Proof of mutual-exclusion and non-starvation of a program: PostgreSQL

Jade Alglave (MSR-Cambridge, UCL, UK)
Patrick Cousot (NYU, Emer. ENS, PSL)

Dagstuhl Seminar 16471

http://www.dagstuhl.de/16471

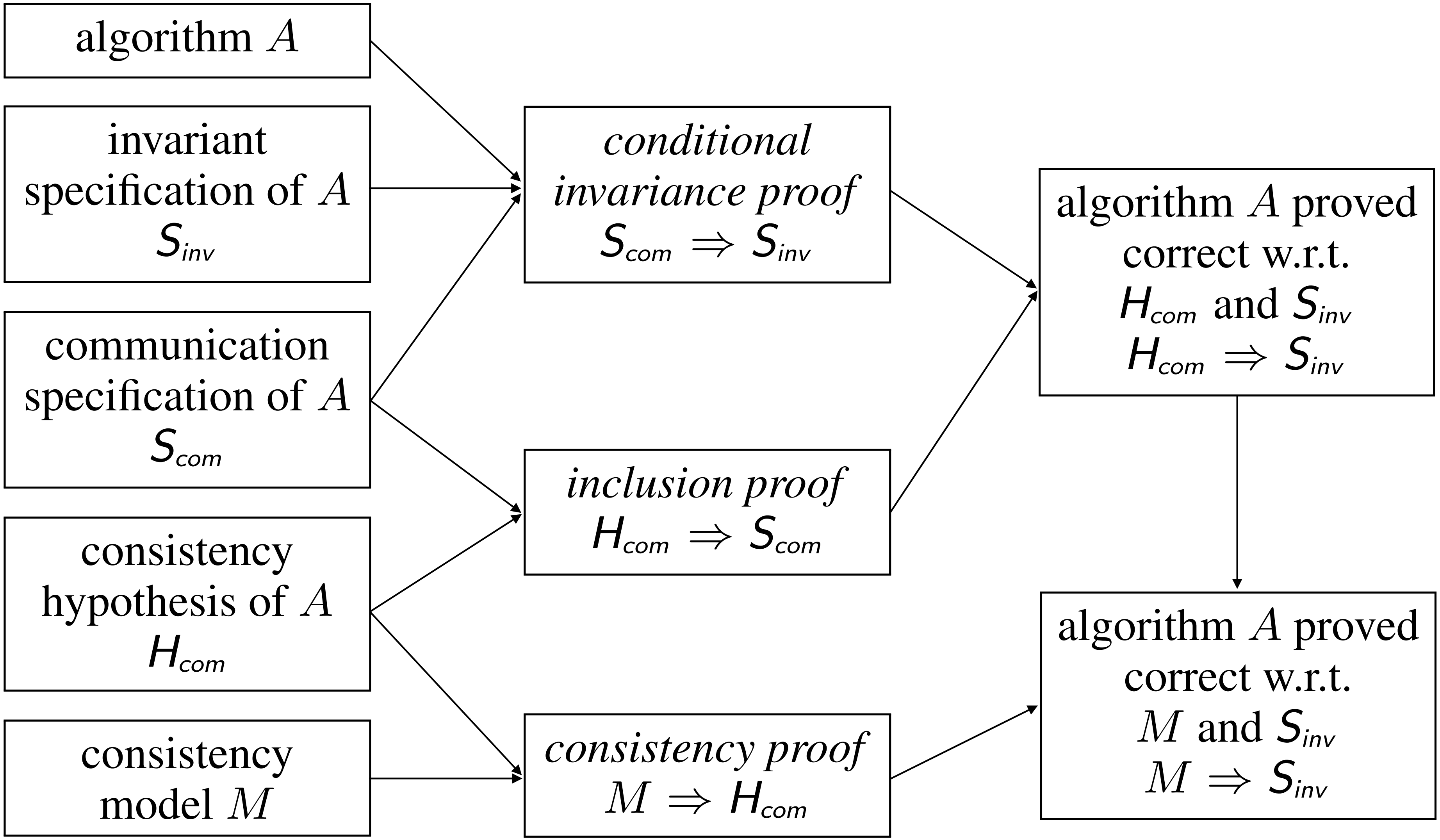
Concurrency with Weak Memory Models: Semantics, Languages, Compilation, Verification, Static Analysis, and Synthesis

November 22 , 2016

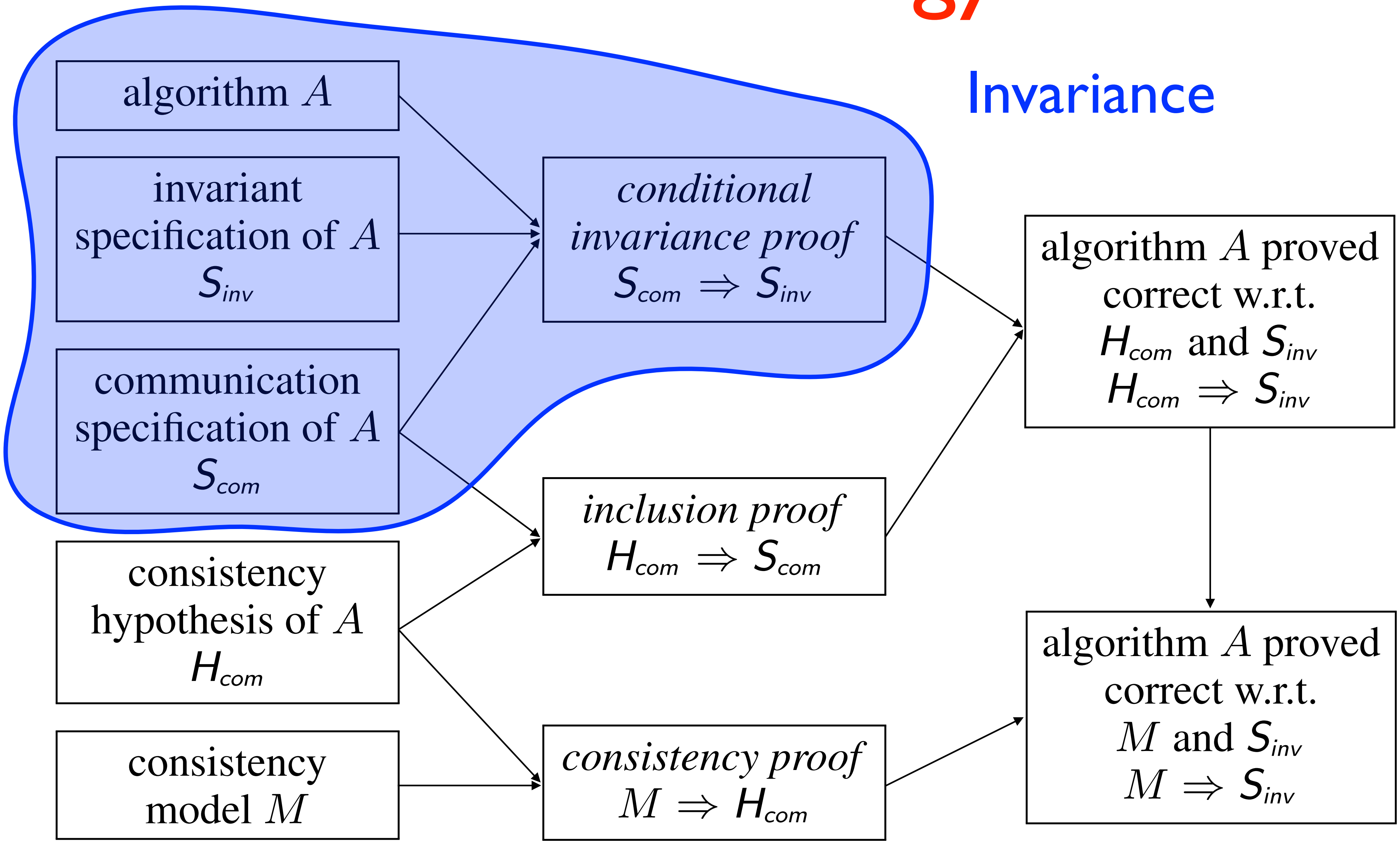
PostgreSQL

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
1: do
2:   do
3:     r[] R10 latch0
4:   while (R10=0)
5:   w[] latch0 0
6:   r[] Rf0 flag0
7:   if (Rf0≠0) then
8:     (* critical section *)
       w[] flag0 0
9:     w[] flag1 1
10:    w[] latch1 1
11:  fi
12:while true
13:
||
21:do
22:  do
23:    r[] R11 latch1
24:  while (R11=0)
25:  w[] latch1 0
26:  r[] Rf1 flag1
27:  if (Rf1≠0) then
28:    (* critical section *)
       w[] flag1 0
29:    w[] flag0 1
30:    w[] latch0 1
31:  fi
32:while true
33:
```

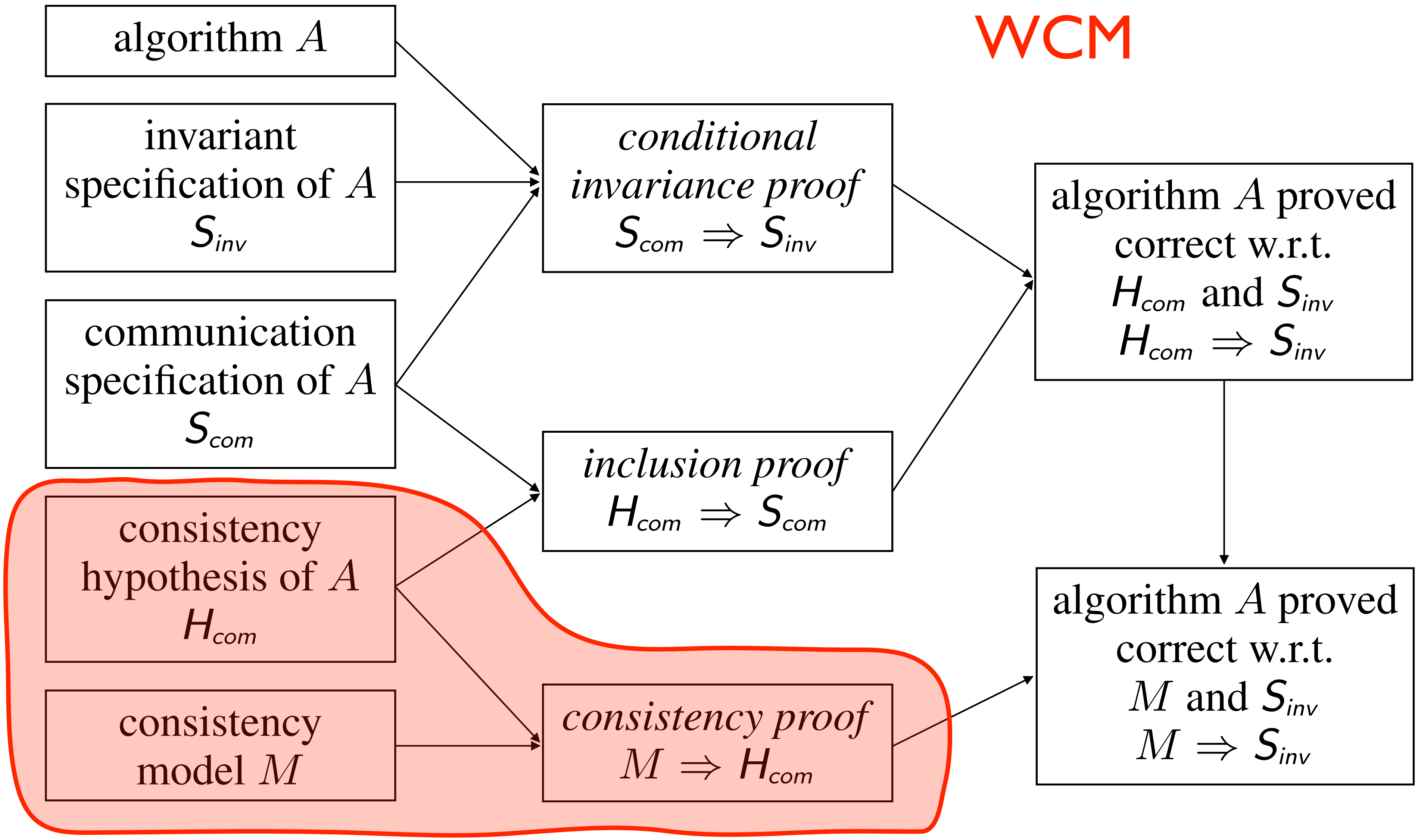
Methodology



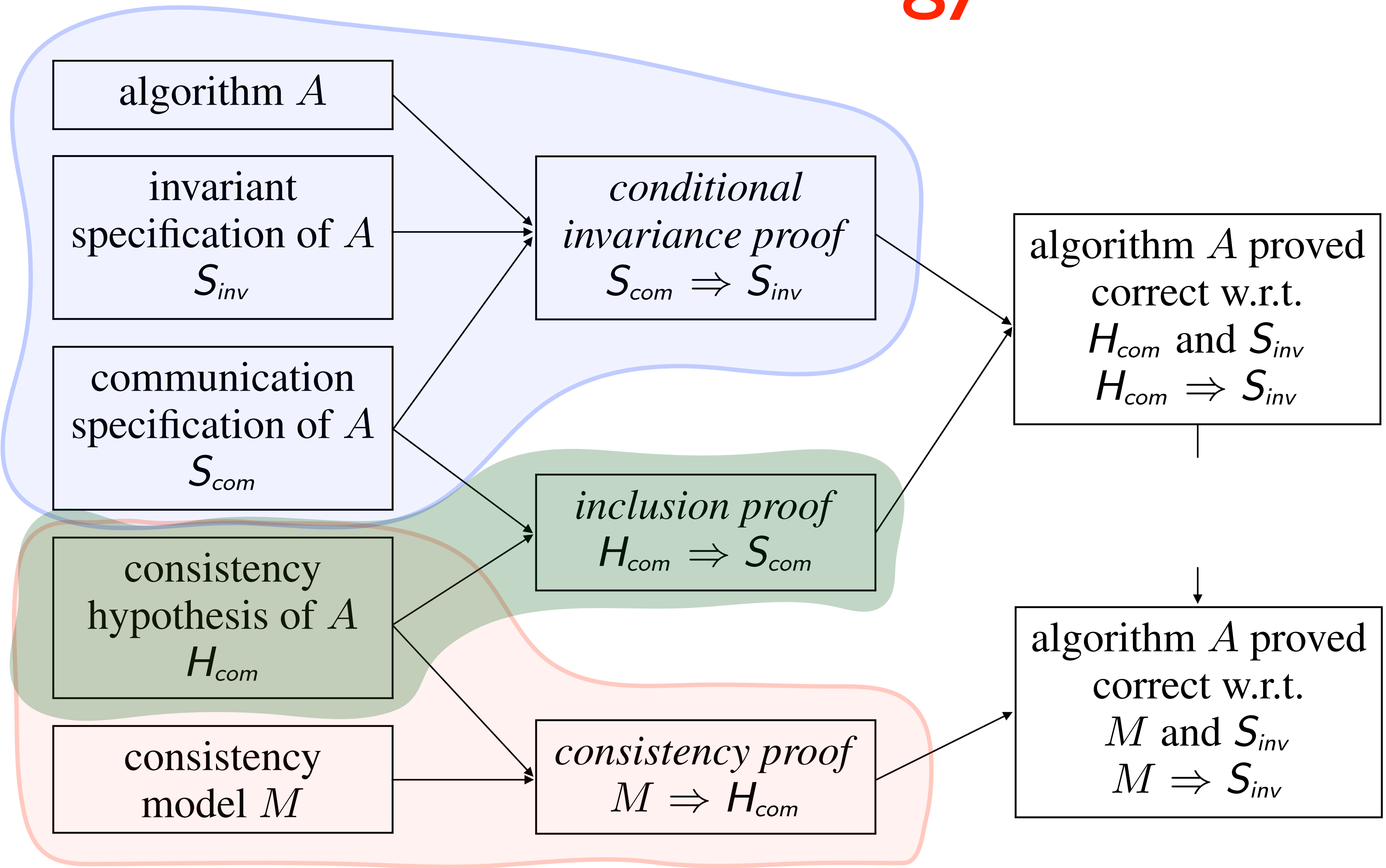
Methodology



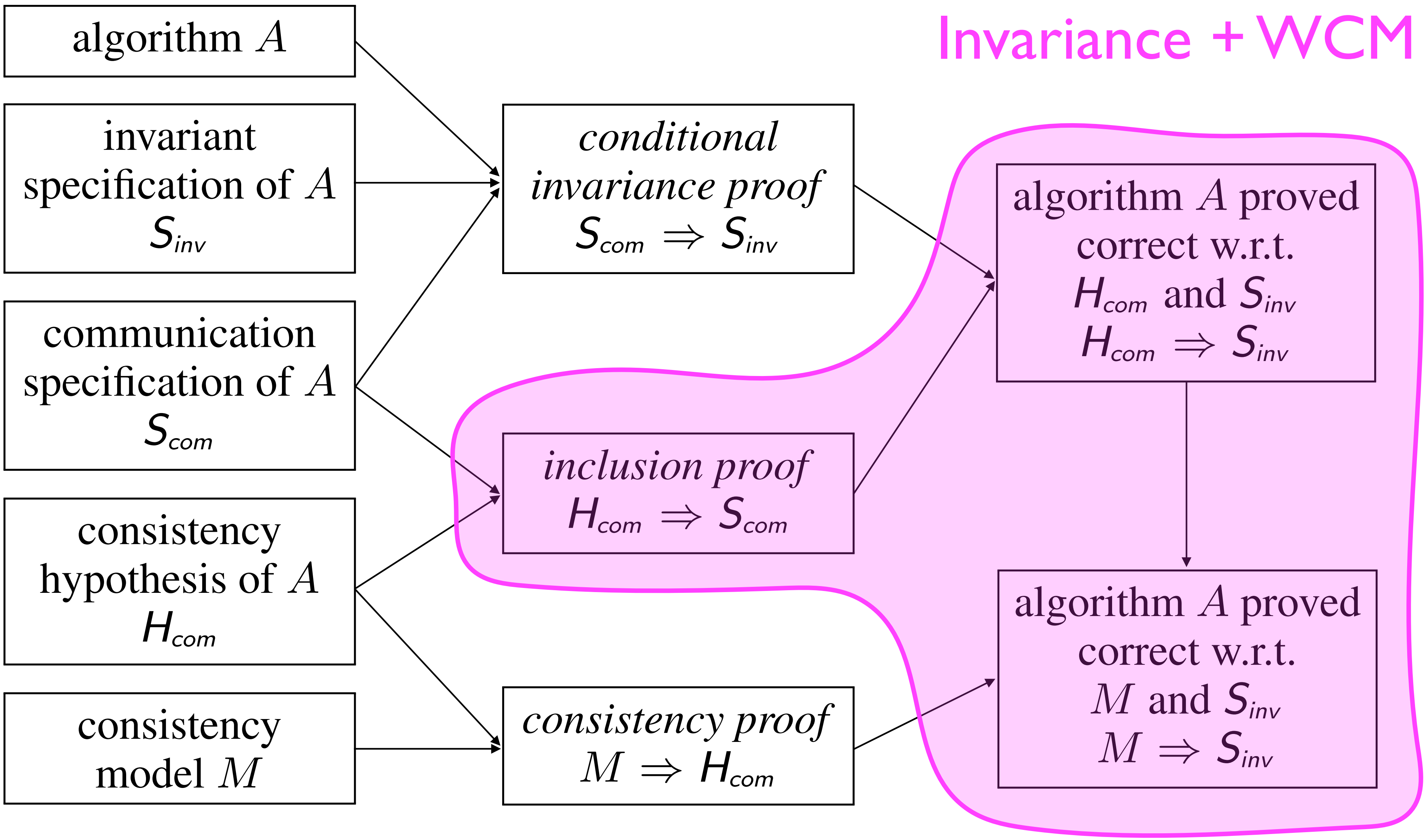
Methodology



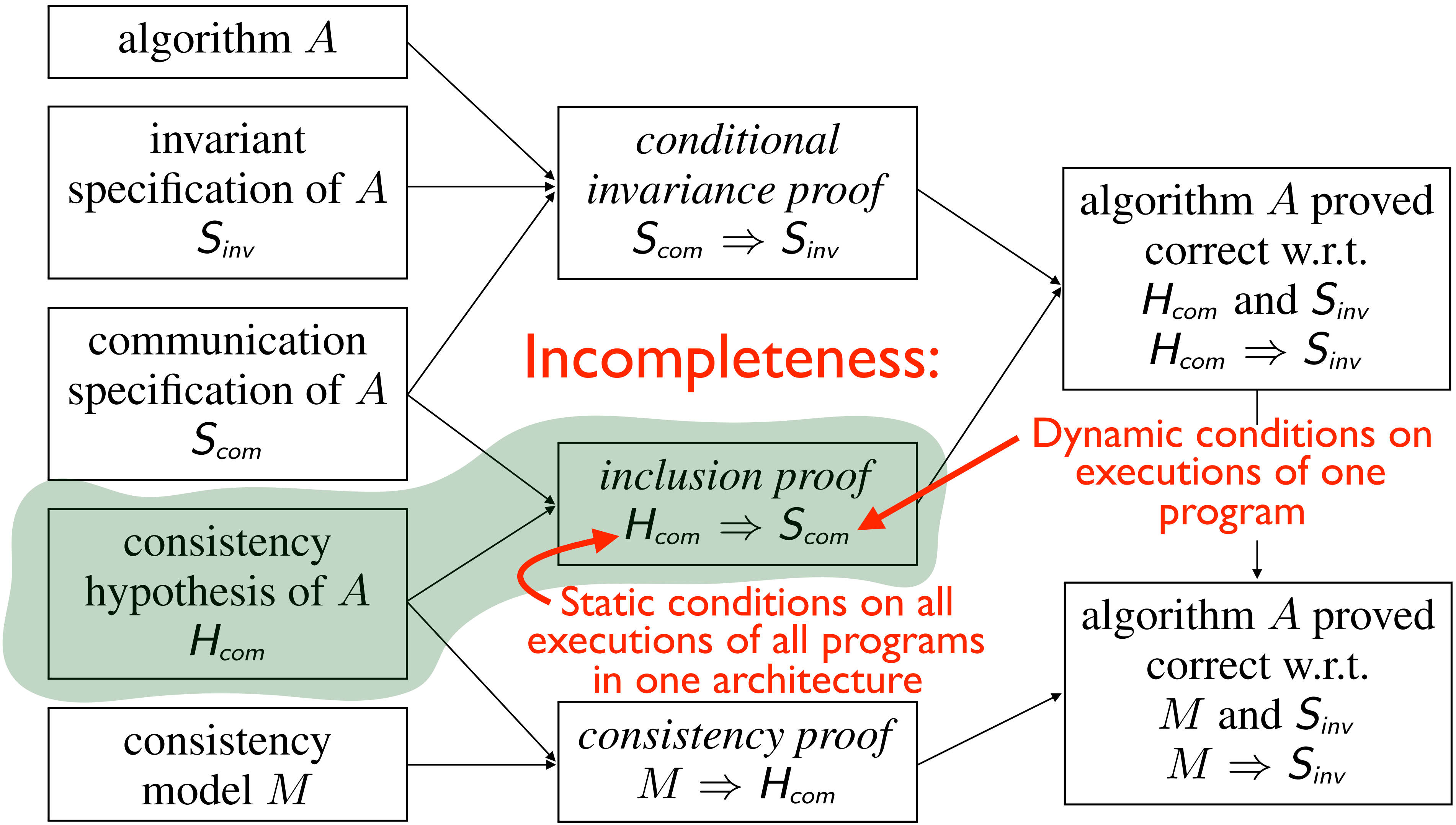
Methodology



Methodology

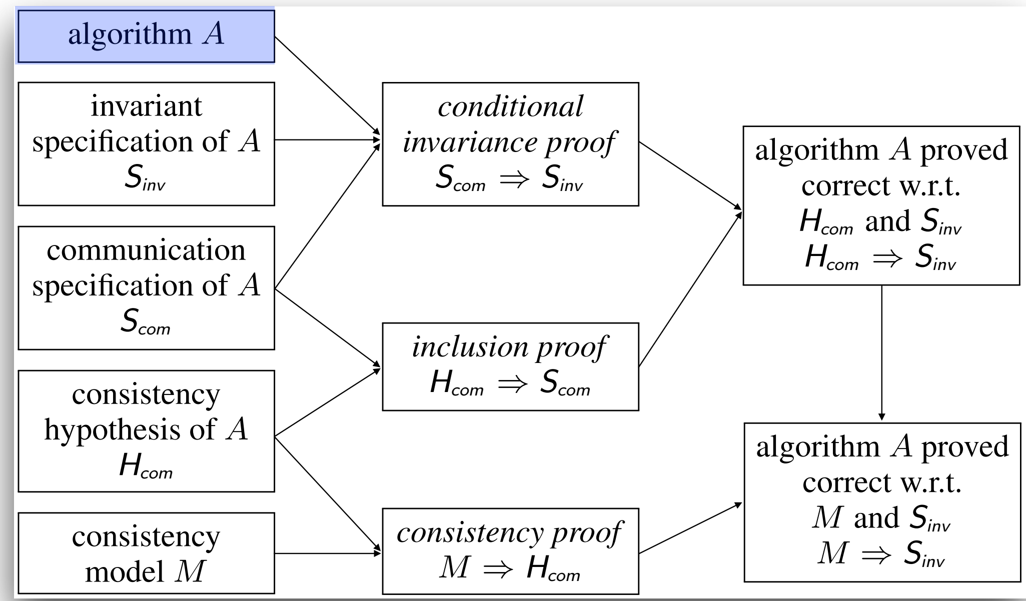


Methodology



Conditional invariance proof: Mutual exclusion

Algorithm



algorithm A
invariant specification of A S_{inv}
communication specification of A S_{com}
consistency hypothesis of A H_{com}
consistency model M
conditional invariance proof $S_{com} \Rightarrow S_{inv}$
inclusion proof $H_{com} \Rightarrow S_{com}$
consistency proof $M \Rightarrow H_{com}$
algorithm A proved correct w.r.t. H_{com} and S_{inv} $H_{com} \Rightarrow S_{inv}$
algorithm A proved correct w.r.t. M and S_{inv} $M \Rightarrow S_{inv}$

PostgreSQL

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
1: do {i}
2:    do {j_i}
3:       r[] R10 latch0 {⇝ L0^i_{j_i}}
4:    while (R10=0) {k_i}
5:    w[] latch0 0
6:    r[] Rf0 flag0 {⇝ F0^i}
7:    if (Rf0≠0) then
8:       (* critical section *)
         w[] flag0 0
9:       w[] flag1 1
10:      w[] latch1 1
11:   fi
12:while true
13:
```

```
21:do {ℓ}
22:   do {m_ℓ}
23:      r[] R11 latch1 {⇝ L1^ℓ_{m_ℓ}}
24:   while (R11=0) {n_ℓ}
25:   w[] latch1 0
26:   r[] Rf1 flag1 {⇝ F1^ℓ}
27:   if (Rf1≠0) then
28:      (* critical section *)
         w[] flag1 0
29:      w[] flag0 1
30:      w[] latch0 1
31:   fi
32:while true
33:
```

