Parameterized Refinement in Abstract-Interpretation-Based Static Analysis

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1. Abstract Interpretation

The Theory of Abstract Interpretation

– A theory of sound approximation of mathematical structures, in particular those involved in the behavior of computer systems
– Systematic derivation of sound methods and algorithms for approximating undecidable or highly complex problems in various areas of computer science
– Main practical application is on the safety and security of complex hardware and software computer systems
– Abstraction: extracting information from a system description that is relevant to proving a property

Applications of Abstract Interpretation

- Static Program Analysis [CC77], [CH78], [CC79] including Dataflow Analysis; [CC79], [CC00], Set-based Analysis [CC95], Predicate Abstraction [Cou03], ...
- Grammar Analysis and Parsing [CC03];
- Hierarchies of Semantics and Proof Methods [CC92b], [Cou02];
- Typing & Type Inference [Cou97];
- (Abstract) Model Checking [CC00];
- Program Transformation (including program optimization, partial evaluation, etc) [CC02];

Applications of Abstract Interpretation (Cont'd)

– Software Watermarking [CC04];
– Bisimulations [RT04, RT06];
– Language-based security [GM04];
– Semantics-based obfuscated malware detection [PCJD07].
– Databases [AGM93, BPC01, BS97]
– Computational biology [Dan07]
– Quantum computing [JP06, Per06]

All these techniques involve sound approximations that can be formalized by abstract interpretation

2. Astrée

Project Members

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[1] Nov. 2001 —— Nov. 2003.

[2] Nov. 2001 —— Aug. 2007.

3. Motivation

The Complexity of Software Design

- The design of complex software is difficult and economically critical
- Example (`www.designnews.com/article/CA6475332.html`):

  Boeing Confirms 787 Delay, Fasteners, Flight Control Software Code Blamed
  John Dodge, Editor-in-Chief – Design News, September 5, 2007

  > Boeing officials confirmed today that a fastener shortage and problems with flight control software have pushed "first flight" of the Boeing 787 Dreamliner to sometime between mid-November and mid-December.
  >
  > ...
  >
  > The software delays involve Honeywell Aerospace, which is responsible for flight control software. The work on this part of the 787 was simply underestimated, said Bair.

Tool-Based Software Design Methods

- New tool-based software design methods will have to emerge to face the unprecedented growth and complexification of critical software
- E.g. FCPC (Flight Control Primary Computer)

4. Problematics

Requirements of Verification Static Analysis [3]

A verifying static program analyzer must be (at least)

– useful (with respect to a correctness proof objective)

– sound (with respect to a concrete semantics)

– conclusive (with respect to a specification)

– non-intrusive (with respect to a system development practice)

– realistic (applicable in an weird industrial environment)

– scalable (to actual industrial code)

3 As opposed to bug-finding static/dynamic analysis

Making Static Analysis Easy (and Ultimately Useless)

Drop any of the requirements

– usefulness

– soundness

– conclusiveness

– non-intrusiveness

– realism

– scalability

Abstract Static Analysis

- Sound unprecise abstraction is mandatory to scale up, but
- Sound precise abstraction is mandatory to be conclusive.

Counter-example: brute force methods (like software model checking) simply fail.

Difficulties of Static Analysis

– Floyd/Naur proof method $\forall P \in \mathcal{L}$, $\forall S \in \mathcal{S}[\![P]\!]$, let $\mathcal{D}[\![P]\!] \supseteq \mathcal{S}[\![P]\!]$, and $F[\![P]\!] \in \mathcal{D}[\![P]\!] \mapsto \mathcal{D}[\![P]\!]$:

$$\mathsf{lfp}^{\subseteq}\, F[\![P]\!] \subseteq S \Leftrightarrow \exists I \in \mathcal{D}[\![P]\!] : F[\![P]\!](I) \subseteq I \wedge I \subseteq S$$

– Abstraction $\langle \mathcal{D}[\![P]\!], \subseteq\rangle \underset{\alpha}{\overset{\gamma}{\longleftrightarrow}} \langle \alpha(\mathcal{D}[\![P]\!]), \sqsubseteq\rangle$:

$$\Leftarrow \exists \bar{I} \in \alpha(\mathcal{D}[\![P]\!]) : \mathsf{lfp}^{\sqsubseteq}\, \alpha \circ F[\![P]\!] \circ \gamma \sqsubseteq \bar{I} \wedge \gamma(\bar{I}) \subseteq S$$

– Main difficulty: in general, there is no inductive invariant $\bar{I}$ in the abstract:

$$\forall \bar{I} \in \alpha(\mathcal{D}[\![P]\!]) : \alpha \circ F[\![P]\!] \circ \gamma(\bar{I}) \not\sqsubseteq \bar{I}$$

5. Astrée Fundamental Choices

Language

Choice of the Language $\forall P \in \mathcal{L}$

Typical choices:

– Deductive methods and model checking: $\mathcal{L} = \{P\}$, for one (model of a) program

– Data flow analysis: $\mathcal{L}$ = C, C++, ..., one programming language

– ASTRÉE: the family of control/command C codes automatically generated from a synchronous specification (SAO/SCADE)[4]

[4] Outside this scope, ASTRÉE is likely not be useful, conclusive, non-intrusive, realistic, and/or scalable!

Programs analysed by ASTRÉE

- Application Domain: large safety critical embedded real-time synchronous software for non-linear control of very complex control/command systems.
- C programs:
  - with
    - basic numeric datatypes, structures and arrays
    - pointers (including on functions),
    - floating point computations
    - tests, loops and function calls
    - limited branching (forward `goto`, `break`, `continue`)

– with (cont'd)
    - `union` [Min06a]
    - pointer arithmetics & casts [Min06a]

– without
    - dynamic memory allocation
    - recursive function calls
    - unstructured/backward branching
    - conflicting side effects
    - C libraries, system calls (parallelism)

Such limitations are quite common for embedded safety-critical software.

The Class of Considered Periodic Synchronous Programs

```
declare volatile input, state and output variables;
initialize state and output variables;
loop forever
    - read volatile input variables,
    - compute output and state variables,
    - write to output variables;
  __ASTREE_wait_for_clock ();
end loop
```

Task scheduling is static:

- Requirements: the only interrupts are clock ticks;
- Execution time of loop body less than a clock tick, as verified by the aiT WCET Analyzers [FHL+01].

Concrete Semantics

Choice of the Concrete Semantics $\mathcal{D}[\![P]\!], F[\![P]\!], P \in \mathcal{L}$

Set of prefix-closed traces for a transition relation defined by

- the international norm of C (ISO/IEC 9899:1999)
- *restricted by* implementation-specific behaviors depending upon the machine and compiler (e.g. representation and size of integers, IEEE 754-1985 norm for floats and doubles)
- *restricted by* user-defined programming guidelines (such as no modular arithmetic for signed integers, even though this might be the hardware choice)
- *restricted by* program specific user requirements (e.g. assert, execution stops on first runtime error [5])

[5] semantics of C unclear after an error, equivalent if no alarm

The Semantics of C is Hard (Ex. 1: Floats)

"*Put* x *in* [m, M] *modulo* (M − m)":

```
x' = x - (int) ((x-m)/(M-m))*(M-m);
```

– The programmer thinks x' $\in$ [m, M]

– But with M $=$ 4095, m $= -$M, IEEE double precision, and x is the greatest float strictly less than M, then x' $=$ m $- \epsilon$ (ϵ very small).

