Understanding Persistent ZLB: Theory and Assessment*

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Abstract

Concerns of prolonged near zero interest rates and below target inflation have become widespread in the advanced world. We build an analytical framework that incorporates two hypotheses of persistent ZLB episodes: *expectations-driven* liquidity traps and *secular stagnation* driven liquidity traps. We estimate the DSGE model with Japanese data from 1998:Q1 to 2012:Q4. Using Bayesian prediction pools, we find that a policymaker faces considerable real-time uncertainty in identifying the dominant narrative. We propose robust policies that eliminate expectations-driven traps and are expansionary under secular stagnation.

Keywords: Expectations-driven trap, secular stagnation, zero lower bound, robust policies.

JEL Classification: E31, E32, E52.

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"I believe that for the euro area there is some risk of *Japanification*, but it is by no means a foregone conclusion." — Mario Draghi (January, 2020).

1. Introduction

Since the financial crisis of 2008-2009, the concerns of prolonged near-zero interest rates and below target inflation have become predominant across much of the advanced world. Such concerns have been dubbed the risk of *Japanification*, where low interest rates and deflation leave central banks unable to fight recessions.[1] Two predominant hypotheses rationalize interest rates near the zero lower bound (ZLB) and inflation below the central bank target. The first hypothesis is the *expectations-driven* trap whereby pessimistic deflationary expectations can become self-fulfilling in the presence of the ZLB constraint on short term nominal interest rates (Benhabib, Schmitt-Grohé and Uribe, 2001, 2002). The second hypothesis is the *secular stagnation* hypothesis that entails a persistently negative natural interest rate constraining the central bank at the ZLB (Hansen, 1939; Summers, 2013). Although it is understood that the nature of stagnation might matter, theory and policy implications have been developed using different frameworks making comparison and quantitative assessments a challenge.[2]

In this paper, we build a theoretical framework to consider the ideas of *expectations-driven* liquidity traps and *secular stagnation* in a unified setting. We analytically derive the contrasting policy implications under each hypothesis and embed our theory in a quantitative New Keynesian model. We use Bayesian methods to estimate our model for both hypothesis using Japanese data, and use the model to construct prediction pools to represent the decision problem of a policy maker trying to distinguish between these competing hypotheses in real time. Our main finding is that discerning these two hypotheses in Japan is difficult, even with extended periods at the zero lower bound. However, our structural analysis provides a framework to undertake such assessments.

Our analytical setup modifies the textbook New Keynesian model by introducing endogenous discounting à la Uzawa-Epstein preferences. The representative agent's intertemporal Euler equation is modified such that the steady state features a negative relation between output and real interest rate, similar to a static IS curve. This modification breaks the tight connection between the natural interest rate and the discount factor, thus allowing

[1] Financial Times, *"Japanification: investors fear malaise is spreading globally,"* August 26 2019. ASSA Annual Meeting Panel Session: *"Japanification, Secular Stagnation, and Fiscal and Monetary Policy Challenges,"* January 2020.

[2] Expectations traps have been investigated using representative agent models (Benhabib et al., 2001; Schmitt-Grohé and Uribe, 2017), while secular stagnation is typically modeled using the overlapping generation framework (Eggertsson and Mehrotra, 2015; Eggertsson, Mehrotra, Singh and Summers, 2016).

for the possibility of a permanently negative natural interest rate. In contrast, in the absence of discounting, the steady-state natural rate is fixed at the inverse of the household's discount factor and thus the model cannot accommodate the *secular stagnation* hypothesis. The *expectations-driven* liquidity trap arises in the presence of a nonlinearity in the monetary policy rule introduced by the zero lower bound constraint on short-term nominal interest rates. Combined with long-run money non-neutrality, a shift in agents inflation expectations becomes self-fulfilling. Inflation pessimism can account for persistent output stagnation, below-target inflation and zero nominal interest rates (Aruoba, Cuba-Borda and Schorfheide, 2018; Schmitt-Grohé and Uribe, 2017).

Using our theory, we build a model that offers the first quantitative assessment of the two hypotheses of stagnation. We discipline our model using data from Japan, a country that has witnessed over two decades of near-zero nominal interest rates. A key advantage of our framework is that we can allow agents in our model to expect persistent stagnation with ZLB episodes of arbitrary duration. This feature stands in contrast to models that use transitory declines in the natural interest rate to generate ZLB episodes where agents expectations have to be consistent with recovery to full-employment steady state in the medium run (Bianchi and Melosi, 2017; Gust, Herbst, López-Salido and Smith, 2017).

Can a policy maker use our framework to tease out the dominant hypothesis? The short answer is yes. To this end, we use linear prediction pools to construct the posterior distribution of time-varying model weights which we interpret as the relative importance of each hypothesis for the policy maker (Geweke and Amissano, 2011; Del Negro, Hasengawa and Schorfheide, 2016). The prediction pools allow us to estimate the model weights in real-time and quantify the uncertainty surrounding the policy maker's views. To illustrate this point, we use Japanese data from 1998:Q1-2012:Q4, a period in which interest rates remained near zero. We find that it takes a considerable amount of time for the data to lean in favor of one of the hypotheses. Moreover, we find substantial uncertainty about the dominant hypothesis throughout our sample.

In our baseline specification, the Japanese data implies roughly similar weights on both hypotheses in the early part of the sample and through the early 2000s. Afterwards, the model weights lean toward the expectations-trap hypothesis. The prediction pool assigns a weight of over 70 percent to the expectations-trap hypothesis in the later part of the sample. We find similar results in a nonlinear version of our model that allows agents to expect a stochastic exit from the liquidity trap. However, in the nonlinear model, the policy maker updates her views sluggishly compared to the model with permanent liquidity traps.

We extend our analysis to explore role of different structural features of the model. For example, selecting a different equilibrium path under the expectations trap hypothesis can shift the model weights toward the secular stagnation hypothesis. Similarly, allowing for correlated shocks, in order to capture potential model misspecification, shifts the policy maker views in favor of the secular stagnation hypothesis. A robust result from our extended analysis is that there is considerable uncertainty in diagnosing the dominant hypothesis.

The real-time uncertainty, that we uncover, and the role of structural assumptions in identifying the dominant narrative suggests a dilemma for policymakers faced with these episodes. Since conventional policy stabilization measures may have contrasting policy implications, there is a need for robust policies to deal with liquidity traps (Bilbiie, 2018). We identify a set of such robust policies that improve outcomes across the liquidity traps by placing a sufficiently high floor on the inflation rate. Imposing a lower bound on inflation rate excludes the possibility of expectations trap. Policies that put a bound on deflation also reduce the severity of a fundamentals-induced liquidity trap due to the paradox of flexibility at the ZLB (Eggertsson and Krugman, 2012). In our benchmark model, these policies are implemented in the form of an appropriate price indexation rule by non-adjusting firms. In models with labor market rigidities, we show that minimum income policies can implement lower bounds on inflation, and emerge as robust policies.

Relation to the Literature. Our work complements the recent analyses of Michaillat and Saez (2019), Michau (2018), and Ono and Yamada (2018) who use the bonds-in-utility assumption to analyze a unique secular stagnation scenario.[3] We distinctly focus on understanding the differences between the two stagnation concepts analytically and quantitatively. These alternate micro-foundations essentially introduce discounting into the Euler equation.

This paper is also related to the work by Mertens and Ravn (2014) and Aruoba et al. (2018), who contrast expectations-driven and fundamental-driven liquidity traps using the standard Euler equation without discounting. Their setup can only accommodate short-lived fundamentals-driven trap, while our modified Euler equation allows the possibility of secular stagnation as a competing hypothesis. Our paper is also complementary to Schmitt-Grohé and Uribe (2017), who analyze the case of permanent expectations-driven traps. We build on their work to show that policies that impose a lower bound on inflation preclude the expectations-driven traps.

Our static prediction pool analysis is related to Lansing (2019) in which a model with endogenous switching regimes generates data from a time-varying mixture of two regimes.

[3] Following Feenstra (1986) and Fisher (2015), a functional equivalence can be shown between using bonds in the utility and endogenous discounting. Pro-cyclical income risk (Acharya and Dogra, 2020), or pro-cyclical bond premium (Caramp and Singh, 2020) can also introduce similar discounting into the Euler equation.

In our setup, a policy maker puts time-varying probability on a mixture of predictive densities coming from two alternative hypotheses. Our paper is also related to Mertens and Williams (2018) that constructs forecast densities of inflation and interest rates in a model with expectations trap and compares them with the options implied distributions for the US. We use the principal-agent decision framework of Del Negro et al. (2016) to combine the predictive densities of output, consumption and inflation. The Bayesian nature of our approach allows us to provide a measure of the policy maker uncertainty about the contrasting hypotheses.

Our robust policies prescribe a flattening of the aggregate supply curve to preclude the existence of expectations-driven liquidity traps. These policies are related to the research and development (R&D) subsidies advocated by Benigno and Fornaro (2018) in an endogenous growth environment. Nakata and Schmidt (2019) argue that a policymaker who puts a sufficiently small weight on government spending stabilization can eliminate expectations traps.[4]

2. Key insights in a two equations setup

We begin with a simple setup that analytically demonstrates our model's ability to entertain expectations driven liquidity trap and fundamentals driven liquidity trap. There are two main takeaways: a) high degree of nominal rigidity may prohibit existence of expectations driven trap and b) a fundamentals driven liquidity trap can exist for an arbitrarily long duration. The model features preferences with endogenous discounting and a particular variant of price setting by monopolistically competitive firms that generates analytical results. We characterize and formally define the different steady states: *targeted inflation* state, *secular stagnation* steady state, and the *expectations-driven trap* (also referred to as the BSGU steady state).

Time is discrete and there is no uncertainty. For now, there is no government spending.

[4]Our minimum wage policy is also related to the work of Glover (2019) which introduces a trade-off for employment stability through an allocative inefficiency.

2.1. Household

A representative agent maximizes the following functional:

$$\max_{\{C_t, b_t\}} \sum_{t=0}^{\infty} \Theta_t \left[\log C_t - \chi h_t\right]$$

$$\Theta_0 = 1;$$

$$\Theta_{t+1} = \hat{\beta}(\tilde{C}_t)\Theta_t \;\forall t \geq 0$$

where Θ_t is an endogenous discount factor (Uzawa, 1968; Epstein and Hynes, 1983; Obstfeld, 1990), C_t is consumption, $\tilde{C}_t$ is average consumption that the household takes as given, and h_t is hours. Such preferences have been prominently used in the small open economy literature (Schmitt-Grohé and Uribe, 2003).[5]

For tractability, we assume a linear functional form for $\hat{\beta}(\cdot) = \delta_t \beta \tilde{C}_t$, where $0 < \beta < 1$ is a parameter, $\tilde{C}_t$ is average consumption that the household takes as given, and $\delta_t > 0$ are exogenous shocks to the discount factor. In contrast to the conventional assumption in the endogenous discounting, we require $\hat{\beta}' > 0$. This is often referred to as *decreasing marginal impatience* (DMI) in the literature.[6]

The household earns wage income $W_t h_t$, interest income on past bond holdings of risk-free government bonds b_{t-1} at gross nominal interest rate R_{t-1}, dividends Φ_t from firms' ownership and makes transfers T_t to the government. Π_t denotes gross inflation rate. The period by period (real) budget constraint faced by the household is given by

$$C_t + b_t = \frac{W_t}{P_t} h_t + \frac{R_{t-1}}{\Pi_t} b_{t-1} + \Phi_t + T_t$$

An interior solution to household optimization yields the consumption Euler equation, and intra-temporal labor supply condition

$$1 = \beta(\tilde{C}_t)\left[\frac{C_t}{C_{t+1}} \frac{R_t}{\Pi_{t+1}}\right]$$

$$\frac{W_t}{P_t} = \chi C_t$$

[5] What will be essential for our purpose is that there is a negative steady state relationship between consumption and real interest rate to generate discounting in the Euler equation. We can show a functional equivalence between preferences with endogenous discounting and a recent approach that employs bonds in utility (Michaillat and Saez, 2019).

[6] Our assumed functional form can be considered a special case of the more general functional: $\hat{\beta}(\cdot) = \delta_t \beta C_t^{\gamma_c}$. When $\gamma_c = 0$, this nests the textbook case of exogenous discounting. We consider $\gamma_c = 1$ for tractability.

