Coupled models of structured contagion processes in human-environment systems

Peter C. Jentsch

Department of Applied Mathematics, University Of Waterloo

2021

Supervised by Chris T. Bauch and Madhur Anand

Background

- Population beliefs about the usage of non-pharmaceutical interventions (NPIs) can have a major effective on the epidemic landscape
- Population age distribution is a key factor in both NPI usage and COVID-19 mortality
- Vaccine rollout important to reducing mortality in the long term [**?**, **?**]

Research questions

How to prioritize vaccination rollout in a population undergoing COVID-19 infection, while both age distribution and social dynamics into account?

How does the effectiveness of vaccination prioritization change with respect to important unknown parameters (such as vaccine availability, population sentiment, vaccination rate, etc.)?

Outline of project

1. Describe age-structured compartmental model of COVID-19 infection in a population, coupled to model of social distancing dynamics.
2. Fit model to data from Ontario, Canada
3. Evaluate outcomes from different vaccination strategies, comparing vaccination of vulnerable populations vs vaccination of susceptible populations

Compartmental model overview

Disease Compartments

$S_i(t)$: Susceptible

$S_{2,i}(t)$: Vaccinated but still susceptible

$V_i(t)$: Vaccinated and immune

$E_i(t)$: Exposed

$P_i(t)$: Pre-symptomatic

$I_{a,i}(T)$: Infectious and asymptomatic

$I_{s,i}(t)$: Infectious and symptomatic

$R_i(t)$: Recovered

where $i = 1 \ldots 16$ comprises age structure

Social compartments

$x(t)$: Uses NPIs

$1 - x(t)$: Does not use NPIs

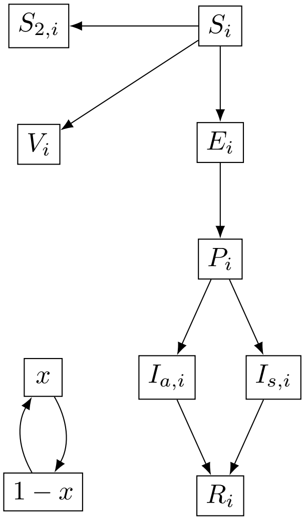


Figure: Compartments

Vaccination strategies

We compare four vaccination strategies

- > 60 first
- < 20 first
- Uniform
- Contact-based

with respect to reduction in cumulative mortality after 5 years.

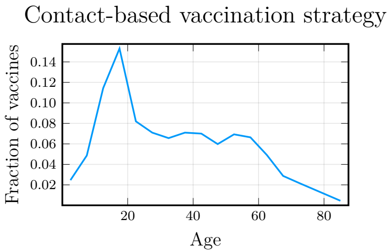
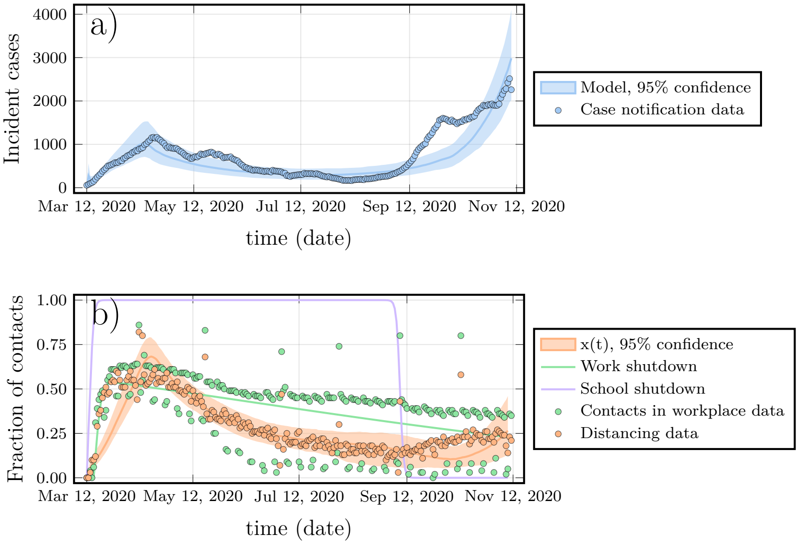


Figure: The contact-based strategy is the normalized leading eigenvector of the sum of the contact matrices