Floats are not real.

ASTRÉE has an abstraction to handle this modulo problem (J. Feret, unpublished)

The Semantics of C is Hard (Ex. 2: Runtime Errors)

What is the effect of out-of-bounds array indexing?

```
% cat unpredictable.c
#include <stdio.h>
int main () { int n, T[1];
 n = 2147483647;
 printf("n = %i, T[n] = %i\n", n, T[n]);
}
```

Yields different results on different machines:

```
n = 2147483647, T[n] = 2147483647     Macintosh PPC
n = 2147483647, T[n] = -1208492044    Macintosh Intel
n = 2147483647, T[n] = -135294988     PC Intel 32 bits
Bus error                             PC Intel 64 bits
```

Execution stops after a runtime error with unpredictable results [6].

[6] Equivalent semantics if no alarm.

Different Classes of Run-time Errors

1. Errors terminating the execution [7]. ASTRÉE warns and continues by taking into account only the executions that did not trigger the error.
2. Errors not terminating the execution with predictable outcome [8]. ASTRÉE warns and continues with worst-case assumptions.
3. Errors not terminating the execution with <u>unpredictable</u> outcome [9]. ASTRÉE warns and continues by taking into account only the executions that did not trigger the error.

$\Rightarrow$ ASTRÉE is sound with respect to C standard, unsound with respect to C implementation, unless no false alarm.

[7] floating-point exceptions e.g. (invalid operations, overflows, NaN, etc.) when traps are activated

[8] e.g. overflows over signed integers resulting in some signed integer.

[9] e.g. memory corruptions.

Why prefix-closed traces?

– Burstall's proof method (using traces) is equivalent to Floyd method (with set of states i.e. invariant) but much easier

```
while (x > 1) {
  if (odd(x)) { x = x + 1; }
  else { x = x / 2; }
}
```

– You can always later abstract sets of (prefix-closed) traces into sets of states

Realistic Semantics: Modulo Arithmetics

In C:

```
% cat -n modulo-c.c
     1 #include <stdio.h>
     2 int main () {
     3 int x,y;
     4 x = -2147483647 / -1;
     5 y = ((-x) -1) / -1;
     6 printf("x = %i, y = %i\n",x,y);
     7 }
     8

% gcc modulo-c.c
% ./a.out
x = 2147483647, y = -2147483648
```

Static Analysis with ASTRÉE

```
% cat -n modulo.c
     1 int main () {
     2 int x,y;
     3 x = -2147483647 / -1;
     4 y = ((-x) -1) / -1;
     5 __ASTREE_log_vars((x,y));
     6 }
     7
% astree -exec-fn main -unroll 0 modulo.c\
  |& egrep -A 1 "(<integers)|(WARN)"
modulo.c:4.4-18::[call#main@1:]: WARN: signed int arithmetic range
  {2147483648} not included in [-2147483648, 2147483647]
  <integers (intv+cong+bitfield+set): y in [-2147483648, 2147483647] /\ Top
   x in {2147483647} /\ {2147483647} >
```

ASTRÉE signals the overflow and goes on with an unkown value.

Realistic Semantics: Floats

```
% cat -n scale.c
 1 int main () {
 2  float x; x = 0.70000001;
 3  while (1) {
 4   x = x / 3.0;
 5   x = x * 3.0;
 6   __ASTREE_log_vars((x));
 7   __ASTREE_wait_for_clock(());
 8  }
 9 }
```

```
% gcc scale.c
% ./a.out
x = 0.699999988079071
```

```
% cat scale.config
 __ASTREE_max_clock((1000000000));
% astree -exec-fn main -config-sem scale.config -unroll 0 scale.c\
 |& grep "x in" | tail -1
direct = <float-interval: x in [0.69999986887, 0.700000047684] >
%
```

Example of accumulation of small rounding errors

```
% cat -n rounding-c.c
 1  #include <stdio.h>
 2  int main () {
 3   int i; double x; x = 0.0;
 4   for (i=1; i<=1000000000; i++) {
 5    x = x + 1.0/10.0;
 6   }
 7  printf("x = %f\n", x);
 8  }
% gcc rounding-c.c
% ./a.out
x = 99999998.745418
%
```

since $(0.1)_{10} = (0.0001100110011001100\ldots)_2$

Static analysis with ASTRÉE

```
% cat -n rounding.c
     1  int main () {
     2   double x; x = 0.0;
     3   while (1) {
     4    x = x + 1.0/10.0;
     5    __ASTREE_log_vars((x));
     6    __ASTREE_wait_for_clock(());
     7   }
     8  }
% cat rounding.config
 __ASTREE_max_clock((1000000000));
% astree -exec-fn main -config-sem rounding.config -unroll 0 rounding.c\
 |& egrep "(x in)|(\|x\|)|(WARN)" | tail -2
direct = <float-interval: x in [0.1, 200000040.938] >
  |x| <= 1.*((0. + 0.1/(1.-1))*(1.)^clock - 0.1/(1.-1)) + 0.1
      <= 200000040.938
```

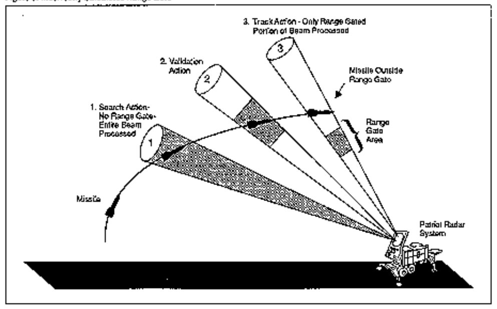
The Patriot missile failure

- "On February 25th, 1991, a Patriot missile … failed to track and intercept an incoming Scud [(*)]."
- The software failure was due to accumulated rounding error [(†)]

(*) This Scud subsequently hit an Army barracks, killing 28 Americans.

(†)
- "Time is kept continuously by the system's internal clock in tenths of seconds"
- "The system had been in operation for over 100 consecutive hours"
- "Because the system had been on so long, the resulting inaccuracy in the time calculation caused the range gate to shift so much that the system could not track the incoming Scud"

Figure 5: Incorrectly Calculated Range Gate


3. Track Action - Only Range Gated Portion of Beam Processed
2. Validation Action
Missile Outside Range Gate
1. Search Action- No Range Gate- Entire Beam Processed
Range Gate Area
Missile
Patriot Radar System
1
2
3

Specification

Choice of the Specification Language $\mathcal{S}[\![P]\!] \subseteq \mathcal{D}[\![P]\!]$

– By the choice of $\mathcal{D}[\![P]\!]$, $\mathcal{S}[\![P]\!]$ can be anything specifying prefix-closed sets of traces (automata, garmmars, synchronous languages, temporal logic, etc.)