In equilibrium, individual and average consumption are identical, i.e. $C_t = \tilde{C}_t$. The Euler equation simplifies to:

$$1 = \delta_t \beta C_t \left[\frac{C_t}{C_{t+1}} \frac{R_t}{\Pi_{t+1}} \right]$$

Discussion of DMI: DMI assumption implies that as agents get richer they become more patient. Several papers in the literature agree it is more realistic and intuitive to assume decreasing rather than increasing impatience (Epstein and Hynes, 1983; Lucas and Stokey, 1984; Obstfeld, 1990; Svensson and Razin, 1983; Uzawa, 1968). Despite the realism, it is conventional to use increasing marginal impatience since it is consistent with boundedness of wealth and stability in environments with exogenous real interest rate. See also Barro and Sala-i Martin (2004). As shown by Das (2003), it is possible to guarantee stability in environments with decreasing marginal impatience as long as the returns to savings diminish at high enough rate.[7]

2.2. Production

A perfectly competitive final-good producing firm combines a continuum of intermediate goods indexed by $j \in [0,1]$ using the CES Dixit-Stiglitz technology: $Y_t = \left(\int_0^1 Y_t(j)^{1-\nu} dj \right)^{\frac{1}{1-\nu}}$, where $1/\nu > 1$ is the elasticity of substitution across varieties. The price of the final good $P_t = \left(\int_0^1 P_t(j)^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}$. Profit maximization gives The demand for intermediate good j can be derived from profit maximization as $Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-1/\nu} Y_t$.

Intermediate good j is produced by a monopolist with a linear production technology: $Y_t(j) = h_t(j)$. Intermediate good producers buy labor services $H_t(j)$ at a nominal price of W_t. Moreover, they face nominal rigidities in terms of price adjustment costs, and maximize profits $\Phi_t(j) = (1+\tau)P_t(j)Y_t(j) - W_t h_t(j)$, where τ is a production subsidy.

We introduce nominal rigidities in price setting of these intermediate producers. In order to derive a tractable Phillips curve, we assume the setup of Bhattarai, Eggertsson and Gafarov (2019). A fraction α_p of the firms set prices flexibly every period to maximize per period profits $(1+\tau)\frac{p_t^*}{P_t} = \frac{1}{1-\nu}\frac{W_t}{P_t}$. We set the production subsidy to eliminate markups so that (imposing $Y_t = C_t$), optimal price becomes $\frac{p_t^*}{P_t} = \chi Y_t$. The remaining fraction $1 - \alpha_p$ index their price p^n to the aggregate price level from previous period $\frac{p_t^n}{P_t} = \Gamma_t \frac{P_{t-1}}{P_t}$ where Γ_t is indexing variable to be defined shortly. The price index then becomes $P_t^{\frac{\nu-1}{\nu}} = \alpha_p \left(p_t^*\right)^{\frac{\nu-1}{\nu}} + (1-\alpha_p)\left(p_t^n\right)^{\frac{\nu-1}{\nu}}$. With choice of $\nu = 1/2$ and $\Gamma_t = \frac{P_t}{Y_t^{-1}(P_t - P_{t-1}) + P_{t-1}}$, we can

[7]Following the insights of Das (2003), it can be shown that decreasing marginal impatience assumption is consistent with existence of a stable equilibrium with capital accumulation. Results are available upon request.

derive the following relationship between gross inflation and aggregate output:[8]

$$\Pi_t = \kappa Y_t + (1 - \kappa \bar{Y})$$

where $\kappa \equiv \frac{\alpha_p}{1-\alpha_p}$ is slope of the Phillips curve and $\bar{Y}$ is steady state output in the absence of no nominal rigidities (or zero price dispersion). We set $\chi = 1$ so as to normalize $\bar{Y} = 1$.

2.3. Government and resource constraint

We close the model by assuming a government that balances budget every period and a monetary authority that sets nominal interest rate on the net zero supply of nominal risk-free one-period bonds using the following Taylor rule $R_t = \max\{1, (1+r^*)\Pi_t^{\phi_\pi}\}$, where $(1+r^*) \equiv \frac{1-\delta_t}{\beta}$ is the natural interest rate, and $\phi_\pi > 1$.[9] We implicitly assumed the (gross) inflation target of the central bank to be one. The zero lower bound (ZLB) constraint on the short-term nominal interest rate introduces an additional non-linearity in the policy rule. Finally, we assume that the resource constraints hold in the aggregate: $C_t = Y_t$, and $b_t = 0$.

2.4. Equilibrium

The competitive equilibrium is given by the sequence of three endogenous processes $\{Y_t, R_t, \Pi_t\}$ that satisfy the conditions 1–3 for a given exogenous sequence of process $\{\delta_t\}_{t=0}^{\infty}$ and initial price level P_{-1}:

$$\begin{aligned} 1 &= \delta_t \beta Y_t \left[\frac{Y_t}{Y_{t+1}} \frac{R_t}{\Pi_{t+1}} \right] && (1) \\ \Pi_t &= \kappa Y_t + (1 - \kappa) && (2) \\ R_t &= \max\{1, (1+r_t^*)\Pi_t^{\phi_\pi}\} && (3) \end{aligned}$$

where the exogenous sequence of natural interest rate is given by $1 + r_t^* \equiv \frac{1}{\delta_t \beta}$.

[8] With $\Gamma_t = 1$, we get the neoclassical Phillips curve (See Ch 3.1 Woodford (2003)). Allowing for indexation to depend on current output allows us to derive Phillips curve that helps us derive insights analytically and make comparison with downward nominal wage rigidity assumption. More generally, the firms that are indexing prices follow an indexation rule $\Gamma(Y_t)$ where $\Gamma'(\cdot) > 0$ i.e. price reduction is increasing in unemployment. Similar analytical results with $\Gamma_t = Y_t$ can be shown.

[9] The natural interest rate is defined as the real interest rate on one-period government bonds that would prevail in the absence of nominal rigidities.

2.5. Non-stochastic steady state

We can represent the steady state equilibrium with an aggregate demand block and an aggregate supply block.

Aggregate Demand (AD) is a relation between output and inflation and is derived by combining the Euler equation and the Taylor rule. Mathematically, the AD curve is given by

$$Y_{AD} = \frac{1}{\beta\delta}\begin{cases}\frac{1}{(1+r^*)\Pi^{\phi\pi-1}}, & \text{if R } > 1,\\ \Pi, & \text{if R } = 1\end{cases} \tag{4}$$

When ZLB is not binding, the AD curve has a strictly negative slope, and it becomes linear and upward sloping when the nominal interest rate is constrained by the ZLB. The kink in the aggregate demand curve occurs at the inflation rate at which monetary policy is constrained by the ZLB.

$$\Pi_{kink} = \left(\frac{1}{(1+r^*)}\right)^{\frac{1}{\phi\pi}}$$

When $1 + r^* > 1$, the kink in the AD curve occurs at an inflation rate below 1. For the natural interest rate to be positive, the patience parameter must be low enough i.e. $\delta < \frac{1}{\beta}$.

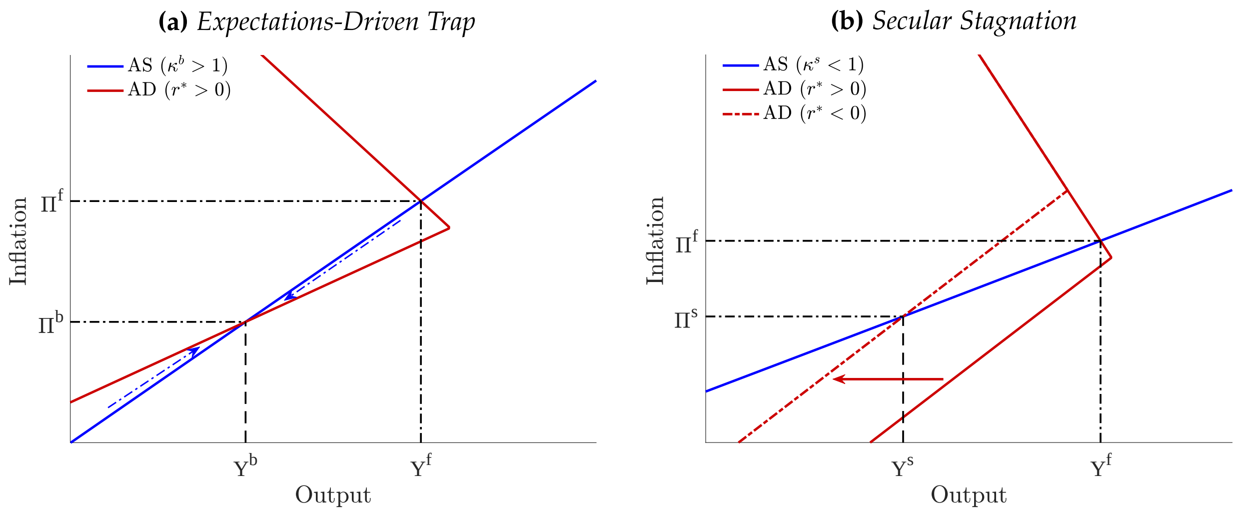
Aggregate Supply (AS) is given by $\Pi = \kappa Y + (1-\kappa)$ in the steady state. When $Y = 1$, $\Pi \geq 1$. In this linear aggregate supply curve, the degree of nominal rigidity κ also determines the lower bound on inflation = $1-\kappa$. In the quantitative section, we will work with the standard forward-looking NK Phillips curve.

In this two-equation representation, we can characterize and prove the existence of different steady state equilibria. Proposition 1 shows that a targeted steady state exists as long as natural interest rate is positive.

Proposition 1. (Targeted Steady State): Let $0 < \delta < \frac{1}{\beta}$.There exists a unique positive interest rate steady state with $Y = 1$, $\Pi = 1$ and $R = \frac{1}{\beta\delta} > 1$. It features output at efficient steady state, and inflation at the policy target. The equilibrium dynamics in this steady state's neighborhood are locally determinate.

A steady state at which the central bank can meet its inflation target is *defined* as the *targeted inflation steady state*. The presence of targeted steady state is contingent on the natural interest rate and the inflation target of the monetary authority. With a unitary inflation target, it must be the case that the natural interest rate be non-negative, which is implied by the assumption of $\delta < \frac{1}{\beta}$. We next show in Proposition 2 that a liquidity trap

Figure 1: *Steady-State Representation*



steady state (à la Schmitt-Grohé and Uribe, 2017) may jointly co-exist with the targeted steady state described above. However, with a flat enough Phillips curve, targeted steady state is the unique steady state in this economy. A high enough nominal rigidity prevents inflation from falling to levels such that self-fulfilling deflationary expectations do not manifest in the steady state.

Proposition 2. (BSGU steady state): Let $0 < \delta < \frac{1}{\beta}$. For $\kappa > 1$ (i.e. $\alpha_p > 0.5$) there exist two steady states:

1. The targeted steady state with $Y = 1, \Pi = 1$ and $R = \frac{1}{\beta\delta} > 1$.
2. (*Expectations-driven* trap) A unique-ZLB steady state with $Y = \frac{1-\kappa}{\beta\delta-\kappa} < 1$, $\Pi = \frac{\beta\delta(1-\kappa)}{\beta\delta-\kappa} < 1$ and $R = 1$. The local dynamics in a neighborhood around this steady state are locally indeterminate.

When prices are rigid enough, i.e. $\kappa < 1$, there exists a unique steady state and it is the targeted inflation steady state. When prices are flexible $\alpha_p = 1$ ($\kappa \to \infty$), there always exist two steady states: a unique deflationary steady state with zero nominal interest rates and a unique targeted inflation steady state.

Panel a) in Figure 1 illustrates the unique targeted-steady state (Y^f, Π^f) and the unique ZLB steady state (Y^b, Π^b) with the modified Euler equation. We define the *expectations-driven* trap as the steady state with positive natural interest rate, positive output gap, and deflation and in whose neighborhood the equilibrium dynamics are *locally indeterminate*. Pessimistic inflationary expectations can push the economy to this steady state without any change in fundamentals. As a shorthand, henceforth, we refer to it as the *BSGU* steady state.