Stamps

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
```

```
1: do {$i$}
2:   do {$j_i$}
3:     r[] Rl0 latch0 {$\rightsquigarrow L0^{i}_{j_i}$}
4:   while (Rl0=0) {$k_i$}
5:   w[] latch0 0
6:   r[] Rf0 flag0 {$\rightsquigarrow F0^{i}$}
7:   if (Rf0≠0) then
8:     (* critical section *)
       w[] flag0 0
9:     w[] flag1 1
10:    w[] latch1 1
11:  fi
12:while true
13:
```

```
21:do {$\ell$}
22:  do {$m_\ell$}
23:    r[] Rl1 latch1 {$\rightsquigarrow L1^{\ell}_{m_\ell}$}
24:  while (Rl1=0) {$n_\ell$}
25:  w[] latch1 0
26:  r[] Rf1 flag1 {$\rightsquigarrow F1^{\ell}$}
27:  if (Rf1≠0) then
28:    (* critical section *)
       w[] flag1 0
29:    w[] flag0 1
30:    w[] latch0 1
31:  fi
32:while true
33:
```

Ensure that events are unique (your choice)

Variables in Hoare logic & L/O-G

- program variables: `int x;`
- in predicates you need to name the value of variable `x` to express properties of this value of x:
  - $\mathrm{valueof}(x)$
  - x
- WCM: no notion of "the" value of a shared variable `x`
- The only way to know something about "the" value of a shared variable `x` is to read it
- Pythia variable: name given to the read value
- Not necessary in the semantics, only in assertions (but we put them in the semantics)

Pythia variables

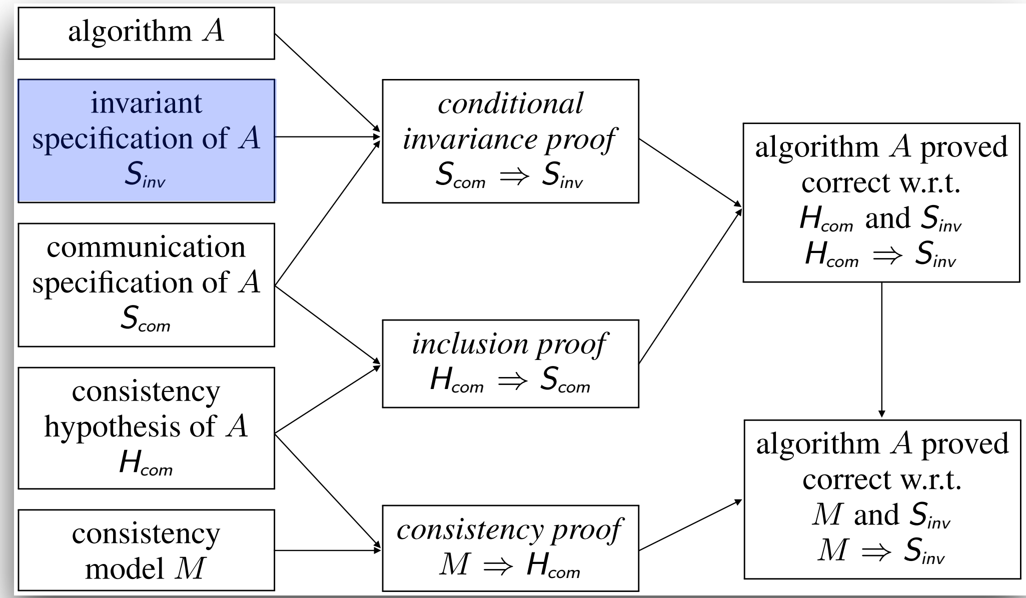
`{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }`

`1: do` $\{i\}$
`2:    do` $\{j_i\}$
`3:       r[] R10 latch0` $\{\leadsto L0^{i}_{j_i}\}$
`4:    while (R10=0)` $\{k_i\}$
`5:    w[] latch0 0`
`6:    r[] Rf0 flag0` $\{\leadsto F0^{i}\}$
`7:    if (Rf0≠0) then`
`8:       (* critical section *)`
`         w[] flag0 0`
`9:       w[] flag1 1`
`10:      w[] latch1 1`
`11:   fi`
`12:while true`
`13:`

$\|$

`21:do` $\{\ell\}$
`22:   do` $\{m_\ell\}$
`23:      r[] R11 latch1` $\{\leadsto L1^{\ell}_{m_\ell}\}$
`24:   while (R11=0)` $\{n_\ell\}$
`25:   w[] latch1 0`
`26:   r[] Rf1 flag1` $\{\leadsto F1^{\ell}\}$
`27:   if (Rf1≠0) then`
`28:      (* critical section *)`
`         w[] flag1 0`
`29:      w[] flag0 1`
`30:      w[] latch0 1`
`31:   fi`
`32:while true`
`33:`

Invariant specification S_{inv}



Mutual exclusion

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
```

```
1: do {i}
2:    do {j_i}
3:      r[] Rl0 latch0 {⇝ L0^i_{j_i}}
4:    while (Rl0=0) {k_i}
5:    w[] latch0 0
6:    r[] Rf0 flag0 {⇝ F0^i}
7:    if (Rf0≠0) then
8:      ¬at{28}
        (* critical section *)
        w[] flag0 0
9:      w[] flag1 1
10:     w[] latch1 1
11:   fi
12:while true
13:
```

||

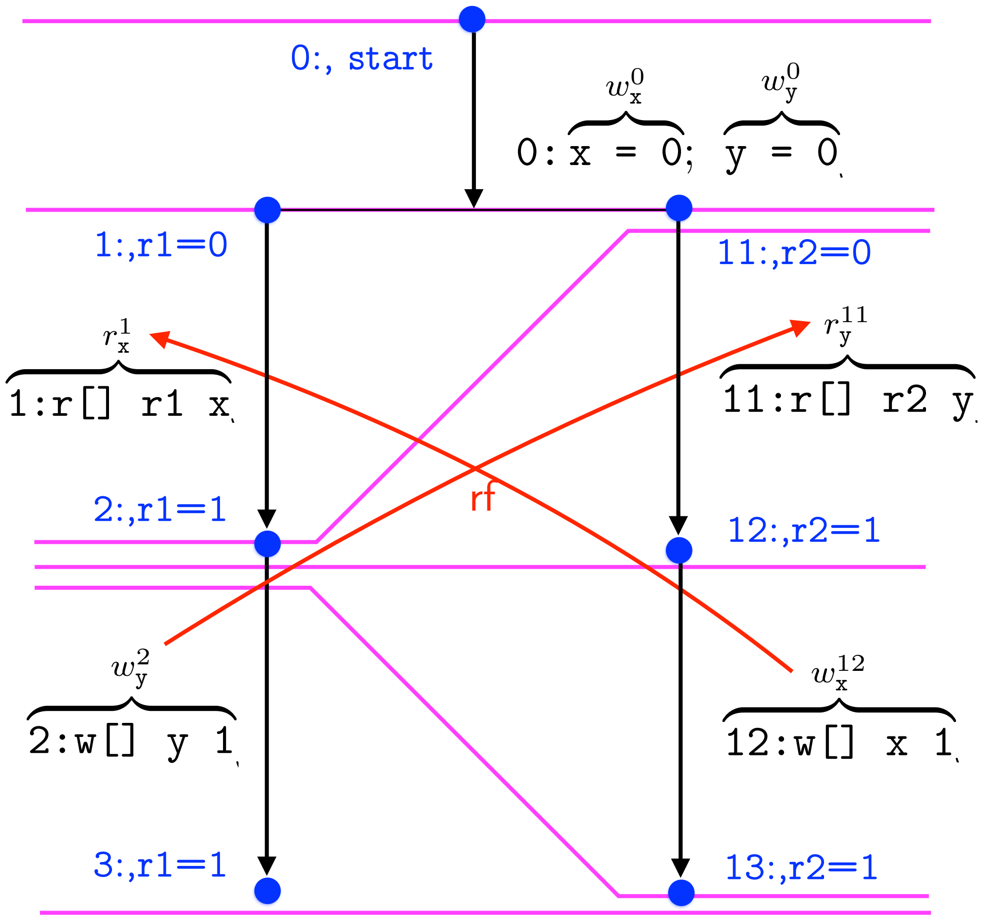
```
21:do {ℓ}
22:   do {m_ℓ}
23:     r[] Rl1 latch1 {⇝ L1^ℓ_{m_ℓ}}
24:   while (Rl1=0) {n_ℓ}
25:   w[] latch1 0
26:   r[] Rf1 flag1 {⇝ F1^ℓ}
27:   if (Rf1≠0) then
28:     ¬at{8}
        (* critical section *)
        w[] flag1 0
29:     w[] flag0 1
30:     w[] latch0 1
31:   fi
32:while true
33:
```

(invariant S_{inv} is elsewhere `true`)

Analytic semantics = Anarchic semantics + communication constraints

Analytics semantics with cuts

```
0:{ x = 0; y = 0; }
P0           || P1           ;
1:r[] r1 x   || 11:r[] r2 y;
2:w[] y 1    || 12:w[] x 1 ;
3:           || 13:        ;
```



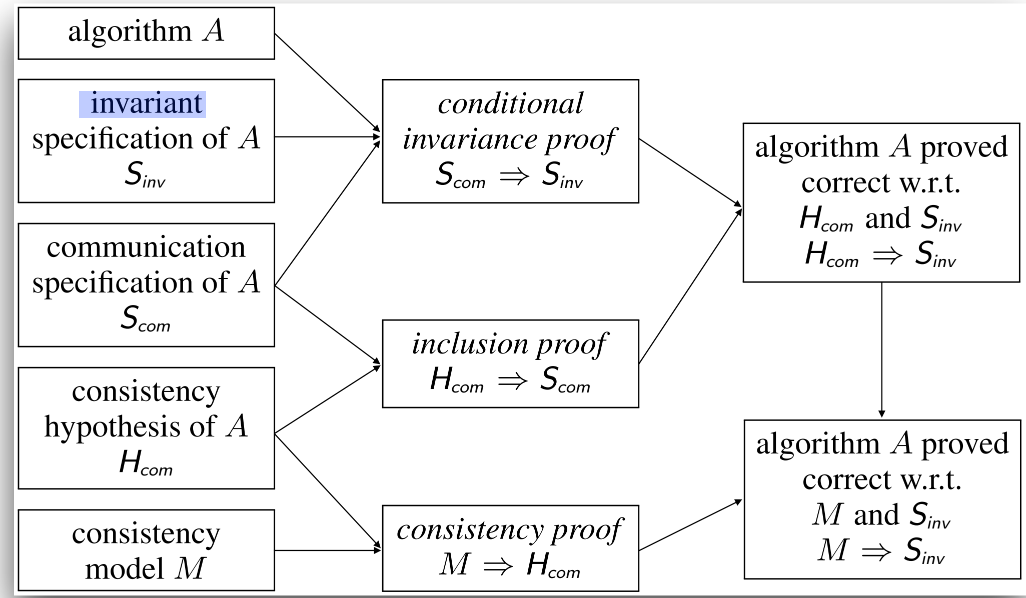
- Anarchic semantics: set of executions:

  $\pi = \varsigma \times \pi \times \mathsf{rf}$

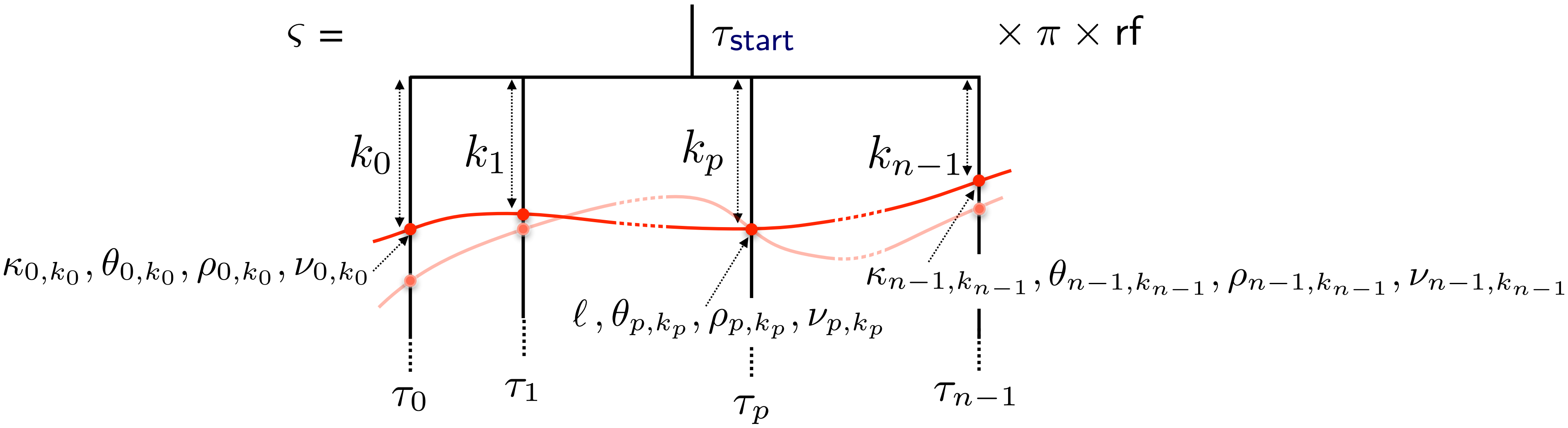
  - ς is the *computation*
  - π is the *cut sequence*
  - rf is the *communication*

- Communication semantics: restrictions on rf in `cat`

Local invariants



Local invariant



- Attached to each program point ℓ of each process p
- Depends on
  - Program points of all other processes κ
  - Stamps θ of all processes
  - Local registers of all processes ρ
  - Pythia variables ν
  - Communications (rf)

Communication relation rf

- rf: relation between write and read events
- Each rf is encoded by $\mathit{\Gamma}$, a set of pairs

$$\mathfrak{rf}\langle x_\theta,\ \langle \ell :,\ \theta',\ v\rangle\rangle$$

Pythia variable of the read event

Program label of the write action

Stamp of the write event

Value write

- $\mathit{\Gamma} \in \Gamma$ (the set of all possible communications rf)

Anarchic communications

Anarchic communications

- Any read can read from any write on the same shared variable (location)

$$\mathrm{RL0}^{i}_{j_i} \triangleq \{\mathfrak{rf}\langle L0^{i}_{j_i},\ \langle 0\!:,\ \_,\ 0\rangle\rangle, \mathfrak{rf}\langle L0^{i}_{j_i},\ \langle 5\!:,\ i_5,\ 0\rangle\rangle, \mathfrak{rf}\langle L0^{i}_{j_i},\ \langle 30\!:,\ \ell_{30},\ 1\rangle\rangle \mid i_5 \in \mathbb{N} \wedge \ell_{30} \in \mathbb{N}\}$$

