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Coupled models of structured contagion processes in human-environment systems

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August 6th, 2021

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Introduction

Contagious processes

- A contagious process is one that is able to propagate itself through a set of hosts
- Contagious processes shape our lives in many important ways
- Current situation has thrust the importance of modeling infections into popular consciousness

Coupled systems

- Coupling can often dramatically affect stability, e.g. coupled oscillators
- Human-environment systems focus on dynamics of humanity and the environment viewed as a whole
- This perspective provides useful answers to important problems

First Project

Background

- The epidemic landscape is a function of population beliefs about the usage of non-pharmaceutical interventions (NPIs)
- Population age distribution is a key factor in both NPI usage and COVID-19 mortality
- Vaccine rollout important to reducing mortality in the long term (Bubar et al., 2020; Buckner, Chowell, and Springborn, 2020)

Outline of project

1. Describe age-structured compartmental model of COVID-19 infection in a population, coupled to model of social distancing dynamics.
2. Fit model to data from Ontario, Canada
3. Evaluate outcomes from different vaccination strategies, comparing vaccination of vulnerable populations vs vaccination of susceptible populations

Compartmental model overview

Disease Compartments

$S_i(t)$: Susceptible

$S_{2,i}(t)$: Vaccinated but still susceptible

$V_i(t)$: Vaccinated and immune

$E_i(t)$: Exposed

$P_i(t)$: Pre-symptomatic

$I_{a,i}(T)$: Infectious and asymptomatic

$I_{s,i}(t)$: Infectious and symptomatic

$R_i(t)$: Recovered

where $i = 1 \ldots 16$ comprises age structure

Social compartments

$x(t)$: Uses NPIs

$1 - x(t)$: Does not use NPIs

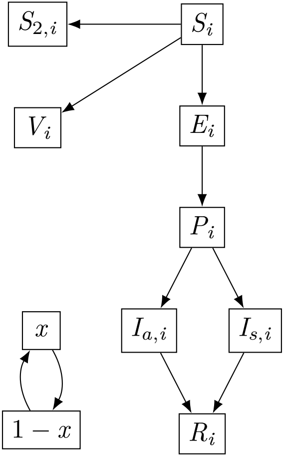


Figure: Compartments

Game theory as a model of NPI adoption

P1 \ P2	use NPI	don't use NPI
use NPI	P2: low risk, NPIs unpleasant; P1: low risk, NPIs unpleasant	P2: med risk; P1: med risk, NPIs unpleasant
don't use NPI	P2: med risk, NPIs unpleasant; P1: med risk	P2: high risk; P1: high risk

Table: NPI adoption as a two-player game (between P1 and P2)

Our model of NPI usage

The full equation for $x(t)$, in this model, is given by

$$\frac{dx}{dt} = \sigma x(1-x)\left(\frac{\sum_{i=1}^{16}\alpha_i(I_{a_i}+I_{s_i})}{\sum_{i=1}^{16} N_i} - cx\right) + p_{ul}(1-2x) \quad (1)$$

- The term $p_{ul}(1-2x)$ accounts for outside influence, where p_{ul} is small.
- α_i denotes the fraction of cases ascertained through testing.
- N_i is the population in age compartment i, $\sum_{i=1}^{16} N_i$ is the total population.
- $x(t)$ interacts with the infection dynamics by reducing the fraction of contacts contributing to the infection rate

Vaccination strategies

We compare four vaccination strategies

- > 60 first
- < 20 first
- Uniform
- Contact-based

with respect to reduction in cumulative mortality after 5 years.

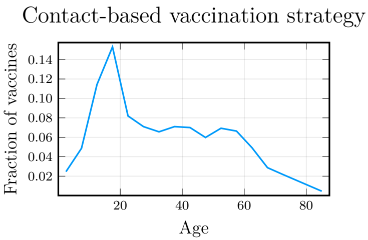


Figure: **Vaccine allocation by age in the contact-based vaccination strategy**. Computed as the normalized leading eigenvector of the sum of the contact matrices.

