

# Static Analysis and Verification of Synchronous Embedded Code by Abstract Interpretation

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# Motivation (1 mn)

# All Computer Scientists Have Experienced Bugs



Ariane 5.01 failure  
(overflow)



Patriot failure  
(float rounding)



Mars orbiter loss  
(unit error)

It is preferable to verify that mission/safety-critical programs do not go wrong before running them.

# Static Analysis by Abstract Interpretation

**Static analysis:** analyze the program at compile-time to verify a program runtime property (e.g. the absence of some categories of bugs)

Undecidability  $\longrightarrow$

**Abstract interpretation:** effectively compute an abstraction/  
**sound approximation** of the program **semantics**,  
– which is **precise** enough to imply the desired property, and  
– coarse enough to be **efficiently computable**.

# Abstract Interpretation, Reminder (10 mn)

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## Reference

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- [POPL '77] P. Cousot and R. Cousot. Abstract interpretation: a unified lattice model for static analysis of programs by construction or approximation of fixpoints. In *4<sup>th</sup> ACM POPL*.
- [Thesis '78] P. Cousot. Méthodes itératives de construction et d'approximation de points fixes d'opérateurs monotones sur un treillis, analyse sémantique de programmes. Thèse ès sci. math. Grenoble, march 1978.
- [POPL '79] P. Cousot & R. Cousot. Systematic design of program analysis frameworks. In *6<sup>th</sup> ACM POPL*.

## Syntax of programs

$X$

variables  $X \in \mathbb{X}$

$T$

types  $T \in \mathbb{T}$

$E$

arithmetic expressions  $E \in \mathbb{E}$

$B$

boolean expressions  $B \in \mathbb{B}$

$D ::= T \ X;$

$\quad | \quad T \ X ; D'$

$C ::= X = E;$

$\quad | \quad \text{while } B \ C'$

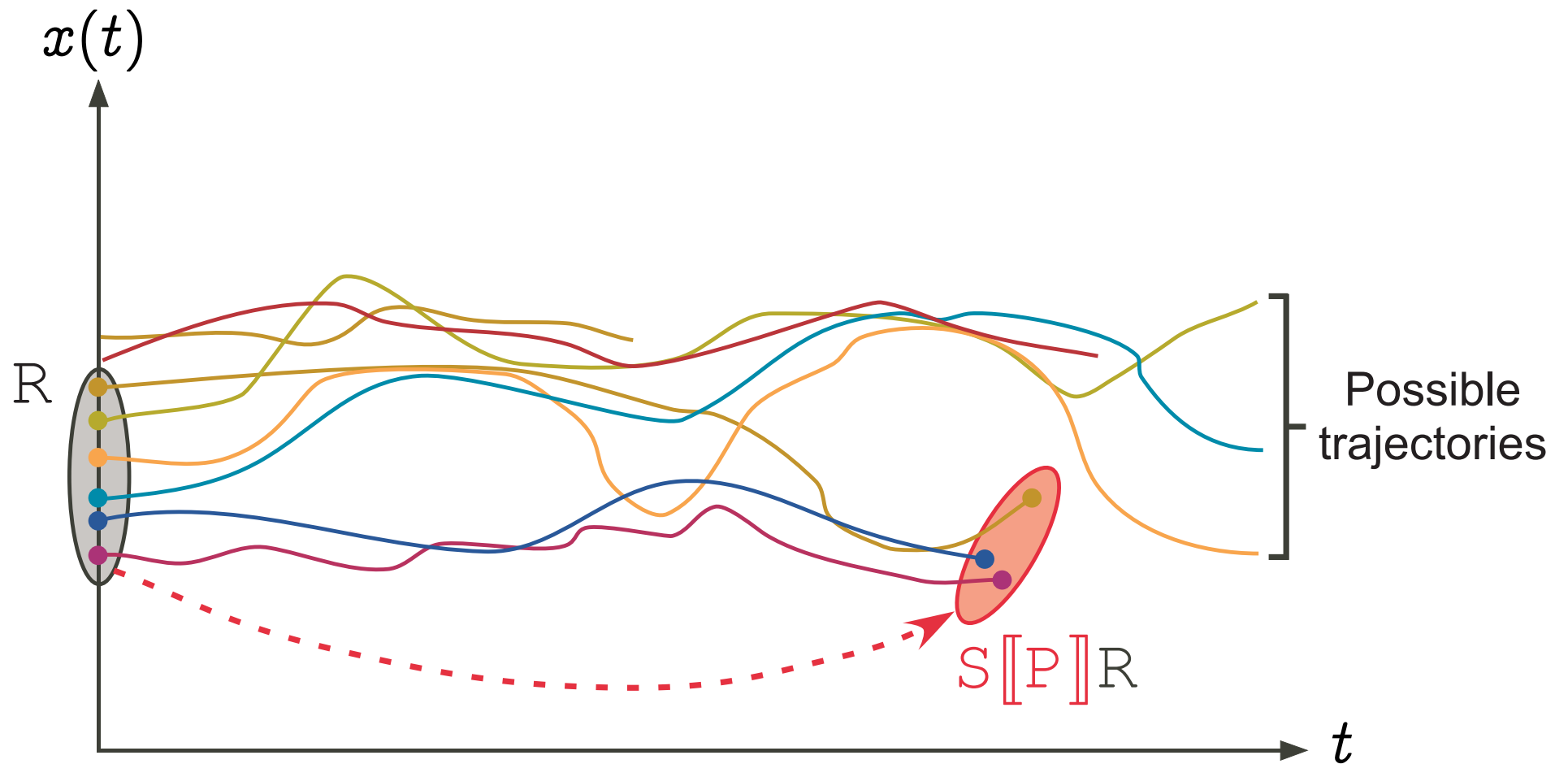
$\quad | \quad \text{if } B \ C' \text{ else } C''$

$\quad | \quad \{ C_1 \dots C_n \}, (n \geq 0)$

$P ::= D \ C$

program  $P \in \mathbb{P}$

## Postcondition semantics



## States

Values of given type:

$\mathcal{V}[[T]]$  : values of type  $T \in \mathbb{T}$

$$\mathcal{V}[[\text{int}]] \stackrel{\text{def}}{=} \{z \in \mathbb{Z} \mid \text{min\_int} \leq z \leq \text{max\_int}\}$$

Program states  $\Sigma[[P]]$ <sup>1</sup>:

$$\Sigma[[D \ C]] \stackrel{\text{def}}{=} \Sigma[[D]]$$

$$\Sigma[[T \ X;]] \stackrel{\text{def}}{=} \{X\} \mapsto \mathcal{V}[[T]]$$

$$\Sigma[[T \ X; \ D]] \stackrel{\text{def}}{=} (\{X\} \mapsto \mathcal{V}[[T]]) \cup \Sigma[[D]]$$

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<sup>1</sup> States  $\rho \in \Sigma[[P]]$  of a program  $P$  map program variables  $X$  to their values  $\rho(X)$



# Concrete Semantic Domain of Programs

Concrete semantic domain for reachability properties:

$$\mathcal{D}[[P]] \stackrel{\text{def}}{=} \wp(\Sigma[[P]]) \quad \text{sets of states}$$

i.e. program properties where  $\subseteq$  is implication,  $\emptyset$  is false,  $\cup$  is disjunction.

