Software Verification by Abstract Interpretation: Current Trends and Perspectives

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Talk Outline

Motivation

All Computer Scientists Have Experienced Bugs

It is preferable to verify that safety-critical programs do not go wrong before running them.

Static Analysis by Abstract Interpretation

Static analysis: analyse the program at compile-time to verify a program runtime property (e.g. the absence of some categories of bugs)

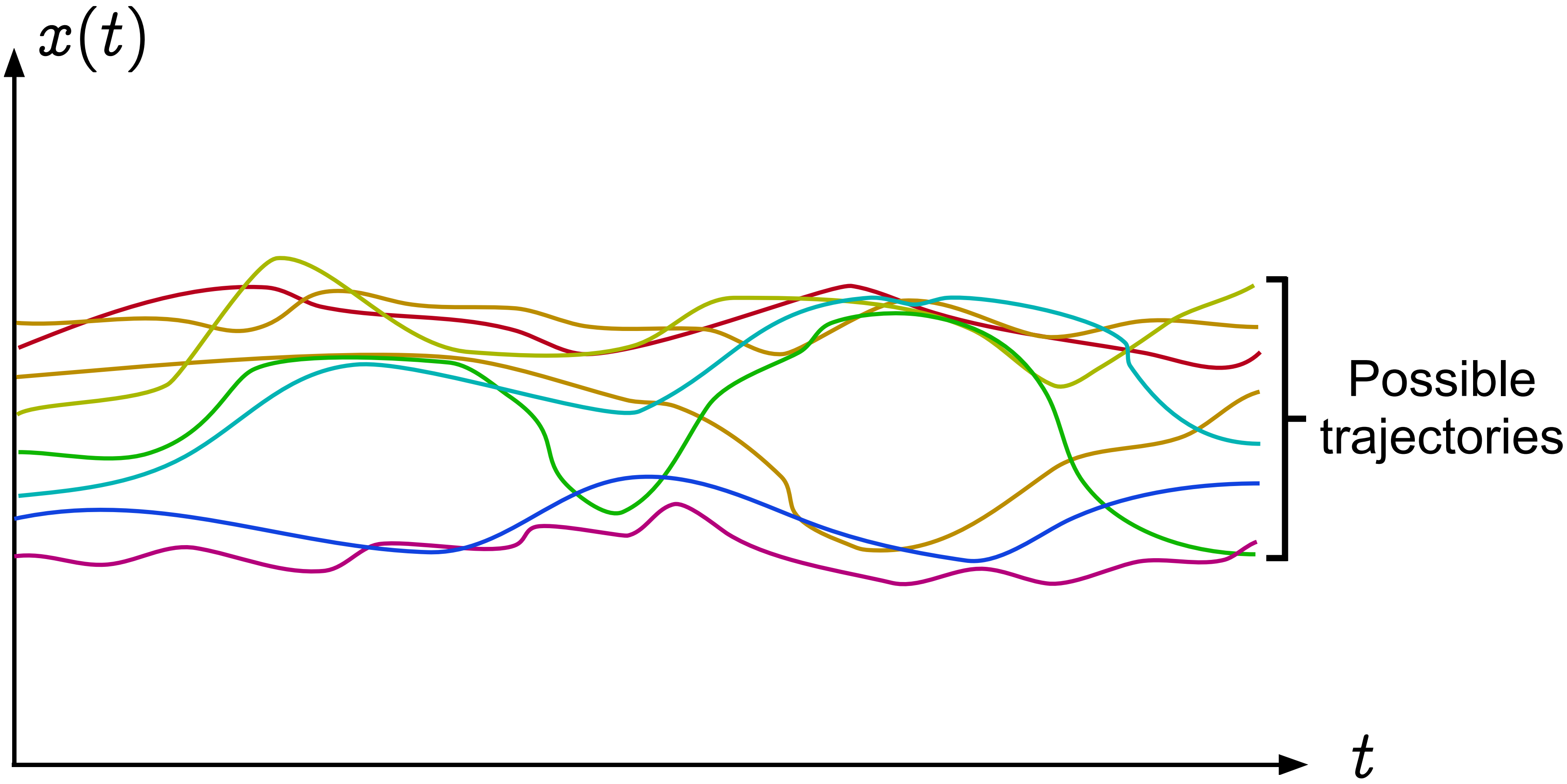
Undecidability $\longrightarrow$

Abstract interpretation: effectively compute an abstraction/ sound approximation of the program semantics,

- which is precise enough to imply the desired property, and
- coarse enough to be efficiently computable.