We now consider the case where adverse fundamentals can push the economy to a permanent liquidity trap. If agents are sufficiently patient $\delta > \frac{1}{\beta}$, i.e. the natural rate of interest is negative, the monetary policy is constrained by the zero lower bound on short-term nominal interest rate. In that case, the nominal interest rate is permanently zero while there is below-potential output and deflation in the economy. We characterize this possibility in Proposition 3.[10]

Proposition 3. (Secular Stagnation): Let $\delta > \frac{1}{\beta}$ and $\kappa < 1$. There exists a unique steady state with $Y = \frac{1-\kappa}{\beta\delta-\kappa} < 1$, $\Pi = \frac{\beta\delta(1-\kappa)}{\beta\delta-\kappa} < 1$ and $R = 1$. It features output below the targeted steady state and deflation, caused by a permanently negative natural interest rate. The equilibrium dynamics in this steady state's neighborhood are locally determinate.

See panel b) in Figure 1 for illustration of this unique steady state. The intersection of the solid red line(AD) with the solid blue line (AS) at (Y^f, Π^f) depicts the result of proposition 1, and the intersection of dashed red and blue lines at (Y^s, Π^s) depicts the liquidity trap steady state in proposition 3. We formally *define* the *secular stagnation steady state* as the steady state featuring positive output gap from efficient steady state, zero nominal interest rate on short-term government bonds and exhibiting *locally determinate* equilibrium dynamics in its neighborhood. This local determinacy property is the main difference between the secular stagnation narrative and the expectations-driven narrative.

Note that the secular stagnation steady state exists in this model because of sufficient discounting in the modified Euler equation. Unlike the textbook new Keynesian model, an arbitrarily long ZLB episode driven by negative natural rate can exist in equilibrium. In the log-linearized textbook NK models, *deflationary black holes* emerge as the duration of temporary liquidity trap is increased, with inflation and output tending to negative infinity (Eggertsson, 2011). The solution remains bounded in our setup as the duration of ZLB is increased.

3. Quantitative DSGE Model

We now present a quantitative analysis based on a stylized small-scale New Keynesian model that has been widely studied in the literature—see An and Schorfheide (2007). The key difference is that the Euler equation in our model features discounting to allow the possibility of arbitrarily long duration of fundamentals-driven liquidity trap. Our model economy is composed of households, intermediate good producers, final good producers and a government. Relative to Section 2, the quantitative model features a forward looking

[10] Note the efficient steady state is always an equilibrium of an economy without nominal rigidities.

Phillips curve and exogenous shocks to government spending, technology growth and markups. Since the model is relatively standard, we only summarize the equilibrium conditions leaving a detailed derivations to Appendix C. In this section we focus on calibration of the key parameters, in particular those that give rise to the two stagnation hypothesis and analyze the steady state implications. We estimate the model in Section 4.

3.1. Equilibrium Conditions and Competitive Equilibrium

Our model economy evolves around a stochastic balanced growth path given by the level of productivity A_t. Let y_t and c_t denote (stationary) output and consumption, R_t and Π_t be the gross nominal interest rate and gross inflation rate respectively.

Definition 1. A stationary competitive equilibrium is given by the sequence of quantities and prices $\{y_t, c_t, R_t, \Pi_{t+1}\}$ which satisfy equations 5–8, given an exogenous sequence for processes $\{g_t, \nu_t, z_t\}$:

$$1 = \beta \left(\frac{c_{t+1}}{c_t}\right)^{-1} \frac{R_t}{z_{t+1}\Pi_{t+1}} + \delta c_t \tag{5}$$

$$\zeta_t = \phi \nu_t \beta \left(\frac{c_{t+1}}{c_t}\right)^{-1} \frac{y_{t+1}}{y_t} \Pi_{t+1} \left(\Pi_{t+1} - \Pi^*\right) \tag{6}$$

$$R_t = \max\left\{1, r^*\Pi^* \left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}\right\} \tag{7}$$

$$c_t = \frac{y_t}{g_t}, \tag{8}$$

where $\zeta_t = 1 - \nu_t - \chi c_t y_t^{1/\eta} + \nu_t \phi \left(\Pi_t - \Pi^*\right) \Pi_t$.

There are three exogenous disturbances in the model: (i) exogenous changes to government expenditure g_t, (ii) exogenous changes to the growth rate of productivity z_t, and (iii) exogenous changes to the inverse demand elasticity for intermediate goods ν_t. We assume that these exogenous components obey an AR(1) process around their deterministic mean (g^*, z^*, ν^*), with persistence equal to $(\rho_g, \rho_z, \rho_\nu)$ and standard deviation $(\sigma_g, \sigma_z, \sigma_\nu)$, respectively.

As in our benchmark model, equation 5 is the modified Euler equation. We chose to use the modified Euler equation that has an additive wedge, instead of the multiplicative wedge considered in Section 2, for three reasons: a) as $\delta \to 0$, this equation nests the textbook Euler equation as a special case; b) similar wedge can be derived from a wealth in utility function argument (Michaillat and Saez, 2019), which is an alternate device to generate a persistent fundamentals-driven liquidity trap; and c) can readily map to models of convenience yield that emphasize flight-to-liquidity aspects of Great Recession (Del Negro, Eggertsson, Ferrero and Kiyotaki, 2017).[11]

Equation (6) is the standard new Keynesian Phillips curve derived with the assumption of Rotemberg (1982) quadratic adjustment costs in price setting. ϕ is the adjustment cost parameter determining the slope of Phillips curve, $1/\nu$ is elasticity of substitution across varieties in the CES Dixit Stiglitz aggregator, η is the Frisch labor supply elasticity, χ is the labor disutility parameter that will be calibrated to normalize targeted steady state output at one, and Π^* is the inflation target of central bank. Equation 7 is the policy rule that states that central bank varies the nominal interest rate more than one to one ($\phi_\pi > 1$) with inflation as long as zero lower bound constraint doesn't bind. r^* is the gross natural interest rate, defined as real interest rate that applies in the absence of price adjustment frictions. Finally, equation 8 is the resource constraint of the economy that specifies a time-varying wedge between consumption and output, which is labeled as exogenous shocks to government spending.[12]

3.2. Calibration

Because of the multiplicity of steady states, we calibrate parameters to match observed empirical moments in the data for Japan. Most of the calibrated parameter values are borrowed from Aruoba et al. (2018), and presented in Table 1. The remaining parameters are set such that the economy is either in the secular stagnation steady state or the BSGU steady state.

The Frisch labor supply elasticity is fixed at 0.85, based on micro-level data based estimates of Kuroda and Yamamoto (2008). The (inverse) elasticity of demand for intermediate goods, ν, is set to a value of 0.1 to generate a steady state markup of 11% for the monopolistically competitive firms. Japan did not officially adopt an inflation target until 2013Q2. Consequently, we choose an inflation target of 1% ($\Pi^* = 1.0025$).[13] The labor disutility parameter χ_H is chosen so as to normalize the output in the targeted inflation steady-state at one. We choose the values of z^* such that the model matches the average output growth over the estimation sample. Steady state government spending ratio is chosen from the consumption output ratio of 58% in the Japanese data.

We fix the discount factor β to 0.942 consistent with structural estimates of Galı and

[11] Some of the analytical results in Section 2 can be shown with this additive wedge. See our earlier draft for those results, which are now relegated to online appendix.

[12] As in Section 2, we assume that government runs a balanced budget every period, and there is net zero supply of government bonds in the economy. Finally, we rebate the adjustment costs to the household to reduce the role of large adjustments costs in driving equilibrium dynamics—see Eggertsson and Singh (2018).

[13] Japan did not officially have an inflation target until January 2013, when it adopted a 2% target. Our results are robust to choosing a zero inflation target as well.

Table 1: *Fixed Parameters*

β	η	ν^*	τ	Π^*	g^*	$100\ln(z^*)$	ϕ_π
Discount factor	Inverse Frisch elasticity	Price s.s. markup	Intertemporal elasticity of substitution	Inflation target	Government spending parameter	TFP growth rate	Taylor rule coefficient
0.942	0.85	0.1	1	1.0025	1.72	0.56	1.50

Table 2: *Calibrated Parameters*

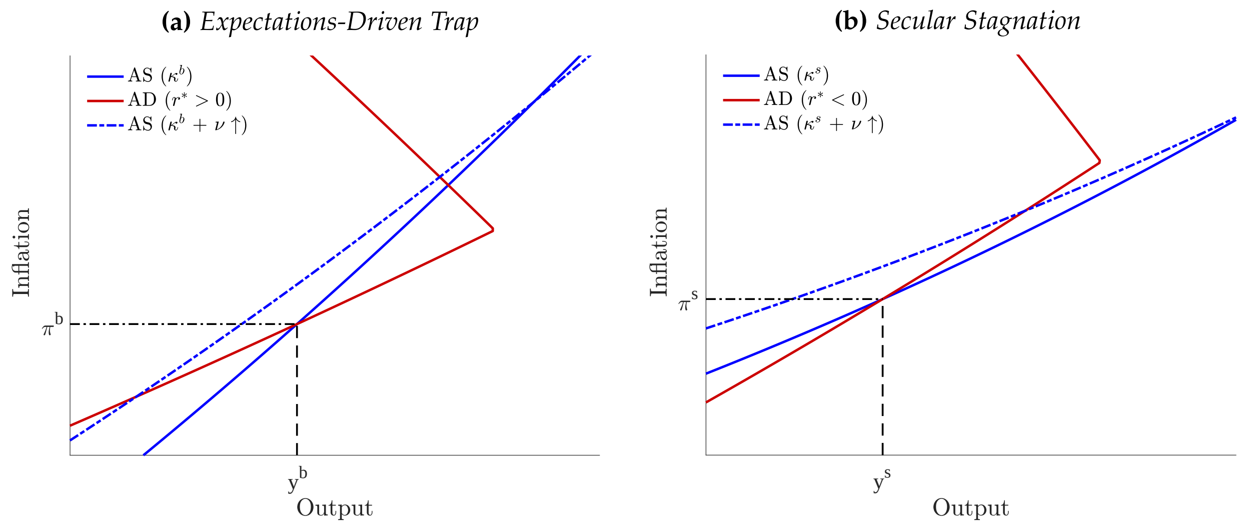
	δ Euler Equation Wedge	ϕ Adjustment cost parameter	
Secular Stagnation	0.1132	4825	
BSGU Trap	0.1088	2524	
	Natural Rate	Inflation	Output Gap
Secular Stagnation	-1.1	-1.06	-7.5
BSGU Trap	1.0	-1.06	-4.5

Notes: The table shows the parameter values of the model for the baseline calibration.

Gertler (1999). While this estimate is lower than the standard calibrated value of 0.99 in the literature, a low β is needed for the model to generate a positive natural interest rate in the presence of a bond premium. In studies that have estimated the discount rate using field and laboratory experiments, the estimates for β are dispersed but point to high discount rates. Surveys of these studies are conducted in Frederick, Loewenstein and O'Donoghue (2002, table 1), and Andersen, Harrison, Lau and Rutström (2014, table 3). Michaillat and Saez (2019) choose an annual discount rate of 43% from the median value of these estimates.

The remaining parameters δ, and ϕ are chosen to jointly match targets for natural interest rate and average inflation in Japan. For the natural rate, we adopt two different targets depending on the regime. Under secular stagnation we choose an annual rate of -1.1%. This is based on two studies by Fujiwara et al. (2016) and Iiboshi et al. (2018) that separately estimate a series for the natural rate of interest in Japan based on Laubach and Williams (2003). They find that the quarterly estimate was often -0.5% since late 1990s, and -2% at the lowest level. In contrast, the BSGU steady state is calibrated to imply a (annualized) long-run real interest rate of 0.75%. The calibration implies an inflation rate of -1.06% for both steady states, which is the average inflation rate in Japan over the period 1998Q4 – 2012Q4 . Furthermore, the calibration implied slope of the log-linearized Phillips curve κ is 0.0036 at the secular stagnation steady state, and 0.0019 when calibrated to the BSGU steady state. These values lie in the range of the conventional estimates found in the literature—see Aruoba, Bocola and Schorfheide (2017). Finally, the implied output gap is close to the estimates of 5% in Hausman and Wieland (2014).