– but

  - Intrusive (who will write the formal specification?)
  - Costly (e.g. to check $\gamma(\bar{I}) \subseteq \mathcal{S}$)

– In Astrée, implicit specification (absence of runtime error) automatically computed from the program text

Implicit Specification: Absence of Runtime Errors

- No violation of the norm of C (e.g. array index out of bounds, division by zero)
- No implementation-specific undefined behaviors (e.g. maximum short integer is 32767, NaN)
- No violation of the programming guidelines (e.g. static variables cannot be assumed to be initialized to 0)
- No violation of the programmer assertions (must all be statically verified).

Example: Dichotomy Search I

```
% cat dichotomy.c
int main () {
   int R[100], X; short lwb, upb, m;
   lwb = 0; upb = 99;
   while (lwb <= upb) {
      m = upb + lwb;
      m = m » 1;
      if (X == R[m]) { upb = m; lwb = m+1; }
      else if (X < R[m]) { upb = m - 1; }
      else { lwb = m + 1; }
   }
   __ASTREE_log_vars((m));
}
% astree -exec-fn main dichotomy.c |& egrep "(WARN)|(m in)"
direct = <integers (intv+cong+bitfield+set): m in [0, 99] /\ Top >
%
```

Example: Dichotomy Search II

```
% diff dichotomy.c dichotomy-bug.c
2,3c2,3
<     int R[100], X; short lwb, upb, m;
<     lwb = 0; upb = 99;
--
>     int R[30000], X; short lwb, upb, m;
>     lwb = 0; upb = 29999;
%
% astree -exec-fn main dichotomy-bug.c |& egrep "WARN" | head -n2
dichotomy-bug.c:5.6-19::[call#main@1:loop@4=2:]: WARN: implicit signed int->signed
short conversion range [14998, 44999] not included in [-32768, 32767]
dichotomy-bug.c:7.15-19::[call#main@1:loop@4=2:]: WARN: invalid dereference:
dereferencing 4 byte(s) at offset(s) [0;4294967295] may overflow the variable R of
byte-size 120000 or mis-aligned pointer (1Z+0) may not a multiple of 4
%
```

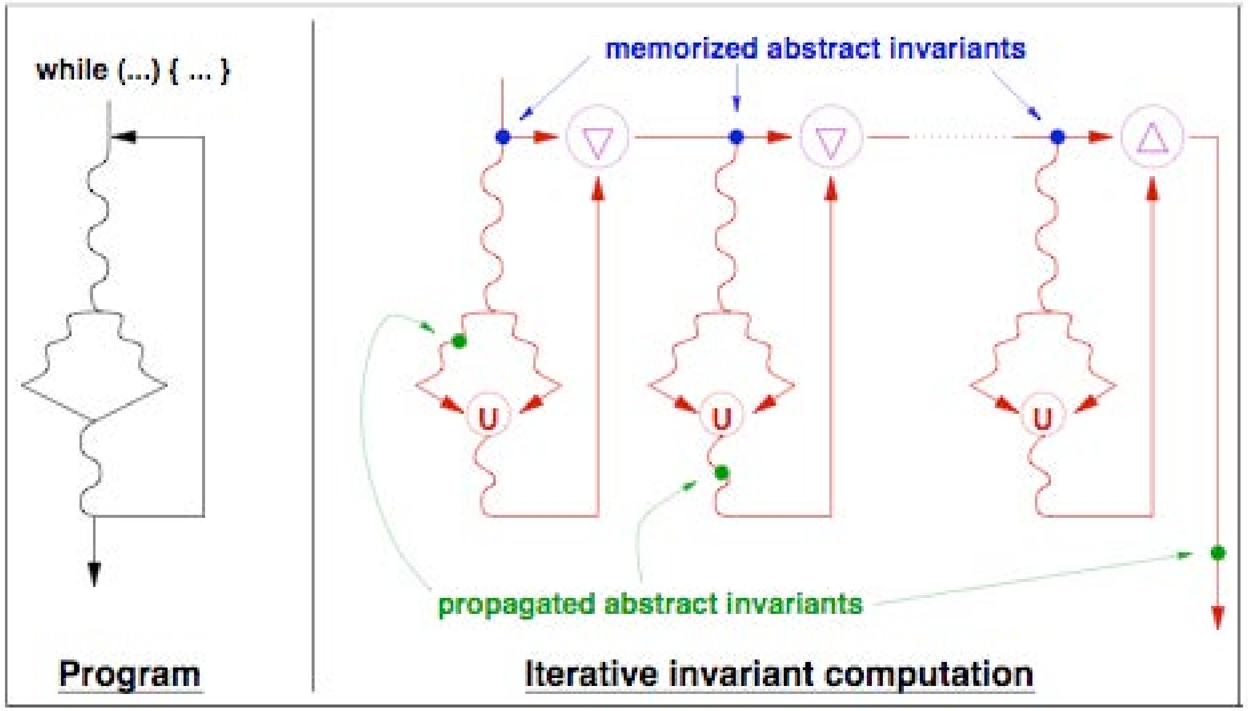
ASTRÉE finds bugs in programs based on algorithms which have been formally proved correct.

Iterator

Choice of the Abstract Iterator $\mathsf{lfp}^{\sqsubseteq} \alpha \circ F[\![P]\!] \circ \gamma$

– Control graph (would loose useful information), or
– ASTRÉE:
  - isomorphic projection of the set of prefix-closed traces to contexts = call stack + program point
  - by structural induction on the abstract syntax tree
    · initialize (empty traces at program entry point)
    · given a prefix-closed set of traces up to the prececessor contexts, extend each trace by one computation step/transition to the next contexts
    · repeat with widening/narrowing until stabilization

Abstract Iterator $\mathsf{lfp}^{\sqsubseteq}\ \alpha \circ F[\![P]\!] \circ \gamma$


while (...) { ... }
memorized abstract invariants
propagated abstract invariants
Program
Iterative invariant computation

Abstraction

Bad ideas on Abstraction

- Abstract exclusively to finite domains (provably worse than infinite domain plus widening [CC92a])
- Uniform abstractions (same abstraction everywhere, everytime, like in dataflow analysis)
- Keep as much disjunctions as possible (e.g. predicate abstraction, abstraction is all about "how to get rid of disjunctions"!)
- Cascaded abstractions, one after the other (provably worse than reduced product)
- . . .

Choice of the Abstraction $\langle\alpha, \gamma\rangle$

– Extremely complex $\rightarrow$ divide and conquer using a reduced product [CC79]

– $\alpha = \rho_{1,\ldots,n}(\alpha_i, \alpha_p, \alpha_m, \alpha_1, \ldots, \alpha_i(\alpha_j) \ldots, \alpha_n)$, where
  - $\rho_{1,\ldots,n}$ is the reduction,
  - α_i is the trace projection (to each context = call stack + program point),
  - α_p is the trace abstraction (trace partitionning [MR05]),
  - α_m is the state abstraction (memory model [Min06a]),
  - $\alpha_1, \ldots, \alpha_n$ are the basic abstractions or abstraction functors on abstract variables $\mathcal{X}$ (mutable, remanent)

