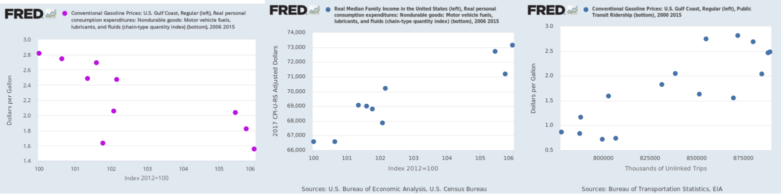
Price and Income Effects

Microeconomics
20851

Does a Tax on Fuel Promote Public Transit ?



(a) price effect (b) income effect (c) cross effect

Figure – FRED Data on fuel price and consumption and public transit

Demand Preferences

The problem

- Fuel (X) and Public Transit (Y)
- Utility $U(X, Y)$
- Budgand constraint : $p_X Y + p_Y Y = I$

Optimizing with a given constraint :

- (X^*, Y^*) such that

$$\frac{dU}{dX} \Bigg/ \frac{dU}{dY} = \frac{p_X}{p_Y} \text{ and } p_X X + p_Y Y = I$$

- At the optimum : the budget is respected and we are indifferent between giving up some X to acquire Y and vice versa.
- Two equations, two variables : we can solve for X^* and Y^* as a function of (p_X, p_Y, I)

Example

- $U(X, Y) = \ln X + 2 \ln Y$

$$\frac{dU}{dX} \Bigg/ \frac{dU}{dY} \quad = \quad \frac{p_X}{p_Y} \iff \frac{1/X}{2/Y} = \frac{p_X}{p_Y}$$
$$\iff \; p_Y Y = 2 p_X X$$

- With $p_X X + p_Y Y = I$ implies

$$X^* = \frac{I}{3p_X} \text{ and } Y^* = \frac{2I}{3p_Y}$$

How X^* and Y^* Vary as a Function of I

Engel's curve

- Individual demands $X(p_X, p_Y, I)$, $Y(p_X, p_Y, I)$.
- Engel's curve for X : how does X^* change when I changes
- Proportion of the budget dedicated to X : $s_X = \frac{p_X X}{I}$

Normal goods

- A good is said to be normal if and only if the demand of the good increases with income (constant prices)
- Example : fuel (luxury car)

Inferior goods

- A good is said to be inferior if demand decreases as income increases (constant prices)
- Example : it depends on initial income levels, but junk food (kraft dinner), lottery tickets, perhaps public transit ?

How the Demands X^* and Y^* Change with Prices

Keep p_Y and I constant. How does the demand adjust to an increase of p_X?

Decomposing Demand Changes

When fuel price p_X increases, two forcess :

- Public transit is more affordable than a car (fuel) : Want to consume more of the cheaper good. This is the **substitution effect**.

$$\frac{U'_X(X,Y)}{U'_Y(X,Y)} = \frac{p_X}{p_Y}$$

- Purchasing power decreases : need more income to by the same basket of goods as before the change : **income effect**.

Objective : Identify price effects and income effects

Compensated Demand

Context

- Reference price (p_X, p_Y), Reference income I, new price $(\hat{p}_X, p_Y)$
- Reference demand, $X(p_X, p_Y, I)$, reference (indirect) utility $V(p_X, p_Y, I)$
- New demand, $X(\hat{p}_X, p_Y, I)$, new (indirect) utility $V(\hat{p}_X, p_Y, I)$.

Compensated Demand

Definition

- Compensated income : income I^{cmp} such that we can achieve the reference utility with the new prices

$$V(p_X, p_Y, I) = V(\hat{p}_X, p_Y, I^{cmp})$$

- Compensated demand $X^{cmp} = X(\hat{p}_X, p_Y, I^{cmp})$
- **Property** : IF $\hat{p}_X > p_X$, then $X^{cmp} < X(p_X, p_Y, I)$
  the compensated demand of X is decreasing in terms of p_X.

