

# Coupled models of structured contagion processes in human-environment systems

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# Introduction

# Contagious processes

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- A contagious process is one that is able to propagate itself through a set of hosts (Peterson (1999))
- Contagious processes shape our lives in many important ways
- Current situation has thrust the importance of modeling infections into popular consciousness

# Coupled systems

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- Coupling can often dramatically affect stability, e.g. Kapitza's pendulum
- Human-environment systems focus on dynamics of humanity and the environment viewed as a whole
- My thesis demonstrates that this perspective provides useful answers to important problems

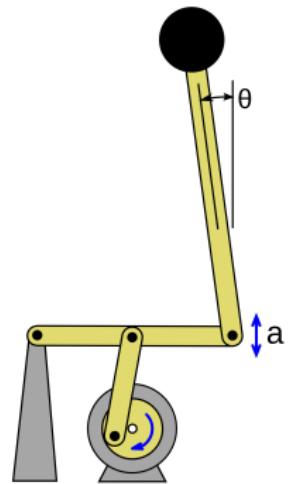


Figure: Kapitza's pendulum

Source: Chris Burks

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# First Project

# Background

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- Population beliefs about the usage of non-pharmaceutical interventions (NPIs) can have a major effect on the epidemic landscape
- Population age distribution is a key factor in both NPI usage and COVID-19 mortality
- Vaccine rollout important to reducing mortality in the long term (Bubar et al., 2020; Buckner, Chowell, and Springborn, 2020)

## Outline of project

- ① Describe age-structured compartmental model of COVID-19 infection in a population, coupled to model of social distancing dynamics.
- ② Fit model to data from Ontario, Canada
- ③ Evaluate outcomes from different vaccination strategies, comparing vaccination of vulnerable populations vs vaccination of susceptible populations

# Compartmental model overview

## Disease Compartments

$S_i(t)$  : Susceptible

$S_{2,i}(t)$  : Vaccinated but still  
susceptible

$V_i(t)$  : Vaccinated and  
immune

$E_i(t)$  : Exposed

$P_i(t)$  : Pre-symptomatic

$I_{a,i}(T)$  : Infectious and  
asymptomatic

$I_{s,i}(t)$  : Infectious and  
symptomatic

$R_i(t)$  : Recovered

where  $i = 1 \dots 16$  comprises age  
structure

## Social compartments

$x(t)$  : Uses NPIs

$1 - x(t)$  : Does not use NPIs

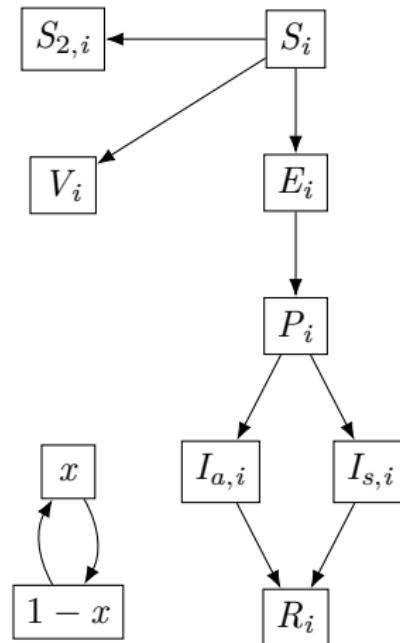


Figure: Compartments

# Game theory as a model of NPI adoption

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P1 \ P2	use NPI	don't use NPI
use NPI	low risk, NPIs unpleasant	med risk
don't use NPI	med risk, NPIs unpleasant	high risk

Table: NPI adoption as a two-player game (between P1 and P2)

# Vaccination strategies

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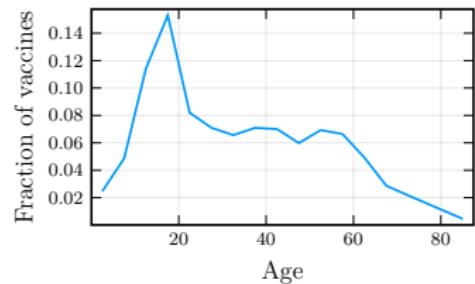
References

We compare four vaccination strategies

- > 60 first
- < 20 first
- Uniform
- Contact-based

with respect to reduction in cumulative mortality after 5 years.