ASTRÉE's Architecture

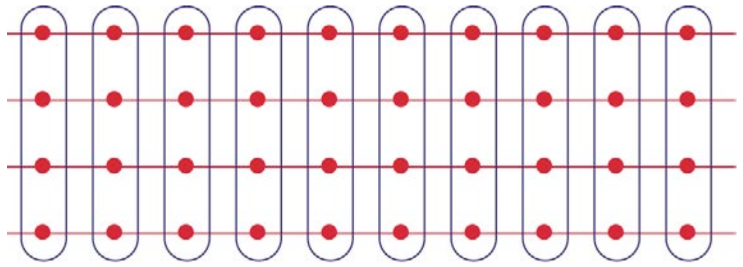
$$\begin{tikzcd}
\text{C-preprocessor} \arrow[d, leftrightarrow] \\
\text{C99 parser} \arrow[d, leftrightarrow] \\
\text{Link editor} \arrow[d, leftrightarrow] \\
\text{Intermediate code generation and typing} \arrow[d, leftrightarrow] \\
\text{Constant propagation and simplification} \arrow[d, leftrightarrow] \\
\text{Local and global dependence analysis} \arrow[d, leftrightarrow] \\
\text{Abstract Interpreter}
\end{tikzcd}$$

The Abstract Interpreter

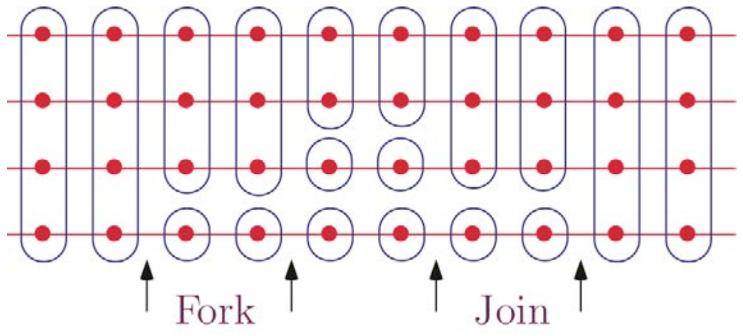
$$\begin{tikzcd}
 & & \text{Iterator } \alpha_i \arrow[d, leftrightarrow] & \\
 & & \text{Trace partitionning } \alpha_p \arrow[d, leftrightarrow] & \\
 & & \text{Memory model and aliases } \alpha_m \arrow[d, leftrightarrow] & \\
 & & \text{Reduced product of numerical abstract domains } \rho_{1,\ldots,n} \arrow[dll, leftrightarrow] \arrow[dl, leftrightarrow] \arrow[d, leftrightarrow] \arrow[dr, leftrightarrow] & \\
\text{Intervals } \alpha_1 & \text{Octagons } \alpha_2 & \text{Decision trees } \alpha_3(\alpha_1) \arrow[d, leftrightarrow] & \ldots \\
 & & \text{intervals } \alpha_1 &
\end{tikzcd}$$

Trace Partitionning Abstraction α_p [MR05]

State-based partitionning at control points:



Trace-based partitionning at control points:



Delaying abstract unions in tests and loops is more precise for non-distributive abstract domains (and much less expensive than disjunctive completion).

Trace Partitioning

Principle:

- Semantic equivalence:

```
if (B) { C1 } else { C2 }; C3
                ⇓
if (B) { C1; C3 } else { C2; C3 };
```

- More precise in the abstract: concrete execution paths are merged later.

Application:

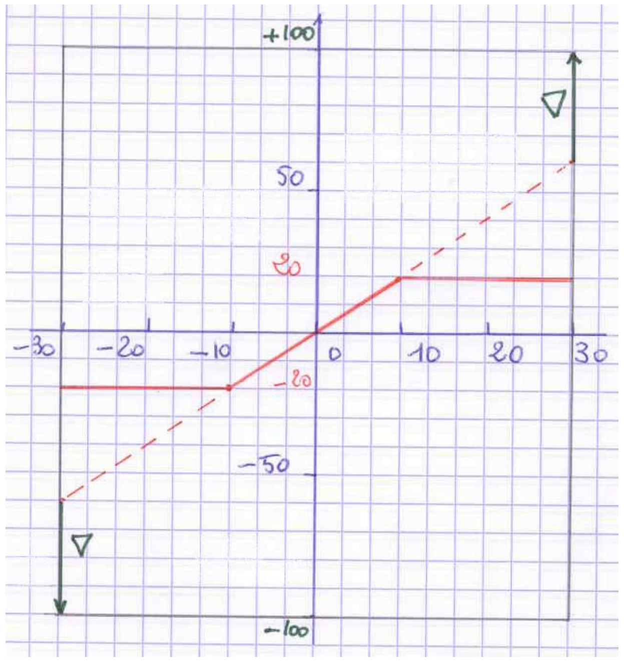
```
if (B)
  { X=0; Y=1; }
else
  { X=1; Y=0; }
R = 1 / (X-Y);
```

cannot result in a division by zero

Case analysis with loop unrolling

– Code Sample:

```
/* trace_partitionning.c */
void main() {
  float t[5] = {-10.0, -10.0, 0.0, 10.0, 10.0};
  float c[4] = {0.0, 2.0, 2.0, 0.0};
  float d[4] = {-20.0, -20.0, 0.0, 20.0};
  float x, r;
  int i = 0;
  __ASTREE_known_fact(((-30.0 <= x) && (x <= 30.0)));
  while ((i < 3) && (x >= t[i+1])) {
    i = i + 1;
  }
  r = (x - t[i]) * c[i] + d[i];
  __ASTREE_log_vars((r));
}
```

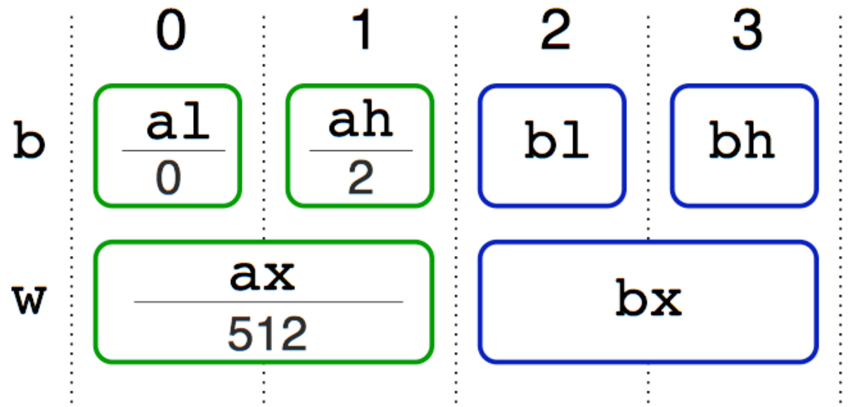


```
% astree -exec-fn main -no-trace -no-relational trace-partitioning.c |& egrep "(WARN)|(r in)"
direct = <float-interval: r in [-20, 20] >
%
% astree -exec-fn main -no-partition -no-trace -no-relational trace-partitioning.c \
  |& egrep "(WARN)|(r in)"
direct = <float-interval: r in [-100, 100] >
%
```

State Abstraction (Memory Model) α_m [Min06a]

The union type, pointer arithmetics and pointer transtyping is handled by allowing aliasing at the byte level [1]:

```
union {
  struct { uint8  al,ah,bl,bh; } b;
  struct { uint16 ax,bx; } w;
} r;
r.w.ax = 0; r.b.ah = 2;
```



– A box (auxiliary variable) in $\mathcal{X}$ for each offset and each scalar type

– intersection semantics for overlapping boxes

Reference

[1] A. Miné. Field-Sensitive Value Analysis of Embedded C Programs with Union Types and Pointer Arithmetics. In *LCTES '2006*, pp. 54–63, June 2006, ACM Press.