Parameterization



Results

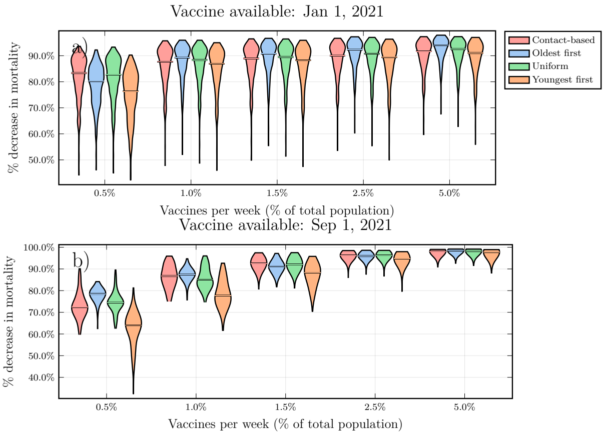
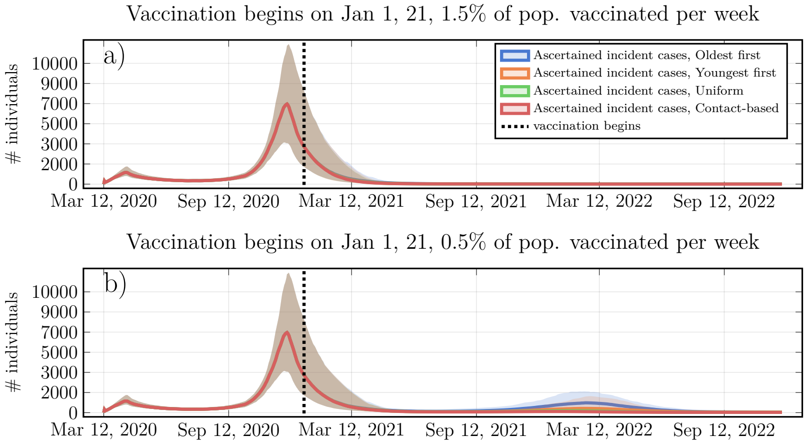
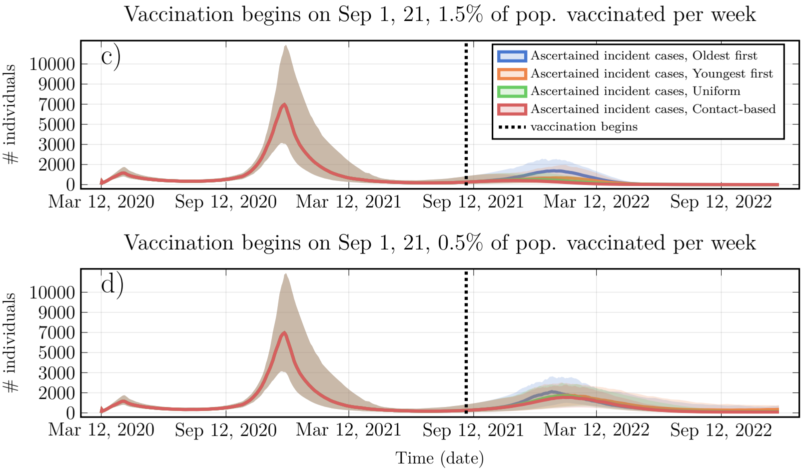


Figure: Percentage reduction in cumulative mortality due to COVID-19 after 5 years with respect to *psi*, expressed as a percentage of the total population per week. Here $v_{D_i} = v_{T_i} = 0.75$, shutdown at 200% of first wave. Percentage reductions are relative to no vaccination.


Vaccination begins on Jan 1, 21, 1.5% of pop. vaccinated per week
a)
individuals
10000
9000
7000
5000
3000
2000
0
Mar 12, 2020
Sep 12, 2020
Mar 12, 2021
Sep 12, 2021
Mar 12, 2022
Sep 12, 2022
Ascertained incident cases, Oldest first
Ascertained incident cases, Youngest first
Ascertained incident cases, Uniform
Ascertained incident cases, Contact-based
vaccination begins
Vaccination begins on Jan 1, 21, 0.5% of pop. vaccinated per week
b)
individuals
10000
9000
7000
5000
3000
2000
0
Mar 12, 2020
Sep 12, 2020
Mar 12, 2021
Sep 12, 2021
Mar 12, 2022
Sep 12, 2022