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
1: do {i}
2:    do {j_i}
3:      r[] R10 latch0 {~> L0^i_{j_i}}
4:    while (R10=0) {k_i}
5:    w[] latch0 0
6:    r[] Rf0 flag0 {~> F0^i}
7:    if (Rf0≠0) then
8:      (* critical section *)
        w[] flag0 0
9:      w[] flag1 1
10:     w[] latch1 1
11:   fi
12:while true
13:
||
21:do {ℓ}
22:   do {m_ℓ}
23:     r[] R11 latch1 {~> L1^ℓ_{m_ℓ}}
24:   while (R11=0) {n_ℓ}
25:   w[] latch1 0
26:   r[] Rf1 flag1 {~> F1^ℓ}
27:   if (Rf1≠0) then
28:     (* critical section *)
        w[] flag1 0
29:     w[] flag0 1
30:     w[] latch0 1
31:   fi
32:while true
33:
```

Anarchic communications

- Possible communications for each read at each stamp (point in the execution):

$$\mathrm{RL0}^i_{j_i} \triangleq \{\mathfrak{rf}\langle L0^i_{j_i},\ \langle \mathtt{0:},\ \_,\ 0\rangle\rangle, \mathfrak{rf}\langle L0^i_{j_i},\ \langle \mathtt{5:},\ i_5,\ 0\rangle\rangle, \mathfrak{rf}\langle L0^i_{j_i},\ \langle \mathtt{30:},\ \ell_{30},\ 1\rangle\rangle \mid i_5 \in \mathbb{N} \wedge \ell_{30} \in \mathbb{N}\}$$
$$\mathrm{RF0}^i \triangleq \{\mathfrak{rf}\langle F0^i,\ \langle \mathtt{0:},\ \_,\ 0\rangle\rangle, \mathfrak{rf}\langle F0^i,\ \langle \mathtt{8:},\ i_8,\ 0\rangle\rangle, \mathfrak{rf}\langle F0^i,\ \langle \mathtt{29:},\ \ell_{29},\ 1\rangle\rangle \mid i_8 \in \mathbb{N} \wedge \ell_{29} \in \mathbb{N}\}$$
$$\mathrm{RL1}^\ell_{m_\ell} \triangleq \{\mathfrak{rf}\langle L1^\ell_{m_\ell},\ \langle \mathtt{0:},\ \_,\ 1\rangle\rangle, \mathfrak{rf}\langle L1^\ell_{m_\ell},\ \langle \mathtt{25:},\ \ell_{25},\ 0\rangle\rangle, \mathfrak{rf}\langle L1^\ell_{m_\ell},\ \langle \mathtt{10:},\ i_{10},\ 1\rangle\rangle \mid \ell_{25} \in \mathbb{N} \wedge i_{10} \in \mathbb{N}\}$$
$$\mathrm{RF1}^\ell \triangleq \{\mathfrak{rf}\langle F1^\ell,\ \langle \mathtt{0:},\ \_,\ 1\rangle\rangle, \mathfrak{rf}\langle F1^\ell,\ \langle \mathtt{28:},\ \ell_{28},\ 0\rangle\rangle, \mathfrak{rf}\langle F1^\ell,\ \langle \mathtt{9:},\ i_9,\ 1\rangle\rangle \mid \ell_{28} \in \mathbb{N} \wedge i_9 \in \mathbb{N}\}$$

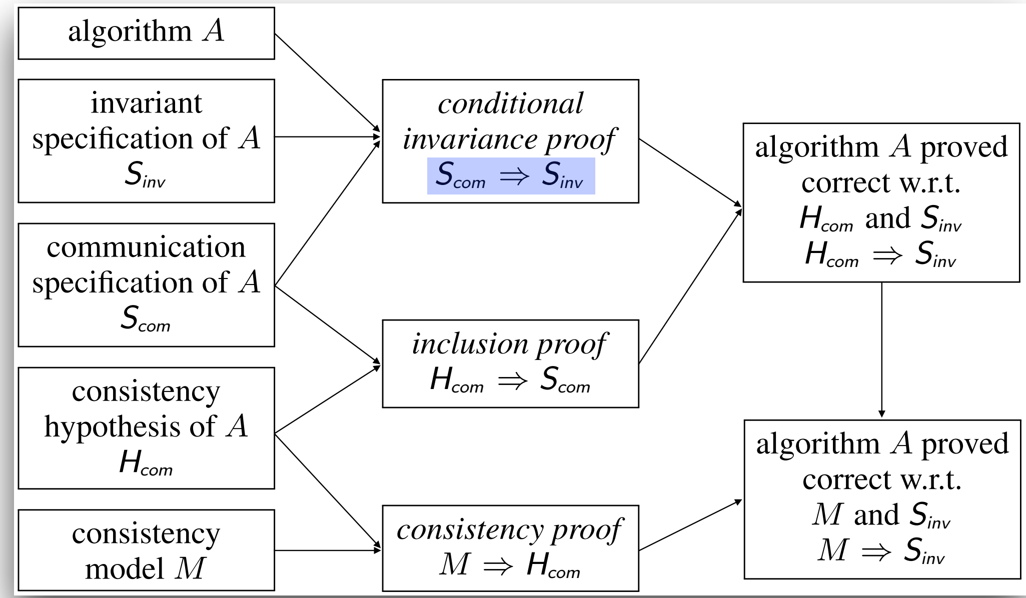
- Anarchic communications:

$$\overline{\Gamma} = \{\{\mathrm{rl0}^i_{j_i}, \mathrm{rf0}^i, \mathrm{rl1}^\ell_{m_\ell}, \mathrm{rf1}^\ell \mid i \in \mathbb{N} \wedge j_i \in [0, k_i] \wedge \ell \in \mathbb{N} \wedge j \in [0, n_\ell]\} \mid \forall i \in \mathbb{N}\ .\ \forall j_i \in [1, k_i]\ .\ \mathrm{rl0}^i_{j_i} \in \mathrm{RL0}^i_{j_i} \wedge \mathrm{rf0}^i \in \mathrm{RF0}^i \wedge \forall \ell \in \mathbb{N}\ .\ \forall m_\ell \in [1, m_\ell]\ .\ \mathrm{rl1}^\ell_{m_\ell} \in \mathrm{RL1}^\ell_{m_\ell} \wedge \mathrm{rf1}^\ell \in \mathrm{RF1}^\ell\}$$

- Anarchic semantics: $\mathit{\Gamma} \in \overline{\Gamma}$

- WCM semantics: $\mathit{\Gamma} \in \Gamma, \Gamma \subseteq \overline{\Gamma}$

Inductive invariant S_{ind}



Inductive invariant

- S_{ind} is inductive under hypothesis S_{com} iff, assuming S_{com}, we have:
  - S_{ind} is true at the beginning of an execution
  - If S_{ind} is true during execution is remains true after one more computation or communication step
- $$\frac{S_{inv} \text{ holds under hypothesis } S_{com} \qquad S_{ind} \Rightarrow S_{inv}}{S_{com} \Rightarrow S_{inv}}$$

Inductive invariant

`{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }`

1: $\{\mathit{\Gamma} \in \Gamma\}$
  `do` $\{i\}$
2: $\{\mathit{\Gamma} \in \Gamma\}$
  `do` $\{j_i\}$
3: $\{\mathit{\Gamma} \in \Gamma\}$
  `r[] Rl0 latch0` $\{\leadsto L0^i_{j_i}\}$
4: $\{\mathit{\Gamma} \in \Gamma \wedge \texttt{Rl0} = L0^i_{j_i} \wedge (\mathrm{r0Rl0}^i_{j_i}[\mathit{\Gamma}] \vee \mathrm{r1Rl0}^i_{j_i}[\mathit{\Gamma}])\}$
  `while (Rl0=0)` $\{k_i\}$
5: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}]\}$
  `w[] latch0 0`
6: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}]\}$
  `r[] Rf0 flag0` $\{\leadsto F0^i\}$
7: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}] \wedge \texttt{Rf0} = F0^i \wedge (\mathrm{r0Rf0}^i[\mathit{\Gamma}] \vee \mathrm{r1Rf0}^i[\mathit{\Gamma}])\}$
  `if (Rf0≠0) then`
8: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
  `(* critical section *)`
  `w[] flag0 0`
9: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
  `w[] flag1 1`
10: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
  `w[] latch1 1`
11: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
  `fi`
12: $\{\mathit{\Gamma} \in \Gamma\}$
  `while true`
13: {false}

21: $\{\mathit{\Gamma} \in \Gamma\}$
  `do` $\{\ell\}$
22: $\{\mathit{\Gamma} \in \Gamma\}$
  `do` $\{m_\ell\}$
23: $\{\mathit{\Gamma} \in \Gamma\}$
  `r[] Rl1 latch1` $\{\leadsto L1^\ell_{m_\ell}\}$
24: $\{\mathit{\Gamma} \in \Gamma \wedge \texttt{Rl1} = L1^\ell_{m_\ell} \wedge (\mathrm{r0Rl1}^\ell_{m_\ell}[\mathit{\Gamma}] \vee \mathrm{r1Rl1}^\ell_{m_\ell}[\mathit{\Gamma}])\}$
  `while (Rl1=0)` $\{n_\ell\}$
25: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}]\}$
  `w[] latch1 0`
26: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}]\}$
  `r[] Rf1 flag1` $\{\leadsto F1^\ell\}$
27: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \texttt{Rf1} = F1^\ell \wedge (\mathrm{r0Rf1}^\ell[\mathit{\Gamma}] \vee \mathrm{r1Rf1}^\ell[\mathit{\Gamma}])\}$
  `if (Rf1≠0) then`
28: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
  `(* critical section *)`
  `w[] flag1 0`
29: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
  `w[] flag0 1`
30: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
  `w[] latch0 1`
31: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
  `fi`
32: $\{\mathit{\Gamma} \in \Gamma\}$
  `while true`
33: {false}

Inductive invariant

`{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }`

1: $\{\mathit{\Gamma} \in \Gamma\}$
`do` $\{i\}$
2: $\{\mathit{\Gamma} \in \Gamma\}$
`do` $\{j_i\}$
3: $\{\mathit{\Gamma} \in \Gamma\}$
`r[] R10 latch0` $\{\leadsto L0^i_{j_i}\}$
4: $\{\mathit{\Gamma} \in \Gamma \wedge \mathtt{R10} = L0^i_{j_i} \wedge (\mathrm{r0R10}^i_{j_i}[\mathit{\Gamma}] \vee \mathrm{r1}$ …
`while (R10=0)` $\{k_i\}$
5: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\mathit{\Gamma}]\}$
`w[] latch0 0`
6: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\mathit{\Gamma}]\}$
`r[] Rf0 flag0` $\{\leadsto F0^i\}$
7: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\mathit{\Gamma}] \wedge \mathtt{Rf0} = F0^i \wedge (\mathrm{r0Rf0}^i[\mathit{\Gamma}] \vee \mathrm{r1Rf0}^i[\mathit{\Gamma}])\}$
`if (Rf0`$\neq$`0) then`
8: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
`(* critical section *)`
`w[] flag0 0`
9: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
`w[] flag1 1`
10: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
`w[] latch1 1`
11: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
`fi`
12: $\{\mathit{\Gamma} \in \Gamma\}$
`while true`
13: {false}

21: $\{\mathit{\Gamma} \in \Gamma\}$
`do` $\{\ell\}$
22: $\{\mathit{\Gamma} \in \Gamma\}$
… $\mathrm{R11}^\ell_{m_\ell}[\mathit{\Gamma}])\}$
`w[] latch1 0`
26: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^\ell_{n_\ell}[\mathit{\Gamma}]\}$
`r[] Rf1 flag1` $\{\leadsto F1^\ell\}$
27: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathtt{Rf1} = F1^\ell \wedge (\mathrm{r0Rf1}^\ell[\mathit{\Gamma}] \vee \mathrm{r1Rf1}^\ell[\mathit{\Gamma}])\}$
`if (Rf1`$\neq$`0) then`
28: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
`(* critical section *)`
`w[] flag1 0`
29: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
`w[] flag0 1`
30: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
`w[] latch0 1`
31: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
`fi`
32: $\{\mathit{\Gamma} \in \Gamma\}$
`while true`
33: {false}

Possible communications

Inductive invariant

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
```

1: $\{\Gamma \in \Gamma\}$
do $\{i\}$
2: $\{\Gamma \in \Gamma\}$
do $\{j_i\}$
3: $\{\Gamma \in \Gamma\}$
r[] Rl0 latch0 $\{\rightsquigarrow L0^i_{j_i}\}$
4: $\{\Gamma \in \Gamma \wedge \texttt{Rl0} = L0^i_{j_i} \wedge (\mathrm{r0Rl0}^i_{j_i}[\Gamma] \vee \mathrm{r1Rl0}^i_{j_i}[\Gamma])\}$
while (Rl0=0) $\{k_i\}$
5: $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\Gamma]\}$
w[] latch0 0
6: $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\Gamma]\}$
r[] Rf0 flag0 $\{\rightsquigarrow F0^i\}$
7: $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\Gamma] \wedge \texttt{Rf0} = F0^i \wedge (\mathrm{r0Rf0}^i[\Gamma] \vee \mathrm{r1Rf0}^i[\Gamma])\}$
if (Rf0≠0) then
8: $\{\Gamma \in$
(* c
w[]
9: $\{\Gamma \in$
w[]
10: $\{\Gamma \in$
w[]
11: $\{\Gamma \in$
fi
12: $\{\Gamma \in \Gamma\}$
while true
13: {false}

21: $\{\Gamma \in \Gamma\}$
do $\{\ell\}$
22: $\{\Gamma \in \Gamma\}$
do $\{m_\ell\}$
23: $\{\Gamma \in \Gamma\}$
r[] Rl1 latch1 $\{\rightsquigarrow L1^\ell_{m_\ell}\}$
24: $\{\Gamma \in \Gamma \wedge \texttt{Rl1} = L1^\ell_{m_\ell} \wedge (\mathrm{r0Rl1}^\ell_{m_\ell}[\Gamma] \vee \mathrm{r1Rl1}^\ell_{m_\ell}[\Gamma])\}$
while (Rl1=0) $\{n_\ell\}$
25: $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\Gamma]\}$
w[] latch1 0
26: $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\Gamma]\}$
r[] Rf1 flag1 $\{\rightsquigarrow F1^\ell\}$
27: $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\Gamma] \wedge \texttt{Rf1} = F1^\ell \wedge (\mathrm{r0Rf1}^\ell[\Gamma] \vee \mathrm{r1Rf1}^\ell[\Gamma])\}$
if (Rf1≠0) then
$\Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\Gamma] \wedge \mathrm{r1Rf1}^\ell[\Gamma]\}$
itical section *)
lag1 0
$\Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\Gamma] \wedge \mathrm{r1Rf1}^\ell[\Gamma]\}$
lag0 1
$\Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\Gamma] \wedge \mathrm{r1Rf1}^\ell[\Gamma]\}$
atch0 1
$\Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\Gamma] \wedge \mathrm{r1Rf1}^\ell[\Gamma]\}$
fi
32: $\{\Gamma \in \Gamma\}$
while true
33: {false}

Register assignment of the Pythia variable after read event

Inductive invariant

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
1:  {Γ ∈ Γ}
    do {i}
2:    {Γ ∈ Γ}
      do {j_i}
3:      {Γ ∈ Γ}
        r[] R10 latch0 {⤳ L0^i_{j_i}}
4:      {Γ ∈ Γ ∧ R10 = L0^i_{j_i} ∧ (r0R10^i_{j_i}[Γ] ∨ r1R10^i_{j_i}[Γ])}
      while (R10=0) {k_i}
5:    {Γ ∈ Γ ∧ r1R10^i_{k_i}[Γ]}
      w[] latch0 0
        w[] flag0 0
9:      {Γ ∈ Γ ∧ r1R10^i_{k_i}[Γ] ∧ r1Rf0^i[Γ]}
        w[] flag1 1
10:     {Γ ∈ Γ ∧ r1R10^i_{k_i}[Γ] ∧ r1Rf0^i[Γ]}
        w[] latch1 1
11:     {Γ ∈ Γ ∧ r1R10^i_{k_i}[Γ] ∧ r1Rf0^i[Γ]}
      fi
12:   {Γ ∈ Γ}
    while true
13:{false}
```