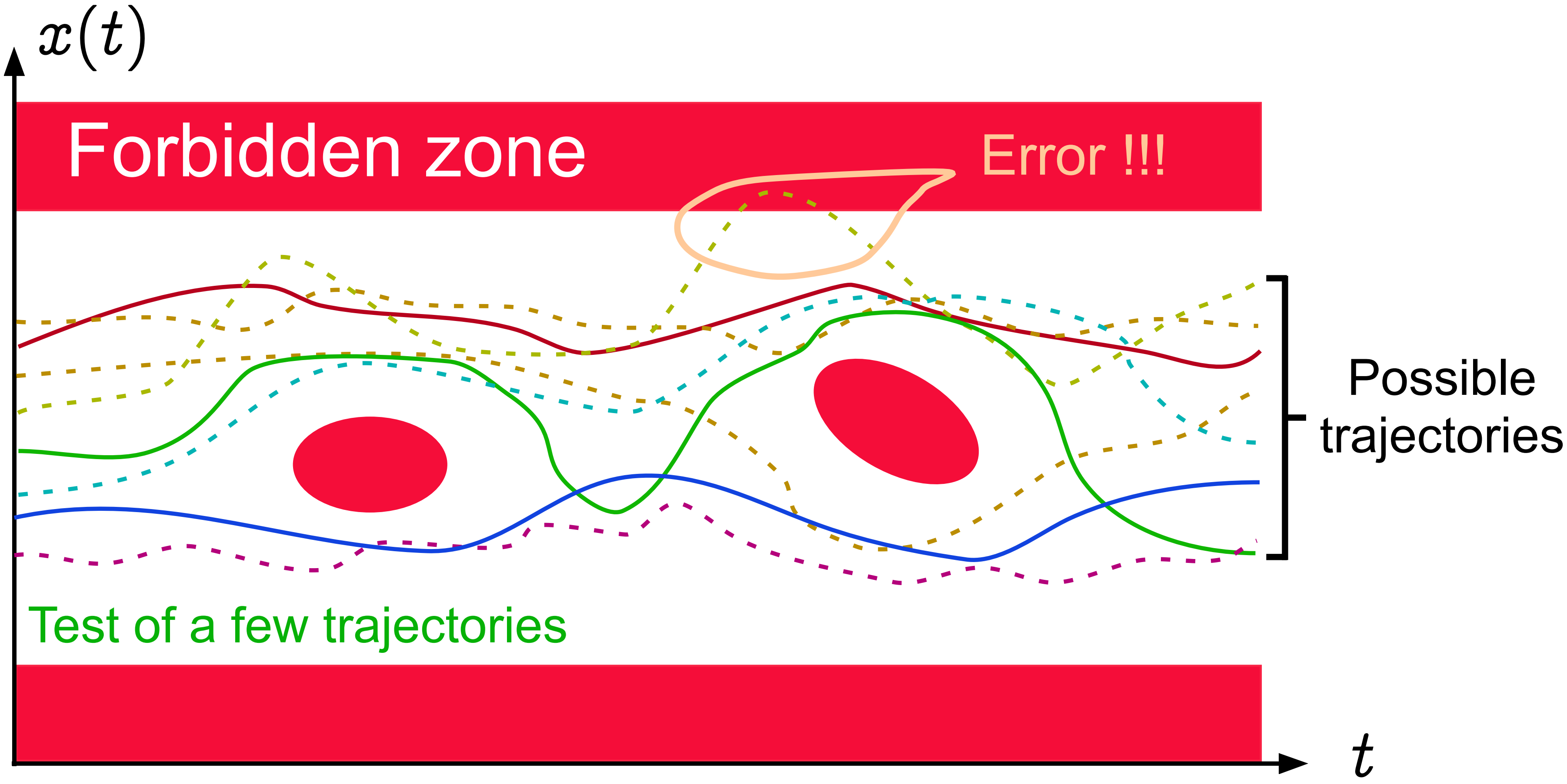
Abstract Interpretation, Informally

Operational Semantics

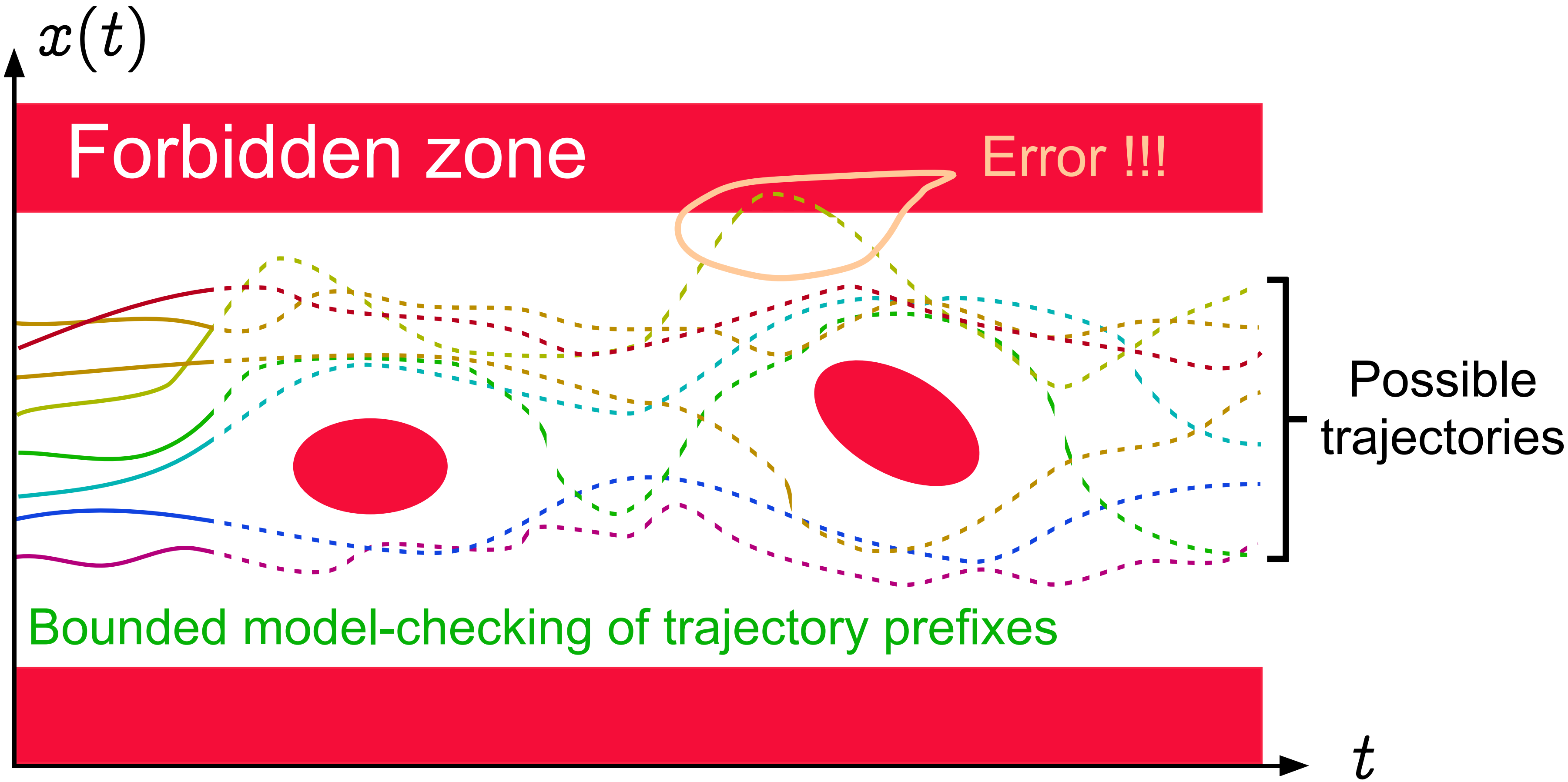


Safety property

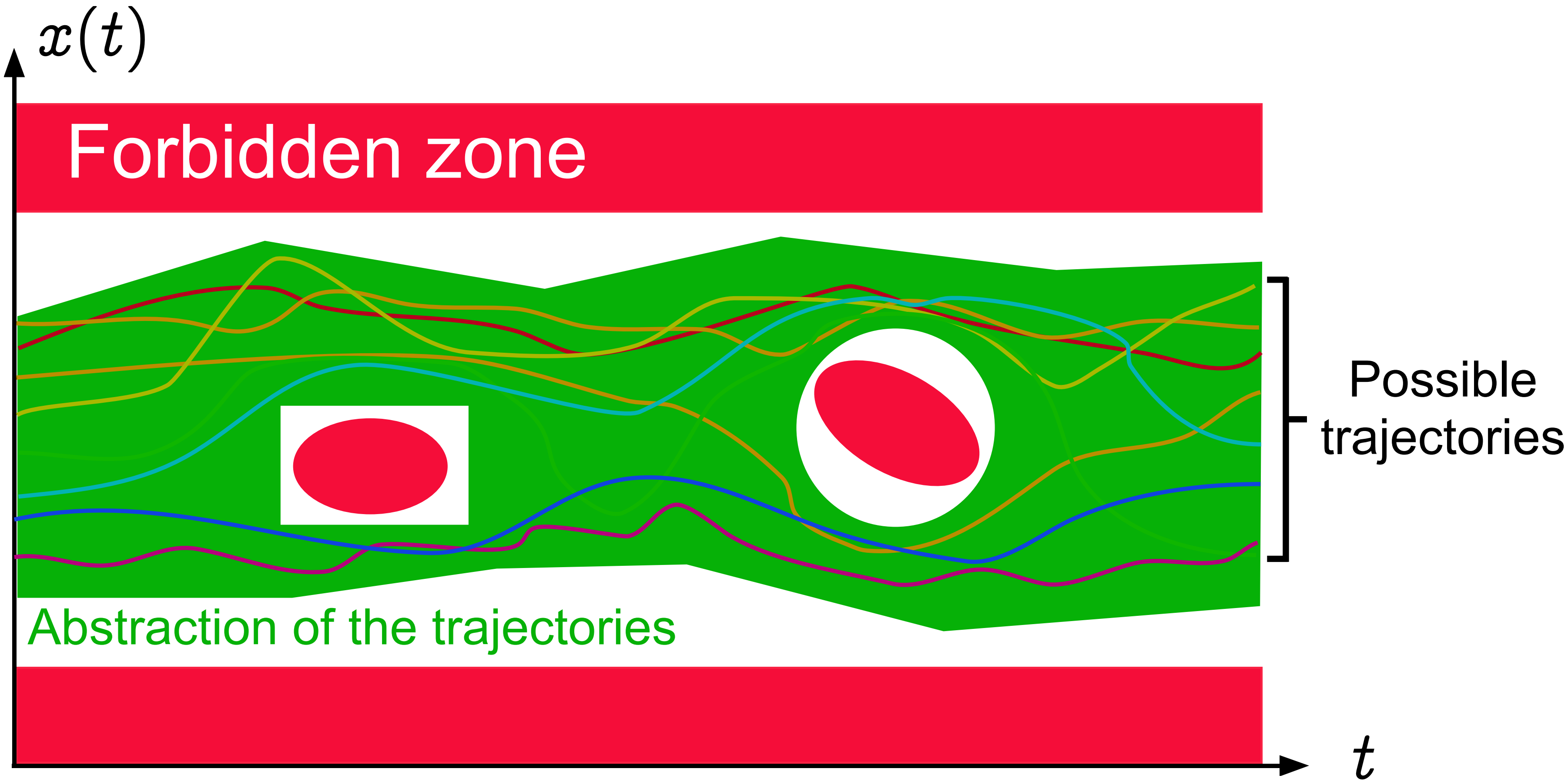
Test/Debugging is Unsafe



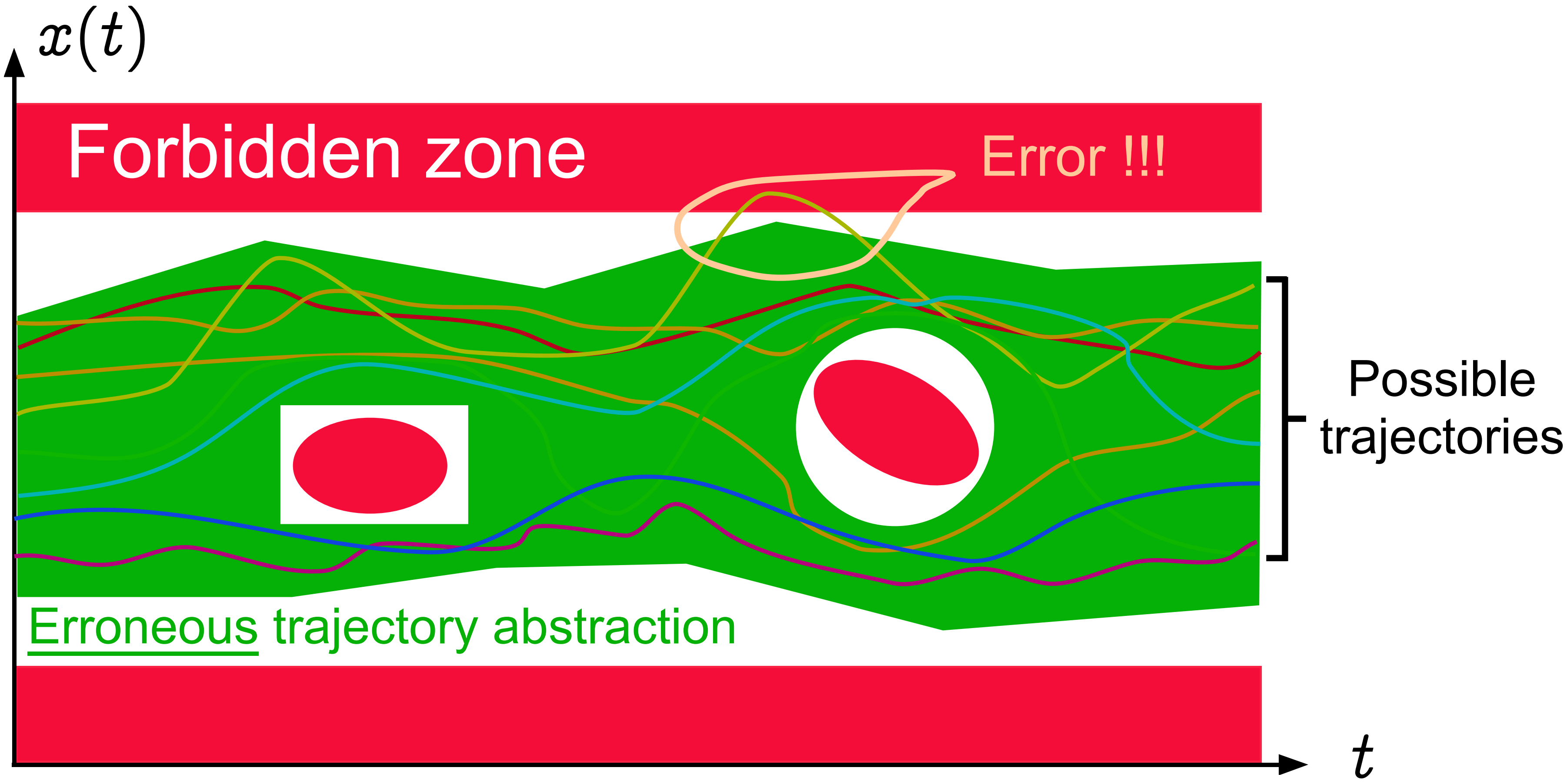
Bounded Model Checking is Unsafe



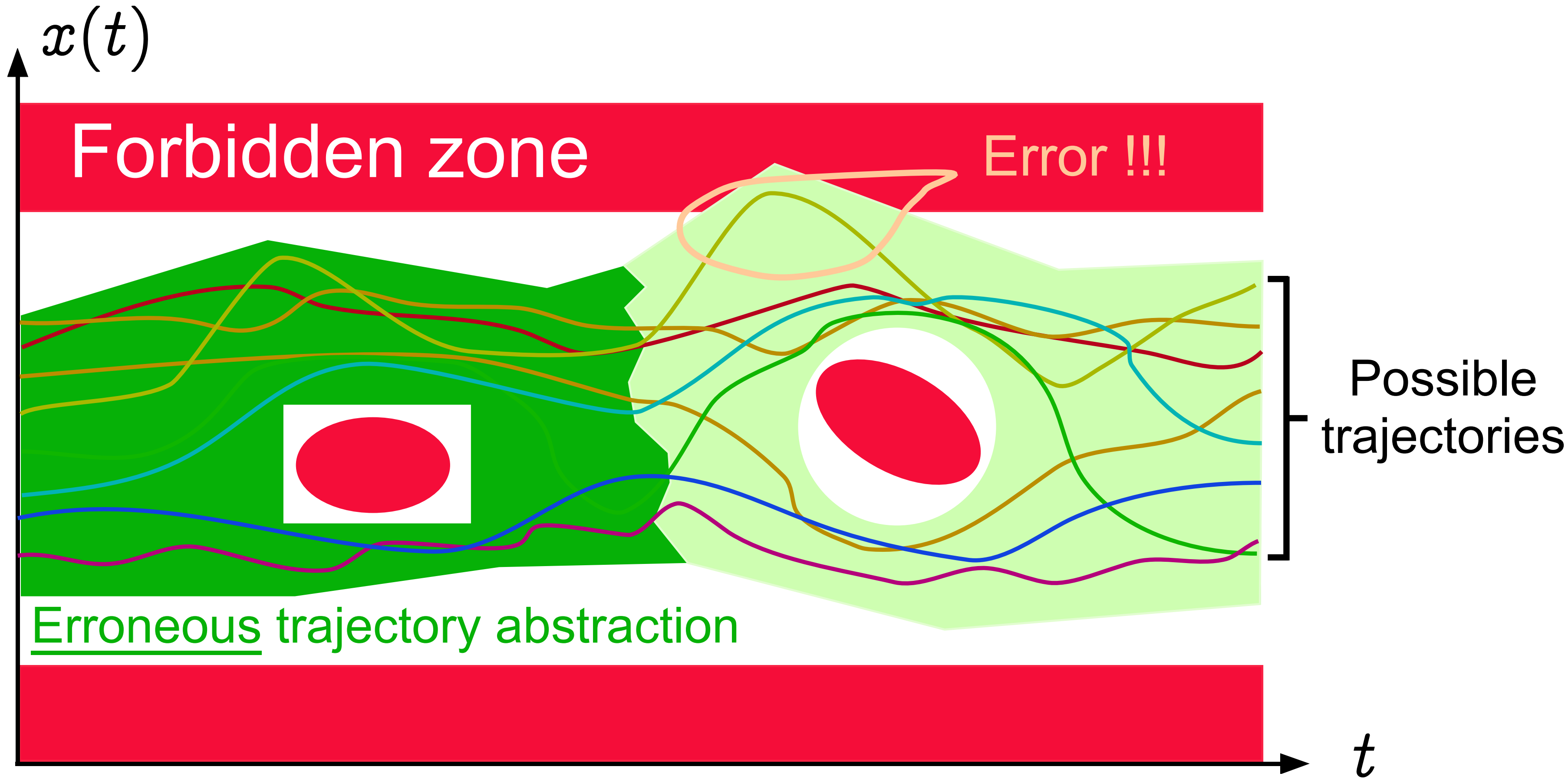
Abstract Interpretation



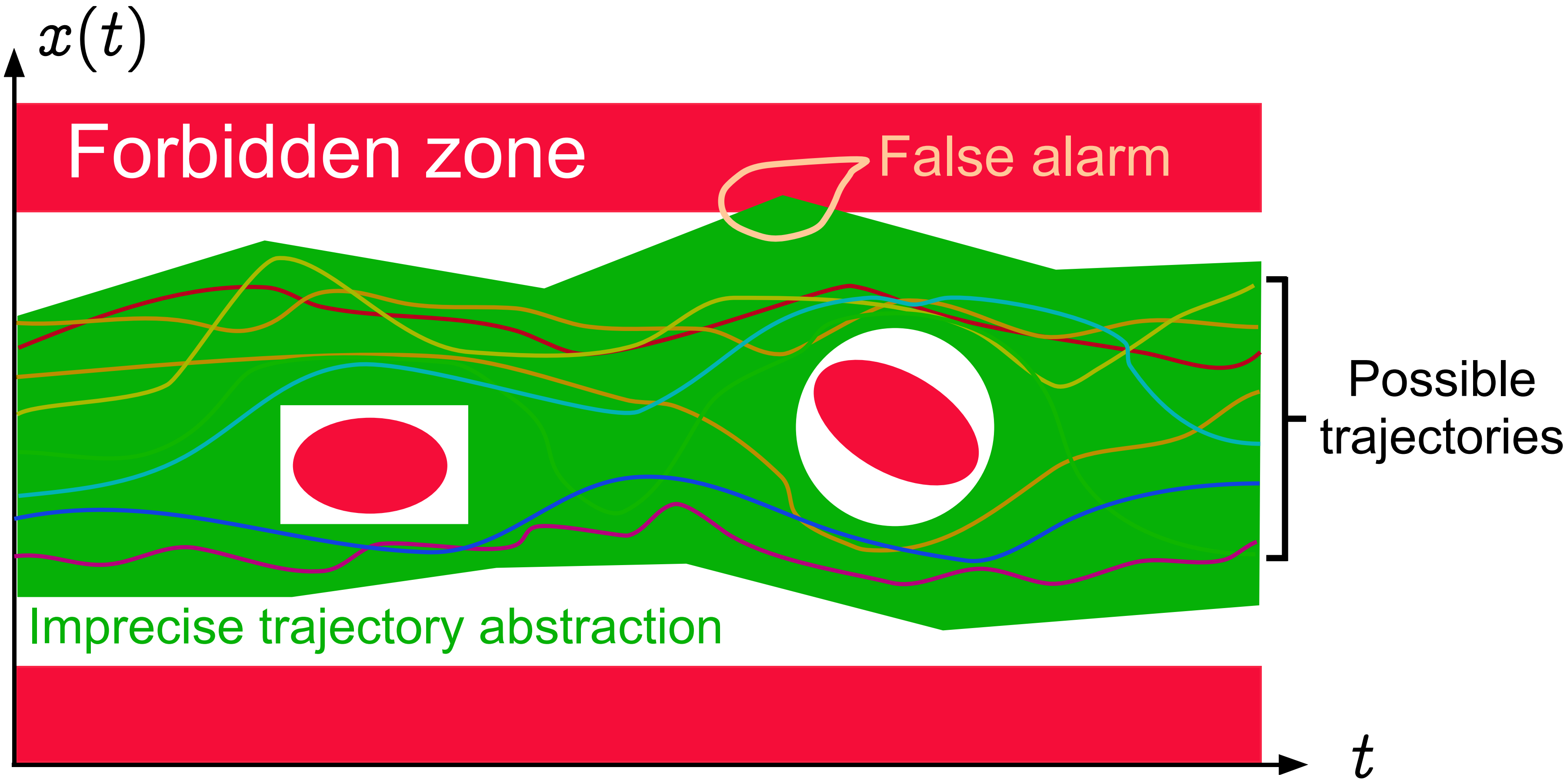
Soundness: Erroneous Abstraction — I



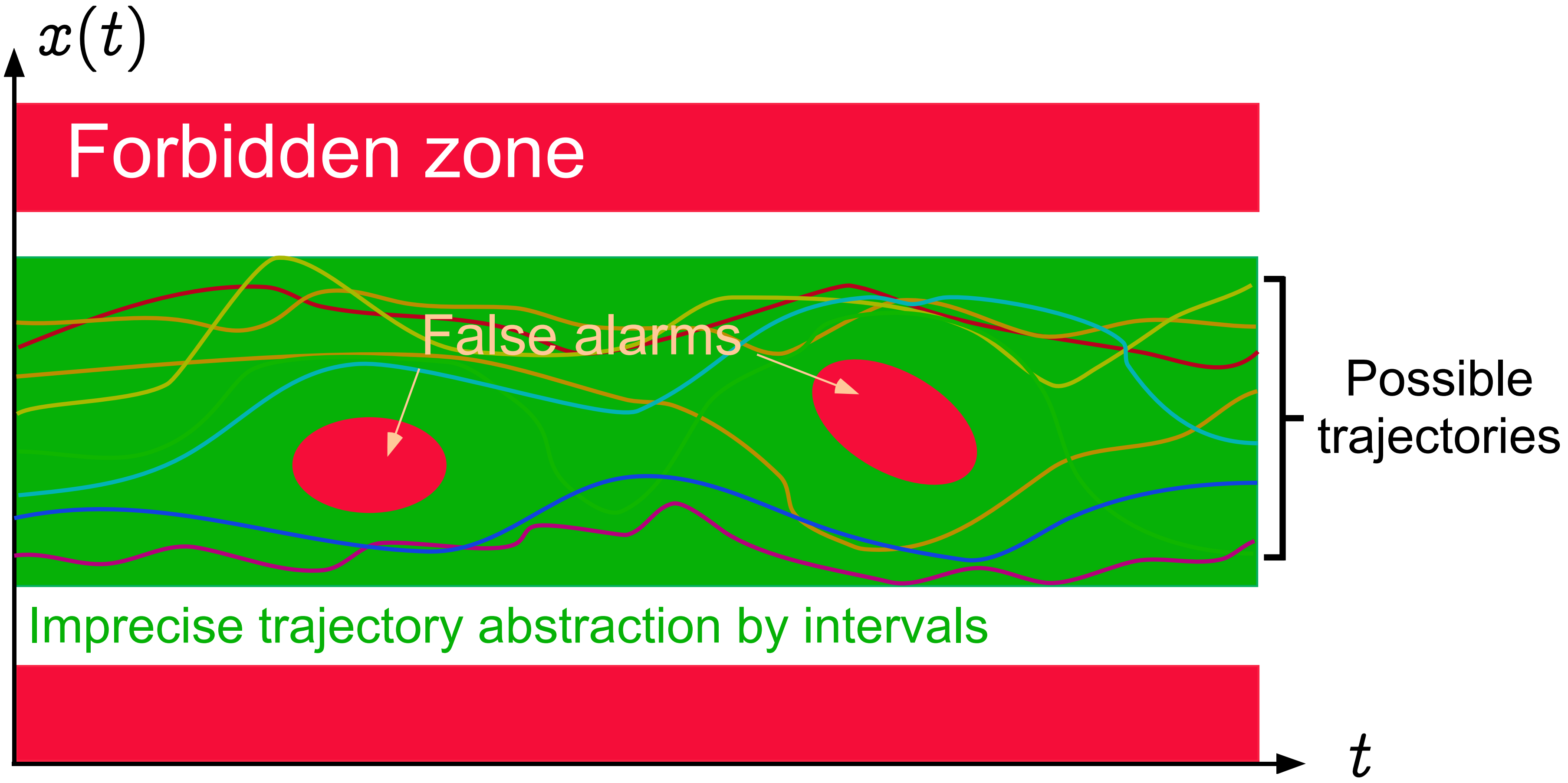
Soundness: Erroneous Abstraction — II



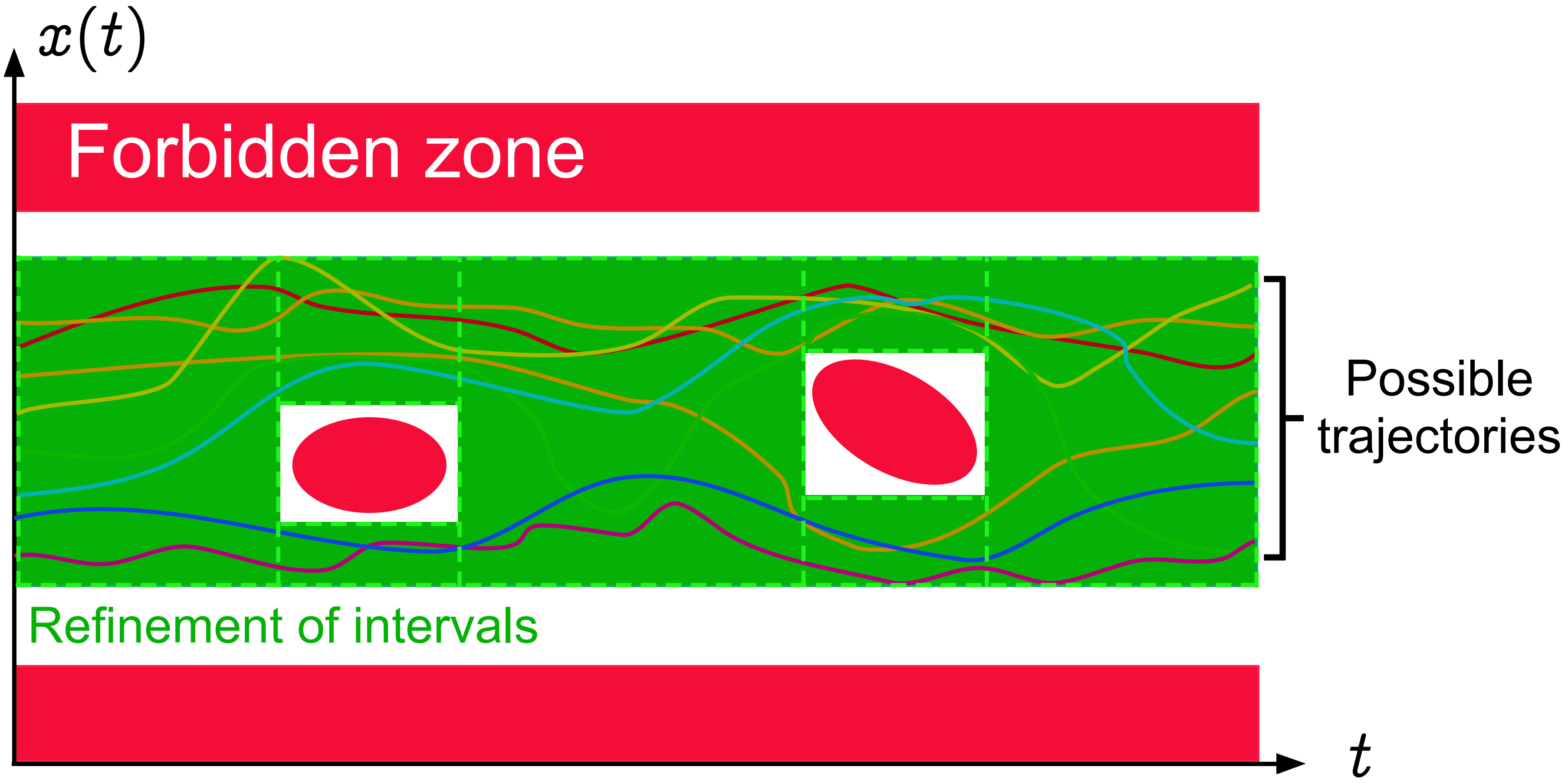
Imprecision $\Rightarrow$ False Alarms



Interval Abstraction $\Rightarrow$ False Alarms



Refinement by Partitionning



Applications of Abstract Interpretation

Applications of Abstract Interpretation

- **Static Program Analysis** [POPL '77], [POPL '78], [POPL '79] including **Dataflow Analysis** [POPL '79], [POPL '00], **Set-based Analysis** [FPCA '95], **Predicate Abstraction** [Manna's festschrift '03], ...
- **Syntax Analysis** [TCS 290(1) 2002]
- **Hierarchies of Semantics (including Proofs)** [POPL '92], [TCS 277(1–2) 2002]
- **Typing & Type Inference** [POPL '97]

Applications of Abstract Interpretation (Cont'd)