Parameterization

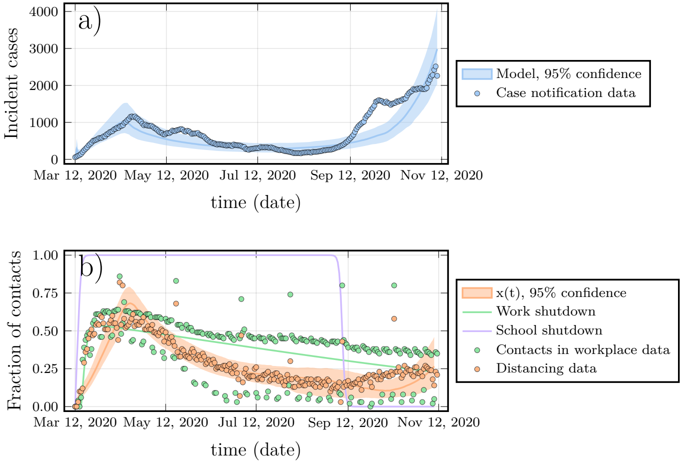


Figure: Model parameterization used, physical distancing matches reported cases.

Results

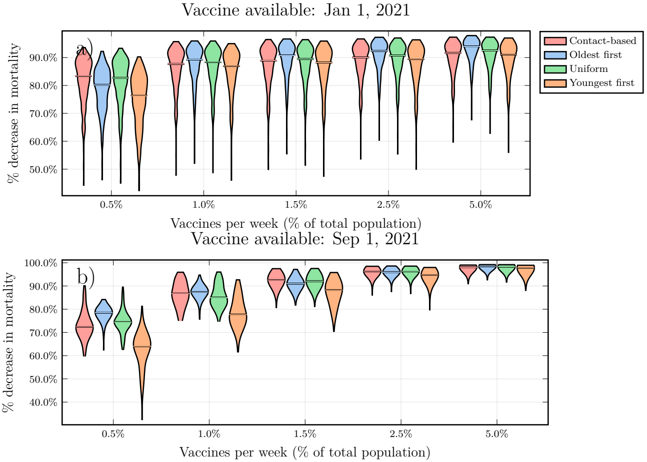


Figure: Percentage reduction in mortality for four strategies depends on vaccination start date and the vaccination rate.

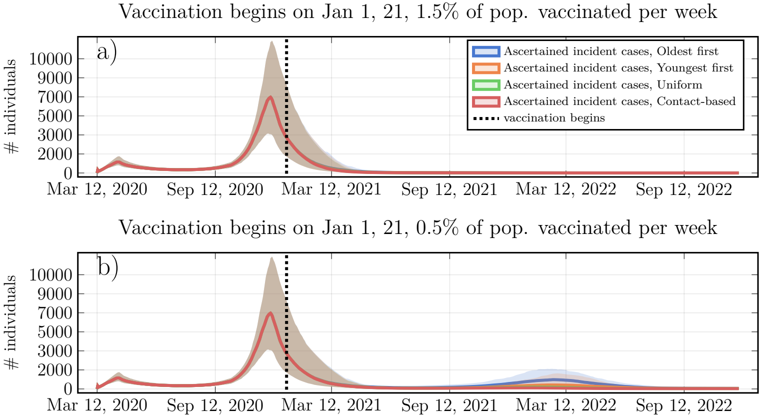


Figure: (a) timely vaccination prevents third wave, (b) partial vaccination and indirect protection help during the third wave