Figure 2: *Permanent increase in markups ν*



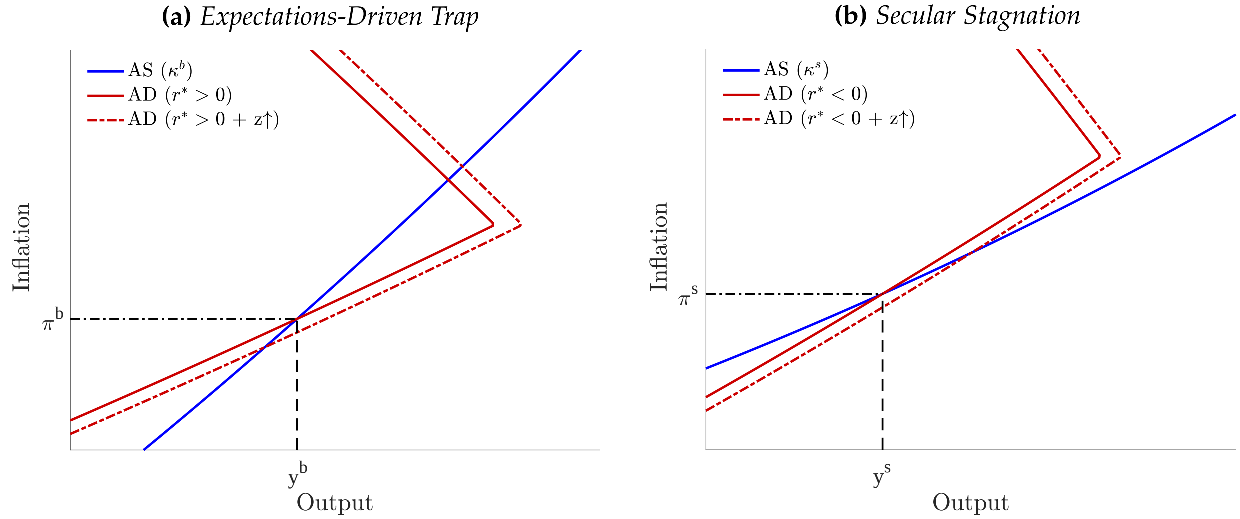
3.3. Comparative Statics: Expectations Trap vs Secular Stagnation

The BSGU and the secular stagnation hypotheses have contrasting implications for shocks and policy. These differences stem from the local determinacy property of these steady states, which translate into differences in slopes of aggregate supply and aggregate demand in our model. We now demonstrate these properties with comparative static experiments. Because of local determinacy of the secular stagnation steady state, the comparative static experiment is well-defined without the need for additional assumptions. With the BSGU steady state, we assume that inflation expectations do not change drastically to push the economy to the full-employment steady state in response to the experiment.

In Figure 2, solid lines plot the steady-state AD-AS representation of the quantitative model under two parametrizations. Annualized inflation deviation relative to the central bank target inflation is on the vertical axes and output gap relative to target-steady state output (in percents) is on the horizontal axes. The coordinates (y^b, π^b) and (y^s, π^s) denote the expectations-driven and fundamentals-driven liquidity trap steady states respectively. The left panel plots AD-AS curves when prices are relatively flexible (κ^b) and the natural rate of interest is positive. The AD-AS intersection depicted at (y^b, π^b) is locally indeterminate, features zero nominal interest rates and output is permanently below potential. At this intersection, the AS curve is steeper than the AD curve. In the right panel, we plot the AS curve with relatively rigid prices (κ^s), and the AD curve with negative natural interest rate, $r^* < 0$. AD intersects AS at the secular stagnation steady state at the coordinate (y^s, π^s).

An upward shift in aggregate supply curve in Figure 2, denoted with dashed blue line, induced by permanent increase in steady state markups, translates into higher output under secular stagnation and lower output under BSGU. Under secular stagnation, the

Figure 3: *Permanent increase in TFP growth rate z*



natural interest rate is too low for the central bank to stabilize the economy. An increase in markups through inflationary pressures helps lower real interest rate, thus reducing the real interest rate gap and expand output. Under BSGU, the problem is of pessimism about inflation expectations. If agents remain pessimistic about inflation undershooting its target, an increase in markups is further contractionary since the resource inefficiencies associated with increased markups dominate the increase in output demand due to higher prices (see also Mertens and Ravn, 2014).

An outward shift in aggregate demand in Figure 3, denoted with dashed red line, induced by permanent increase in steady state TFP growth, translates to higher output under secular stagnation but lower output under BSGU. Higher TFP growth signals higher income for households and leads to increased consumption demand. This increased impatience translates into higher output under secular stagnation. Under BSGU, in contrast, the increased TFP growth translates into higher reduction in prices by firms, which dominates the increased demand by households. As a result, there is lower output and inflation under BSGU.

Similarly, a neo-Fisherian exit policy of raising interest rates at the ZLB is contractionary under secular stagnation as it increases the real interest rate gap from natural rate, but it is expansionary at the BSGU steady state equilibrium (Schmitt-Grohé and Uribe, 2017) . Furthermore, an increase in government expenditure (financed by lumpsum taxes) or a permanent reduction in short term interest rates below the ZLB has inflationary effects under secular stagnation but deflationary effects under BSGU.[14]

[14] We model the neo-Fisherian policy as a permanent change in the intercept of the Taylor rule, a: $R^{new} = \max\{1 + a, a + R^* \left(\frac{\Pi}{\Pi^*}\right)^{\phi_\pi}\} = a + R$. where a is increased to a positive number from zero. This policy

These disparate policy implications raise the question whether it is possible to distinguish these two different kinds of liquidity traps in the data. We turn to this question next.

4. Estimating the Model for Japan

We estimate the model for Japan with zero nominal interest rates using standard linear rational expectations (LRE) methods (Herbst and Schorfheide, 2016). Because we consider a long period in which the ZLB was binding in Japan, we assume that agents expect to be in the liquidity trap permanently. The assumption of a permanent liquidity trap might be extreme, but it allows us to highlight the difference in the transmission mechanism during expectations traps and secular stagnation episodes. Moreover, our framework allows us to analyze the dynamics of long-lasting liquidity traps, something that is not possible in the absence of discounting. We extend our analysis to allow for exit dynamics and recurrent liquidity traps in the nonlinear model of Section 6.

4.1. Data

In Section 3.2, we calibrated the parameters $[\beta, \tau, \eta, z^*, \nu^*, g^*, \delta, \phi]'$, to construct the secular stagnation or BSGU steady state. Conditional on these parameters we are left to estimate the parameters governing the shock processes to match the data. We use quarterly data on output growth, consumption growth and annualized GDP deflator inflation in Japan during the period 1998:Q1 to 2012:Q4. We focus on this sample for two reasons. First, from 1995 to 1998 the Bank of Japan (BOJ) held the monetary policy rate at 0.5%, while struggling to boost the economy amidst turmoil in domestic and international financial markets (Ito and Mishkin, 2004). We start our analysis in 1998 to parallel the assumption in our model in which the economy starts at the ZLB and agents expect low interest rates to last for a prolonged period.[15] Second, in 2013 the BOJ introduced new monetary policy program that included an explicit inflation target, asset and bond purchase programs as well as considering negative nominal interest rates (Gertler, 2017). Most of these policies

simultaneously increases the lower bound on nominal interest rate and thus does not have any effect on the placement of the kink in the aggregate demand curve. Given the inflation rate, an increase in a lowers output demanded. At the secular stagnation steady state, this induces deflationary pressures that increases the real interest rate gap and causes a further drop in output. In contrast, during a BSGU trap, an increase in nominal interest rate anchors agents' expectations to higher levels of inflation, thus obtaining the neo-Fisherian results (Schmitt-Grohé and Uribe, 2017). The effects of increased government spending on output are somewhat ambiguous because of elastic labor supply that also causes changes in the aggregate supply curve.

[15] The BOJ lowered its policy rate to zero in the first quarter of 1999 and it remained between 0% and 0.5% through 2016.

cannot be modeled with the admittedly simple feedback rule in our quantitative model.

4.2. Model

We log-linearize the equilibrium conditions around the secular stagnation and the BSGU steady state separately. In both cases, the dynamical system can be summarized with the following equations:[16]

$$\hat{y}_t = \tau \mathbb{E}_t \tilde{\mathcal{D}}(\hat{y}_{t+1} - \hat{g}_{t+1}) + \tilde{\mathcal{D}} \mathbb{E}_t \left(\hat{\pi}_{t+1} + \hat{z}_{t+1} \right) + \hat{g}_t \tag{9}$$

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \tilde{\lambda} \nu_t + \tilde{\kappa}(\hat{y}_t - \hat{g}_t) + \frac{\tilde{\kappa}}{\eta \tau} \hat{y}_t + \tilde{\Gamma} \mathbb{E}_t \left(\hat{y}_{t+1} - \hat{y}_t \right) - \tau \tilde{\Gamma} \mathbb{E}_t (\hat{c}_{t+1} + \hat{c}_t) \tag{10}$$

$$\hat{c}_t = \hat{y}_t - \hat{g}_t \tag{11}$$

Equations (9) resembles the usual dynamic IS equation of the log-linearized New Keynesian model, but is modified by the discounting coefficient $\tilde{\mathcal{D}} = \frac{\beta}{(\beta + \tilde{\pi} z^* \delta \tilde{c}^{\tau})}$, which shows that when $\delta > 0$ the IS equation exhibits discounting. Equation 10 is the forward-looking Phillips curve, where $\tilde{\kappa}$ is the slope of the Phillips curve and we obtain two additional terms that relate current inflation to expected future output and consumption growth. These extra terms commonly arise around the full employment steady state in the presence of inflation indexation (Ascari, Castelnuovo and Rossi, 2011). The coefficient $\tilde{\Gamma} = \beta \frac{(\tilde{\pi} - \bar{\pi})}{2\tilde{\pi} - \bar{\pi}}$, shows that in the case $\tilde{\pi} = \bar{\pi}$ we obtain the well known expression for the Phillips equation. Equation 11, is the log-linearized resource constraint. Because we assume the economy is permanently constrained by the ZLB, the nominal interest rate is pegged at its theoretical lower bound of $R_t = 1$.[17]

4.3. Solution and parameter estimation

Under secular stagnation, the LRE system 9–11 satisfies the Blanchard-Khan determinacy conditions and a unique dynamic solution can be constructed using standard methods. Under an expectations trap, the dynamics of the LRE system exhibit equilibrium indeterminacy, or the existence of multiple stable rational expectation solutions. Because of this multiplicity, it is possible to construct a continuum of equilibria in which the model dynamics are affected by non-fundamental sunspot shocks (Lubik and Schorfheide, 2003,

[16]For any endogenous variable x_t, we denote all steady state related variables by $\tilde{x}$ in the deflation steady state associated with the expectations trap and secular stagnation models. We reserve the notation x for values associated with the full employment steady state. Appendix C provides the derivation of the log-linearized equations

[17]It is possible to incorporate the nominal rate in our estimation restricting the parameter $0 < \phi_\pi < 1$ such that the Taylor principle doesn't hold (Hirose, 2018). We achieve the same result assuming $R_t = 1$.

2004). We construct a solution under indeterminacy using the approach proposed in Bianchi and Nicolo (2017) by augmenting the LRE system with the equation:

$$\omega_t = \left(\frac{1}{\alpha}\right)\omega_{t-1} + \zeta_t - \left(\hat{\pi}_t - \mathbb{E}_{t-1}\hat{\pi}_t\right), \quad \zeta \sim \mathcal{N}(0, \sigma_\zeta) \tag{12}$$

We set $\alpha < 1$, such that we introduce an additional explosive root into the system and allow the expectation error in inflation $(\pi_t - \mathbb{E}_{t-1}\pi_t)$ to be driven by the sunspot shock ζ_t.

For both specifications we estimate the persistence and standard deviations of the fundamental shocks. For the case of secular stagnation, we estimate the covariance matrix of the fundamental shocks $\mathbb{E}\left[\varepsilon_t\varepsilon_t'\right] = \Omega_{\varepsilon\varepsilon}$, assuming that this matrix is diagonal implying uncorrelated fundamental shocks, and explore the implications of relaxing this assumption in Section 5. For the expectation traps model, the vector of shocks becomes $\hat{\varepsilon}_t = (\varepsilon_t^z, \varepsilon_t^g, \varepsilon_t^v, \zeta_t)'$, and we estimate the extended covariance matrix $\mathbb{E}\left[\hat{\varepsilon}_t\hat{\varepsilon}_t'\right] = \Omega_{\hat{\varepsilon}\hat{\varepsilon}}$, such that we let the data speak about which of the multiple equilibrium paths fit the data better. We assume fundamental shocks are orthogonal to each other, but allow fundamental and sunspot shocks to be correlated. To match the model to the data, we construct model implied output and consumption growth measured in quarter-on-quarter percentages $(\Delta y_t^Q, \Delta c_t^Q)$, and inflation measured in annualized percentages (π_t^A). The system of measurement equations is described in the appendix together with the description of the random walk Metropolis-Hastings (RWMH) algorithm used to explore the posterior distribution of model parameters.