Compensated Demand

Exercise A : Calculate the compensated income and demand for X if $U(X, Y) = XY$ and $p_X X + p_Y Y \leq I$ for a price change $\hat{p}_X > p_X$.

Substitution and Income Effects

Substitution effect

- Change in demand caused by relative price change, keeping utility constant :
- Substitution Effect = Compensated demand - Reference demand

$$\Delta X^{cmp} = X(\hat{p}_X, p_Y, I^{cmp}) - X(p_X, p_Y, I)$$

Income effect

- A change in demand caused by a change in purchasing power keeping prices constant
- Income effect = New demand - Compensated demand
$$\Delta X^I = X(\hat{p}_X, p_Y, I) - X(\hat{p}_X, p_Y, I^{cmp})$$

Approximating the Compensated Income

Consider a small price change $\hat{p}_X = p_X + \Delta p_X$. To keep notation simple : $X^* = X(p_X, p_X, I)$, $Y^* = Y(p_X, p_Y, I)$

We define $I^{cmp} = I + \Delta I^{cmp}$, $X^{cmp} = X^* + \Delta X^{cmp}$ and $Y^{cmp} = Y^* + \Delta Y^{cmp}$.

$$
\begin{aligned}
I^{cmp} &= \hat{p}_X X^{cmp} + p_Y Y^{cmp} \\
&= (p_X + \Delta p_X)(X^* + \Delta X^{cmp}) + p_Y(Y^* + \Delta Y^{cmp}) \\
&= \underbrace{p_X X^* + p_Y Y^*}_{=I} + \underbrace{\Delta p_X \Delta X^{cmp}}_{\simeq 0} + \Delta p_X X^* \\
&\qquad + \underbrace{p_X \Delta X^{cmp} + p_Y \Delta Y^{cmp}}_{=0} \\
&\simeq I + \Delta p_X X^* \\
\Delta I^{cmp} &\simeq \Delta p_X X^*
\end{aligned}
$$

A Trick to identify Compensated Income

Why does $p_X \Delta X^{cmp} + p_Y \Delta Y^{cmp} = 0$?

1. (X^*, Y^*) and (X^{cmp}, Y^{cmp}) are on the same indifference curve, which implies

$$\frac{\Delta Y^{cmp}}{\Delta X^{cmp}} = MRS_{X \to Y}$$

2. (X^*, Y^*) is optimal at the prices p_X, p_Y, which implies $MRS_{X \to Y} = -\frac{p_X}{p_Y}$.
3. Therefore, $p_X \Delta X^{cmp} + p_Y \Delta Y^{cmp} = 0$.

Exercise B : Check if this approximation works for $U(X, Y) = XY$ with reference prices and income $(p_X, p_Y, I) = (1, 1, 100)$ and $\Delta p_X = 1$ and $\Delta p_X = 0.1$.

The Slutsky Equation

This equation comes from the decomposition of the price elasticity of demand.

To keep notation simple, consider

$$X^* = X(p_X, p_Y, I), \qquad X(p_X + \Delta p_X, p_Y, I) = X^* + \Delta X^*,$$
$$X(p_X + \Delta p_X, p_Y, I) = X^{cmp} + \Delta X^I$$

We get

$$\underbrace{\Delta X^*}_{\text{Total effect}} = \underbrace{\Delta X^{cmp}}_{\text{Substitution effect}} + \underbrace{\Delta X^I}_{\text{Income effect}}$$

Exercise D : Find the income and substitution effects in exercise C.