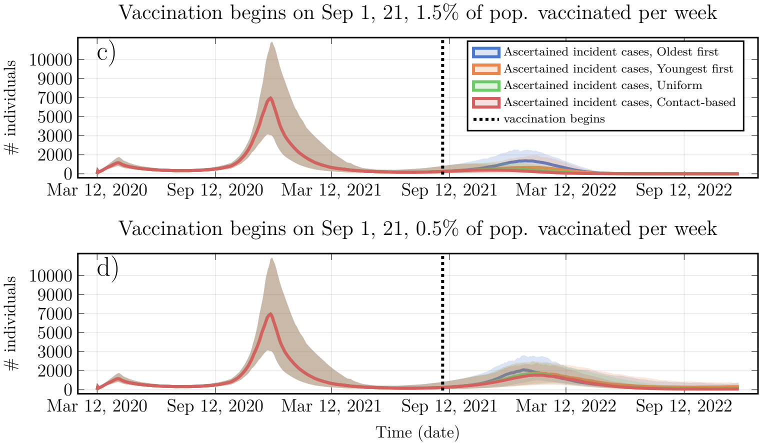


Figure: c) with a later start date but higher vaccination rate, partial vaccination and indirect protection help during the third wave as in b), (d) slow and late vaccination fails to prevent third wave.

Results

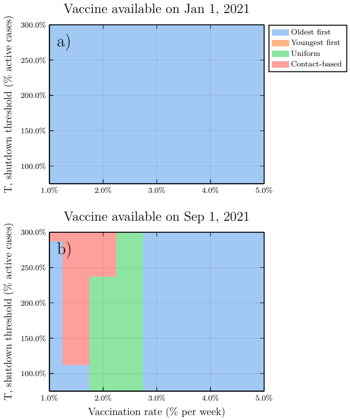


Figure: Later start to vaccination favours transmission-interrupting strategies for moderate vaccination rates. Each parameter pair is colored according to the strategy that prevents most deaths on average, over all realizations of the model.

Results

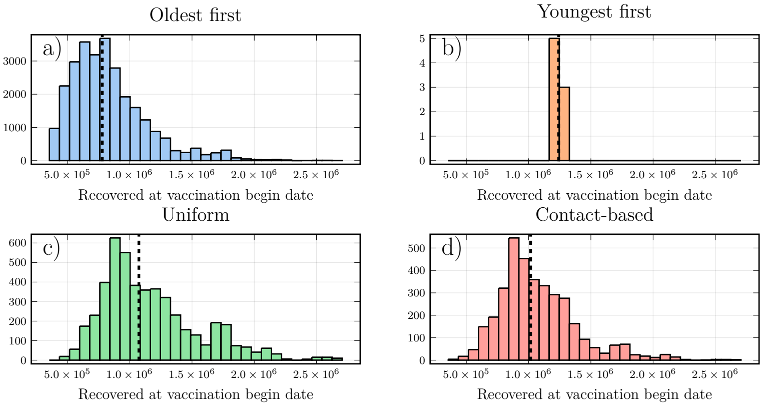


Figure: More pre-existing natural immunity makes transmission-interrupting strategies more effective. Histogram of no. recovered at vaccination begin date, according to best strategy for that realization, over all parameter values in sensitivity analysis. Vertical lines are the median.

Chapter Conclusion

- We described an age structured compartmental model of Sars-CoV-2 infection and vaccination coupled to a social model
- Showed that sometimes transmission interrupting strategies can be more effective
- Depends on the pre-existing immunity in the population

Second Project

Project 2

- Invasive forest pests cause incredible damage to ecosystems and lumber resources
- Evidence shows that movement of firewood is a major long distance vector (Koch et al. (2014))
- Education and awareness is a major way we try to reduce this vector

Figure: an Emerald Ash Borer, which devastated Ash populations in North America

Outline of project

1. Adapt a model such as Barlow et al. (2014) to a larger, more realistic network
2. Use model to compare three possible prevention measures
   - Education/awareness
   - Inspection of moved firewood
   - Quarantine of highly susceptible forest patches
3. Assess measures across a range of parameter values and time horizons

Model

$$\frac{dS_i}{dt} = \underbrace{rS_i\left(1 - \frac{(S_i + I_i)}{K}\right)}_{\text{Logistic Growth Of Forest}} - \underbrace{AS_i(I_i + B_i)\theta_k(I_i - I_a)}_{\text{Infestation term}} \quad (2)$$

$$\frac{dI_i}{dt} = \underbrace{-\gamma I_i}_{\substack{\text{Death of}\\ \text{infested trees}}} + \underbrace{AS_i(I_i + B_i)\theta_k(I_i - I_a)}_{\text{Susceptibles become infested}} \underbrace{- d\sum_{j=1, j\neq i}^{N} P_{j,i}(1 - C_e)(1 - L_i)I_i}_{\substack{\text{Total infested wood}\\ \text{leaving due to transport}}} \quad (3)$$

