Characterizing evolutionary dynamics on a broader scale: a strain-space model for SARS-CoV-2

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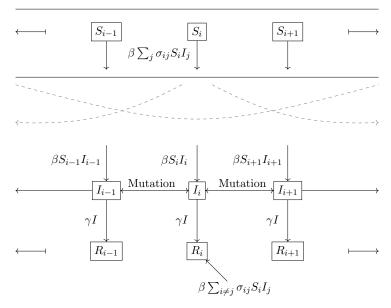
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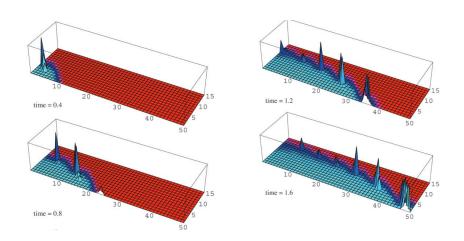
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September 25, 2022

Full strain lattice model in 1 dimension

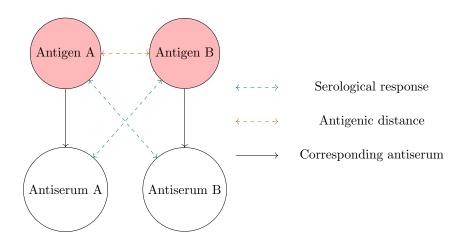


Extending the strain lattice to 2 dimensions

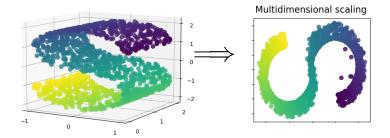


[Gog and Grenfell, 2002]

Quantifying antigenicity between viruses

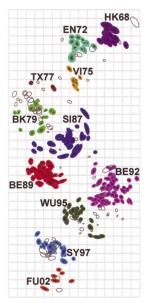


Projecting to low dimensions while preserving distances

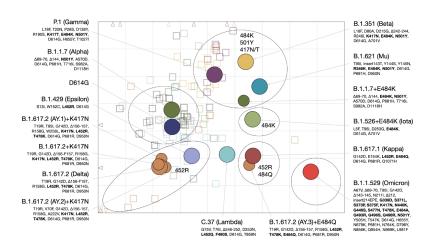


[Pedregosa et al., 2011]

Approximating genomic data in 2 dimesions

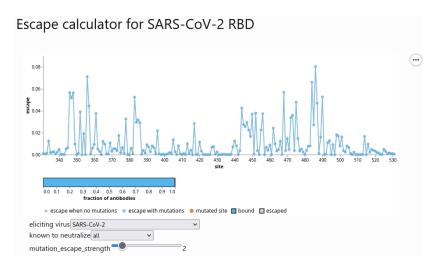


Mapping SARS-CoV-2 in 2D



[Wilks et al., 2022]

Methods of adding more genomes: polyclonal antibody binding studies



[Greaney et al., 2022]

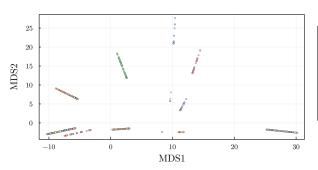
- 1. Find the closest lineages in existing antigenic map to g_i , and g_j , and corresponding points x_i and x_j
- 2.
- 3.
- 4.

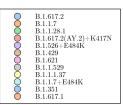
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- 2. Determine difference in polyclonal binding affinity $B(g_i, g_j)$
- 3.
- 4.

- 1. Find the closest lineages in existing antigenic map to g_i , and g_j , and corresponding points x_i and x_j
- 2. Determine difference in polyclonal binding affinity $B(g_i, g_j)$
- 3. Distance between g_i and g_j is $||x_i x_j|| + B(g_i, g_j)$
- 4.

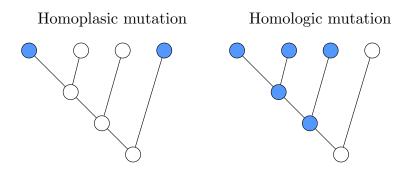
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- 4. Use these distances as inputs to multidimensional scaling

Antibody Binding map



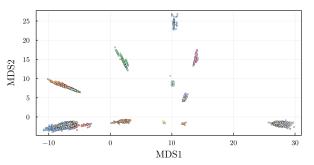


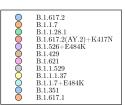
Methods of adding more genomes: homoplasic sites



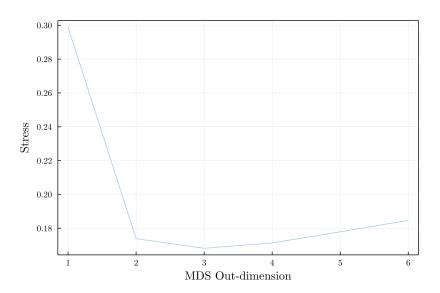
[Page and Holmes, 2009]

Homoplasic mutations map





Evaluating the MDS approximation



Model parameters/variables

Symbol	Description
\overline{N}	Size of variant grid
K	Total population
S_{ij}	Population susceptible to variant $(i, j) \in [0, N]^2$
I_{ij}	Population infected by variant $(i, j) \in [0, N]^2$
R_{ij}	Recovered from variant $(i, j) \in [0, N]^2$
V_{ij}	Immune by vaccination to variant $(i, j) \in [0, N]^2$
σ_{ijkl}	Probability that exposure to variant (i, j) causes
	immunity
	to variant (k, l)
eta_{ij}	Transmission rate of variant (i, j)
$v_{ij}(t)$	vaccination rate at time t against variant (i, j)
s(t)	stringency at t
ξ	Recovery rate of all strains
γ	Rate of immunity loss of all strains

Table of symbols for Model $2\,$

Model Equations

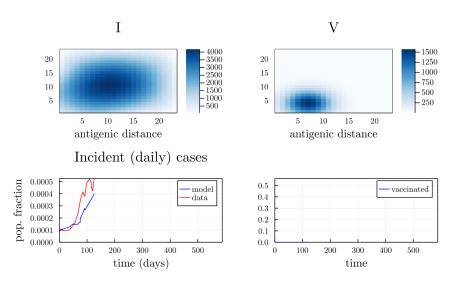
$$\frac{S_{ij}}{dt} = -\sum_{kl} s(t)\beta_{kl}\sigma_{ijkl}S_{ij}I_{kl} + \gamma(R_{ij} + V_{ij}) - v_{ij}(t)S \qquad (1)$$

$$\frac{I_{ij}(t)}{dt} = s(t)\beta_{ij}S_{ij}I_{ij} - \xi I_{ij} + M\left(-4I_{ij} + I_{i-1,j} + I_{i+1,j} + I_{i,j-1} + I_{i,j+1}\right)$$
(2)

$$\frac{R_{ij}(t)}{dt} = \sum_{kl \neq ij} s(t)\beta_{kl}\sigma_{ijkl}S_{ij}I_{kl} + \xi I_{ij} - \gamma R_{ij} + v_i j(t)S$$
 (3)

$$\frac{V_{ij}(t)}{dt} = v_{ij}(t)S - \gamma V_{ij} \tag{4}$$

Model Dynamics



Case importation

- Evolution of Omicron represents a huge shift in antigenic space
- Intermediate steps not represented in data
- We add imports to solve this problem

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Mutation homoplasy

