Inflation Targeting under Fiscal Fragility

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This version:
April 12, 2021

Abstract

We model the intertemporal trade-off between fiscal and monetary policy under an inflation targeting regime. An indebted and altruistic policymaker chooses public expenditure and current inflation. Private agents choose consumption, debt purchase, and form expected inflation. Debt level determines target credibility: low debt levels make target fully assured, and high levels make target announcement innocuous. For an endogenous interval of intermediate debt level, named fiscal fragility zone (FFZ), expected inflation may converge to the target or not. Within FFZ, self-confirmed inflation may surge, making debt rollover too expensive and inflating the debt away inevitable. We show that fiscal austerity to gradually reduce debt and to prevent coordination failure is an optimal policy. Moreover, within FFZ, policymakers should (i) increase the target to sustain higher debt levels out of the FFZ and (ii) reduce the share of inflation-indexed bonds to reduce potential inflation overshoot. Cross-country facts support our results.

Keywords: Monetary Policy, Fiscal Policy, Debt Policy

JEL - Classification: E52, E62, E63

*We thank Christopher Sims and other participants of the Lames-Lacea Guayaquil on November 2018, Giancarlo Corsetti and other participants of the Ridge Workshop on International Macro on December 2018, and participants of the International Reading Group at the University of Rochester, the 4th REAP Seminar on 2018, the 40th SBE seminar on 2018, the Macro Group from FGV EPGE, the RBE 70^{th} Anniversary Celebration, and the Seminários SBE de Economia Aplicada. We also thank comments of Gaston Chaumont, Bernardo Guimarães, Tim Kehoe, Ricardo Sabbadini, Gustavo Serra, Gian Soaves, and Henry Thompson. This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001.

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1 Introduction

In a seminal paper Kydland & Prescott (1977) post a general claim against discretionary policies arguing that rules are a better way to coordinate expectations. Regarding monetary policy, they conclude that a policymaker "doing what is best, given the current situation, results in an excessive level of inflation, but unemployment is no lower than it would be if inflation (possibly deflation or price stability) were at the socially optimal rate." During the 1980s and 90s several monetary policymakers adopted their prescription. Since then, independence and the inflation targeting regime have been the cornerstone of central bank coordination of inflation expectations. However, inflation target ranges are missed frequently, both in advanced and emerging economies. There are common episodes of coordination failures in which inflation expectations suddenly lose their anchor and diverge from announced targets.[1] Most of these episodes lack the changes in fundamentals that would explain the shift in expectations, raising questions on the limits of inflation targeting to anchor short term inflation expectations.

We propose a model that rationalizes these observable episodes of coordination failures and self-fulfilling inflation. The heart of our argument is the idea that the fiscal side of the economy is fundamental to understand the capacity of inflation targeting to coordinate inflation expectations. We model a closed economy in which two types of agents, an altruistic policymaker and private agents, form rational expectations with complete information. The policymaker acts as a fiscal authority and central bank that targets inflation, choosing current inflation and financing government expenditures by selling debt. We assume that the policymaker is not perfectly committed to the inflation target and might deviate from it to make fiscal room for spending. The decision is the solution of the trade-off between inflating public debt away and keeping inflation on target to avoid the economic costs of deviating. Private agents choose how much debt to hold and form expectations about next period inflation. Our framework builds on Cole & Kehoe (1996, 2000) and Araujo *et al.* (2013) expanding the analysis to a monetary policy setting.

In our model, target failures happen when the public debt level exceeds an endogenous threshold limit and enters the fiscal fragility zone (FFZ) marked by high debt and limited fiscal space for public spending. In that zone, adverse inflation expectations decrease debt prices and expose the policymaker to confidence loss in their commitment to the target in a self-fulfilling target failure. Accordingly, when the fiscal space for public spending decreases, debt rollover becomes too expensive and makes partial default on maturing debt through

[1] Roger & Stone (2005) notice that targets are often missed (40% in their sample) and sometimes "by substantial amounts and for prolonged periods." Based on our updated data set, used in section 5, we concluded targets are still frequently missed (26%).

inflation the best policy response. Fiscal soundness defined as low public-debt shuts the door to confidence crisis and supports the inflation targeting regime. The indebted policymaker should gradually reduce the public debt. Moreover, while inside the fiscal fragility region, keeping higher targets and reducing the share of inflation-indexed bonds are valuable tools to anchor expectations and mitigate the risk of inflation overshooting..

The intuition of why a higher inflation target helps to anchor expectations is simple. We show the amount of partial default available to the policymaker decreases in the level of the inflation target. Higher targets decrease temptation for indebted policymakers and increase its credibility. We also show (i) the size of the target deviation decreases by increasing the target and reducing debt, and (ii) the probability to overshoot the target increases with debt and decreases with the target level. The intuition for avoiding indexed bonds is similar. The share of non-indexed bonds provides room for partial default and reduces ex-post inflation level. Therefore, wider room lowers the level of expected inflation.

To test our model predictions, we use a panel dataset of 20 countries with at least 15 years of inflation targeting. We find evidence that deviations from the target and the probability of overshooting are negatively related to the target level. We also find evidence that deviations from the target are positively related to the debt level.

Our results have implications for the conduct of monetary policy. It seems naïve to choose a 2% inflation target without considering fiscal fundamentals as countries eventually do. Our model suggests raising the inflation target to help coordinate expectations in the short term. The call for indebted economies to not support low target levels and seek targets compatible with their fundamentals also holds under imperfect information when private agents disagree about inflation forecast (Araujo *et al.*, 2016). In addition, pre-fixed interest rate securities improve credibility of the central bank by reducing the level of inflation necessary to restore fiscal space during the crisis.

These policy prescriptions reflect some emerging economies practice as they are more prone to crises and usually have higher inflation targets than advanced economies. Recently, they are gaining importance as the fiscal limits of inflation targets have been tested by developed and emerging countries as they increase their debt levels to provide fiscal response against the Covid-19 pandemic and shutdowns.

Others have raised concerns about the high debt levels recently observed in advanced economies. Taylor *et al.* (2020) highlight that the US debt level is expected to continue growing and should reach 192% in 2050. The fiscal deficits in the US are a structural problem and a challenge to inflation expectation coordination. Sims (2020) argues that the ratio of debt service cost to total tax receipts is critical to understanding the temptation to inflate the debt away. Debt services increase with the debt and/or the interest rate. Sims notices that the interest rate is a positive function of the ratio of debt to GDP with no guarantee it will be forever low. A sudden increase in inflation expectation and consequently in the nominal interest rate as in our model could trigger an inflation episode.

Our model also rationalizes the response to the inflationary pressures that happened in Brazil at the end of 2002. In that period, it became clear that the presidential candidate who would win the election could arrive with a new policy framework. Inflation expectations exceeded the upper bound of the target as seen in Figure 1 indicating a target confidence crisis. In response to rising inflation expectations, Brazilian policymakers twice increased the target for 2003, first at an extra meeting held in June 2002 and again in January 2003, and gradually reduced the share of inflation-indexed bonds in the public debt. This response of policymakers is in line with the prediction of our model.

[Figure 1 about here.]

The message on the fiscal limits of monetary policy achievements and the interdependence between fiscal discipline and price stability is amply addressed in the literature. Sargent & Wallace (1981) show the importance of the fiscal side to understand inflation control, followed by Sargent & Wallace (1981), Leeper (1991), Sims (1994, 2011), Woodford (1995), Araujo & Leon (2002), Leeper & Leith (2016), Araujo *et al.* (2016), and Cochrane (2017). We are adding to this literature by formalizing some policy prescriptions for the indebted policymakers in an inflation-targeting regime. Our policy prescription for bond-type selection has been examined in a different framework by Fischer (1983).

We believe our messages are novel to the inflation targeting literature. We innovate by bringing to a simple DSGE a strategic policymaker who may use inflation to partially default. Our approach closely follows papers on confidence crises in debt markets as in Cole & Kehoe (1996, 2000), Calvo (1988), and Arellano *et al.* (2019). However, we detach from a policymaker modeled by fiscal and monetary rules as in Christiano *et al.* (2005) and Smets & Wouters (2007). Papers exploring debt crises and their relation to monetary policy include Uribe (2006), Aguiar *et al.* (2013), Corsetti & Dedola (2016), Bacchetta *et al.* (2018), and Arellano *et al.* (2019). But none of these consider the relation with inflation target coordination. To the best of our knowledge, the literature combining inflation targeting and debt crises is scarce.

In section 2 we set out the model and derive the recursive form defining the equilibrium. In section 3, we specify functional forms and parameter values in a quantitative analysis to match the situation in Brazil in 2002. We then move to analyze the results from our model. In section 4, we analyze the 2002 confidence crisis in Brazil and the subsequent policy responses. In section 5, we test the predictions of our model using a sample of 20 countries. Finally, the last section presents some remarks.

2 Model

We consider a closed economy with two types of agents: a policymaker and private agents. Each agent lives infinite periods and forms rational expectations with complete information. The policymaker acts as a mix of fiscal and monetary authority, choosing current inflation and selling one-period debt to finance itself. In our setup, inflation choice reduces to a discrete choice each period whether to deviate from the target. We assume the policymaker is altruistic and maximizes private agent welfare. Private agents receive a stream of fixed endowments. Each period they choose how much debt to hold and form expectations about next-period inflation taking into account the exogenous announced inflation target and the current debt level. When multiple equilibria are possible a sunspot variable determines the equilibrium.

2.1 Basic Setup

Policymaker

We assume an altruistic policymaker who chooses both fiscal and monetary policies to maximize private agents utility. As a monetary authority, the policymaker chooses the inflation rate π_t and as a fiscal authority next period's debt D_{t+1}:

$$\max_{\pi_t, D_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, g_t) \tag{1}$$

where c_t is private agents consumption in period t, g_t is government spending on public goods, and β is the inter-temporal discount rate $0 < \beta < 1$. Consumption and public goods are non-negative. We further assume the utility function $u(c, g)$ is continuously differentiable and monotonically increasing, linear in c, and strictly concave in g satisfying $\lim_{g \to 0^+} u(g) = -\infty$.

Linearity in consumption is a strong assumption and deserves further comments. First it allows us to define the real interest rate as a risk neutral pricing formula. It also simplifies the problem making the debt stationary outside the crises zone that remains to be defined. Finally, it readily makes the marginal utility of public goods higher (lower) than the marginal utility of consumption for high (low) values of debt.

Each period, non-negative spending g_t and the repayments on previous period obligations are financed through taxes τ on a deterministic endowment e and the issuance of new debt D_{t+1}. The government's budget constraint is given by,

$$g_t + (1 + r_t)D_t \leq D_{t+1} + \alpha_t \tau e \tag{2}$$

where D_t is last period debt. The usual no-Ponzi condition is imposed on debt holdings. If the policymaker deviates from inflation target π_a, the economy suffers a permanent negative shock to endowments through α_t such that deviation from the target with $\pi_t > \pi_a$ implies $0 < \alpha_t < 1$. The productivity factor α_t should be understood as a reduced form capturing the cost of deviating from the inflation target on economic activity. We assume that $\alpha_t = \alpha(\pi, \pi^a, \alpha_{-1})$, i.e. the penalty is a function of the inflation target π^a, current inflation π_t, and its past value α_{-1}. We further restrict the penalty to be a function of the deviation of current inflation from the inflation target, that is $\pi - \pi^a$, and assume $\alpha(\pi - \pi^a)$ is continually differentiable, strictly decreasing, and $0 < \alpha_t \leq 1$. This structure gives us an increasing cost in deviations as opposed to a fixed cost as in Araujo *et al.* (2016).

Our approach of assuming an exogenous function form for the "cost of deviating" are in line with the literature. Exogenous penalty functions are also assumed in self-fulfilling debt crises models as in Cole & Kehoe (1996, 2000) and in sovereign default models as in Arellano (2008). We interpret the penalty function as a reduced and parsimonious form of capturing negative impacts of inflation on the economic activity.

Given a linear utility in c, in equilibrium the ex-post real interest rate[2] will be given by:

$$r_t = \frac{1 + \pi_t^e}{1 + \pi_t} \frac{1}{\beta} - 1 \tag{3}$$

where $\pi_t^e = \mathbb{E}(\pi_t | t - 1)$ is the expected inflation for period t formed by private agents in period $t - 1$ and where π_t is the current inflation.

In each period, the policymaker can satisfy the budget constraint by: i) adjusting expenditures, ii) issuing new debt D_{t+1}; iii) partially defaulting on debt through an inflationary surprise ($\pi_t > \pi_t^e$) and rolling over the remaining debt. When the current inflation rate is equal to expectation $\pi_t = \pi_t^e$ the ex-post real interest rate will be equal to the inverse of the inter-temporal discount rate, $1/\beta$. An inflationary surprise defined by $\pi_t > \pi_t^e$ reduces the ex-post real interest rate and consequently payments the policymaker makes on its debt. Such a partial default offers additional fiscal room for government spending.[3]

[2] This is the real interest rate after the realization of inflation at period t.