Maximal Abstraction α_1

– The verification condition (ultimate phase of ASTRÉE) includes the test

$$\exists \bar{I} \in \alpha(\mathcal{D}[\![P]\!]) : \ldots \wedge \gamma(\bar{I}) \subseteq S$$

(in the abstract) and so the abstract domain $\alpha(\mathcal{D}[\![P]\!])$ should contain all possible $S \in \mathcal{S}[\![P]\!]$

– In ASTRÉE $\mathcal{S}[\![P]\!]$ is the abstract domain of intervals [CC76] (plus $\neq 0$)

Choice of abstractions $\alpha_2, \ldots, \alpha_n$ in ASTRÉE

The other abstract domains $\alpha_2, \ldots, \alpha_n$ can be chosen thanks to parameters when launching ASTRÉE, for example:

```
/* Launching the forward abstract interpreter */
/* Domains:  Guard domain, and Boolean packs (based on Absolute
value equality relations, and Symbolic constant propagation
(max_depth=20), and Linearization, and Integer intervals, and
congruences, and bitfields, and finite integer sets, and Float
intervals), and Octagons, and High_passband_domain(10), and
Second_order_filter_domain (with real roots)(10), and
Second_order_filter_domain (with complex roots)(10), and
Arithmetico-geometric series, and new clock, and Dependencies
(static), and Equality relations, and Modulo relations, and
Symbolic constant propagation (max_depth=20), and Linearization,
and Integer intervals, and congruences, and bitfields, and
finite integer sets, and Float intervals.  */
```

Reduction [CC79, CCF$^+$08]

Example: reduction of intervals [CC76] by simple congruences [Gra89]

```
% cat -n congruence.c
     1 /* congruence.c */
     2 int main()
     3 { int X;
     4   X = 0;
     5   while (X <= 128)
     6     { X = X + 4; };
     7   __ASTREE_log_vars((X));
     8 }
% astree congruence.c -no-relational -exec-fn main |& egrep "(launched)|(WA
direct = <integers (intv+cong+bitfield+set): X in {132} >
```

Intervals : $X \in [129, 132]$ + congruences : $X = 0 \mod 4 \Longrightarrow X \in \{132\}$.

Refinement Strategies

Cost/Precision Ratio Adjustment

– We prefer coarse abstractions (for scalability, this excludes e.g. polyedra)
– We anticipate the need for necessary refinements (for precision)

Abstraction/Refinement

– Parameterized refinement: choose abstractions which precision can be refined/coarsened thanks to
    - manual parametrization
    - manual directives
    - automated directives
– Unexpected refinement: add a new abstract domain (and reduction)

Parameterized Refinement

Termination

SLAM uses CEGAR and does not terminate[10] on

```
% cat slam.c
int main() { int x, y;
  x = 0; y = 0;
  while (x < 2147483647)
     { x = x + 1; y = y + 1; }
  __ASTREE_assert((x == y));
}
```

whereas ASTRÉE uses widening/narrowing-based extrapolation techniques to prove the assertion

```
% astree -exec-fn main slam.c |& egrep "WARN"
%
```

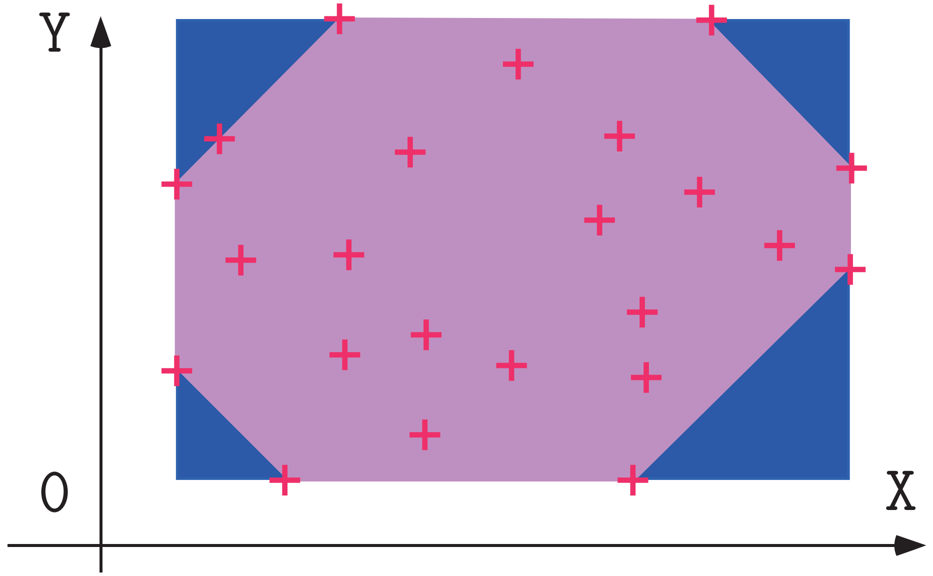
[10] CEGAR cannot generate the invariant `y = x - 1` so produces all counter examples $x = i + 1 \wedge y = i$, $i = 0, 1, 2, 3, \ldots$

Parameterized Abstraction e.g. Array Smashing

`--smash-threshold` n (400 by default)

smash elements of arrays of size $> n$, otherwise individualize array elements (each handled as a simple variable).

Parameterized Abstract Domains: Intervals and Octagons



Intervals [CC76]:

$$\begin{cases} 1 \leq x \leq 9 \\ 1 \leq y \leq 20 \end{cases}$$

Octagons [Min01]:

$$\begin{cases} 1 \leq x \leq 9 \\ x + y \leq 77 \\ 1 \leq y \leq 20 \\ x - y \leq 07 \end{cases}$$

Difficulties: many global variables, arrays (smashed or not), IEEE 754 floating-point arithmetic (in program and analyzer) [CC77, Min01, Min04a]

Parameterized Widening e.g. Intervals

Thresholds for integer widening:

```
let widening_sequence =
  [ of_int 0; of_int 1; of_int 2; of_int 3; of_int 4; of_int 5;
    of_int 32767; of_int 32768; of_int 65535; of_int 65536;
    of_string "2147483647"; of_string "2147483648"; of_string "4294967295" ]
```

Thresholds for float widening:

```
let widening_sequence =
  [ neg 1.;neg 0.15;neg 0.1;neg 0.01;neg 0.001;neg 0.000001;0.;0.000001;0.001;0.01;1.;
    1e1;1e2;1e3;1e4; 70000.25;1e5;1e6;1.5e6;2e6;2.5e6;3e6;3.5e6;4e6;4.5e6;5e6;5.5e6;
    6e6;6.5e6;7e6;7.5e6;8e6;8.5e6;9e6;9.5e6;1e7; 10000020.; 1.5e7;2e7;2.5e7;3e7;3.5e7;
    4e7;4.5e7;5e7;5.5e7;6e7;6.5e7;7e7;7.5e7;8e7;8.5e7;9e7;9.5e7;1e8;1e9;1e10;1e11;1e12;
    1e15;1e18;1e20;1e22;1e25;1e28;1e30;1e32]
```

Delayed widenings: `--forced-union-iterations-at-beginning` n (2 by default)

Enforced widenings: `--forced-widening-iterations-after` n (250 by default), ...), etc.

