

Automatic Verification of Avionic Synchronous Safety Critical Embedded Software

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Talk Outline

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Motivation



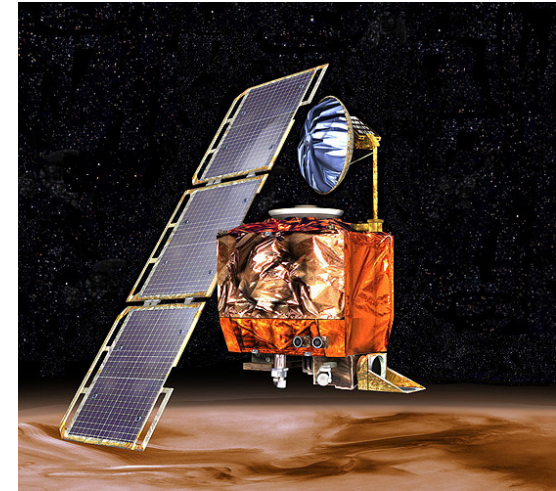
All Computer Scientists Have Experienced Bugs



Ariane 5.01 failure
(overflow)



Patriot failure
(float rounding)



Mars orbiter loss
(unit error)

It is preferable to verify that mission/safety-critical programs do not go wrong before running them.



Static Analysis by Abstract Interpretation

Static analysis: analyse the program at compile-time to verify a program runtime property (e.g. the absence of some categories of bugs)

Undecidability \longrightarrow

Abstract interpretation: effectively compute an abstraction/
sound approximation of the program semantics,
– which is precise enough to imply the desired property, and
– coarse enough to be efficiently computable.



Abstract Interpretation, Reminder

Reference

- [POPL '77] P. Cousot and R. Cousot. Abstract interpretation: a unified lattice model for static analysis of programs by construction or approximation of fixpoints. In *4th ACM POPL*.
- [Thesis '78] P. Cousot. Méthodes itératives de construction et d'approximation de points fixes d'opérateurs monotones sur un treillis, analyse sémantique de programmes. Thèse ès sci. math. Grenoble, march 1978.
- [POPL '79] P. Cousot & R. Cousot. Systematic design of program analysis frameworks. In *6th ACM POPL*.



Syntax of programs

X

variables $X \in \mathbb{X}$

T

types $T \in \mathbb{T}$

E

arithmetic expressions $E \in \mathbb{E}$

B

boolean expressions $B \in \mathbb{B}$

$D ::= T \ X;$

$\quad | \quad T \ X ; D'$

$C ::= X = E;$

$\quad | \quad \text{while } B \ C'$

$\quad | \quad \text{if } B \ C' \text{ else } C''$

$\quad | \quad \{ C_1 \dots C_n \}, (n \geq 0)$

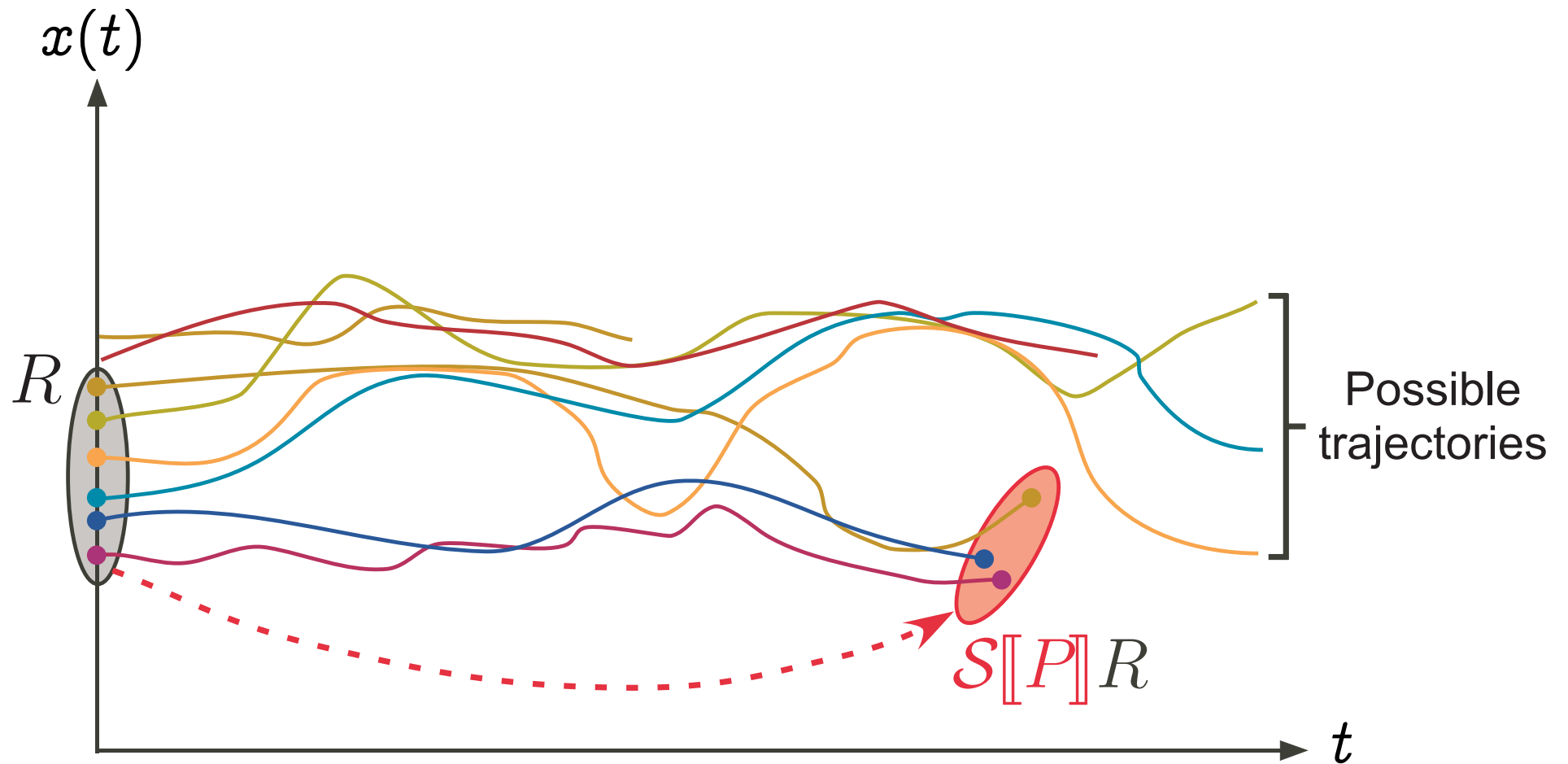
$P ::= D \ C$

commands $C \in \mathbb{C}$

program $P \in \mathbb{P}$



Postcondition semantics



States

Values of given type:

$\mathcal{V}[[T]]$: values of type $T \in \mathbb{T}$

$$\mathcal{V}[[\text{int}]] \stackrel{\text{def}}{=} \{z \in \mathbb{Z} \mid \text{min_int} \leq z \leq \text{max_int}\}$$

Program states $\Sigma[[P]]$ ¹:

$$\Sigma[[D \ C]] \stackrel{\text{def}}{=} \Sigma[[D]]$$

$$\Sigma[[T \ X;]] \stackrel{\text{def}}{=} \{X\} \mapsto \mathcal{V}[[T]]$$

$$\Sigma[[T \ X; \ D]] \stackrel{\text{def}}{=} (\{X\} \mapsto \mathcal{V}[[T]]) \cup \Sigma[[D]]$$

¹ States $\rho \in \Sigma[[P]]$ of a program P map program variables X to their values $\rho(X)$



Concrete Semantic Domain of Programs

Concrete semantic domain for reachability properties:

$$\mathcal{D}[[P]] \stackrel{\text{def}}{=} \wp(\Sigma[[P]]) \quad \text{sets of states}$$

i.e. program properties where \subseteq is implication, \emptyset is false, \cup is disjunction.



Concrete Reachability Semantics of Programs

$$S[X = E;]R \stackrel{\text{def}}{=} \{\rho[X \leftarrow \mathcal{E}[E]\rho] \mid \rho \in R \cap \text{dom}(E)\}$$

$$\rho[X \leftarrow v](X) \stackrel{\text{def}}{=} v, \quad \rho[X \leftarrow v](Y) \stackrel{\text{def}}{=} \rho(Y)$$

$$S[\text{if } B \text{ } C']R \stackrel{\text{def}}{=} S[C'](\mathcal{B}[B]R) \cup \mathcal{B}[\neg B]R$$

$$\mathcal{B}[B]R \stackrel{\text{def}}{=} \{\rho \in R \cap \text{dom}(B) \mid B \text{ holds in } \rho\}$$

$$S[\text{if } B \text{ } C' \text{ else } C'']R \stackrel{\text{def}}{=} S[C'](\mathcal{B}[B]R) \cup S[C''](\mathcal{B}[\neg B]R)$$

$$S[\text{while } B \text{ } C']R \stackrel{\text{def}}{=} \text{let } \mathcal{W} = \text{lfp}_{\emptyset}^{\subseteq} \lambda \mathcal{X}. R \cup S[C'](\mathcal{B}[B]\mathcal{X}) \\ \text{in } (\mathcal{B}[\neg B]\mathcal{W})$$

$$S[\{\}]R \stackrel{\text{def}}{=} R$$

$$S[\{C_1 \dots C_n\}]R \stackrel{\text{def}}{=} S[C_n] \circ \dots \circ S[C_1] \quad n > 0$$

$$S[D \text{ } C]R \stackrel{\text{def}}{=} S[C](\Sigma[D]) \quad (\text{uninitialized variables})$$

Not computable (undecidability).



Abstract Semantic Domain of Programs

$$\langle \mathcal{D}^\# \llbracket P \rrbracket, \sqsubseteq, \perp, \sqcup \rangle$$

such that:

$$\langle \mathcal{D}, \sqsubseteq \rangle \overset{\gamma}{\underset{\alpha}{\rightleftarrows}} \langle \mathcal{D}^\# \llbracket P \rrbracket, \sqsubseteq \rangle$$

hence $\langle \mathcal{D}^\# \llbracket P \rrbracket, \sqsubseteq, \perp, \sqcup \rangle$ is a complete lattice such that $\perp = \alpha(\emptyset)$ and $\sqcup X = \alpha(\cup \gamma(X))$



Reduced Product of Abstract Domains

To combine abstractions

$$\langle \mathcal{D}, \sqsubseteq \rangle \xrightleftharpoons[\alpha_1]{\gamma_1} \langle \mathcal{D}_1^\#, \sqsubseteq_1 \rangle \text{ and } \langle \mathcal{D}, \sqsubseteq \rangle \xrightleftharpoons[\alpha_2]{\gamma_2} \langle \mathcal{D}_2^\#, \sqsubseteq_2 \rangle$$

the reduced product is

$$\alpha(X) \stackrel{\text{def}}{=} \sqcap \{ \langle x, y \rangle \mid X \subseteq \gamma_1(X) \wedge X \subseteq \gamma_2(X) \}$$

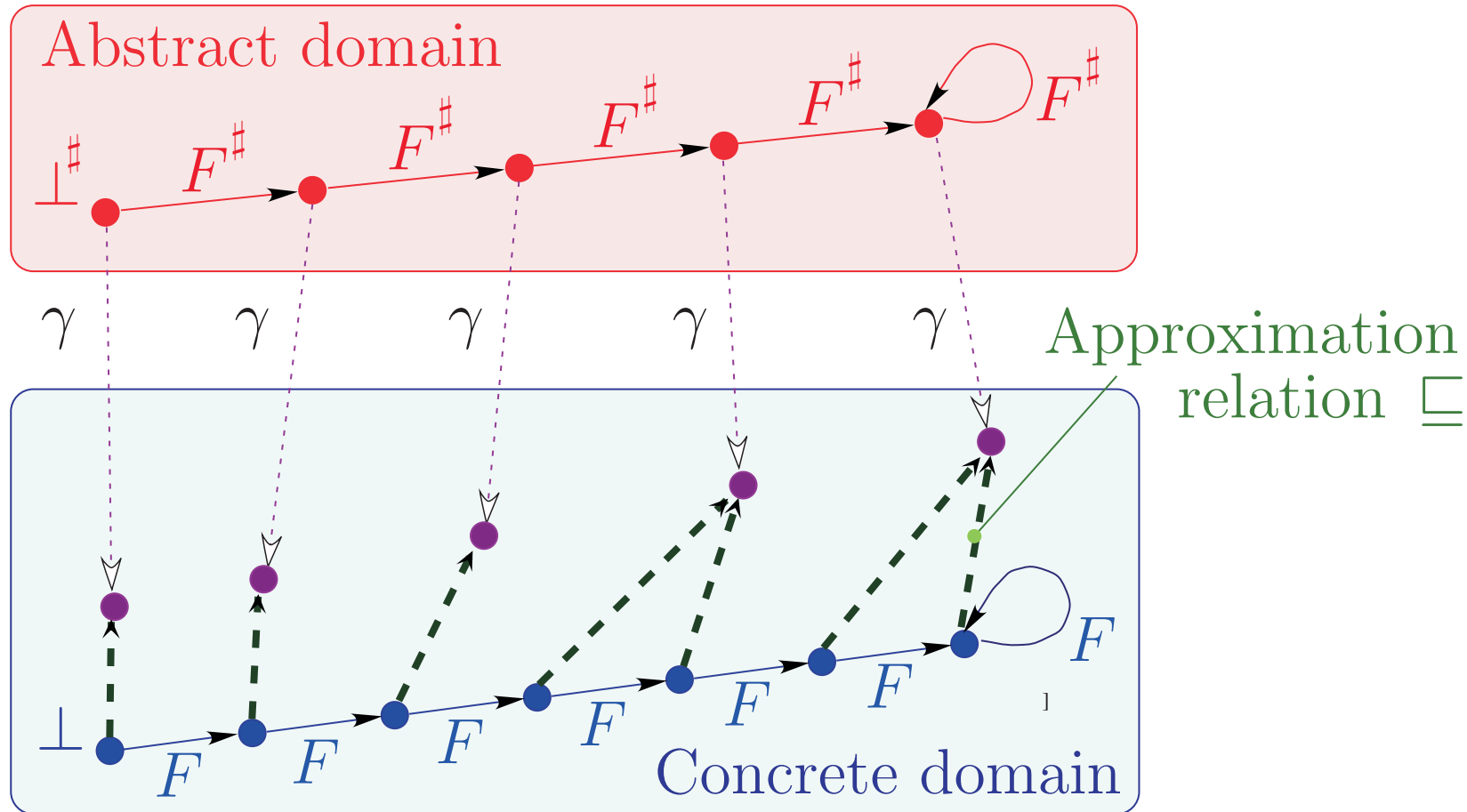
such that $\sqsubseteq \stackrel{\text{def}}{=} \sqsubseteq_1 \times \sqsubseteq_2$ and