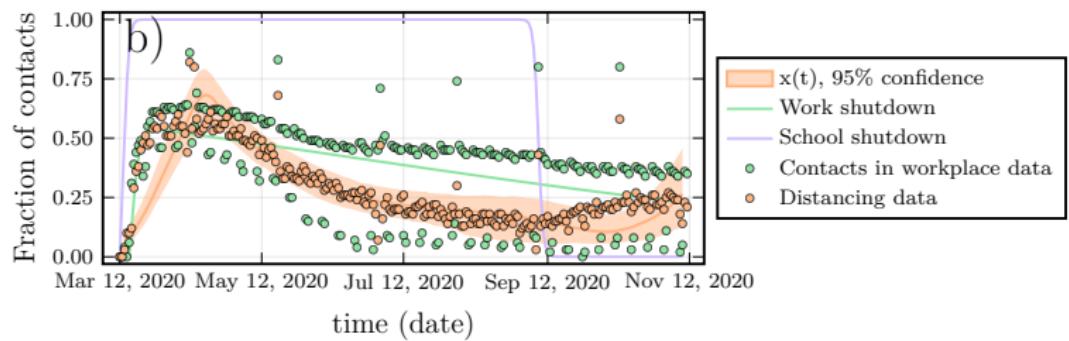
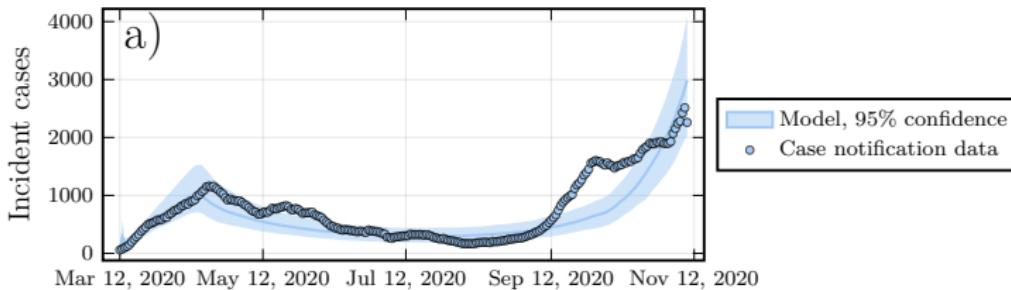
Contact-based vaccination strategy



**Figure:** The contact-based strategy is the normalized leading eigenvector of the sum of the contact matrices

# Parameterization

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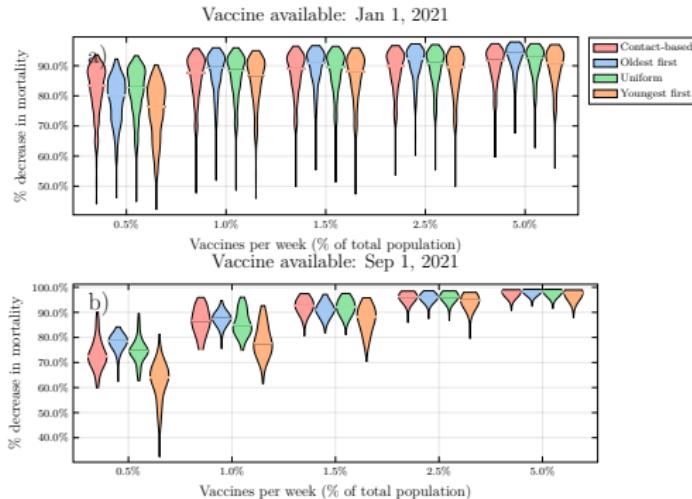
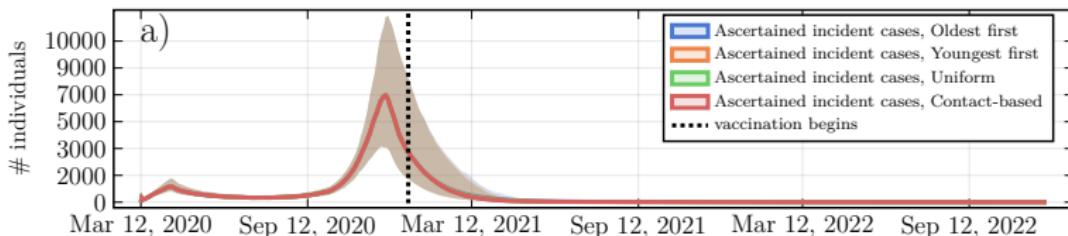
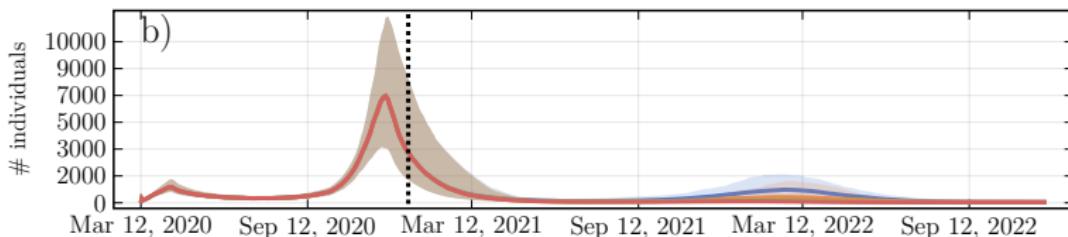