$$\frac{dB_i}{dt} = \underbrace{-\gamma B_i}_{\text{Decay of firewood}} + \underbrace{d\sum_{j=1, j\neq i}^{N} P_{i,j}(1 - C_e)(1 - L_j)I_j}_{\text{Import of fallen wood}} \quad (4)$$

$$\frac{dL_i}{dt} = \sigma L_i(1-L_i)\Big(\underbrace{U}_{\substack{\text{Net cost to}\\ \text{transport firewood}}} + \underbrace{s(2L_i - 1)}_{\text{Social influence term}} + \underbrace{fI_i}_{\text{Impact of infestation}}\Big) \quad (5)$$

We use $T_i(t)$, computed from equation 5 to be the total number of infested trees in patch i up to time t.

$$\frac{dT_i}{dt} = AS_i(I_i + B_i)\theta_k(I_i - I_a) \tag{6}$$

Define $T(t) = \frac{1}{N}\sum_{i=1}^{N} T_i(t)$ to be the average total number of infested trees up to t.

Data

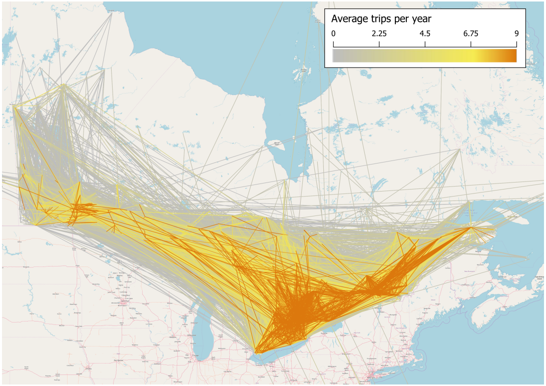


Figure: Travel network used to weight edges in firewood transport network

Results

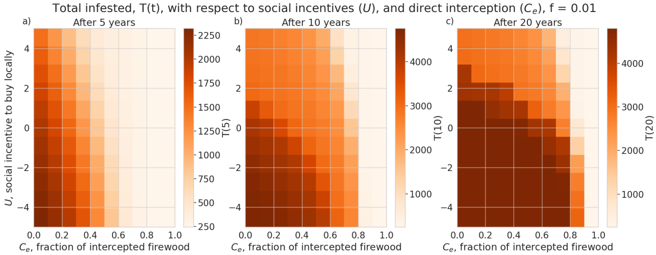


Figure: Neither increasing U nor C_e are effective at long time scales. Total number of infested trees per node over 5 (a), 10 (b), and 20 (c) years

Results

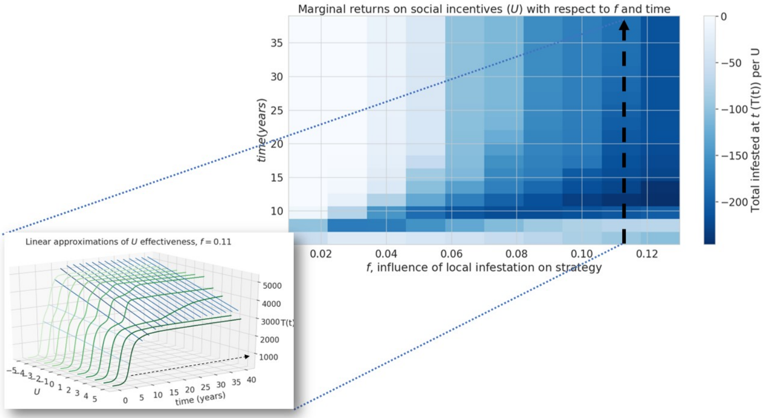


Figure: The influence of infestation on transport strategy, f, can hinder the intervention by public outreach, in the long-term (after approximately 20 years). Efficacy of social incentives on infestation after time T. Inset graph shows an example of cross-section along the line $f = 0.11$