The marginal prior and posterior distributions for the estimated parameters are tabulated in the appendix. The posterior estimates for the common parameters are remarkably similar across model specifications. For the expectation traps model, the standard deviation of the sunspot shock is statistically different from zero and with a magnitude similar to that of the technology shock. In this specification, the estimated correlation between the fundamental and sunspot shocks varies substantially. The data favors a high correlation between the markup shock and the sunspot shock while picking up a small correlation of the sunspot shock with the other two fundamental shocks.

4.4. Transmission of shocks

We now illustrate the difference in dynamics of expectation traps and secular stagnation through impulse responses. Figure 4 shows the output and inflation impulse response functions after a one-time unanticipated shock to government expenditure, aggregate productivity growth and price markups. Around the secular stagnation steady state, there is no crowding out of consumption in response to an increase in government expenditure.

This is because there is no endogenous monetary policy reaction that counteracts the inflationary pressures of increased government spending. As a result, the government expenditure shock is expansionary. Similarly, technology growth shocks in the model act as positive news shocks about the level of future productivity and lead to an expected increase in future and current demand, which translate into higher inflation. Finally, markup shocks that create exogenous shifts in the Philips curve have an expansionary effect on output. Higher markups put upward pressure on nominal wages and because the economy remains at the zero bound this reduces the real interest rate and increases consumption. This result echoes the implications of transitory increases in productivity that are contractionary (*paradox of toil*) in temporary liquidity traps driven by shocks to the discount rate (Eggertsson, 2010).

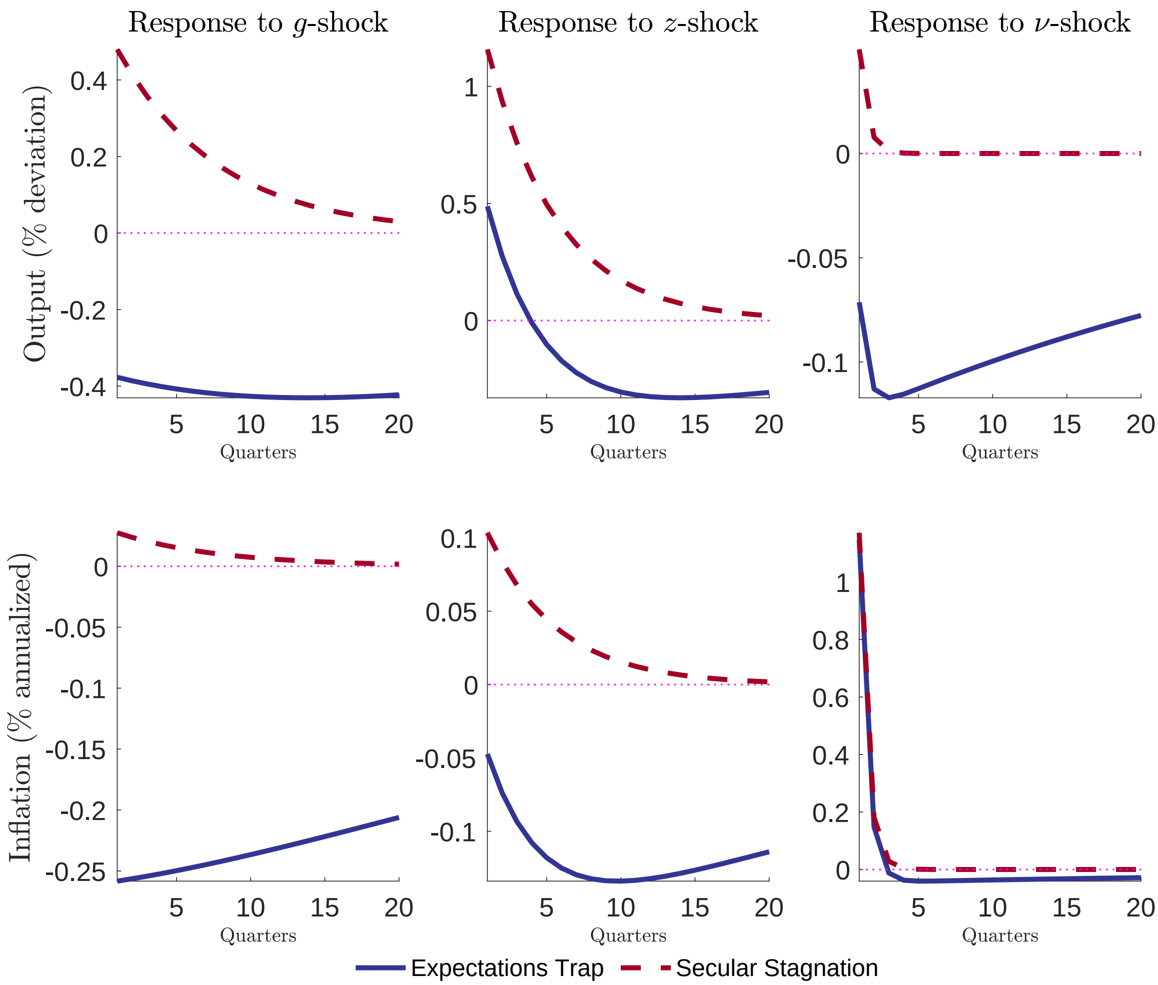
The dynamics in the neighborhood of BSGU steady state are depicted by the solid blue lines in Figure 4. In the presence of deflationary expectations, higher government expenditure signals weaker demand which further reduces inflation. Because the interest rate is pegged to its zero lower bound, lower expected inflation raises the real rate and consumption and output contract. These results are similar to dynamic responses documented in Mertens and Ravn (2014) and Aruoba et al. (2018). Technology growth shocks raise output temporarily but as higher productivity lowers prices inflation and output contract. Lastly, higher markups boost inflation on impact, but the effect quickly reverses turning into deflation as agents expect to remain in the deflationary steady state in the long run. These deflationary expectations are associated with an increase in the real interest rate and contraction in output.

In the next section we exploit the different transmission of shocks under expectation traps and secular stagnation to make inference about which hypotheses is more likely to account for the dynamics of consumption, output and inflation in Japan.

5. Expectations traps or secular stagnation?

How can we assess the importance of the two competing hypothesis of persistent stagnation? Are differences in the transmission of shocks identify expectation-driven traps from secular stagnation episodes? We answer these questions using static prediction pools as in Geweke and Amissano (2011) and Del Negro et al. (2016) that rely on predictive densities to construct recursive estimates of model weights that represent a policy maker views on the most relevant model.

Figure 4: *Impulse Responses: Expectations Traps vs Secular Stagnation*



Notes: Impulse responses to one standard deviation shocks. All responses are computed at the posterior mean of the estimated parameters. The blue solid line corresponds to the expectations-driven traps model. The red dashed line corresponds to the secular stagnation model.

5.1. Model comparison

We follow the principal-agent framework described in Del Negro et al. (2016), with a policymaker confronted with two different models: $\mathcal{M}_b$ that corresponds to the expectations trap model and $\mathcal{M}_s$ that corresponds to the secular stagnation model. For each model the policymaker has access to the sequence of one-period-ahead predictive densities $p\left(y_t|y_{1:t-1}, \mathcal{M}_j\right)$.[18] We are interested in constructing an estimate of the model weight, λ,

[18]The predictive density is constructed sampling from the posterior distribution of the DSGE parameters of the baseline model of Section 4 and averaging the predictive densities across draws.

that pools the information of each individual model:

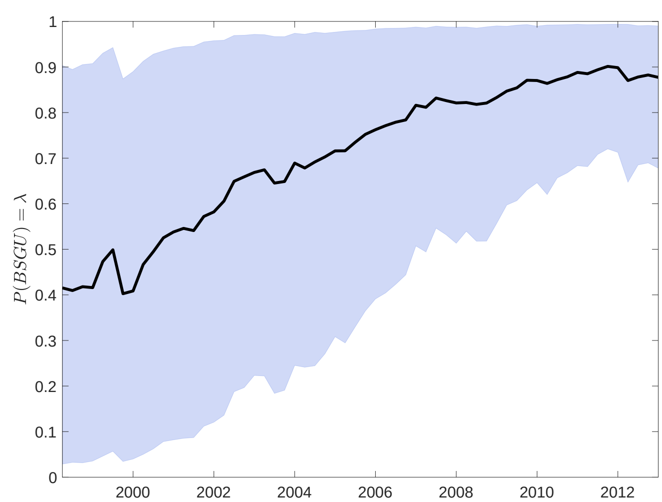
$$p\left(y_t|\lambda,\mathcal{P}\right)=\lambda p\left(y_t|y_{1:t-1},\mathcal{M}_b\right)+(1-\lambda)p\left(y_t|y_{1:t-1},\mathcal{M}_s\right),\quad 0\leq\lambda\leq 1 \tag{13}$$

Where $p\left(y_t|\lambda,\mathcal{P}\right)$ is the predictive density obtained by pooling the two competing models for a given weight λ and pool $\mathcal{P}=\{\mathcal{M}_b,\mathcal{M}_s\}$. The policymaker is Bayesian and has a prior density $p(\lambda|\mathcal{P})$ of the weight assigned to each model in the pool. The posterior distribution of the model weights, $p(\lambda|\mathcal{I}_t^{\mathcal{P}},\mathcal{P})$, can be updated recursively conditional on the information available to the pool in the previous period $\mathcal{I}_{t-1}^{\mathcal{P}}$:

$$p(\lambda|\mathcal{I}_t^{\mathcal{P}},\mathcal{P})\propto p\left(y_t|\lambda,\mathcal{P}\right)p(\lambda|\mathcal{I}_{t-1}^{\mathcal{P}},\mathcal{P}) \tag{14}$$

We estimate the posterior distribution in equation 14 recursively, starting in 1998:Q1 when the ZLB became binding in Japan, and use it to construct a time-varying measure of the weight a policy maker would attach to our two competing hypothesis for stagnation. The estimated path of model weights is shown in Figure 5 together with posterior credible sets to capture uncertainty about this parameter. The Japanese data implies roughly similar weights on both models in the early part of the sample and through the early 2000s. Afterwards, the data leans in favor of the specification $\mathcal{M}_b$ indicating a better fit of the expectation trap hypothesis. Uncertainty about the posterior distribution of the model weight is substantial but it decreases later in the sample as more information favoring the expectation trap accumulates.

Figure 5: *Model Weights: Expectations Traps vs Secular Stagnation*



Notes: The solid black line is the posterior mean of λ estimated recursively over the period 1998:Q1-2012:Q4. The shaded areas correspond to the 90 percent credible set of the posterior distribution.

What feature in the data helps distinguish between the two hypothesis? The difference in the transmission mechanism shown in Figure 4 suggests that the correlation between output and inflation will differ across models. It is clear that this correlation will be positive under secular stagnation, while it might be weaker or even negative under expectation traps, In fact, using model simulated data we verify that the correlation between model implied output growth (Δy_t^Q) and annualized inflation (π_t^A) is -0.02 in the expectation trap model and 0.12 in the secular stagnation model. The data counterpart for this unconditional moment is -0.15, thus suggesting that the expectation trap model will be preferred from a likelihood perspective. Wieland (2017) shows that both in an stylized and a medium-scale New Keynesian model, negative supply shocks at the ZLB generate a positive correlation between inflation and output and finds this to be inconsistent with the empirical response of Japanese data during the 2011 earthquake and global oil supply shocks. We highlight that the unconditional correlation between inflation and output during a liquidity trap provides useful information to distinguish between competing mechanisms of persistent stagnation.