Figure: Percentage reduction in cumulative mortality due to COVID-19 after 5 years with respect to  $\psi$ , expressed as a percentage of the total population per week. Here  $v_{D_i} = v_{T_i} = 0.75$ , shutdown at 200% of first wave. Percentage reductions are relative to no vaccination.

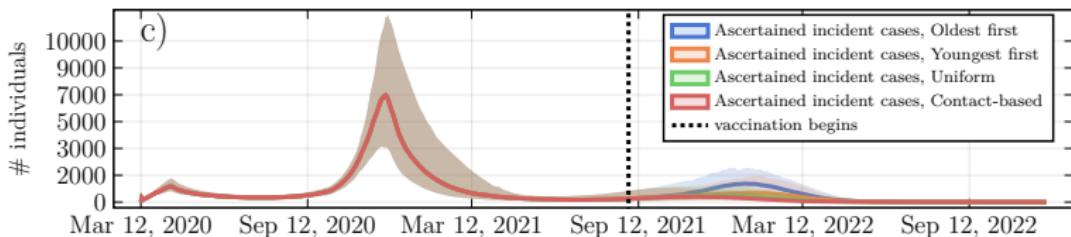
Vaccination begins on Jan 1, 21, 1.5% of pop. vaccinated per week



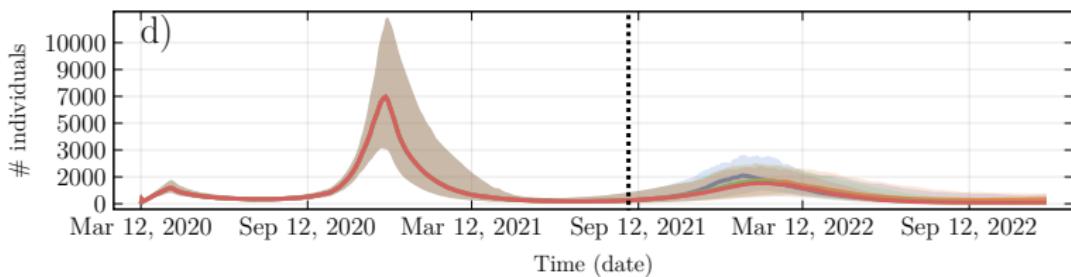
Vaccination begins on Jan 1, 21, 0.5% of pop. vaccinated per week



Vaccination begins on Sep 1, 21, 1.5% of pop. vaccinated per week



Vaccination begins on Sep 1, 21, 0.5% of pop. vaccinated per week



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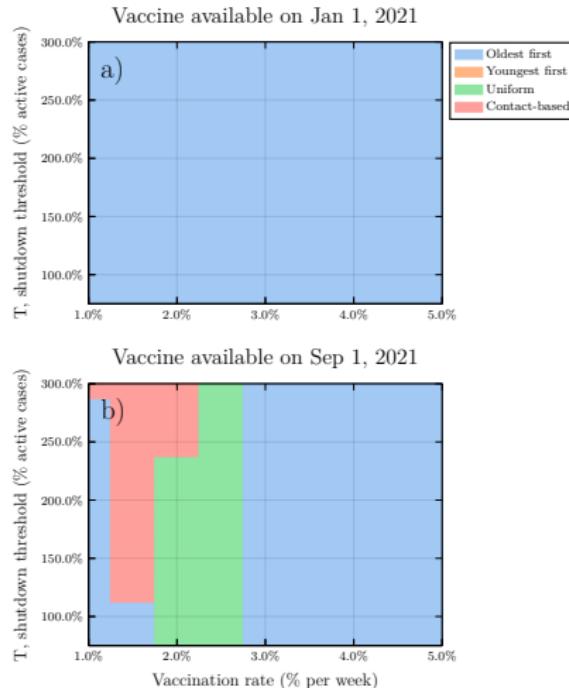


Figure: Each parameter pair is colored according to the strategy that prevents most deaths on average, over all realizations of the model.