Results

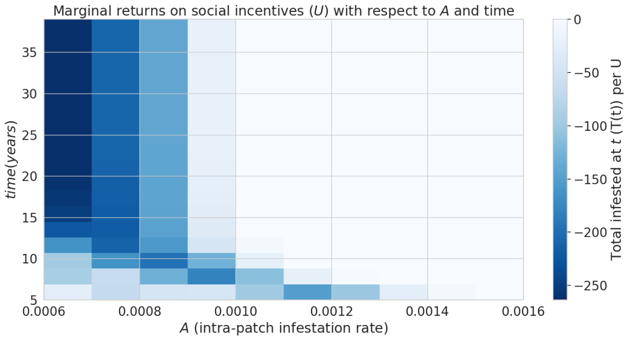


Figure: Increasing U becomes ineffective over time if A is sufficiently large. Efficacy of social incentives on infestation after time period T with respect to A, the intra-patch infestation parameter.

Results

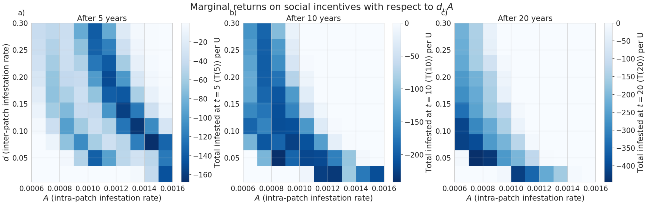


Figure: The social incentive to not transport firewood, U, is more effective with lower pest spread rates. Efficacy of social incentives on infestation after time T intra-patch spreading rate A, affects infestation outcomes.

Peter C. Jentsch

Results

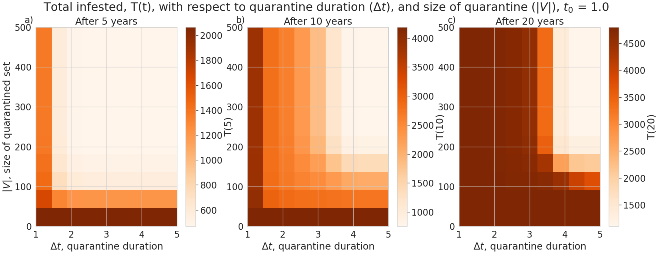


Figure: Quarantine needs to include many patches for long term effects. Average total infested trees ($T(t)$) after 5, 10 and 15 years (panels a), b), and c) respectively), assuming the quarantine begins one year after the pest is introduced.

Discussion

- Firewood inspection not likely to be effective in implementation
- Education able to decrease infection in the short term, but dependent on pest-specific parameters
- Patch quarantine effective enough patches are isolated, and the pest is detected early

Third Project

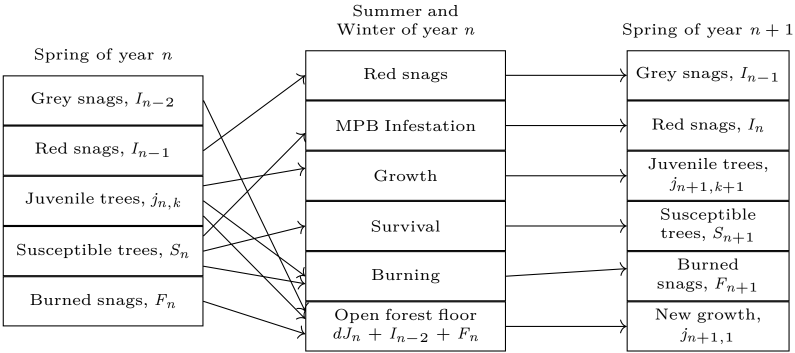
Background

- Wildfire and bark beetles are disturbances integral to coniferous forest ecosystems in the western cordillera of North America (Kaufmann et al., 2008)
- Bark beetle outbreaks have always been destructive, but seem to be worse in recent decades
- Literature on causal relationship between bark beetle outbreaks and wildfire is extensive but inconclusive (Axelson, Alfaro, and Hawkes (2009))
- Existing modelling of these two coupled disturbances is sparse