Vaccination begins on Sep 1, 21, 1.5% of pop. vaccinated per week
c)
individuals
Ascertained incident cases, Oldest first
Ascertained incident cases, Youngest first
Ascertained incident cases, Uniform
Ascertained incident cases, Contact-based
vaccination begins
10000
9000
7000
5000
3000
2000
0
Mar 12, 2020
Sep 12, 2020
Mar 12, 2021
Sep 12, 2021
Mar 12, 2022
Sep 12, 2022
Vaccination begins on Sep 1, 21, 0.5% of pop. vaccinated per week
d)
individuals
Time (date)

Results

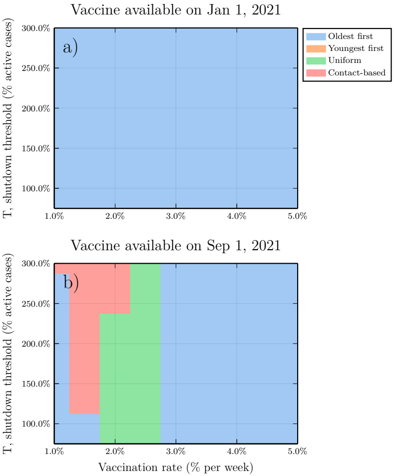


Figure: Each parameter pair is colored according to the strategy that prevents most deaths on average, over all realizations of the model.

Results

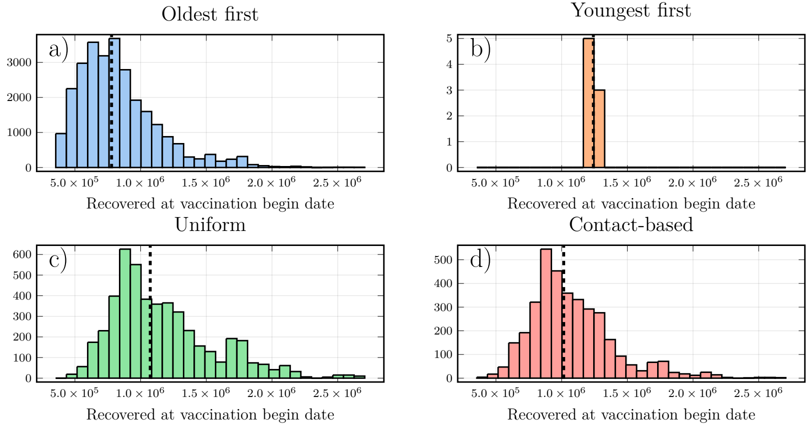


Figure: Histogram of no. recovered at vaccination begin date, according to best strategy for that realization, over all parameter values in sensitivity analysis. Vertical lines are the median.

Chapter Conclusion

- We described an age structured compartmental model of Sars-CoV-2 infection and vaccination coupled to a social model
- Showed that sometimes transmission interrupting strategies can be more effective
- Depends on the pre-existing immunity in the population

Project 2

- Invasive forest pests cause incredible damage to ecosystems and lumber resources
- Evidence shows that movement of firewood is a major long distance vector
- Education and awareness is a major way we try to reduce this vector

Figure: an Emerald Ash Borer, which devastated Ash populations in North America

Research questions

Which methods are effective in reducing forest pest spread?

When is each method most effective with respect to pest attributes?

Outline of project

1. Adapt a model such as Barlow et al. (CITE) to a larger, more realistic network
2. Use model to compare three possible prevention measures
   - Education/awareness
   - Inspection of moved firewood
   - Quarantine of highly susceptible forest patches
3. Assess measures across a range of parameter values and time horizons

Model

$$\frac{dS_i}{dt} = \underbrace{rS_i\left(1 - \frac{(S_i + I_i)}{K}\right)}_{\text{Logistic Growth Of Forest}} - \underbrace{AS_i(I_i + B_i)\theta_k(I_i - I_a)}_{\text{Infestation term}} \tag{1}$$