$$\langle \mathcal{D}, \sqsubseteq \rangle \xrightleftharpoons[\alpha]{\gamma_1 \times \gamma_2} \langle \alpha(\mathcal{D}), \sqsubseteq \rangle$$

Example: $x \in [1, 9] \wedge x \bmod 2 = 0$ reduces to $x \in [2, 8] \wedge x \bmod 2 = 0$



Approximate Fixpoint Abstraction



$$F \circ \gamma \sqsubseteq \gamma \circ F^\# \Rightarrow \text{lfp } F \sqsubseteq \gamma(\text{lfp } F^\#)$$



Abstract Reachability Semantics of Programs

$$S^\sharp \llbracket X = E; \rrbracket R \stackrel{\text{def}}{=} \alpha(\{\rho[X \leftarrow \mathcal{E} \llbracket E \rrbracket \rho] \mid \rho \in \gamma(R) \cap \text{dom}(E)\})$$

$$S^\sharp \llbracket \text{if } B \text{ } C' \rrbracket R \stackrel{\text{def}}{=} S^\sharp \llbracket C' \rrbracket (\mathcal{B}^\sharp \llbracket B \rrbracket R) \sqcup \mathcal{B}^\sharp \llbracket \neg B \rrbracket R$$

$$\mathcal{B}^\sharp \llbracket B \rrbracket R \stackrel{\text{def}}{=} \alpha(\{\rho \in \gamma(R) \cap \text{dom}(B) \mid B \text{ holds in } \rho\})$$

$$S^\sharp \llbracket \text{if } B \text{ } C' \text{ else } C'' \rrbracket R \stackrel{\text{def}}{=} S^\sharp \llbracket C' \rrbracket (\mathcal{B}^\sharp \llbracket B \rrbracket R) \sqcup S^\sharp \llbracket C'' \rrbracket (\mathcal{B}^\sharp \llbracket \neg B \rrbracket R)$$

$$S^\sharp \llbracket \text{while } B \text{ } C' \rrbracket R \stackrel{\text{def}}{=} \text{let } \mathcal{W} = \text{lfp}_{\perp}^{\sqsubseteq} \lambda \mathcal{X}. R \sqcup S^\sharp \llbracket C' \rrbracket (\mathcal{B}^\sharp \llbracket B \rrbracket \mathcal{X}) \\ \text{in } (\mathcal{B}^\sharp \llbracket \neg B \rrbracket \mathcal{W})$$

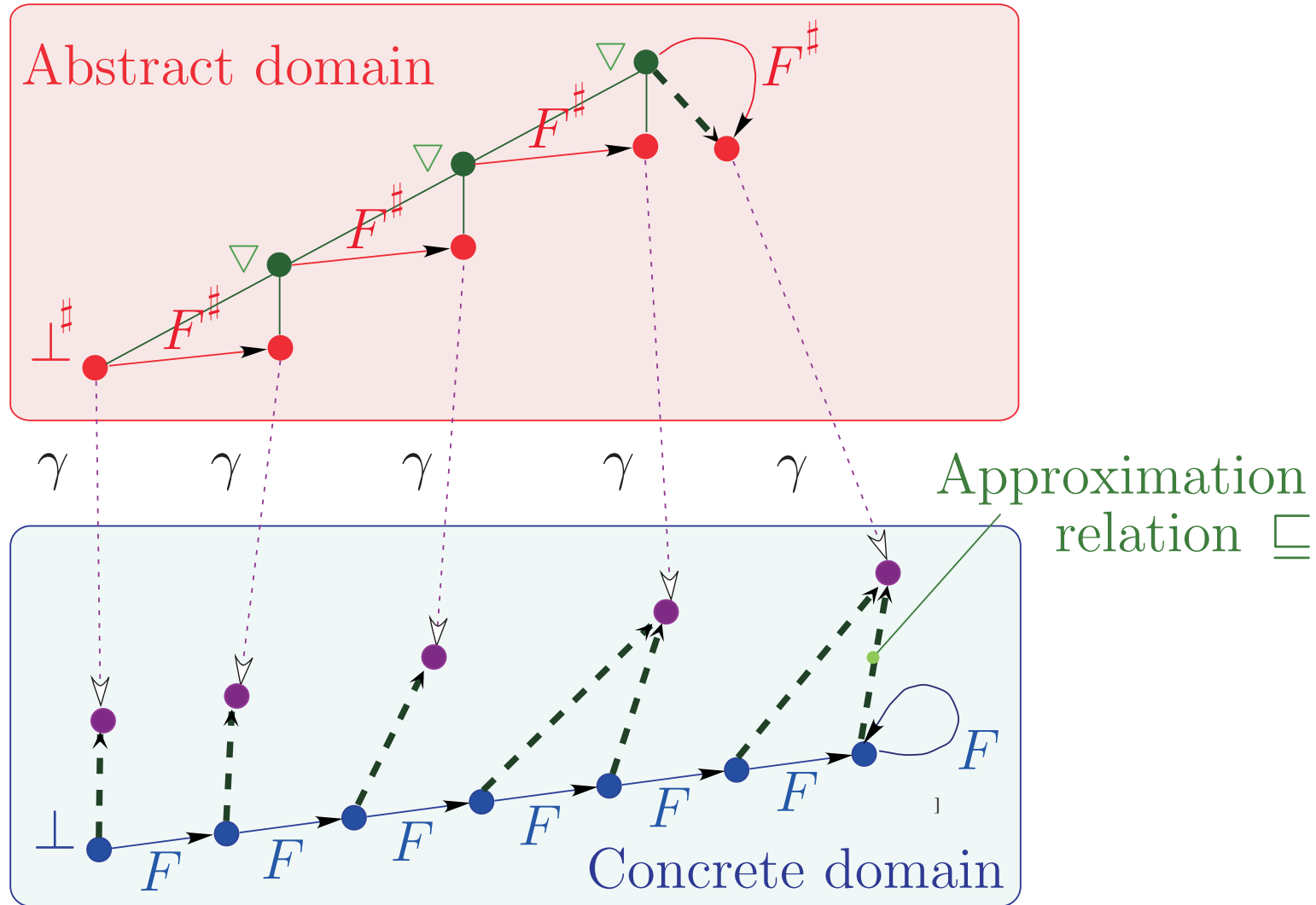
$$S^\sharp \llbracket \{\} \rrbracket R \stackrel{\text{def}}{=} R$$