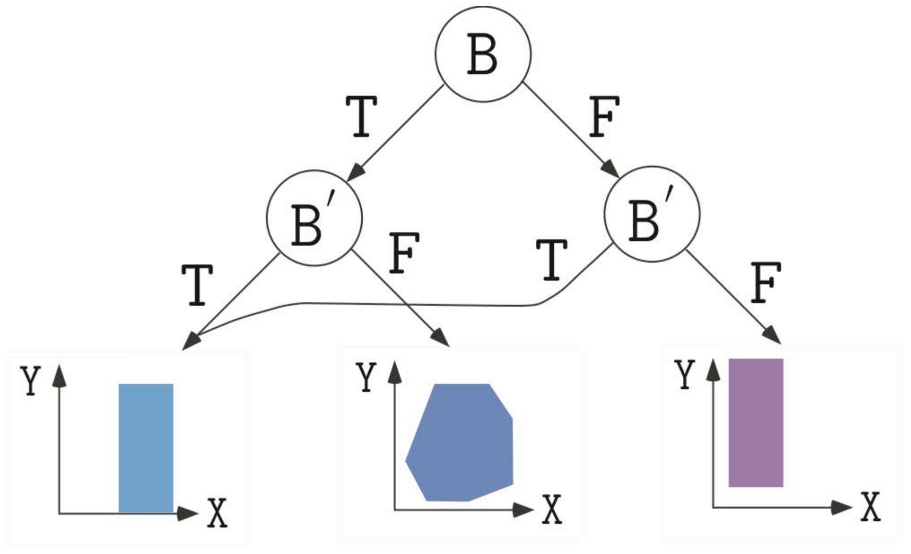
Parameterized Octagons

- Using octagons on all numerical variables would not scale up
- The analysis is parameterized by "packs of variables" stating which independent subsets of the variables should be related (everywhere, at which program points, in which context, ...)
- Automatic packing by another analysis (e.g. pre-analysis, on the fly, etc). In ASTRÉE pre-analysis at the block level.
- Parameters can modify the choice of packs globally (e.g. `--fewer-oct`: no packs at the function level, `--max-array-size-in-octagons` n: unsmashed array elements of size $> n$ don't go to octagons packs)
- Directives can modify the choice of packs locally: (`__ASTREE_octagon_pack((`V_1,...,V_n`));`)

Decision Trees for Boolean Control

– Code Sample:

```
/* boolean.c */
typedef enum {F=0,T=1} BOOL;
BOOL B;
void main () {
  unsigned int X, Y;
  while (1) {
    ...
    B = (X == 0);
    ...
    if (!B) {
      Y = 1 / X;
    }
    ...
  }
}
```



The boolean relation abstract domain is parameterized by the height of the decision tree (an analyzer option) and the abstract domain at the leafs

Parameterized Decision Trees

– Using decision trees on all variables would not scale up
– The analysis is parameterized by "packs of variables" stating which booleans go in nodes and numerical variables in leaves
– Automatic packing by a simple dependence analysis: Candidates for packing in a decision tree are the boolean variables to which a boolean expression is assigned or which are involved in a test as well as the non-volatile and non-constant variables which depend directly or indirectly on such a boolean
– Parameters can modify the choice of packs globally (e.g. `--max-bool-var` n, $n = 3$ by default)
– Directives can modify the choice of packs locally to state which boolean variables to put in internals nodes and numerical variables to put in abstract domains at the leaves `__ASTREE_boolean_pack(`
– TODO: partition on small values (other than booleans)

Example of directive

```
% cat repeat1.c
typedef enum {FALSE=0,TRUE=1} BOOL;
int main () {
  int x = 100; BOOL b = TRUE;

  while (b) {
    x = x - 1;
    b = (x > 0);
  }
}
% astree -exec-fn main repeat1.c |& egrep "WARN"
repeat1.c:5.8-13::[call#main@2:loop@4>=4:]: WARN: signed int arithmetic
range [-2147483649, 2147483646] not included in [-2147483648, 2147483647]
%
```

Example of directive (Cont'd)

```
% cat repeat2.c
typedef enum {FALSE=0,TRUE=1} BOOL;
int main () {
  int x = 100; BOOL b = TRUE;
  __ASTREE_boolean_pack((b,x));
  while (b) {
    x = x - 1;
    b = (x > 0);
  }
}
% astree -exec-fn main repeat2.c |& egrep "WARN"
%
```

The insertion of this directive could be automated in ASTRÉE (if the considered family of programs has "repeat" loops).

Parameterized Loop Partitionning

- No loop unrolling a priori
- Unrolling is controlled by parameters `--unroll` u and directives

`__ASTREE_partition_control((`p`)) while (B) {C}; C'; __ASTREE_partition_merge(());`

so that the analysis is semantically equivalent to:

$$\bigcup_{i=0}^{p} \left((\mathtt{B};\mathtt{C})^i; \neg\mathtt{B}; \mathtt{C}' \right)$$

partitionning of the first p iterations

$$\cup \left(\bigcup_{i=0}^{p} (\mathtt{B};\mathtt{C})^i \right); \left(\bigcup_{i=p+1}^{u} \left((\mathtt{B};\mathtt{C})^i; \neg\mathtt{B} \right) \right); \mathtt{C}'$$

semantic unrolling of the next $u - p$ iterations

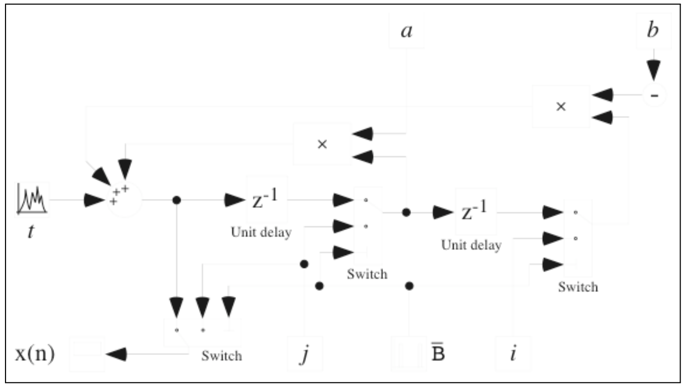
$$\cup \quad \left(\bigcup_{i=0}^{u} (\mathtt{B};\mathtt{C})^i \right); \left(\mathop{\mathit{\Delta}}_{i=u+1}^{+\infty} (\mathtt{B};\mathtt{C})^i \right); \neg\mathtt{B}; \mathtt{C}'$$

next iterations with widening

Unexpected Refinement

Ellipsoid Abstract Domain for Filters

2^d Order Digital Filter:

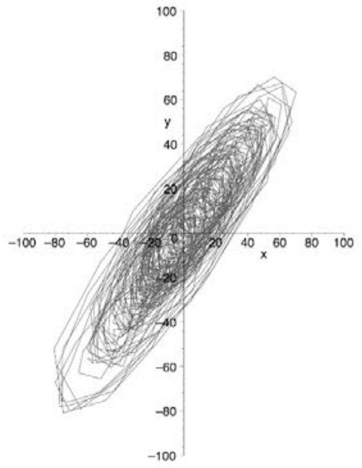


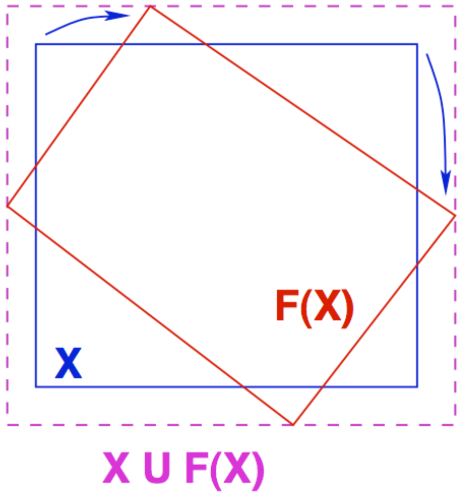
– Computes $X_n = \begin{cases} \alpha X_{n-1} + \beta X_{n-2} + Y_n \\ I_n \end{cases}$

– The concrete computation is bounded, which must be proved in the abstract.

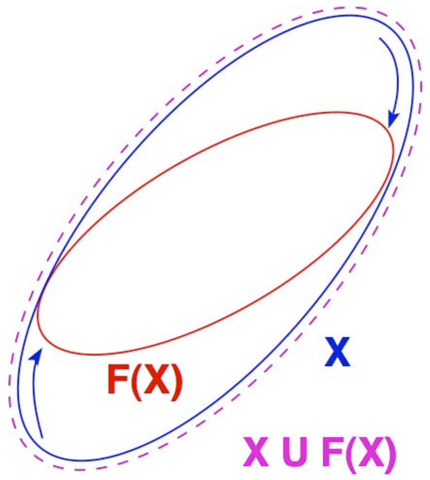
– There is no stable interval or octagon.

– The simplest stable surface is an ellipsoid.


execution trace



unstable interval



stable ellipsoid

Filter Example [Fer04]

```
typedef enum {FALSE = 0, TRUE = 1} BOOLEAN;
BOOLEAN INIT; float P, X;
void filter () {
  static float E[2], S[2];
  if (INIT) { S[0] = X; P = X; E[0] = X; }
  else { P = (((((0.5 * X) - (E[0] * 0.7)) + (E[1] * 0.4))
              + (S[0] * 1.5)) - (S[1] * 0.7)); }
  E[1] = E[0]; E[0] = X; S[1] = S[0]; S[0] = P;
  /* S[0], S[1] in [-1327.02698354, 1327.02698354] */
}
void main () { X = 0.2 * X + 5; INIT = TRUE;
  while (1) {
    X = 0.9 * X + 35; /* simulated filter input */
    filter (); INIT = FALSE; }
}
```

Arithmetic-Geometric Progressions [Fer05] Example 1

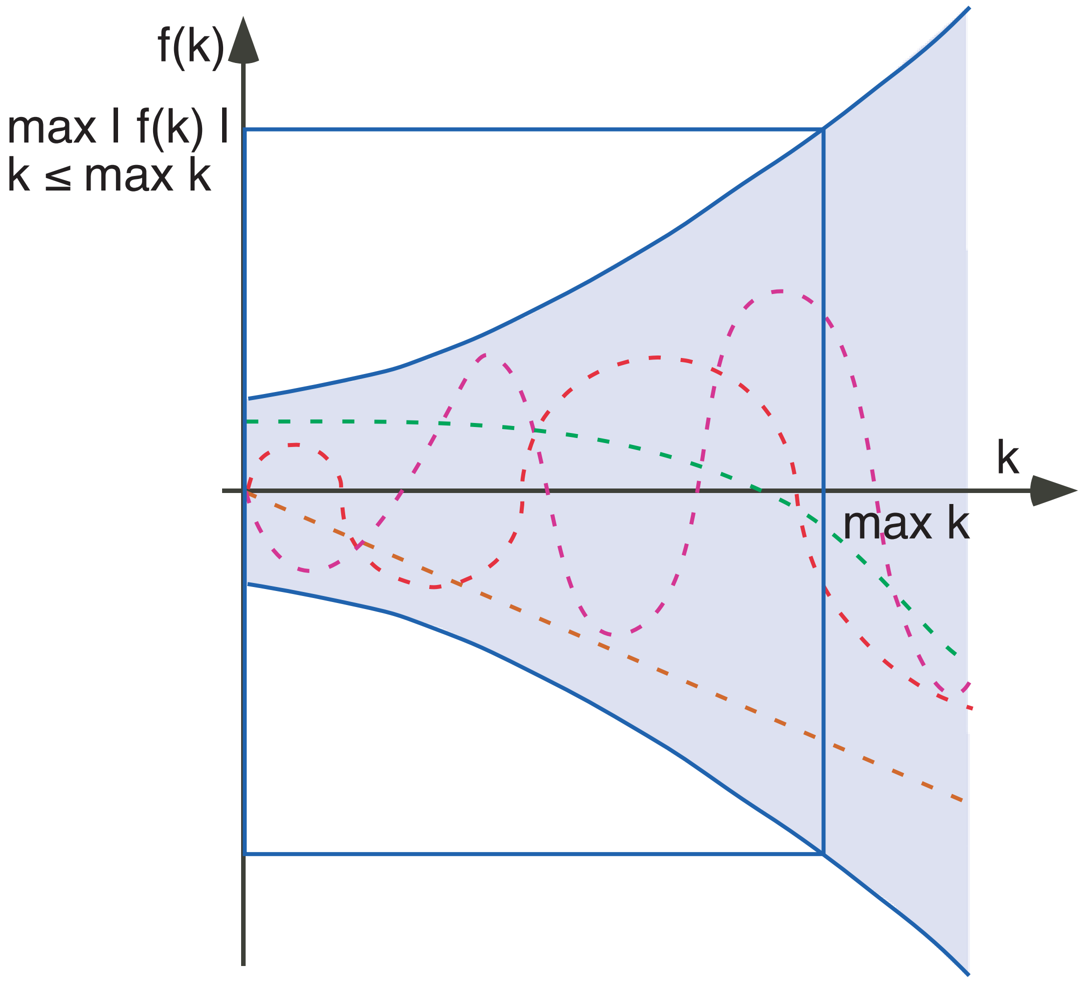
```
% cat retro.c
typedef enum {FALSE=0, TRUE=1} BOOL;
BOOL FIRST;
volatile BOOL SWITCH;
volatile float E;
float P, X, A, B;

void dev( )
{ X=E;
  if (FIRST) { P = X; }
  else
    { P =  (P - ((((2.0 * P) - A) - B)
            * 4.491048e-03)); };
  B = A;
  if (SWITCH) {A = P;}
  else {A = X;}
}
```

```
void main()
{ FIRST = TRUE;
  while (TRUE) {
    dev( );
    FIRST = FALSE;
    __ASTREE_wait_for_clock(());
  }}
% cat retro.config
__ASTREE_volatile_input((E [-15.0, 15.0]));
__ASTREE_volatile_input((SWITCH [0,1]));
__ASTREE_max_clock((3600000));
|P| <= (15.  + 5.87747175411e-39
/ 1.19209290217e-07) * (1
+ 1.19209290217e-07)^clock
- 5.87747175411e-39 /
1.19209290217e-07 <= 23.0393526881
```

