Question 1

A) Contour and surface plots available in Figure 2. There are 4 local minima.

B) The optimizer is successful in finding a minimum, but which minimum it finds depends on the starting guess. For instance, starting at (0,0) leads Julia to find (3,2) as the minimizer, while starting at (-1, -1) results in Julia returning (-3.78, -3.28).

C) With Nelder-Mead, the number of iterations starting from (0,0) goes from 7 to 57. The elapsed time goes from about 6e-5 to 7e-5 seconds.

Question 2)
A) Contour and surface plots available in Figure 3. The global minimum is zero.

B) This function is tricky to optimize. Very close starting guesses to (0,0) typically result in LBFGS getting the right answer, but I've even seen Nelder-Mead fail with a starting guess of (1,1).

Question 3)
A) See figure 4. The global minimum is zero, and this holds for any arbitrary n since the x_i

factor into the function additively separatively.

B) See figure 6.

C) LBFGS() seems to be pretty robust at finding the global minimum for the Rastrigin function. Nelder-Mead is not – even (1,1) fails here!

Question 5

A) See Figure 1. The approximation error is largest where the curvature of the function is the strongest.

C) Unsurprisingly, Nelder-Mead places a larger density of points toward zero, where the curvature of the function is strongest. The total approximation error decreases from 50.4 to about 11. See Figure 2 for plots.

D) With cubic splines, the approximation error drops from 41.3 to about 8.923 — not all that different from what you can get with linear interpolation.

Figure 1: Question 1)

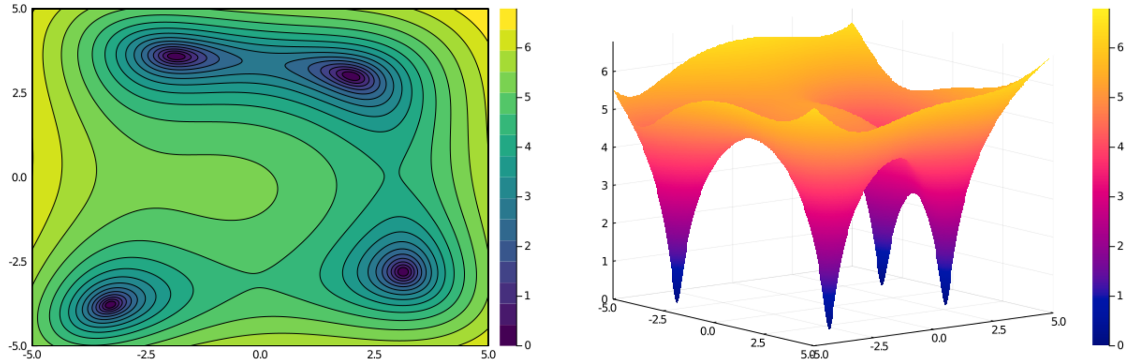

Figure 2: Question 2)