5.2. Equilibrium selection and model misspecification

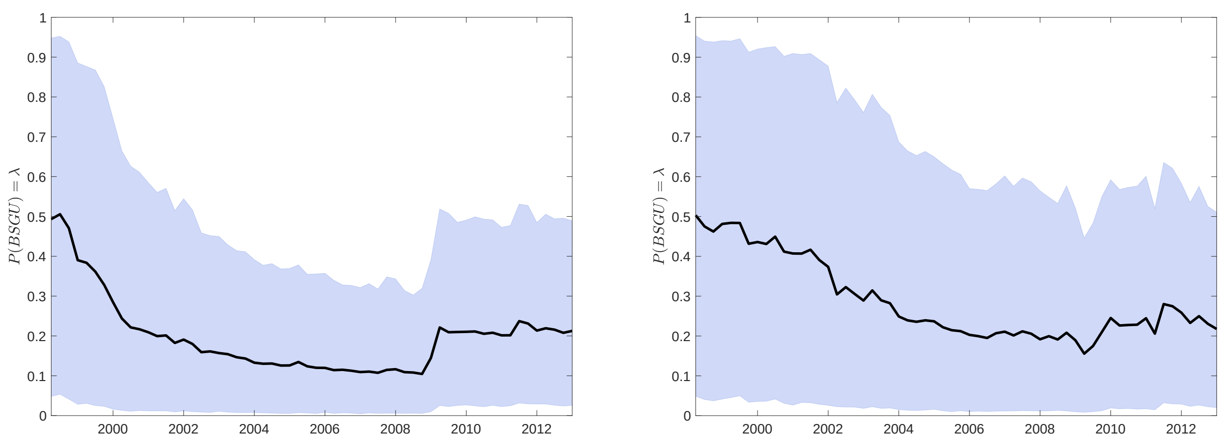
What explains the better fit of the expectations-trap hypothesis? We explore features of the equilibrium dynamics that influence the ability of each model to fit Japanese data.

It is well know that equilibrium dynamics under indeterminacy have two distinguishing features relative to the determinate counterpart (Lubik and Schorfheide, 2004; Canova and Gambetti, 2010). One is the presence of an additional state variable, that captures the effect of time $t-1$ inflation expectations $(\mathbb{E}_{t-1}\hat{\pi}_t)$ on endogenous variables at time t. The second component is the presence of non-fundamental or sunspot shock (ζ_t) as an additional driver of equilibrium dynamics. To isolate the effect of these two components we construct the *Minimum State Variable* solution corresponding to our BSGU model, and label it $\mathcal{M}_b^{msv}$. In this case, we restrict the local dynamics of inflation and output to be functions of the vector of fundamental shocks. This solution concept is commonly used in models featuring expectation traps and equilibrium indeterminacy (McCallum, 2003; Aruoba et al., 2018; Lansing, 2019).

Panel (a) in Figure 6 shows the estimated model weight for $\mathcal{P} = \{\mathcal{M}_b^{msv}, \mathcal{M}_s\}$. The weight on the expectation-trap model declines sharply early in the sample falling to near 15% by 2002, and then increases to about 20% after 2008. Uncertainty about the policy maker's view on the most relevant hypothesis has a similar pattern as in our baseline result, it is high at the begging of the sample but declines as more information becomes available. However, the policy maker's model uncertainty increases somewhat after 2008 as seen by the wider credible set surrounding the mean estimate of λ. This result suggest that the propagation of shocks through the additional state variable related to inflation expectations and the presence of sunspot shocks are key elements for the fit of the expectation-trap hypothesis.

Turning to the converse of our question, we explore why the secular stagnation hypothesis is outperformed in terms of fitting the Japanese experience. Because equilibrium dynamics under secular stagnation are determinate, we investigate the role of model misspecification. Rather than enriching the baseline model with additional structural features or incorporating non-structural shocks to detect misspecification (Den Haan and Drechsel, 2020) we make the stochastic process more flexible by allowing cross-correlation in the fundamental disturbances (Curdia and Reis, 2010). We refer to this specification as $\mathcal{M}_s^{corr}$.

Figure 6: *Model Weights: Alternative Model Specifications*



(a) $\mathcal{M}_b^{msv}$ *vs* $\mathcal{M}_s$ **(b)** $\mathcal{M}_b$ *vs* $\mathcal{M}_s^{corr}$

Notes: In both panels, the solid black line is the posterior mean of λ estimated recursively over the period 1998:Q1-2012:Q4. The shaded areas correspond to the 90 percent credible set of the posterior distribution. In panel (b), the cross-shock correlations in $\mathcal{M}_s^{corr}$ are $\rho(\varepsilon_z, \varepsilon_g) = -0.7109$, $\rho(\varepsilon_z, \varepsilon_v) = -0.2662$, $\rho(\varepsilon_g, \varepsilon_v) = 0.1978$.

Panel (b) in Figure 6 shows the estimated model weights for $\mathcal{P} = \{\mathcal{M}_b, \mathcal{M}_s^{corr}\}$. As in the previous exercise, the policy maker starts with split views with respect to the best fitting model. However, as information arrives, the weight on the expectations trap hypothesis declines slowly to around 20 percent. Uncertainty about the model weight is substantial and declines less than in the baseline specification. Overall, this result suggests that a more flexible specification of the secular stagnation improves the fit to Japanese data and shifts the policy maker's view about the most relevant model during the sample. It also points to the role of model misspecification as a key factor when comparing alternative models of persistent liquidity traps.

6. Nonlinear model

The previous sections focused on the assumption of a permanent liquidity trap. We now explore the role of global dynamics when we allow agents to form expectations about returning to full employment. For this analysis, we log-linearize the equilibrium system (equations 5 - 8) around $\pi_t = \pi^*$, $y_t = 1$, $R_t = r\pi^*$ and set $r = 1.0025$ which corresponds to a 1% annualized natural interest rate, with an associated value for $\delta = 0.1088$.

$$\hat{y}_t = \tau \mathbb{E}_t \mathcal{D} \left(\hat{y}_{t+1} - \hat{g}_{t+1} \right) - \mathcal{D} \mathbb{E}_t \left(\hat{R}_t - \hat{\pi}_{t+1} - \hat{z}_{t+1} \right) + \hat{g}_t - \mu \hat{\delta}_t \tag{15}$$

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \lambda \hat{v}_t + \kappa \left(\hat{y}_t - \hat{g}_t \right) + \frac{\kappa}{\eta \tau} \hat{y}_t \tag{16}$$

$$\hat{R}_t = \max \left\{ -\log(r\pi^*), \psi \hat{\pi}_t \right\} \tag{17}$$

Equations 15 – 17 are the discounted Euler equation, the Philips curve equations, and the non-linear monetary policy rule that enforces the ZLB constraint, respectively. Relative to the permanent liquidity trap model of Section 4, the discounting term $\mathcal{D} = \frac{R\beta}{(R\beta + \pi^* z^* \delta c^\tau)}$, includes the term $R = \frac{z^*\left(1 - \delta g^{*-\tau}\right)}{\beta} \pi^*$ which shows how δ affects the central bank's intercept for the long-run value of the nominal rate. We set $\psi = 1.5$ to illustrate our experiment.[19]

6.1. Expectation traps

For the expectations trap model, we follow Aruoba et al. (2018) and model fluctuations in agents confidence with an extrinsic sunspot variable that follows a two-state Markov switching process $s_t \in \{0, 1\}$, with transition probabilities $\mathbb{P}\left(s_{t+1} = 0 | s_t = 0\right) = \rho_0 < 1$ and $\mathbb{P}\left(s_{t+1} = 1 | s_t = 1\right) = \rho_1 < 1$. The state $s_t = 0$ is the low-confidence state associated with the expectations-driven liquidity trap, while $s_t = 1$ is the high-confidence state in which the economy evolves around the full employment steady state. We refer to this specification of the model as $\mathcal{M}_b^N$.

6.2. Secular stagnation

To model secular stagnation episodes, we assume the fundamental shock $\hat{\delta}$ in equation 15 follows a two-state Markov switching process $\hat{\delta}_t \in \{0, \delta_H\}$, with transition probabilities $\mathbb{P}\left(\hat{\delta}_{t+1} = 0 | \hat{\delta}_t = 0\right) = \rho_0 < 1$ and $\mathbb{P}\left(\hat{\delta}_{t+1} = \delta_H | \hat{\delta}_t = \delta_H\right) = \rho_1 < 1$. The state $\hat{\delta}_t = 0$, corresponds to the full employment steady state in which there is no change in the marginal utility for bonds. In this case the ZLB constraint might bind because of other fundamental

[19] Appendix C spells the derivation of the coefficients of the log-linearized system.

shocks lower inflation and the nominal interest rate. The state $\hat{\delta}_t = \delta_H > 0$, is associated with a transitory increase in the preference for bonds as in Fisher (2015) and we interpret this state as a secular stagnation episode. We set δ_H to ensure that, absent other shocks, the ZLB constraint becomes binding when the economy transitions to secular stagnation. This does not rule out the possibility of observing positive nominal interest rate in response to fundamental shocks. We refer to this specification of the model as $\mathcal{M}_s^N$.

6.3. Solution and Inference

We calibrate the Markov switching parameter $p_0 = 0.98$. This implies an expected duration of 15 years for either an expectations-driven or secular stagnation liquidity trap. In other words, we put both theories on the same footing to reproduce the observed Japanese experience in our sample. We set $p_1 = 0.99$, such that we allow the full employment state under both hypothesis to be highly persistent with an expected duration of 250 years. We keep all other parameters similar to those presented in Table 2. Because agents form expectations about returning to full employment after a liquidity trap and vice-versa, average inflation during the trap differs from its steady state value. To mirror the calibration of Section 4, we adjust the degree of nominal rigidity such that when we simulate the model, average inflation in a liquidity trap is at -1.06%, consistent with our baseline calibration.

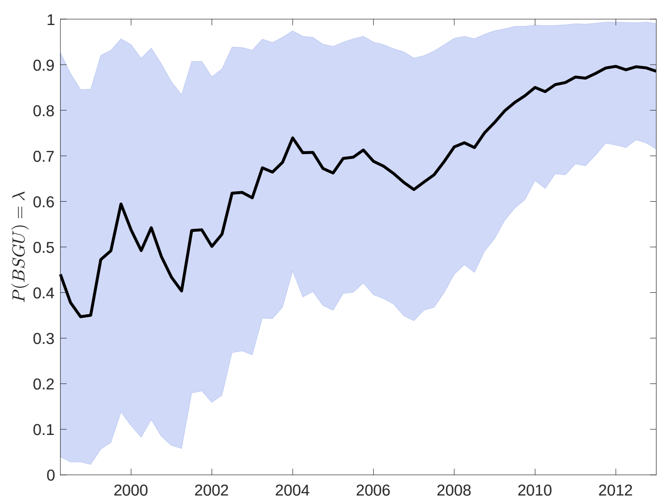
Our model features two forms of non-linearity. The first is the ZLB constraint that imposes a kink in the decision rules of the model. Second, the two-state Markov switching structure to model fluctuations between full employment and expectation traps and full employment and secular stagnation, introduces discrete jumps in the dynamics of the model variables despite the semi-log linear nature of the system (equations 15 – 17). Two tackle both non-itineraries, we implement the solution algorithm of Aruoba et al. (2018) in which decision rules are approximated using piece-wise smooth combinations of Chebyshev polynomials. Because our model specification does not include endogenous state variables, we can easily construct a deterministic solution grid to solve the model. This has the advantage that we only need to solve the model once and can do so within a few seconds. The solution procedure finds the coefficients of the Chebyshev polynomials by minimizing the sum of squared residuals implied by the equilibrium conditions over the grids $\mathcal{G}_b$ and $\mathcal{G}_s$, respectively. We provided a detailed implementation of our solution procedure in Appendix C.

The last step is to construct the predictive densities $p\left(y_t|y_{1:t-1}, \mathcal{M}_b^N\right)$ and $p\left(y_t|y_{1:t-1}, \mathcal{M}_s^N\right)$. For each of our two non-linear specifications, we use a particle filter as in Aruoba et al. (2018) to match Japanese data on output growth, consumption growth and inflation during the period 19981:Q1-2012:Q4. We do not incorporate the nominal interest rate as an observable in order to keep the policy maker information set similar to our exercise in Section 5. Using the filtered shocks from each model, we verified that the implied nominal interest rate remained at or near the ZLB for the vast majority of this period.

6.4. Results

Figure 7 shows the posterior distribution of model weights when the policy maker compares $\mathcal{M}_b^N$ vs $\mathcal{M}_s^N$. The model weight fluctuates around 0.5 in the first part of the sample and tilts in favor of the expectation trap hypothesis from 2005 onward. Similar to the analysis of our permanent stagnation models of Section 5, uncertainty about the model weight is substantial and persists for many years before the data provides enough evidence in favor of one of the competing hypothesis.

