Question 1: Write a function to solve the following problem using dynamic programming:

Given a value N, if we want to make change for N cents, and we have an infinite supply of each $S = \{S1, S2, ..., S_m\}$ valued coins, how many ways can we make the change? The order of the coins does not matter. For example, if $N = 10$ and $S = \{2, 5, 3, 6\}$, there are five solutions.

Hint: All solutions can be divided into the set of solutions that contain *at least one* of the mth coin and those that do not.

Question 2: Write a function to solve the following problem using dynamic programming:

Given a rod of length n inches and an array $\boldsymbol{P}$ of prices that contains prices of all pieces of size $s \leq n$, determine the maximum value obtainable by cutting up the rod in even inch increments and selling the pieces. For example, if rod is 8 inches and the piece length values are given by $\boldsymbol{P} = \{1, 5, 8, 9, 10, 17, 17, 20\}$, then the maximum obtainable value is 22 by cutting pieces of length 6 and 2.

Hint: Call the function $V(n)$ the maximum value obtainable for a rod of length n. Then

$$V(n) = \max_{c \in \{0,1,...n-1\}} \{\boldsymbol{P}[c] + V(n-c)\}$$

Question 3: Write a function to solve the following problem using dynamic programming:

Given weights $\boldsymbol{W}$ and values $\boldsymbol{V}$ of n items, put these items in a knapsack of capacity C to get the maximum total value in the knapsack. As an example, if $\boldsymbol{W} = \{10, 20, 30\}$ and $\boldsymbol{V} = \{60, 100, 120\}$ and $C = 50$, then the maximum value is 220, obtained by selecting the second and third item. **Bonus:** argue that this problem is a generalization of the problem in Question 2.

Hint: Possible subsets of items can be split up into including the nth item or not including the nth item.

Question 4
Write down an economic problem that interests you specifically and express it as a dynamic programming problem. Solve the problem on your computer and report things about the results that you find interesting — this could be plots of decision rules, simulated results of agents solving the problem, or whatever else comes to mind.