Model

- Discrete time MPB model extended from Duncan et al. (2015)
- Assume trees are susceptible only after 50 generations
- Seedlings germinate in open canopy space immediately
- Assume fire risk is proportional to the sum of the unburned land from previous years, weighted by a negative exponential term

Diagram of compartments



Model

$$j_{n+1,1} = dJ_n + I_{n-2} + F_n \tag{7a}$$

$$j_{n+1,k} = (1-d)j_{n,k-1} - \frac{\alpha_1}{T} P_n j_{n,k-1}, \qquad k = 2 \ldots K-1, K \tag{7b}$$

$$S_{n+1} = S_n + (1-d)j_{n,K} - \left(I_n + \frac{\alpha_2}{T} P_n I_n\right) - \frac{\alpha_2}{T} P_n \left(S_n + (1-d)j_{n,K}\right) - \sigma_F \tag{7c}$$

$$I_{n+1} = r_1 I_n e^{-\beta_1 (T - S_{n+1})} - \frac{\alpha_2}{T} P_n I_n + \sigma_I \xi_n \tag{7d}$$

$$F_{n+1} = P_n \left[\frac{\alpha_1}{T} \sum_{k=1}^{K-1} j_{n,k} + \frac{\alpha_2}{T} \left(S_n + (1-d)j_{n,K}\right) + \frac{\alpha_2}{T} I_n\right] + \sigma_F \gamma_n \tag{7e}$$

$$P_n = \max\left(0, T - \sum_{i=1}^{n} F_i e^{-\kappa(n-i)}\right) \tag{7f}$$

Results

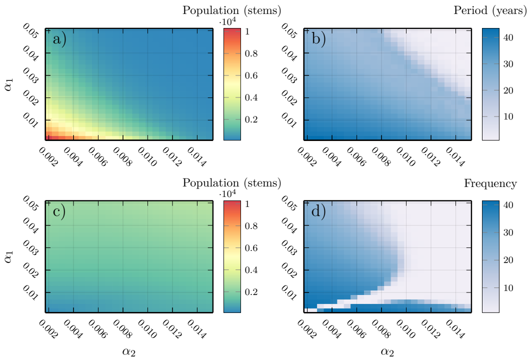


Figure: Increasing burning rates α_1,α_2 reduce MPB outbreak size, cause stability in fire regimes. Panels: a) Average size of largest MPB population, b) Average frequency of MPB outbreaks, c) Average size of largest fire season, d) Average frequency of severe fire seasons. All measured at equilibrium.

Results

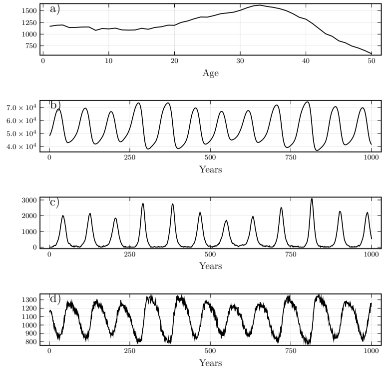


Figure: Without forest trimming protocol (FTP), $\alpha_1 = 0.02, \alpha_2 = 0.0025$, large even aged stands visible in juvenile age distribution (top panel).

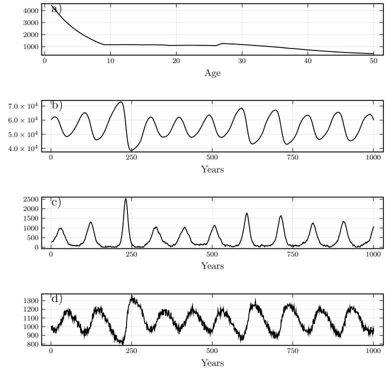


Figure: With forest trimming protocol (FTP), $\alpha_1 = 0.02, \alpha_2 = 0.0025$, $\tau = 0.15, m = 8$, with greater heterogeneity in juvenile age structure (top panel).