[3] For a really higher level of debt, the policymaker has an empty choice set $\{\pi^t, D_{t+1}\}$. We restrict our analysis to an initial debt levels that leaves the policymaker with a non-empty set of feasible choices. The need to impose this restriction comes from the positive spending and consumption restrictions together with a non-Ponzi condition on debt. For very high initial debt levels the policymaker could be left with no option. To see this, suppose that debt servicing costs are higher than tax revenues, $\left(\frac{1}{\beta} - 1\right) D_0 > \tau e$, leaving no space for spending. Even if the policymaker were to partially default on debt payments, it would still not be able to meet future positive spending restrictions due to the high future debt servicing costs and the inability to make use of inflationary surprises again.

Private-agents

We assume a continuum of infinitely lived private agents who choose consumption and savings to maximize their expected utility:

$$\max_{c_t, d_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, g_t) \tag{4}$$

Each period, private agents receive a deterministic endowment e and receive payments on their bond holdings. The endowment is taxed at a constant rate τ by the government. The private agents budget constraint is given by:

$$c_t + d_{t+1} \leq (1 + r_t)d_t + \alpha_t(1 - \tau)e \tag{5}$$

where d_{t+1} are one-period bonds bought in t and d_t are the previous period bond holdings paying the interest rate $(1+r_t)$. Private agents also form inflation expectation π_t^e. Their form the expectation will depend on the timing of actions assumed and will be properly defined when we present the timing of actions.

Discretionary Inflation

We motivate the existence of deviations from the inflation target by modeling an altruistic policymaker who might choose a higher inflation level than the inflation target to transfer resources for increasing public spending in the face of high debt servicing costs.[4] In each period the policymaker may choose to deviate from the exogenously set inflation target π^a, and private agents understand this when forming their expectations π_t^e. The discretionary inflation chosen by the policymaker is the result of a trade-off between increasing spending today against the costs of reducing consumption and the loses due to the costs of deviating from the inflation target. Let π^D be the endogenous and optimal level of discretionary inflation chosen at the time T of the deviation.

When deviating from the inflation target, the simplifying assumption is that the policymaker will roll over debt. We assume that if the policymaker decides to deviate at time T from the inflation target π^a then debt will be rolled over such that $D_t = D_T, \forall t > T$. This assumption makes the problem tractable and limits the use of inflation to increasing current spending.

Once the policymaker deviates from the inflation target, private agents lose confidence in the commitment of the policymaker to the inflation target. Private agents update the

[4] We do not model other mechanisms of partial default on local currency domestic debt other than inflation, although governments have opted for alternatives such as reduction of principal or lower coupons Reinhart & Rogoff (2008).

probability of the policymaker deviating next period setting it equal to 1. Consequently, after the policymaker deviates the economy enters a steady state as there is no longer any uncertainty to be resolved. The optimal fiscal policy is to maintain constant debt as proved in the online appendix B.2.1. Finally, the penalty function takes the value $\alpha_t = \alpha(\pi, \pi^a, \alpha_T) = \alpha(\pi - \pi^a)$ when deviating and remains so thereafter. The problem the policymaker resolves when defining the level of discretionary inflation, can be written:

$$\pi^D = \underset{\pi}{\operatorname{argmax}}\ u(c_T, g_T) + \frac{\beta}{1-\beta} u(c, g)$$

subject to

$$\begin{aligned} g_T &= D\left(1 - \frac{1+\pi_T^e}{1+\pi}\frac{1}{\beta}\right) + \alpha(\pi - \pi^a)\tau e \\ g &= D\left(1 - \frac{1}{\beta}\right) + \alpha(\pi - \pi^a)\tau e \\ c_T &= \left(\frac{1+\pi_T^e}{1+\pi}\frac{1}{\beta} - 1\right) D + \alpha(\pi - \pi^a)(1-\tau)e \\ c &= \left(\frac{1}{\beta} - 1\right) D + \alpha(\pi - \pi^a)(1-\tau)e \end{aligned} \tag{6}$$

Observe that $u(c_T, g_T)$ is possibly increasing in π but $\frac{\beta}{1-\beta}u(c, g)$ is necessarily decreasing in π. Given rational expectations, in equilibrium π^D is optimal given π^e and vice versa. We numerically solve this problem by writing it as a fixed point. First assume an initial $\pi_0^e = \pi^a$ and then find the optimal π_1^D computing the new π_1^e using π_1^D. If $\pi_1^e \neq \pi_0^e$ the problem is iterated to find the new the optimal π_2^D given π_1^e. We continue this process until $|\pi_{i-1}^e - \pi_i^e| < \epsilon$ is a small number. In the online appendix B.2.2 we show this problem has a solution.

To gain intuition, consider the first order condition of the policymaker problem for choosing discretionary inflation. The first order condition is,

$$\begin{aligned} u_{c_T}\left(-\frac{1+\pi_T^e}{(1+\pi)^2}\frac{1}{\beta}D + \alpha'(1-\tau)e\right) + u_{g_T}\left(\frac{1+\pi_T^e}{(1+\pi)^2}\frac{1}{\beta}D + \alpha'\tau e\right) \\ + \frac{\beta}{1-\beta}\left(u_c\alpha'(1-\tau)e + u_g\alpha'\tau e\right) = 0 \end{aligned} \tag{7}$$

where α' is the first order derivative of the penalty function with respect to the deviation $\pi - \pi^a$. The first term represents the short-term marginal loss stemming from reduced consumption caused by the inflationary surprise and the productivity shock to endowments. The second term that may be positive is the marginal benefits of higher spending through the inflationary surprise after discounting marginal losses due to lower tax revenues. Finally, the last term illustrates the lasting effects of the productivity shock to the economy causing lost consumption and government spending.

The policymaker will choose discretionary inflation levels above the inflation target when the benefits outweigh the costs. If there were no costs to transferring resources through inflation, a benevolent policymaker would strive to equate both the marginal utility of consumption to the marginal utility for government spending. A higher marginal utility for government spending would be necessary and sufficient for the policymaker to choose discretionary inflation above the inflation target. However, since there are productivity costs associated with such a deviation this condition is only necessary. Proposition 1 gives the conditions for the policymaker to choose $\pi^D > \pi^A$. Intuitively, whether a higher marginal utility for public spending g is sufficient will depend on the marginal cost of deviating. Lower marginal costs make the policymaker more likely to choose discretionary inflation to reduce debt servicing costs. A higher debt level will furthermore pressure the available space for spending and increase the marginal utility for g given the strictly concave utility of private agents in g.

Proposition 1 *Suppose the utility function, penalty, and initial debt level satisfy the stated assumptions. When the ratio between the benefit of deviating and the cost of losing endowment in the long run is higher than the nominal interest rate when agents expect the inflation target, the policymaker will choose discretionary inflation higher than the inflation target.*

Proof: see appendix A.1.

An altruistic policymaker, maximizing private agent welfare, may choose to deviate from the inflation target when it has limited fiscal room to finance public spending.

Timing

At the beginning of each period, the policymaker implements the actual inflation rate π_t and chooses how much debt D_{t+1} to sell. Thereafter, privateagents form their expectations π^e_{t+1} about next period inflation and decide how much debt d_{t+1}. Finally, next period uncertainty is resolved through the realization of the sunspot variable ζ_{t+1} selecting next period equilibrium according to:

1^{st} Policymaker chooses actual inflation π_t;

2^{nd} Policymaker chooses next debt level D_{t+1};

3^{rd} Private agents form next-period inflation expectation π^e_{t+1} and choose the amount of next period debt d_{t+1} to hold;

4^{th} Next-period sunspot variable ζ_{t+1} is realized.

Rational expectations govern the strategic interactions between the policymaker and private agents. As in Cole & Kehoe (2000), self fulfilling multiple equilibria may occur. Conditional on the debt level the best response from the policymaker perspective may depend on the expectations of private agents. If private agents expect a deviation from the target, the best response will be to deviate. If they expect no deviations, the best response will be to keep inflation on target. In this case we consider an exogenous sunspot variable ζ_t to determine the selection of the equilibrium.

Given this timing, private agents may face uncertainty about which equilibrium will be selected next period when forming their inflation expectations. Private agents will form expectations over the probability of each outcome. Inflation expectations will therefore be of the form $\pi_t^e = f\pi^D + (1-f)\pi^a$ where f is the probability of the policymaker deciding to deviate from the inflation target due to an adverse situation, a negative sunspot.

2.2 Recursive Equilibrium

We define a recursive equilibrium where the policymaker and private agents choose their actions sequentially. At the beginning of each period, the aggregate state $s = (D, \pi^e, \zeta, \alpha_{-1})$ is public since the aggregate debt D, the expected inflation π^e for the current period, the realization of the sunspot variable ζ, and the past penalty α_{-1} have all been determined in the previous period. The policy choices (π, D'), the expected inflation $(\pi^{e\prime})$ for next period, and the individual debt holdings d' for next period determine the equilibrium jointly with s, We denote by $\pi(.)$ and $D(.)$ the inflation and debt policy functions, and by $\pi^e(.)$ the inflation expectation function, all yet to be defined.

To define a recursive equilibrium, we work the timing of actions in each period backward. We start the definition of a recursive equilibrium with private agents as they move last. When forming expectation $\pi^{e\prime}$ at the end of any period, private agents know all the parameters of the economy, his individual public debt holding d, the aggregate state s, policymaker offer of new debt D', the current period inflation π, and the policymaker optimal policy functions. The private agent's value function is defined by the following functional equation:

$$
\begin{aligned}
V^{pa}(s, d, \pi, D') = &\max_{c,d'} u(c,g) + \beta \mathbb{E} V^{pa}(s', d', \pi', D'') \\
&\text{subject to} \\
&\qquad c + d' \leq \frac{1+\pi^e}{1+\pi}\frac{1}{\beta} d + \alpha(\pi, \pi^a, \alpha_{-1})(1-\tau)e \\
&\qquad s' = \Big(D', \pi^e(s, d, \pi, D'), \alpha(\pi, \pi^a, \alpha_{-1}), \zeta'\Big) \\
&\qquad \pi' = \pi(s') \\
&\qquad D'' = D(s') \\
&\qquad c \geq 0 \\
&\qquad d' \geq -A
\end{aligned}
\tag{8}
$$

in which $A > 0$ rules out Ponzi schemes in favor of private agents but does not bind in equilibrium for any positive initial debt condition. The penalty function $\alpha(.)$ is a function of its previous value α_{-1}, the inflation target π^a, and current inflation π.

Each period after the policymaker decided how much debt D' to offer and defined inflation π, private agents decide how much debt to hold. Let $d'(s, d, \pi, D')$ be their debt policy function. When forming inflation expectations, private agents determine the nominal interest rate for next period. In the absence of multiple equilibria, π is perfectly anticipated and the real return is always $1/\beta$. If multiple equilibria are possible, private agents do not know and what the policymaker will opt to do.

When forming inflation expectations private agents look at what the policymaker could do next period. Their expectations can be written as $\pi^e(s, d, \pi, D') = \mathbb{E}\pi(s')$. When forming expectations, the set $(D', \pi^{e\prime}, \alpha) \in s'$ is known to private agents. Hence, the only unknown variable on which private agents form their expectations is the realization of the sunspot variable ζ'. Integrating out the sunspot variable commonly known distribution,

$$\mathbb{E}\pi(s') = \begin{cases} f \times \pi^D(D', \pi^{e\prime}, \alpha) + (1-f) \times \pi^a & \text{if multiple eq.} \\ \pi^D(D', \pi^{e\prime}, \alpha) & \text{if deviating unique eq.} \\ \pi^a & \text{if not deviating unique eq.} \end{cases} \tag{9}$$

where f is the exogenous probability of the adverse equilibrium occurring.

The policymaker chooses at the beginning of the period inflation π and debt issuance D' given state s. The policymaker knows that next period debt level affects the private agents inflation expectations and resolves the following problem:

$$\begin{aligned} V^p(s) = &\max_{\pi, D'} u\Big(c(s, d, \pi, D'), g\Big) + \beta \mathbb{E} V^p(s') \\ \text{subject to} & \\ & g + \frac{1+\pi^e}{1+\pi}\frac{1}{\beta} D \leq D' + \alpha(\pi, \pi^a, \alpha_{-1})\tau e \\ & s' = (D', \pi^e(s, d, \pi, D'), \alpha(\pi, \pi^a, \alpha_{-1}), \zeta') \\ & g \geq 0 \end{aligned} \tag{10}$$

We can now define a recursive equilibrium for our model economy. An equilibrium is a list of value functions V for the representative private agent V^{pa} and for the policymaker V^p; functions $c()$ and $d'()$ for the private agents consumption and saving decisions; functions $\pi()$ and $D'()$ for the policymaker inflation and debt decisions; an inflation expectation function $\pi^e()$; and an equation of motion for the aggregate debt level D' such that the following holds:

- Given D' and π, V^{pa} is the value function for the solution to the representative private-agent problem with c, d' and $\pi^{e\prime}$ the maximizing choices when $d' = D'$;

- Given π^e, V^p is the value function for the solution to the policymaker problem, and both D' and π are the maximizing choices;
- $D'(s)$ equals $d'(s, d, \pi, D')$.