```
21:{Γ ∈ Γ}
    do {ℓ}
22:   {Γ ∈ Γ}
      do {m_ℓ}
23:     {Γ ∈ Γ}
        r[] R11 latch1 {⤳ L1^ℓ_{m_ℓ}}
24:     {Γ ∈ Γ ∧ R11 = L1^ℓ_{m_ℓ} ∧ (r0R11^ℓ_{m_ℓ}[Γ] ∨ r1R11^ℓ_{m_ℓ}[Γ])}
      while (R11=0) {n_ℓ}
25:   {Γ ∈ Γ ∧ r1R11^ℓ_{n_ℓ}[Γ]}
      w[] latch1 0
        w[] flag1 0
29:     {Γ ∈ Γ ∧ r1R11^ℓ_{n_ℓ}[Γ] ∧ r1Rf1^ℓ[Γ]}
        w[] flag0 1
30:     {Γ ∈ Γ ∧ r1R11^ℓ_{n_ℓ}[Γ] ∧ r1Rf1^ℓ[Γ]}
        w[] latch0 1
31:     {Γ ∈ Γ ∧ r1R11^ℓ_{n_ℓ}[Γ] ∧ r1Rf1^ℓ[Γ]}
      fi
32:   {Γ ∈ Γ}
    while true
33:{false}
```

Possible values of Pythia variables depending on communications

$$\mathrm{r0R10}^i_{j_i}[\mathit{\Gamma}] \triangleq (\mathfrak{rf}\langle L0^i_{j_i},\ \langle 0:,\ \_,\ 0\rangle\rangle \in \mathit{\Gamma} \wedge L0^i_{j_i} = 0) \vee (\exists i_5 \in \mathbb{N}\ .\ \mathfrak{rf}\langle L0^i_{j_i},\ \langle 5:,\ i_5,\ 0\rangle\rangle \in \mathit{\Gamma} \wedge L0^i_{j_i} = 0)$$

$$\mathrm{r1R10}^i_{j_i}[\mathit{\Gamma}] \triangleq (\exists \ell_{30} \in \mathbb{N}\ .\ \mathfrak{rf}\langle L0^i_{j_i},\ \langle 30:,\ \ell_{30},\ 1\rangle\rangle \in \mathit{\Gamma} \wedge L0^i_{j_i} = 1)$$

Communicated values

- Notation: $\underbrace{\mathrm{r}}_{\text{read}}\underbrace{(0|1)}_{\text{value 0 or 1}}\underbrace{\mathrm{R}(\mathrm{l},\mathrm{f})(0|1)}_{\text{register}}$

latch

fetch

process

$$\mathrm{r0Rl0}^i_{j_i}[\varGamma] \triangleq (\mathfrak{rf}\langle L0^i_{j_i},\ \langle \mathtt{0:},\ \_,\ 0\rangle\rangle \in \varGamma \wedge L0^i_{j_i} = 0) \vee (\exists i_5 \in \mathbb{N}\ .\ \mathfrak{rf}\langle L0^i_{j_i},\ \langle \mathtt{5:},\ i_5,\ 0\rangle\rangle \in \varGamma \wedge L0^i_{j_i} = 0)$$

$$\mathrm{r1Rl0}^i_{j_i}[\varGamma] \triangleq (\exists \ell_{30} \in \mathbb{N}\ .\ \mathfrak{rf}\langle L0^i_{j_i},\ \langle \mathtt{30:},\ \ell_{30},\ 1\rangle\rangle \in \varGamma \wedge L0^i_{j_i} = 1)$$

$$\mathrm{r0Rf0}^i[\varGamma] \triangleq (\mathfrak{rf}\langle F0^i,\ \langle \mathtt{0:},\ \_,\ 0\rangle\rangle \in \varGamma \wedge F0^i = 0) \vee (\exists i_8 \in \mathbb{N}\ .\ \mathfrak{rf}\langle F0^i,\ \langle \mathtt{8:},\ i_8,\ 0\rangle\rangle \in \varGamma \wedge F0^i = 0)$$

$$\mathrm{r1Rf0}^i[\varGamma] \triangleq (\exists \ell_{29} \in \mathbb{N}\ .\ \mathfrak{rf}\langle F0^i,\ \langle \mathtt{29:},\ \ell_{29},\ 1\rangle\rangle \in \varGamma \wedge F0^i = 1)$$

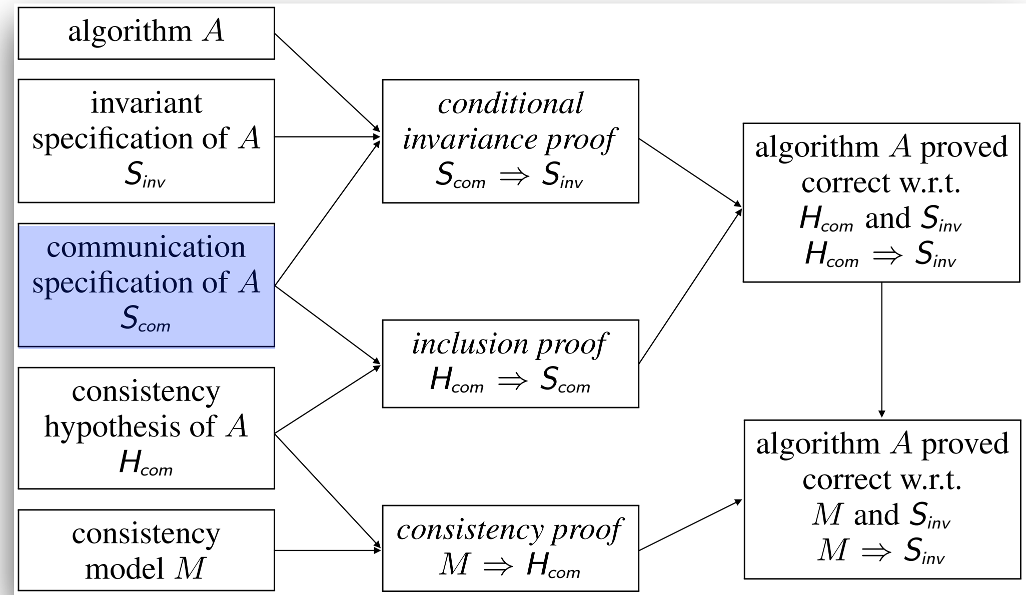
$$\mathrm{r0Rl1}^\ell_{m_\ell}[\varGamma] \triangleq (\exists \ell_{25} \in \mathbb{N}\ .\ \mathfrak{rf}\langle L1^\ell_{m_\ell},\ \langle \mathtt{25:},\ \ell_{25},\ 0\rangle\rangle \in \varGamma \wedge L1^\ell_{m_\ell} = 0)$$

$$\mathrm{r1Rl1}^\ell_{m_\ell}[\varGamma] \triangleq (\mathfrak{rf}\langle L1^\ell_{m_\ell},\ \langle \mathtt{0:},\ \_,\ 1\rangle\rangle \in \varGamma \wedge L1^\ell_{m_\ell} = 1) \vee (\exists i_{10} \in \mathbb{N}\ .\ \mathfrak{rf}\langle L1^\ell_{m_\ell},\ \langle \mathtt{10:},\ i_{10},\ 1\rangle\rangle \in \varGamma \wedge L1^\ell_{m_\ell} = 1)$$

$$\mathrm{r0Rf1}^\ell[\varGamma] \triangleq (\exists m_{28} \in \mathbb{N}\ .\ \mathfrak{rf}\langle F1^\ell,\ \langle \mathtt{28:},\ m_{28},\ 0\rangle\rangle \in \varGamma \wedge F1^\ell = 0)$$

$$\mathrm{r1Rf1}^\ell[\varGamma] \triangleq (\mathfrak{rf}\langle F1^\ell,\ \langle \mathtt{0:},\ \_,\ 1\rangle\rangle \in \varGamma \wedge F1^\ell = 1) \vee (\exists i_9 \in \mathbb{N}\ .\ \mathfrak{rf}\langle F1^\ell,\ \langle \mathtt{9:},\ i_9,\ 1\rangle\rangle \in \varGamma \wedge F1^\ell = 1)$$

Communication specification



Calculational design of the communication specification

$$
\begin{array}{ll}
& (\neg S_{inv}(\varGamma, \Gamma)) \wedge S_{ind}(\varGamma, \Gamma) \\
\triangleq & \mathsf{at}\{8\} \wedge \mathsf{at}\{28\} \wedge S_{ind}(\varGamma, \Gamma) \qquad \text{(def. invariance specification } S_{inv}\text{)} \\
\Rightarrow & \mathsf{at}\{8\} \wedge \mathsf{at}\{28\} \wedge (\exists i, k_i, \ell, n_\ell \in \mathbb{N} \;.\; \varGamma \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\varGamma] \wedge \\
& \mathrm{r1Rf0}^i[\varGamma] \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\varGamma] \wedge \mathrm{r1Rf1}^\ell[\varGamma]) \qquad \text{(by invariant } S_{ind}(\varGamma, \Gamma)\text{)} \\
\Rightarrow & \mathsf{at}\{8\} \wedge \mathsf{at}\{28\} \wedge \big(\exists i, k_i, \ell, n_\ell, \ell_{30}, \ell_{29} \in \mathbb{N} \;.\; \varGamma \in \Gamma \wedge (\mathfrak{rf}\langle L0^i_{k_i}, \\
& \langle \mathtt{30:},\ \ell_{30},\ 1\rangle\rangle \in \varGamma) \wedge (\mathfrak{rf}\langle F0^i,\ \langle \mathtt{29:},\ \ell_{29},\ 1\rangle\rangle \in \varGamma) \wedge (\mathfrak{rf}\langle L1^\ell_{n_\ell}, \\
& \langle \mathtt{0:},\ \_,\ 1\rangle\rangle \in \varGamma) \wedge (\mathfrak{rf}\langle F1^\ell,\ \langle \mathtt{0:},\ \_,\ 1\rangle\rangle \in \varGamma)\big) \vee \\
& \big(\exists i, k_i, \ell, n_\ell, \ell_{30}, \ell_{29}, i_9 \in \mathbb{N} \;.\; \varGamma \in \Gamma \wedge (\mathfrak{rf}\langle L0^i_{k_i},\ \langle \mathtt{30:},\ \ell_{30}, \\
& 1\rangle\rangle \in \varGamma) \wedge (\mathfrak{rf}\langle F0^i,\ \langle \mathtt{29:},\ \ell_{29},\ 1\rangle\rangle \in \varGamma) \wedge (\mathfrak{rf}\langle L1^\ell_{n_\ell},\ \langle \mathtt{0:},\ \_, \\
& 1\rangle\rangle \in \varGamma) \wedge (\mathfrak{rf}\langle F1^\ell,\ \langle \mathtt{9:},\ i_9,\ 1\rangle\rangle \in \varGamma)\big) \vee \\
& \big(\exists i, k_i, \ell, n_\ell, \ell_{30}, \ell_{29}, i_{10} \in \mathbb{N} \;.\; \varGamma \in \Gamma \wedge (\mathfrak{rf}\langle L0^i_{k_i},\ \langle \mathtt{30:},\ \ell_{30}, \\
& 1\rangle\rangle \in \varGamma) \wedge (\mathfrak{rf}\langle F0^i,\ \langle \mathtt{29:},\ \ell_{29},\ 1\rangle\rangle \in \varGamma) \wedge (\mathfrak{rf}\langle L1^\ell_{n_\ell},\ \langle \mathtt{10:},\ i_{10}, \\
& 1\rangle\rangle \in \varGamma) \wedge (\mathfrak{rf}\langle F1^\ell,\ \langle \mathtt{0:},\ \_,\ 1\rangle\rangle \in \varGamma)\big) \vee \\
& \big(\exists i, k_i, \ell, n_\ell, \ell_{30}, \ell_{29}, i_{10}, i_9 \in \mathbb{N} \;.\; \varGamma \in \Gamma \wedge (\mathfrak{rf}\langle L0^i_{k_i},\ \langle \mathtt{30:},\ \ell_{30}, \\
& 1\rangle\rangle \in \varGamma) \wedge (\mathfrak{rf}\langle F0^i,\ \langle \mathtt{29:},\ \ell_{29},\ 1\rangle\rangle \in \varGamma) \wedge (\mathfrak{rf}\langle L1^\ell_{n_\ell},\ \langle \mathtt{10:},\ i_{10}, \\
& 1\rangle\rangle \in \varGamma) \wedge (\mathfrak{rf}\langle F1^\ell,\ \langle \mathtt{9:},\ i_9,\ 1\rangle\rangle \in \varGamma)\big) \\
& \quad \text{(def. } \mathrm{r1Rl0}^i_{k_i}[\varGamma], \mathrm{r1Rf0}^i[\varGamma], \mathrm{r1Rl1}^\ell_{n_\ell}[\varGamma]\text{, and } \mathrm{r1Rf1}^\ell[\varGamma], \mathfrak{rf}\langle x_\theta, \\
& \quad \langle \ell\mathtt{:},\ \theta',\ v\rangle\rangle \text{ implies that } x_\theta = v,\ A \wedge (B \vee C) = (A \wedge B) \vee \\
& \quad (A \wedge C), \exists \text{ distributes over } \vee\text{, and } (\exists x \;.\; A(x)) \wedge B = \exists x \;. \\
& \quad (A(x) \wedge B) \text{ when } x \text{ is not free in } B\text{)} \\
\Rightarrow & \mathsf{at}\{8\} \wedge \mathsf{at}\{28\} \wedge (\neg S_{com_1}(\varGamma, \Gamma) \vee \neg S_{com_2}(\varGamma, \Gamma) \vee \neg S_{com_3}(\varGamma, \Gamma) \vee \\
& \neg S_{com_4}(\varGamma, \Gamma)) \\
\Rightarrow & \neg S_{com}(\varGamma, \Gamma)
\end{array}
$$

Calculational design of the communication specification

- where

$S_{com}(\Gamma,\overline{\Gamma}) \triangleq (\mathsf{at}\{8\} \wedge \mathsf{at}\{28\}) \Longrightarrow (S_{com_1}(\Gamma,\overline{\Gamma}) \wedge S_{com_2}(\Gamma,\overline{\Gamma}) \wedge S_{com_3}(\Gamma,\overline{\Gamma}) \wedge S_{com_4}(\Gamma,\overline{\Gamma}))$

$$S_{com_1} \triangleq \neg(\exists i, k_i, \ell, n_\ell, \ell_{30}, \ell_{29} \in \mathbb{N} \;.\; \mathit{\Gamma} \in \Gamma \wedge \mathfrak{rf}\langle L0^i_{k_i},\ \langle 30{:},\ \ell_{30},\ 1\rangle\rangle \in \mathit{\Gamma} \wedge \mathfrak{rf}\langle F0^i,\ \langle 29{:},\ \ell_{29},\ 1\rangle\rangle \in \mathit{\Gamma} \wedge \mathfrak{rf}\langle L1^\ell_{n_\ell},\ \langle 0{:},\ \_,\ 1\rangle\rangle \in \mathit{\Gamma} \wedge \mathfrak{rf}\langle F1^\ell,\ \langle 0{:},\ \_,\ 1\rangle\rangle \in \mathit{\Gamma}$$

$$S_{com_2} \triangleq \neg(\exists i, k_i, \ell, n_\ell, \ell_{30}, \ell_{29}, i_9 \in \mathbb{N} \;.\; \mathit{\Gamma} \in \Gamma \wedge \mathfrak{rf}\langle L0^i_{k_i},\ \langle 30{:},\ \ell_{30},\ 1\rangle\rangle \in \mathit{\Gamma} \wedge \mathfrak{rf}\langle F0^i,\ \langle 29{:},\ \ell_{29},\ 1\rangle\rangle \in \mathit{\Gamma} \wedge \mathfrak{rf}\langle L1^\ell_{n_\ell},\ \langle 0{:},\ \_,\ 1\rangle\rangle \in \mathit{\Gamma} \wedge \mathfrak{rf}\langle F1^\ell,\ \langle 9{:},\ i_9,\ 1\rangle\rangle \in \mathit{\Gamma}$$

$$S_{com_3} \triangleq \neg(\exists i, k_i, \ell, n_\ell, \ell_{30}, \ell_{29}, i_{10} \in \mathbb{N} \;.\; \mathit{\Gamma} \in \Gamma \wedge \mathfrak{rf}\langle L0^i_{k_i},\ \langle 30{:},\ \ell_{30},\ 1\rangle\rangle \in \mathit{\Gamma} \wedge \mathfrak{rf}\langle F0^i,\ \langle 29{:},\ \ell_{29},\ 1\rangle\rangle \in \mathit{\Gamma} \wedge \mathfrak{rf}\langle L1^\ell_{n_\ell},\ \langle 10{:},\ i_{10},\ 1\rangle\rangle \in \mathit{\Gamma} \wedge \mathfrak{rf}\langle F1^\ell,\ \langle 0{:},\ \_,\ 1\rangle\rangle \in \mathit{\Gamma}$$

$$S_{com_4} \triangleq \neg(\exists i, k_i, \ell, n_\ell, \ell_{30}, \ell_{29}, i_{10}, i_9 \in \mathbb{N} \;.\; \mathit{\Gamma} \in \Gamma \wedge \mathfrak{rf}\langle L0^i_{k_i},\ \langle 30{:},\ \ell_{30},\ 1\rangle\rangle \in \mathit{\Gamma} \wedge \mathfrak{rf}\langle F0^i,\ \langle 29{:},\ \ell_{29},\ 1\rangle\rangle \in \mathit{\Gamma} \wedge \mathfrak{rf}\langle L1^\ell_{n_\ell},\ \langle 10{:},\ i_{10},\ 1\rangle\rangle \in \mathit{\Gamma} \wedge \mathfrak{rf}\langle F1^\ell,\ \langle 9{:},\ i_9,\ 1\rangle\rangle \in \mathit{\Gamma}$$

- This proves S_{com} sufficient for correctness
- Counter-examples prove S_{com} necessary $\Rightarrow$ S_{com} is the weakest WCM requirement for correctness

Example of counter-example to S_{com_1}

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
```

```
1:
   do {i}
2:
     do {j_i}
3:
       r[] Rl0 latch0 {⇝ L0^i_{j_i}}
4:

     while (Rl0=0) {k_i}
5:
     w[] latch0 0
6:
     r[] Rf0 flag0 {⇝ F0^i}
7:

     if (Rf0≠0) then
8:
       (* critical section *)
       w[] flag0 0
9:
       w[] flag1 1
10:
       w[] latch1 1
11:
     fi
12:
   while true
13:
```

```
21:
   do {ℓ}
22:
     do {m_ℓ}
23:
       r[] Rl1 latch1 {⇝ L1^ℓ_{m_ℓ}}
24:

     while (Rl1=0) {n_ℓ}
25:
     w[] latch1 0
26:
     r[] Rf1 flag1 {⇝ F1^ℓ}
27:

     if (Rf1≠0) then
28:
       (* critical section *)
       w[] flag1 0
29:
       w[] flag0 1
30:
       w[] latch0 1
31:
     fi
32:
   while true
33:
```

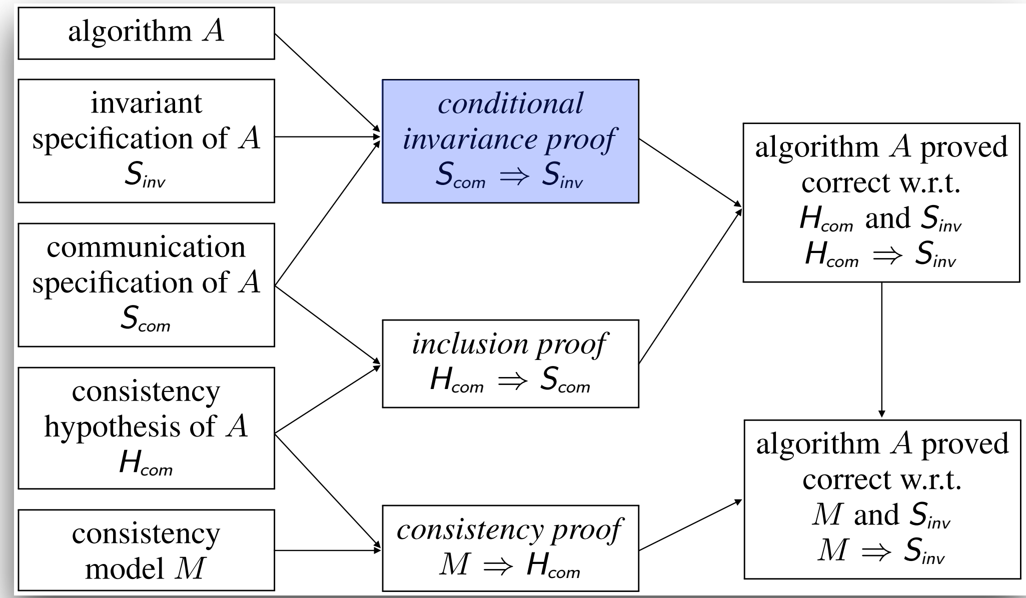
cut

Proof of mutual exclusion

- S_{com} implies mutual exclusion (for any Γ)

$$
\begin{array}{rll}
& (\neg S_{inv}(\Gamma,\Gamma) \wedge S_{ind}(\Gamma,\Gamma)) \Longrightarrow \neg(S_{com}(\Gamma,\Gamma)) & \\
\Longrightarrow & S_{com}(\Gamma,\Gamma) \Longrightarrow (S_{inv}(\Gamma,\Gamma) \vee \neg S_{ind}(\Gamma,\Gamma)) & \wr\text{contraposition}\wr \\
\Longrightarrow & S_{com}(\Gamma,\Gamma) \Longrightarrow (S_{ind}(\Gamma,\Gamma) \Longrightarrow S_{inv}(\Gamma,\Gamma)) & \wr\text{implication}\wr \\
\Longrightarrow & (S_{com}(\Gamma,\Gamma) \wedge S_{ind}(\Gamma,\Gamma)) \Longrightarrow S_{inv}(\Gamma,\Gamma) & \wr\text{implication}\wr \\
\Longrightarrow & S_{com}(\Gamma,\overline{\Gamma}) \Rightarrow S_{inv}(\Gamma,\overline{\Gamma}) & \wr\text{since } S_{com}(\Gamma,\overline{\Gamma}) \Rightarrow S_{ind}(\Gamma,\overline{\Gamma})\wr
\end{array}
$$

Conditional invariance proof



Sequential proof $\ell = \kappa$ and $p = q$

`{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }`

1: $\{\mathit{\Gamma} \in \Gamma\}$
`do` $\{i\}$
2: $\{\mathit{\Gamma} \in \Gamma\}$
`do` $\{j_i\}$
3: $\{\mathit{\Gamma} \in \Gamma\}$
`r[] R10 lat`
4: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{R1O}$
`while (R10=0)`
5: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^{i}_{k_i}[\mathit{\Gamma}]\}$
`w[] latch0 0`
6: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^{i}_{k_i}[\mathit{\Gamma}]\}$
`r[] Rf0 flag0` $\{\rightsquigarrow F0^i\}$
7: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^{i}_{k_i}[\mathit{\Gamma}] \wedge \mathtt{Rf0} = F0^i$
$\wedge (\mathrm{r0Rf0}^i[\mathit{\Gamma}] \vee \mathrm{r1Rf0}^i[\mathit{\Gamma}])\}$
`if (Rf0≠0) then`
8: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^{i}_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
`(* critical section *)`
`w[] flag0 0`
9: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^{i}_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
`w[] flag1 1`
10: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^{i}_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
`w[] latch1 1`
11: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^{i}_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
`fi`
12: $\{\mathit{\Gamma} \in \Gamma\}$
`while true`
13: $\{\mathsf{false}\}$

For a ***read instruction*** κ : `r[`ts`] R x` κ': (read)

$$\mathsf{PRE}^{\ell,\kappa}_{p,r}[\theta_r, \rho_r, \nu_r, \mathsf{rf}] \wedge \mathit{rf}[\mathfrak{w}(\langle q,\ \ell',\ \mathtt{w[}ts\mathtt{]\ x}\ \textit{r-value},\ \theta'\rangle, v),\ \mathfrak{r}(\langle r,\ \ell,\ \mathtt{r[}ts\mathtt{]\ R\ x},\ \theta_r\rangle, \mathtt{x}_{\theta_r})] \in \mathsf{rf}$$

$$\Rightarrow \mathsf{POST}^{\ell,\kappa'}_{p,r}[\rho_r \leftarrow \rho_r[\mathtt{R} := \mathtt{x}_{\theta_r}], \nu_r \leftarrow \nu_r[\mathtt{x}_{\theta_r} := v]]$$

21: $\{\mathit{\Gamma} \in \Gamma\}$
25: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^{\ell}_{n_\ell}[\mathit{\Gamma}]\}$
`w[] latch1 0`
26: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^{\ell}_{n_\ell}[\mathit{\Gamma}]\}$
`r[] Rf1 flag1` $\{\rightsquigarrow F1^\ell\}$
27: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \mathtt{Rf1} = F1^\ell$
$\wedge (\mathrm{r0Rf1}^\ell[\mathit{\Gamma}] \vee \mathrm{r1Rf1}^\ell[\mathit{\Gamma}])\}$
`if (Rf1≠0) then`
28: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
`(* critical section *)`
`w[] flag1 0`
29: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
`w[] flag0 1`
30: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
`w[] latch0 1`
31: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
`fi`
32: $\{\mathit{\Gamma} \in \Gamma\}$
`while true`
33: $\{\mathsf{false}\}$

Sequential proof $\ell = \kappa$ and $p = q$

`{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }`

1: $\{\mathit{\Gamma} \in \Gamma\}$
  `do` $\{i\}$
2: $\{\mathit{\Gamma} \in \Gamma\}$
  `do` $\{j_i\}$
3: $\{\mathit{\Gamma} \in \Gamma\}$
  `r[] R10 lat`
4: $\{\mathit{\Gamma} \in \Gamma \wedge \mathtt{R1O}$
  `while (R10=0)`
5: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}$
  `w[] latch0 0`
6: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^{i}_{k_i}[\mathit{\Gamma}]\}$
  `r[] Rf0 flag0` $\{\rightsquigarrow F0^{i}\}$
7: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^{i}_{k_i}[\mathit{\Gamma}] \wedge \mathtt{Rf0} = F0^{i} \wedge (\mathrm{r0Rf0}^{i}[\mathit{\Gamma}] \vee \mathrm{r1Rf0}^{i}[\mathit{\Gamma}])\}$
  `if (Rf0≠0) then`
8: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^{i}_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^{i}[\mathit{\Gamma}]\}$
  `(* critical section *)`
  `w[] flag0 0`
9: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^{i}_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^{i}[\mathit{\Gamma}]\}$
  `w[] flag1 1`
10: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^{i}_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^{i}[\mathit{\Gamma}]\}$
  `w[] latch1 1`
11: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^{i}_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^{i}[\mathit{\Gamma}]\}$
  `fi`
12: $\{\mathit{\Gamma} \in \Gamma\}$
  `while true`
13: {false}

21: $\{\mathit{\Gamma} \in \Gamma\}$
  `do` $\{\ell\}$
22: $\{\mathit{\Gamma} \in \Gamma\}$
  `do` $\{m_\ell\}$
26: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^{\ell}_{n_\ell}[\mathit{\Gamma}]\}$
  `r[] Rf1 flag1` $\{\rightsquigarrow F1^{\ell}\}$
27: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \mathtt{Rf1} = F1^{\ell} \wedge (\mathrm{r0Rf1}^{\ell}[\mathit{\Gamma}] \vee \mathrm{r1Rf1}^{\ell}[\mathit{\Gamma}])\}$
  `if (Rf1≠0) then`
28: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^{\ell}[\mathit{\Gamma}]\}$
  `(* critical section *)`
  `w[] flag1 0`
29: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^{\ell}[\mathit{\Gamma}]\}$
  `w[] flag0 1`
30: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^{\ell}[\mathit{\Gamma}]\}$
  `w[] latch0 1`
31: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^{\ell}[\mathit{\Gamma}]\}$
  `fi`
32: $\{\mathit{\Gamma} \in \Gamma\}$
  `while true`
33: {false}

For a ***test instruction*** $\kappa : \mathtt{b}\,[ts]$ *operation* $l_t\ \kappa'$: (test)

$$\mathsf{PRE}^{\ell,\kappa}_{p,r}[\rho_r, \nu_r] \wedge \mathsf{sat}(E[\![operation]\!](\rho_r, \nu_r) \neq 0) \Rightarrow \mathsf{POST}^{\ell,l_t}_{p,r}$$

$$\mathsf{PRE}^{\ell,\kappa}_{p,r}[\rho_r, \nu_r] \wedge \mathsf{sat}(E[\![operation]\!](\rho_r, \nu_r) = 0) \Rightarrow \mathsf{POST}^{\ell,\kappa'}_{p,r}$$

Sequential proof $\ell = \kappa$ and $p = q$

`{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }`

1: $\{\mathit{\Gamma} \in \Gamma\}$
`do` $\{i\}$
2: $\{\mathit{\Gamma} \in \Gamma\}$
`do` $\{j_i\}$
3: $\{\mathit{\Gamma} \in \Gamma\}$
`r[] R10 latch0` $\{\leadsto L0^i_{j_i}\}$
4: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{R10} = L0^i_{j_i} \wedge (\mathrm{r0R10}^i_{j_i}[\mathit{\Gamma}] \vee \mathrm{r1R10}^i_{j_i}[\mathit{\Gamma}])\}$
`while (R10=0)`
5: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}$
`w[] latch0 0`
6: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}$
`r[] Rf0 flag0`
7: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{Rf0} = F0$
$\wedge (\mathrm{r0Rf0}^i[\mathit{\Gamma}] \vee \mathrm{r1Rf0}^i[\mathit{\Gamma}])\}$
`if (Rf0≠0) then`
8: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
`(* critical section *)`
`w[] flag0 0`
9: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
`w[] flag1 1`
10: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
`w[] latch1 1`
11: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
`fi`
12: $\{\mathit{\Gamma} \in \Gamma\}$
`while true`
13: $\{\mathsf{false}\}$