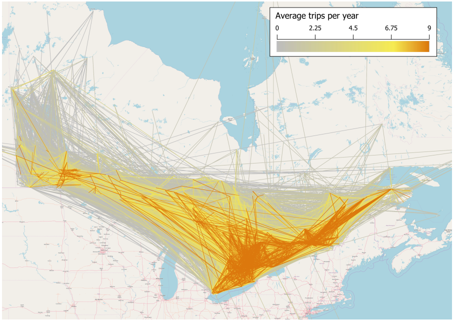
$$\frac{dI_i}{dt} = \underbrace{-\gamma I_i}_{\substack{\text{Death of}\\ \text{infested trees}}} + \underbrace{AS_i(I_i + B_i)\theta_k(I_i - I_a)}_{\text{Susceptibles become infested}} - \underbrace{d\sum_{j=1, j\neq i}^{N} P_{j,i}(1 - C_e)(1 - L_i)I_i}_{\substack{\text{Total infested wood}\\ \text{leaving due to transport}}} \tag{2}$$

$$\frac{dB_i}{dt} = \underbrace{-\gamma B_i}_{\text{Decay of firewood}} + \underbrace{d\sum_{j=1, j\neq i}^{N} P_{i,j}(1 - C_e)(1 - L_j)I_j}_{\text{Import of fallen wood}} \tag{3}$$

$$\frac{dL_i}{dt} = \sigma L_i(1 - L_i)(\underbrace{U}_{\substack{\text{Net cost to}\\ \text{transport firewood}}} + \underbrace{s(2L_i - 1)}_{\text{Social influence term}} + \underbrace{fI_i}_{\text{Impact of infestation}}) \tag{4}$$

Data

- sdf



Results

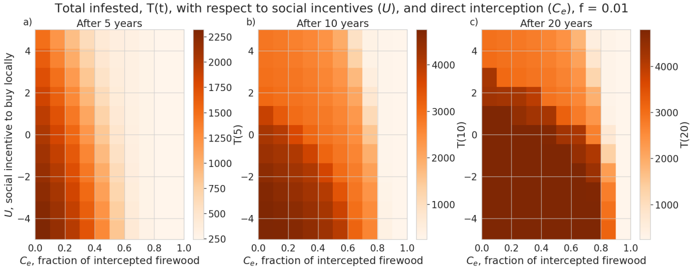


Figure: Total number of infested trees per node over 5 (a), 10 (b), and 20 (c) years

Results

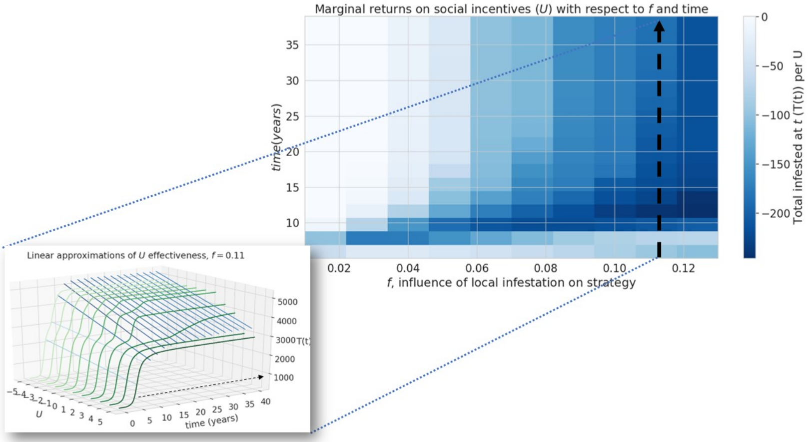


Figure: Efficacy of social incentives on infestation after time T. Inset graph shows an example of cross-section along the line $f = 0.11$

Results

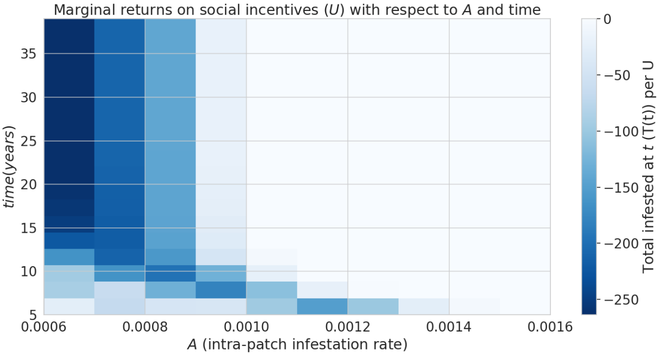


Figure: Efficacy of social incentives on infestation after time period T with respect to A, the intra-patch infestation parameter.

Results

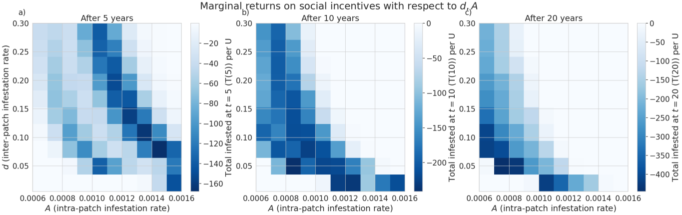


Figure: Efficacy of social incentives on infestation after time T intra-patch spreading rate A, affects infestation outcomes.