## Concrete Reachability Semantics of Programs

$$S[X = E;]R \stackrel{\text{def}}{=} \{\rho[X \leftarrow \mathcal{E}[E]\rho] \mid \rho \in R \cap \text{dom}(E)\}$$

$$\rho[X \leftarrow v](X) \stackrel{\text{def}}{=} v, \quad \rho[X \leftarrow v](Y) \stackrel{\text{def}}{=} \rho(Y)$$

$$S[\text{if } B \text{ } C']R \stackrel{\text{def}}{=} S[C'](\mathcal{B}[B]R) \cup \mathcal{B}[\neg B]R$$

$$\mathcal{B}[B]R \stackrel{\text{def}}{=} \{\rho \in R \cap \text{dom}(B) \mid B \text{ holds in } \rho\}$$

$$S[\text{if } B \text{ } C' \text{ else } C'']R \stackrel{\text{def}}{=} S[C'](\mathcal{B}[B]R) \cup S[C''](\mathcal{B}[\neg B]R)$$

$$S[\text{while } B \text{ } C']R \stackrel{\text{def}}{=} \text{let } \mathcal{W} = \text{lfp}_{\emptyset}^{\subseteq} \lambda \mathcal{X}. R \cup S[C'](\mathcal{B}[B]\mathcal{X}) \\ \text{in } (\mathcal{B}[\neg B]\mathcal{W})$$

$$S[\{\}]R \stackrel{\text{def}}{=} R$$

$$S[\{C_1 \dots C_n\}]R \stackrel{\text{def}}{=} S[C_n] \circ \dots \circ S[C_1] \quad n > 0$$

$$S[D \text{ } C]R \stackrel{\text{def}}{=} S[C](\Sigma[D]) \quad (\text{uninitialized variables})$$

Not computable (undecidability).

# Abstract Semantic Domain of Programs

$$\langle \mathcal{D}^\# \llbracket P \rrbracket, \sqsubseteq, \perp, \sqcup \rangle$$

such that:

$$\langle \mathcal{D} \llbracket P \rrbracket, \subseteq \rangle \xrightleftharpoons[\alpha]{\gamma} \langle \mathcal{D}^\# \llbracket P \rrbracket, \sqsubseteq \rangle$$

i.e.

$$\forall X \in \mathcal{D} \llbracket P \rrbracket, Y \in \mathcal{D}^\# \llbracket P \rrbracket : \alpha(X) \sqsubseteq Y \iff X \subseteq \gamma(Y)$$

hence  $\langle \mathcal{D}^\# \llbracket P \rrbracket, \sqsubseteq, \perp, \sqcup \rangle$  is a complete lattice such that  $\perp = \alpha(\emptyset)$  and  $\sqcup X = \alpha(\cup \gamma(X))$

## Example 1 of Abstraction

**Traces:** set of finite or infinite maximal sequences of states for the operational transition semantics

$\xrightarrow{\alpha}$  **Strongest liberal postcondition:** final states  $s$  reachable from a given precondition  $P$

$$\alpha(X) = \lambda P. \{s \mid \exists \sigma_0 \sigma_1 \dots \sigma_n \in X : \sigma_0 \in P \wedge s = \sigma_n\}$$

We have ( $\Sigma$ : set of states,  $\dot{\subseteq}$  pointwise):

$$\langle \wp(\Sigma^\infty), \subseteq \rangle \xrightleftharpoons[\alpha]{\gamma} \langle \wp(\Sigma) \xrightarrow{\cup} \wp(\Sigma), \dot{\subseteq} \rangle$$

## Example 2 of Abstraction

**Traces:** set of finite or infinite maximal sequences of states for the operational transition semantics

$\xrightarrow{\alpha_1}$  **Set of reachable states:** set of states appearing at least once along one of these traces (global invariant)

$$\alpha_1(X) = \{\sigma_i \mid \sigma \in X \wedge 0 \leq i < |\sigma|\}$$

$\xrightarrow{\alpha_2}$  **Partitionned set of reachable states:** project along each control point (local invariant)

$$\alpha_2(\{\langle c_i, \rho_i \rangle \mid i \in \Delta\}) = \lambda c. \{\rho_i \mid i \in \Delta \wedge c = c_i\}$$

$\alpha_3$   
→ Partitionned cartesian set of reachable states: project along each program variable (relationships between variables are now lost)

$$\alpha_3(\lambda c. \{\rho_i \mid i \in \Delta_c\}) = \lambda c. \lambda x. \{\rho_i(x) \mid i \in \Delta_c\}$$

$\alpha_4$   
→ Partitionned cartesian interval of reachable states: take min and max of the values of the variables<sup>2</sup>

$$\alpha_4(\lambda c. \lambda x. \{v_i \mid i \in \Delta_{c,x}\}) = \lambda c. \lambda x. \langle \min\{v_i \mid i \in \Delta_{c,x}\}, \max\{v_i \mid i \in \Delta_{c,x}\} \rangle$$

$\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$  and  $\alpha_4$ , whence  $\alpha_4 \circ \alpha_3 \circ \alpha_2 \circ \alpha_1$  are lower-adjoints of Galois connections

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<sup>2</sup> assuming these values to be totally ordered.

### Example 3: Reduced Product of Abstract Domains

To combine abstractions

$$\langle \mathcal{D}, \sqsubseteq \rangle \xrightleftharpoons[\alpha_1]{\gamma_1} \langle \mathcal{D}_1^\#, \sqsubseteq_1 \rangle \text{ and } \langle \mathcal{D}, \sqsubseteq \rangle \xrightleftharpoons[\alpha_2]{\gamma_2} \langle \mathcal{D}_2^\#, \sqsubseteq_2 \rangle$$

the reduced product is

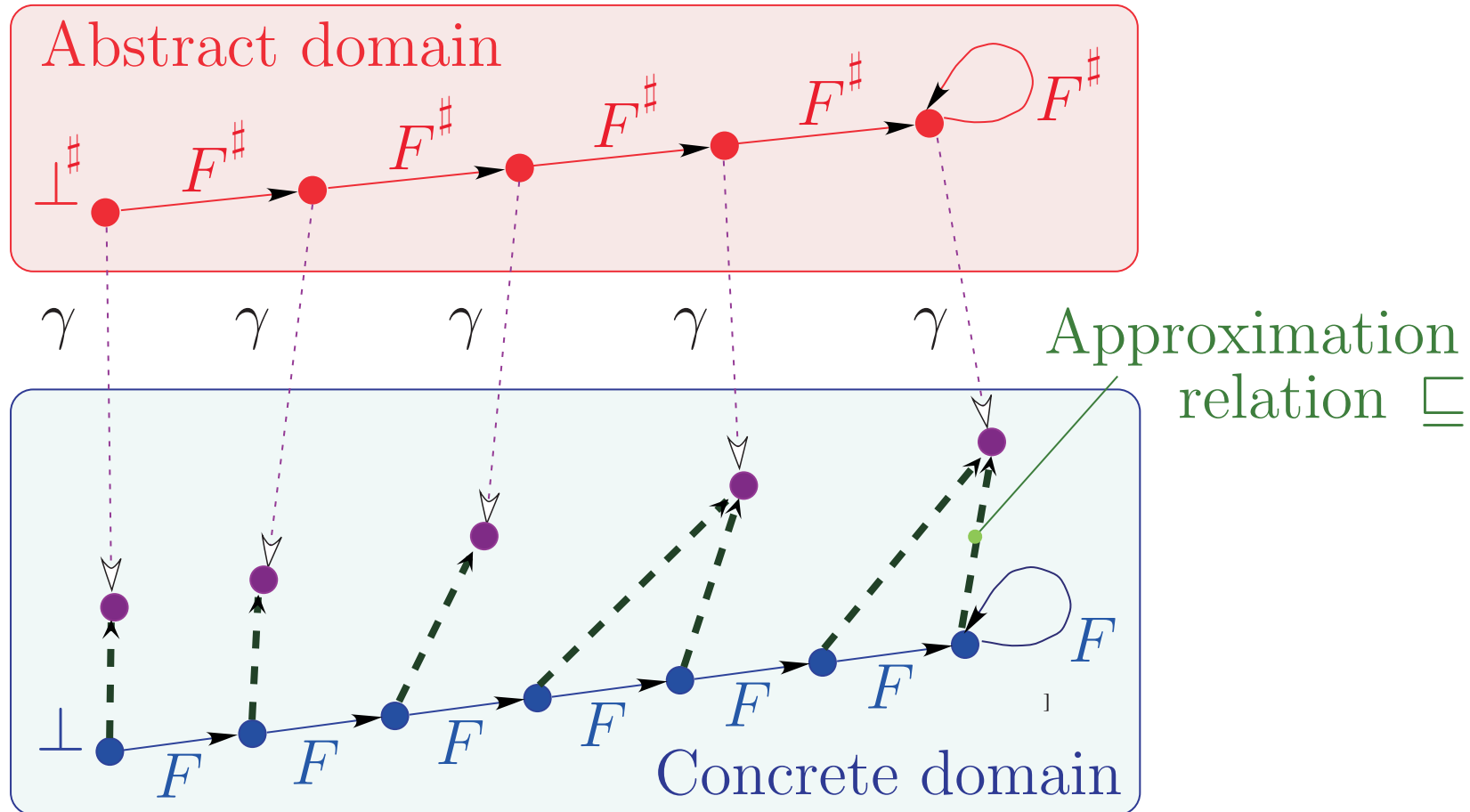
$$\alpha(X) \stackrel{\text{def}}{=} \sqcap \{ \langle x, y \rangle \mid X \sqsubseteq \gamma_1(x) \wedge X \sqsubseteq \gamma_2(y) \}$$