21: $\{\mathit{\Gamma} \in \Gamma\}$
`do` $\{\ell\}$
22: $\{\mathit{\Gamma} \in \Gamma\}$
`do` $\{m_\ell\}$
23: $\{\mathit{\Gamma} \in \Gamma\}$
`r[] Rl1 latch1` $\{\leadsto L1^\ell_{m_\ell}\}$
24: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{Rl1} = L1^\ell_{m_\ell} \wedge (\mathrm{r0Rl1}^\ell_{m_\ell}[\mathit{\Gamma}] \vee \mathrm{r1Rl1}^\ell_{m_\ell}[\mathit{\Gamma}])\}$
27: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{Rf1} = F1$
$\wedge (\mathrm{r0Rf1}^\ell[\mathit{\Gamma}] \vee \mathrm{r1Rf1}^\ell[\mathit{\Gamma}])\}$
`if (Rf1≠0) then`
28: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
`(* critical section *)`
`w[] flag1 0`
29: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
`w[] flag0 1`
30: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
`w[] latch0 1`
31: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
`fi`
32: $\{\mathit{\Gamma} \in \Gamma\}$
`while true`
33: $\{\mathsf{false}\}$

For local side-effect free ***marker instructions*** κ : *instr* κ'
where *instr* = `f [`*ts*`]` $\left[\{l_1^0 \ldots l_1^m\}\ \{l_2^0 \ldots l_2^q\}\right]$, `w [`*ts*`] x` *r-value*,
`beginrmw [`*ts*`] x`, `endrmw [`*ts*`] x`: (marker)

$$\mathsf{PRE}^{\ell,\kappa}_{p,r} \Rightarrow \mathsf{POST}^{\ell,\kappa'}_{p,r}$$

Non-interference proof

{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }

> The local invariants of process p depend only on Γ and local registers or Pythia variables unchanged by a step in the other process

1: $\{\mathit{\Gamma} \in \Gamma\}$
do $\{i\}$
2: $\{\mathit{\Gamma} \in \Gamma\}$
do $\{j_i\}$
3: $\{\mathit{\Gamma} \in \Gamma\}$
r[] R10 latch0 $\{\rightsquigarrow L0$
4: $\{\mathit{\Gamma} \in \Gamma \wedge \mathtt{R10} = L0^i_{j_i} \wedge (\mathrm{r}$
while (R10=0) $\{k_i\}$
5: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}]\}$
w[] latch0 0
6: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}]\}$
r[] Rf0 flag0 $\{\rightsquigarrow F0^i\}$
7: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}] \wedge \mathtt{Rf0} = F0^i \wedge (\mathrm{r0Rf0}^i[\mathit{\Gamma}] \vee \mathrm{r1Rf0}^i[\mathit{\Gamma}])\}$
if (Rf0≠0) then
8: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
(* critical section *)
w[] flag0 0
9: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
w[] flag1 1
10: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
w[] latch1 1
11: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
fi
12: $\{\mathit{\Gamma} \in \Gamma\}$
while true
13: {false}

26: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}]\}$
r[] Rf1 flag1 $\{\rightsquigarrow F1^\ell\}$
27: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathtt{Rf1} = F1^\ell \wedge (\mathrm{r0Rf1}^\ell[\mathit{\Gamma}] \vee \mathrm{r1Rf1}^\ell[\mathit{\Gamma}])\}$
if (Rf1≠0) then
28: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
(* critical section *)
w[] flag1 0
29: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
w[] flag0 1
30: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
w[] latch0 1
31: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
fi
32: $\{\mathit{\Gamma} \in \Gamma\}$
while true
33: {false}

Communication proof

`{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }`

1: $\{\mathit{\Gamma} \in \Gamma\}$
  `do` $\{i\}$
2: $\{\mathit{\Gamma} \in \Gamma\}$
  `do` $\{j_i\}$ • • •
3:   $\{\mathit{\Gamma} \in \Gamma\}$
   `r[] R10 latch0` $\{\rightsquigarrow\ L0$
4:   $\{\mathit{\Gamma} \in \Gamma \wedge \mathtt{R10} = L0^i_{j_i} \wedge (\mathrm{r}$
  `while (R10=0)` $\{k_i\}$
5: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\mathit{\Gamma}]\}$
  `w[] latch0 0`
6: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\mathit{\Gamma}]\}$
  `r[] Rf0 flag0` $\{\rightsquigarrow\ F0^i\}$
7: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\mathit{\Gamma}] \wedge \mathtt{Rf0} = F0^i \wedge (\mathrm{r0Rf0}^i[\mathit{\Gamma}] \vee \mathrm{r1Rf0}^i[\mathit{\Gamma}])\}$
  `if (Rf0`$\neq$`0) then`
8:   $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
   `(* critical section *)`
   `w[] flag0 0`
9:   $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
   `w[] flag1 1`
10:   $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
   `w[] latch1 1`
11:   $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
  `fi`
12: $\{\mathit{\Gamma} \in \Gamma\}$
  `while true`
13: $\{\mathsf{false}\}$

- *Communication condition*
  $$\mathsf{COM}^{\ell}_{p}[\mathsf{rf}] \triangleq S_{ind\ p}(\ell)[\mathsf{rf}] \wedge S_{comp}{}_{p}(\ell)[\mathsf{rf}]$$
- A read event can read from only one write event.
  $$\mathsf{COM}^{\ell}_{p}[\mathsf{rf}] \wedge \mathit{rf}[r, w_1] \in \mathsf{rf} \wedge \mathit{rf}[r, w_2] \in \mathsf{rf} \qquad \text{(singleness)}$$
  $$\Rightarrow w_1 = w_2 \ .$$

26: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\mathit{\Gamma}]\}$
  `r[] Rf1 flag1` $\{\rightsquigarrow\ F1^{\ell}\}$
27: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \mathtt{Rf1} = F1^{\ell} \wedge (\mathrm{r0Rf1}^{\ell}[\mathit{\Gamma}] \vee \mathrm{r1Rf1}^{\ell}[\mathit{\Gamma}])\}$
  `if (Rf1`$\neq$`0) then`
28: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^{\ell}[\mathit{\Gamma}]\}$
   `(* critical section *)`
   `w[] flag1 0`
29: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^{\ell}[\mathit{\Gamma}]\}$
   `w[] flag0 1`
30: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^{\ell}[\mathit{\Gamma}]\}$
   `w[] latch0 1`
31: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^{\ell}[\mathit{\Gamma}]\}$
  `fi`
32: $\{\mathit{\Gamma} \in \Gamma\}$
  `while true`
33: $\{\mathsf{false}\}$

Communication proof

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
1:  {Γ ∈ Γ}
   do {i}
2:    {Γ ∈ Γ}
      do {j_i}
3:       {Γ ∈ Γ}
         r[] R10 latch0 {⇝ L0
4:       {Γ ∈ Γ ∧ R10 = L0^i_{j_i} ∧ (r
      while (R10=0) {k_i}
5:    {Γ ∈ Γ ∧ r1Rl0^i_{k_i}[Γ]}
      w[] latch0 0
6:    {Γ ∈ Γ ∧ r1Rl0^i_{k_i}[Γ]}
      r[] Rf0 flag0 {⇝ F0^i}
7:    {Γ ∈ Γ ∧ r1Rl0^i_{k_i}[Γ] ∧ Rf0 = F0^i
                          ∧ (r0Rf0^i[Γ] ∨ r1Rf0^i[Γ])}
      if (Rf0≠0) then
8:       {Γ ∈ Γ ∧ r1Rl0^i_{k_i}[Γ] ∧ r1Rf0^i[Γ]}
         (* critical section *)
         w[] flag0 0
9:       {Γ ∈ Γ ∧ r1Rl0^i_{k_i}[Γ] ∧ r1Rf0^i[Γ]}
         w[] flag1 1
10:      {Γ ∈ Γ ∧ r1Rl0^i_{k_i}[Γ] ∧ r1Rf0^i[Γ]}
```

- All process read instructions ℓ : `r[`ts`] R x` ℓ' must read either from an initial or a reachable program write, allowed by the communication hypothesis ($\exists$ P$[X_1, \ldots, X_m]$ means that all free variables in

$$\mathsf{COM}^{\ell}_{p}[\theta_p, \mathsf{rf}] \wedge \mathsf{rf} \neq \emptyset \Rightarrow \exists\, \mathit{rf}[\mathfrak{w}(\langle q,\ \ell_q,\ \texttt{w[}ts\texttt{] x}\ \textit{r-value},\ \theta'\rangle, v),$$

$$\mathfrak{r}(\langle p,\ \ell,\ \texttt{r[}ts\texttt{] R x},\ \theta_p\rangle, \mathtt{x}_{\theta_p})] \in \mathsf{rf}\ . \qquad \text{(satisfaction)}$$

$$((q \in \mathbb{P}\mathbb{i} \wedge \exists\, \mathsf{PRE}^{\ell_q}_{q}[\theta_q \leftarrow \theta', \mathsf{rf}]) \vee (q = \mathsf{start} \wedge v = 0))\ .$$

```
26:   {Γ ∈ Γ ∧ r1Rl1^ℓ_{n_ℓ}[Γ]}
      r[] Rf1 flag1 {⇝ F1^ℓ}
27:   {Γ ∈ Γ ∧ r1Rl1^ℓ_{n_ℓ}[Γ] ∧ Rf1 = F1^ℓ
                          ∧ (r0Rf1^ℓ[Γ] ∨ r1Rf1^ℓ[Γ])}
      if (Rf1≠0) then
28:      {Γ ∈ Γ ∧ r1Rl1^ℓ_{n_ℓ}[Γ] ∧ r1Rf1^ℓ[Γ]}
         (* critical section *)
         w[] flag1 0
29:      {Γ ∈ Γ ∧ r1Rl1^ℓ_{n_ℓ}[Γ] ∧ r1Rf1^ℓ[Γ]}
         w[] flag0 1
30:      {Γ ∈ Γ ∧ r1Rl1^ℓ_{n_ℓ}[Γ] ∧ r1Rf1^ℓ[Γ]}
```

$$\mathrm{r0Rf0}^i[\Gamma] \triangleq (\mathfrak{rf}\langle F0^i,\ \langle 0{:},\ \_,\ 0\rangle\rangle \in \Gamma \wedge F0^i = 0) \vee (\exists i_8 \in \mathbb{N}\ .\ \mathfrak{rf}\langle F0^i,\ \langle 8{:},\ i_8,\ 0\rangle\rangle \in \Gamma \wedge F0^i = 0)$$

$$\mathrm{r1Rf0}^i[\Gamma] \triangleq (\exists \ell_{29} \in \mathbb{N}\ .\ \mathfrak{rf}\langle F0^i,\ \langle 29{:},\ \ell_{29},\ 1\rangle\rangle \in \Gamma \wedge F0^i = 1)$$

```
12:   {Γ ∈ Γ}
   while true
13:{false}

32:   {Γ ∈ Γ}
   while true
33:{false}
```

Communication proof

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
1: {Γ ∈ Γ}
   do {i}
2:   {Γ ∈ Γ}
     do {j_i}
3:     {Γ ∈ Γ}
       r[] R10 latch0 {⇝ L0
4:     {Γ ∈ Γ ∧ R10 = L0^i_{j_i} ∧ (r
     while (R10=0) {k_i}
5:   {Γ ∈ Γ ∧ r1R10^i_{k_i}[Γ]}
     w[] latch0 0
6:   {Γ ∈ Γ ∧ r1R10^i_{k_i}[Γ]}
     r[] Rf0 flag0 {⇝ F0^i}
7:   {Γ ∈ Γ ∧ r1R10^i_{k_i}[Γ] ∧ Rf0 = F0^i
                    ∧ (r0Rf0^i[Γ] ∨ r1Rf0^i[Γ])}
     if (Rf0≠0) then
8:     {Γ ∈ Γ ∧ r1R10^i_{k_i}[Γ] ∧ r1Rf0^i[Γ]}
       (* critical section *)
       w[] flag0 0
9:     {Γ ∈ Γ ∧ r1R10^i_{k_i}[Γ] ∧ r1Rf0^i[Γ]}
       w[] flag1 1
10:    {Γ ∈ Γ ∧ r1R10^i_{k_i}[Γ] ∧ r1Rf0^i[Γ]}
```

```
26:  {Γ ∈ Γ ∧ r1R11^ℓ_{n_ℓ}[Γ]}
     r[] Rf1 flag1 {⇝ F1^ℓ}
27:  {Γ ∈ Γ ∧ r1R11^ℓ_{n_ℓ}[Γ] ∧ Rf1 = F1^ℓ
                    ∧ (r0Rf1^ℓ[Γ] ∨ r1Rf1^ℓ[Γ])}
     if (Rf1≠0) then
28:    {Γ ∈ Γ ∧ r1R11^ℓ_{n_ℓ}[Γ] ∧ r1Rf1^ℓ[Γ]}
       (* critical section *)
       w[] flag1 0
29:    {Γ ∈ Γ ∧ r1R11^ℓ_{n_ℓ}[Γ] ∧ r1Rf1^ℓ[Γ]}
       w[] flag0 1
30:    {Γ ∈ Γ ∧ r1R11^ℓ_{n_ℓ}[Γ] ∧ r1Rf1^ℓ[Γ]}
```

- The values v allowed to be read by the communication hypothesis must originate from reachable program write instructions ℓ : `w[`ts`] x` *r-value* ℓ':

$$\forall \mathsf{rf} \,.\, \forall\, rf[\mathfrak{w}(\langle q,\ \ell_q,\ \mathtt{w[}ts\mathtt{]\ x}\ \textit{r-value},\ \theta_p\rangle, v), r] \in \mathsf{rf}\ \text{(match)}$$
$$\mathsf{COM}^{\ell}_{p}[\theta_q, \rho_q, \nu_q, \mathsf{rf}] \Rightarrow v = E[\![\textit{r-value}]\!](\rho_q, \nu_q)$$

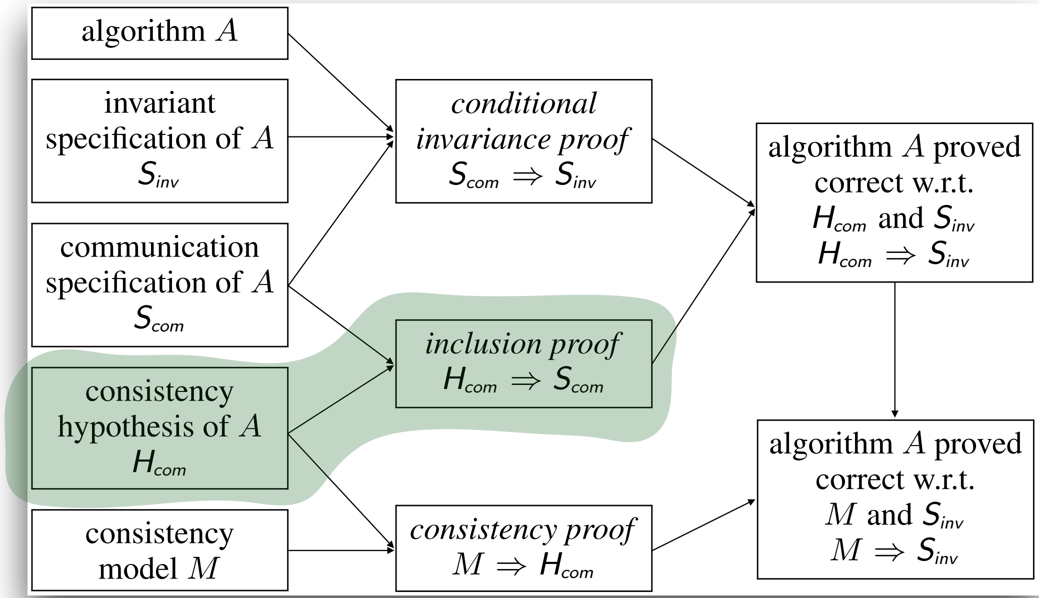
$$\mathrm{r0Rf0}^i[\Gamma] \triangleq (\mathfrak{rf}\langle F0^i,\ \langle 0\!:,\ \_,\ 0\rangle\rangle \in \Gamma \wedge F0^i = 0) \vee (\exists i_8 \in \mathbb{N} \,.\, \mathfrak{rf}\langle F0^i,\ \langle 8\!:,\ i_8,\ 0\rangle\rangle \in \Gamma \wedge F0^i = 0)$$

$$\mathrm{r1Rf0}^i[\Gamma] \triangleq (\exists \ell_{29} \in \mathbb{N} \,.\, \mathfrak{rf}\langle F0^i,\ \langle 29\!:,\ \ell_{29},\ 1\rangle\rangle \in \Gamma \wedge F0^i = 1)$$

```
12:  {Γ ∈ Γ}
   while true
13:{false}
```

```
32:  {Γ ∈ Γ}
   while true
33:{false}
```

Inclusion proof



Method

- The communication specification is

$$S_{com}(\Gamma,\overline{\Gamma}) \triangleq (\mathsf{at}\{8\} \wedge \mathsf{at}\{28\}) \Longrightarrow (S_{com_1}(\Gamma,\overline{\Gamma}) \wedge S_{com_2}(\Gamma,\overline{\Gamma}) \wedge S_{com_3}(\Gamma,\overline{\Gamma}) \wedge S_{com_4}(\Gamma,\overline{\Gamma}))$$

- The consistency specification must satisfy

$$H_{com}(\Gamma,\overline{\Gamma}) \Rightarrow S_{com}(\Gamma,\overline{\Gamma}) \qquad \textbf{i.e.} \quad \neg S_{com}(\Gamma,\overline{\Gamma}) \Rightarrow \neg H_{com}(\Gamma,\overline{\Gamma})$$

- So the design of $H_{com}(\Gamma,\overline{\Gamma})$ must forbid the erroneous communications specified by the communication specification

$$\left(\mathsf{at}\{8\} \wedge \mathsf{at}\{28\} \wedge \bigvee_{i=1}^{4} \neg S_{com_i}(\Gamma,\overline{\Gamma})\right) \Longrightarrow \bigvee_{i=1}^{4} \neg H_{com\,i}(\Gamma,\overline{\Gamma})$$

$$\mathcal{S}_{com_1} \triangleq \neg(\exists i, k_i, \ell, n_\ell, \ell_{30}, \ell_{29} \in \mathbb{N} \,.\, \Gamma \in \Gamma \wedge \mathfrak{rf}\langle L0^i_{k_i},\ \langle 30\!:,\ \ell_{30},\ 1\rangle\rangle \in \Gamma \wedge \mathfrak{rf}\langle F0^i,\ \langle 29\!:,\ \ell_{29},\ 1\rangle\rangle \in \Gamma \wedge \mathfrak{rf}\langle L1^\ell_{n_\ell},\ \langle 0\!:,\ \_,\ 1\rangle\rangle \in \Gamma \wedge \mathfrak{rf}\langle F1^\ell,\ \langle 0\!:,\ \_,\ 1\rangle\rangle \in \Gamma$$

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
1: do {i}                                  || 21:do {ℓ}
2:   do {j_i}                              || 22:   do {m_ℓ}
3:     r[] Rl0 latch0 {⇝ L0^i_{j_i}}       || 23:     r[] Rl1 latch1 {⇝ L1^ℓ_{m_ℓ}}
4:   while (Rl0=0) {k_i}                   || 24:   while (Rl1=0) {n_ℓ}
5:   w[] latch0 0                          || 25:   w[] latch1 0
6:   r[] Rf0 flag0 {⇝ F0^i}                || 26:   r[] Rf1 flag1 {⇝ F1^ℓ}
7:   if (Rf0≠0) then                       || 27:   if (Rf1≠0) then
8:     (* critical section *)              || 28:     (* critical section *)
       w[] flag0 0                         ||         w[] flag1 0
9:     w[] flag1 1                         || 29:     w[] flag0 1
10:    w[] latch1 1                        || 30:     w[] latch0 1
11:  fi                                    || 31:   fi
12:while true                              || 32:while true
13:                                        || 33:
```

cut

no prophecy beyond cut during execution

$$S_{com_2} \triangleq \neg(\exists i, k_i, \ell, n_\ell, \ell_{30}, \ell_{29}, i_9 \in \mathbb{N} \,.\, \Gamma \in \Gamma \wedge \mathfrak{rf}\langle L0^i_{k_i},\ \langle 30{:},\ \ell_{30},\ 1\rangle\rangle \in \Gamma \wedge \mathfrak{rf}\langle F0^i,\ \langle 29{:},\ \ell_{29},\ 1\rangle\rangle \in \Gamma \wedge \mathfrak{rf}\langle L1^\ell_{n_\ell},\ \langle 0{:},\ \_,\ 1\rangle\rangle \in \Gamma \wedge \mathfrak{rf}\langle F1^\ell,\ \langle 9{:},\ i_9,\ 1\rangle\rangle \in \Gamma$$

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
```

```
1: do {i}
2:   do {j_i}
3:     r[] Rl0 latch0 {⇝ L0^i_{j_i}}
4:   while (Rl0=0) {k_i}
5:   w[] latch0 0
6:   r[] Rf0 flag0 {⇝ F0^i}
7:   if (Rf0≠0) then
8:     (* critical section *)
       w[] flag0 0
9:     w[] flag1 1
10:    w[] latch1 1
11:  fi
12:while true
13:
```

```
21:do {ℓ}
22:  do {m_ℓ}
23:    r[] Rl1 latch1 {⇝ L1^ℓ_{m_ℓ}}
24:  while (Rl1=0) {n_ℓ}
25:  w[] latch1 0
26:  r[] Rf1 flag1 {⇝ F1^ℓ}
27:  if (Rf1≠0) then
28:    (* critical section *)
       w[] flag1 0
29:    w[] flag0 1
30:    w[] latch0 1
31:  fi
32:while true
33:
```

cut

no prophecy beyond cut during execution

$$S_{com_3} \triangleq \neg(\exists i, k_i, \ell, n_\ell, \ell_{30}, \ell_{29}, i_{10} \in \mathbb{N} \,.\, \Gamma \in \Gamma \wedge \mathfrak{rf}\langle L0^i_{k_i}, \langle 30\!:, \ell_{30},\ 1\rangle\rangle \in \Gamma \wedge \mathfrak{rf}\langle F0^i, \langle 29\!:, \ell_{29},\ 1\rangle\rangle \in \Gamma \wedge \mathfrak{rf}\langle L1^\ell_{n_\ell}, \langle 10\!:, i_{10},\ 1\rangle\rangle \in \Gamma \wedge \mathfrak{rf}\langle F1^\ell, \langle 0\!:, \_,\ 1\rangle\rangle \in \Gamma$$

