Partial Differential Equations - MSO 203B

Assignment 1 - Fourier Series

Tutorial Problems

1. Find the Fourier series of $f(x) = \begin{cases} x;\ 0 \leq x \leq \pi \\ 0;\ -\pi \leq x < 0 \end{cases}$ and f is extended periodically to whole of $\mathbf{R}$.

2. Find the Fourier series of $f(x) = \begin{cases} 1;\ 0 \leq x < \frac{1}{2} \\ 0;\ \frac{1}{2} \leq x < 1 \\ -1;\ -\frac{1}{2} \leq x < 0 \\ 0;\ -1 \leq x < \frac{1}{2} \end{cases}$ and f is extended periodically to whole of $\mathbf{R}$.

3. An elastic string of length 4m with fixed ends is raised by 2m and then released from rest. Assume the displacement satisfies $u_{tt} - 25u_{xx} = 0$ for $0 \leq x \leq 4;\ t \geq 0$. Describe the motion of the string in terms of a Fourier series solution.

4. Prove that $\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\ dx = 2a_0^2 + \Sigma_{n\in\mathbf{N}}(a_n^2 + b_n^2)$ where a_i and b_i are Fourier coefficients of the square summable function f.

5. Comment on the convergence of $\Sigma_{n\in\mathbf{N}}\frac{(-1)^n}{n^2}$ using the Fourier series of $f(x) = x^2$ on $[-\frac{1}{2}, \frac{1}{2}]$.

Practice Problems

1. Show the convergence of the series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ...$ using the Fourier Convergence theorem on the function $g(x) = \begin{cases} -k;\ -\pi < x < 0 \\ +k;\ 0 < x < \pi \end{cases}$

with $g(x + 2\pi) = g(x)$.

2. Prove that all functions of period π forms a vector space.

3. Find the Fourier series of $\cos x$ in $[-\pi, \pi]$.

4. Find the Fourier Series of the sawtooth wave function which is 2π periodic and is given by $f(x) = x + \pi;\ -\pi < x < \pi$

5. Let f be an odd function, write the Fourier series of such a function in terms of only b_n's.

6. Find the Fourier half range expansion of the function $f(x) = e^x$ on $0 < x < 1$ (Either one of sine/cosine is OK).

7. Find the Fourier sine series for the function $f(x) = \begin{cases} x;\ 0 < x < 1 \\ 2 - x;\ 1 < x < 2 \end{cases}$

END