- **(Abstract) Model Checking** [POPL '00]
- **Program Transformation** [POPL '02]
- **Software Watermarking** [POPL '04]
- **Bisimulations** [RT-ESOP '04]

All these techniques involve sound approximations that can be formalized by abstract interpretation

A Practical Application of Abstract Interpretation to the Verification of Safety Critical Embedded Control-Command Software

Reference

[1] B. Blanchet, P. Cousot, R. Cousot, J. Feret, L. Mauborgne, A. Miné, D. Monniaux, and X. Rival. Design and implementation of a special-purpose static program analyzer for safety-critical real-time embedded software. *The Essence of Computation: Complexity, Analysis, Transformation. Essays Dedicated to Neil D. Jones*, LNCS 2566, pages 85–108. Springer, 2002.

[2] B. Blanchet, P. Cousot, R. Cousot, J. Feret, L. Mauborgne, A. Miné, D. Monniaux, and X. Rival. A static analyzer for large safety-critical software. PLDI'03, San Diego, June 7–14, ACM Press, 2003.

ASTRÉE: A Sound, Automatic, Specializable, Domain-Aware, Parametric, Modular, Efficient and Precise Static Program Analyzer

www.astree.ens.fr

- C programs:
  - with
    - pointers (including on functions), structures and arrays
    - floating point computations