such that  $\sqsubseteq \stackrel{\text{def}}{=} \sqsubseteq_1 \times \sqsubseteq_2$  and

$$\langle \mathcal{D}, \sqsubseteq \rangle \xrightleftharpoons[\alpha]{\gamma_1 \times \gamma_2} \langle \alpha(\mathcal{D}), \sqsubseteq \rangle$$

Example:  $x \in [1, 9] \wedge x \bmod 2 = 0$  reduces to  $x \in [2, 8] \wedge x \bmod 2 = 0$

# Approximate Fixpoint Abstraction



$$F \circ \gamma \sqsubseteq \gamma \circ F^\# \Rightarrow \text{lfp } F \sqsubseteq \gamma(\text{lfp } F^\#)$$



## Abstract Reachability Semantics of Programs

$$S^\sharp \llbracket X = E; \rrbracket R \stackrel{\text{def}}{=} \alpha(\{\rho[X \leftarrow \mathcal{E} \llbracket E \rrbracket \rho] \mid \rho \in \gamma(R) \cap \text{dom}(E)\})$$

$$S^\sharp \llbracket \text{if } B \text{ } C' \rrbracket R \stackrel{\text{def}}{=} S^\sharp \llbracket C' \rrbracket (\mathcal{B}^\sharp \llbracket B \rrbracket R) \sqcup \mathcal{B}^\sharp \llbracket \neg B \rrbracket R$$

$$\mathcal{B}^\sharp \llbracket B \rrbracket R \stackrel{\text{def}}{=} \alpha(\{\rho \in \gamma(R) \cap \text{dom}(B) \mid B \text{ holds in } \rho\})$$

$$S^\sharp \llbracket \text{if } B \text{ } C' \text{ else } C'' \rrbracket R \stackrel{\text{def}}{=} S^\sharp \llbracket C' \rrbracket (\mathcal{B}^\sharp \llbracket B \rrbracket R) \sqcup S^\sharp \llbracket C'' \rrbracket (\mathcal{B}^\sharp \llbracket \neg B \rrbracket R)$$

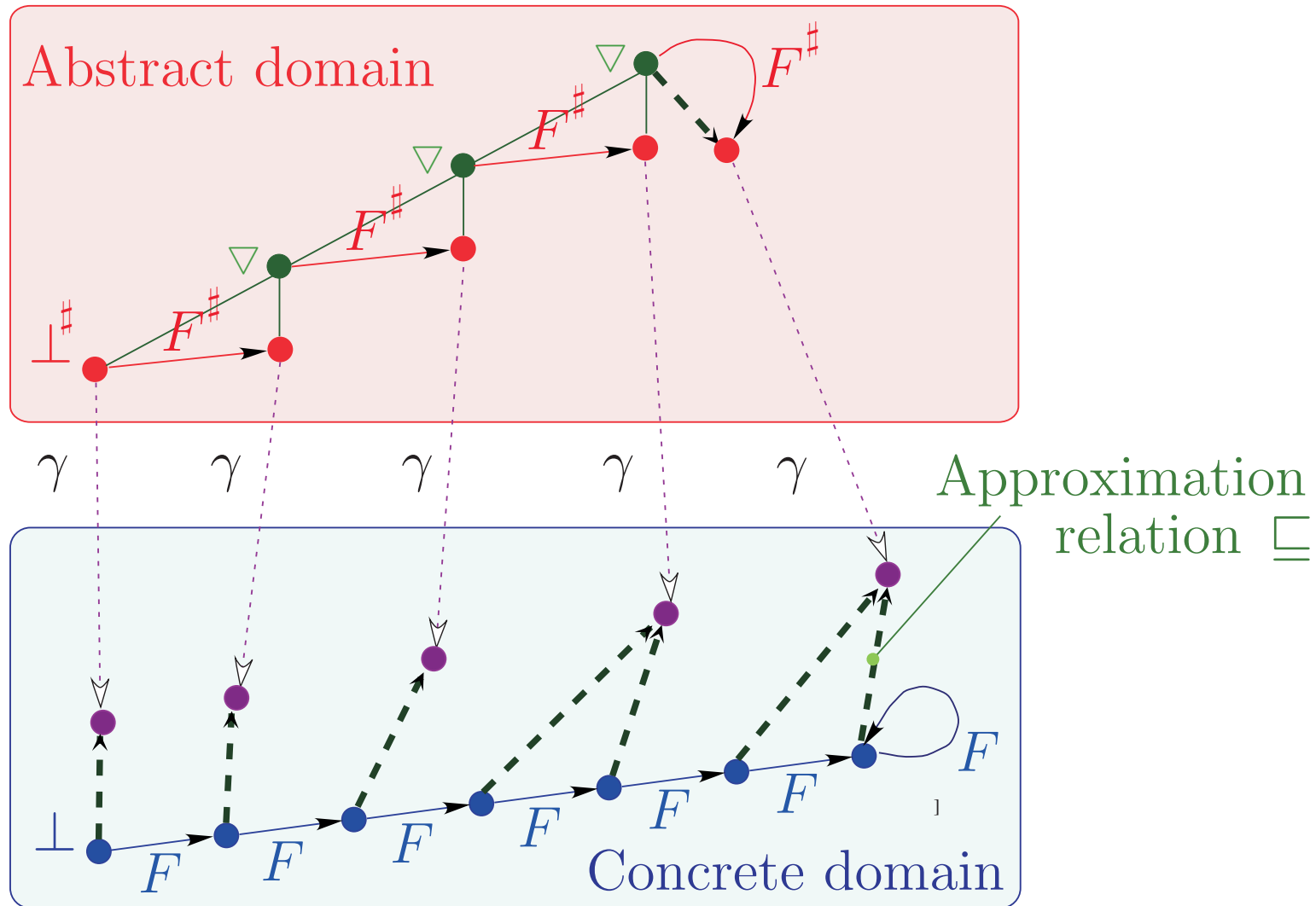
$$S^\sharp \llbracket \text{while } B \text{ } C' \rrbracket R \stackrel{\text{def}}{=} \text{let } \mathcal{W} = \text{lfp}_{\perp}^{\sqsubseteq} \lambda \mathcal{X}. R \sqcup S^\sharp \llbracket C' \rrbracket (\mathcal{B}^\sharp \llbracket B \rrbracket \mathcal{X}) \\ \text{in } (\mathcal{B}^\sharp \llbracket \neg B \rrbracket \mathcal{W})$$

$$S^\sharp \llbracket \{\} \rrbracket R \stackrel{\text{def}}{=} R$$

$$S^\sharp \llbracket \{C_1 \dots C_n\} \rrbracket R \stackrel{\text{def}}{=} S^\sharp \llbracket C_n \rrbracket \circ \dots \circ S^\sharp \llbracket C_1 \rrbracket \quad n > 0$$

$$S^\sharp \llbracket D \text{ } C \rrbracket R \stackrel{\text{def}}{=} S^\sharp \llbracket C \rrbracket (\top) \quad (\text{uninitialized variables})$$