Our definition of an equilibrium is similar to that of Cole & Kehoe (1996) and Cole & Kehoe (2000) and is restricted to a Markov equilibrium. Future conditional plans of the agent can be derived from their policy functions.

2.3 The Fiscal Fragility Zone

The ability of the policymaker to effectively target inflation is restricted by debt levels. Assuming inflation has always been on target, three different scenarios can be drawn. When conditioning the policymaker value function on private agents expectations and on its own choice of whether to deliver the target, we define the following scenarios:

- $V^p(D|\pi = \pi^a, \pi^e = \pi^D) \geq V^p(D|\pi = \pi^D, \pi^e = \pi^D) \rightarrow \pi^e = \pi = \pi^a$
- $\pi \in \{\pi^a, \pi^D\}$ depends on the sunspot
- $V^p(D|\pi = \pi^a, \pi^e = \pi^a) \leq V^p(D|\pi = \pi^D, \pi^e = \pi^a) \rightarrow \pi^e = \pi = \pi^D$

In the first case, the policymaker finds it better to keep inflation on target even when private agents think it will not. Consequently, there is only one equilibrium possible where private agents have faith in the policymaker delivering on target inflation. Conditional on (π^e, α_{-1}) and given the sunspot ζ is simply disregarded, the only important variable defining the policymaker value function V^p is the debt level D. The same holds for the third case when the only equilibrium is the policymaker always deviating from the inflation target.

The more interesting scenario is multiple equilibria akin to self-fulfilling target failures. If private agents believe the target will be delivered, then the policymaker will indeed prefer to do so. On the contrary, in the face of adverse expectations the policymaker chooses to deviate. The interval of debt $\left(\underline{\mathrm{D}}\ \overline{\mathrm{D}}\right)$ for which there exist multiple equilibria is the fiscal fragility zone. In this zone, private agents have doubts about the commitment of the monetary authority to the target. For debt levels under $\underline{\mathrm{D}}$ the target is perfectly supported.

Higher inflation targets change the tradeoffs the policymaker has to make. To see this let us recall the real interest rate on bonds from equation 3, $r_t = (1+\pi_t^e)/\beta(1+\pi_t) - 1$. In the fiscal fragility zone, inflation expectations will be given by $\pi_t^e = f\pi_t^D + (1-f)\pi^a$. The real interest rate in the fiscal fragility zone when the policymaker deviates will be given by:

$$r_t = \frac{(1 + f\pi_t^D + (1-f)\pi^a)}{1+\pi_t^D}\frac{1}{\beta} - 1 \tag{11}$$

The Inflation Target Coordination Role

The marginal ability of the policymaker to transfer resources through inflation decreases in the target, changing the tradeoff determining discretionary inflation. The marginal benefit of discretionary inflation will be reduced given the lower marginal capacity to transfer resources. Given certainty properties of the utility and penalty functions, increasing the target can therefore help reduce deviations as stated in proposition 2.

Proposition 2 *Let $u(c, g)$ satisfy the stated assumptions and be separable and linear in c. Let the penalty function $\alpha(\pi - \pi^a)$ satisfy the stated assumptions and $\alpha'' \leq 0$. Then for some reasonable but sufficiently high probability of deviation f and for a marginal utility in spending that does not decrease too fast, increasing the inflation target π^A decrease the deviation, $\partial\pi/\partial\pi^A < 1$.*

Proof: See appendix A.2.

A policymaker with a higher inflation target will choose a smaller deviation from the target when private agents start doubting the target. Consequently, as deviations decrease, the policymaker will face a lower real interest rate on its bonds in the fiscal fragility zone. To see this last point, let us look at the implication on the real interest rate in the fiscal fragility zone . The real interest rate on bonds is given by:

$$r = \frac{1+\pi^e}{1+\pi}\frac{1}{\beta} - 1$$

where $\pi = \pi^A$ and $\pi^e = f\pi^D + (1-f)\pi^A$ in the fiscal fragility zone. By proposition 2, we know that $\frac{\partial\pi^D}{\partial\pi^A} < 1$. Therefore, it is also true that $\frac{\partial r}{\partial\pi^A} < 0$,

$$\begin{aligned}
\frac{\partial r}{\partial \pi^A} &= \frac{1}{\beta}\left(\frac{f\frac{\partial\pi^D}{\partial\pi^A} + (1-f)}{1+\pi^A} - \frac{1+f\pi^D+(1-f)\pi^A}{(1+\pi^A)^2}\right) \\
&< \frac{1}{\beta}\left(\frac{1}{1+\pi^A} - \frac{1+f\pi^D+(1-f)\pi^A}{(1+\pi^A)^2}\right) \\
&= \frac{1}{\beta}\left(\frac{f(\pi^A-\pi^D)}{(1+\pi^A)^2}\right) \\
&< 0
\end{aligned}$$

since $\pi^A - \pi^D < 0$.

2.4 Inflation-Indexed Debt

It is not unusual for governments to issue inflation indexed bonds. We will here look at the implications of changing the nature of the bonds. To achieve such indexed bonds within the framework of our model, we change the action timing to give private agents all the needed information to perfectly anticipate policymaker decisions. By allowing private agents to know the realization of the sunspot variable when forming their inflation expectations, bonds will simply pay real interest rate $1/\beta$ in all states of nature.

1^{st} Policymaker chooses actual inflation π_t;

2^{nd} Policymaker chooses next debt level D_{t+1};

3^{rd} Next period sunspot variable ζ_{t+1} is realized;

4^{th} Private agents form next period inflation expectation π^e_{t+1} and choose the amount of next period debt d_{t+1} to hold.

With this new timing, private agents information sets are given by $(s, d, \pi, D', \zeta') = s'$. Inflation expectations π^e given information set s' will be such that $\pi^e(s') = \pi(s')$ as the policymaker choice of inflation for next period. As policymaker choices are anticipated, it is no longer possible to transfer resources from private agents in the event of a bad sunspot. It is important to notice that in equilibrium the discretionary inflation could be different from the announced target $\pi^D \neq \pi^a$. Recall that π^D is optimal given π^e and vice versa. We solve this problem using a fixed-point algorithm. The different timing only change equation 6 by how we update inflation expectation π^e[5]. We will exploit the differences between indexed bonds and nominal bonds in next section.

3 Quantitative Analysis

In this section we calibrate the model based on the 2002 confidence crisis in Brazil. The presidential election of 2002 is an interesting case study in that the candidate most likely to win was running on a platform seem to deteriorate the fiscal situation. Professional forecasters surveyed by the central bank saw inflation overshooting the target for all horizons. This loss of credibility of the inflation target in the face of a perceived fiscally fragile situation is the type of event our model aims to capture.

3.1 Functional Forms

We first start by defining the functional forms of both private agents utility and the penalty functions. We define private agents utility as a weighted average of a linear consumption

[5]The only way the discretionary inflation could equal the announced target $\pi^D = \pi^a$ would be if inflation expectation equal to the inflation target $\pi^e = \pi^a$. Given π^e, the policymaker would choose $\pi^D = \pi^e$ in the discretionary equilibrium.

and logarithmic government spending utility similar to Cole & Kehoe (2000). The weights are defined by the parameter $\rho \in (0, 1)$ that can be interpreted as a relative preference for consumption:

$$u(c_t, g_t) = \rho c_t + (1 - \rho) log(g_t) \tag{12}$$

The penalty function for deviating is a reduced form that captures the impacts of inflation on economic activity. We furthermore define a lower bound for the penalty, captured by the parameter κ, to keep the size of the economy above $(1 - \kappa)e$ as $\lim_{\pi \to \infty} \alpha(\pi, .) = (1 - \kappa)$. The penalty α_t is defined as:

$$\alpha_t = \begin{cases} 1 & \text{if } \alpha_{t-1} = 1, \pi_t = \pi_a \\ (1 - \kappa) + \kappa e^{-(c_0 + c_1(\pi_t - \pi_a)^2)} & \text{if } \alpha_{t-1} = 1, \pi_t \neq \pi_a \\ \alpha_{t-1} & \text{otherwise} \end{cases} \tag{13}$$

As typical in the literature on defaults we assume a perpetual penalty after deviating from the target. Hence, the penalty is conditional on its previous value: if $\alpha_{t-1} \neq 1$ then $\alpha_t = \alpha_{t-1}$.

3.2 Calibration

Our model is calibrated on yearly data to match the usual time frame targeted by central banks. The inflation target is set to the official target 3.5% at the time. For the discount factor, we use the historical average of the ex-post real interest rate for the period 1996-2019 [6]. The 2002 general government revenue over GDP is used as a proxy for the imposition rate on the endowments. The exogenous crisis probability is calibrated on country risk captured by the EMBI + Brazil around election time. The endowments e are chosen to represent a relatively poor government looking to increase spending. For the baseline exercises, we choose the neutral value for consumption preference $\rho = 1/2$. The remaining parameters referring to the penalty function are chosen so as to obtain a crisis zone around 70% of debt matching gross debt level in 2002 with reasonable levels of discretionary inflation π^D.

[Table 1 about here.]

3.3 Results

An indebted and altruistic policymaker choosing inflation optimally may decide to deviate from the target in the event of an expectation shock. With this calibrated model the policymaker becomes subject to such shocks after it reaches a debt level of 70%. Discretionary

[6] Using inflation indexed bonds, such as Brazilian Bonds NTN-C or NTN-B, around 2002 would give similar results.

inflation spans the interval $\pi^D \in [4, 20]$. Up to 70% of debt/GDP, the policymaker always prefers to keep inflation on target. For debt levels exceeding this lower-bound, multiple equilibria are possible as the policymaker may decide to deviate. Taking this probability into account, private agents will demand higher nominal interest rates on government bonds once the policymaker exceeds a certain debt level. Finally, for debt levels exceeding 175% of GDP the policymaker will always deviate from the target.

Optimal Fiscal Policy

The policymaker optimal debt path depends upon the initial value of its debt stock. Outside the fiscal fragility zone, it prefers to maintain debt levels constant as shown in the online appendix B.2.1. Within the fiscal fragility zone, it might either: i) choose fiscal responsibility and run down its debt to avoid the costs of an adverse equilibrium; ii) maintain constant debt levels; or iii) increase its debt to maintain a given spending level. In Figure 2, we plot next period debt as a function of current debt. The three possible responses of the policymaker are seen within the fiscal fragility zone. Those results are similar to Cole & Kehoe (1996).

[Figure 2 about here.]

The policymaker chooses a fiscally responsible debt path to avoid the expected endowment loss from deviating from the inflation target in the eventuality of adverse inflation expectation. Nevertheless, the fiscal room available to the policymaker may shrink to an extend where it becomes more interesting to run-up debt to maintain spending. As it runs up debt, the policymaker eventually suffers an adverse shock and loses credibility. By opting to run up debt the policymaker will ultimately fail to give the needed fiscal support to the inflation target.

Coordinating Expectations Through the Target

Higher inflation targets may improve credibility of monetary policy increasing the costs of deviating to attain a given inflationary transfer of resources. The level of the inflation target can help coordinate private agents expectations. First, the cost of deviating depends on the target level. Second, private agents use the inflation target to form expectations in the fiscal fragility zone. The target functions as a nominal anchor for expectations. Proposition 2 holds that under some conditions deviations $\pi^D - \pi^A$ decrease in the inflation target reducing the ex-post real interest rate in the fiscal fragility zone. In Figure 3 we show a sensitivity analysis of the deviations to changes in the inflation target. We plot $\pi^D - \pi^A$ for three different inflation targets (0%, 10%, 20%) keeping the other parameters at their baseline.

[Figure 3 about here.]

For baseline parameters, deviations $\pi^D - \pi^A$ decrease in the inflation target reducing the ex-post real interest rate in the fiscal fragility zone. Let us now look at what happens to the lower bound $\underline{\mathrm{D}}$ of the fiscal fragility zone defined by $V^p(\underline{\mathrm{D}}|\pi = \pi^a, \pi^e = \pi^D) = V^p(\underline{\mathrm{D}}|\pi = \pi^D, \pi^e = \pi^D)$. For initial debt levels below $\underline{\mathrm{D}}$ the policymaker will have a perfectly credible target preferring to keep inflation on target regardless of the private agent expectations. Above the lower bound, private agents may doubt its commitment. As deviations decrease in the target, it becomes less costly to keep inflation controlled for a given level of debt. This effect in turn increases the credibility of the inflation target as it remains fully assured up to higher levels of debt as shown in Figure 4. We plot next-period debt for the different inflation targets.

[Figure 4 about here.]

Inflation Indexed Debt

Indexed debt was defined by taking away the uncertainty about which equilibrium would be selected next period by revealing the sunspot variable to private agents. Private agents are able to correctly anticipate inflation and obtain a constant real interest rate on their bond holdings. We show that indexed debt so defined comes with higher inflation.