    - tests, loops and function calls
    - limited branching (forward `goto`, `break`, `continue`)

- without
  - `union`
  - dynamic memory allocation
  - recursive function calls
  - backward branching
  - conflict side effects
  - C libraries
- Application Domain: safety critical embedded real-time synchronous software for non-linear control of very complex control/command systems.

Concrete Operational Semantics

- International norm of C (ISO/IEC 9899:1999)
- *restricted by* implementation-specific behaviors depending upon the machine and compiler (e.g. representation and size of integers, IEEE 754-1985 norm for floats and doubles)
- *restricted by* user-defined programming guidelines (such as no modular arithmetic for signed integers, even though this might be the hardware choice)
- *restricted by* program specific user requirements (e.g. assert)

Abstract Semantics

- Trace-based refinement of the reachable states for the concrete operational semantics
- Volatile environment is specified by a *trusted* configuration file.

Implicit Specification: Absence of Runtime Errors

- No violation of the norm of C (e.g. array index out of bounds)
- No implementation-specific undefined behaviors (e.g. maximum short integer is 32767)
- No violation of the programming guidelines (e.g. static variables cannot be assumed to be initialized to 0)
- No violation of the programmer assertions (must all be statically verified).

Example application

- Primary flight control software of the Airbus A340/A380 fly-by-wire system

- C program, automatically generated from a proprietary high-level specification (à la Simulink/SCADE)
- A340 family: 132,000 lines, 75,000 LOCs after preprocessing, 10,000 global variables, over 21,000 after expansion of small arrays
- A380: $\times$ 3

The Class of Considered Periodic Synchronous Programs

declare `volatile input, state and output variables;`
`initialize state and output variables;`
loop forever

```
- read volatile input variables,
- compute output and state variables,
- write to volatile output variables;
```

wait_for_clock ();
end loop

- Requirements: the only interrupts are clock ticks;
- Execution time of loop body less than a clock tick [3].

Reference

[3] C. Ferdinand, R. Heckmann, M. Langenbach, F. Martin, M. Schmidt, H. Theiling, S. Thesing, and R. Wilhelm. Reliable and precise WCET determination for a real-life processor. *ESOP (2001)*, LNCS 2211, 469–485.