The Slutsky Equation

Since

$$\Delta X^{I} = -\frac{\partial X}{\partial I}\Delta I^{cmp} = -\frac{\partial X}{\partial I}\Delta p_X X^*$$

Then,

$$\Delta X^* = \underbrace{\Delta X^{cmp}}_{\leq 0} - \underbrace{\frac{\partial X}{\partial I} \times \Delta p_X X^*}_{\geq 0 \text{ if normal, } <0 \text{ if inferior}}$$

As an elasticity,

$$\frac{\Delta X^*}{\Delta p_X}\frac{p_X}{X^*} = \frac{\Delta X^{cmp}}{\Delta p_X}\frac{p_X}{X^*} - \frac{\partial X}{\partial I}\Delta p_X X^* \times \frac{p_X}{\Delta p_X X^*}\frac{I}{I}$$

The Slutsky equation is given by :

$$\eta_{X,p} and = \eta_{X,p}^{cmp} - \eta_{X,I} \times s_X$$

The nature of goods

The goods X and Y are :

- Substitutes : if the cross effect is $\frac{\partial X^{cmp}}{\partial p_Y} > 0$
- Compléments : if the cross effect is $\frac{\partial X^{cmp}}{\partial p_Y} < 0$

Properties

- Homogeneity of degree 0

$$X(\lambda p_X, \lambda p_Y, \lambda I) = X(p_X, p_Y, I)$$

- Symetry :

$$\frac{\partial X^{cmp}}{\partial p_Y} = \frac{\partial Y^{cmp}}{\partial p_X}$$

- Additivity :

$$p_X \frac{\partial X(p_X, p_Y, I)}{\partial I} + p_Y \frac{\partial Y(p_X, p_Y, I)}{\partial I} = 0$$

- Negativity :

$$\frac{\partial X^{cmp}}{\partial p_X} < 0, \frac{\partial Y^{cmp}}{\partial p_Y} < 0$$

Giffen Goods

Direction of income and wealth effects

- When indifference curves are convex, the compensated demand for X decreases as p_X increases
- Income effects depend on whether the good is normal or inferior at reference income and prices.
- If normal good, price increase causes a negative income effect (same direction as price effect)
- If inferior good, price increase causes a positive income effect (opposite direction)

Giffen Goods

- If the income effect is larger than the substitution effect, as the price p_X increases, the demand for X increases.
- Classic example : Potatoes in Ireland (circa 1850, according to legend).

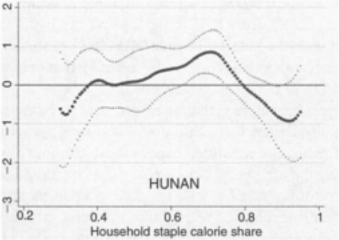
Chinese Rice Subsidy

American Economic Review 2008, 98:4, 1553–1577
http://www.aeaweb.org/articles.php?doi=10.1257/aer.98.4.1553

Giffen Behavior and Subsistence Consumption

By ROBERT T. JENSEN AND NOLAN H. MILLER*

This paper provides the first real-world evidence of Giffen behavior, i.e., upward sloping demand. Subsidizing the prices of dietary staples for extremely poor households in two provinces of China, we find strong evidence of Giffen behavior for rice in Hunan, and weaker evidence for wheat in Gansu. The data provide new insight into the consumption behavior of the poor, who act as though maximizing utility subject to subsistence concerns. We find that their elasticity of demand depends significantly, and nonlinearly, on the severity of their poverty. Understanding this heterogeneity is important for the effective design of welfare programs for the poor. (*JEL* D12, O12)



Doctors

How can a wage increase cause a drop in work hours ?

docteurs.png

Figure – TVA Nouvelles

Price and Costs of Living Indexes

To measure changes in costs of living, we often use consumption price indices. A very common one is the Laspeyres index :

$$\pi_L = \frac{\hat{p}_X X + \hat{p}_Y Y}{p_X X + p_Y Y}$$

- The Quebec Pension Plan (QPP) and private pension plans are often indexed using this kind of index.
- Is this a good index to capture an increase in the cost of living ?

The Ideal Price Index

Need account for behavioral changes. Therefore, a price increases implies substitution.

- Following a price increase for the good X, the necessary compensation to keep welfare constant is

$$\pi_I = \frac{I^{cmp}}{I}$$

.

- In a Cobb-Douglas situation $u(X, Y) = X^{\alpha} Y^{1-\alpha}$:

$$\pi_I = \frac{I^{cmp}}{I} = \left(\frac{\hat{p}_X}{p_X}\right)^{\alpha}$$