$$S^\sharp \llbracket \{C_1 \dots C_n\} \rrbracket R \stackrel{\text{def}}{=} S^\sharp \llbracket C_n \rrbracket \circ \dots \circ S^\sharp \llbracket C_1 \rrbracket \quad n > 0$$

$$S^\sharp \llbracket D \text{ } C \rrbracket R \stackrel{\text{def}}{=} S^\sharp \llbracket C \rrbracket (\top) \quad (\text{uninitialized variables})$$



Convergence Acceleration with Widening



Abstract Semantics with Convergence Acceleration²

$$S^\sharp[X = E;]R \stackrel{\text{def}}{=} \alpha(\{\rho[X \leftarrow \mathcal{E}[E]\rho] \mid \rho \in \gamma(R) \cap \text{dom}(E)\})$$

$$S^\sharp[\text{if } B \text{ } C']R \stackrel{\text{def}}{=} S^\sharp[C'](\mathcal{B}^\sharp[B]R) \sqcup \mathcal{B}^\sharp[\neg B]R$$

$$\mathcal{B}^\sharp[B]R \stackrel{\text{def}}{=} \alpha(\{\rho \in \gamma(R) \cap \text{dom}(B) \mid B \text{ holds in } \rho\})$$

$$S^\sharp[\text{if } B \text{ } C' \text{ else } C'']R \stackrel{\text{def}}{=} S^\sharp[C'](\mathcal{B}^\sharp[B]R) \sqcup S^\sharp[C''](\mathcal{B}^\sharp[\neg B]R)$$

$$S^\sharp[\text{while } B \text{ } C']R \stackrel{\text{def}}{=} \text{let } \mathcal{F}^\sharp = \lambda \mathcal{X}. \text{let } \mathcal{Y} = R \sqcup S^\sharp[C'](\mathcal{B}^\sharp[B]\mathcal{X}) \\ \text{in if } \mathcal{Y} \sqsubseteq \mathcal{X} \text{ then } \mathcal{X} \text{ else } \mathcal{X} \nabla \mathcal{Y}$$

$$\text{and } \mathcal{W} = \text{lfp}_{\perp}^{\sqsubseteq} \mathcal{F}^\sharp \quad \text{in } (\mathcal{B}^\sharp[\neg B]\mathcal{W})$$

$$S^\sharp[\{\}]R \stackrel{\text{def}}{=} R$$

$$S^\sharp[\{C_1 \dots C_n\}]R \stackrel{\text{def}}{=} S^\sharp[C_n] \circ \dots \circ S^\sharp[C_1] \quad n > 0$$

$$S^\sharp[D \text{ } C]R \stackrel{\text{def}}{=} S^\sharp[C](\top) \quad (\text{uninitialized variables})$$

² Note: \mathcal{F}^\sharp not monotonic!



Applications of Abstract Interpretation



Applications of Abstract Interpretation

- **Static Program Analysis** [POPL '77], [POPL '78], [POPL '79]
including **Dataflow Analysis** [POPL '79], [POPL '00], **Set-based Analysis** [FPCA '95], **Predicate Abstraction** [Manna's festschrift '03], ...
- **Syntax Analysis** [TCS 290(1) 2002]
- **Hierarchies of Semantics (including Proofs)** [POPL '92], [TCS 277(1–2) 2002]
- **Typing & Type Inference** [POPL '97]



Applications of Abstract Interpretation (Cont'd)

- (Abstract) Model Checking [POPL '00]
- Program Transformation [POPL '02]
- Software Watermarking [POPL '04]
- Bisimulations [RT-ESOP '04]

All these techniques involve **sound approximations** that can be formalized by **abstract interpretation**



A Practical Application of Abstract Interpretation to the Verification of Safety Critical Embedded Control-Command Software

Reference

[1] <http://www.astree.ens.fr/>



ASTRÉE: A Sound, Automatic, Specializable, Domain-Aware, Parametric, Modular, Efficient and Precise Static Program Analyzer

www.astree.ens.fr

– C programs:

– with

- pointers (including on functions), structures and arrays
- floating point computations
- tests, loops and function calls
- limited branching (forward goto, break, continue)



– without

- union
- dynamic memory allocation
- recursive function calls
- backward branching
- conflict side effects
- C libraries

– **Application Domain:** safety critical embedded real-time synchronous software for non-linear control of very complex control/command systems.



Concrete Operational Semantics

- International **norm of C** (ISO/IEC 9899:1999)
- *restricted by* **implementation-specific behaviors** depending upon the machine and compiler (e.g. representation and size of integers, IEEE 754-1985 norm for floats and doubles)
- *restricted by* user-defined **programming guidelines** (such as no modular arithmetic for signed integers, even though this might be the hardware choice)
- *restricted by* program specific **user requirements** (e.g. assert)



Abstract Semantics

- Reachable states for the concrete trace operational semantics
- Volatile environment is specified by a *trusted* configuration file.

Requirements:

- Soundness: absolutely essential
- Precision: few or no false alarm (full certification)
- Efficiency: rapid analyses and fixes during development



Implicit Specification: Absence of Runtime Errors

- No violation of the **norm of C** (e.g. array index out of bounds, division by zero)
- **No** implementation-specific **undefined behaviors** (e.g. maximum short integer is 32767, NaN)
- No violation of the **programming guidelines** (e.g. static variables cannot be assumed to be initialized to 0)
- No violation of the **programmer assertions** (must all be statically verified).



Example application

- Primary flight control software of the Airbus A340 family/A380 fly-by-wire system



- C program, automatically generated from a proprietary high-level specification (à la Simulink/SCADE)
- A340 family: 132,000 lines, 75,000 LOCs after preprocessing, 10,000 global variables, over 21,000 after expansion of small arrays
- A380: $\times 3$



The Class of Considered Periodic Synchronous Programs

```
declare volatile input, state and output variables;  
initialize state and output variables;  
loop forever  
  - read volatile input variables,  
  - compute output and state variables,  
  - write to volatile output variables;  
  wait_for_clock ();  
end loop
```

- Requirements: the only interrupts are clock ticks;
- Execution time of loop body less than a clock tick
[EMSOFT '01].



