Merton’s Jump-Diffusion model: Delta hedging and model calibration

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Merton's Jump-Diffusion model

$$dS_t = (\mu + r)S_t dt + \sigma S_t dW_t + (Y - 1)S_t dN_t$$

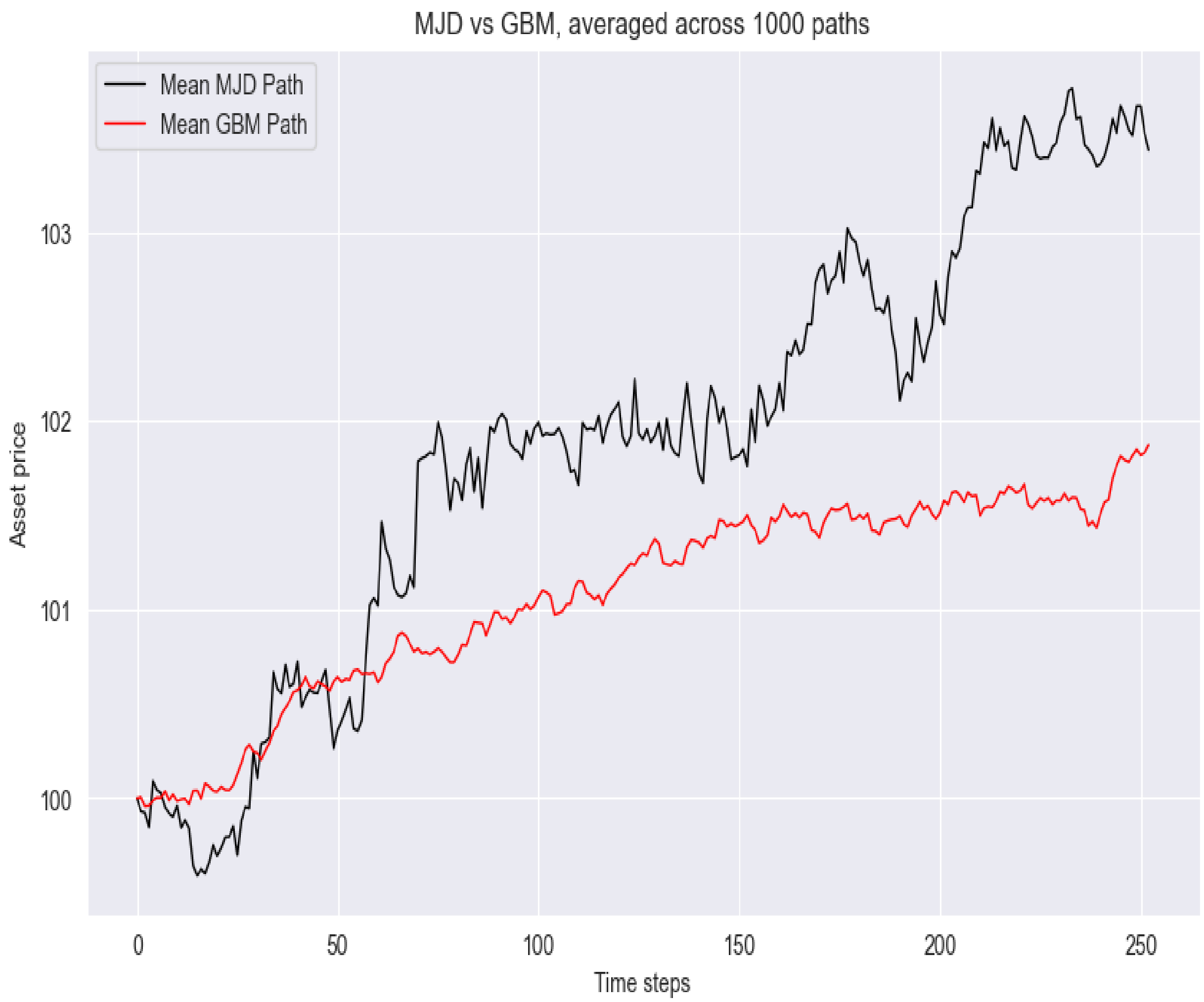
- Asset pricing model involving Geometric Brownian Motion with "kicks" to describe sudden market crashes/shocks.