# Results

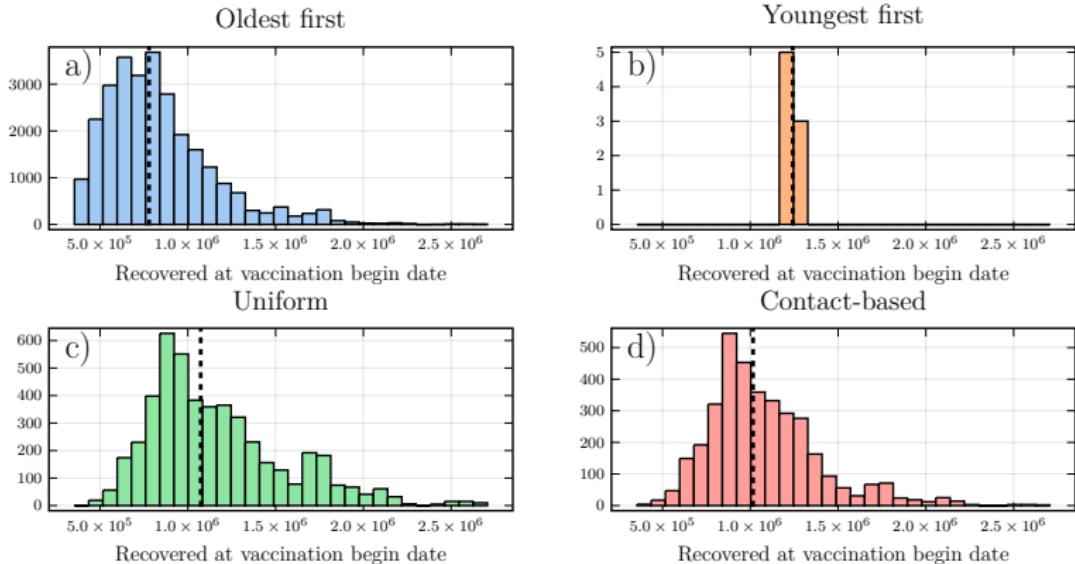
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**Figure:** Histogram of no. recovered at vaccination begin date, according to best strategy for that realization, over all parameter values in sensitivity analysis. Vertical lines are the median.

# Chapter Conclusion

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- We described an age structured compartmental model of Sars-CoV-2 infection and vaccination coupled to a social model
- Showed that sometimes transmission interrupting strategies can be more effective
- Depends on the pre-existing immunity in the population

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## Second Project

## Project 2

- Invasive forest pests cause incredible damage to ecosystems and lumber resources
- Evidence shows that movement of firewood is a major long distance vector (Koch et al. (2014))
- Education and awareness is a major way we try to reduce this vector



Figure: an Emerald Ash Borer, which devastated Ash populations in North America

# Outline of project

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References

- ① Adapt a model such as Barlow et al. (2014) to a larger, more realistic network
- ② Use model to compare three possible prevention measures
  - Education/awareness
  - Inspection of moved firewood
  - Quarantine of highly susceptible forest patches
- ③ Assess measures across a range of parameter values and time horizons

# Model

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$$\frac{dS_i}{dt} = \underbrace{rS_i \left(1 - \frac{(S_i + I_i)}{K}\right)}_{\text{Logistic Growth Of Forest}} - \underbrace{AS_i(I_i + B_i)\theta_k(I_i - I_a)}_{\text{Infestation term}} \quad (1)$$

$$\frac{dI_i}{dt} = \underbrace{-\gamma I_i}_{\text{Death of infested trees}} + \underbrace{AS_i(I_i + B_i)\theta_k(I_i - I_a)}_{\text{Susceptibles become infested}} - \underbrace{d \sum_{j=1, j \neq i}^N P_{j,i}(1 - C_e)(1 - L_j)I_j}_{\text{Total infested wood leaving due to transport}} \quad (2)$$

$$\frac{dB_i}{dt} = \underbrace{-\gamma B_i}_{\text{Decay of firewood}} + d \underbrace{\sum_{j=1, j \neq i}^N P_{i,j}(1 - C_e)(1 - L_j)I_j}_{\text{Import of fallen wood}} \quad (3)$$

$$\frac{dL_i}{dt} = \sigma L_i(1 - L_i) \left( \underbrace{U}_{\text{Net cost to transport firewood}} + \underbrace{s(2L_i - 1)}_{\text{Social influence term}} + \underbrace{fI_i}_{\text{Impact of infestation}} \right) \quad (4)$$