# Convergence Acceleration with Widening



## Abstract Semantics with Convergence Acceleration<sup>3</sup>

$$S^\sharp[X = E;]R \stackrel{\text{def}}{=} \alpha(\{\rho[X \leftarrow \mathcal{E}[E]\rho] \mid \rho \in \gamma(R) \cap \text{dom}(E)\})$$

$$S^\sharp[\text{if } B \text{ } C']R \stackrel{\text{def}}{=} S^\sharp[C'](\mathcal{B}^\sharp[B]R) \sqcup \mathcal{B}^\sharp[\neg B]R$$

$$\mathcal{B}^\sharp[B]R \stackrel{\text{def}}{=} \alpha(\{\rho \in \gamma(R) \cap \text{dom}(B) \mid B \text{ holds in } \rho\})$$

$$S^\sharp[\text{if } B \text{ } C' \text{ else } C'']R \stackrel{\text{def}}{=} S^\sharp[C'](\mathcal{B}^\sharp[B]R) \sqcup S^\sharp[C''](\mathcal{B}^\sharp[\neg B]R)$$

$$S^\sharp[\text{while } B \text{ } C']R \stackrel{\text{def}}{=} \text{let } \mathcal{F}^\sharp = \lambda \mathcal{X}. \text{let } \mathcal{Y} = R \sqcup S^\sharp[C'](\mathcal{B}^\sharp[B]\mathcal{X}) \\ \text{in if } \mathcal{Y} \sqsubseteq \mathcal{X} \text{ then } \mathcal{X} \text{ else } \mathcal{X} \nabla \mathcal{Y} \\ \text{and } \mathcal{W} = \text{lfp}_{\perp}^{\sqsubseteq} \mathcal{F}^\sharp \quad \text{in } (\mathcal{B}^\sharp[\neg B]\mathcal{W})$$

$$S^\sharp[\{\}]R \stackrel{\text{def}}{=} R$$

$$S^\sharp[\{C_1 \dots C_n\}]R \stackrel{\text{def}}{=} S^\sharp[C_n] \circ \dots \circ S^\sharp[C_1] \quad n > 0$$

$$S^\sharp[D \text{ } C]R \stackrel{\text{def}}{=} S^\sharp[C](\top) \quad (\text{uninitialized variables})$$

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<sup>3</sup> Note:  $\mathcal{F}^\sharp$  not monotonic!

# Applications of Abstract Interpretation (2 mn)

# Applications of Abstract Interpretation

- **Static Program Analysis** [POPL '77], [POPL '78], [POPL '79]  
including
  - **Dataflow Analysis** [POPL '79], [POPL '00],
  - **Set-based Analysis** [FPCA '95],
  - **Predicate Abstraction** [Manna's festschrift '03],
  - ...
- **Grammar Analysis and Parsing** [TCS 290(1) 2002],  
[Wilhelm's festschrift '07]

## Applications of Abstract Interpretation (Cont'd)

- Hierarchies of Semantics (including Proofs) [POPL '92], [TCS 277(1–2) 2002]
- Typing & Type Inference [POPL '97]
- (Abstract) Model Checking [POPL '00]
- Bisimulations [RT-ESOP '04]
- Non-interference [POPL '04]
- Models of Security Protocols [IPL'05]

## Applications of Abstract Interpretation (Cont'd)

- Program Transformation [POPL '02] including
  - Software Watermarking [POPL '04]
  - (Semantic/Abstract) Slicing [14]
  - Code Obfuscation [DPG-ICALP '05]
- Malware Detection [POPL '07]
- ...

All these techniques involve sound approximations that can be formalized by abstract interpretation

# Application to the Astrée Static Analyzer (15 mn)

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## Reference

[1] <http://www.astree.ens.fr/>



# Project Members



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<sup>4</sup> Nov. 2001 — Nov. 2003.

## Programs analysed by Astrée

- **Application Domain:** large safety critical embedded real-time synchronous software for non-linear control of very complex control/command systems.
- **C programs:**
  - with
    - basic numeric datatypes, structures and arrays
    - pointers (including on functions),
    - floating point computations
    - tests, loops and function calls
    - limited branching (forward goto, break, continue)

- with (cont'd)
  - union **NEW** [LCTES '06]
  - pointer arithmetics & casts **NEW** [LCTES '06]
- without
  - dynamic memory allocation
  - recursive function calls
  - unstructured/backward branching
  - conflicting side effects
  - C libraries, system calls (parallelism)

*Such limitations are quite common for embedded safety-critical software.*

## Concrete Operational Semantics

- International **norm of C** (ISO/IEC 9899:1999)
- *restricted by* **implementation-specific behaviors** depending upon the machine and compiler (e.g. representation and size of integers, IEEE 754-1985 norm for floats and doubles)
- *restricted by* user-defined **programming guidelines** (such as no modular arithmetic for signed integers, even though this might be the hardware choice)
- *restricted by* program specific **user requirements** (e.g. assert, execution stops on first runtime error<sup>5</sup>)

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<sup>5</sup> semantics of C unclear after an error, equivalent if no alarm

## Abstract Semantics

- Reachable states for the concrete trace operational semantics
- Volatile environment is specified by a *trusted* configuration file.

### Requirements:

- Soundness: absolutely essential
- Precision: few or no false alarm<sup>6</sup> (full certification)
- Efficiency: rapid analyses and fixes during development

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<sup>6</sup> Potential runtime error signaled by the analyzer due to overapproximation but impossible in any actual program run.

## Implicit Specification: Absence of Runtime Errors

- No violation of the **norm of C** (e.g. array index out of bounds, division by zero)
- **No** implementation-specific **undefined behaviors** (e.g. maximum short integer is 32767, NaN)
- No violation of the **programming guidelines** (e.g. static variables cannot be assumed to be initialized to 0)
- No violation of the **programmer assertions** (must all be statically verified).

## Example application

- Primary flight control software of the Airbus A340 family/A380 fly-by-wire system



- C program, automatically generated from a proprietary high-level specification (à la Simulink/SCADE)
- A340 family: 132,000 lines, 75,000 LOCs after preprocessing, 10,000 global variables, over 21,000 after expansion of small arrays, now  $\times 2$
- A380:  $\times 3/7$

## The Class of Considered Periodic Synchronous Programs

```
declare volatile input, state and output variables;  
initialize state and output variables;  
loop forever  
  - read volatile input variables,  
  - compute output and state variables,  
  - write to output variables;  
  __ ASTREE_wait_for_clock ();  
end loop
```

Task scheduling is static:

- Requirements: the only interrupts are clock ticks;
- Execution time of loop body less than a clock tick,  
as verified by the aiT WCET Analyzers [EMSOF'T '01].

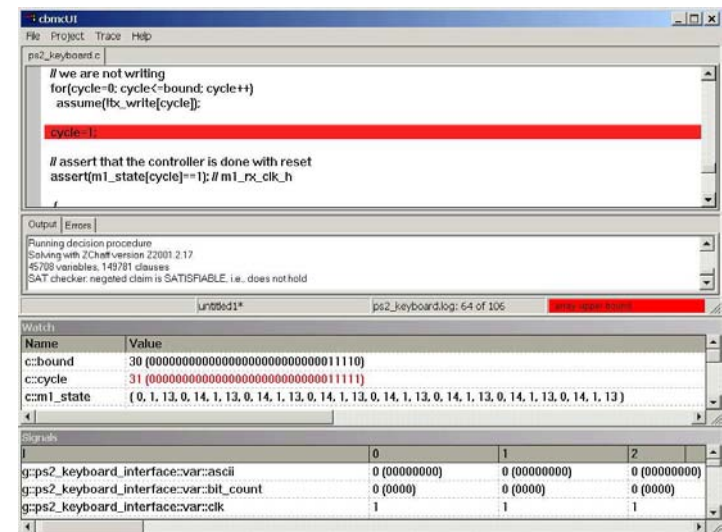


## Challenging aspects

- Size: 100/1000 kLOC, 10/150 000 global variables
- Floating point computations  
including interconnected networks of filters, non linear control with feedback, interpolations...
- Interdependencies among variables:
  - Stability of computations should be established
  - Complex relations should be inferred among numerical and boolean data
  - Very long data paths from input to outputs