Challenging aspects

- Size: > 100 kLOC, $> 10\,000$ variables
- Floating point computations
including filtering, non linear control with feedback, interpolations...
- Interdependencies among variables:
 - Stability of computations should be established
 - Complex relations should be inferred among numerical and boolean data
 - Very long data paths from input to outputs



Characteristics of the **ASTRÉE** Analyzer

Static: compile time analysis (\neq run time analysis **Rational Purify**, **Parasoft Insure++**)

Program Analyzer: analyzes programs not micromodels of programs (\neq **PROMELA** in **SPIN** or **Alloy** in the **Alloy Analyzer**)

Automatic: no end-user intervention needed (\neq **ESC Java**, **ESC Java 2**)

Sound: covers the whole state space (\neq **MAGIC**, **CBMC**) so never omit potential errors (\neq **UNO**, **CMC** from **coverity.com**) or sort most probable ones (\neq **Splint**)



Characteristics of the **ASTRÉE** Analyzer (Cont'd)

- Multiabstraction:** uses many numerical/symbolic abstract domains (\neq symbolic constraints in **Bane** or the canonical abstraction of **TVLA**)
- Infinitary:** all abstractions use infinite abstract domains with widening/narrowing (\neq model checking based analyzers such as **VeriSoft**, **Bandera**, **Java PathFinder**)
- Efficient:** always terminate (\neq counterexample-driven automatic abstraction refinement **BLAST**, **SLAM**)



Characteristics of the **ASTRÉE** Analyzer (Cont'd)

Specializable: can easily incorporate new abstractions (and reduction with already existing abstract domains) (\neq general-purpose analyzers **PolySpace Verifier**)

Domain-Aware: knows about control/command (e.g. digital filters) (as opposed to specialization to a mere programming style in **C Global Surveyor**)

Parametric: the precision/cost can be tailored to user needs by options and directives in the code



Characteristics of the **ASTRÉE** Analyzer (Cont'd)

Automatic Parametrization: the generation of parametric directives in the code can be programmed (to be specialized for a specific application domain)

Modular: an analyzer instance is built by selection of **O-CAML** modules from a collection each implementing an abstract domain

Precise: very few or no false alarm when adapted to an application domain \longrightarrow **it is a VERIFIER!**



Example of Analysis Session

Visualizer

Search string: Next Previous First Last / Goto line:

Program points: Current

Context

- ▼ filtre2.c:126
 - ▼ Call main @ filtre2.c:205
 - ▼ While @ filtre2.c:232
 - Iter = 2
 - ▼ Call filtre2 @ filtre2.c:254
 - 0
 - Iter = 3
 - ▼ Call filtre2 @ filtre2.c:254
 - 0
 - Iter = 4
 - ▼ Call filtre2 @ filtre2.c:254
 - 0

Sources

filtre2.c

```
typedef enum {FALSE = 0, TRUE = 1} BOOLEAN;
BOOLEAN INIT;
float P, X;

void filtre2 () {
  static float E[2], S[2];
  if (INIT) {
    S[0] = X;
    P = X;
    E[0] = X;
  } else {
    P = (((0.4677826 * X) - (E[0] * 0.7700725)) + (E[1] * 0.4344376)) + (S[0] * 1.5419) - (S[1] * 0.6740476);
  }
  E[1] = E[0];
  E[0] = X;
  S[1] = S[0];
  S[0] = P;
}

void main () {
  X = 0.2 * X + 5;
  INIT = TRUE;
  while (TRUE) {
    X = 0.9 * X + 35;
    filtre2 ();
    INIT = FALSE;
  }
}
```

Location: filtre2.c:126[call#main@20:loop@23>=4:call#filtre2@25]

Variables: P (1)

invariant:

<interval: P in [-1252.84, 1252.84] inter [-3362.7, 3491.96]>clock inter [-3362.7, 3491.96]>-clock>

Filtre d'ordre 2

Var_entree 1	:E[0]
Var_entree 2	:E[1]
Var_sortie	:P
Var_sortie_pred	:S[1]
coef_e1	:0.4677826
coef_e2	:-0.7700725
coef_e3	:0.4344376
coef_a	:1.5419
coef_b	:-0.6740476

Egalite des entrees a l'origine!!

Nb de deroulement : 38

plus_grande entree : <= 935.935061096

erreur en entree : <= 0.00246160101051

gain leres sorties : <= 1.33715602022

gain last entrees : <= 1.3366487752

gain autres entrees : <= 0.00213381749462

erreur_sortie : <= 0.0400176854152

sortie_max : <= 1259.02359782

<octagon:

filtre2.c@12@5=

{ -5430.9504421651563462 <= P <= 39396.917979075267795,

info

/* Analyzer launched at 2004/ 3/16 20:41:58

Command line was "/Volumes/PB_cousot_PGP/Projet/absinthe2/analyzer.opt --exec-fn main filtre2.c --export-invariant-stat filtre2.bin "

Launched by "cousot" on "PB-64-Patrick-COUSOT.local"

filtre2.c — line 12 — column 6 — character 193



Benchmarks (Airbus A340 Primary Flight Control Software)

- 132,000 lines, 75,000 LOCs after preprocessing
- Comparative results (commercial software):
 - 4,200 (false?) alarms,
 - 3.5 days;
- Our results:
 - 0 alarms,
 - 40mn on 2.8 GHz PC,
 - 300 Megabytes
 - A world première!



(Airbus A380 Primary Flight Control Software)

- 450,000 lines
- 0 alarms (Nov. 2004),
7h³ on 2.8 GHz a PC,
1 Gigabyte
→ A world grand première!

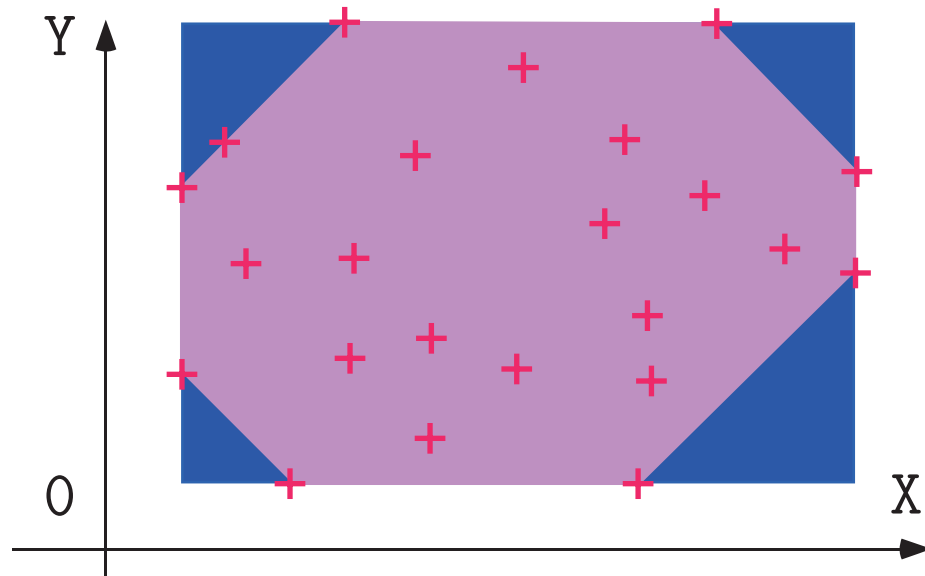
³ It would be possible to favour computation costs rather than precision, and this should go down. For example, the A340 analysis went up to 5 h, before being reduced by requiring less precision while still getting no false alarm.