We use  $T_i(t)$ , computed from equation 5 to be the total number of infested trees in patch  $i$  up to time  $t$ .

$$\frac{dT_i}{dt} = AS_i(I_i + B_i)\theta_k(I_i - I_a) \quad (5)$$

Define  $T(t) = \frac{1}{N} \sum_{i=1}^N T_i(t)$  to be the average total number of infested trees up to  $t$ .

# Data

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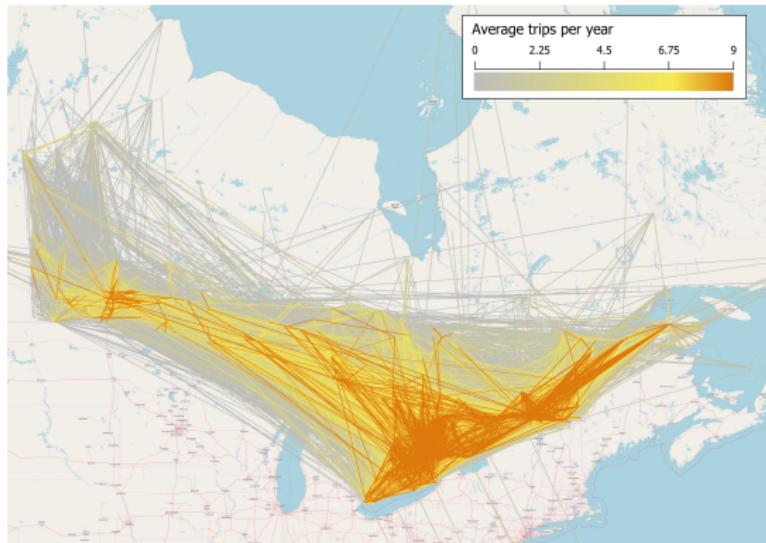


Figure: Travel network used to weight edges in firewood transport network

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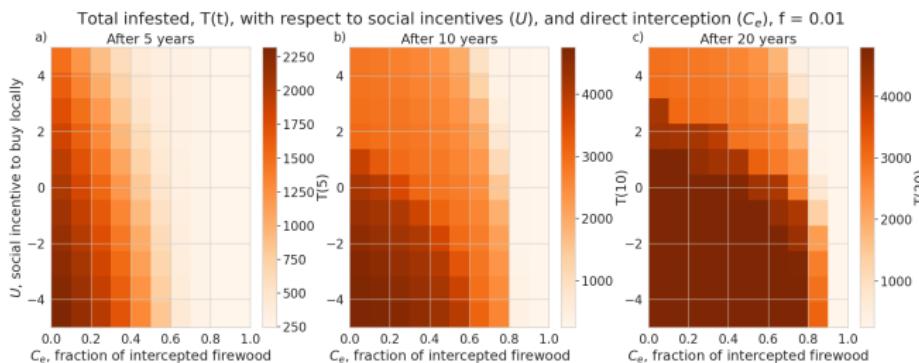


Figure: Total number of infested trees per node over 5 (a), 10 (b), and 20 (c) years

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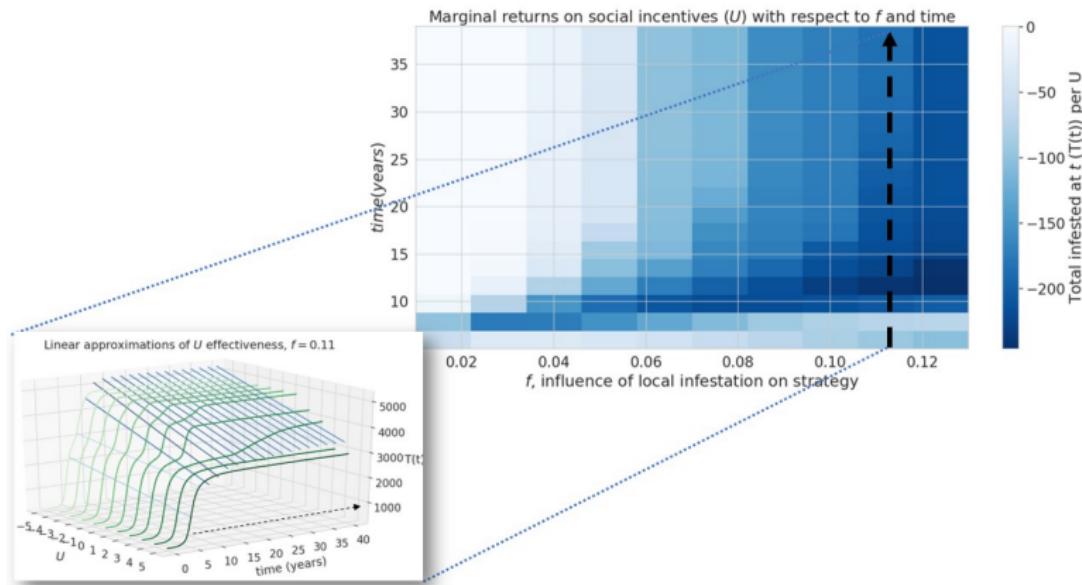