Results

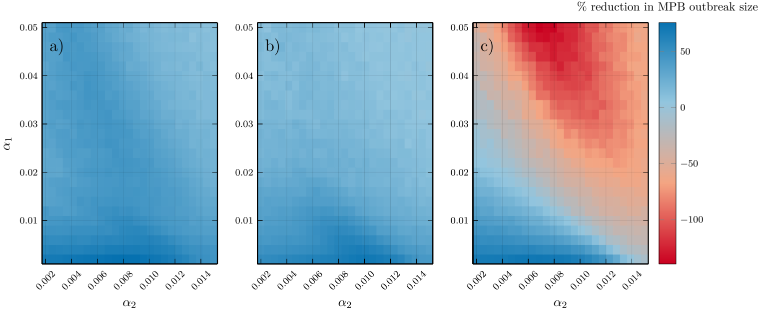


Figure: FTP most effective with small α_1, Controlled Burning Protocol (CBP) most effective with small α_1, α_2. a) $\tau = 0.15, m = 8$, b) with $\tau = 0.15, m = 8$ applied every 5 years, c) controlled burning with $\tau = 0.15, m = 8$, with respect to burning rates α_1, α_2.

Chapter Conclusion

- We show that increasing fire prevalence is able to dampen MPB outbreaks by increasing stand heterogeneity
- Stand heterogeneity can also be increased through forest thinning or prescribed burning, which also dampens MPB outbreaks.
- These results are consistent with ecological evidence (Seidl et al. (2016) and Kaufmann et al. (2008))

Overall conclusion

- Coupled human-environment systems introduced new modeling possibilities
- Compared methods of mitigation for respective coupled infectious processes
- Results obtained from these answer a gap identified in literature

Thank you for listening!

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Kaufmann, Merrill R et al. (2008). "The status of our scientific understanding of lodgepole pine and mountain pine beetles: a focus on forest ecology and fire behavior". In.

Koch, Frank H et al. (2014). "Using a network model to assess risk of forest pest spread via recreational travel". In: *PloS one* 9.7, e102105.

Seidl, Rupert et al. (2016). "Spatial variability in tree regeneration after wildfire delays and dampens future bark beetle outbreaks". In: *Proceedings of the National Academy of Sciences*, p. 201615263.

Model Equations

$$\frac{dS_i^1}{dt} = -r\rho_i s(t) S_i^1 \sum_{j=1}^{16} C_{ij}(t) \left(\frac{I_{s_j} + I_{a_j} + P_j}{N_j} \right) - \tau S_i^1 \tag{8}$$

$$\frac{dS_i^2}{dt} = -r\rho_i s(t) S_i^2 \sum_{j=1}^{16} C_{ij}(t) \left(\frac{I_{s_j} + I_{a_j} + P_j}{N_j} \right) - \tau S_i^2 \tag{9}$$

$$\frac{dE_i}{dt} = r_i s(t)(S_i^1 + S_i^2) \sum_{j=1}^{16} C_{ij}(t) \left(\frac{I_{s_j} + I_{a_j} + P_j}{N_j} \right) - \sigma_0 E_i + \tau(S_i^1 + S_i^2) \tag{10}$$

$$\frac{dP_i}{dt} = \sigma_0 E_i - \sigma_1 P_i \tag{11}$$

$$\frac{dI_{a_i}}{dt} = \eta\sigma_1 P_i - \gamma_a I_{a_i} \tag{12}$$

$$\frac{dI_{s_i}}{dt} = (1-\eta)\sigma_1 P_i - \gamma_s I_{s_i} \tag{13}$$

$$\frac{dR_i}{dt} = \gamma_a I_{a_i} + \gamma_s I_{s_i} \tag{14}$$

$$\frac{dx}{dt} = \kappa x(1-x) \left(\frac{\sum_{i=1}^{16} \alpha_i (I_{a_i} + I_{s_i})}{\sum_{i=1}^{16} N_i} - cx \right) + p_{ul}(1-2x) \tag{15}$$

$$C_{ij}(t,x) = C_{ij}^W(t) + C_{ij}^S(t) + (1-\epsilon_P x) \left(C_{ij}^O + C_{ij}^H \right) \tag{16}$$