Examples of Abstractions



General-Purpose Abstract Domains: Intervals and Octagons



Intervals:

$$\begin{cases} 1 \leq x \leq 9 \\ 1 \leq y \leq 20 \end{cases}$$

Octagons [10]:

$$\begin{cases} 1 \leq x \leq 9 \\ x + y \leq 77 \\ 1 \leq y \leq 20 \\ x - y \leq 04 \end{cases}$$

Difficulties: many global variables, arrays (smashed or not), IEEE 754 floating-point arithmetic (in program and analyzer) [10, 11]



Floating-Point Computations

```
/* float-error.c */
int main () {
    float x, y, z, r;
    x = 1.000000019e+38;
    y = x + 1.0e21;
    z = x - 1.0e21;
    r = y - z;
    printf("%f\n", r);
}
% gcc float-error.c
% ./a.out
0.000000
```

```
/* double-error.c */
int main () {
    double x; float y, z, r;
    /* x = ldexp(1.,50)+ldexp(1.,26); */
    x = 1125899973951488.0;
    y = x + 1;
    z = x - 1;
    r = y - z;
    printf("%f\n", r);
}
% gcc double-error.c
% ./a.out
134217728.000000
```

$$(x + a) - (x - a) \neq 2a$$



Floating-Point Computations

```
/* float-error.c */
int main () {
    float x, y, z, r;
    x = 1.000000019e+38;
    y = x + 1.0e21;
    z = x - 1.0e21;
    r = y - z;
    printf("%f\n", r);
}
% gcc float-error.c
% ./a.out
0.000000
```

```
/* double-error.c */
int main () {
    double x; float y, z, r;
    /* x = ldexp(1.,50)+ldexp(1.,26); */
    x = 1125899973951487.0;
    y = x + 1;
    z = x - 1;
    r = y - z;
    printf("%f\n", r);
}
% gcc double-error.c
% ./a.out
0.000000
```

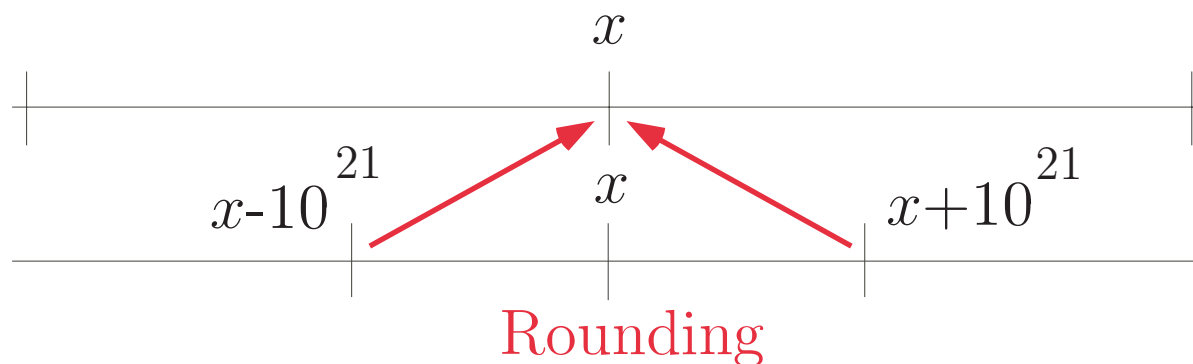
$$(x + a) - (x - a) \neq 2a$$



Explanation of the huge rounding error

(1) Floats

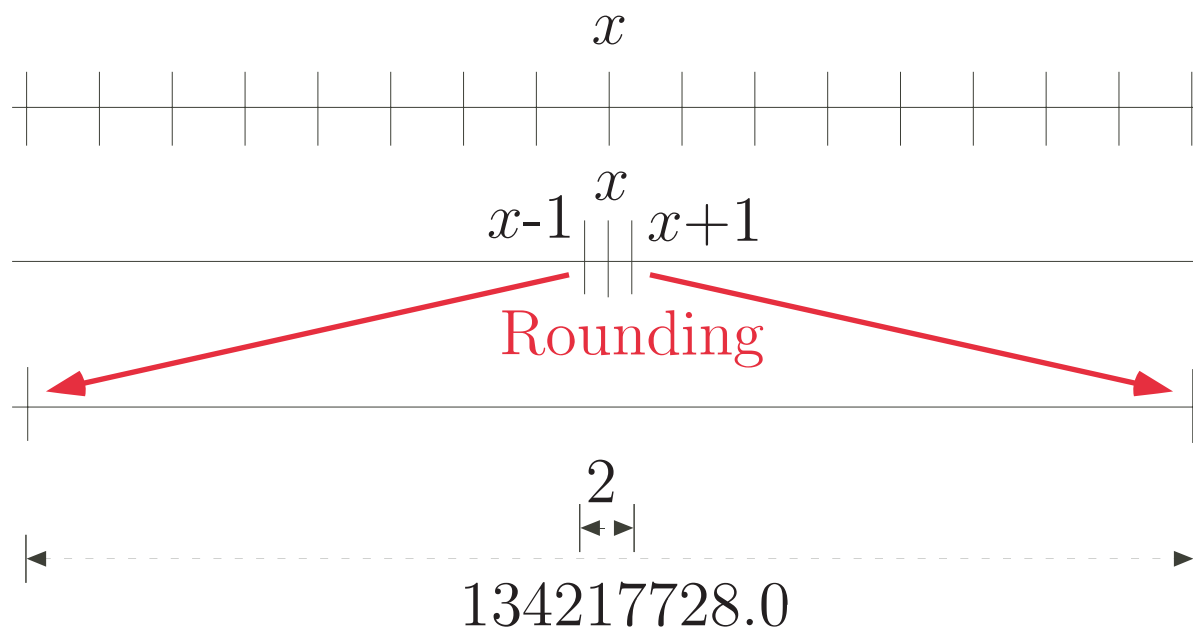
Reals



(2) Doubles

Reals

Floats



Symbolic abstract domain

- Interval analysis: if $x \in [a, b]$ and $y \in [c, d]$ then $x - y \in [a - d, b - c]$ so if $x \in [0, 100]$ then $x - x \in [-100, 100]$!!!
- The symbolic abstract domain propagates the symbolic values of variables and performs simplifications;
- Must maintain the maximal possible rounding error for float computations (overestimated with intervals);

```
% cat -n x-x.c
```

```
1 void main () { int X, Y;  
2     __ASTREE_known_fact(((0 <= X) && (X <= 100)));  
3     Y = (X - X);  
4     __ASTREE_log_vars((Y));  
5 }
```

```
astree -exec-fn main -no-relational x-x.c
```

```
Call main@x-x.c:1:5-x-x.c:1:9:
```

```
<interval: Y in [-100, 100]>
```

```
astree -exec-fn main x-x.c
```

```
Call main@x-x.c:1:5-x-x.c:1:9:
```

```
<interval: Y in {0}> <symbolic: Y = (X -i X)>
```