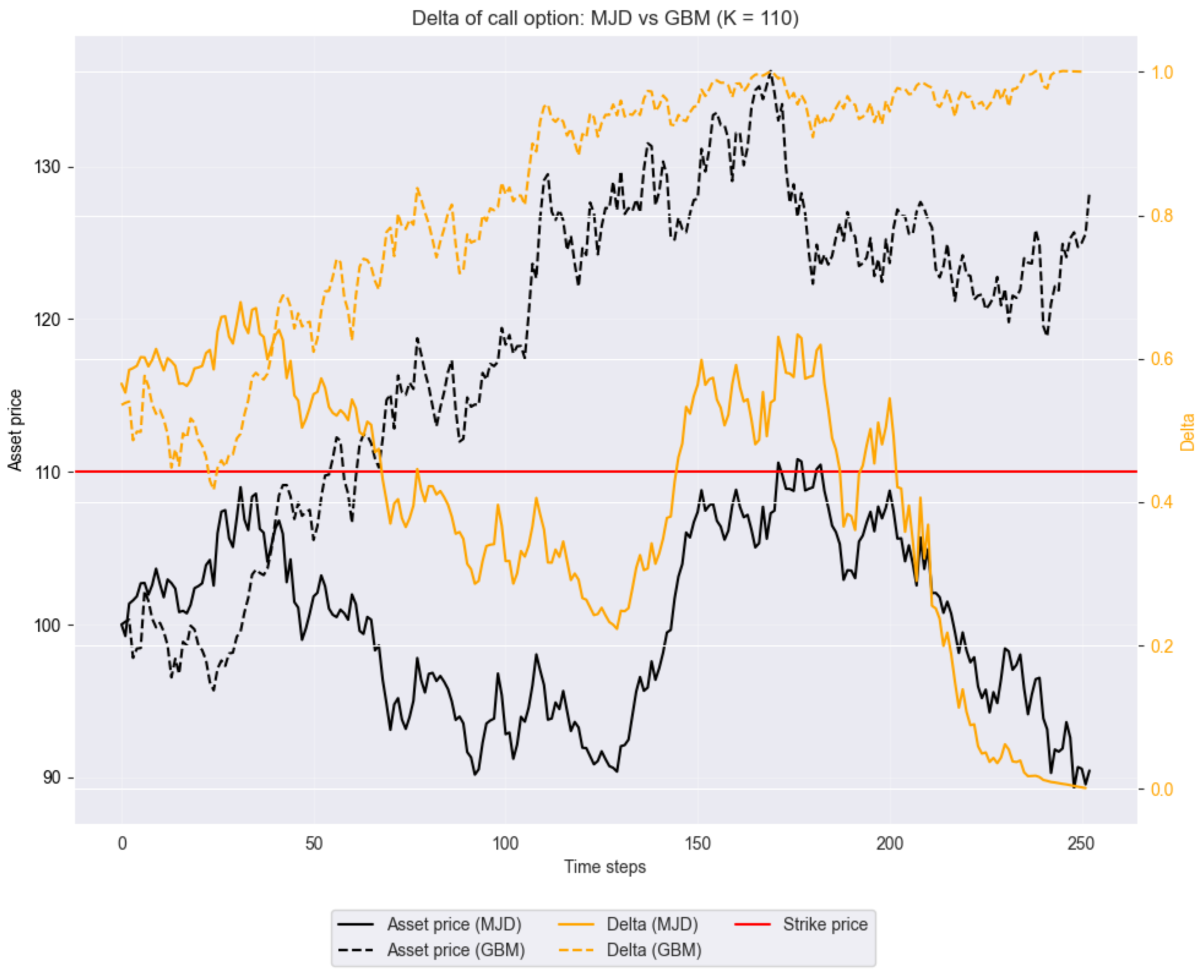
- Drift-Diffusion: The standard GBM smooth path.
- Jump process: Poisson process that models the *frequency* (λ) of jumps, and log-normal process for the *size* (α, δ) of jumps.