Figure: Efficacy of social incentives on infestation after time  $T$ .  
Inset graph shows an example of cross-section along the line  
 $f = 0.11$

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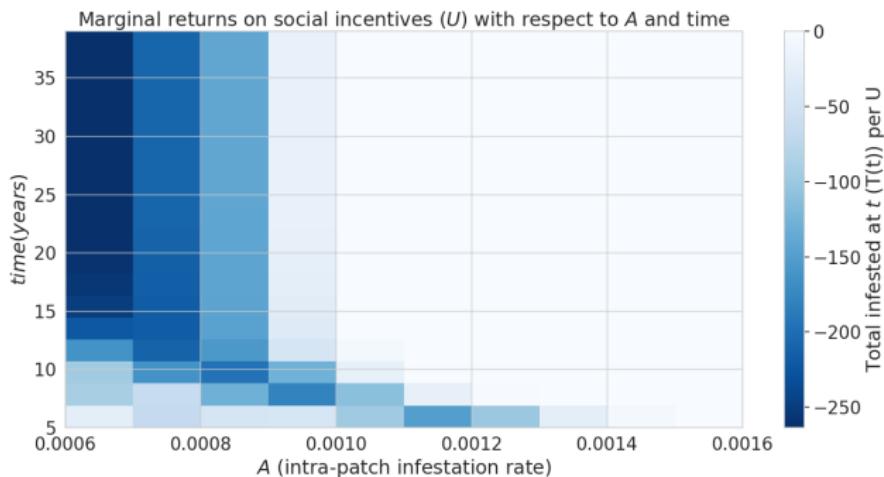


Figure: Efficacy of social incentives on infestation after time period  $T$  with respect to  $A$ , the intra-patch infestation parameter.

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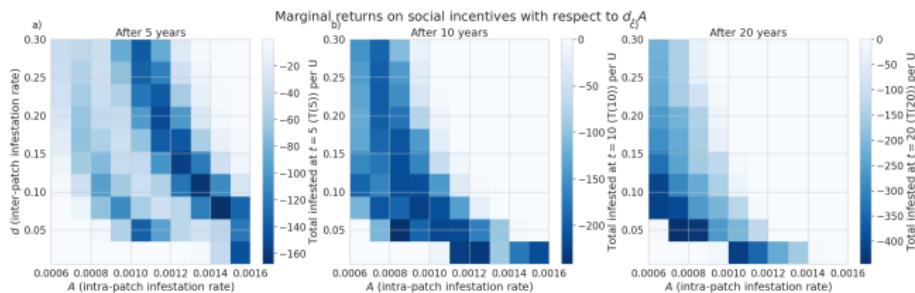


Figure: Efficacy of social incentives on infestation after time  $T$   
intra-patch spreading rate  $A$ , affects infestation outcomes.

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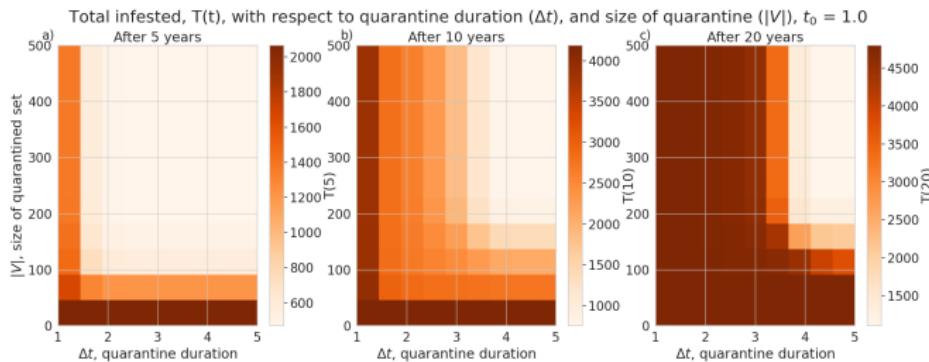


Figure: Average total infested trees ( $T(t)$ ) after 5, 10 and 15 years (panels a), b), and c) respectively), assuming the quarantine begins one year after the pest is introduced.

