Cross-property DEM Toolbox Version 1.0
Release Notes

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Summary

These release notes provide an overview of the package Cross-Property DEM Version 1.0. The MATLAB scripts and functions contained in the package are used to perform operations demonstrated by Cilli and Chapman (2021). Namely, to:

- Solve for cross-property DEM parameter, equivalent pore aspect ratio (EPAR)
- Model elastic moduli from electrical conductivity
- Model V_p/V_s ratios from electrical conductivity
- Invert for electrical conductivity from elastic measurements

This research is ongoing and both scripts and documentation are expected to be updated at times.

Key concepts for cross-property DEM modelling

The electrical-elastic cross-property differential effective medium (DEM) model of Cilli and Chapman (2021) is 2 strongly non-linear coupled differential equations which forward model elastic moduli from electrical conductivity. We do not currently know whether they can be solved analytically; here they are solved numerically. The equations are derived from inclusion modelling theory (e.g., Eshelby (1957)) where ellipsoidal inclusions are embedded into a background material and the effective properties of that composite material are approximated using potential theory. Specifically, Cilli and Chapman (2021) combine an electrical DEM approximation (Torquato and Haslach Jr, 2002) and an elastic DEM approximation (Berryman, 1992) using the chain

Table 1: EPAR for mixed, clean, and clay-bearing sandstones (Cilli and Chapman, 2021).

Data set	α_K	α_μ
All samples	16.4	12.8
Clean (clay $< 3.5\%$)	16.6	12.3
Clay-bearing (clay $> 3.5\%$)	16.3	12.9

rule to derive the model, which has no porosity terms as it is rendered a dummy variable in the process.

All modelling of real data using the cross-property DEM model needs an awareness of bulk and shear modulus EPARs,

Theoretical inclusion modelling (e.g., Eshelby (1957)) uses a single aspect ratio as the model parameter. When modelling real data, however, two aspect ratios are needed: Bulk modulus EPAR (K-EPAR a.k.a. α_K) and shear modulus EPAR (mu-EPAR a.k.a. α_μ). Parameters α_K and α_μ are the aspect ratios which minimise the misfit in the modelled and measured bulk and shear moduli respectively. This is discussed more in Cilli and Chapman (2021) for cross-property inclusion modelling. In all these scripts, make sure you know which EPAR you are using and make sure modFlag is set accordingly.

The modelling in these packages can be performed using either geometrical function $\bar{m}$ (Cilli and Chapman, 2021) or R (Berryman, 1995). It is equivalent and both options are available in the package. Functions P and Q are from Berryman (1980).

Scripts and functions

Four example scripts are given showing how to use the toolbox's functions. To model moduli, V_p/V_s, and invert for conductivity, EPAR must be known. So if you have all the data and parameters required to invert for EPAR this should be done first. Or you can use the parameters solved by Cilli and Chapman (2021) on the data of Han et al. (2011) for mixed, clean, and clay-bearing sandstones (Table 1) as a guide. Cilli and Chapman (2021) found prolate spheroids ("Needles": EPAR > 1) fit the data better than oblate spheroids ("Disks": EPAR < 1).

A function for the electrical-elastic Hashin-Shtrikman bounds (Carcione et al., 2007) is also included. This function assumes the fluid phase doesn't support shear stress so the shear modulus lower bound is hard-coded to zero.

Options for the functions tend to include whether the bulk modulus or shear modulus EPAR is being used in the calculation, whether the l_1- or l_2-norm is used in optimisation cost functions (l_2 is default), and whether the model parameters are single values or vectors with a model parameter value for every measurement.

The core of the toolbox is the cpdem_Forward function, which relies on MATLAB's ode45 differential equation solver.

Limitations

Theoretical limitations

The cross-property DEM model is derived for a theoretical composite material which:

- is infinitely expansive
- has two isotropic constituent phases
- has spheroidal inclusions with at least two equal semi-major axes
- has randomly oriented inclusions and hence isotropic effective properties
- has a DC electrical or elastic source at it boundary, infinitely far from any inclusion
- has inclusions so disperse in the infinite material that the perturbation of the electric or elastic field due to each inclusion is not influenced by the presence of any other inclusion
- is linearly elastic.

The model does not theoretically account for the double-layer effect or surface effects (e.g., Waxman and Smits (1968)), which can be a significant factor in the electrical modelling of clay-bearing rocks. This issue is discussed by Cilli and Chapman (2021).

Computation limitations

The scripts in this package are tested only on MATLAB R2019b. They have not been tested on Octave. There are a few MATLAB functions in the package such as ode45.

Disclaimer

Although the author *thinks* everything in this toolbox is coded correctly, they take no responsibility for the performance and accuracy of its scripts and functions, and do not provide any warranties. Ultimately, the toolbox is used solely at the user's risk.

Licence

References

Berryman, J. G. (1980). Long-wavelength propagation in composite elastic media II. Ellipsoidal inclusions. *The Journal of the Acoustical Society of America*, 68(6):1820–1831.

Berryman, J. G. (1992). Single-scattering approximations for coefficients in Biot's equations of poroelasticity. *The Journal of the Acoustical Society of America*, 91(2):551–571.

Berryman, J. G. (1995). Mixture theories for rock properties. *Rock physics and phase relations: A handbook of physical constants*, 3:205–228.

Carcione, J. M., Ursin, B., and Nordskag, J. I. (2007). Cross-property relations between electrical conductivity and the seismic velocity of rocks. *Geophysics*, 72(5):E193–E204.

Cilli, P. A. and Chapman, M. (2021). Linking elastic and electrical properties of rocks using cross-property DEM. *Geophysical Journal International*.

Eshelby, J. D. (1957). The determination of the elastic field of an ellipsoidal inclusion, and related problems. *Proceedings of the Royal Society of London A*, 241(1226):376–396.

Han, T., Best, A. I., Sothcott, J., and MacGregor, L. M. (2011). Joint elastic-electrical properties of reservoir sandstones and their relationships with petrophysical parameters. *Geophysical Prospecting*, 59(3):518–535.

Torquato, S. and Haslach Jr, H. (2002). Random heterogeneous materials: microstructure and macroscopic properties. *Appl. Mech. Rev.*, 55(4):B62–B63.

Waxman, M. H. and Smits, L. J. M. (1968). Electrical conductivities in oil-bearing shaly sands. *Society of Petroleum Engineers Journal*, 8(02):107–122.