Characteristics of the ASTRÉE Analyzer

Static: compile time analysis ($\neq$ run time analysis Rational Purify, Parasoft Insure++)

Program Analyzer: analyzes programs not micromodels of programs ($\neq$ PROMELA in SPIN or Alloy in the Alloy Analyzer)

Automatic: no end-user intervention needed ($\neq$ ESC Java, ESC Java 2)

Sound: covers the whole state space ($\neq$ MAGIC, CBMC) so never omit potential errors ($\neq$ UNO, CMC from coverity.com) or sort most probable ones ($\neq$ Splint)

Characteristics of the ASTRÉE Analyzer (Cont'd)

Multiabstraction: uses many numerical/symbolic abstract domains ($\neq$ symbolic constraints in Bane or the canonical abstraction of TVLA)

Infinitary: all abstractions use infinite abstract domains with widening/narrowing ($\neq$ model checking based analyzers such as VeriSoft, Bandera, Java PathFinder)

Efficient: always terminate ($\neq$ counterexample-driven automatic abstraction refinement BLAST, SLAM)

Characteristics of the ASTRÉE Analyzer (Cont'd)

Specializable: can easily incorporate new abstractions (and reduction with already existing abstract domains) ($\neq$ general-purpose analyzers PolySpace Verifier)

Domain-Aware: knows about control/command (e.g. digital filters) (as opposed to specialization to a mere programming style in C Global Surveyor)

Parametric: the precision/cost can be tailored to user needs by options and directives in the code

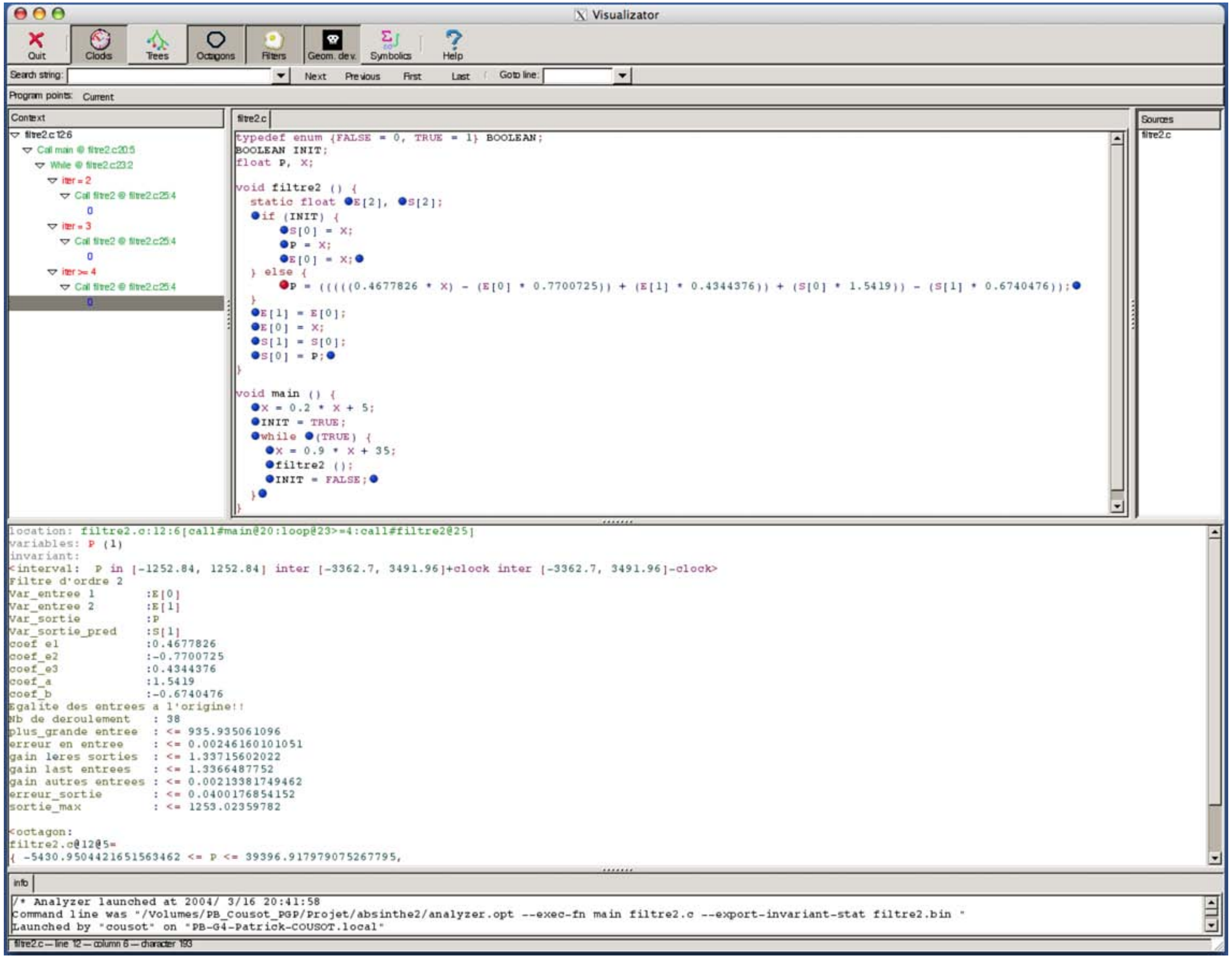
Characteristics of the ASTRÉE Analyzer (Cont'd)

Automatic Parametrization: the generation of parametric directives in the code can be programmed (to be specialized for a specific application domain)

Modular: an analyzer instance is built by selection of O-CAML modules from a collection each implementing an abstract domain

Precise: very few or no false alarm when adapted to an application domain $\longrightarrow$ it is a VERIFIER!

Example of Analysis Session



Benchmarks (Airbus A340 Primary Flight Control Software)

- 132,000 lines, 75,000 LOCs after preprocessing
- Comparative results (commercial software):

  4,200 (false?) alarms,

  3.5 days;

- Our results, November 2003:

  $\underline{\underline{\mathbf{0}}}$ alarms,

  40mn on 2.8 GHz PC,

  300 Megabytes

  $\longrightarrow$ A world première!

(Airbus A380 Primary Flight Control Software)

- 350,000 lines
- **0** alarms (mid-October 2004!),

  7h [1] on 2.8 GHz PC,

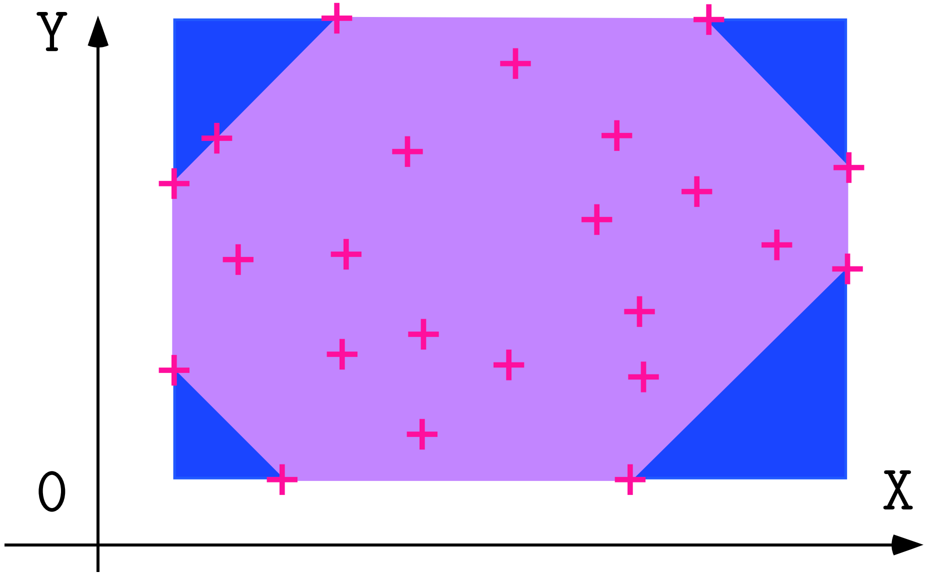
  1 Gigabyte

  $\longrightarrow$ A world grand première!

[1] We are still in a phase where we favour precision rather than computation costs, and this should go down. For example, the A340 analysis went up to 5 h, before being reduced by requiring less precision while still getting no false alarm.