- MJD provides a more accurate simulation of asset prices, especially the "fat tails" seen in reality.

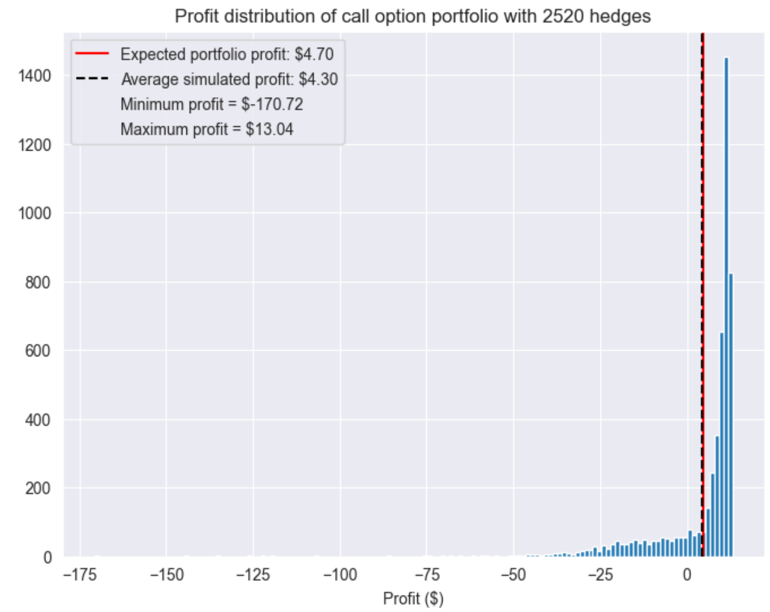
$$S_t = S_0 e^{\left(\mu - \lambda k + r - \frac{\sigma^2}{2}\right)t + N(n\alpha, \sigma^2 t + n\delta^2)}$$