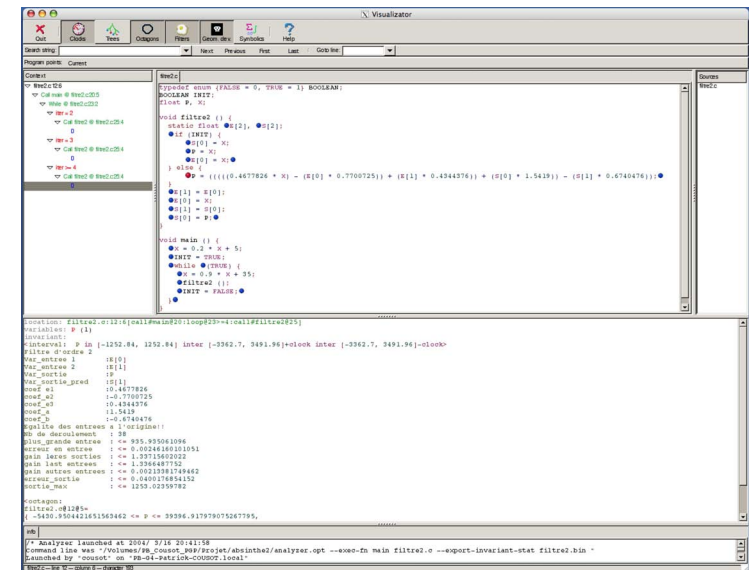
## Example 1: CBMC

- **CBMC** is a **Bounded Model Checker** for ANSI-C programs (started at CMU in 1999).
- Allows verifying array bounds (buffer overflows), pointer safety, exceptions and user-specified assertions.
- Aimed for embedded software, also supports dynamic memory allocation using malloc.
- Done by unwinding the loops in the program and passing the resulting equation to a SAT solver.
- **Problem (a.o.): does not scale up!**



## Example 2: Astrée

- **ASTRÉE** is an abstract interpretation-based **static analyzer** for ANSI-C programs (started at ENS in 2001).
- Allows verifying array bounds (buffer overflows), pointer safety, exceptions and user-specified assertions.
- Aimed for embedded software, does not support dynamic memory allocation.
- Done by abstracting the reachability fixpoint equations for the program operational semantics.
- **Advantage (a.o.): does scale up!**



## Characteristics of the Astrée Analyzer

- Sound: –ASTRÉE is a **bug eradicator**: finds all bugs in a well-defined class (runtime errors)
- ASTRÉE is not a **bug hunter**: finding some bugs in a well-defined class (e.g. by *bug pattern detection* like **FindBugs™**, **PREfast** or **PMD**)
  - ASTRÉE is **exhaustive**: covers the whole state space ( $\neq$  **MAGIC**, **CBMC**)
  - ASTRÉE is **comprehensive**: never omits potential errors ( $\neq$  **UNO**, **CMC** from [coverity.com](http://coverity.com)) or sort most probable ones to avoid overwhelming messages ( $\neq$  **Splint**)

## Characteristics of the Astrée Analyzer (Cont'd)

**Static:** compile time analysis ( $\neq$  run time analysis **Rational Purify**, **Parasoft Insure++**)

**Program Analyzer:** analyzes programs not micromodels of programs ( $\neq$  **PROMELA** in **SPIN** or **Alloy** in the **Alloy Analyzer**)

**Automatic:** no end-user intervention needed ( $\neq$  **ESC Java**, **ESC Java 2**), or **PREfast** (annotate functions with intended use)

## Characteristics of the Astrée Analyzer (Cont'd)

**Multiabstraction:** uses many numerical/symbolic abstract domains ( $\neq$  symbolic constraints in **Bane** or the canonical abstraction of **TVLA**)

**Infinitary:** all abstractions use infinite abstract domains with widening/narrowing ( $\neq$  model checking based analyzers such as **Bandera**, **Bogor**, **Java PathFinder**, **Spin**, **VeriSoft**)

**Efficient:** always terminate ( $\neq$  counterexample-driven automatic abstraction refinement **BLAST**, **SLAM**)

## Characteristics of the Astrée Analyzer (Cont'd)

**Extensible/Specializable:** can easily incorporate new abstractions (and reduction with already existing abstract domains) ( $\neq$  general-purpose analyzers **PolySpace Verifier**)

**Domain-Aware:** knows about control/command (e.g. digital filters) (as opposed to specialization to a mere programming style in **C Global Surveyor**)

**Parametric:** the precision/cost can be tailored to user needs by options and directives in the code

## Characteristics of the Astrée Analyzer (Cont'd)

**Automatic Parametrization:** the generation of parametric directives in the code can be programmed (to be specialized for a specific application domain)

**Modular:** an analyzer instance is built by selection of **O-CAML** modules from a collection each implementing an abstract domain

**Precise:** very few or no false alarm when adapted to an application domain  $\longrightarrow$  it is a **VERIFIER!**



# Example of Analysis Session

Visualizer

Search string:  Next Previous First Last Goto line:

Program points: Current

Context

- ▼ filtre2.c:126
  - ▼ Call main @ filtre2.c:205
    - ▼ While @ filtre2.c:232
      - ▼ Iter = 2
        - ▼ Call filtre2 @ filtre2.c:254
          - 0
      - ▼ Iter = 3
        - ▼ Call filtre2 @ filtre2.c:254
          - 0
      - ▼ Iter >= 4
        - ▼ Call filtre2 @ filtre2.c:254
          - 0

filtre2.c

```
typedef enum {FALSE = 0, TRUE = 1} BOOLEAN;
BOOLEAN INIT;
float P, X;

void filtre2 () {
  static float E[2], S[2];
  if (INIT) {
    S[0] = X;
    P = X;
    S[1] = X;
  } else {
    P = (((0.4677826 * X) - (E[0] * 0.7700725)) + (E[1] * 0.4344376)) + (S[0] * 1.5419) - (S[1] * 0.6740476);
    E[1] = E[0];
    E[0] = X;
    S[1] = S[0];
    S[0] = P;
  }
}

void main () {
  X = 0.2 * X + 5;
  INIT = TRUE;
  while (TRUE) {
    X = 0.9 * X + 35;
    filtre2 ();
    INIT = FALSE;
  }
}
```

Location: filtre2.c:126[call#main@20:loop@23>=4:call#filtre2@25]

Variables: P (1)

Invariant:

<interval: P in [-1252.84, 1252.84] inter [-3362.7, 3491.96]+clock inter [-3362.7, 3491.96]-clock>

Filtre d'ordre 2

Var_entree 1	:E[0]
Var_entree 2	:E[1]
Var_sortie	:P
Var_sortie_pred	:S[1]
coef_e1	:0.4677826
coef_e2	:-0.7700725
coef_e3	:0.4344376
coef_a	:1.5419
coef_b	:-0.6740476

Egalite des entrees a l'origine!!