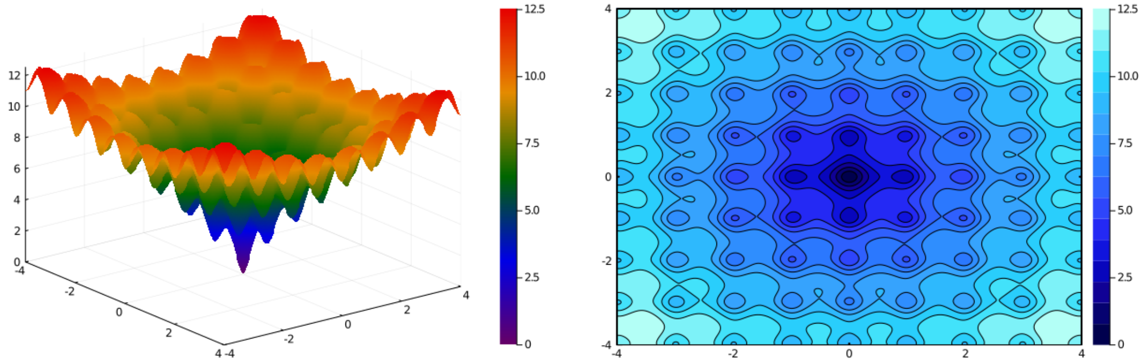

Figure 3: Question 3a

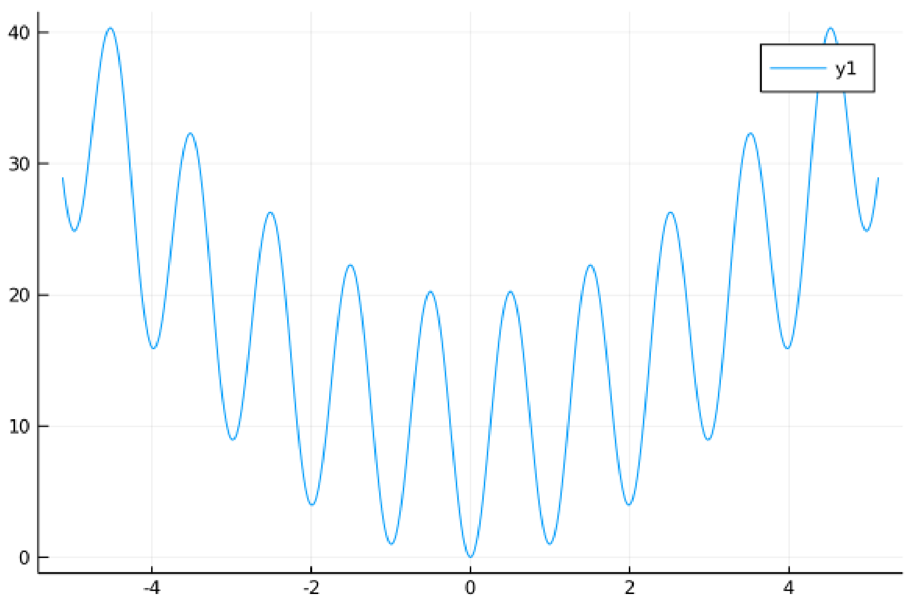

Figure 4: Question 3)

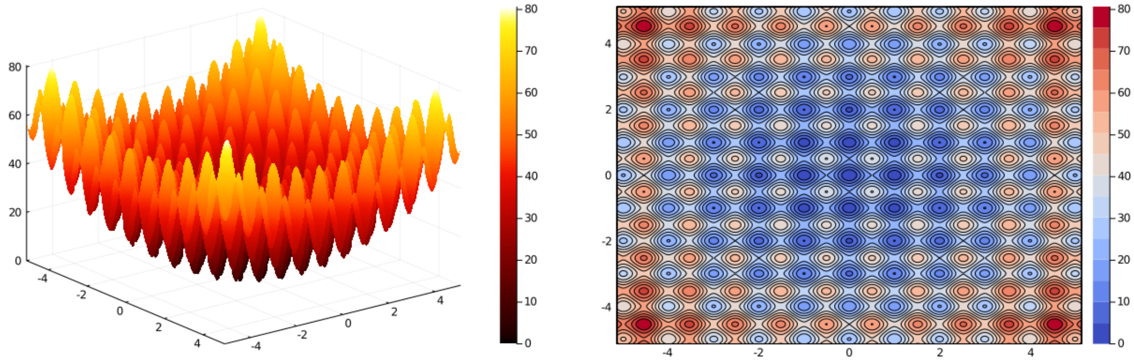

Figure 5: Question 5a)

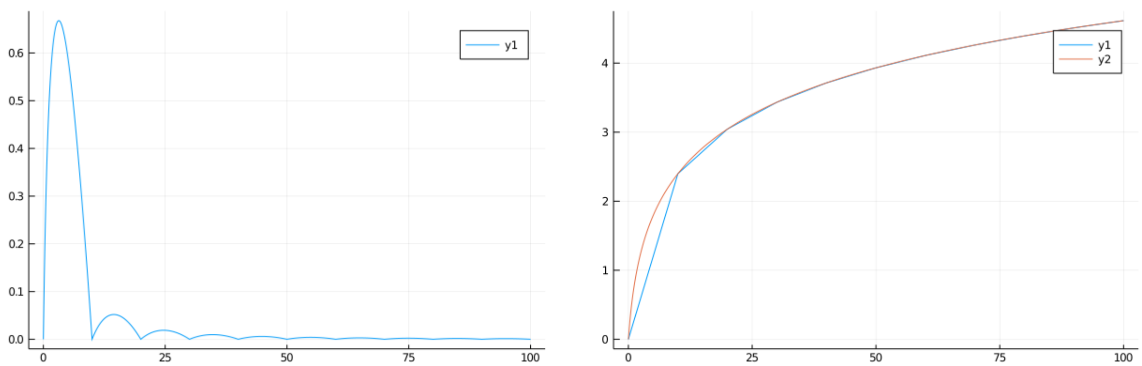

Figure 6: Question 5c)