Recall that we find discretionary inflation solving for the best inflation given expectations and a discretionary policymaker, that is given π^e we find the optimal π^D. The difference between the two timing assumptions is in the formation of inflation expectations. In the discretionary world with nominal debt, the inflation expectation π^e is equal to $f\pi^D + (1-f)\pi^a$. With indexed debt it equals π^D. With nominal debt, agents forming expectations account for the probability of the policymaker delivering the target With indexed debt they do not. Intuitively, the policymaker unable to use inflation to partially default when subjected to a negative expectations shock attempts to transfer resources. Private agents adapt their expectations, leading to higher levels of discretionary inflation. The optimal inflation chosen by the policymaker when its debt stock is nominal versus inflation indexed is depicted in figure 5.

[Figure 5 about here.]

The higher discretionary inflation resulting from this timing may change the credibility of the inflation target for an initial debt stock, as the cost of maintaining the target increases in discretionary inflation under adverse expectations. The policymaker facing higher inflation expectations would have to pay a higher real interest rate to keep inflation on target. Debt levels $D \in \{D : V^p(D|\pi = \pi^a, \pi^e = \pi^D) \geq V^p(D|\pi = \pi^D, \pi^e = \pi^D)\}$ support the inflation target with certainty. The cost of keeping inflation on target when private agents expect a deviation is much higher. The reduced fiscal room with the policymaker facing higher debt servicing costs results in lower spending. Given the setup and calibration, we have higher marginal utility for spending than for consumption when the policymaker deviates. A decrease in spending would mean a decrease in value $V^p(D|\pi = \pi^a, \pi^e = \pi^D)$. This effect is much more pronounced than the loss of welfare related to the higher productivity shock from the higher deviation. For any D, the value $V^p(D|\pi = \pi^D, \pi^e = \pi^D)$ falls but by less than $V^p(D|\pi = \pi^a, \pi^e = \pi^D)$. The lower bound of the fiscal fragility zone $\underline{\mathrm{D}}$ decreases when debt is indexed indicating a reduced credibility of the target as it is fully supported by a reduced set of initial debt levels.

[Figure 6 about here.]

Preference for Spending

A shock to preferences can connect our model to the situation observed in Brazil during the 2002 confidence crisis. Suppose policymaker preferences shift towards giving more weight to public spending. Based on the utility function in 12 decreasing ρ would be tantamount to increasing the weight of public spending. This shift changes marginal utilities and the optimal allocation of resources increasing the share going to public spending. The altruistic policymaker chooses higher discretionary inflation levels. For a given stock of debt accounting for 70% of GDP, Figure 7 depicts the change of discretionary inflation in the preference parameter ρ.

[Figure 7 about here.]

Given a debt level, relatively higher preference for public spending increases the level of discretionary inflation. For initial debt, a preference shock could push the policymaker into the fiscal fragility zone. A sufficiently high shock to ρ could result in the loss of credibility of the target under adverse expectations. Private agents would adapt their inflation expectations. A non-null probability assigned to an adverse event would increase expectations compared to a scenario where the target is perfectly assured. Such a preference shock explains how expectations can suddenly overshoot the target as happened in Brazil in 2002.

Changing preferences have an impact on optimal fiscal policy and consequently on the debt level that supports monetary policy. The optimal fiscal policy is to run down debt for a range of initial debt. The upper limit of that range shifts downwards as policymaker preference shifts toward more public spending. In Figure 8 we show that this upper limit that can be thought of as an inflection point decreases in the preference for public spending.

[Figure 8 about here.]

3.4 Robustness: Welfare Cost of Inflation

Up to this point we have made two strong assumptions regarding penalty α, namely that it is a function of deviations from the inflation target and that inflation only has cost when the policymaker deviates. These choices allow us to focus on coordination dynamics around the target. However, due to the lack of inflation cost in "normal times" our model has the counterfactual implication that a extremely high target is welfare improving. As an example, in our baseline it is optimal to raise the inflation target to a high number such as 100%, ruling out crisis in the debt levels observable in Brazil. In this section, we show that our results are robust to the inclusion of an inflation cost for all states, not only for when the policymaker deviates. This additional cost allows us to analyze the optimal inflation target and perform some welfare comparisons.

We will use the measure of welfare costs of inflation derived by Cysne (2009) and estimated for Brazil by Campos & Cysne (2018)[7] who find a 0.24% cost of GDP for an average annual inflation of 6.63%. This percentage is in line with Aiyagari *et al.* (1998) for the US who find a cost of 0.5% for a 10% nominal interest rate. A linear approximation to the cost of inflation from Campos & Cysne (2018) leads to

$$cost(\pi_t) = 0.0272\pi_t + 0.0007 \tag{14}$$

The linear function $cost(\pi_t)$ so defined is not bounded. Nevertheless, for limited positive inflation rates this might be an adequate approximation. Combining the cost of inflation with the cost of deviating, the penalty function becomes

$$\tilde{\alpha}_t(\pi_t, \pi^a, \alpha_{t-1}) = \alpha_t(\pi_t - \pi^a, \alpha_{t-1}) + cost(\pi_t) \tag{15}$$

where α_t is defined in 13. In the calibrated model, the results and the various sensitivity analyses continue to hold. In Figure 9 the dynamics of the optimal fiscal policy remain similar. The lower bound of the fiscal fragility zone shifted upwards due to the higher costs of choosing discretionary inflation, making the target fully credible up to higher levels of debt. Within the fiscal fragility zone, the optimal fiscal policy continues to depend on the initial debt stock. For sufficiently low levels within the fiscal fragility zone, the policymaker will prefer to run down debt to exit the crisis zone. For high initial stocks, the optimal policy is to slowly increase debt up to a confidence crisis. Such debt levels do not support the inflation target.[8]

[Figure 9 about here.]

[7]Expanding on the work of Bailey (1956) and Lucas (2000), Cysne (2009) shows that Bailey's measure provides a measure of the welfare costs of inflation derived from an intertemporal general-equilibrium model.

[8]Increasing the inflation target increases the lower bound of the fiscal fragility zone as depicted in figure 4. These results are available by request.

By including the welfare costs of inflation it is possible to make a welfare analysis about the optimal inflation target for an initial debt range. By including welfare costs, there exists a tradeoff between costs associated with higher levels of inflation against the benefits of a more credible higher target. Let us define the debt interval $D_0 \in [80, 110]$ that includes the lower bounds of the fiscal fragility zone for inflation targets $\pi^a \in [0, 20\%]$. Let W be private agents welfare on the discrete debt interval

$$W = \sum_{D=80\%GDP}^{110\%GDP} V^p(s) \tag{16}$$

where $s = (D, \pi^e, \zeta, \alpha_{-1})$. The value function $V^p(s)$ has been previously defined in 10. Private agents welfare defined on the initial debt interval is depicted in Figure 10. Given the linear costs in inflation, welfare is upward bounded. Maximizing welfare on the interval with respect to a pre-announced inflation target would therefore yield an optimal choice.

[Figure 10 about here.]

4 2002 Confidence Crisis in Brazil

In 2002 and 2003 Brazilian policymakers faced inflationary pressures when it became clear the left presidential candidate would win. The perception was that his victory would mean implementation of a new policy framework that could undermine the previous reduction of inflation. Consequently, inflation expectations overshoot the target's upper bounds at all horizons relevant to the central bank as shown in Figure 1. We map this event in our model as a shock to the preference for spending in the parameter ρ. Sensitivity analysis in Section 3.3 shows that for a given initial debt stock the target could lose credibility after a preference shock. By favoring more public spending the policymaker could become vulnerable to adverse shocks that would make it deviate from the inflation target. Private agents taking this probability into account when forming expectations would increase their forecasts of future inflation, exactly as observed in 2002.

In response to rising inflation expectations, the outgoing and new administrations took several steps. To coordinate inflation expectations in the short run, they increased the target for 2003 in an extra meeting held in June 2002 and unofficially again in January 2003., In 2003 responsible macroeconomic policies were sustained by public debt reduction. Changes in the debt mix away from indexed bonds especially foreign exchange were implemented. Ultimately inflation expectations converged back to the target. These policy responses closely mirror the prescriptions suggested by the various sensitivity analyses performed. Consider these policies in further detail.

Fiscal Policy

After the 2002 election gross public debt was gradually reduced. Gross debt went down from almost 80% of GDP in 2002 to close to 70% in 2004. Furthermore, the government continued to run primary surpluses to meet its debt obligations in a signal of fiscal responsibility. The primary surplus went up form 2.16% of GDP in 2001 to 2.70% in 2004. From the perspective of our model, such fiscal policy is compatible with the policymaker trying to exit the fiscal fragility zone and give the needed fiscal support to its inflation target.

Inflation Target

Before the October elections, the 2003 target was exceptionally revised upwards from a previously announced 3.25% to 4%. Similarly, the upper and lower bounds widened from -/+ 2% to 2.5%. In January 2003, the Ministry of Finance sent a letter stating that the adjusted target would be of 8.5% in 2003 and 5.5% in 2004. The latter was confirmed by the national monetary committee as the inflation target for 2004 in June 2003 as can be seen in Table 2. From the perspective of our model an indebted policymaker with a higher inflation target might be more credible. Serving as a nominal anchor, the higher and more credible inflation target makes private agents readjust their inflation expectations.

[Table 2 about here.]

Debt Management

The largest portion of Brazilian public securities was indexed to some benchmark prior to 2002. Such benchmarks include consumer price inflation, exchange rates, and the targeted policy interest rate. On one hand, debt indexed to inflation and exchange rates accounted for 56% of outstanding debt in 2002 and were gradually reduced to 33% in 2005. On the other hand, pre-fixed securities and those indexed to the overnight interest rate (Selic) were close to 44% in 2002 and increased to over 67% in 2005. The latter type of debt can be thought of as debt on which partial defaults are possible.[9] Using this classification as a proxy for the nominal and indexed debt denominations in the model, we suggest that reducing indexed debt could provide support to the inflation target.

5 Empirical Results

The calibrated model leads to the conclusions that i) the size of the deviation could be reduced by increasing the target and reducing debt and ii) the probability to overshoot the

[9]Pre-fixed securities when up from 1.5% up 2002 to 23.6% in 2005. Selic indexed securities were 42.4% in 2002 and 43.9% in 2005.

target would increase with debt and decrease with higher target levels. The present section questions whether there is empirical evidence for the predictions based on our model. We construct a dataset includes 20 countries with at least 15 years of inflation targeting[10] covering the period 2000 to 2019. Targets are those reported by the respective central banks that were manually collected from each central bank web page. Inflation and gross debt and revenue to GDP statistics are from the IMF. With regards to inflation, end-of-year consumer price inflation is the target benchmark. Some general statistics are reported in Table 3. The variables present both inter and intra-country variability. In the case of CPI targets, 55% of our sample changed the target at least once. Most of the changes are in middle income countries.[11]

[Table 3 about here.]

Real Effective Exchange Rate (Reer) and GDP gap estimates enter robustness checks. When Reer statistics were not available from the IMF, other sources were accessed.[12] GDP gap estimates are constructed using quarterly seasonally adjusted GDP volume statistics from the IMF. When not available, the unadjusted equivalent are seasonally adjusted with the Arima X-11 procedure.[13] The quarterly GDP gap statistics are obtained applying an HP filter with a smoothing parameter of 1600. To mitigate the endpoint bias of the filter at the beginning of each series, we estimate the gap for the longer 1996Q1 - 2020Q1 period. Finally, the yearly GDP gap is defined as the average gap over the relevant period.

Deviations from the Target

The first order condition of the discretionary inflation problem from 6 relates the deviation of inflation $\pi_{i,t}$ from the inflation target $\pi^A_{i,t}$ to observable and latent variables for each country i. We estimate the following model,[14]

$$\pi_{i,t} - \pi^A_{i,t} = \beta_1 \text{revenue}_{i,t} + \beta_2 \text{debt}_{i,t} + \beta_3 \pi^A_{i,t} + \beta_4 \text{revenue}_{i,t} * \text{debt}_{i,t} + c_i + u_{i,t} \qquad (17)$$

where the idiosyncratic error $u_{i,t}$ satisfies $\mathbb{E}(u_{i,t}|X_{i,1}, ..., X_{i,T}, c_i) = 0,\ t = 1, ..., T$ with $X_{i,t}$ being a vector of the observable regressors at time t and for country i. The variables and parameters of the model are mapped into both observed series and latent variables. We map

[10] The countries in the sample are Australia, Brazil, Canada, Chile, Colombia, Czech Republic, Iceland, Indonesia, Israel, Mexico, New Zealand, Norway, Peru, Philippines, Poland, South Africa, Sweden, Thailand, Turkey, and the United Kingdom.

[11] We used the World Bank classification.

[12] BIS for Peru, Indonesia, and Turkey. Bank of Thailand for Thailand.

[13] This was the case for Peru and Turkey.