Examples of Abstractions

General-Purpose Abstract Domains: Intervals and Octagons



Intervals:

$$\begin{cases} 1 \leq x \leq 9 \\ 1 \leq y \leq 20 \end{cases}$$

Octagons [4]:

$$\begin{cases} 1 \leq x \leq 9 \\ x + y \leq 77 \\ 1 \leq y \leq 20 \\ x - y \leq 04 \end{cases}$$

Difficulties: many global variables, arrays (smashed or not), IEEE 754 floating-point arithmetic (in program <u>and</u> analyzer) [5]

Reference

[4] A. Miné. A New Numerical Abstract Domain Based on Difference-Bound Matrices. In *PADO'2001*, LNCS 2053, Springer, 2001, pp. 155–172.

[5] A. Miné. Relational abstract domains for the detection of floating-point run-time errors. In *ESOP'04*, Barcelona, LNCS 2986, pp. 1—17, Springer, 2004.

Floating-Point Computations

- Code Sample:

```
/* float-error.c */
int main () {
  float x, y, z, r;
  x = 1.000000019e+38;
  y = x + 1.0e21;
  z = x - 1.0e21;
  r = y - z;
  printf("%f\n", r);
} % gcc float-error.c
% ./a.out
0.000000
```

$$(x + a) - (x - a) \neq 2a$$

```
/* double-error.c */
int main () {
double x; float y, z, r;
/* x = ldexp(1.,50)+ldexp(1.,26); */
x = 1125899973951488.0;
y = x + 1;
z = x - 1;
r = y - z;
printf("%f\n", r);
}
% gcc double-error.c
% ./a.out
134217728.000000
```

Symbolic abstract domain

- Interval analysis: if $x \in [a, b]$, $y \in [c, d]$ & $a, c \geq 0$ then $x - y \in [a - d, b - c]$ so if $x \in [0, 100]$ then $x - x \in [-100, 100]$!!!
- The symbolic abstract domain propagates the symbolic values of variables and performs simplifications;
- Must maintain the maximal possible rounding error for float computations (overestimated with intervals);

```
% cat -n x-x.c
  void main () { int X, Y;
          __ASTREE_known_fact(((0 <= X) && (X <= 100)));
          Y = (X - X);
          __ASTREE_log_vars((Y)); }
```

```
astree -exec-fn main -no-relational x-x.c
Call main@x-x.c:1:5-x-x.c:1:9:
<interval: Y in [-100, 100]>
```

```
astree -exec-fn main x-x.c
Call main@x-x.c:1:5-x-x.c:1:9:
<interval: Y in {0}> <symbolic: Y = (X -i X)>
```

Clock Abstract Domain for Counters

- Code Sample:

```
R = 0;
while (1) {
  if (I)
    { R = R+1; }
  else
    { R = 0; }
  T = (R>=n);
  wait_for_clock ();
}
```

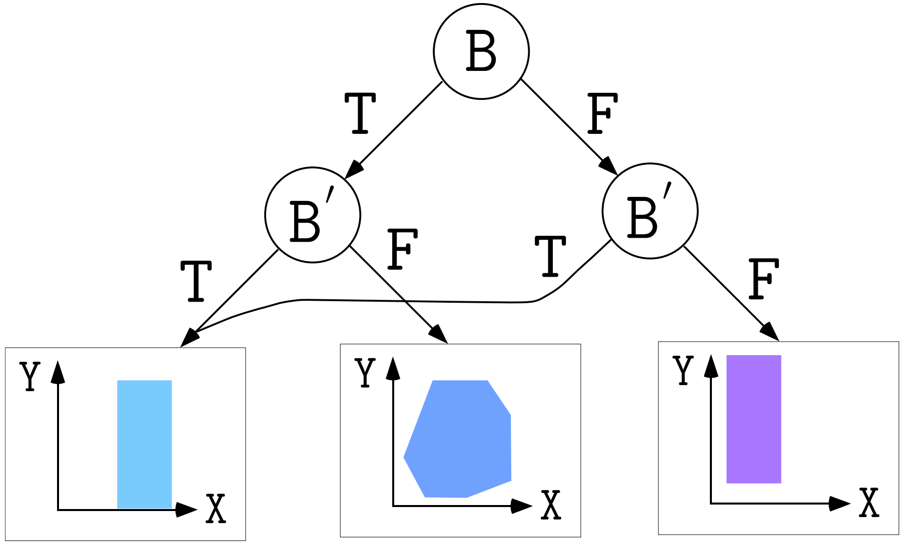
  - Output T is true iff the volatile input I has been true for the last n clock ticks.
  - The clock ticks every s seconds for at most h hours, thus R is bounded.
  - To prove that R cannot overflow, we must prove that R cannot exceed the elapsed clock ticks *(impossible using only intervals)*.

- Solution:
  - We add a phantom variable clock in the concrete user semantics to track elapsed clock ticks.
  - For each variable X, we abstract three intervals: X, X+clock, and X-clock.
  - If X+clock or X-clock is bounded, so is X.

Boolean Relations for Boolean Control

- Code Sample:

```
/* boolean.c */
typedef enum {F=0,T=1} BOOL;
BOOL B;
void main () {
  unsigned int X, Y;
  while (1) {
    ...
    B = (X == 0);
    ...
    if (!B) {
      Y = 1 / X;
    }
    ...
  }
}
```



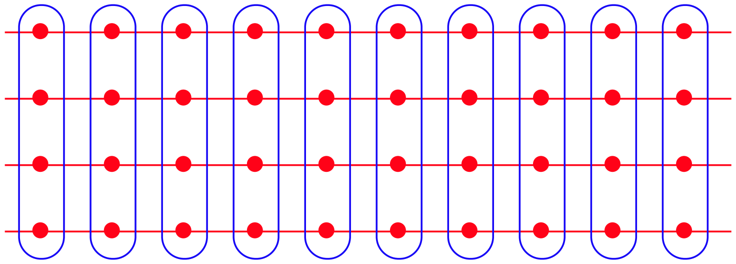
The boolean relation abstract domain is parameterized by the height of the decision tree (an analyzer option) and the abstract domain at the leafs

Control Partitionning for Case Analysis

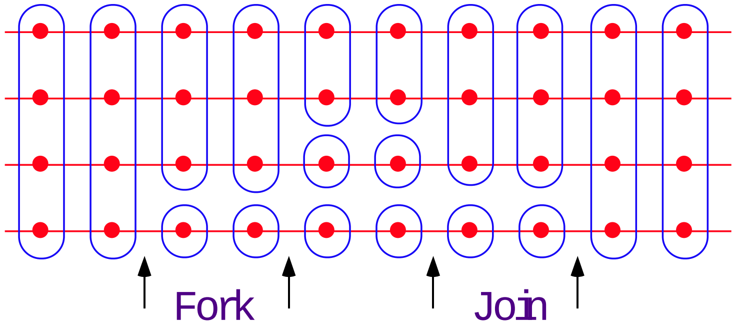
- Code Sample:

```
/* trace_partitionning.c */
void main() {
  float t[5] = {-10.0, -10.0, 0.0, 10.0, 10.0};
  float c[4] = {0.0, 2.0, 2.0, 0.0};
  float d[4] = {-20.0, -20.0, 0.0, 20.0};
  float x, r;
  int i = 0;

  ... found invariant −100 ≤ x ≤ 100 ...

  while ((i < 3) && (x >= t[i+1])) {
    i = i + 1;
  }
  r = (x - t[i]) * c[i] + d[i];
}
```

Control point partitionning:



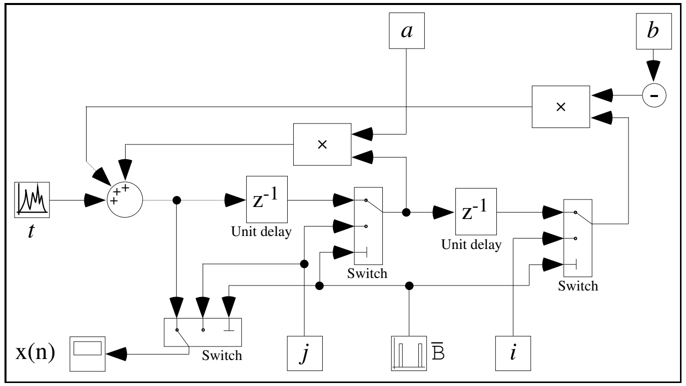
Trace partitionning:



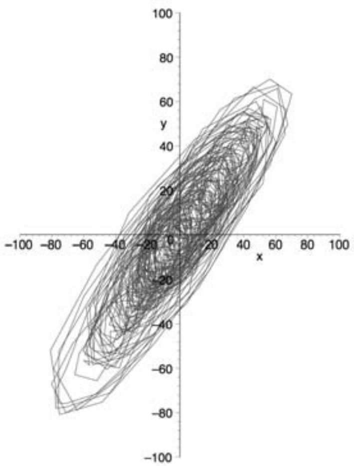
Delaying abstract unions in tests and loops is more precise for non-distributive abstract domains (and much less expensive than disjunctive completion).