Nb de deroulement : 38

plus\_grande entree : <= 935.935061096

erreur en entree : <= 0.00246160101051

gain leres sorties : <= 1.33715602022

gain last entrees : <= 1.3366487752

gain autres entrees : <= 0.00213381749462

erreur\_sortie : <= 0.0400176854152

sortie\_max : <= 1253.02359782

<octagon:

filtre2.c@12@5=

{ -5430.9504421651563462 <= P <= 39396.917979075267795,

info

/\* Analyzer launched at 2004/ 3/16 20:41:58

Command line was "/Volumes/PB\_Cousot\_PGP/Projet/absinthe2/analyzer.opt --exec-in main filtre2.c --export-invariant-stat filtre2.bin "

Launched by "cousot" on "PB-64-Patrick-COUSOT.local"

filtre2.c — line 12 — column 6 — character 193

## Benchmarks (Airbus A340 Primary Flight Control Software) <sup>7</sup>

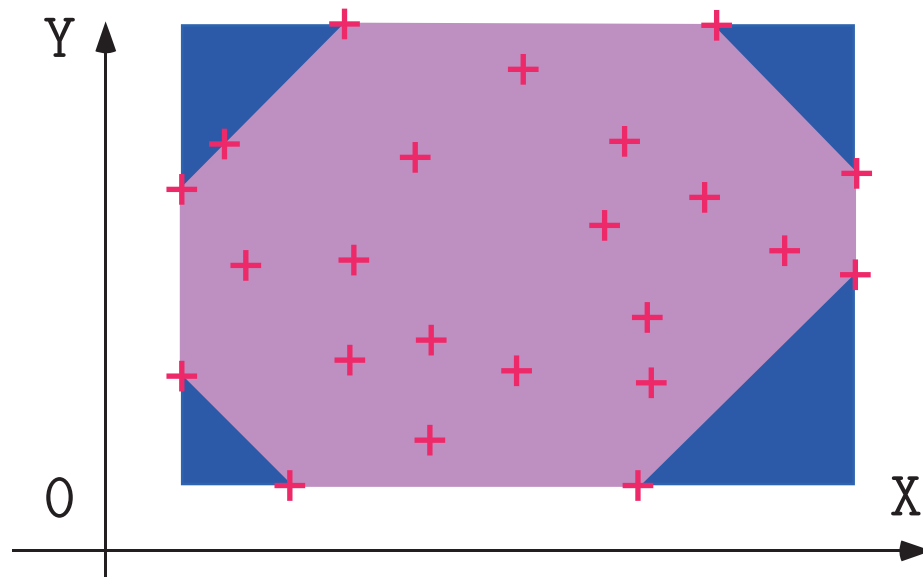
- 132,000 lines, 75,000 LOCs after preprocessing
- Comparative results (commercial software):
  - 4,200 (false?) alarms, 3.5 days;
- Our results:
  - 0 alarms,
  - 40mn on 2.8 GHz PC, 300 Megabytes
  - A world première in Nov. 2003!

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<sup>7</sup> “Flight Control and Guidance Unit” (FCGU) running on the “Flight Control Primary Computers” (FCPC). The A340 electrical flight control system is placed between the pilot’s controls (sidesticks, rudder pedals) and the control surfaces of the aircraft, whose movement they control and monitor.

# Examples of Abstractions in Astrée (15 mn)

# General-Purpose Abstract Domains: Intervals and Octagons



Intervals:

$$\begin{cases} 1 \leq x \leq 9 \\ 1 \leq y \leq 20 \end{cases}$$

Octagons [11]:

$$\begin{cases} 1 \leq x \leq 9 \\ x + y \leq 77 \\ 1 \leq y \leq 20 \\ x - y \leq 04 \end{cases}$$

**Difficulties:** many global variables, arrays (smashed or not), IEEE 754 floating-point arithmetic (in program and analyzer) [POPL '77, 11, 12]

# Floating-Point Computations

```
/* float-error.c */
int main () {
    float x, y, z, r;
    x = 1.000000019e+38;
    y = x + 1.0e21;
    z = x - 1.0e21;
    r = y - z;
    printf("%f\n", r);
}
% gcc float-error.c
% ./a.out
0.000000
```

```
/* double-error.c */
int main () {
    double x; float y, z, r;
    /* x = ldexp(1.,50)+ldexp(1.,26); */
    x = 1125899973951488.0;
    y = x + 1;
    z = x - 1;
    r = y - z;
    printf("%f\n", r);
}
% gcc double-error.c
% ./a.out
134217728.000000
```

$$(x + a) - (x - a) \neq 2a$$

# Floating-Point Computations

```
/* float-error.c */
int main () {
    float x, y, z, r;
    x = 1.000000019e+38;
    y = x + 1.0e21;
    z = x - 1.0e21;
    r = y - z;
    printf("%f\n", r);
}
% gcc float-error.c
% ./a.out
0.000000
```

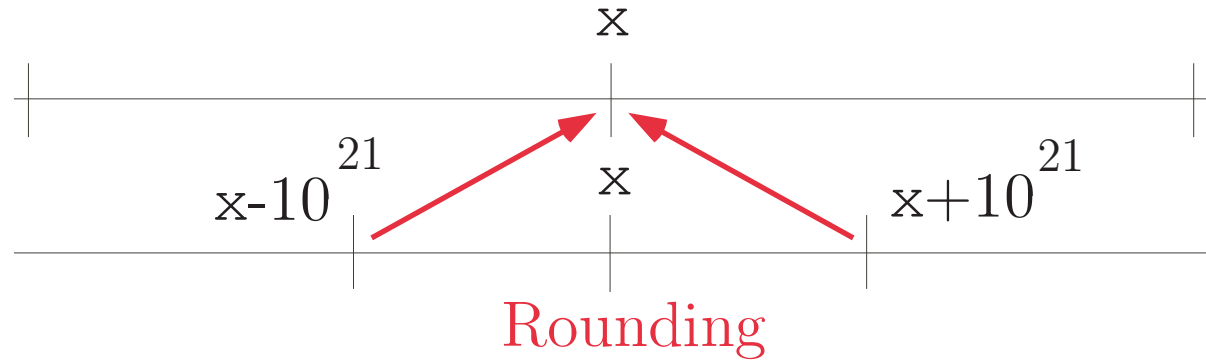
```
/* double-error.c */
int main () {
    double x; float y, z, r;
    /* x = ldexp(1.,50)+ldexp(1.,26); */
    x = 1125899973951487.0;
    y = x + 1;
    z = x - 1;
    r = y - z;
    printf("%f\n", r);
}
% gcc double-error.c
% ./a.out
0.000000
```

$$(x + a) - (x - a) \neq 2a$$

# Explanation of the huge rounding error

(1) Floats

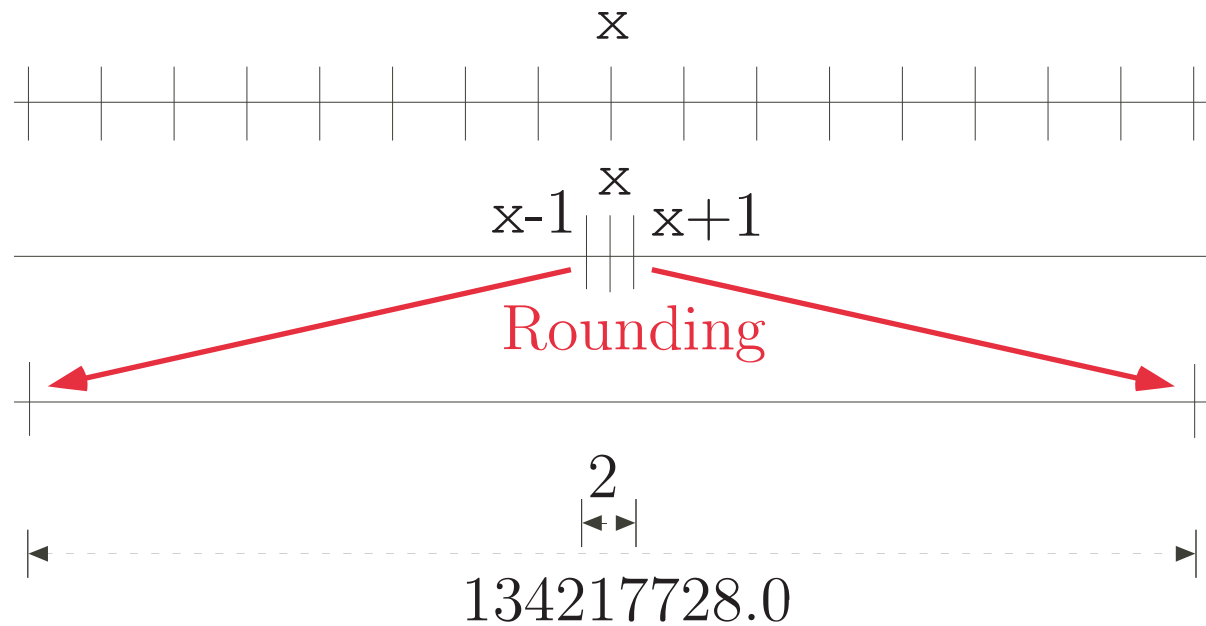
Reals



(2) Doubles

Reals

Floats



## Floating-point linearization [12, 13]

- Approximate arbitrary expressions in the form

$$[a_0, b_0] + \sum_k ([a_k, b_k] \times V_k)$$

- Example:

$Z = X - (0.25 * X)$  is linearized as

$$Z = ([0.749 \dots, 0.750 \dots] \times x) + (2.35 \dots 10^{-38} \times [-1, 1])$$

- Allows **simplification** even in the interval domain

if  $X \in [-1, 1]$ , we get  $|Z| \leq 0.750 \dots$  instead of  $|Z| \leq 1.25 \dots$

- Allows using a **relational abstract domain** (octagons)

- Example of good compromise between cost and precision



## Symbolic abstract domain [12, 13]

- Interval analysis: if  $x \in [a, b]$  and  $y \in [c, d]$  then  $x - y \in [a - d, b - c]$  so if  $x \in [0, 100]$  then  $x - x \in [-100, 100]$ !!!
- The symbolic abstract domain propagates the symbolic values of variables and performs simplifications;
- Must maintain the maximal possible rounding error for float computations (overestimated with intervals);

```
% cat -n x-x.c
```

```
1 void main () { int X, Y;  
2     __ASTREE_known_fact(((0 <= X) && (X <= 100)));  
3     Y = (X - X);  
4     __ASTREE_log_vars((Y));  
5 }
```

```
astree -exec-fn main -no-relational x-x.c
```

```
Call main@x-x.c:1:5-x-x.c:1:9:
```

```
<interval: Y in [-100, 100]>
```

```
astree -exec-fn main x-x.c
```

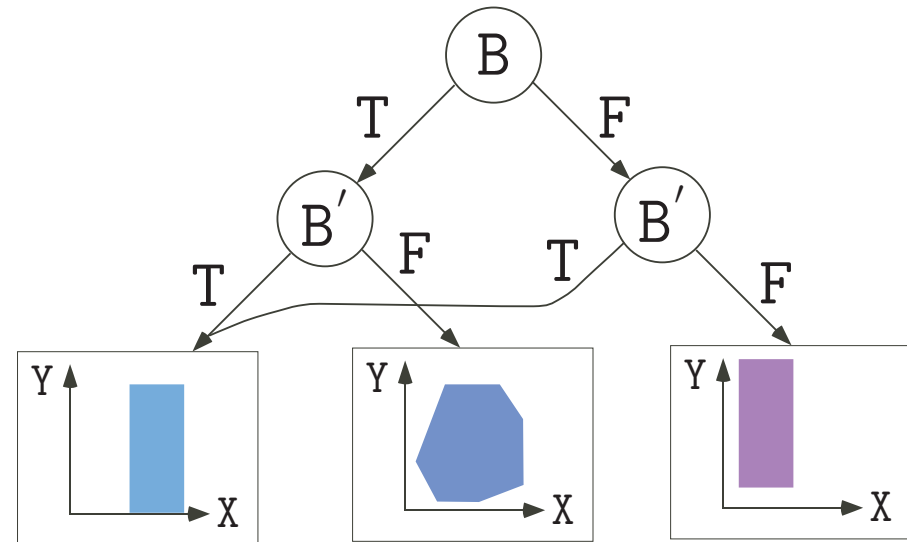
```
Call main@x-x.c:1:5-x-x.c:1:9:
```

```
<interval: Y in {0}> <symbolic: Y = (X -i X)>
```

# Boolean Relations for Boolean Control

## – Code Sample:

```
/* boolean.c */
typedef enum {F=0,T=1} BOOL;
BOOL B;
void main () {
    unsigned int X, Y;
    while (1) {
        ...
        B = (X == 0);
        ...
        if (!B) {
            Y = 1 / X;
        }
        ...
    }
}
```



The boolean relation abstract domain is parameterized by the height of the decision tree (an analyzer option) and the abstract domain at the leafs

# Control Partitionning for Case Analysis

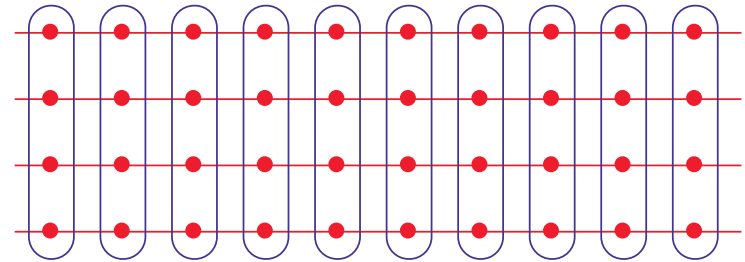
## —Code Sample:

```
/* trace_partitionning.c */
void main() {
  float t[5] = {-10.0, -10.0, 0.0, 10.0, 10.0};
  float c[4] = {0.0, 2.0, 2.0, 0.0};
  float d[4] = {-20.0, -20.0, 0.0, 20.0};
  float x, r;
  int i = 0;

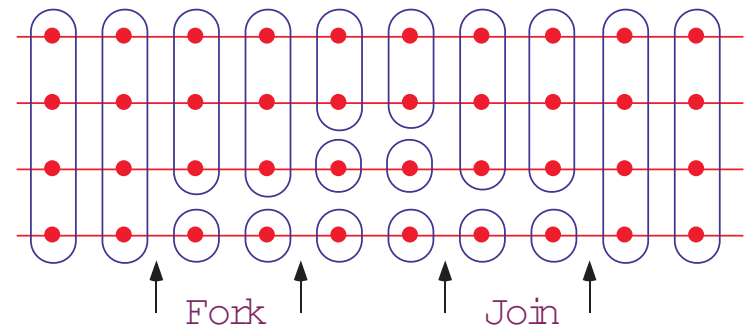
  ... found invariant  $-100 \leq x \leq 100$  ...

  while ((i < 3) && (x >= t[i+1])) {
    i = i + 1;
  }
  r = (x - t[i]) * c[i] + d[i];
}
```

## Control point partitionning:

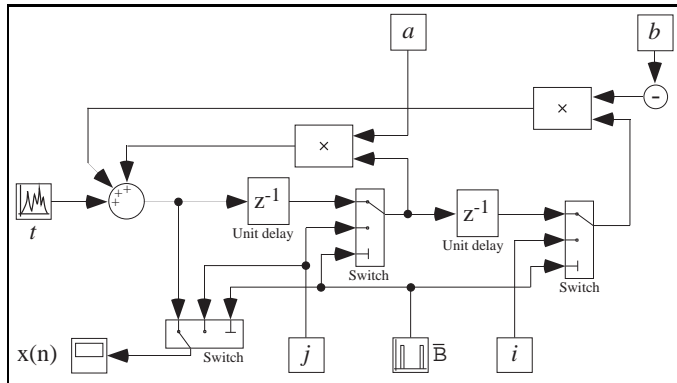


## Trace partitionning:



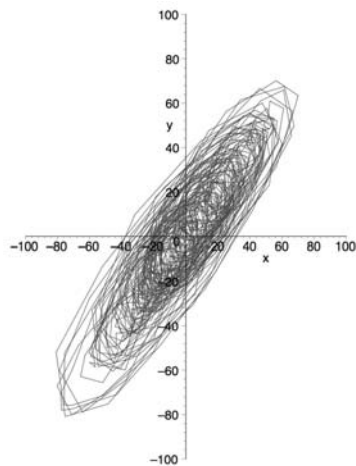
Delaying abstract unions in tests and loops is more precise for non-distributive abstract domains (and much less expensive than disjunctive completion).