## Discussion

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- Firewood inspection not likely to be effective in implementation
- Education, represented as social incentives, are able to decrease infection in the short term, but dependent on pest-specific parameters
- Patch quarantine can be effective if sufficiently many patches are isolated, and the pest is detected early

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## Third Project

# Background

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- Wildfire and bark beetles are disturbances integral to coniferous forest ecosystems in the western cordillera of North America (Kaufmann et al., 2008)
- Bark beetle outbreaks have always been destructive, but seem to be worse in recent decades
- Literature on causal relationship between bark beetle outbreaks and wildfire is extensive but inconclusive (Axelson, Alfaro, and Hawkes (2009))
- Existing modelling of these two coupled disturbances is sparse

# Project Outline

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- Extend existing model of Duncan et al. (2015) to include wildfires
- Explore parameter regime of extended model
- Introduce forest stand thinning procedures to reduce MPB outbreaks
- Show that these stand thinning procedures are able to work due to increased stand heterogeneity

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$$j_{n+1,1} = dJ_n + I_{n-2} + F_n \quad (6a)$$

$$j_{n+1,k} = (1-d)j_{n,k-1} - \frac{\alpha_1}{T} P_n j_{n,k-1}, \quad k = 2 \dots K-1, K \quad (6b)$$

$$S_{n+1} = S_n + (1-d)j_{n,K} - \left( I_n + \frac{\alpha_2}{T} P_n I_n \right) - \frac{\alpha_2}{T} P_n (S_n + (1-d)j_{n,K}) - \sigma_F \xi_n \quad (6c)$$

$$I_{n+1} = r_1 I_n e^{-\beta_1(T-S_{n+1})} - \frac{\alpha_2}{T} P_n I_n + \sigma_I \xi_n \quad (6d)$$

$$F_{n+1} = P_n \left[ \frac{\alpha_1}{T} \sum_{k=1}^{K-1} j_{n,k} + \frac{\alpha_2}{T} (S_n + (1-d)j_{n,K}) + \frac{\alpha_2}{T} I_n \right] + \sigma_F \gamma_n \quad (6e)$$

$$P_n = T - \sum_{i=1}^n F_i e^{-\kappa(n-i)} \quad (6f)$$

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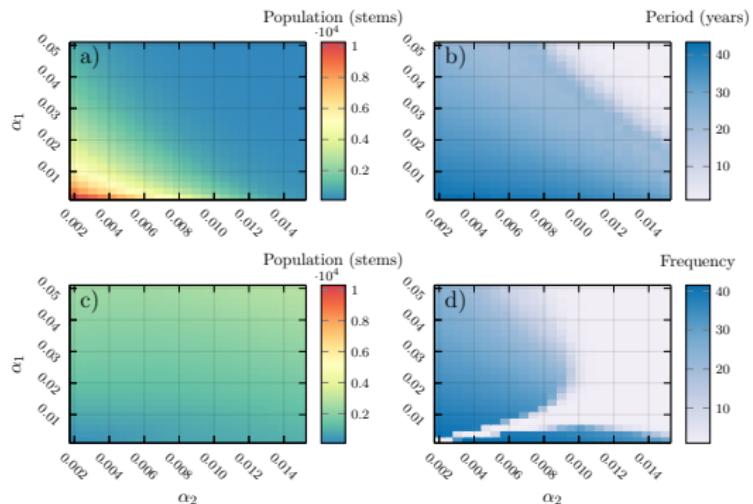
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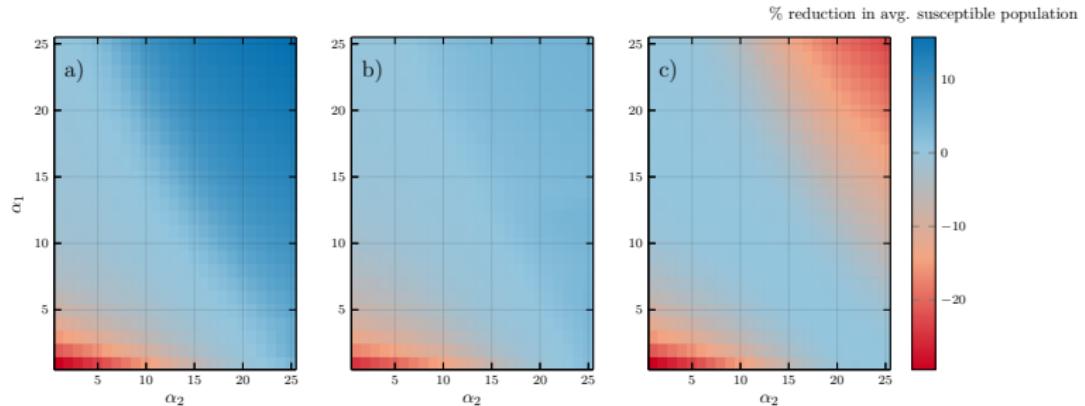
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**Figure:** Panels: a) Average size of largest MPB population, b) Average frequency of MPB outbreaks, c) Average size of largest fire season, d) Average frequency of severe fire seasons. All measured at equilibrium.

