Supplement_3: Formulation for grandparent-grandchild pair probabilities.

In this supplement, we describe the probability that two individuals, selected at random, are a grandparent-grandchild pair conditional on relevant covariates. For purposes of these calculations, covariates consist of ages (and applied birthdates), sex, date of harvest, and mtDNA haplotype. To our knowledge, this is the first such calculation to be conducted.

We shall assume that mtDNA haplotype diversity is high enough that a grandparent and grandchild will only share mtDNA if (1) the potential grandparent is female, and (2) the unobserved parent of the grandchild (the grandparent's direct offspring) is female. This dynamic can be visualized in Figure S2a.

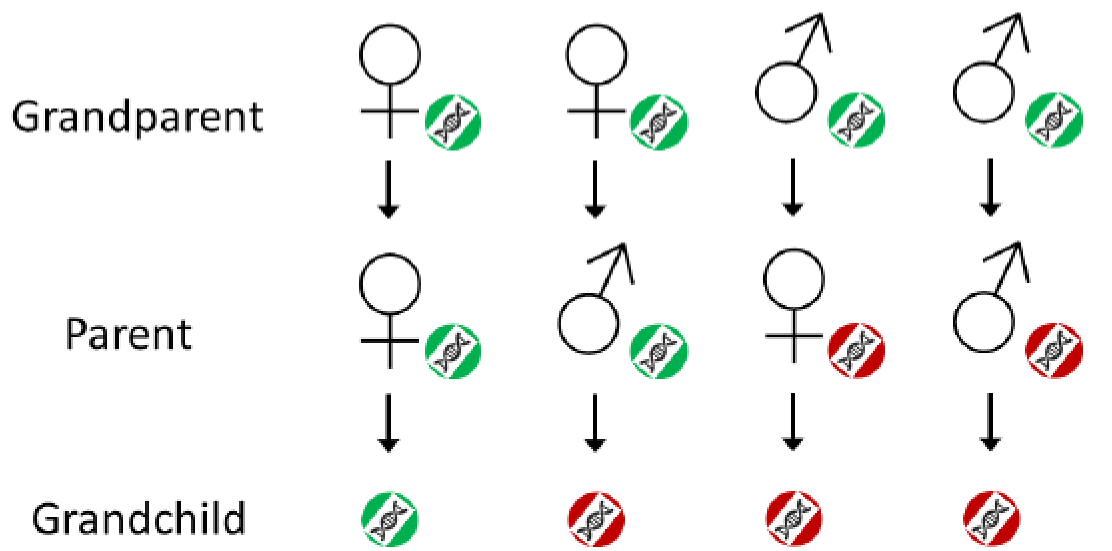

Figure S2a

Here, the green DNA bits show inheritance of mtDNA, with green mtDNA being identical to that possessed by the grandparent. Importantly, if the potential grandparent is male, the chance that the GGP shares mtDNA is negligible (this would not be the case in populations with low mtDNA haplotype diversity).

There are four different possibilities, associated with (1) the sex of the parent, and (2) whether or not the two individuals compared share mitochondrial DNA. We shall describe each of these cases separately, letting s_i denote sex of the older seal (with $s_i = 1$ if i is male), and m_{ij} be a binary random variable that takes on the value 1 if individuals i and j share mtDNA.

Case 1: $s_i = 0$, $m_{ij} = 1$

We'll denote the probability of a GGP sharing mtDNA when the potential grandparent is female as $\Pr(GGP, m_{ij} = 1 | s_i = 0, d_i, b_i, b_j)$. Note that this expression depends on the time of death of the potential grandparent, d_i (for instance, if it dies before it was old enough to have potentially reproduced, it is clearly not a grandparent), and birth years of the potential grandparent and grandchild (b_i and b_j, respectively). It is also dependent on female fecundity-at-age (f_a), year- and age-specific adult abundance ($N_{t,a}$), and age-specific survival probability (ϕ_a). Note also that we are assuming (by virtue of high mtDNA haplotype diversity) that a grandparent and grandchild can only share mtDNA if the unobserved parent is female. Accordingly,

$$Pr(GGP, m_{ij} = 1 | s_i = 0, d_i, b_i, b_j) = \sum_{t=b_i}^{\min(d_i, b_j)} \frac{f_{t-b_i} N_{t,0}}{\sum_a f_a N_{t,a}} \frac{\{\prod_{k=t}^{b_j-1} \phi_{k-t}\} 2 f_{b_j-t}}{\sum_a f_a N_{b_j,a}} = \sum_{t=b_i}^{\min(d_i, b_j)} \frac{f_{t-b_i} N_{b_j, b_j-t}}{\sum_a f_a N_{t,a}} \frac{2 f_{b_j-t}}{\sum_a f_a N_{b_j,a}}$$

Here, relative reproductive success is conditional on the unknown age of the parent, so we must sum over the possible years (t) of the mother's birth. The (N_{b_j, b_j-t}) in the numerator arises because the potential parent can be any of the females born in year t that survive to the year of j's birth. For seals, many of the f_a values are zero for low ages, so practically speaking we must have a sufficient birth gap (and late enough time of death for the potential grandparent) to enable $Pr(GGP) > 0$. As in previous calculations, this formulation requires a number of things to hold like equal male:female sex ratios, equal survival among sexes, etc.

Case 2: $s_i = 0$, $m_{ij} = 0$

The only way for a grandmother and grandchild not to share mtDNA is if the unobserved parent is male, so our answer will be similar but will involve male maturity-at-age indexed to the year before j's birth:

$$Pr(GGP, m_{ij} = 0 | s_i = 0, d_i, b_i, b_j) = \sum_{t=b_i}^{\min(d_i, b_j-1)} \frac{f_{t-b_i}}{\sum_a f_a N_{t,a}} \frac{N_{b_j-1, b_j-t-1} 2 m_{b_j-t-1}}{\sum_a m_a N_{b_j-1,a}}$$

Case 3: $s_i = 1$, $m_{ij} = 1$

By assumption, $\Pr(GGP, m_{ij} = 1 | s_i = 1, d_i, b_i, b_j) = 0$ for reasons stated previously.

Case 4: $s_i = 1$, $m_{ij} = 0$

This case can happen whether the offspring of i is male or female, so we must account for both. Fortunately, it is very similar to what we have written already, though it involves male maturity in the year before the birth of the prospective parent:

$$
\begin{aligned}
Pr(GGP, m_{ij} = 0 | s_i = 1, d_i, b_i, b_j) = & \sum_{t=b_i+1}^{\min(d_i, b_j)} \frac{m_{t-b_i-1}}{\sum_a m_a N_{t-1,a}} \frac{N_{b_j, b_j-t} 2 f_{b_j-t}}{\sum_a f_a N_{b_j,a}} \\
& + \sum_{t=b_i+1}^{\min(d_i, b_j)} \frac{m_{t-b_i-1}}{\sum_a m_a N_{t-1,a}} \frac{N_{b_j-1, b_j-t-1} 2 m_{b_j-t-1}}{\sum_a m_a N_{b_j-1,a}}
\end{aligned}
$$