Partial Differential Equations - MSO 203B

Assignment 2 - Sturm-Liouville BVP

Tutorial Problems

1. Which of the following domain admits a continuously differentiable boundary:
   - $\{(x, y) : \ x^2 + y^2 < 1\}$.
   - $\{(x, y); \ \max(|x|, |y|) < 1\}$.
   - $\{(x, y); \ |x| + |y| = 1\}$
2. Classify the following based on their linearity/semilinearity/quasilinearity and fully nonlineraity.
   - $u_x + u_y = 1$
   - $u_x + xu_y = u^2$
   - $u_x + uu_y = 0$
   - $\mathrm{div}(|\nabla u|^{p-2}\nabla u) = 0$ for $p > 2$.
   - $\det(D^2 u) = 1$
3. Solve the following PDE:
   - $u_{xx} = u$
   - $u_{xy} + u_x = 0$
4. Show that for smooth functions u, v in a smooth, bounded domain one has the following

$$\int_{\Omega} (u\Delta v - v\Delta u) \ dx = \int_{\partial\Omega} (u\frac{\partial v}{\partial \gamma} - v\frac{\partial u}{\partial \gamma}) \ dS$$

   where γ is the unit outward normal vector.

5. Verify that $u(x, y) = x^2+t^2$ solves the wave equation given by $u_{tt} = u_{xx}$.
6. Show that the solutions $z(x, y)$ of $yz_x = xz_y$ represents surface of revolution.

END