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
```

```
1: do {i}
2:    do {j_i}
3:       r[] R10 latch0 {⇝ L0^i_{j_i}}
4:    while (R10=0) {k_i}
5:    w[] latch0 0
6:    r[] Rf0 flag0 {⇝ F0^i}
7:    if (Rf0≠0) then
8:       (* critical section *)
         w[] flag0 0
9:       w[] flag1 1
10:      w[] latch1 1
11:   fi
12:while true
13:
```

```
21:do {ℓ}
22:   do {m_ℓ}
23:      r[] Rl1 latch1 {⇝ L1^ℓ_{m_ℓ}}
24:   while (Rl1=0) {n_ℓ}
25:   w[] latch1 0
26:   r[] Rf1 flag1 {⇝ F1^ℓ}
27:   if (Rf1≠0) then
28:      (* critical section *)
         w[] flag1 0
29:      w[] flag0 1
30:      w[] latch0 1
31:   fi
32:while true
33:
```

cut

no prophecy beyond cut during execution

$$S_{com_4} \triangleq \neg(\exists i, k_i, \ell, n_\ell, \ell_{30}, \ell_{29}, i_{10}, i_9 \in \mathbb{N} \;.\; \Gamma \in \Gamma \wedge \mathfrak{rf}\langle L0^i_{k_i}, \langle 30\!:, \ell_{30}, 1\rangle\rangle \in \Gamma \wedge \mathfrak{rf}\langle F0^i, \langle 29\!:, \ell_{29}, 1\rangle\rangle \in \Gamma \wedge \mathfrak{rf}\langle L1^\ell_{n_\ell}, \langle 10\!:, i_{10}, 1\rangle\rangle \in \Gamma \wedge \mathfrak{rf}\langle F1^\ell, \langle 9\!:, i_9, 1\rangle\rangle \in \Gamma$$

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
```

```
1:  do {i}
2:    do {j_i}
3:      r[] Rl0 latch0 {⇝ L0^i_{j_i}}
4:    while (Rl0=0) {k_i}
5:    w[] latch0 0
6:    r[] Rf0 flag0 {⇝ F0^i}
7:    if (Rf0≠0) then
8:      (* critical section *)
        w[] flag0 0
9:      w[] flag1 1
10:     w[] latch1 1
11:   fi
12:while true
13:
```

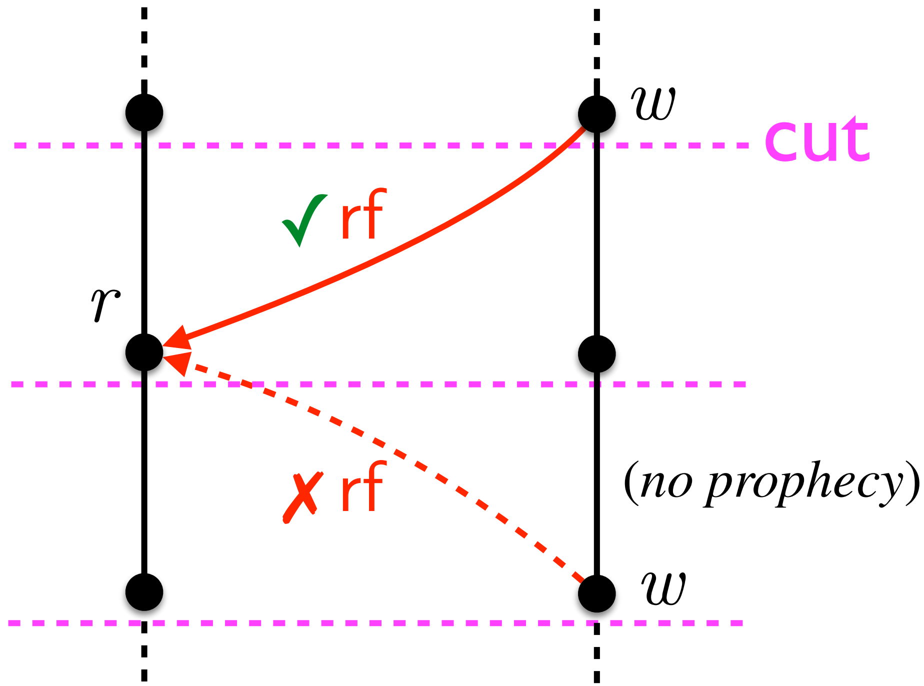
```
21:do {ℓ}
22:   do {m_ℓ}
23:     r[] Rl1 latch1 {⇝ L1^ℓ_{m_ℓ}}
24:   while (Rl1=0) {n_ℓ}
25:   w[] latch1 0
26:   r[] Rf1 flag1 {⇝ F1^ℓ}
27:   if (Rf1≠0) then
28:     (* critical section *)
        w[] flag1 0
29:     w[] flag0 1
30:     w[] latch0 1
31:   fi
32:while true
33:
```

cut

no prophecy beyond cut during execution

Conclusion on mutual exclusion

- PostgreSQL is correct on architectures satisfying the ``no prophecy beyond cut during execution'' property



- Intuition on necessity: when waiting for a spinlock, you should look at its current value, not at later ones!

in cat

- A static condition to impose a dynamic condition:

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
1: do {i}
2:   do {j_i}
3:     r[] R10 latch0 {⇝ L0^i_{j_i}}
4:   while (R10=0) {k_i}
5:   w[] latch0 0
6:   r[] Rf0 flag0 {⇝ F0^i}
7:   if (Rf0≠0) then
8:     f[cut]
       (* critical section *)
       w[] flag0 0
9:     w[] flag1 1
10:    w[] latch1 1
11:  fi
12:while true
13:
||
21:do {ℓ}
22:  do {m_ℓ}
23:    r[] R11 latch1 {⇝ L1^ℓ_{m_ℓ}}
24:  while (R11=0) {n_ℓ}
25:  w[] latch1 0
26:  r[] Rf1 flag1 {⇝ F1^ℓ}
27:  if (Rf1≠0) then
28:    f[cut]
       (* critical section *)
       w[] flag1 0
29:    w[] flag0 1
30:    w[] latch0 1
31:  fi
32:while true
33:
```

po

cut

rf

po

```
enum fences = 'cut
instructions F[{'cut}]
let cut = (tag2events('cut) * tag2events('cut)) & ext
irreflexive rf; po; cut; po
```

Prevents valid executions

- 2, 4, po, cut, rf, po, 1, 3 — Invalid

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
1: do {i}
2:   do {j_i}
3:     r[] R10 latch0 {⇝ L0^i_{j_i}}
4:   while (R10=0) {k_i}
5:   w[] latch0 0
6:   r[] Rf0 flag0 {⇝ F0^i}
7:   if (Rf0≠0) then
8:     f[cut]
       (* critical section *)
       w[] flag0 0
9:     w[] flag1 1
10:    w[] latch1 1
11:  fi
12:while true
13:
||
21:do {ℓ}
22:  do {m_ℓ}
23:    r[] R11 latch1 {⇝ L1^ℓ_{m_ℓ}}
24:  while (R11=0) {n_ℓ}
25:  w[] latch1 0
26:  r[] Rf1 flag1 {⇝ F1^ℓ}
27:  if (Rf1≠0) then
28:    f[cut]
       (* critical section *)
       w[] flag1 0
29:    w[] flag0 1
30:    w[] latch0 1
31:  fi
32:while true
33:
```

- 3, 4, po, cut, rf, po, 1, 2 — Valid

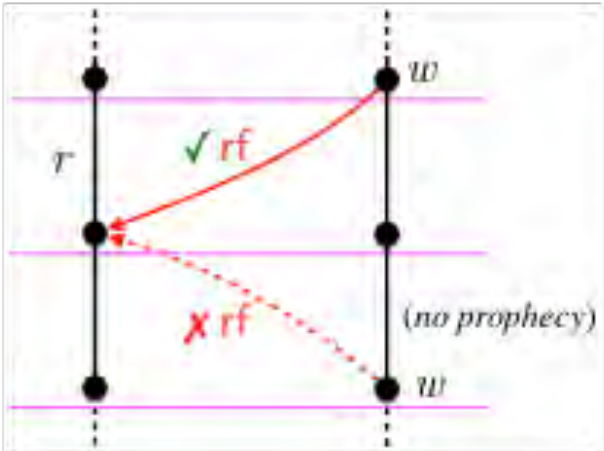
```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
1: do {i}
2:   do {j_i}
3:     r[] R10 latch0 {⇝ L0^i_{j_i}}
4:   while (R10=0) {k_i}
5:   w[] latch0 0
6:   r[] Rf0 flag0 {⇝ F0^i}
7:   if (Rf0≠0) then
8:     f[cut]
       (* critical section *)
       w[] flag0 0
9:     w[] flag1 1
10:    w[] latch1 1
11:  fi
12:while true
13:
||
21:do {ℓ}
22:  do {m_ℓ}
23:    r[] R11 latch1 {⇝ L1^ℓ_{m_ℓ}}
24:  while (R11=0) {n_ℓ}
25:  w[] latch1 0
26:  r[] Rf1 flag1 {⇝ F1^ℓ}
27:  if (Rf1≠0) then
28:    f[cut]
       (* critical section *)
       w[] flag1 0
29:    w[] flag0 1
30:    w[] latch0 1
31:  fi
32:while true
33:
```

```
irreflexive rf; po; cut; po
```

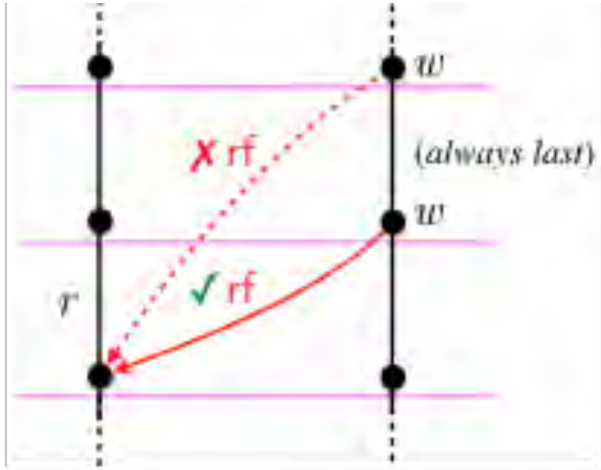
Non-starvation

Difference with Lamport/Owicki-Gries

- The communications in L/O-G are fixed in the semantics (SC) for <u>all</u> executions:



(a) No prophecy beyond cuts



(b) Read from last write

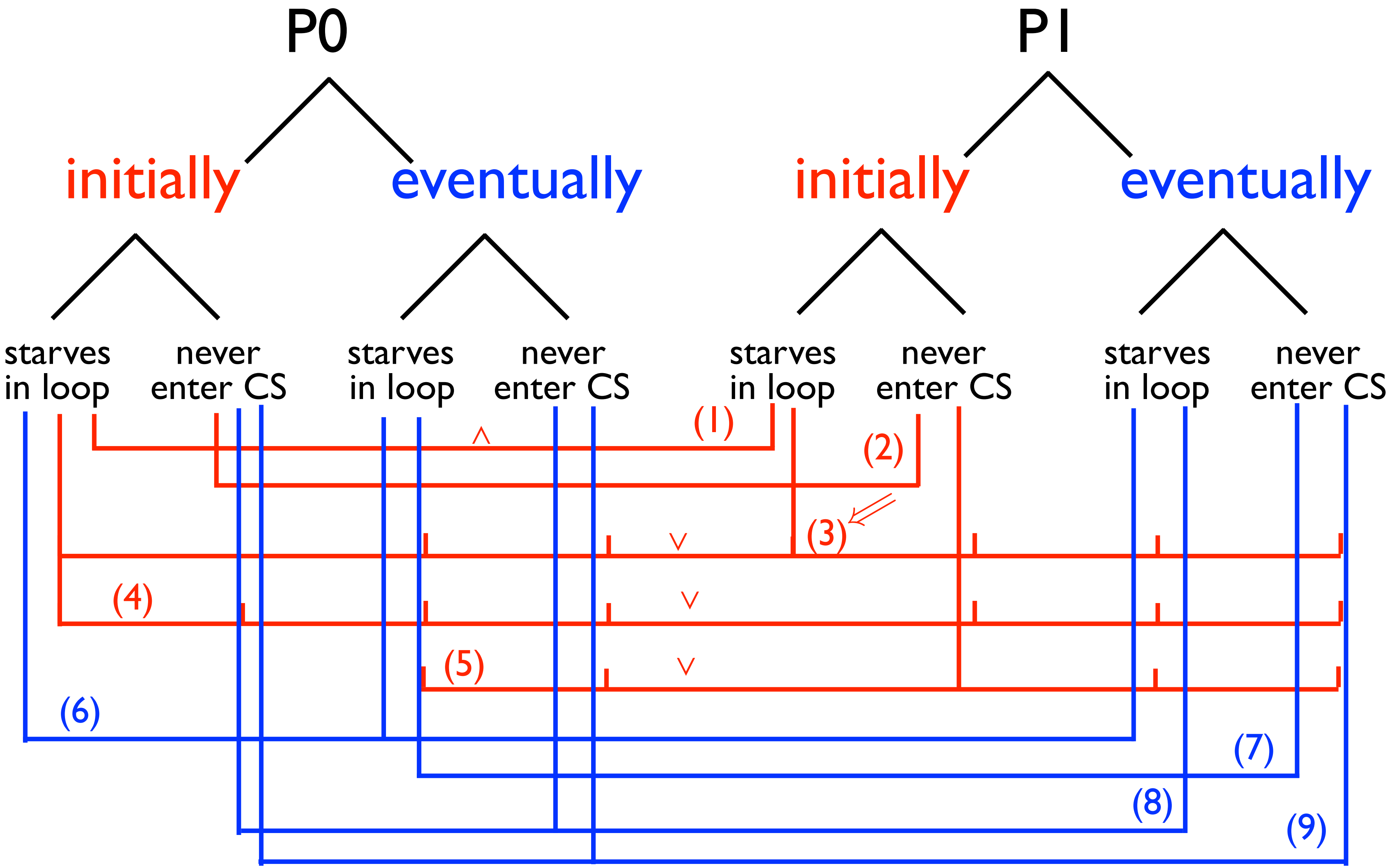
$\Longrightarrow$ entangled with the verification conditions

$\Longrightarrow$ impossible to reason on one execution trace only

Reasoning on only one execution

- An execution is entirely determined by its read-from relation rf
- The verification conditions depend on a set Γ of verification conditions
- By choosing Γ = {rf}, we can reason on this execution
- This execution satisfies the inductive invariant $S_{ind}(\{\text{rf}\})$
- To prove that this execution is impossible it is sufficient to prove that $S_{ind}(\{\text{rf}\})$ cannot hold (according to the verification conditions)
- Since the method is sound, if the verification conditions are not satisfied, the execution is excluded by the semantics

9 cases of starvation



(1) Both processes starve in spin loops

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
1: {true}
   do {i}
2:    {true}
      do {j_i}
3:       {true}
         r[] R10 latch0 {⇝ L0^i_{j_i}}
4:       {R10 = L0^i_{j_i} ∧
           (r0R10^i_{j_i}[Γ_rf] ∨ r1R10^i_{j_i}[Γ_rf])}
      while (R10=0) {k_i}
5:    {r1R10^i_{k_i}[Γ_rf]}
      w[] latch0 0
6:    {r1R10^i_{k_i}[Γ_rf]}
      r[] Rf0 flag0 {⇝ F0^i}
7:    {r1R10^i_{k_i}[Γ_rf] ∧ Rf0 = F0^i ∧
        (r0Rf0^i[Γ_rf] ∨ r1Rf0^i[Γ_rf])}
      if (Rf0≠0) then
8:       {r1R10^i_{k_i}[Γ_rf] ∧ r1Rf0^i[Γ_rf]}
false    (* critical section *)
         w[] flag0 0
9:       {r1R10^i_{k_i}[Γ_rf] ∧ r1Rf0^i[Γ_rf]}
         w[] flag1 1
10:      {r1R10^i_{k_i}[Γ_rf] ∧ r1Rf0^i[Γ_rf]}
         w[] latch1 1
11:      {r1R10^i_{k_i}[Γ_rf] ∧ r1Rf0^i[Γ_rf]}
      fi
12:   {true}
   while true
13:{false}
```

