Exercices on Consumer Theory

20851-W2020

1. Two consumers have the preferences represented by $u(X,Y) = \frac{1}{4}\ln X + \frac{3}{4}\ln Y$ and $v(X,Y) = XY^3$. Do they have the same preferences ? Answer : Yes, given that $v(X,Y) = f(u) = e^{4u}$.
2. What is the MRS for $u(X,Y) = (X-\gamma_X)^\alpha (Y-\gamma_Y)^{1-\alpha}$? Answer : $MRS = -\frac{\alpha}{1-\alpha}\frac{Y-\gamma_Y}{X-\gamma_X}$.
3. Preferences are given by $u(X,Y) = \min(X, 2Y)$ and the budget constraint is $p_X X + p_Y Y \leq I$. At $(p_X, p_Y, I) = (2, 1, 10)$, is the allocation $(X,Y) = (4,2)$ maximizing utility ? Answer : Yes, demand for $X^* = \frac{I}{p_X + \frac{1}{2}p_Y}$. At $(2,1,10)$, $X^* = 4$ and by budget constraint $Y^* = 2$.
4. If $u(X,Y) = X^{2/3}Y^{1/3}$ and $p_X X + p_Y Y \leq I$, what is the compensation that keeps utility constant when $\Delta p_X = -1$ and $(p_X, p_Y, I) = (2, 1, 60)$? Answer : Find indirect utility. Then compensated income I^{cmp}. This yields for this price change $\Delta I^{cmp} = -22.20$.
5. What is the compensated price elasticity for X if utility is $u(X,Y) = \alpha \ln X + Y$ and the budget constraint is $p_X X + p_Y Y \leq I$? Answer = By Slutsky, and seeing that income elasticity is zero with quasi-linear preferences, one can show that $\eta_{cmp} = -1$.
6. What is the utility function that has an indirect utility function

$$V(p_X, p_Y, I) = \alpha^\alpha (1-\alpha)^{1-\alpha} \frac{I}{p_X^\alpha p_Y^{1-\alpha}}$$

when $p_X X + p_Y \leq I$ is the budget constraint ? Answer : Using Roy's Identity and recognizing the common Cobb-Douglas form of demand functions, we can identify $u(X,Y) = X^\alpha Y^{1-\alpha}$ as the utility function.