[14] In the online appendix B.1 we show how 17 is related to the first order condition of the discretionary inflation problem from 7.

the model variables D, τe, and π^A to gross debt (%GDP), revenue (%GDP), and the inflation target. The unobservables variables e, f, c_1, c_2 are mapped into a country fixed effect c_i that captures the time-constant individual heterogeneity between countries. We use a fixed effect estimator as it seems reasonable to assume that their choices of debt, revenue and inflation target are related to the unobserved characteristics of each country c_i. In other words, we cannot assume $\mathbb{E}(X_{i,t}c_i) = 0 \ \forall t$ as required for a random effect estimator.[15]

In terms of interpretation, the net impact of debt should be positive. Given higher levels of debt, the policymaker will have more incentive for discretionary inflation. Furthermore, discretionary inflation increases in debt. Hence, the deviation to increase in debt levels as the policymaker will be more likely to deviate and will choose higher discretionary inflation when doing so. Given an interaction term in 17 one would have to look at the joint impact captured by β_2 and β_4 for a given level of revenue to GDP. We also expect the coefficient on the inflation target to be negative as the policymaker could help coordinate private agents expectations by adopting a more credible (higher) inflation target in given situations. Were inflation perfectly anchored, changing the target would not result in changes in expected deviation. In other words, the coefficient β_3 would equal zero. Finally, higher revenue means the policymaker has more fiscal room for spending. This room decreases the incentives to transfer resources through discretionary inflation leading to a negative net impact of revenue. Given the interaction term between debt and revenue, the joint impact captured by β_1 and β_4 should be negative for a given level of debt.

Estimation I in Table 4 is the basic model from 17. The remaining estimations, II-V, are robustness checks.

[Table 4 about here.]

In estimation I, deviations from the target are on average negatively related to the target level. In the case of perfectly anchored inflation, the coefficient should not be statistically different from zero. We also have a positive coefficient on debt and a negative coefficient for the interaction term between debt and revenues. This can be interpreted as higher debt implying higher deviations for countries with limited revenues. For revenues no higher than 35% of GDP, the net impact of debt is positive. This result applies to the middle income countries in our sample. The result goes in the direction of what the theoretical model predicted as both the probability of deviating and deviations from the target are positively related to debt levels. On average countries with higher debt levels have higher deviations from their inflation target.

The coefficient on revenue is positive in all settings although not always significant. Given the interaction term with debt, the net impact of revenue is positive up to debt levels of

[15] A Hausman test between a fixed and random effect estimator similarly suggests the use of the former.

88%, above the maximum in our sample. Hence the impact of higher revenue is to increase deviations from the inflation target. Although this goes against what was expected from the theoretical model, one could argue that higher revenue could be correlated to preferences for public spending that in turn could lead to inflationary pressure.

The results remain accounting for different types of shocks and variables usually associated with inflation dynamics. In estimation II, we include a time fixed effect in order to account for global shocks such as commodity prices. In our sample 2008 stands out as many countries overshot their inflation targets after the financial crisis. The time dummies are meant to take such global co-movements in inflation in account. Estimation III also includes shocks to the real effective exchange rate. Estimation IV adds the impact of deviations from potential GDP on inflation.

Probability of Overshooting the Target

The policymaker overshoots the inflation target when end-of-year inflation exceeds the upper bound of the target.[16] In the theoretical model, the policymaker had more incentive to overshoot the target when it had limited fiscal space due to high debt servicing cost. We estimate a similar equation to 17, but with regard to the probability of overshooting the target:

$$I_{\pi_{i,t}>\overline{\pi}^A_{i,t}} = \beta_1 \text{revenue}_{i,t} + \beta_2 \text{debt}_{i,t} + \beta_3 \text{target}_{i,t} + \beta_4 \text{revenue}_{i,t} * \text{debt}_{i,t} + c_i + u_{i,t} \tag{18}$$

where $\overline{\pi}^A_{i,t}$ is the upper bound of the inflation target for country i at time t. The indicator $I_{\pi_{i,t}>\overline{\pi}^A_{i,t}} = 1$ when inflation $\pi_{i,t}$ overshoots the upper bound of the inflation target $\overline{\pi}^A_{i,t}$. The idiosyncratic error $u_{i,t}$ satisfies $\mathbb{E}(u_{i,t}|X_{i,1}, ..., X_{i,T}, c_i) = 0,\ t = 1, ..., T$. The probability of overshooting the target will then be a logistic function:

$$Pr(I_{\pi_{i,t}>\overline{\pi}^A_{i,t}} = 1|X_{i,t}, c_i) = \frac{1}{1 + e^{-X'_{i,t}\beta - c_i}}, \quad t = 1, ..., T \tag{19}$$

The expected results and dynamics are quite similar to those in the previous section with an expected net positive impact of debt, negative impact of the inflation target, and negative impact of revenue on the probability of overshooting the target. Each year in the sample at least two countries overshoot their respective inflation target. The years 2007 and 2008 stand out as over half of of the countries overshoot their inflation target. A time dummy is likely to capture this effect. Also, virtually all countries except two overshoot their target at least once with some countries such as Turkey close to being serial over shooters. Overall, middle

[16] Some countries adopt pointwise targets instead of tolerance bounds. This is for instance the case of the UK and Norway. In such cases we used the average upper tolerance limit from the rest of the sample (1.2%).

income countries overshoot the target more often than high-income countries. Nevertheless, high income countries overshoot the target 39 times.

The first column of Table 5 is the baseline model while the remaining columns represent robustness checks similar in spirit to the previous section. When looking at the net impact of debt on the probability to overshoot the target the coefficients have similar signs as the previous estimates with regards to deviations from the target. Estimation I has the most restrictive condition for a net positive effect of debt. For revenues over 30% of GDP the net effect of debt stops being positive. Not all middle-income countries in our sample have revenue below this level. However, the effects are not statistically significant in any of the settings.

[Table 5 about here.]

The net impact of revenue remains positive for debt levels in the sample, not in the same direction as predicted by the theoretical model. The predicted negative impact of debt is based on increased fiscal room provided by higher revenue, decreasing incentive to use discretionary inflation to transfer resources away from private agents debt. However, another channel is possible. Revenue might be correlated with some other factors such as a higher preference for government spending, what would increases incentives to use inflation for transfer of resources. This channel could explain our results.

The probability of overshooting the target is negatively related to the target level and significant at the 5% level in all settings. Our interpretation is that some countries might have inflation targets that are too low, leaving the door open to overshoot the target more often. Those counties could improve their ability to keep inflation on target by adopting higher targets. The results remain little changed when including shocks to exchange rates, the output gap, or a time dummy. Changes in the real effective exchange rate seem to be an important factor in causing policymakers to overshoot their inflation target. The output gap is not significant.

6 Conclusion

High public debt opens the door to inflation due to target coordination failure, depressing private consumption and GDP. We propose a model to describe the inter-temporal tradeoff between fiscal policy and monetary policy when forward-looking and rational private agents finance an altruistic policymaker. Indebted policymakers have a limited budget and are subject to expectation shocks forcing to them either to accept a higher interest rate with inflation on the pre-announced target or accept higher inflation. Our results endorse fiscal austerity to gradually lower the public debt to prevent coordination failure and self-confirmed inflation. If debt is high, the policymaker should avoid both an excessively low inflation target and inflation indexed securities as the possibility of partial default limits inflation.

For central bankers that might face doubts about their credibility to sustain an inflation target, a set of tools based on our model has been successfully implemented in Brazil during the 2002 confidence crisis. Our results are consistent with data from a panel of inflation targeting countries.

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A Proofs

A.1 Above-Target Discretionary Inflation

We want to show that there exists a discretionary inflation π^D such that $\pi^D > \pi^a$ under certain initial conditions. From the first order conditions of the policymaker's problem given by equation 7 we have:

$$u_{c_T}\left(-\frac{1+\pi_T^e}{(1+\pi)^2}\frac{1}{\beta}D + \alpha'(1-\tau)e\right) + u_{g_T}\left(\frac{1+\pi_T^e}{(1+\pi)^2}\frac{1}{\beta}D + \alpha'\tau e\right)$$
$$+ \frac{\beta}{1-\beta}\left(u_c\alpha'(1-\tau)e + u_g\alpha'\tau e\right) = 0$$

We define $\kappa_1 \equiv -u_{c_T}\alpha'(1-\tau)e - u_{g_T}\alpha'\tau e - \frac{\beta}{1-\beta}\left(u_c\alpha'(1-\tau)e + u_g\alpha'\tau e\right) > 0$. The assumption on α gives us $\alpha' < 0$ and assumption on $u(c,g)$ gives us $u_c, u_g > 0$. Then,

$$(1+\pi)^2 = \frac{1+\pi_T^e}{\kappa_1}\frac{1}{\beta}D(u_{g_T} - u_{c_T})$$

We know that π_T^e satisfies $\pi_T^e = f\pi + (1-f)\pi^a$ where $f \in [0,1]$ and suppose that $u_{g_T} - u_{c_T} > 0$. We can write the equation above as the following quadratic problem:

$$\pi^2 + \pi\left(2 - \frac{f}{\kappa_1}\frac{1}{\beta}D(u_{g_T} - u_{c_T})\right) + 1 - \frac{1+(1-f)\pi^a}{\kappa_1}\frac{1}{\beta}D(u_{g_T} - u_{c_T}) = 0$$

Simplifying notation once again:

$$\pi^2 + \pi(2 - f\kappa_2) + 1 - (1 + \pi^a - f\pi^a)\kappa_2 = 0$$

Which gives the solution:

$$\pi = \frac{-(2-f\kappa_2) \pm \sqrt{(2-f\kappa_2)^2 - 4(1-(1+\pi^a - f\pi^a)\kappa_2)}}{2}$$
$$= \frac{-(2-f\kappa_2) \pm \sqrt{4\kappa_2(1-f)(1+\pi^a) + (f\kappa_2)^2}}{2}$$

Observe that $4\kappa_2(1-f)(1+\pi^a) + (f\kappa_2)^2 > 0$ since $\kappa_2 > 0$. For π to be higher than the inflation target π^a we must have:

$$\pi = \frac{-(2-f\kappa_2) \pm \sqrt{4\kappa_2(1-f)(1+\pi^a) + (f\kappa_2)^2}}{2} > \pi^a$$

Rewriting:

$$\pm\sqrt{4\kappa_2(1-f)(1+\pi^a) + (f\kappa_2)^2} > 2\pi^a + (2 - f\kappa_2)$$

If $2\pi^a + (2 - f\kappa_2) < 0$ than there is always at least one solution such that $\pi > \pi^a$ since $\sqrt{4\kappa_2(1-f)(1+\pi^a) + (f\kappa_2)^2} > 0 > 2\pi^a + (2 - f\kappa_2)$. If, however, $2\pi^a + (2 - f\kappa_2) \geq 0$, for there to be at most one possible solution we need to prove that $\sqrt{4\kappa_2(1-f)(1+\pi^a) + (f\kappa_2)^2} > 2\pi^a + (2 - f\kappa_2)$. Since both terms are defined on $\mathbb{R}_+$ we can use the square operator and keep the inequality preserved:

$$4\kappa_2(1-f)(1+\pi^a) + (f\kappa_2)^2 > (2\pi^a + (2 - f\kappa_2))^2$$

Which can be simplifyed to $\kappa_2 > 1 + \pi^a$ and, replacing κ_2,

$$\frac{D(u_{g_T} - u_{c_T})}{-u_{c_T}\alpha'(1-\tau)e - u_{g_T}\alpha'\tau e - \frac{\beta}{1-\beta}\left(u_c\alpha'(1-\tau)e + u_g\alpha'\tau e\right)} > \beta(1+\pi^a)$$

In the denominator, it is the benefit of deviating - increasing public spending and decreasing private spending. In the numerator, it is the cost of losing endowment in the long run. This ratio must be greater than the nominal interest rate when agents expect the inflation target.

A.2 Lower Real Interest Rates through Higher Target

We first show the conditions under which deviations decrease in the inflation target. The first order condition of the discretionary inflation problem is given by:

$$u_c^T\left[\left(-\frac{1+\pi_T^e}{(1+\pi)^2}\frac{1}{\beta}\right)D+\alpha'(1-\tau)e\right]+u_g^T\left[\left(\frac{1+\pi_T^e}{(1+\pi)^2}\frac{1}{\beta}\right)D+\alpha'\tau e\right]$$
$$+\frac{\beta}{1-\beta}\left[u_c\alpha'(1-\tau)e+u_g\alpha'\tau e\right]=0$$

where the subscript T in u_c^T and u_g^T indicates that the marginal utilities are evaluated at time T when the policymaker chooses to deviate from the target. After the policymaker chooses discretionary inflation the economy goes to a steady-state and we therefore drop all time subscripts.