Results

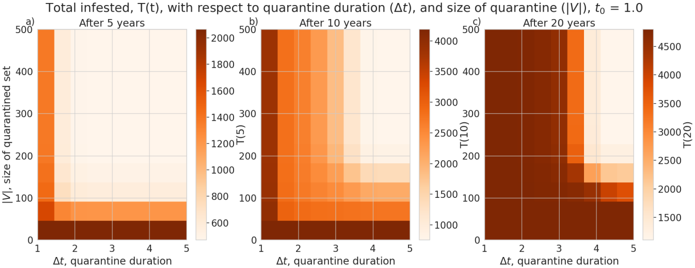


Figure: Average total infested trees ($T(t)$) after 5, 10 and 15 years (panels a), b), and c) respectively), assuming the quarantine begins one year after the pest is introduced.

Discussion

- Firewood inspection not likely to be effective in implementation
- Education, represented as social incentives, are able to decrease infection in the short term, but dependent on pest-specific parameters
- Patch quarantine can be effective if sufficiently many patches are isolated, and the pest is detected early

Chapter 3

- Wildfire and bark beetles are disturbances integral to coniferous forest ecosystems in the western cordillera of North America
- Bark beetle outbreaks have always been destructive, but seem to be worse in recent decades
- Literature on causal relationship between bark beetle outbreaks and wildfire is extensive but inconclusive
- Existing modelling of these two coupled disturbances is sparse

Research questions

How do changes in fire prevalence affect MPB dynamics?

How can we exploit stand structure to dampen MPB outbreaks?

Can we create a simple model that replicates major features of the Fire-MPB system?

Project Outline

- Extend existing model of Duncan et al. to include wildfires
- Explore parameter regime of extended model
- Introduce forest stand thinning procedures to reduce MPB outbreaks
- Show that these stand thinning procedures are able to work due to increased stand heterogeneity

Model

$$j_{n+1,1} = dJ_n + I_{n-2} + F_n \tag{5a}$$

$$j_{n+1,k} = (1-d)j_{n,k-1} - \frac{\alpha_1}{T} P_n j_{n,k-1}, \qquad k = 2 \ldots K-1, K \tag{5b}$$

$$S_{n+1} = S_n + (1-d)j_{n,K} - \left(I_n + \frac{\alpha_2}{T} P_n I_n\right) - \frac{\alpha_2}{T} P_n \left(S_n + (1-d)j_{n,K}\right) - \sigma_F \gamma_n \tag{5c}$$

$$I_{n+1} = r_1 I_n e^{-\beta_1 (T - S_{n+1})} - \frac{\alpha_2}{T} P_n I_n + \sigma_I \xi_n \tag{5d}$$

$$F_{n+1} = P_n \left[\frac{\alpha_1}{T} \sum_{k=1}^{K-1} j_{n,k} + \frac{\alpha_2}{T} \left(S_n + (1-d)j_{n,K}\right) + \frac{\alpha_2}{T} I_n\right] + \sigma_F \gamma_n \tag{5e}$$

$$P_n = T - \sum_{i=1}^{n} F_i e^{-\kappa(n-i)} \tag{5f}$$

Results

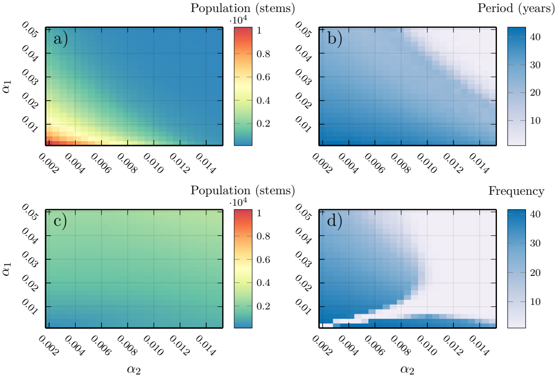


Figure: Panels: a) Average size of largest MPB population, b) Average frequency of MPB outbreaks, c) Average size of largest fire season, d) Average frequency of severe fire seasons. All measured at equilibrium.