```
21:{true}
   do {ℓ}
22:   {true}
      do {m_ℓ}
23:      {true}
         r[] R11 latch1 {⇝ L1^ℓ_{m_ℓ}}
24:      {R11 = L1^ℓ_{m_ℓ} ∧
           (r0R11^ℓ_{m_ℓ}[Γ_rf] ∨ r1R11^ℓ_{m_ℓ}[Γ_rf])}
      while (R11=0) {n_ℓ}
25:   {r1R11^ℓ_{n_ℓ}[Γ_rf]}
      w[] latch1 0
26:   {r1R11^ℓ_{n_ℓ}[Γ_rf]}
      r[] Rf1 flag1 {⇝ F1^ℓ}
27:   {r1R11^ℓ_{n_ℓ}[Γ_rf] ∧ Rf1 = F1^ℓ ∧
        (r0Rf1^ℓ[Γ_rf] ∨ r1Rf1^ℓ[Γ_rf])}
      if (Rf1≠0) then
28:      {r1R11^ℓ_{n_ℓ}[Γ_rf] ∧ r1Rf1^ℓ[Γ_rf]}
         (* critical section *)                false
         w[] flag1 0
29:      {r1R11^ℓ_{n_ℓ}[Γ_rf] ∧ r1Rf1^ℓ[Γ_rf]}
         w[] flag0 1
30:      {r1R11^ℓ_{n_ℓ}[Γ_rf] ∧ r1Rf1^ℓ[Γ_rf]}
         w[] latch0 1
31:      {r1R11^ℓ_{n_ℓ}[Γ_rf] ∧ r1Rf1^ℓ[Γ_rf]}
      fi
32:   {true}
   while true
33:{false}
```

- let rf be the communication for such a trace (encoded in Γ_{rf})
- invariant false after both spin loops
- so `latch1` in 23: can only be read from initialization
- so `latch1` is 1 not 0, a contradiction

(2) Both processes never enter their critical section

`{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }`

1: $\{\text{true}\}$
  `do` $\{i\}$
2: $\{\text{true}\}$
   `do` $\{j_i\}$
3:   $\{\text{true}\}$
    `r[] R10 latch0` $\{\leadsto L0^{i}_{j_i}\}$
4:   $\{\mathtt{R10} = \mathtt{L0}^{\mathrm{i}}_{\mathrm{j_i}} \wedge$
     $(\mathrm{r0R10}^{\mathrm{i}}_{\mathrm{j_i}}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1R10}^{\mathrm{i}}_{\mathrm{j_i}}[\Gamma_{\mathsf{rf}}])\}$
   `while (R10=0)` $\{k_i\}$
5: $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}]\}$
   `w[] latch0 0`
6: $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}]\}$
   `r[] Rf0 flag0` $\{\leadsto F0^{i}\}$
7: $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathtt{Rf0} = \mathtt{F0}^{\mathrm{i}} \wedge$
   $(\mathrm{r0Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}])\}$
   `if (Rf0`$\neq$`0) then`
8:   $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
    `(* critical section *)`
    `w[] flag0 0`
9:   $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
    `w[] flag1 1`
10:   $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
    `w[] latch1 1`
11:   $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
   `fi`
12: $\{\text{true}\}$
  `while true`
13: $\{\text{false}\}$

||

21: $\{\text{true}\}$
  `do` $\{\ell\}$
22: $\{\text{true}\}$
   `do` $\{m_\ell\}$
23:   $\{\text{true}\}$
    `r[] Rl1 latch1` $\{\leadsto L1^{\ell}_{m_\ell}\}$
24:   $\{\mathtt{Rl1} = \mathtt{L1}^{\ell}_{\mathrm{m}_\ell} \wedge$
     $(\mathrm{r0Rl1}^{\ell}_{\mathrm{m}_\ell}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1Rl1}^{\ell}_{\mathrm{m}_\ell}[\Gamma_{\mathsf{rf}}])\}$
   `while (Rl1=0)` $\{n_\ell\}$
25: $\{\mathrm{r1Rl1}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}]\}$
   `w[] latch1 0`
26: $\{\mathrm{r1Rl1}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}]\}$
   `r[] Rf1 flag1` $\{\leadsto F1^{\ell}\}$
27: $\{\mathrm{r1Rl1}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathtt{Rf1} = \mathtt{F1}^{\ell} \wedge$
   $(\mathrm{r0Rf1}^{\ell}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}])\}$
   `if (Rf1`$\neq$`0) then`
28:   $\{\mathrm{r1Rl1}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
    `(* critical section *)`
    `w[] flag1 0`
29:   $\{\mathrm{r1Rl1}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
    `w[] flag0 1`
30:   $\{\mathrm{r1Rl1}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
    `w[] latch0 1`
31:   $\{\mathrm{r1Rl1}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
   `fi`
32: $\{\text{true}\}$
  `while true`
33: $\{\text{false}\}$

- let rf be the communication for such a trace (encoded in Γ_{rf})

(2) Both processes never enter their critical section

`{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }`

1: {true}
  `do` $\{i\}$
2:   {true}
   `do` $\{j_i\}$
3:     {true}
     `r[] R10 latch0` $\{\rightsquigarrow L0^{i}_{j_i}\}$
4:     $\{\mathtt{R10} = \mathtt{L0}^{i}_{j_i} \wedge (\mathrm{r0R10}^{i}_{j_i}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1R10}^{i}_{j_i}[\Gamma_{\mathsf{rf}}])\}$
   `while (R10=0)` $\{k_i\}$
5:   $\{\mathrm{r1R10}^{i}_{k_i}[\Gamma_{\mathsf{rf}}]\}$
   `w[] latch0 0`
6:   $\{\mathrm{r1R10}^{i}_{k_i}[\Gamma_{\mathsf{rf}}]\}$
   `r[] Rf0 flag0` $\{\rightsquigarrow F0^{i}\}$
7:   $\{\mathrm{r1R10}^{i}_{k_i}[\Gamma_{\mathsf{rf}}] \wedge \mathtt{Rf0} = \mathtt{F0}^{i} \wedge (\mathrm{r0Rf0}^{i}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1Rf0}^{i}[\Gamma_{\mathsf{rf}}])\}$
   `if (Rf0`$\neq$`0) then`
8:     $\{\mathrm{r1R10}^{i}_{k_i}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{i}[\Gamma_{\mathsf{rf}}]\}$
     `(* critical section *)`
     `w[] flag0 0`
9:     $\{\mathrm{r1R10}^{i}_{k_i}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{i}[\Gamma_{\mathsf{rf}}]\}$
false
     `w[] flag1 1`
10:     $\{\mathrm{r1R10}^{i}_{k_i}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{i}[\Gamma_{\mathsf{rf}}]\}$
     `w[] latch1 1`
11:     $\{\mathrm{r1R10}^{i}_{k_i}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{i}[\Gamma_{\mathsf{rf}}]\}$
   `fi`
12:   {true}
  `while true`
13: {false}

||

21: {true}
  `do` $\{\ell\}$
22:   {true}
   `do` $\{m_\ell\}$
23:     {true}
     `r[] Rl1 latch1` $\{\rightsquigarrow L1^{\ell}_{m_\ell}\}$
24:     $\{\mathtt{Rl1} = \mathtt{L1}^{\ell}_{m_\ell} \wedge (\mathrm{r0Rl1}^{\ell}_{m_\ell}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1Rl1}^{\ell}_{m_\ell}[\Gamma_{\mathsf{rf}}])\}$
   `while (Rl1=0)` $\{n_\ell\}$
25:   $\{\mathrm{r1Rl1}^{\ell}_{n_\ell}[\Gamma_{\mathsf{rf}}]\}$
   `w[] latch1 0`
26:   $\{\mathrm{r1Rl1}^{\ell}_{n_\ell}[\Gamma_{\mathsf{rf}}]\}$
   `r[] Rf1 flag1` $\{\rightsquigarrow F1^{\ell}\}$
27:   $\{\mathrm{r1Rl1}^{\ell}_{n_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathtt{Rf1} = \mathtt{F1}^{\ell} \wedge (\mathrm{r0Rf1}^{\ell}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}])\}$
   `if (Rf1`$\neq$`0) then`
28:     $\{\mathrm{r1Rl1}^{\ell}_{n_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
     `(* critical section *)`
     `w[] flag1 0`
29:     $\{\mathrm{r1Rl1}^{\ell}_{n_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
     `w[] flag0 1`   false
30:     $\{\mathrm{r1Rl1}^{\ell}_{n_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
     `w[] latch0 1`
31:     $\{\mathrm{r1Rl1}^{\ell}_{n_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
   `fi`
32:   {true}
  `while true`
33: {false}

- let rf be the communication for such a trace (encoded in Γ_{rf})
- the invariant inside critical sections must be false

(2) Both processes never enter their critical section

`{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }`

1: $\{\mathsf{true}\}$
  do $\{i\}$
2:   $\{\mathsf{true}\}$
   do $\{j_i\}$
3:     $\{\mathsf{true}\}$
     `r[] R10 latch0` $\{\leadsto L0^{i}_{j_i}\}$
4:     $\{\mathtt{R10} = \mathtt{L0}^{\mathrm{i}}_{\mathrm{j_i}} \wedge$
      $(\mathrm{r0R10}^{\mathrm{i}}_{\mathrm{j_i}}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1R10}^{\mathrm{i}}_{\mathrm{j_i}}[\Gamma_{\mathsf{rf}}])\}$
   `while (R10=0)` $\{k_i\}$
5:   $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}]\}$
   `w[] latch0 0`
6:   $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}]\}$
   `r[] Rf0 flag0` $\{\leadsto F0^{i}\}$
7:   $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathtt{Rf0} = \mathtt{F0}^{\mathrm{i}} \wedge$
    $(\mathrm{r0Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}] \vee \cancel{\mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]})\}$
   `if (Rf0≠0) then`
8:     $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
     `(* critical section *)`
     `w[] flag0 0`
9:     $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
false
     `w[] flag1 1`
10:     $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
     `w[] latch1 1`
11:     $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
   `fi`
12:   $\{\mathsf{true}\}$
  `while true`
13: $\{\mathsf{false}\}$

||

21: $\{\mathsf{true}\}$
  do $\{\ell\}$
22:   $\{\mathsf{true}\}$
   do $\{m_\ell\}$
23:     $\{\mathsf{true}\}$
     `r[] R11 latch1` $\{\leadsto L1^{\ell}_{m_\ell}\}$
24:     $\{\mathtt{R11} = \mathtt{L1}^{\ell}_{\mathrm{m_\ell}} \wedge$
      $(\mathrm{r0R11}^{\ell}_{\mathrm{m_\ell}}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1R11}^{\ell}_{\mathrm{m_\ell}}[\Gamma_{\mathsf{rf}}])\}$
   `while (R11=0)` $\{n_\ell\}$
25:   $\{\mathrm{r1R11}^{\ell}_{\mathrm{n_\ell}}[\Gamma_{\mathsf{rf}}]\}$
   `w[] latch1 0`
26:   $\{\mathrm{r1R11}^{\ell}_{\mathrm{n_\ell}}[\Gamma_{\mathsf{rf}}]\}$
   `r[] Rf1 flag1` $\{\leadsto F1^{\ell}\}$
27:   $\{\mathrm{r1R11}^{\ell}_{\mathrm{n_\ell}}[\Gamma_{\mathsf{rf}}] \wedge \mathtt{Rf1} = \mathtt{F1}^{\ell} \wedge$
    $(\mathrm{r0Rf1}^{\ell}[\Gamma_{\mathsf{rf}}] \vee \cancel{\mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]})\}$
   `if (Rf1≠0) then`
28:     $\{\mathrm{r1R11}^{\ell}_{\mathrm{n_\ell}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
     `(* critical section *)`
     `w[] flag1 0`
29:     $\{\mathrm{r1R11}^{\ell}_{\mathrm{n_\ell}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
     `w[] flag0 1`   false
30:     $\{\mathrm{r1R11}^{\ell}_{\mathrm{n_\ell}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
     `w[] latch0 1`
31:     $\{\mathrm{r1R11}^{\ell}_{\mathrm{n_\ell}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
   `fi`
32:   $\{\mathsf{true}\}$
  `while true`
33: $\{\mathsf{false}\}$

- let rf be the communication for such a trace (encoded in Γ_{rf})
- the invariant inside critical sections must be false
- tests (`Rf0≠0`) and (`Rf1≠0`) must be false (written ~~xxx~~)

(2) Both processes never enter their critical section

`{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }`

1: $\{\mathsf{true}\}$
  `do` $\{i\}$
2:   $\{\mathsf{true}\}$
   `do` $\{j_i\}$
3:     $\{\mathsf{true}\}$
     `r[] R10 latch0` $\{\leadsto L0^i_{j_i}\}$
4:     $\{\mathtt{R10} = \mathtt{L0}^{\mathtt{i}}_{\mathtt{j_i}} \wedge (\mathrm{r0R10}^{\mathrm{i}}_{\mathrm{j_i}}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1R10}^{\mathrm{i}}_{\mathrm{j_i}}[\Gamma_{\mathsf{rf}}])\}$
   `while (R10=0)` $\{k_i\}$
5:   $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}]\}$
   `w[] latch0 0`
6:   $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}]\}$
   `r[] Rf0 flag0` $\{\leadsto F0^i\}$
7:   $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathtt{Rf0} = \mathtt{F0}^{\mathtt{i}} \wedge (\mathrm{r0Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}] \vee \cancel{\mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]})\}$
   `if (Rf0≠0) then`
8:     $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
     `(* critical section *)`
     `w[] flag0 0`
rf
9:     $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
false
     `w[] flag1 1`
10:     $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
     `w[] latch1 1`
11:     $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
   `fi`
12:   $\{\mathsf{true}\}$
  `while true`
13: $\{\mathsf{false}\}$

||

21: $\{\mathsf{true}\}$
  `do` $\{\ell\}$
22:   $\{\mathsf{true}\}$
   `do` $\{m_\ell\}$
23:     $\{\mathsf{true}\}$
     `r[] R11 latch1` $\{\leadsto L1^\ell_{m_\ell}\}$
24:     $\{\mathtt{R11} = \mathtt{L1}^{\ell}_{\mathtt{m_\ell}} \wedge (\mathrm{r0R11}^{\ell}_{\mathrm{m_\ell}}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1R11}^{\ell}_{\mathrm{m_\ell}}[\Gamma_{\mathsf{rf}}])\}$
   `while (R11=0)` $\{n_\ell\}$
25:   $\{\mathrm{r1R11}^{\ell}_{\mathrm{n_\ell}}[\Gamma_{\mathsf{rf}}]\}$
   `w[] latch1 0`
26:   $\{\mathrm{r1R11}^{\ell}_{\mathrm{n_\ell}}[\Gamma_{\mathsf{rf}}]\}$
   `r[] Rf1 flag1` $\{\leadsto F1^\ell\}$
27:   $\{\mathrm{r1R11}^{\ell}_{\mathrm{n_\ell}}[\Gamma_{\mathsf{rf}}] \wedge \mathtt{Rf1} = \mathtt{F1}^{\ell} \wedge (\mathrm{r0Rf1}^{\ell}[\Gamma_{\mathsf{rf}}] \vee \cancel{\mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]})\}$
   `if (Rf1≠0) then`
28:     $\{\mathrm{r1R11}^{\ell}_{\mathrm{n_\ell}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
     `(* critical section *)`
rf
     `w[] flag1 0`
29:     $\{\mathrm{r1R11}^{\ell}_{\mathrm{n_\ell}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
     `w[] flag0 1`
false
30:     $\{\mathrm{r1R11}^{\ell}_{\mathrm{n_\ell}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
     `w[] latch0 1`
31:     $\{\mathrm{r1R11}^{\ell}_{\mathrm{n_\ell}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
   `fi`
32:   $\{\mathsf{true}\}$
  `while true`
33: $\{\mathsf{false}\}$

- let rf be the communication for such a trace (encoded in Γ_{rf})
- the invariant inside critical sections must be false
- tests (`Rf0≠0`) and (`Rf1≠0`) must be false (written ~~xxx~~)
- so read of `Rf0` and `Rf1` is 0 from a reachable write

(2) Both processes never enter their critical section

`{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }`

1: {true}
  do $\{i\}$
2: {true}
  do $\{j_i\}$
3: {true}
  r[] R10 latch0 $\{\rightsquigarrow L0^{i}_{j_i}\}$
4: $\{\mathtt{R10} = \mathtt{L0}^{i}_{j_i} \wedge (\mathrm{r0R10}^{i}_{j_i}[\Gamma_{\mathrm{rf}}] \vee \mathrm{r1R10}^{i}_{j_i}[\Gamma_{\mathrm{rf}}])\}$
  while (R10=0) $\{k_i\}$
5: $\{\mathrm{r1R10}^{i}_{k_i}[\Gamma_{\mathrm{rf}}]\}$
  w[] latch0 0
6: $\{\mathrm{r1R10}^{i}_{k_i}[\Gamma_{\mathrm{rf}}]\}$
  r[] Rf0 flag0 $\{\rightsquigarrow F0^{i}\}$
7: $\{\mathrm{r1R10}^{i}_{k_i}[\Gamma_{\mathrm{rf}}] \wedge \mathtt{Rf0} = \mathtt{F0}^{i} \wedge (\mathrm{r0Rf0}^{i}[\Gamma_{\mathrm{rf}}] \vee \cancel{\mathrm{r1Rf0}^{i}[\Gamma_{\mathrm{rf}}]})\}$
  if (Rf0≠0) then
8: $\{\mathrm{r1R10}^{i}_{k_i}[\Gamma_{\mathrm{rf}}] \wedge \mathrm{r1Rf0}^{i}[\Gamma_{\mathrm{rf}}]\}$
  (* critical section *)
  w[] flag0 0
9: $\{\mathrm{r1R10}^{i}_{k_i}[\Gamma_{\mathrm{rf}}] \wedge \mathrm{r1Rf0}^{i}[\Gamma_{\mathrm{rf}}]\}$
  w[] flag1 1
false
10: $\{\mathrm{r1R10}^{i}_{k_i}[\Gamma_{\mathrm{rf}}] \wedge \mathrm{r1Rf0}^{i}[\Gamma_{\mathrm{rf}}]\}$
  w[] latch1 1
11: $\{\mathrm{r1R10}^{i}_{k_i}[\Gamma_{\mathrm{rf}}] \wedge \mathrm{r1Rf0}^{i}[\Gamma_{\mathrm{rf}}]\}$
  fi
12: {true}
  while true
13: {false}

rf

||

21: {true}
  do $\{\ell\}$
22: {true}
  do $\{m_\ell\}$
23: {true}
  r[] R11 latch1 $\{\rightsquigarrow L1^{\ell}_{m_\ell}\}$
24: $\{\mathtt{R11} = \mathtt{L1}^{\ell}_{m_\ell} \wedge (\mathrm{r0R11}^{\ell}_{m_\ell}[\Gamma_{\mathrm{rf}}] \vee \cancel{\mathrm{r1R11}^{\ell}_{m_\ell}[\Gamma_{\mathrm{rf}}]})\}$
  while (R11=0) $\{n_\ell\}$
25: $\{\cancel{\mathrm{r1R11}^{\ell}_{n_\ell}[\Gamma_{\mathrm{rf}}]}\}$
  w[] latch1 0
26: $\{\cancel{\mathrm{r1R11}^{\ell}_{n_\ell}[\Gamma_{\mathrm{rf}}]}\}$
  r[] Rf1 flag1 $\{\rightsquigarrow F1^{\ell}\}$
27: $\{\mathrm{r1R11}^{\ell}_{n_\ell}[\Gamma_{\mathrm{rf}}] \wedge \mathtt{Rf1} = \mathtt{F1}^{\ell} \wedge (\cancel{\mathrm{r0Rf1}^{\ell}[\Gamma_{\mathrm{rf}}]} \vee \cancel{\mathrm{r1Rf1}^{\ell}[\Gamma_{\mathrm{rf}}]})\}$
  if (Rf1≠0) then
28: $\{\mathrm{r1R11}^{\ell}_{n_\ell}[\Gamma_{\mathrm{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathrm{rf}}]\}$
  (* critical section *)
rf
  w[] flag1 0
29: $\{\mathrm{r1R11}^{\ell}_{n_\ell}[\Gamma_{\mathrm{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathrm{rf}}]\}$
  w[] flag0 1
false
30: $\{\mathrm{r1R11}^{\ell}_{n_\ell}[\Gamma_{\mathrm{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathrm{rf}}]\}$
  w[] latch0 1
31: $\{\mathrm{r1R11}^{\ell}_{n_\ell}[\Gamma_{\mathrm{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathrm{rf}}]\}$
  fi
32: {true}
  while true
33: {false}

- let rf be the communication for such a trace (encoded in Γ_{rf})
- the invariant inside critical sections must be false
- tests (Rf0≠0) and (Rf1≠0) must be false (written ~~xxx~~)
- so read of Rf0 and Rf1 is 0 from a reachable write
- impossible for Rf1 so loop 23—24 is never exited

⟹ we are in case (3), P1 stuck in spin loop

(3) Process P1 stuck in spin loop (no hypothesis on P0)

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
1: {true}
   do {i}
2:   {true}
     do {j_i}
3:     {true}
       r[] Rl0 latch0 {⇝ L0^i_{j_i}}
4:     {Rl0 = L0^i_{j_i} ∧
        (r0Rl0^i_{j_i}[Γrf] ∨ r1Rl0^i_{j_i}[Γrf])}
     while (Rl0=0) {k_i}
5:   {r1Rl0^i_{k_i}[Γrf]}
     w[] latch0 0
6:   {r1Rl0^i_{k_i}[Γrf]}
     r[] Rf0 flag0 {⇝ F0^i}
7:   {r1Rl0^i_{k_i}[Γrf] ∧ Rf0 = F0^i ∧
      (r0Rf0^i[Γrf] ∨ r1Rf0^i[Γrf])}
     if (Rf0≠0) then
8:     {r1Rl0^i_{k_i}[Γrf] ∧ r1Rf0^i[Γrf]}
       (* critical section *)
       w[] flag0 0
9:     {r1Rl0^i_{k_i}[Γrf] ∧ r1Rf0^i[Γrf]}
 false w[] flag1 1
10:    {r1Rl0^i_{k_i}[Γrf] ∧ r1Rf0^i[Γrf]}
       w[] latch1 1
11:    {r1Rl0^i_{k_i}[Γrf] ∧ r1Rf0^i[Γrf]}
     fi
12:  {true}
   while true
13:{false}
||
21:{true}
   do {ℓ}
22:  {true}
     do {m_ℓ}
23:    {true}
       r[] Rl1 latch1 {⇝ L1^ℓ_{m_ℓ}}
24:    {Rl1 = L1^ℓ_{m_ℓ} ∧
        (r0Rl1^ℓ_{m_ℓ}[Γrf] ∨ ~~r1Rl1^ℓ_{m_ℓ}[Γrf]~~)}
     while (Rl1=0) {n_ℓ}
25:  {r1Rl1^ℓ_{n_ℓ}[Γrf]}
     w[] latch1 0
26:  {r1Rl1^ℓ_{n_ℓ}[Γrf]}
     r[] Rf1 flag1 {⇝ F1^ℓ}
27:  {r1Rl1^ℓ_{n_ℓ}[Γrf] ∧ Rf1 = F1^ℓ ∧
      (r0Rf1^ℓ[Γrf] ∨ r1Rf1^ℓ[Γrf])}
     if (Rf1≠0) then
28:    {r1Rl1^ℓ_{n_ℓ}[Γrf] ∧ r1Rf1^ℓ[Γrf]}
       (* critical section *)
       w[] flag1 0                              false
29:    {r1Rl1^ℓ_{n_ℓ}[Γrf] ∧ r1Rf1^ℓ[Γrf]}
       w[] flag0 1
30:    {r1Rl1^ℓ_{n_ℓ}[Γrf] ∧ r1Rf1^ℓ[Γrf]}
       w[] latch0 1
31:    {r1Rl1^ℓ_{n_ℓ}[Γrf] ∧ r1Rf1^ℓ[Γrf]}
     fi
32:  {true}
   while true
33:{false}
```

- let rf be the communication for such a trace (encoded in Γ_{rf})
- the invariant after 25: must be false
- read of `latch1` in 23: must be a 0
- only possibility if from 25:
- A contradiction since 25: is unreachable