Assuming that utility is linear in c and using the expected inflation $\pi_T^e = f\pi+(1-f)\pi^A$, can be rewritten as:

$$f\frac{1}{1+\pi}\frac{1}{\beta}D\left(u_g^T-1\right)+(1-f)\frac{1+\pi^A}{(1+\pi)^2}\frac{1}{\beta}D\left(u_g^T-1\right)$$
$$+u_g^T\alpha'\tau e+\frac{1}{1-\beta}\alpha'(1-\tau)e+\frac{\beta}{1-\beta}u_g\alpha'\tau e=0$$

We can find $\frac{\partial\pi}{\partial\pi^A}$ by taking the implicit derivative of the above equation with respect to π^A.

$$\frac{\partial\pi}{\partial\pi^A}\Bigg[-f\frac{D}{\beta}\frac{(u_g^T-1)}{(1+\pi)^2}-(1-f)\frac{D}{\beta}\frac{2(1+\pi^A)(u_g^T-1)}{(1+\pi)^3}+\alpha''\left(u_g^T\tau e+\frac{1}{1-\beta}(1-\tau)e+\frac{\beta}{1-\beta}u_g\tau e\right)$$
$$+u_{gg}^T\left(\frac{(1-f)(1+\pi^A)}{(1+\pi)^2}\frac{D}{\beta}+\alpha'\tau e\right)\left(f\frac{D}{\beta}\frac{1}{1+\pi}+(1-f)\frac{D}{\beta}\frac{1+\pi^A}{(1+\pi)^2}+\alpha'\tau e\right)+\frac{\beta}{1-\beta}(\alpha'\tau e)^2u_{gg}\Bigg]$$
$$-u_{gg}^T\left(\alpha'\tau e+\frac{1-f}{1+\pi}\frac{1}{\beta}D\right)\left(f\frac{D}{\beta}\frac{1}{1+\pi}+(1-f)\frac{D}{\beta}\frac{1+\pi^A}{(1+\pi)^2}+\alpha'\tau e\right)$$
$$-\frac{\beta}{1-\beta}(\alpha'\tau e)^2u_{gg}+(1-f)\frac{D}{\beta}\frac{(u_g^T-1)}{(1+\pi)^2}-\alpha''\left(\tau eu_g^T+\frac{1}{1-\beta}(1-\tau)e+\frac{\beta}{1-\beta}\tau eu_g\right)=0$$

For the sake of notation let us write:

$$\frac{\partial\pi}{\partial\pi^A}\left[\text{term 1}\right]=\text{term 2}$$

In order to find the conditions under which $\frac{\partial\pi}{\partial\pi^A}<1$ we have to prove that $\frac{\text{term 2}}{\text{term 1}}<1$.

Suppose for now that term $1 < 0$, then we have:

$$\text{term } 1 < \text{term } 2$$

Replacing terms:

$$\begin{aligned}
&- f\frac{D}{\beta}\frac{(u_g^T-1)}{(1+\pi)^2} - (1-f)\frac{D}{\beta}\frac{2(1+\pi^A)(u_g^T-1)}{(1+\pi)^3} + \alpha''\left(u_g^T\tau e + \frac{1}{1-\beta}(1-\tau)e + \frac{\beta}{1-\beta}u_g\tau e\right)\\
&+u_{gg}^T\left(\frac{(1-f)(1+\pi^A)}{(1+\pi)^2}\frac{D}{\beta}+\alpha'\tau e\right)\left(f\frac{D}{\beta}\frac{1}{1+\pi}+(1-f)\frac{D}{\beta}\frac{1+\pi^A}{(1+\pi)^2}+\alpha'\tau e\right)+\frac{\beta}{1-\beta}(\alpha'\tau e)^2u_{gg}\\
&<u_{gg}^T\left(\alpha'\tau e+\frac{1-f}{1+\pi}\frac{1}{\beta}D\right)\left(f\frac{D}{\beta}\frac{1}{1+\pi}+(1-f)\frac{D}{\beta}\frac{1+\pi^A}{(1+\pi)^2}+\alpha'\tau e\right)\\
&+\frac{\beta}{1-\beta}(\alpha'\tau e)^2u_{gg}-(1-f)\frac{D}{\beta}\frac{(u_g^T-1)}{(1+\pi)^2}+\alpha''\left(\tau e u_g^T+\frac{1}{1-\beta}(1-\tau)e+\frac{\beta}{1-\beta}\tau e u_g\right)
\end{aligned}$$

Eliminating terms on both sides, rearranging, and isolating f,

$$f\left(\frac{2(1+\pi^A)}{1+\pi}-2\right) < \frac{2(1+\pi^A)}{1+\pi}-1$$

If $\pi > \pi^A$ then:

$$f > \frac{1+2\pi^A-\pi}{2(\pi^A-\pi)}$$

To see that the restriction on the probability of default is reasonable, suppose that discretionary inflation is below 100% and that the inflation target is low. Then the restriction $f > \frac{1+2\pi^A-\pi}{2(\pi^A-\pi)}$ would not be binding. Only for higher levels of discretionary inflation would this restriction become binding. At the limit, when $\pi \to \infty$, the restriction requires $f > \frac{1}{2}$.

We supposed that term $1 < 0$. Let us look at the conditions necessary for this to be true.

$$\begin{aligned}
\text{term } 1 = &- f\frac{D}{\beta}\frac{(u_g^T-1)}{(1+\pi)^2} - (1-f)\frac{D}{\beta}\frac{2(1+\pi^A)(u_g^T-1)}{(1+\pi)^3} + \frac{\beta}{1-\beta}(\alpha'\tau e)^2u_{gg}\\
&+\alpha''\left(u_g^T\tau e+\frac{1}{1-\beta}(1-\tau)e+\frac{\beta}{1-\beta}u_g\tau e\right)\\
&+u_{gg}^T\left(\frac{(1-f)(1+\pi^A)}{(1+\pi)^2}\frac{D}{\beta}+\alpha'\tau e\right)\left(f\frac{D}{\beta}\frac{1}{1+\pi}+\frac{D}{\beta}\frac{1-f)(1+\pi^A)}{(1+\pi)^2}+\alpha'\tau e\right)
\end{aligned}$$

The first line is negative. The only uncertainty about the sign of term 1 therefore comes from the last two lines. Suppose that $\alpha'' \leq 0$, we then either want $\frac{(1-f)(1+\pi^A)}{(1+\pi)^2}\frac{D}{\beta} + \alpha'\tau e$ and $f\frac{D}{\beta}\frac{1}{1+\pi} + \frac{D}{\beta}\frac{1-f)(1+\pi^A)}{(1+\pi)^2} + \alpha'\tau e$ to be of the same sign or u_{gg}^T to be limited in such a way for term 1 to remain negative. The former is true when

$$\alpha' \in \mathbb{R}_- \setminus \left(-\frac{D}{\beta \tau e} \left(f \frac{1}{1+\pi} + (1-f) \frac{1+\pi^A}{(1+\pi)^4} \right), -\frac{D}{\beta \tau e} (1-f) \frac{(1+\pi^A)}{(1+\pi)^2} \right)$$

Therefore, there exists $\epsilon < 0$ such that $\alpha'' \leq 0$ and $\epsilon < u_{gg}^T$ imply that term 1 is negative.

Summarizing, if i) $\alpha'' \leq 0$ and ii) $f > \frac{1+2\pi^A - \pi}{2(\pi^A - \pi)}$, then there exists some $\epsilon < 0$ such that $\epsilon < u_{gg}^T$ implies that the deviation is decreasing in the target, $\frac{\partial \pi}{\partial \pi^A} < 1$.

B Online appendix

B.1 Testing the FOC

The first order condition of the discretionary inflation problem from equation 7 when assuming linear utility in consumption is given by:

$$\frac{1+\pi_T^e}{(1+\pi_T^D)^2}\frac{1}{\beta}D(u_g^T-1)+u_g^T\alpha'\tau e+\frac{1}{1-\beta}\alpha'(1-\tau)e+\frac{\beta}{1-\beta}u_g\alpha'\tau e=0$$

where u_g^T is the marginal utility of spending when deviating at time T and u_g is the ensuing steady state marginal utility. π_T^e are the private agents' expectations at time T, π_T^D is the optimal discretionary inflation chosen by the policymaker when deviating from the target at time T, D is the level of debt, τe the policymaker's revenues and α' is the marginal productivity shock when deviating.

The equation can be rewritten as:

$$D\left(u_g^T-1\right)\left(\frac{1+\pi_T^e}{(1+\pi^D)^2}\right)=\tau e\alpha'\left[\frac{1}{1-\beta}\frac{1-\tau}{\tau}+\frac{\beta}{1-\beta}u_g+u_g^T\right]$$

Taking logs we obtain:

$$\begin{aligned}&d+\log(u_g^T-1)+\log(1+\pi_T^e)-2\log(1+\pi^D)\\&=\log(\tau e)+\log(\alpha')+\log\left(\frac{1}{1-\beta}\frac{1-\tau}{\tau}+\frac{\beta}{1-\beta}u_g+u_g^T\right)\end{aligned}$$

where $d=\log(D)$. Replacing expectations by $\pi_T^e=f\pi^D+(1-f)\pi^A$ and using the approximation for $\log(1+x)\simeq x$ for small x, we have $\log(1+\pi_T^e)-2\log(1+\pi^D)=(2-f)(\pi^A-\pi^D)-\pi^A$. Hence:

$$\pi^D-\pi^A=-\frac{\log(\tau e)}{2-f}+\frac{d}{2-f}-\frac{\pi^A}{2-f}-\frac{c}{2-f}$$

Where $c=\log(\alpha')-\log(u_g^T-1)+\log(\frac{1}{1-\beta}\frac{1-\tau}{\tau}+\frac{\beta}{1-\beta}u_g+u_g^T)$ will also capture effects of debt levels d and revenue τe through the marginal utility of government spending. This unfortunately makes the coefficients less straightforward to interpret without any prior calibration and initial conditions. We propose to model the relationship for country i as follows:

$$\pi_{i,t}-\pi_{i,t}^A=\beta_{0,i}+\beta_1\text{revenue}_{i,t}+\beta_2\text{debt}_{i,t}+\beta_3\text{target}_{i,t}+\beta_4\text{revenue}_{i,t}*\text{debt}_{i,t}+u_{i,t}$$

where the interaction term between revenue and public debt is meant to capture the dynamics of the marginal utility of government spending at time t. The idiosyncratic error term $u_{i,t}$ satisfies $\mathbb{E}(u_{i,t}|X_{i,1},...,X_{i,T},c_i) = 0, t = 1,...,T$. Coefficient $\beta_{0,i}$ captures a country fixed effect while all other coefficients are common to all countries.

B.2 Proofs

B.2.1 Optimal Debt Policy Outside the Fiscal Fragility Zone

Outside of the fiscal fragility zone there is only a unique inflation equilibrium making it perfectly anticipated. The policymaker's problem can be reduced to the following:

$$\begin{aligned} &\max_{D_{t+1}} \sum_t \beta^t u(c_t, g_t) \\ &\text{s.t. } c_t = \frac{1}{\beta} D_t + \alpha(1-\tau)e - D_{t+1} \\ &\qquad g_t = D_{t+1} + \alpha\tau e - \frac{1}{\beta} D_t \end{aligned}$$

The first order condition for D_{t+1} gives:

$$u_{g_t} - u_{c_t} = u_{g_{t+1}} - u_{c_{t+1}}$$

and, giving the assumption of linear in c, $u_{g_{t+1}} = u_{g_t}$. Given $u(c, g)$ strictly concave in g, we must have $g_{t+1} = g_t$. Replacing g_t and g_{t+1} by the government budget equation, iterating forward and taking limits we obtain:

$$\lim_{t\to\infty} D_t = \sum_{i=0}^{\infty} \left(\frac{1}{\beta}\right)^i (D_{t+1} - D_t) + D_{t+1}$$

Suppose that $D_{t+1} \neq D_t$, then the policymaker will either run up infinite debt or credit and violate the no Ponzi condition. Hence, outside of the fiscal fragility zone we must have that $D_{t+1} = D_t$.

B.2.2 Existence Solution Discretionary Inflation

Proposition 3 *Let the the utility function $u(c, g)$ and the penalty function $\alpha(\pi - \pi^a)$ be such that they satisfy the already stated assumptions. If the universe of possible inflation choices is defined on the compact set $[0, \overline{\pi}]$ where $\overline{\pi} > 0$ is some upper limit. Then there exists a discretionary inflation level π^D such that π^D is optimal given private agents' inflation expectations π^e and vice versa.*

Proof: In order to prove that there exists a discretionary inflation level π^D such that π^D is optimal given π^e, and vice versa, we will use Brouwer's fixed point theorem. Since we are only interested in the universe of limited inflation we state that $\pi^D \in [0, \overline{\pi}]$ where $\overline{\pi} > 0$ is an upper limit for the possible inflation levels. Let $\pi : [0, \overline{\pi}] \to [0, \overline{\pi}]$ be the function mapping private agents expectations into the policymaker's inflation choice as defined by the discretionary inflation problem in equation 6.

Let us now define the auxiliary function $\tilde{\pi}(\pi^D) := \pi(f\pi^D + (1 - f)\pi^a) = \pi(\pi^e)$. Since $\tilde{\pi} : [0, \overline{\pi}] \to [0, \overline{\pi}]$ maps a compact interval on $\mathbb{R}$ into itself, we only need to prove that it is continuous to use Brouwer's theorem for the existence of a fixed point.