## 2<sup>d</sup> Order Digital Filter:

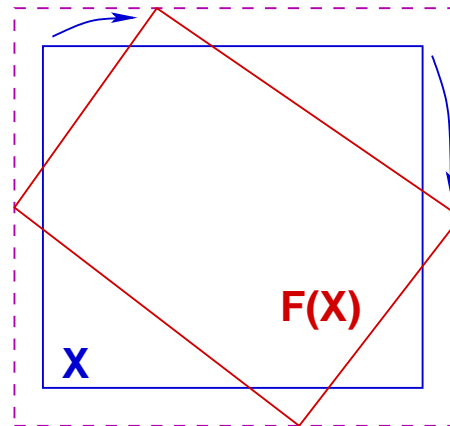


## Ellipsoid Abstract Domain for Filters

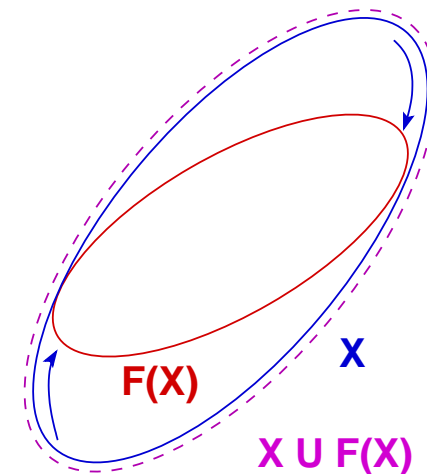
- Computes  $X_n = \begin{cases} \alpha X_{n-1} + \beta X_{n-2} + Y_n \\ I_n \end{cases}$
- The concrete computation is **bounded**, which must be proved in the abstract.
- There is **no stable interval or octagon**.
- The simplest stable surface is an **ellipsoid**.



execution trace



$X \cup F(X)$   
unstable interval



$X \cup F(X)$   
stable ellipsoid

## Filter Example [8]

```
typedef enum {FALSE = 0, TRUE = 1} BOOLEAN;
BOOLEAN INIT; float P, X;

void filter () {
    static float E[2], S[2];
    if (INIT) { S[0] = X; P = X; E[0] = X; }
    else { P = (((((0.5 * X) - (E[0] * 0.7)) + (E[1] * 0.4))
                + (S[0] * 1.5)) - (S[1] * 0.7)); }
    E[1] = E[0]; E[0] = X; S[1] = S[0]; S[0] = P;
    /* S[0], S[1] in [-1327.02698354, 1327.02698354] */
}

void main () { X = 0.2 * X + 5; INIT = TRUE;
    while (1) {
        X = 0.9 * X + 35; /* simulated filter input */
        filter (); INIT = FALSE; }
}
```

## Arithmetic-geometric progressions<sup>8</sup> [9]

– Abstract domain:  $(\mathbb{R}^+)^5$

– Concretization:

$$\gamma \in (\mathbb{R}^+)^5 \longmapsto \wp(\mathbb{N} \mapsto \mathbb{R})$$

$$\gamma(M, a, b, a', b') = \{f \mid \forall k \in \mathbb{N} : |f(k)| \leq (\lambda x. ax + b \circ (\lambda x. a'x + b')^k)(M)\}$$

i.e. any function bounded by the arithmetic-geometric progression.

---

<sup>8</sup> here in  $\mathbb{R}$

# Arithmetic-Geometric Progressions (Example 1)

```
% cat count.c
typedef enum {FALSE = 0, TRUE = 1} BOOLEAN;
volatile BOOLEAN I; int R; BOOLEAN T;
void main() {
    R = 0;
    while (TRUE) {
        __ASTREE_log_vars((R));
        if (I) { R = R + 1; }
        else { R = 0; }
        T = (R >= 100);
        __ASTREE_wait_for_clock(());
    }
}
```

← potential overflow!

```
% cat count.config
__ASTREE_volatile_input((I [0,1]));
__ASTREE_max_clock((3600000));
% astree -exec-fn main -config-sem count.config count.c|grep '|R|'
|R| <= 0. + clock *1. <= 3600001.
```

## Arithmetic-geometric progressions (Example 2)

```
% cat retro.c
typedef enum {FALSE=0, TRUE=1} BOOL;
BOOL FIRST;
volatile BOOL SWITCH;
volatile float E;
float P, X, A, B;

void dev( )
{ X=E;
  if (FIRST) { P = X; }
  else
    { P = (P - (((2.0 * P) - A) - B)
          * 4.491048e-03)); };
  B = A;
  if (SWITCH) {A = P;}
  else {A = X;}
}
```

```
void main()
{ FIRST = TRUE;
  while (TRUE) {
    dev( );
    FIRST = FALSE;
    __ASTREE_wait_for_clock();
  }}

% cat retro.config
__ASTREE_volatile_input((E [-15.0, 15.0]));
__ASTREE_volatile_input((SWITCH [0,1]));
__ASTREE_max_clock((3600000));

|P| <= (15.  + 5.87747175411e-39
/ 1.19209290217e-07) * (1
+ 1.19209290217e-07)^clock
- 5.87747175411e-39 /
1.19209290217e-07 <=
23.0393526881
```



## (Automatic) Parameterization

- All abstract domains of ASTRÉE are **parameterized**, e.g.
  - variable packing for octagones and decision trees,
  - partition/merge program points,
  - loop unrollings,
  - thresholds in widenings, ...;
- End-users can either **parameterize by hand** (analyzer options, directives in the code), or
- choose the **automatic parameterization** (default options, directives for pattern-matched predefined program schemata).

## The main loop invariant for the A340

A textual file over 4.5 Mb with

- 6,900 boolean interval assertions ( $x \in [0; 1]$ )
- 9,600 interval assertions ( $x \in [a; b]$ )
- 25,400 clock assertions ( $x + \text{clk} \in [a; b] \wedge x - \text{clk} \in [a; b]$ )
- 19,100 additive octagonal assertions ( $a \leq x + y \leq b$ )
- 19,200 subtractive octagonal assertions ( $a \leq x - y \leq b$ )
- 100 decision trees
- 60 ellipse invariants, etc ...

involving over 16,000 floating point constants (only 550 appearing in the program text)  $\times$  75,000 LOCs.

## Possible origins of imprecision and how to fix it

In case of false alarm, the imprecision can come from:

- **Abstract transformers** (not best possible)  $\longrightarrow$  improve algorithm;
- **Automatized parametrization** (e.g. variable packing)  $\longrightarrow$  improve pattern-matched program schemata;
- **Iteration strategy** for fixpoints  $\longrightarrow$  fix widening <sup>9</sup>;
- **Inexpressivity** i.e. indispensable local inductive invariant are inexpressible in the abstract  $\longrightarrow$  add a **new abstract domain** to the reduced product (e.g. filters).

---

<sup>9</sup> This can be very hard since at the limit only a precise infinite iteration might be able to compute the proper abstract invariant. In that case, it might be better to design a more refined abstract domain.

# Demo of Astrée (10mn)

# Conclusion (2mn)

## Conclusion

- Most applications of abstract interpretation **tolerate a small rate** (typically 5 to 15%) **of false alarms**:
  - Program transformation → do not optimize,
  - Typing → reject some correct programs, etc,
  - WCET analysis → overestimate;
- Some applications **require no false alarm** at all:
  - **Program verification**.
- **Theoretically possible** [SARA '00], **practically feasible** [PLDI '03]

---

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# The Future & Grand Challenges

Forthcoming (1/2 years):

- Industrialization

Future (5 years):

- Asynchronous concurrency (for less critical software)
- Functional properties (reactivity)

Grand challenge:

- Verification from specifications to machine code (verifying compiler)
- Verification of systems (quasi-synchrony, distribution)

# THE END, THANK YOU

More references at URL [www.di.ens.fr/~cousot](http://www.di.ens.fr/~cousot)  
[www.astree.ens.fr](http://www.astree.ens.fr).



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