Problem Statement

Find the per-person rate that utilizes the available funding while respecting caps on the total amount of funds that can be awarded.

Definitions

$$x_i = The\ number\ of\ people\ served\ by\ center\ i$$

$$x_i \in Z^+$$

$$y_i = Whether\ center\ i\ received\ an\ incentive\ for\ operating\ in\ unincorporated\ Pierce\ County$$

$$y_i \in \{0, 1\}$$

$$TF = Scalar\ for\ total\ funding$$

$$BF = Scalar\ for\ base\ funding$$

$$UF = Scalar\ for\ unincorporated\ incentive\ funding$$

$$Cap_i = Scalar\ for\ total\ amount\ of\ funding\ an\ individual\ center\ can\ recieve$$

$$PP = Per\ person\ funding\ rate$$

$$TF, BF, UF, Cap, PP \in R^+$$

Problem statements

$$\boldsymbol{min_{PP}} \left| \boldsymbol{TF} - (\sum_{\boldsymbol{i=1}}^{\boldsymbol{n}} \boldsymbol{min}(\boldsymbol{BF} + \boldsymbol{UF} * \boldsymbol{y_i} + \boldsymbol{PP} * \boldsymbol{x_i},\ \ \boldsymbol{cap_i})) \right|$$

Such that: $PP \geq 0$

Methods

The following formulation represents a non-convex, monotonic, constrained optimization problem. The problem is non-convex because we need to take the minimum of two affine functions. Due to non-convexity, straightforward applications of optimization algorithms such as Simplex will not work. Potential solutions are reformulating the problem into a convex optimization, modified gradient descent (modified because the minimum function isn't strictly differentiable), or bisection. However, given that the search space on the per-person rate is relatively small, we can simply perform enumeration and pick the per-person rate that minimizes the objective function. In this case, we search the space of [0-$4,000] over every cent resulting in 400,001 searches. If the search space becomes greatly enlarged in a future application new methods will need to be used.