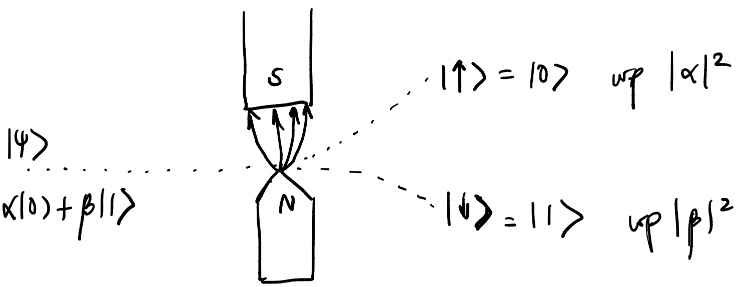
important to note

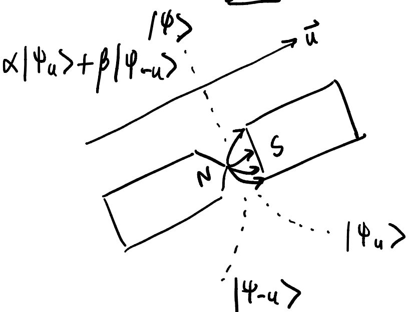
in 2D $|0\rangle$ & $|1\rangle$ are orthogonal

in 3D $|0\rangle$ & $|1\rangle$ are $180°$ (π) distance from each other

[STEARN- GERLACH]

spin of electron creates magnetic moment





USING BLOCH SPHERE

where $$|\psi_u\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

$$|\psi_{-u}\rangle = \sin\frac{\theta}{2}|0\rangle - e^{i\phi}\cos\frac{\theta}{2}|1\rangle$$

[PAULI SPIN MATRICES]

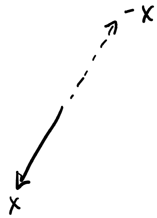
$|\uparrow\rangle = |0\rangle \longrightarrow 1$

$|\downarrow\rangle = |1\rangle \longrightarrow -1$

eigenvectors

eigenvalues

$\sigma_z = Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ observable!



$|\psi_x\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle = 1$

$|\psi_{-x}\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle = -1$

eigenvalues

eigenvectors

$\sigma_x = X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ observable

$|\psi_y\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle = 1$

$|\psi_{-y}\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle = -1$

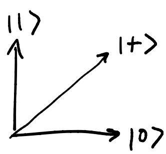
$\sigma_y = Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ observable

checking answer:

$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix} = \overset{\lambda = 1}{\downarrow}\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix}$$

[LARMOR PRECESSION]

$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$



flip across 45° (bloch sphere)

Interesting Topics

Quantum Hamiltonian Complexity
- interaction b/t quantum complexity theory & condensed matter physics