Clock Abstract Domain for Counters

– Code Sample:

```
R = 0;
while (1) {
  if (I)
    { R = R+1; }
  else
    { R = 0; }
  T = (R>=n);
  wait_for_clock ();
}
```

- Output T is true iff the volatile input I has been true for the last *n* clock ticks.
- The clock ticks every *s* seconds for at most *h* hours, thus *R* is bounded.
- To prove that *R* cannot overflow, we must prove that *R* cannot exceed the elapsed clock ticks (*impossible using only intervals*).

– Solution:

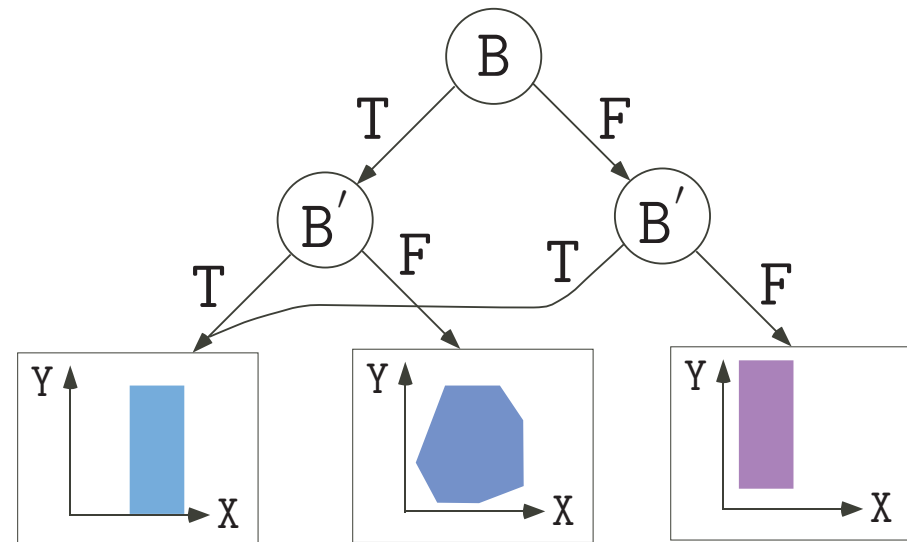
- We add a phantom variable *clock* in the concrete user semantics to track elapsed clock ticks.
- For each variable *X*, we abstract *three intervals*: *X*, *X+clock*, and *X-clock*.
- If *X+clock* or *X-clock* is bounded, so is *X*.



Boolean Relations for Boolean Control

– Code Sample:

```
/* boolean.c */
typedef enum {F=0,T=1} BOOL;
BOOL B;
void main () {
    unsigned int X, Y;
    while (1) {
        ...
        B = (X == 0);
        ...
        if (!B) {
            Y = 1 / X;
        }
        ...
    }
}
```



The boolean relation abstract domain is parameterized by the height of the decision tree (an analyzer option) and the abstract domain at the leafs



Control Partitionning for Case Analysis

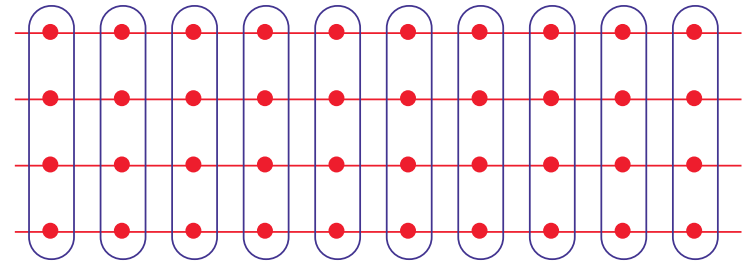
—Code Sample:

```
/* trace_partitionning.c */
void main() {
  float t[5] = {-10.0, -10.0, 0.0, 10.0, 10.0};
  float c[4] = {0.0, 2.0, 2.0, 0.0};
  float d[4] = {-20.0, -20.0, 0.0, 20.0};
  float x, r;
  int i = 0;

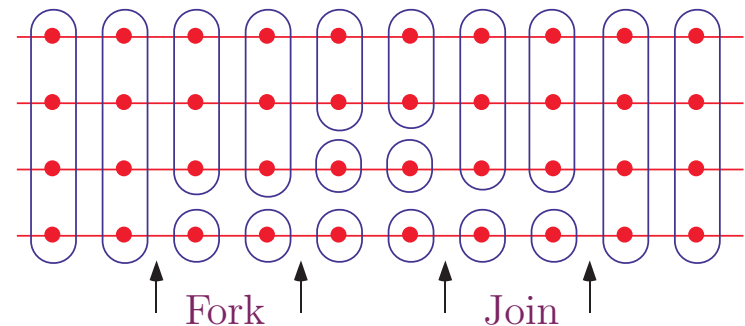
  ... found invariant  $-100 \leq x \leq 100$  ...

  while ((i < 3) && (x >= t[i+1])) {
    i = i + 1;
  }
  r = (x - t[i]) * c[i] + d[i];
}
```

Control point partitionning:



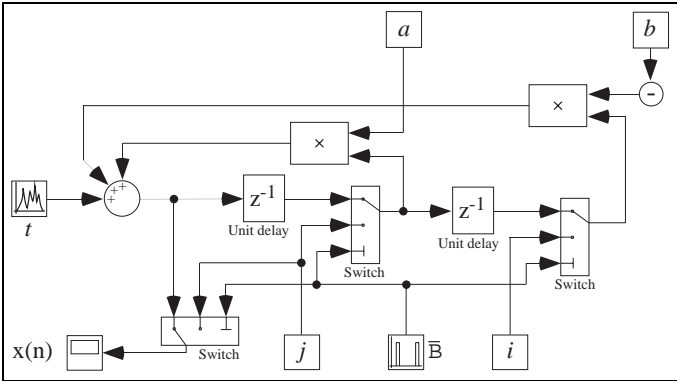
Trace partitionning:



Delaying abstract unions in tests and loops is more precise for non-distributive abstract domains (and much less expensive than disjunctive completion).