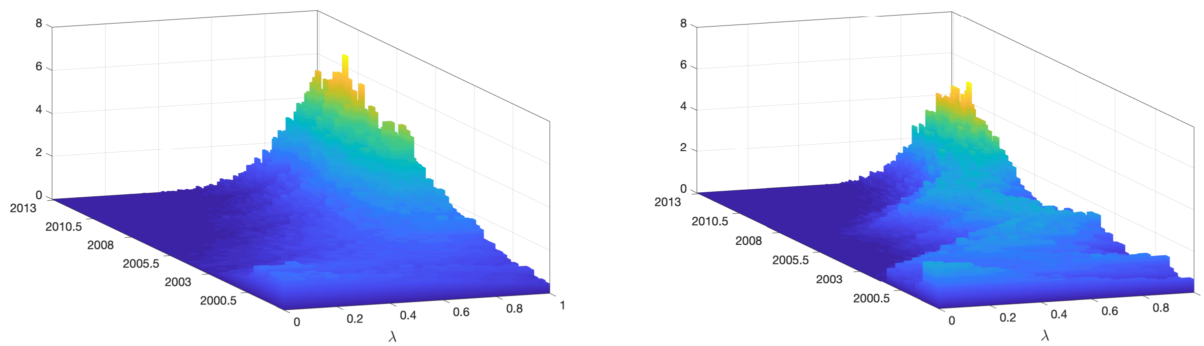
Figure 7: *Model Weights with Expectations of ZLB Exit*



Notes: The solid black line is the posterior mean of λ estimated recursively over the period 1998:Q1-2012:Q4. The shaded areas correspond to the 90 percent credible set of the posterior distribution.

To provide additional insights about the difference between our specification in which agents expect a permanent liquity and the specification of our non-linear model in which agents understand that liquidity traps can be recurrent and of arbitrary duration, we compare the entire posterior distributions of model weights for both models. The left panel of Figure 8 corresponds to the permanent liquidity trap models $p(\lambda|\mathcal{I}_t^{\mathcal{P}}, \{\mathcal{M}_b, \mathcal{M}_s\})$ and the right panel corresponds to transitory but persistent traps $p(\lambda|\mathcal{I}_t^{\mathcal{P}}, \{\mathcal{M}_b^N, \mathcal{M}_s^N\})$. Three features arise from inspecting the posterior distributions. First, consistent with the results in Figure 5 and 7, there is substantial uncertainty in the first years of our sample, with model weights all over the interval $[0, 1]$. Second, the mass of the posterior distribution shifts towards $\lambda = 1$ after 2005, and suggest that a maximum posterior probability estimate would favor the expectation driven hypothesis regardless of the nature of expectations about the duration of the trap. Third, the mass of the distribution $p(\lambda|\mathcal{I}_t^{\mathcal{P}}, \{\mathcal{M}_b^N, \mathcal{M}_s^N\})$ moves more slowly towards $\lambda = 1$ than the distribution $p(\lambda|\mathcal{I}_t^{\mathcal{P}}, \{\mathcal{M}_b, \mathcal{M}_s\})$. This is consistent with the fact that when agents form expectations about exiting the liquidity trap, they need to accumulate more information in order to move their priors in favor of any of the competing hypothesis.

Figure 8: *Posterior Distribution of Model Weights: Permanent vs Transitory Liquidity Traps*



(a) $\mathcal{M}_b$ *vs* $\mathcal{M}_s$

(b) $\mathcal{M}_b^N$ *vs* $\mathcal{M}_s^N$

Notes: The panels show the posterior distributions $p(\lambda|\mathcal{I}_t^{\mathcal{P}}, \{\mathcal{M}_b, \mathcal{M}_s\})$ (permanent liquidity trap) and $p(\lambda|\mathcal{I}_t^{\mathcal{P}}, \{\mathcal{M}_b^N, \mathcal{M}_s^N\})$ (transitory but persistent liquidity trap), for t =1998:Q1-2012:Q4.

7. Discussion

7.1. A robust policy: appropriate price indexation

Our quantitative analysis for Japan highlights the challenges in disentangling the source of liquidity trap, especially in real time. This motivates the need for developing policies that are robust to the source of recession. We develop one such set of policy prescriptions to tackle these recessions. In models with temporary liquidity traps, inflationary pressures of a temporary increase in price markups can be expansionary (Eggertsson, 2012). We use this insight to show that an appropriate price indexation rules can increase output under secular stagnation and also eliminate the expectations-driven trap.

In the context of the simple model of Section 2, we now prove that an appropriate indexation scheme can eliminate expectations-driven liquidity trap while also improving the output under secular stagnation. Recount that a fraction $1 - \alpha_p$ of firm index their price

p^n to the aggregate price level from previous period $\frac{p_t^n}{P_t} = \Gamma_t \frac{P_{t-1}}{P_t}$. We now allow Γ_t to be somewhat general:

$$\Gamma_t = \frac{P_t}{Y_t^{-1}(P_t - \lambda P_{t-1}) + P_{t-1}}; \quad \lambda > 0.$$

The price Phillips curve is given by:

$$\Pi_t = \kappa Y_t + (\lambda - \kappa \bar{Y})$$

We now consider λ to be a policy tool that requires adjusting firms to index prices in a particular manner.

Proposition 4. Consider an indexation rule where the non-resetters index their prices to last period's price level with indexation coefficient: $\Gamma_t = \frac{P_t}{Y_t^{-1}(P_t - \lambda P_{t-1}) + P_{t-1}}$. There does not exist expectations-driven liquidity trap $\forall\ \lambda > \kappa$. Output and inflation under secular stagnation are increasing in λ.

An increase in λ acts like a price-markup shock that lasts for the duration of the trap and therefore increases output under secular stagnation. A policy of setting high enough $\lambda > \kappa$ installs a lower bound on deflation, and hence eliminates the expectations driven trap altogether. Other policies that flatten the Phillips curve by strengthening labor unions during recessions can also preclude the possibility of BSGU trap. A converse implication of this finding is that structural reforms that increase downward flexibility in prices make the economy vulnerable to swings in pessimistic expectations. We label this result as the *curse of flexibility*.[20]

7.2. Results with downward nominal wage rigidity

We briefly discuss the robustness of our theoretical results with downward nominal wage rigidity.[21] In particular, we discuss that minimum wage policies can also act like robust policies. Furthermore, we show that expectations-driven trap can also emerge in the over-

[20]In a recent work, Benigno and Fornaro (2018) construct a model with an expectations-driven trap similar to the BSGU trap in terms of TFP growth and nominal interest rate. They find that R&D subsidies that install lower bounds on TFP growth can preclude stagnation trap. Thus, their suggested fiscal policy is analogous to price indexation policy in our environment. Our analysis does not imply that imposing a lower bound on deflation is enough to preclude a BSGU-type steady state in more general settings. For example, in Benigno and Fornaro (2018) there is perfect downward nominal rigidity, but endogenous growth opens up the possibility of a stagnation trap. Similarly, Heathcote and Perri (2018) model an economy with perfect downward nominal rigidity, precautionary savings and a liquid asset in net positive supply. Despite perfectly downward rigid wages, a BSGU steady state exists in their model because of precautionary savings motive.

[21]For more elaborate discussion, we refer the reader to February 2019 working paper version of our paper available as FRB International Finance Discussion Paper 1243.

lapping generations model of Eggertsson, Mehrotra and Robbins (2019) with appropriate wage flexibility.

7.2.1 Robust minimum wage policy

We make two changes to the model presented in Section 2. One, we assume inelastic labor supply with time endowment of one. Second, we assume nominal wages are downwardly rigid as in Schmitt-Grohé and Uribe (2017) i.e. $W_t \geq \gamma(h_t)W_{t-1}$, where $\gamma(0) = 1 - \kappa$ is a constant, $\gamma'(\cdot) > 0$, and $\gamma(1) > \delta\beta$. By choosing $\kappa < 1$, a policy maker can eliminate the expectations-driven liquidity trap. Since this policy lever is about the level of wages paid when employment approaches zero (an off-equilibrium limit), our model implies that policies along the lines of universal basic income policy can be used to combat these expectations-driven recessions. We provide a formal proof for these statements in the Appendix D.

7.2.2 Expectations trap in an OLG model

The degree of nominal rigidities also plays a key role in eliminating the locally indeterminate stagnation steady state in the overlapping generations model of Eggertsson et al. (2019) (EMR). We outline the key message here while referring the reader to EMR for detailed model. Agents live for three periods: young, middle and old. Young are borrowing constrained and derive no income. Middle supply labor inelastically to perfectly competitive firms for wages and save for retirement. Old consume the savings made when middle. Supply and demand for savings results in the following bond market clearing condition : $1 + r_t = \frac{1+\beta}{\beta}\frac{D_t}{Y_t - D_{t-1}}$, where D is the exogenous debt limit faced by the young borrowers. It is further assumed that households do not accept nominal wages below a particular wage norm i.e. $W_t = \max\{\bar{W}_t, W_t^{flex}\}$ where $\bar{W}_t = \gamma W_{t-1} + (1-\gamma)W_t^{flex}$ and $W_t^{flex} = P_t\alpha$. Perfectly competitive firms hire workers to produce final output using production function $Y_t = h_t^\alpha$, taking wages as given. The policy rule is same as in our baseline model in Section 2.

Given inflation target $\Pi^* = 1$, the aggregate demand and the aggregate supply blocks in the steady state are given by:

$$Y_{AD} = D + \begin{cases} \frac{1+\beta}{\beta}\frac{D}{\Gamma^*}\Pi^{1-\alpha_\pi}, & \text{if R } > 1, \\ \frac{1+\beta}{\beta}D\Pi, & \text{if R } = 1 \end{cases} \tag{18}$$

where $\Gamma^* = (1 + r^*)^{-1}$.

$$Y_{AS} = \begin{cases} 1, & \text{if } \Pi \geq 1, \\ \left(\frac{1-\frac{\gamma}{\Pi}}{1-\gamma}\right)^{\frac{\alpha}{1-\alpha}}, & \text{if } \Pi < 1 \end{cases} \tag{19}$$

If $\Pi^* = 1$, $r^* > 0$, and $\gamma < 0$, then there exists a unique liquidity trap steady state with positive unemployment, deflation and zero nominal interest rate. The dynamics around this steady state are locally indeterminate. A negative γ implies that nominal wages are allowed to fall increasingly with unemployment as in Schmitt-Grohé and Uribe (2017).

7.3. Comparison with the textbook Euler equation

We provide a brief comparison of results for the reader with the textbook Euler equation (Woodford, 2003). This serves to illustrate the use of two concepts in our framework - a) the modified Euler equation and b) bounds on deflation.

In the textbook model, the natural interest rate is always fixed at $\frac{1}{\beta} > 1$. As a result, the aggregate demand relationship is a horizontal line at $\Pi = \beta < 1$ when the nominal interest rate is constrained by the ZLB. However, the existence of an *unintended* deflationary steady state is contingent on the assumptions regarding the supply side of the economy. Schmitt-Grohé and Uribe (2017) who implicitly assume a zero intercept for inflation in their downward nominal wage rigidity functional form. If the y-intercept of the aggregate supply curve is large enough, then there does not exist a deflationary steady state.[22] Setting this y-intercept is analogous to a minimum wage policy discussed in Section 7.2.1.

While the modification to the Euler equation we proposed is not essential to eliminating the BSGU trap, it is essential for the model to generate a secular stagnation steady state. The modified Euler equation with an endogenous long-run natural rate of interest, opens up the possibility of a secular stagnation steady state. This steady state cannot arise in the standard model because of violation of the transversality condition of the representative household.

An alternative way to derive this modification in the Euler equation is through the use of bonds in the utility function, as developed recently by Michaillat and Saez (2019), Michau (2018), and Ono and Yamada (2018) to analyze a unique persistent stagnation scenario. Fisher (2015) proves an observational equivalence of this assumption with risk-premium shocks assumed by Smets and Wouters (2007) in the budget constraint of the household.

[22] In the notation of Schmitt-Grohé and Uribe 2017, if $\gamma(1) > \tilde{\beta}$ there does not exist an unemployment steady state in their baseline model.