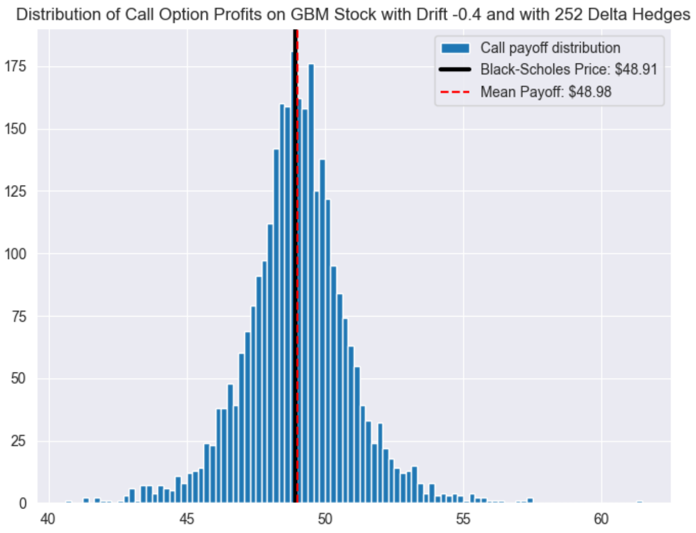
MJD vs GBM: S_t and Δ_{C_0}

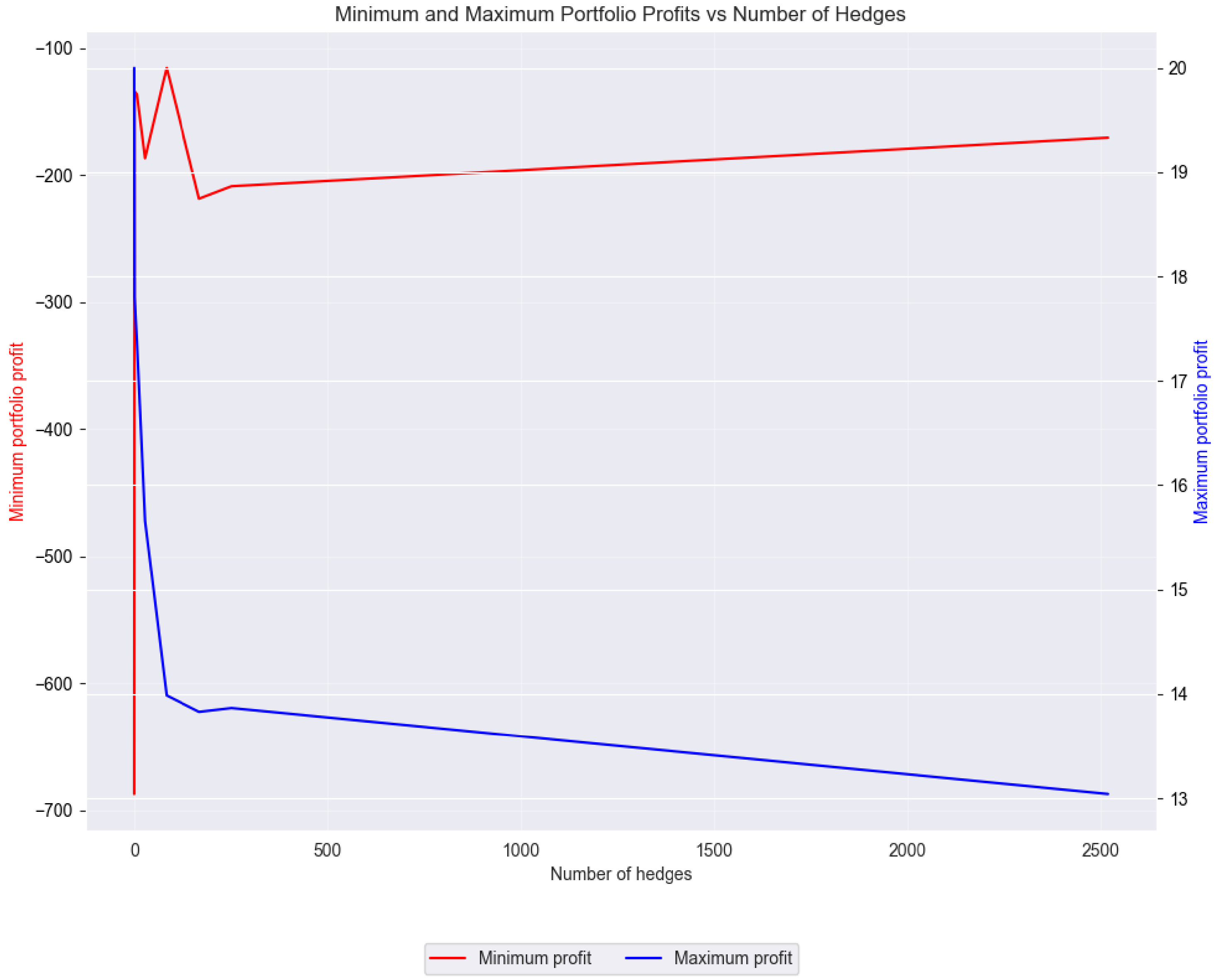




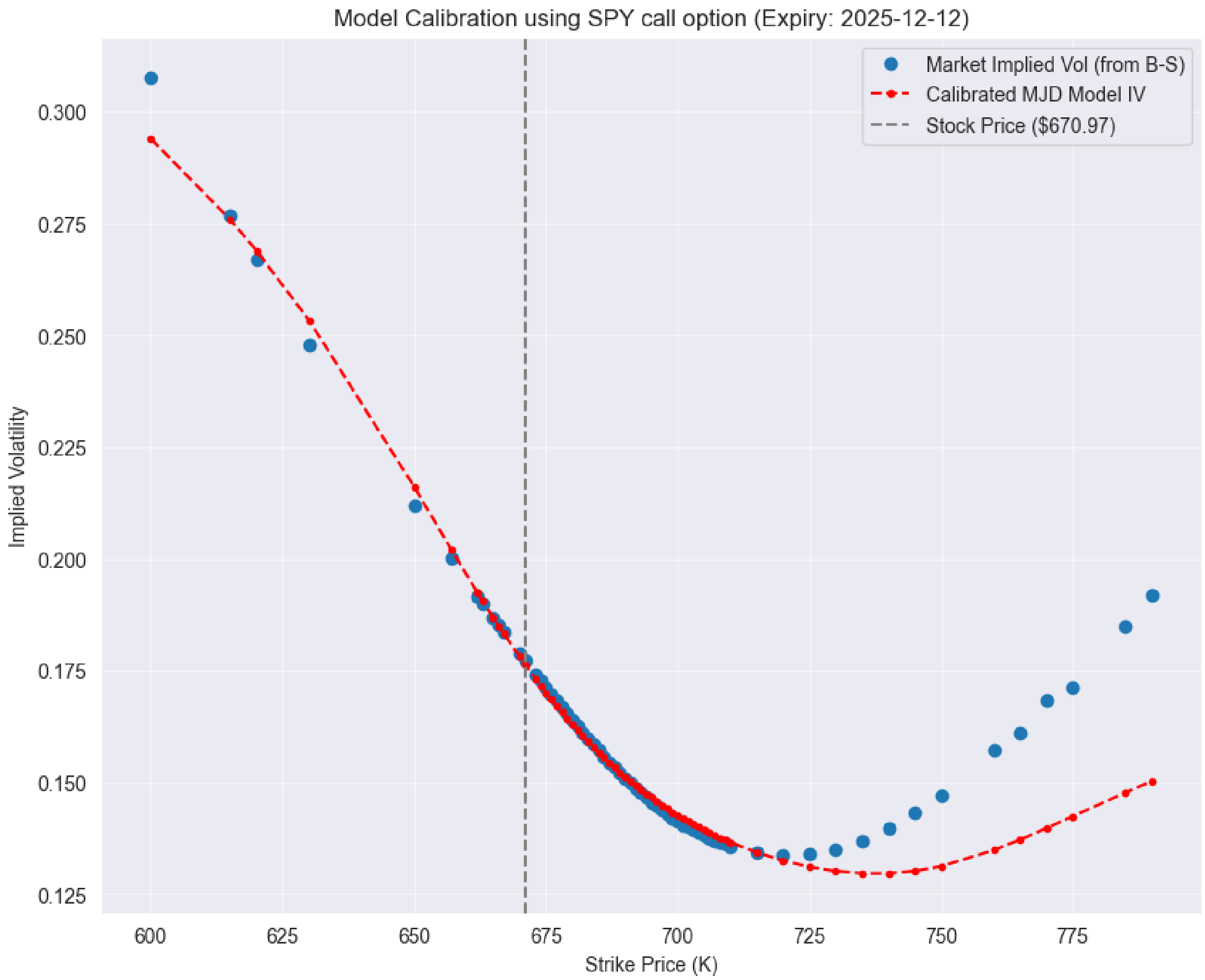
MJD and Delta Hedging

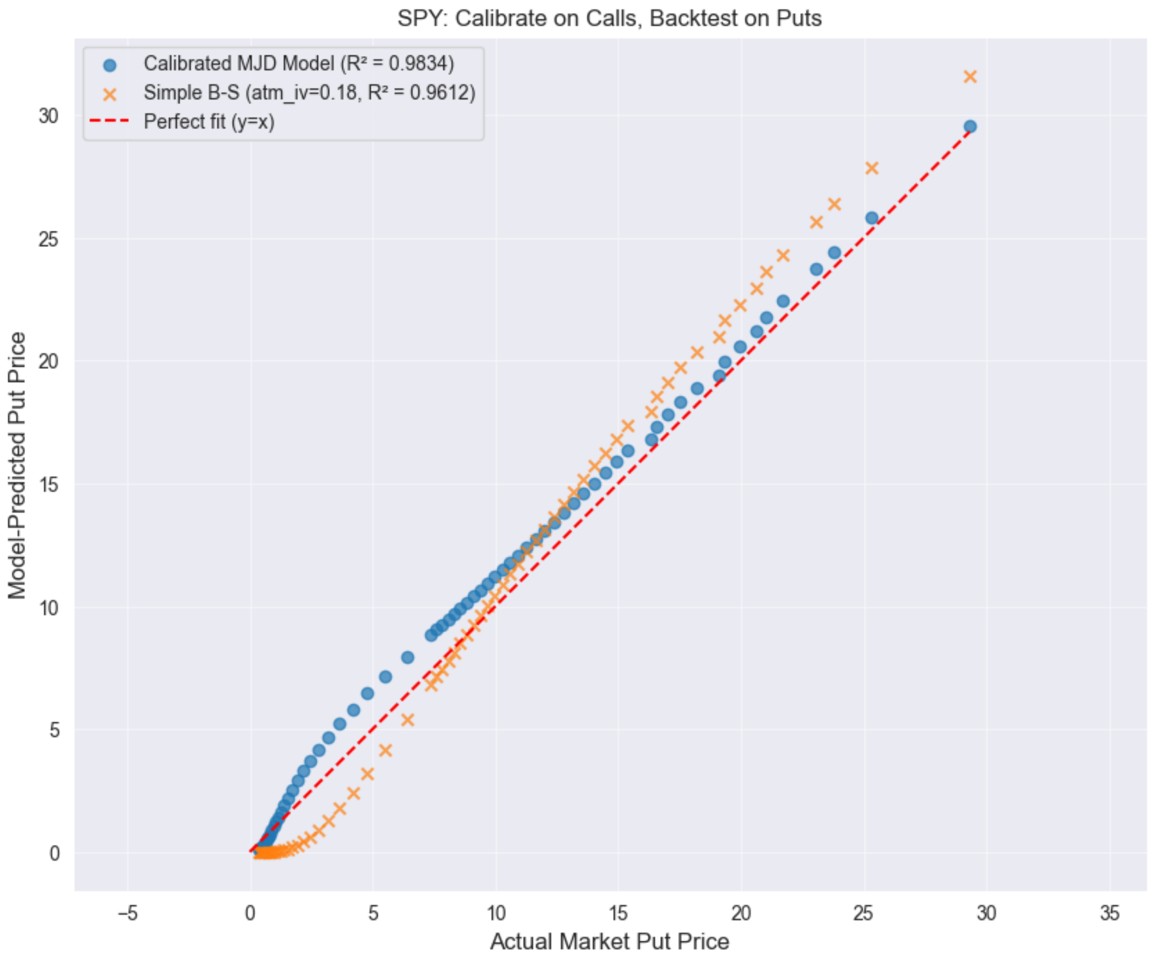






Calibration and Backtesting to market data





Conclusions & Hedging Implications

- **Pricing:** MJD is a superior model for pricing European options as it successfully captures the volatility smile, which is a persistent feature of real markets.
- **Prediction:** The model demonstrates strong predictive power, accurately pricing unseen put options after being trained only on calls.

- **Hedging:**
    - While Delta hedging is possible, it is imperfect under MJD.
    - The model's "jump risk" is systemic and cannot be hedged away simply by trading the underlying asset.
    - Provides a theoretical explanation for why real-world "delta-neutral" portfolios still experience large, sudden losses and gains (i.e., "fat tails").