

Partial Differential Equations - MSO 203B

Assignment 5 - Classification of 2nd Order PDE

Tutorial Problems

1. Classify the following PDE as per their ellipticity, parabolicity or hyperbolicity.
 - $u_{xx} \pm (\operatorname{sech}^4 x)u_{yy} = 0$.
 - $u_{xx} + (2 \operatorname{cosec} y)u_{xy} + (\operatorname{cosec}^2 y)u_{yy} = 0$.
 - $4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2$.
2. Show that the behaviour of a PDE based on ellipticity, parabolicity or hyperbolicity depends on the domain.
3. Reduce $y^2u_{xx} - x^2u_{yy} = 0$ into its canonical form.
4. Reduce the PDE $u_{xx} + x^2u_{yy} = 0$ into its canonical form.
5. Reduce $u_{xx} - 4u_{xy} + 4u_{yy} = \exp y$ into its canonical form and solve it.

Practice Problems

1. Reduce all the equations in problem 1 to its canonical form.
2. Solve the problem $u_{tt} - u_{xx} = 0$.

END