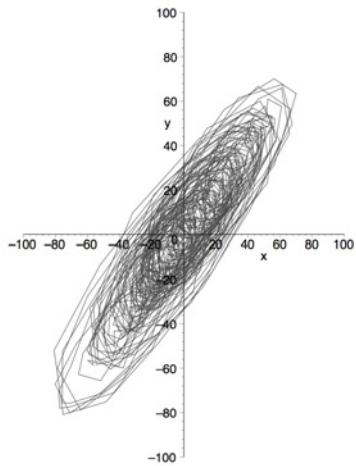


2^d Order Digital Filter:

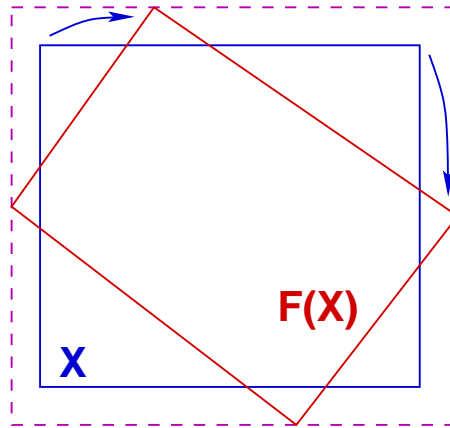


Ellipsoid Abstract Domain for Filters

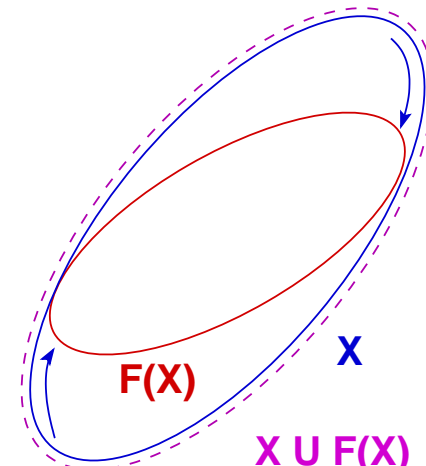
- Computes $X_n = \begin{cases} \alpha X_{n-1} + \beta X_{n-2} + Y_n \\ I_n \end{cases}$
- The concrete computation is **bounded**, which must be proved in the abstract.
- There is **no stable interval or octagon**.
- The simplest stable surface is an **ellipsoid**.



execution trace



$X \cup F(X)$
unstable interval



stable ellipsoid



Filter Example [7]

```
typedef enum {FALSE = 0, TRUE = 1} BOOLEAN;
BOOLEAN INIT; float P, X;

void filter () {
    static float E[2], S[2];
    if (INIT) { S[0] = X; P = X; E[0] = X; }
    else { P = (((((0.5 * X) - (E[0] * 0.7)) + (E[1] * 0.4))
                + (S[0] * 1.5)) - (S[1] * 0.7)); }
    E[1] = E[0]; E[0] = X; S[1] = S[0]; S[0] = P;
    /* S[0], S[1] in [-1327.02698354, 1327.02698354] */
}

void main () { X = 0.2 * X + 5; INIT = TRUE;
    while (1) {
        X = 0.9 * X + 35; /* simulated filter input */
        filter (); INIT = FALSE; }
}
```



Arithmetic-geometric progressions

```
% cat retro.c
typedef enum {FALSE=0, TRUE=1} BOOL;
BOOL FIRST;
volatile BOOL SWITCH;
volatile float E;
float P, X, A, B;

void dev( )
{ X=E;
  if (FIRST) { P = X; }
  else
    { P = (P - (((2.0 * P) - A) - B)
           * 4.491048e-03)); };
  B = A;
  if (SWITCH) {A = P;}
  else {A = X;}
}
```

```
void main()
{ FIRST = TRUE;
  while (TRUE) {
    dev( );
    FIRST = FALSE;
    __ASTREE_wait_for_clock();
  }}

% cat retro.config
__ASTREE_volatile_input((E [-15.0, 15.0]));
__ASTREE_volatile_input((SWITCH [0,1]));
__ASTREE_max_clock((3600000));

|P| <= (15. + 5.87747175411e-39
/ 1.19209290217e-07) * (1 +
1.19209290217e-07)^clock -
5.87747175411e-39 / 1.19209290217e-07
<= 23.0393526881
```



(Automatic) Parameterization

- All abstract domains of ASTRÉE are **parameterized**, e.g.
 - variable packing for octagones and decision trees,
 - partition/merge program points,
 - loop unrollings,
 - thresholds in widenings, ...;
- End-users can either **parameterize by hand** (analyzer options, directives in the code), or
- choose the **automatic parameterization** (default options, directives for pattern-matched predefined program schemata).



The main loop invariant for the A340

A textual file over 4.5 Mb with

- 6,900 boolean interval assertions ($x \in [0; 1]$)
- 9,600 interval assertions ($x \in [a; b]$)
- 25,400 clock assertions ($x + \text{clk} \in [a; b] \wedge x - \text{clk} \in [a; b]$)
- 19,100 additive octagonal assertions ($a \leq x + y \leq b$)
- 19,200 subtractive octagonal assertions ($a \leq x - y \leq b$)
- 100 decision trees
- 60 ellipse invariants, etc ...

involving over 16,000 floating point constants (only 550 appearing in the program text) \times 75,000 LOCs.



Possible origins of imprecision and how to fix it

In case of false alarm, the imprecision can come from:

- **Abstract transformers** (not best possible) \longrightarrow improve algorithm;
- **Automatized parametrization** (e.g. variable packing) \longrightarrow improve pattern-matched program schemata;
- **Iteration strategy** for fixpoints \longrightarrow fix widening ⁴;
- **Inexpressivity** i.e. indispensable local inductive invariant are inexpressible in the abstract \longrightarrow add a **new abstract domain** to the reduced product (e.g. filters).

⁴ This can be very hard since at the limit only a precise infinite iteration might be able to compute the proper abstract invariant. In that case, it might be better to design a more refined abstract domain.



Conclusion



Conclusion

- Most applications of abstract interpretation **tolerate a small rate** (typically 5 to 15%) **of false alarms**:
 - Program transformation → do not optimize,
 - Typing → reject some correct programs, etc,
 - WCET analysis → overestimate;
- Some applications **require no false alarm** at all:
 - **Program verification**.
- **Theoretically possible** [SARA '00], **practically feasible** [PLDI '03]

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The Future & Grand Challenges

Forthcoming (1 year):

- More general memory model (union)

Future (5 years):

- Asynchronous concurrency (for less critical software)
- Functional properties (reactivity)
- Industrialization

Grand challenge:

- Verification from specifications to machine code (verifying compiler)
- Verification of systems (quasi-synchrony, distribution)



THE END, THANK YOU

More references at URL www.di.ens.fr/~cousot
www.astree.ens.fr.



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