2[d] Order Digital Filter:

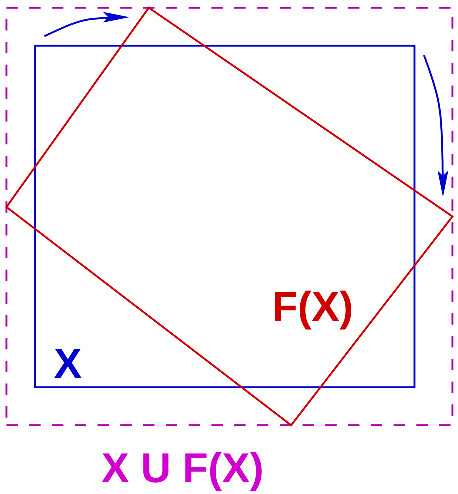
Ellipsoid Abstract Domain for Filters



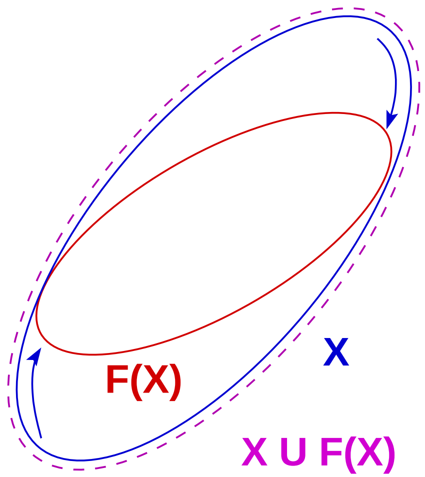
- Computes $X_n = \begin{cases} \alpha X_{n-1} + \beta X_{n-2} + Y_n \\ I_n \end{cases}$
- The concrete computation is bounded, which must be proved in the abstract.
- There is no stable interval or octagon.
- The simplest stable surface is an ellipsoid.



execution trace



unstable interval



stable ellipsoid

Filter Example [6]

```
typedef enum {FALSE = 0, TRUE = 1} BOOLEAN;
BOOLEAN INIT; float P, X;
void filter () {
  static float E[2], S[2];
  if (INIT) { S[0] = X; P = X; E[0] = X; }
  else { P = (((((0.5 * X) - (E[0] * 0.7)) + (E[1] * 0.4))
             + (S[0] * 1.5)) - (S[1] * 0.7)); }
  E[1] = E[0]; E[0] = X; S[1] = S[0]; S[0] = P;
  /* S[0], S[1] in [-1327.02698354, 1327.02698354] */
}
void main () { X = 0.2 * X + 5; INIT = TRUE;
  while (1) {
    X = 0.9 * X + 35; /* simulated filter input */
    filter (); INIT = FALSE; }
}
```


Reference

[6] J. Feret. Static analysis of digital filters. In *ESOP'04*, Barcelona, LNCS 2986, pp. 33—-48, Springer, 2004.

Arithmetic-Geometric Progressions

```
% cat retro.c
typedef enum {FALSE=0, TRUE=1} BOOL;
BOOL FIRST;
volatile BOOL SWITCH;
volatile float E;
float P, X, A, B;

void dev( )
{ X=E;
  if (FIRST) { P = X; }
  else
    { P =  (P - ((((2.0 * P) - A) - B)
            * 4.491048e-03)); };
  B = A;
  if (SWITCH) {A = P;}
  else {A = X;}
}
```

```
void main()
{ FIRST = TRUE;
  while (TRUE) {
    dev( );
    FIRST = FALSE;
    __ASTREE_wait_for_clock(());
  }}
% cat retro.config
__ASTREE_volatile_input((E [-15.0, 15.0]));
__ASTREE_volatile_input((SWITCH [0,1]));
__ASTREE_max_clock((3600000));
|P| <= (15.  + 5.87747175411e-39
/ 1.19209290217e-07) * (1 +
1.19209290217e-07)^clock -
5.87747175411e-39 / 1.19209290217e-07
<= 23.0393526881
```


Reference

[7] J. Feret. The Arithmetic-Geometric Progression Abstract Domain. To appear in *VMCAI'05*, Paris, January 17—19, 2005, LNCS, Springer.

(Automatic) Parameterization

- All abstract domains of ASTRÉE are parameterized, e.g.
  - variable packing for octagones and decision trees,
  - partition/merge program points,
  - loop unrollings,
  - thresholds in widenings, ...;
- End-users can either parameterize by hand (analyzer options, directives in the code), or
- choose the automatic parameterization (default options, directives for pattern-matched predefined program schemata).

The main loop invariant for the A340

A textual file over 4.5 Mb with

- 6,900 boolean interval assertions ($x \in [0; 1]$)
- 9,600 interval assertions ($x \in [a; b]$)
- 25,400 clock assertions ($x + \mathrm{clk} \in [a; b] \wedge x - \mathrm{clk} \in [a; b]$)
- 19,100 additive octagonal assertions ($a \leq x + y \leq b$)
- 19,200 subtractive octagonal assertions ($a \leq x - y \leq b$)
- 100 decision trees
- 60 ellipse invariants, etc ...

involving over 16,000 floating point constants (only 550 appearing in the program text) $\times$ 75,000 LOCs.

Possible origins of imprecision and how to fix it

In case of false alarm, the imprecision can come from:

- Abstract transformers (not best possible) $\longrightarrow$ improve algorithm;
- Automatized parametrization (e.g. variable packing) $\longrightarrow$ improve pattern-matched program schemata;
- Iteration strategy for fixpoints $\longrightarrow$ fix widening [2];
- Inexpressivity i.e. indispensable local inductive invariant are inexpressible in the abstract $\longrightarrow$ add a new abstract domain to the reduced product (e.g. filters).

---

[2] This can be very hard since at the limit only a precise infinite iteration might be able to compute the proper abstract invariant. In that case, it might be better to design a more refined abstract domain.

Conclusion

Conclusion

- Most applications of abstract interpretation tolerate a small rate (typically 5 to 15%) of false alarms:
  - Program transformation → do not optimize,
  - Typing → reject some correct programs, etc,
  - WCET analysis → overestimate;
- Some applications require no false alarm at all:
  - Program verification.
- Theoretically possible [SARA '00], practically feasible [PLDI '03]

Reference

[SARA '00] P. Cousot. Partial Completeness of Abstract Fixpoint Checking, invited paper. In 4^{th} *Int. Symp. SARA '2000*, LNAI 1864, Springer, pp. 1–25, 2000.

[PLDI '03] B. Blanchet, P. Cousot, R. Cousot, J. Feret, L. Mauborgne, A. Miné, D. Monniaux, and X. Rival. A static analyzer for large safety-critical software. PLDI'03, San Diego, June 7–14, ACM Press, 2003.

The Future & Grand Challenges

Forthcoming (1 year):

- More gereral memory model (`union`)

Future (5 years):

- Asynchronous concurrency (for less critical software)
- Functional properties (reactivity)
- Industrialization

Grand challenge:

- Verification from specifications to machine code (verifying compiler)
- Verification of systems (quasi-synchrony, distribution)

THE END, THANK YOU

More references at URL `www.di.ens.fr/~cousot`
`www.astree.ens.fr`.

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