Arithmetic-Geometric Progressions [Fer05] (Example 2)

```
% cat count.c
typedef enum {FALSE = 0, TRUE = 1} BOOLEAN;
volatile BOOLEAN I; int R; BOOLEAN T;
void main() {
  R = 0;
  while (TRUE) {
    __ASTREE_log_vars((R));
    if (I) { R = R + 1; }          ← potential overflow!
    else { R = 0; }
    T = (R >= 100);
    __ASTREE_wait_for_clock(());
  }}
% cat count.config
__ASTREE_volatile_input((I [0,1]));
__ASTREE_max_clock((3600000));
% astree -exec-fn main -config-sem count.config count.c|grep '|R|'
|R| <= 0. + clock *1. <= 3600001.
```

Overapproximation with an Arithmetic-Geometric Progression



Arithmetic-geometric progressions [11] [Fer05]

– Abstract domain: $(\mathbb{R}^+)^5$

– Concretization:

$$\gamma \in (\mathbb{R}^+)^5 \longmapsto \wp(\mathbb{N} \mapsto \mathbb{R})$$

$$\gamma(M, a, b, a', b') =$$
$$\{f \mid \forall k \in \mathbb{N} : |f(k)| \leq \left(\boldsymbol{\lambda}\, x \cdot ax + b \circ (\boldsymbol{\lambda}\, x \cdot a'x + b')^k\right)(M)\}$$

i.e. any function bounded by the arithmetic-geometric progression.

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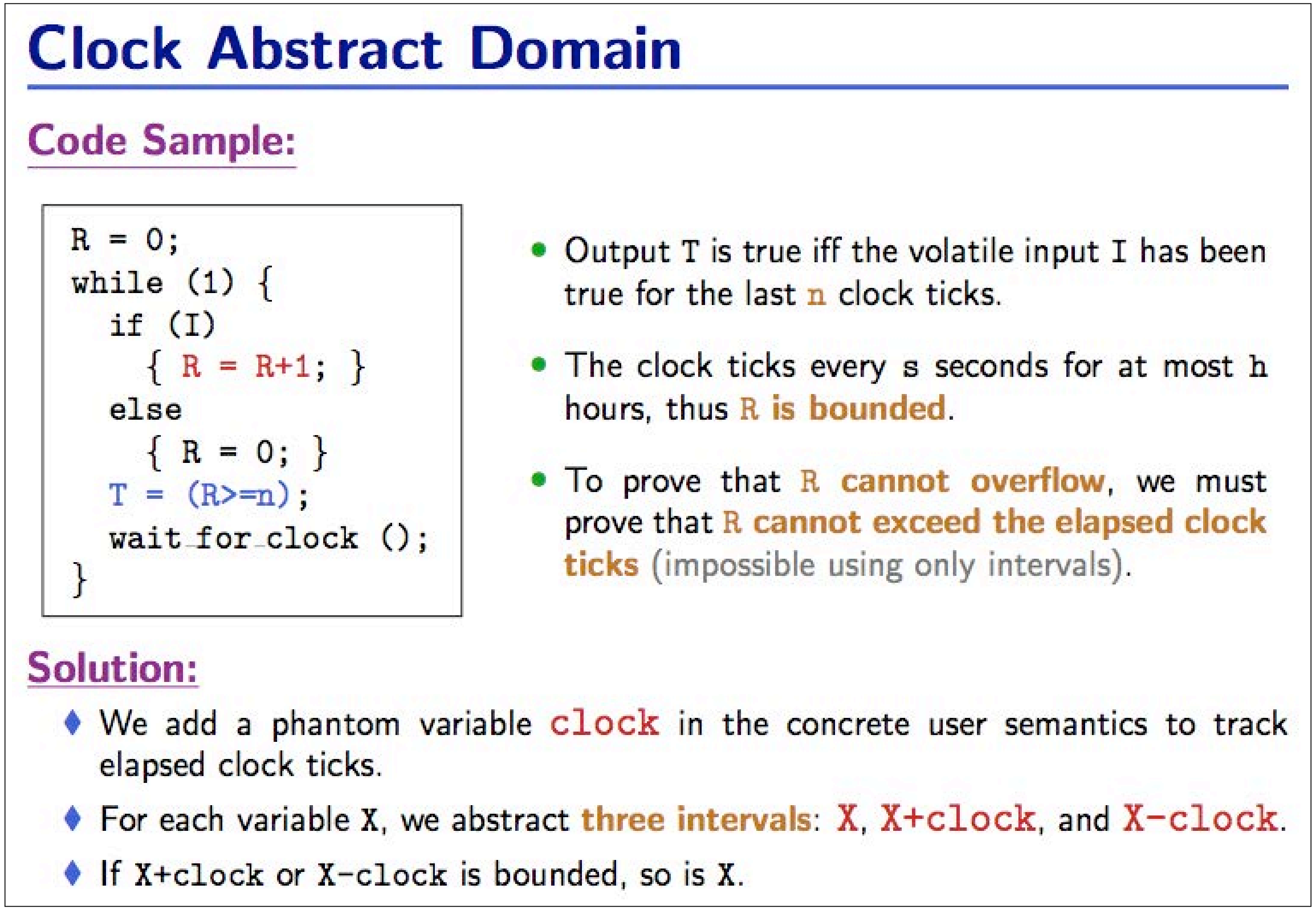
[2] J. Feret. The arithmetic-geometric progression abstract domain. In *VMCAI'05*, Paris, LNCS 3385, pp. 42–58, Springer, 2005.

11 here in $\mathbb{R}$

Obsolete Abstraction



Clock Abstract Domain

Code Sample:

```
R = 0;
while (1) {
  if (I)
    { R = R+1; }
  else
    { R = 0; }
  T = (R>=n);
  wait_for_clock ();
}
```

- Output T is true iff the volatile input I has been true for the last n clock ticks.
- The clock ticks every s seconds for at most h hours, thus R **is bounded**.
- To prove that R **cannot overflow**, we must prove that R **cannot exceed the elapsed clock ticks** (impossible using only intervals).

Solution:

- We add a phantom variable clock in the concrete user semantics to track elapsed clock ticks.
- For each variable X, we abstract **three intervals**: X, X+clock, and X-clock.
- If X+clock or X-clock is bounded, so is X.

Incompleteness

ASTRÉE does not know that

$$\forall x, y \in \mathbb{Z} : 7y^2 - 1 \neq x^2$$

so on the following program

```
void main() { int x, y;
  if ((-4681 < y) && (y < 4681) && (x < 32767) && (-32767 < x) && ((7*y*y - 1) == x*x))
     { y = 1 / x; };
}
```

it produces a false alarm (surely forever in this irrealistic program!)

```
% astree -exec-fn main false-alarm.c |& egrep "WARN"
false-alarm.c:5.9-14::[call#main@1:]: WARN: integer division by zero ([-32766, 32766]
and {1} / Z)
%
```

THE END, THANK YOU

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