First, by hypothesis we know that the penalty function $\alpha : [0, \overline{\pi}] \to (0, 1)$ mapping discretionary inflation into total factor productivity is continuous. Hence, the consumption choice will also be continuous. The same holds for government spending.

Second, the utility function $u : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ mapping government spending and private consumption into a utility scale is also continuous by hypothesis.

Combining the mapping of discretionary inflation $[0, \overline{\pi}]$ into consumption and spending $\mathbb{R}_+ \times \mathbb{R}_+$ and the mapping of consumption and spending $\mathbb{R}_+ \times \mathbb{R}_+$ into a utility scale $\mathbb{R}$, it is easy to see that the map of discretionary inflation $[0, \overline{\pi}]$ into a utility scale $\mathbb{R}$ will also be continuous. Finally, given that the argmax operator, mapping $[0, \overline{\pi}]$ into $[0, \overline{\pi}]$, maintains those properties, we have that $\tilde{\pi} : [0, \overline{\pi}] \to [0, \overline{\pi}]$ is continuous. Which is what we wanted to show.

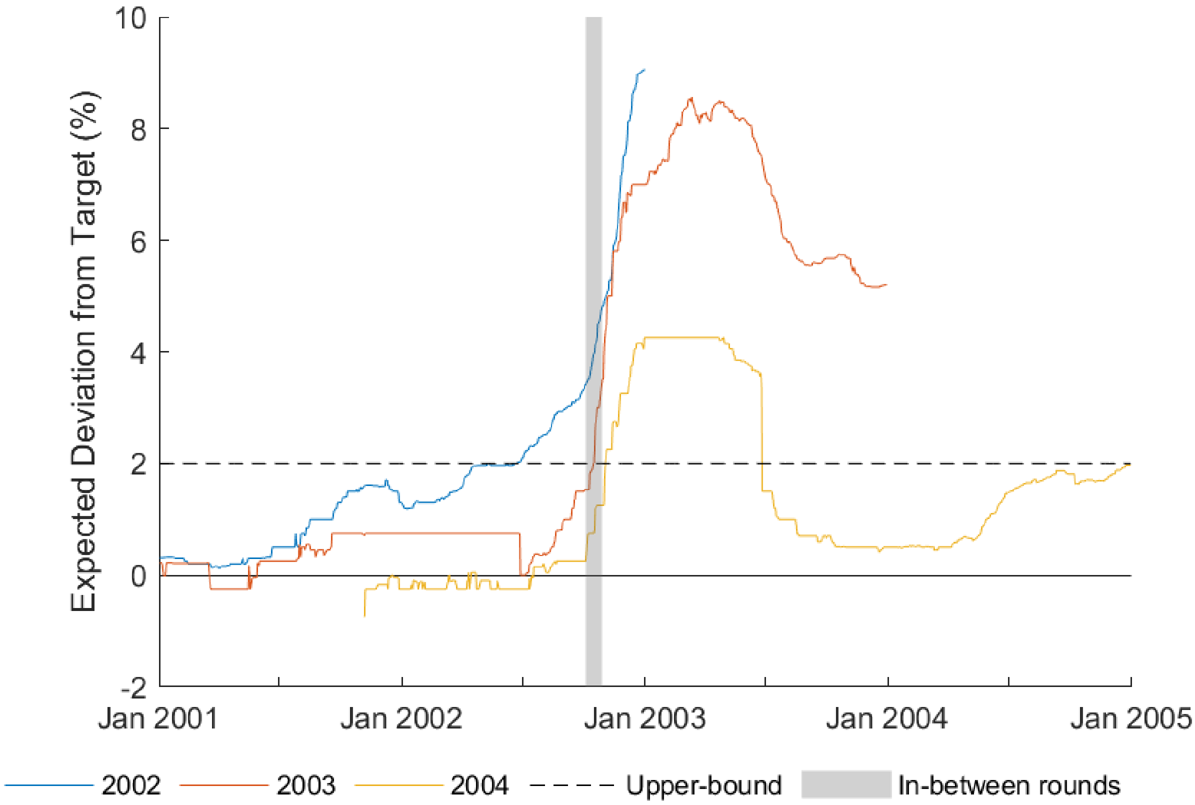

Figure 1: Expectation Crisis in Brazil

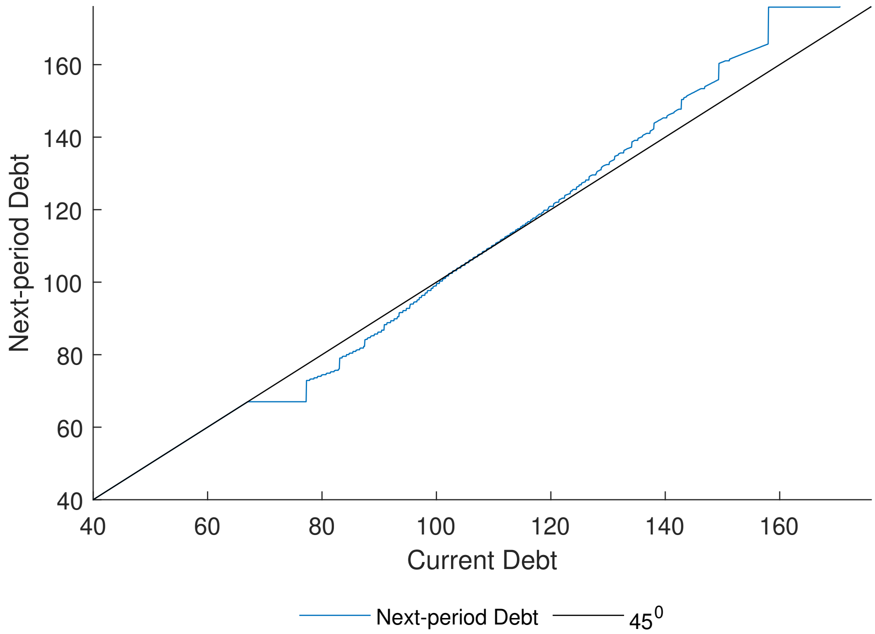

Figure 2: Debt Policy Function

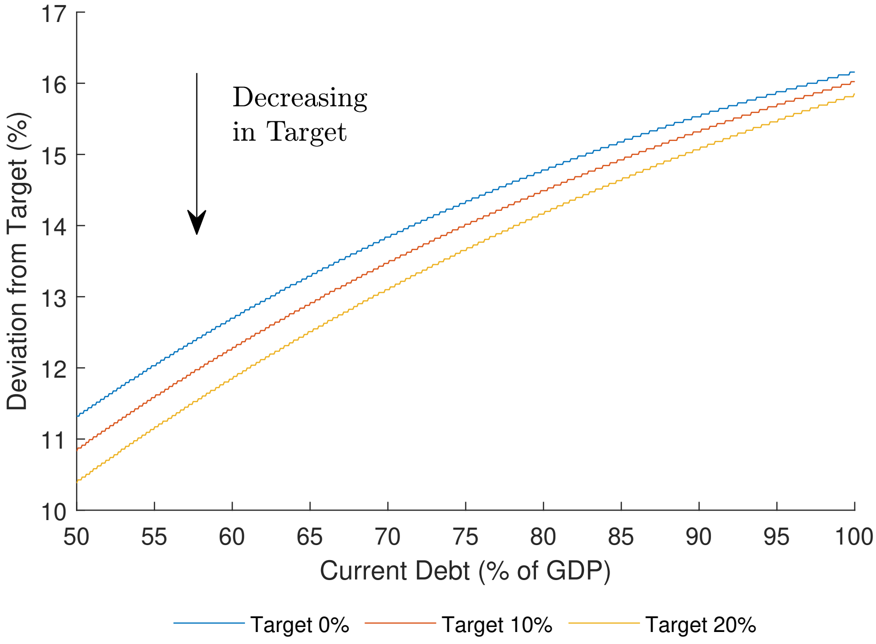

Figure 3: Sensitivity: Deviations to Inflation Target

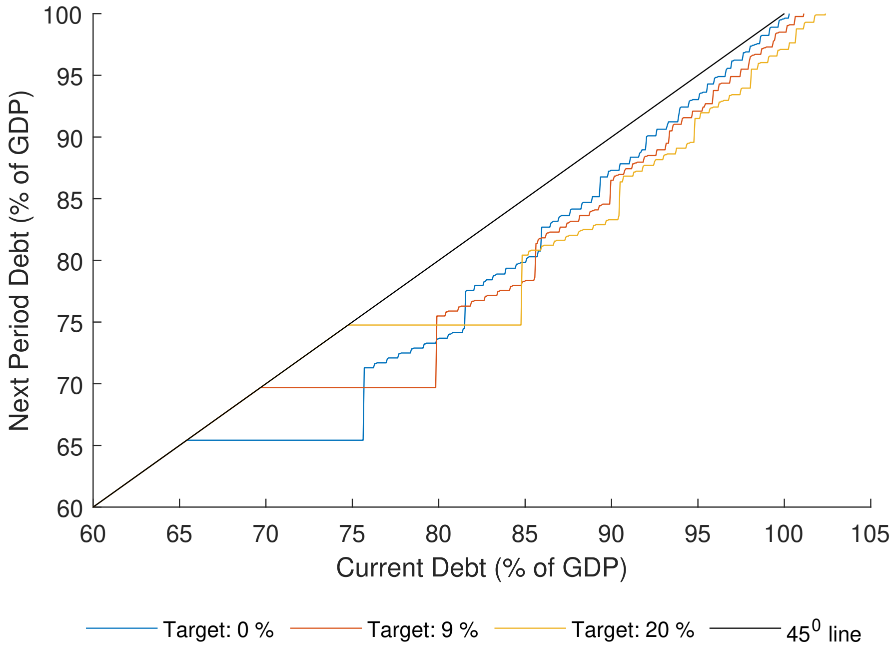

Figure 4: Sensitivity: Inflation Target and the Fiscal Fragility Zone

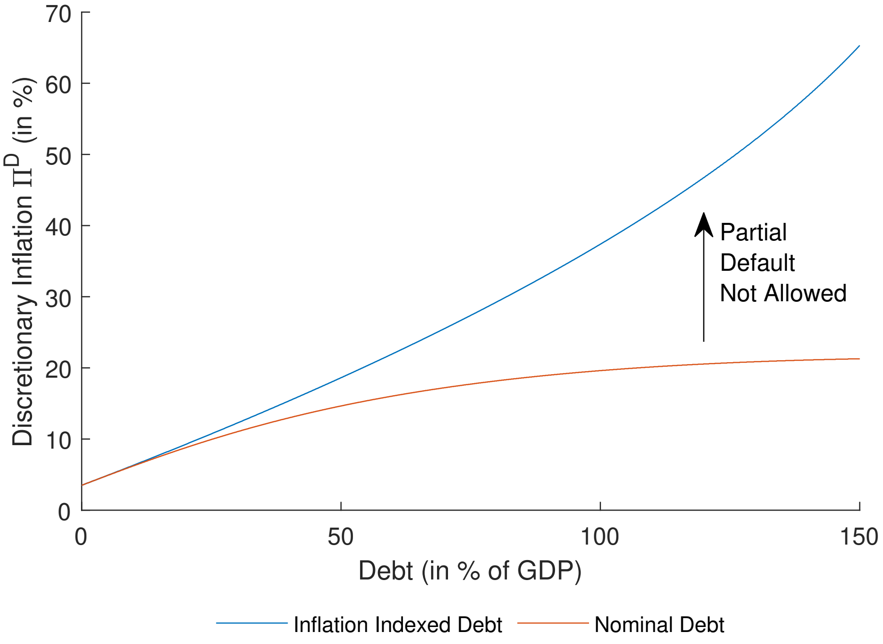

Figure 5: Discretionary Inflation

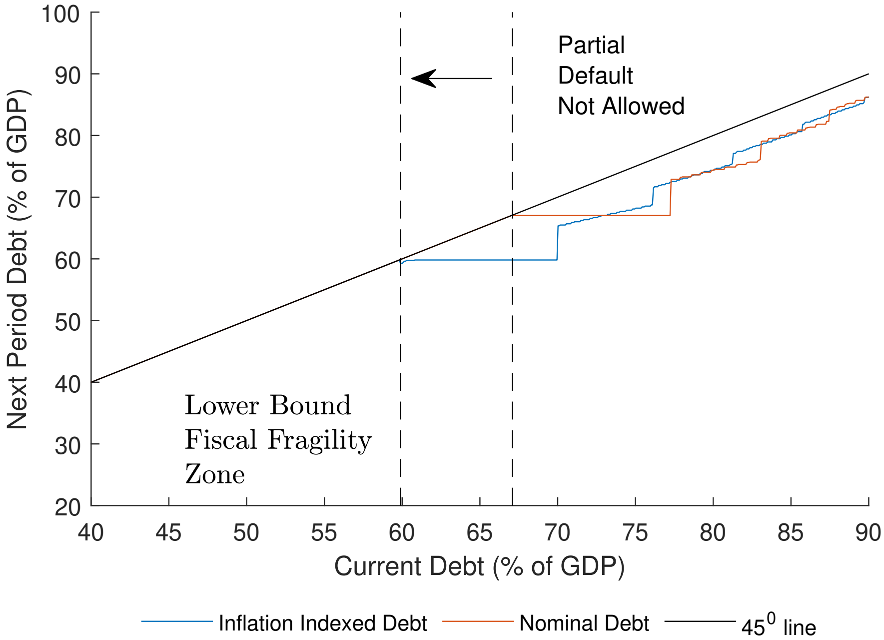

Figure 6: Sensitivity: Lower Bounds

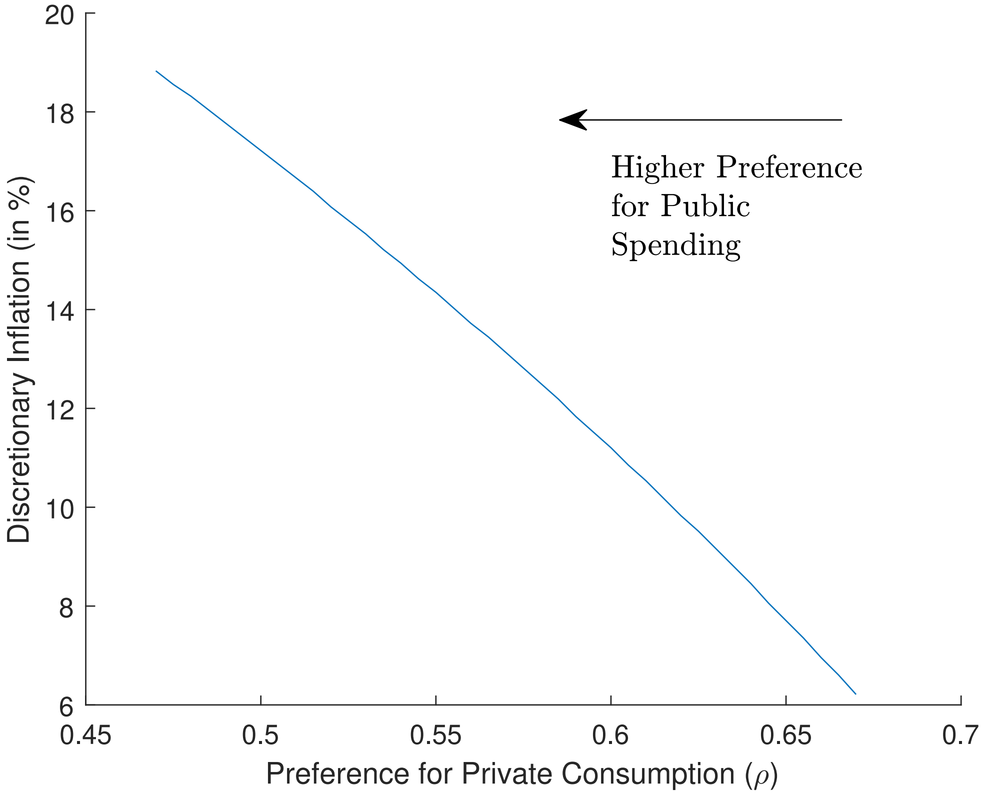

Figure 7: Sensitivity: Preference for Public Spending

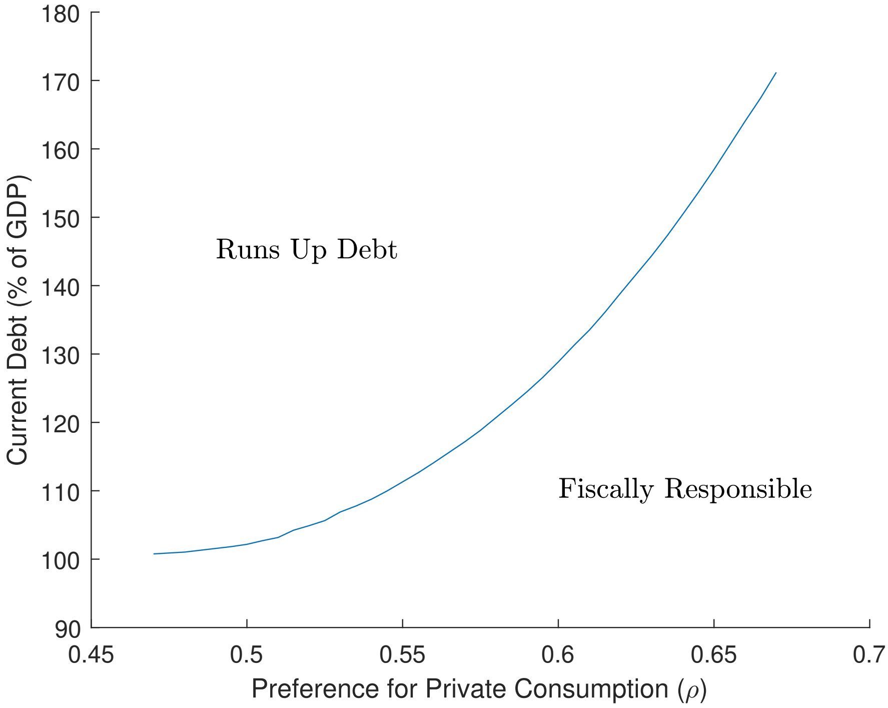

Figure 8: Debt Policy Function Inflection Point

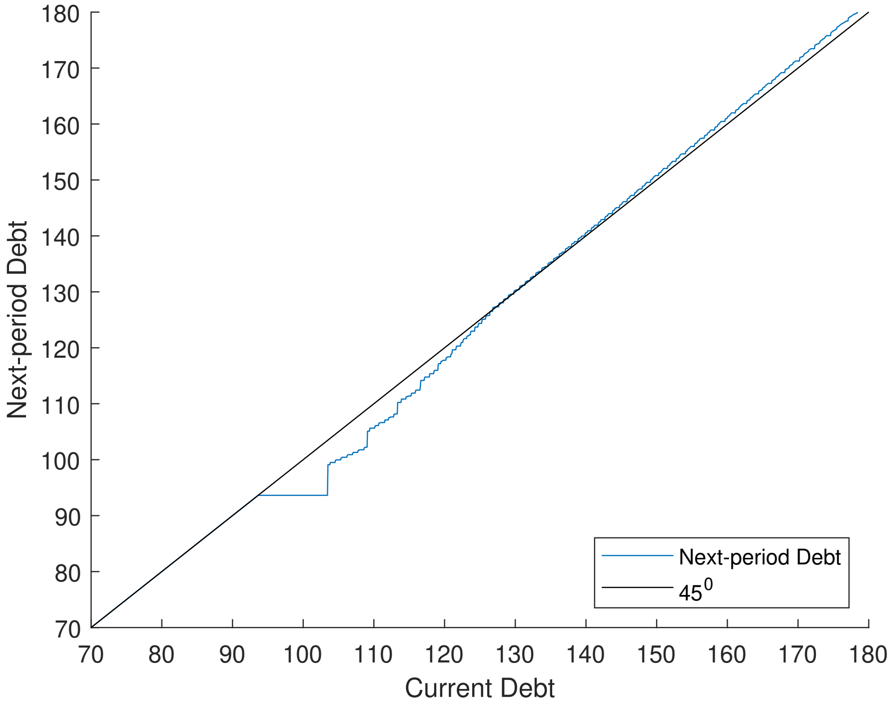

Figure 9: Optimal Fiscal policy

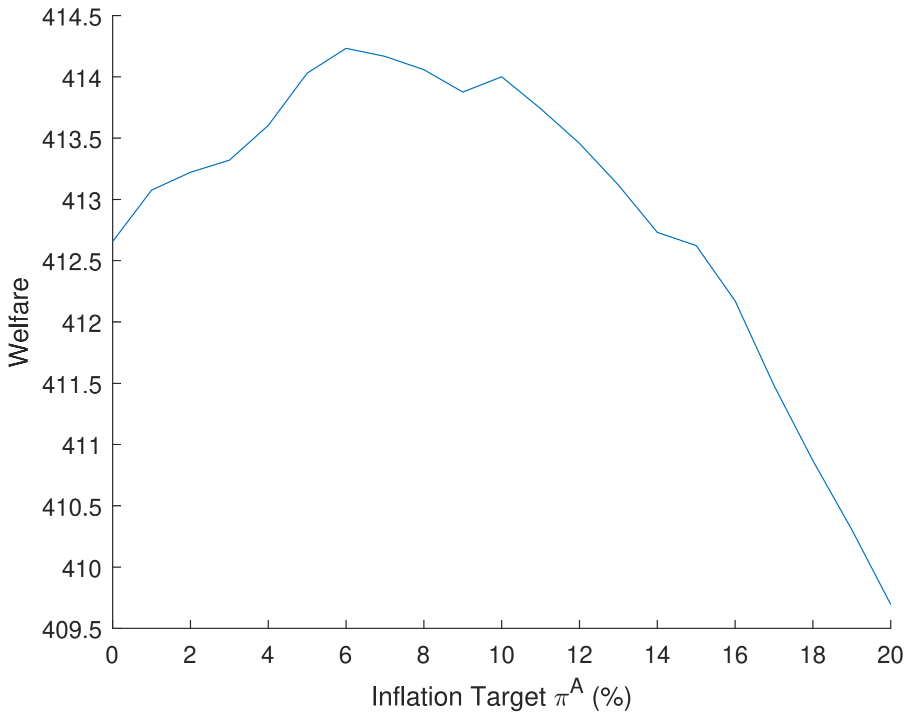

Figure 10: Sensitivity: Welfare Around Fiscal Fragility Zone

Parameter	Value	Meaning	Calibration
β	.915	Discount factor	Ex-post 1996-2019 real interest rate
τ	35%	Tax rate	General gov. revenue in % of GDP