Results

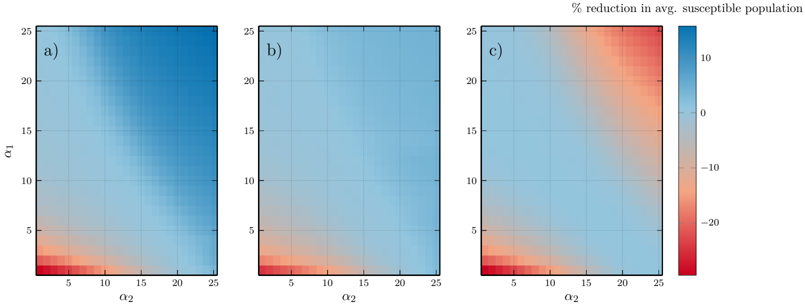


Figure: Percentage change in average susceptible (mature) forest population compared to no FTP with a) $\tau = 0.15, m = 8$, b) with $\tau = 0.15, m = 8$ applied every 5 years, c) controlled burning with $\tau = 0.15, m = 8$, with respect to burning rates α_1, α_2.

Results

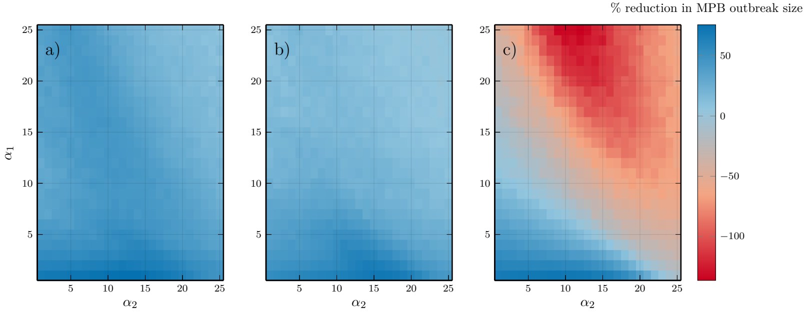


Figure: Percentage change in maximum MPB infestation size within 500 year period under FTP with a) $\tau = 0.15, m = 8$, b) with $\tau = 0.15, m = 8$ applied every 5 years, c) controlled burning with $\tau = 0.15, m = 8$, with respect to burning rates α_1, α_2.

Chapter Conclusion

- We show that increasing fire prevalence is able to dampen MPB outbreaks by increasing stand heterogeneity
- Stand heterogeneity can also be increased through forest thinning or prescribed burning, which also dampens MPB outbreaks.
- These results are consistent with ecological evidence [**?**, **?**]

Model Equations

$$
\begin{aligned}
\frac{dS_i^1}{dt} &= -r\rho_i s(t) S_i^1 \sum_{j=1}^{16} C_{ij}(t) \left(\frac{I_{s_j} + I_{a_j} + P_j}{N_j} \right) - \tau S_i^1 && (6) \\
\frac{dS_i^2}{dt} &= -r\rho_i s(t) S_i^2 \sum_{j=1}^{16} C_{ij}(t) \left(\frac{I_{s_j} + I_{a_j} + P_j}{N_j} \right) - \tau S_i^2 && (7) \\
\frac{dE_i}{dt} &= r_i s(t)(S_i^1 + S_i^2) \sum_{j=1}^{16} C_{ij}(t) \left(\frac{I_{s_j} + I_{a_j} + P_j}{N_j} \right) - \sigma_0 E_i + \tau(S_i^1 + S_i^2) && (8) \\
\frac{dP_i}{dt} &= \sigma_0 E_i - \sigma_1 P_i && (9) \\
\frac{dI_{a_i}}{dt} &= \eta\sigma_1 P_i - \gamma_a I_{a_i} && (10) \\
\frac{dI_{s_i}}{dt} &= (1-\eta)\sigma_1 P_i - \gamma_s I_{s_i} && (11) \\
\frac{dR_i}{dt} &= \gamma_a I_{a_i} + \gamma_s I_{s_i} && (12) \\
\frac{dx}{dt} &= \kappa x(1-x) \left(\frac{\sum_{i=1}^{16} \alpha_i (I_{a_i} + I_{s_i})}{\sum_{i=1}^{16} N_i} - cx \right) + p_{ul}(1-2x) && (13) \\
C_{ij}(t,x) &= C_{ij}^W(t) + C_{ij}^S(t) + (1 - \epsilon_P x)\left(C_{ij}^O + C_{ij}^H \right) && (14)
\end{aligned}
$$