(4) Process `P0` starves in spin loop, no hypothesis on `P1`

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
```

P0

1: $\{\text{true}\}$
do $\{i\}$
2: $\{\text{true}\}$
do $\{j_i\}$
3: $\{\text{true}\}$
`r[] R10 latch0` $\{\rightsquigarrow L0^i_{j_i}\}$
4: $\{\mathtt{R10} = \mathtt{L0}^i_{\mathtt{j_i}} \wedge (\mathtt{r0R10}^i_{\mathtt{j_i}}[\Gamma_{\mathsf{rf}}] \vee \cancel{\mathtt{r1R10}^i_{\mathtt{j_i}}[\Gamma_{\mathsf{rf}}]})\}$
`while (R10=0)` $\{k_i\}$
5: $\{\mathtt{r1R10}^i_{\mathtt{k_i}}[\Gamma_{\mathsf{rf}}]\}$
`w[] latch0 0`
6: $\{\mathtt{r1R10}^i_{\mathtt{k_i}}[\Gamma_{\mathsf{rf}}]\}$
`r[] Rf0 flag0` $\{\rightsquigarrow F0^i\}$
7: $\{\mathtt{r1R10}^i_{\mathtt{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathtt{Rf0} = \mathtt{F0}^i \wedge (\mathtt{r0Rf0}^i[\Gamma_{\mathsf{rf}}] \vee \mathtt{r1Rf0}^i[\Gamma_{\mathsf{rf}}])\}$
`if (Rf0≠0) then`
8: $\{\mathtt{r1R10}^i_{\mathtt{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathtt{r1Rf0}^i[\Gamma_{\mathsf{rf}}]\}$
false
`(* critical section *)`
`w[] flag0 0`
9: $\{\mathtt{r1R10}^i_{\mathtt{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathtt{r1Rf0}^i[\Gamma_{\mathsf{rf}}]\}$
`w[] flag1 1`
10: $\{\mathtt{r1R10}^i_{\mathtt{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathtt{r1Rf0}^i[\Gamma_{\mathsf{rf}}]\}$
`w[] latch1 1`
11: $\{\mathtt{r1R10}^i_{\mathtt{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathtt{r1Rf0}^i[\Gamma_{\mathsf{rf}}]\}$
`fi`
12: $\{\text{true}\}$
`while true`
13: $\{\text{false}\}$

co

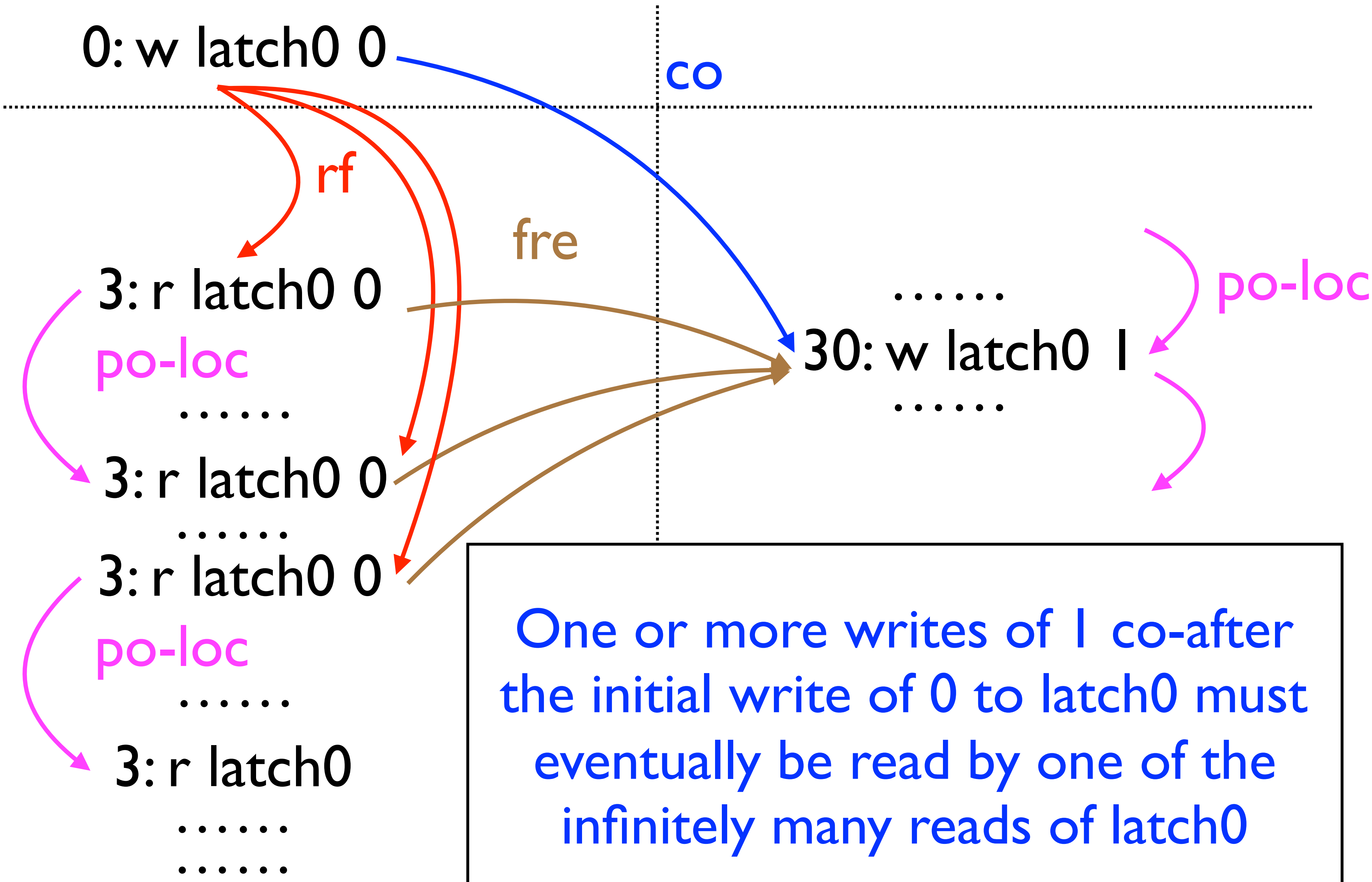
fr

P1

21: $\{\text{true}\}$
do $\{\ell\}$
22: $\{\text{true}\}$
do $\{m_\ell\}$
23: $\{\text{true}\}$
`r[] R11 latch1` $\{\rightsquigarrow L1^\ell_{m_\ell}\}$
24: $\{\mathtt{R11} = \mathtt{L1}^\ell_{\mathtt{m_\ell}} \wedge (\mathtt{r0R11}^\ell_{\mathtt{m_\ell}}[\Gamma_{\mathsf{rf}}] \vee \mathtt{r1R11}^\ell_{\mathtt{m_\ell}}[\Gamma_{\mathsf{rf}}])\}$
`while (R11=0)` $\{n_\ell\}$
25: $\{\mathtt{r1R11}^\ell_{\mathtt{n_\ell}}[\Gamma_{\mathsf{rf}}]\}$
`w[] latch1 0`
26: $\{\mathtt{r1R11}^\ell_{\mathtt{n_\ell}}[\Gamma_{\mathsf{rf}}]\}$
`r[] Rf1 flag1` $\{\rightsquigarrow F1^\ell\}$
27: $\{\mathtt{r1R11}^\ell_{\mathtt{n_\ell}}[\Gamma_{\mathsf{rf}}] \wedge \mathtt{Rf1} = \mathtt{F1}^\ell \wedge (\mathtt{r0Rf1}^\ell[\Gamma_{\mathsf{rf}}] \vee \mathtt{r1Rf1}^\ell[\Gamma_{\mathsf{rf}}])\}$
`if (Rf1≠0) then`
28: $\{\mathtt{r1R11}^\ell_{\mathtt{n_\ell}}[\Gamma_{\mathsf{rf}}] \wedge \mathtt{r1Rf1}^\ell[\Gamma_{\mathsf{rf}}]\}$
`(* critical section *)`
`w[] flag1 0`
29: $\{\mathtt{r1R11}^\ell_{\mathtt{n_\ell}}[\Gamma_{\mathsf{rf}}] \wedge \mathtt{r1Rf1}^\ell[\Gamma_{\mathsf{rf}}]\}$
`w[] flag0 1`
30: $\{\mathtt{r1R11}^\ell_{\mathtt{n_\ell}}[\Gamma_{\mathsf{rf}}] \wedge \mathtt{r1Rf1}^\ell[\Gamma_{\mathsf{rf}}]\}$
`w[] latch0 1`
31: $\{\mathtt{r1R11}^\ell_{\mathtt{n_\ell}}[\Gamma_{\mathsf{rf}}] \wedge \mathtt{r1Rf1}^\ell[\Gamma_{\mathsf{rf}}]\}$
`fi`
32: $\{\text{true}\}$
`while true`
33: $\{\text{false}\}$

- let rf be the communication for such a trace (encoded in Γ_{rf})
- the invariant after 5: must be false so P0 never enters its critical section
- read of `latch0` in 3: must be a 0, with 2 possibilities
- cannot be from write at 5: which is unreachable
- so is from initial write 0:
- but P1 enters its critical section (otherwise see case 1)
- so `w[] latch0 1` will be executed later in co order
- so all `3:r[] R10 latch0` are fr to all 30: w[] latch0 1
- by fairness of communications, this write of 1 to latch0 will eventually be read at 3:
- in contradiction with always reading 0

(4) Process P0 starves in spin loop, P1 does not



One or more writes of 1 co-after the initial write of 0 to latch0 must eventually be read by one of the infinitely many reads of latch0

Communication fairness hypothesis(*)

- All writes eventually hit the memory:
  - If, at a cut of the execution, all the processes infinitely often write the same value v to a shared variable `x` and only that value v
  - and from a later cut point of that execution, a process infinitely often repeats reads to that variable `x`
  - then the reads will end up reading that value v

(*) The SPARC Architecture Manual, Version 8, Section K2, p. 283: ``if one processor does an *S* , and another processor repeatedly does *L* 's to the same location, then there is an *L* that will be after the *S*".

(5) Process P1 never enters its CS

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
1: {true}
   do {i}
2:   {true}
     do {j_i}
3:     {true}
       r[] R10 latch0 {⇝ L0^i_{j_i}}
4:     {R10 = L0^i_{j_i} ∧
        (r0R10^i_{j_i}[Γ_rf] ∨ r1R10^i_{j_i}[Γ_rf])}
     while (R10=0) {k_i}
5:   {r1R10^i_{k_i}[Γ_rf]}
     w[] latch0 0
6:   {r1R10^i_{k_i}[Γ_rf]}
     r[] Rf0 flag0 {⇝ F0^i}
7:   {r1R10^i_{k_i}[Γ_rf] ∧ Rf0 = F0^i ∧
      (r0Rf0^i[Γ_rf] ∨ r1Rf0^i[Γ_rf])}
     if (Rf0≠0) then
8:     {r1R10^i_{k_i}[Γ_rf] ∧ r1Rf0^i[Γ_rf]}
       (* critical section *)
       w[] flag0 0
9:     {r1R10^i_{k_i}[Γ_rf] ∧ r1Rf0^i[Γ_rf]}
       w[] flag1 1
10:    {r1R10^i_{k_i}[Γ_rf] ∧ r1Rf0^i[Γ_rf]}
       w[] latch1 1
11:    {r1R10^i_{k_i}[Γ_rf] ∧ r1Rf0^i[Γ_rf]}
     fi
12:  {true}
   while true
13:{false}
```

```
21:{true}
   do {ℓ}
22:  {true}
     do {m_ℓ}
23:    {true}
       r[] Rl1 latch1 {⇝ L1^ℓ_{m_ℓ}}
24:    {Rl1 = L1^ℓ_{m_ℓ} ∧
        (r0Rl1^ℓ_{m_ℓ}[Γ_rf] ∨ r1Rl1^ℓ_{m_ℓ}[Γ_rf])}
     while (Rl1=0) {n_ℓ}
25:  {r1Rl1^ℓ_{n_ℓ}[Γ_rf]}
     w[] latch1 0
26:  {r1Rl1^ℓ_{n_ℓ}[Γ_rf]}
     r[] Rf1 flag1 {⇝ F1^ℓ}
27:  {r1Rl1^ℓ_{n_ℓ}[Γ_rf] ∧ Rf1 = F1^ℓ ∧
      (r0Rf1^ℓ[Γ_rf] ∨ r1Rf1^ℓ[Γ_rf])}
     if (Rf1≠0) then
28:    {r1Rl1^ℓ_{n_ℓ}[Γ_rf] ∧ r1Rf1^ℓ[Γ_rf]}
       (* critical section *)
       w[] flag1 0
29:    {r1Rl1^ℓ_{n_ℓ}[Γ_rf] ∧ r1Rf1^ℓ[Γ_rf]}
       w[] flag0 1
30:    {r1Rl1^ℓ_{n_ℓ}[Γ_rf] ∧ r1Rf1^ℓ[Γ_rf]}
       w[] latch0 1
31:    {r1Rl1^ℓ_{n_ℓ}[Γ_rf] ∧ r1Rf1^ℓ[Γ_rf]}
     fi
32:  {true}
   while true
33:{false}
```

rf

false

- let rf be the communication for such a trace (encoded in Γ_{rf})
- `P1` exits loop 23:–24: (else see cases (1) or (3))
- must read `Rl1` = 1 from 0: or 10:
- read of `Rf1` at 26: must be 0
- only possibility is from 28:
- impossible from unreachable code

(5) Process P0 leaves spin loop but always fails entering its CS

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
1:  {true}
    do {i}
2:    {Γrf}
      do {j_i}
3:      {true}
        r[] R10 latch0 {⇝ L0^i_{j_i}}
4:      {R10 = L0^i_{j_i} ∧
          (r0R10^i_{j_i}[Γrf] ∨ r1R10^i_{j_i}[Γrf])}
      while (R10=0) {k_i}
5:    {r1R10^i_{k_i}[Γrf]}
      w[] latch0 0
      {r1R10^i_{k_i}[Γrf]}
      f[fdep] {3} {6}
6:    {r1R10^i_{k_i}[Γrf]}
      r[] Rf0 flag0 {⇝ F0^i}
7:    {r1R10^i_{k_i}[Γrf] ∧ Rf0 = F0^i ∧
       (r0Rf0^i[Γrf] ∨ r1Rf0^i[Γrf])}
      if (Rf0≠0) then
8:      {r1R10^i_{k_i}[Γrf] ∧ r1Rf0^i[Γrf]}
        (* critical section *)
        w[] flag0 0
9:      {r1R10^i_{k_i}[Γrf] ∧ r1Rf0^i[Γrf]}
        w[] flag1 1
10:     {r1R10^i_{k_i}[Γrf] ∧ r1Rf0^i[Γrf]}
        w[] latch1 1
11:     {r1R10^i_{k_i}[Γrf] ∧ r1Rf0^i[Γrf]}
      fi
12:   {true}
    while true
13: {false}
```

```
21: {true}
    do {ℓ}
22:   {true}
      do {m_ℓ}
23:     {true}
        r[] R11 latch1 {⇝ L1^ℓ_{m_ℓ}}
24:     {R11 = L1^ℓ_{m_ℓ} ∧
         (r0R11^ℓ_{m_ℓ}[Γrf] ∨ r1R11^ℓ_{m_ℓ}[Γrf])}
      while (R11=0) {n_ℓ}
25:   {r1R11^ℓ_{n_ℓ}[Γrf]}
      w[] latch1 0
26:   {r1R11^ℓ_{n_ℓ}[Γrf]}
      r[] Rf1 flag1 {⇝ F1^ℓ}
27:   {r1R11^ℓ_{n_ℓ}[Γrf] ∧ Rf1 = F1^ℓ ∧
       (r0Rf1^ℓ[Γrf] ∨ r1Rf1^ℓ[Γrf])}
      if (Rf1≠0) then
28:     {r1R11^ℓ_{n_ℓ}[Γrf] ∧ r1Rf1^ℓ[Γrf]}
        (* critical section *)
        w[] flag1 0
29:     {r1R11^ℓ_{n_ℓ}[Γrf] ∧ r1Rf1^ℓ[Γrf]}
        w[] flag0 1
        {r1R11^ℓ_{n_ℓ}[Γrf] ∧ r1Rf1^ℓ[Γrf]}
        f[flw] {29} {30}
30:     {r1R11^ℓ_{n_ℓ}[Γrf] ∧ r1Rf1^ℓ[Γrf]}
        w[] latch0 1
31:     {r1R11^ℓ_{n_ℓ}[Γrf] ∧ r1Rf1^ℓ[Γrf]}
      fi
32:   {true}
    while true
33: {false}
```

fences, co, rf, fre, false, fences

- let rf be the communication for such a trace (encoded in Γ_{rf})
- loop 2:–4: exited
- read of R10 = 1 at 3: is from 30:
- invariant false in critical section 8:–11:
- read of Rf0 = 0 at 6: is from 0: (8: not reachable)

```
withco
let l-fencerel(S) =
        ((po&(_*S));po)&fromto(S)
let Fdep = F & tag2events('fdep)
let deps = l-fencerel(Fdep) & (R*_)
let Flw = F & tag2events('flw)
let flw = l-fencerel(Flw)
let fences = deps | flw
let fre = (rf^-1;co) & ext
irreflexive fre;fences;rfe;fences
```

In TSO there is no need for a fence since it is MP. For weaker than PSO, a fence is needed.

(6) Both processes eventually starve in spin loop

```
   {0:  w latch0 0;                     w latch1 1;
        w flag0 0;                      w flag1 1;}
                              ||
   ... ...                    ||  ... ...
   3:  r R10 latch0 1         ||  23: r R11 latch1 1
   5:  w latch0 0             ||  25: w latch1 0
   6:  r Rf0 flag0 1          ||  26: r Rf1 flag1 1
   8:  (* critical section *) ||  28: (* critical section *)
       w flag0 0              ||      w flag1 0
                              ||      f[bar] {25:} {29:}
   9:  w flag1 1              ||  29: w flag0 1
       f[bar] {5:} {10:}   co ||
   10: w latch1 1             ||  30: w latch0 1
bar... ...                    ||  ... ...              bar
   3:  r R10 latch0 1         ||  23: r R11 latch1 1
   5:  w latch0 0             ||  25: w latch1 0
   6:  r Rf0 flag0 1          ||  26: r Rf1 flag1 1
   8:  (* critical section *) ||  28: (* critical section *)
       w flag0 0              ||      w flag1 0
                              ||      f[bar] {25:} {29:}
   9:  w flag1 1              ||  29: w flag0 1
       f[bar] {5:} {10:}      ||
   10: w latch1 1             ||  30: w latch0 1
   ... ...                    ||  ... ...
   3:  r R10 latch0 0         ||  23: r R11 latch1 0
   3:  r R10 latch0 0         ||  23: r R11 latch1 0
   ... ...                    ||  ... ...
```

- let rf be the communication for such a trace (encoded in Γ_{rf})
- so `latch0` is always 0 and `latch1` is always 0
- so `latch0` in 23 is always read from 25:
- so `10: w latch1 1` was co-before (since otherwise by the communication hypothesis it would be eventually read)
- and `3: R10 latch0 0` is from 0: or 5:
- so `30: w latch0 1` is co-before them (since otherwise by the communication hypothesis it would be eventually read)
- impossible by fences
- irreflexive co; bar; co; bar

(7) Eventually, P0 starves in spin loop, P1 never enters its CS

Process P0 enters & exits CS multiple times

then, never exits the waiting loop

```
{0: w latch0 0;              w latch1 1;
    w flag0 0;               w flag1 1;}
... ...                      ... ...
3:  r Rl0 latch0 1           23: r Rl1 latch1 1
5:  w latch0 0               25: w latch1 0
6:  r Rf0 flag0 1            26: r Rf1 flag1 1
8:  (* critical section *)   28: (* critical section *)
    w flag0 0                    w[] flag1 0
9:  w flag1 1                29: w[] flag0 1
10: w latch1 1               30  w[] latch0 1
... ...                      ... ...
3:  r Rl0 latch0 1           23: r Rl1 latch1 1
5:  w latch0 0               25: w latch1 0
6:  r Rf0 flag0 1            26: r Rf1 flag1 0
8:  (* critical section *)   ... ...
    w flag0 0                23: r Rl1 latch1 1
9:  w flag1 1                25: w latch1 0
10: w latch1 1               26: r Rf1 flag1 0
... ...
3:  r Rl0 latch0 0           ... ...
3:  r Rl0 latch0 0
3:  r Rl0 latch0 0           ... ...
... ...
```

last CS entr-ance

- P1 does not eventually starves in spin loop (otherwise case 6)
- case P1 eventually never starves and never enters its critical section
- P1 then does a last write of 1 to latch0
- P0 eventually makes infinitely many reads of latch0
- A contradiction (since otherwise by the communication hypothesis, this 1 would be eventually read)

(8) Eventually, `P1` starves in spin loop, `P0` never enters its CS

symmetric of (7)

(9) P0 and P1 always leave spin loop and never enter their CS

```
{0:  w[] latch0 0;
     w[] flag0 0;
... ...
3:  r[] Rl0 latch0 1
5:  w[] latch0 0
6:  r[] Rf0 flag0 1
8:  (* critical section *)
    w[] flag0 0
9:  w[] flag1 1
10: w[] latch1 1
... ...
3:  r Rl0 latch0 1
5:  w[] latch0 0
6:  r[] Rf0 flag0 1
8:  (* critical section *)
    w[] flag0 0
9:  w[] flag1 1
10: w[] latch1 1
... ...
3:  r[] Rl0 latch0 1
5:  w[] latch0 0
6:  r[] Rf0 flag0 0
... ...
3:  r[] Rl0 latch0 1
5:  w[] latch0 0
6:  r[] Rf0 flag0 0
... ...
3:  r[] Rl0 latch0 1
5:  w[] latch0 0
6:  r[] Rf0 flag0 0
... ...
```

```
    w[] latch1 1;
    w[] flag1 1;}
... ...
23: r[] Rl1 latch1 1
25: w[] latch1 0
26: r[] Rf1 flag1 1
28: (* critical section *)
    w[] flag1 0
29: w[] flag0 1
30  w[] latch0 1
... ...
23: r[] Rl1 latch1 1
25: w[] latch1 0
26: r[] Rf1 flag1 0
28: (* critical section *)
23: w[] flag1 0
29: w[] flag0 1
30: w[] latch0 1
... ...
23: r[] Rl1 latch1 1
25: w[] latch1 0
26: r[] Rf1 flag1 0
... ...
23: r[] Rl1 latch1 1
25: w[] latch1 0
26: r[] Rf1 flag1 0
... ...
23: r[] Rl1 latch1 1
25: w[] latch1 0
26: r[] Rf1 flag1 0
... ...
```

rf

...

- P0 and P1 eventually never starve and never enter their critical sections
- They both have a last entrance in their critical sections
- This last write of 1 to the latches will, by communication fairness, eventually reach the memory
- Then we only have infinitely many writes of 0 to the latches
- So the read of the latches in the spin loops will eventually always read 0
- So from then on, by communication fairness, all the reads will be from 0, in reads of the latch will be zero
- In contradiction with the fact that the spin loop is always exited
- The barrier prevents infinitely postponing the write 0 actions

Conclusion

Conclusion

- The proof method is parameterized by consistency hypotheses, expressed in
  - Invariance form: S_{com}
  - Consistency form: H_{com} (e.g. in `cat`)
- Program not logic/architecture/consistency model dependent (hence the proof is portable)
- Can reason on *arbitrary* subsets of anarchic executions (hence flexible e.g. non-starvation)

Proposed design methodology

1. Design the algorithm A and its specification S_{inv} (e.g. in the sequential consistency model of parallelism)
2. Consider the anarchic semantics of algorithm A
3. Add communication specifications S_{com} to restrict anarchic communications and ensure the correctness of A with respect to specification S_{inv}
4. Do the invariance proof under WCM with S_{com}
5. Infer H_{com} (in `cat`) from invariant S_{com}
6. Prove that the machine memory model M in `cat` implies H_{cm}

Challenges

- Modern machines have complex memory models

$\Rightarrow$ portability has a price (refencing)

$\Rightarrow$ debugging is very hard/quasi-impossible

$\Rightarrow$ proofs are much harder than with sequential consistency (but still feasible?, mechanically?)

$\Rightarrow$ static analysis parameterized by a WCM will be a challenge

$\Rightarrow$ but we can start with $\mathcal{S}_{com}$

Thanks

- *Patrick Cousot thanks Luc Maranget for his precious help at Dagstuhl on the non-starvation part.*

The End, Thank You