π_a	3.5%	Inflation target	2002 BCB target
f	20%	Crisis prob.	EMBI+ Brazil on 10/2002
e	1.5	Endowment	Expansive gov.
ρ	.5	Pref. for consumpt.	Neutral value
κ	20%	Limit to TFP cost	Crisis zone at a 70% debt level
c_0	.05	Fixed cost	Crisis zone at a 70% debt level
c_1	.5	Variable cost	Crisis zone at a 70% debt level

Table 1: Parameters of the Baseline Model

Year	Date When Set	Target	Bounds
2002	28/6/2000	3.5	2
2003	28/6/2001	3.25	2
	27/6/2002	4	2.5
	21/1/2003	8.5	
2004	27/6/2002	3.75	2.5
	21/1/2003	5.5	
	25/6/2003	5.5	2.5
2005	25/6/2003	4.5	2.5

Table 2: Brazil - Official Inflation Targets

	Debt/GDP	Revenue/GDP	CPI EOY	CPI target
Average	45.2	32.9	3.9	3.2
Min	13.4	16.4	1.5	1.5
Max	80.8	56.1	15.4	8.2

Table 3: Data - Description Timeseries

	I	II	III	IV	V
Revenue	0.171**	0.098	0.063	0.125*	0.087
	(0.076)	(0.076)	(0.078)	(0.070)	(0.072)
Debt	0.069*	0.074**	0.073**	0.062**	0.058*
	(0.035)	(0.034)	(0.034)	(0.031)	(0.032)
Debt * Revenue/100	−0.194*	−0.168*	-0.149	−0.163*	-0.136
	(0.099)	(0.096)	(0.096)	(0.088)	(0.089)
Target	−0.403***	−0.458***	−0.441***	−0.360***	−0.342***
	(0.063)	(0.062)	(0.062)	(0.059)	(0.058)
GDP Gap			0.363***		0.342***
			(0.102)		(0.095)
Reer YoY				−13.956***	−13.645***
				(1.648)	(1.653)
FE	Country	Country & Time	Country & Time	Country & Time	Country & Time
R^2	0.290	0.408	0.433	0.515	0.537
Num. obs.	382	382	374	372	364

Note: *p<0.1; **p<0.05; ***p<0.01

Table 4: Results - Deviations from the Inflation Target

	I	II	III	IV	V
Revenue	0.145	0.108	0.084	0.115	0.082
	(0.091)	(0.105)	(0.109)	(0.110)	(0.113)
Debt	0.034	0.055	0.053	0.050	0.042
	(0.044)	(0.052)	(0.053)	(0.053)	(0.054)
Debt*Revenue/100	-0.114	-0.107	-0.088	-0.121	-0.085
	(0.125)	(0.149)	(0.151)	(0.151)	(0.154)
Target	-0.624^{**}	-1.242^{***}	-1.207^{***}	-0.990^{**}	-0.936^{**}
	(0.263)	(0.376)	(0.376)	(0.390)	(0.386)
GDP Gap			0.206		0.218
			(0.158)		(0.167)
Reer YoY				-10.493^{***}	-10.262^{***}
				(3.062)	(3.101)
Num. obs.	377	377	369	368	360
Log Likelihood	−178.526	−151.367	−149.281	−139.619	−137.954

Note: *p<0.1; **p<0.05; ***p<0.01

Table 5: Results - Probability of Overshooting the Target