Partial Differential Equations - MSO 203B

Assignment 2 - Sturm-Liouville BVP

Tutorial Problems

1. Solve the equation $y''(x) = x\ ; x \in (0, \pi)$ using Fourier Series subject to the boundary condition $y(0) = y(\pi) = 0$.
2. Show that the Chebychev's equation given by $(1-x^2)y'' - xy' + n^2y = 0$ in $[-1, 1]$ is a singular SL-BVP.
3. Let (λ_1, y_1) and (λ_2, y_2) be the two distinct eigenpair of a regular SL-BVP such that $\lambda_1 < \lambda_2$. Show that y_2 admits a zero in between two consecutive zeroes of y_1.
4. Reduce the problem $y^{''} + xy' + \lambda y = 0$ into an appropriate SL-BVP e.q., in self-adjoint form.
5. Comment on the eigen pairs of the problem $y^{''} + \lambda y = 0$; $\lambda > 0$ subject to the boundary condition $y'(0) = h_1 y(0)$ and $y'(l) = -h_2 y(l)$ where $h_1, h_2 > 0$.

Practice Problems

1. Solve the problem $y^{''} + \lambda y = 0\ ; y(0) = y(l) = 0$.
2. True/ False: There exists a continuous spectrum of eigenvalue for any regular SL-BVP.
3. True/ False: There exists a continuous spectrum of eigenvalue for some singular SL-BVP.
4. Expand $f(x) = x^2$ using the eigen functions of $y^{''} + \lambda y = 0$ subject to $y(0) = y(\pi); y'(0) = y'(\pi)$.

5. TRUE/FALSE: Any regular SL-BVP admits an unique solution.

END