# Results



**Figure:** Percentage change in average susceptible (mature) forest population compared to no FTP with a)  $\tau = 0.15, m = 8$ , b) with  $\tau = 0.15, m = 8$  applied every 5 years, c) controlled burning with  $\tau = 0.15, m = 8$ , with respect to burning rates  $\alpha_1, \alpha_2$ .

# Results

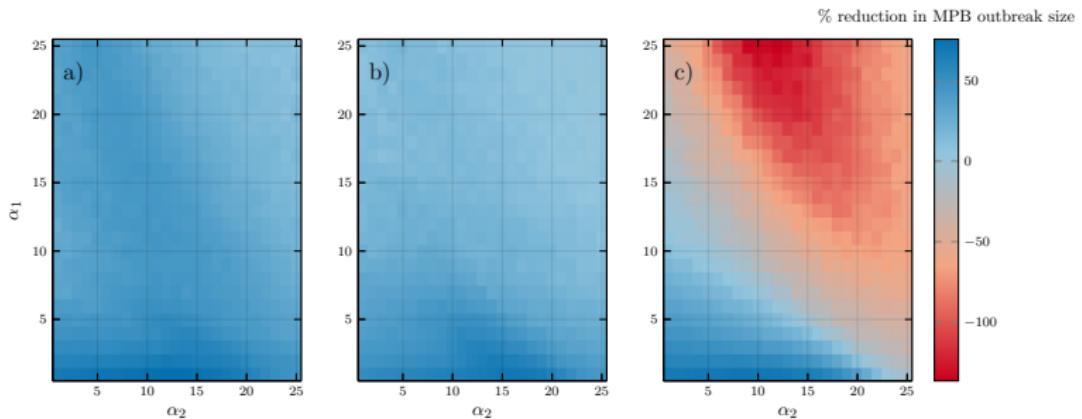
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**Figure:** Percentage change in maximum MPB infestation size within 500 year period under FTP with a)  $\tau = 0.15, m = 8$ , b) with  $\tau = 0.15, m = 8$  applied every 5 years, c) controlled burning with  $\tau = 0.15, m = 8$ , with respect to burning rates  $\alpha_1, \alpha_2$ .

# Chapter Conclusion

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- We show that increasing fire prevalence is able to dampen MPB outbreaks by increasing stand heterogeneity
- Stand heterogeneity can also be increased through forest thinning or prescribed burning, which also dampens MPB outbreaks.
- These results are consistent with ecological evidence (Seidl et al. (2016) and Kaufmann et al. (2008))

# Overall conclusion

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- Chapter 1: developed a disease-behaviour model of COVID-19 to address questions about vaccine prioritization
- Chapter 2: we used a socio-ecological model of forest pest spread to compare the efficacy of measures to prevent invasive pest spread
- Chapter 3: we created a model of coupled MPB and wildfire dynamics to shed light on the stand dynamics of this ecosystem

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References

Thank you!

-  Axelson, Jodi N, René I Alfaro, and Brad C Hawkes (2009). “Influence of fire and mountain pine beetle on the dynamics of lodgepole pine stands in British Columbia, Canada”. In: *Forest Ecology and Management* 257.9, pp. 1874–1882.
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