Another interpretation of the shocks to δ is that these capture the *flight to liquidity* episode of the recent financial crisis (Krishnamurthy and Vissing-Jorgensen, 2012). A similar wedge in the Euler equation can be associated with the deterioration in liquidity properties of AAA-rated corporate bonds in contrast to Treasury securities during the 2008-09 financial crisis (Del Negro et al., 2017).

While these different interpretations follow naturally from an extensive literature, we also view this wedge in the Euler equation as a reduced form for fundamental factors such as aging, savings glut, reserve accumulation, inequality, or debt deleveraging microfounded in the secular stagnation literature (Eggertsson et al. 2016, 2019; Auclert and Rognlie 2018). Such a reduced form modification allows the researchers to employ existing tools to include secular stagnation as a complementary explanation in the estimated DSGE models, much like "β shock" is used in the temporary liquidity trap literature.[23]

8. Conclusion

In this paper, we developed a framework to formally distinguish two hypotheses of stagnation: expectations-driven and fundamentals-driven stagnation. The two hypotheses differ in the local determinacy of the equilibrium. Secular stagnation is defined as the locally determinate equilibrium with zero nominal interest rates, below potential output and below target inflation rate. Expectations-driven trap is defined as the locally indeterminate equilibrium with zero nominal interest rates, below potential output and below target inflation rate. The local determinacy property is the source of different policy implications in the two episodes.

We conduct a Bayesian estimation of a DSGE model for Japan which has experienced twenty years of near zero interest rates. Using techniques from the static prediction pools literature, we construct a time-varying measure of the weight a policymaker would attach to the two competing hypothesis of the liquidity trap. The Japanese data implies roughly similar weights on both models in the early part of the sample and through the early 2000s. We show that this model uncertainty is considerable even when considering issues of model misspecification or equilibrium selection.

Given real-time uncertainty in the inference of the model weights, and the role of structural assumptions in identifying the dominant narrative, we argue that there is a need for robust policies to deal with liquidity traps. We proposed one set of such policies that place a sufficiently high floor on inflation. These lower bounds on inflation exclude the

[23]In the recent literature that augments DSGE models with endogenous growth, (mean zero) shocks to preference for bonds are added to get co-movement of investment and consumption as well to derive the *divine coincidence* benchmark (Garga and Singh, 2016).

possibility of expectations trap as well as reduce the severity of a secular stagnation induced liquidity traps. In our benchmark model, these policies are implemented in the form of an appropriate price indexation rule by non-adjusting firms in the Calvo model. In models with labor market rigidities, we show that minimum income policies can help place this lower bounds on inflation. While our paper took a first step in quantifying the model uncertainty, an extensive study of robust policies and their implementation is an important agenda for future research.

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A. Proofs for Propositions in Section 2

Proposition (Proposition 1: Targeted Steady State)**.** Let $0 < \delta < \frac{1}{\beta}$.There exists a unique positive interest rate steady state with $Y = 1$, $\Pi = 1$ and $R = \frac{1}{\beta\delta} > 1$. It features output at efficient steady state, and inflation at the policy target. The equilibrium dynamics in this steady state's neighborhood are locally determinate.

Proof. The downward sloping portion of aggregate demand always goes through $Y = 1$, $\Pi = 1$. When $\delta < \frac{1}{\beta}$, $1 + r^* > 1$. The kink in the AD curve occurs at $\Pi_{kink} = \left(\frac{1}{(1+r^*)}\right)^{\frac{1}{\phi_\pi}} < 1$ and $Y_{AD,kink} = (1 + r^*)^{1-\frac{1}{\phi_\pi}} > 1$. There always exists an intersection between the AS and the AD at $Y = 1$ and $\Pi = 1$. To show that there does not exist another steady state at positive interest rates, note the AS curve is linear and upward sloping. For $\Pi > 1$, $Y_{AD} < 1 < Y_{AS}$. And for $\Pi_{kink} \leq \Pi < 1$, $Y_{AD} > 1 > Y_{AS}$. There does not exist another steady state at positive nominal interest rate.

To prove local-determinacy, log-linearize the equilibrium conditions (1) - (3) around the unique non-stochastic steady state $Y = 1$, $\Pi = 1$ and $R = \frac{1}{\beta\delta} > 1$. The system of equations can be simplified to:

$$\hat{Y}_t = \frac{1+\kappa}{2+\phi_\pi\kappa}\mathbb{E}_t\hat{Y}_{t+1} + \hat{r}^n_t$$

where hat variables represent log-deviations from steady state. Given, $\kappa > 0$ and $\phi_\pi > 1$, this system satisfies Blanchard-Kahn conditions for determinacy of linear rational-expectations equilibria. □

Proposition (Proposition 2: BSGU steady state)**.** Let $0 < \delta < \frac{1}{\beta}$. For $\kappa > 1$ (i.e. $\alpha_p > 0.5$) there exist two steady states:

1. The targeted steady state with $Y = 1$, $\Pi = 1$ and $R = \frac{1}{\beta\delta} > 1$.
2. (*Expectations-driven* trap) A unique-ZLB steady state with $Y = \frac{1-\kappa}{\beta\delta-\kappa} < 1$, $\Pi = \frac{\beta\delta(1-\kappa)}{\beta\delta-\kappa} < 1$ and $R = 1$. The local dynamics in a neighborhood around the unemployment steady state are locally in-determinate.

When prices are rigid enough, i.e. $\kappa < 1$, there exists a unique steady state and it is the targeted inflation steady state. When prices are flexible $\alpha_p = 1$ ($\kappa \to \infty$), there always exist two steady states: a unique deflationary steady state with zero nominal interest rates and a unique targeted inflation steady state.

Proof. For $\kappa > 1$:

When $\delta < \frac{1}{\beta}$, $1 + r^* > 1$. The kink in the AD curve occurs at $\Pi_{kink} = \left(\frac{1}{(1+r^*)}\right)^{\frac{1}{\phi_\pi}} < 1$ and

$Y_{AD,kink} = (1+r^*)^{1-\frac{1}{\phi_\pi}} > 1$. There always exists an intersection between the AS and the AD at $Y = 1$ and $\Pi = 1$. To show that there doesn't exist another steady state at positive interest rates, note that the AS curve is linear and upward sloping. For $\Pi > 1$, $Y_{AD} < 1 < Y_{AS}$. And for $\Pi_{kink} \leq \Pi < 1$, $Y_{AD} > 1 > Y_{AS}$. There does not exist another steady state at positive nominal interest rate. The proof for local-determinacy of this targeted steady state follows similar steps as in Proposition 1.

To prove that there exists a unique intersection at zero nominal interest rates, we note that AS and AD are linear for $\Pi < \Pi_{kink}$. When $\Pi = \Pi_{kink}$, $Y_{AD} > 1 > Y_{AS}$. And when $\Pi = 0$, $Y_{AD} = 0 < Y_{AS} = \frac{\kappa-1}{\kappa} > 0$. This is because of the assumption that $\alpha_p > 0.5$. Hence, there exists a unique intersection at zero nominal interest rates. To prove local-indeterminacy, log-linearize the equilibrium conditions (1) - (3) around the unique non-stochastic steady state $Y_S = \frac{1-\kappa}{\beta\delta-\kappa} < 1$, $\Pi_S = \frac{\beta\delta(1-\kappa)}{\beta\delta-\kappa} < 1$ and $R_S = 1$. The system of equations can be simplified to:

$$\hat{Y}_t = \frac{1+\kappa_b}{2}\mathbb{E}_t\hat{Y}_{t+1} + \hat{r}_t^n$$

where hat variables represent log-deviations from steady state and $\kappa_b \equiv \frac{\kappa Y_S}{\kappa Y_S+1-\kappa} > 1$ (because $1-\kappa < 0$ and $\beta\delta < 1$). Hence, this system does not have a unique bounded rational expectations equilibrium.

For $\kappa < 1$:

That there exists a unique non-ZLB steady state follows from Proposition 1. Remains to show that there does not exist a ZLB steady state. Note that, AD is linear and upward sloping when ZLB is binding and AS is always linear. Furthermore for $\Pi_{kink} \leq \Pi < 1$, $Y_{AD} > 1 > Y_{AS}$. And for $\Pi = 1-\kappa$, $Y_{AS} = 0 < Y_{AD}$. Thus there does not exist a steady state with zero nominal interest rate.

□

Proposition (Proposition 3: Secular Stagnation)**.** Let $\delta > \frac{1}{\beta}$ and $\kappa < 1$. There exists a unique steady state with $Y = \frac{1-\kappa}{\beta\delta-\kappa} < 1$, $\Pi = \frac{\beta\delta(1-\kappa)}{\beta\delta-\kappa} < 1$ and $R = 1$. It features output below the targeted steady state and deflation, caused by a permanently negative natural interest rate. The equilibrium dynamics in this steady state's neighborhood are locally determinate.

Proof. When $\delta > \frac{1}{\beta}$, $1 + r^* < 1$, thus, the kink in the AD occurs at $\Pi_{kink} > 1, Y_{AD,kink} < 1$. For $\Pi > \Pi_{kink}$, $Y_{AD} < 1 < Y_{AS}$. Thus, no steady state exists at positive nominal interest rates. When $\Pi = \Pi_{kink}$, $Y_{AD} < 1 < Y_{AS}$. For $\Pi < \Pi_{kink}$, AS and AD are both and downward sloping. At $\Pi = 1-\kappa < 1$, $Y_{AS} = 0 < Y_{AD}$. Hence there exists a unique steady state of the economy with nominal rigidities.

To prove local-determinacy, log-linearize the equilibrium conditions (1) - (3) around the unique non-stochastic steady state $Y_S = \frac{1-\kappa}{\beta\delta-\kappa} < 1$, $\Pi_S = \frac{\beta\delta(1-\kappa)}{\beta\delta-\kappa} < 1$ and $R_S = 1$. The system of equations can be simplified to:

$$\hat{Y}_t = \frac{1+\kappa_a}{2}\mathbb{E}_t\hat{Y}_{t+1} + \hat{r}_t^n$$

where hat variables represent log-deviations from steady state and $\kappa_a \equiv \frac{\kappa Y_S}{\kappa Y_S + 1 - \kappa} < 1$. Given, $0 < \kappa_a < 1$, this system satisfies Blanchard-Kahn conditions for determinacy of linear rational-expectations equilibria. $\square$

B. Derivation of the price indexation scheme

As shown in Section 2.2, the price index is given by

$$P_t^{\frac{\nu-1}{\nu}} = \alpha\left(p_t^*\right)^{\frac{\nu-1}{\nu}} + (1-\alpha)\left(p_t^n\right)^{\frac{\nu-1}{\nu}}$$

It can be rewritten as

$$\begin{aligned} 1 &= \alpha\left(\frac{p_t^*}{P_t}\right)^{\frac{\nu-1}{\nu}} + (1-\alpha)\left(\frac{p_t^n}{P_t}\right)^{\frac{\nu-1}{\nu}} \\ &= \alpha\left(\frac{Y_t}{\bar{Y}}\right)^{\frac{\nu-1}{\nu}} + (1-\alpha)\left(\Gamma_t\frac{P_{t-1}}{P_t}\right)^{\frac{\nu-1}{\nu}} \end{aligned}$$

If $\nu = 1/2$, we get:

$$\begin{aligned} 1 = \alpha\left(\frac{Y_t}{\bar{Y}}\right)^{-1} + (1-\alpha)\left(\Gamma_t\frac{P_{t-1}}{P_t}\right)^{-1} \\ (1-\alpha)\left(\Gamma_t^{-1}\Pi_t - 1\right) = \alpha\left(\frac{Y_t-\bar{Y}}{Y_t}\right) \end{aligned}$$

Define $\Gamma_t = \frac{P_t}{Y_t^{-1}(P_t - \lambda P_{t-1}) + P_{t-1}}$, to get

$$(\Pi_t - \lambda) = \frac{\alpha}{1-\alpha}(Y_t - \bar{Y})$$

When $\lambda = 1$, the price Phillips curve simplifies to

$$(\Pi_t - 1) = \frac{\alpha}{1-\alpha}(Y_t - \bar{Y})$$

$\forall\ \lambda > \kappa > 1$, there does not exist expectations trap. An indexation to Γ_t of yesterday's price with $\lambda > \kappa$ can eliminate expectations trap.

Another indexation in price setting that we can assume is $\Gamma_t = Y_t$. This gives rise to the following relationship between gross inflation and output:

$$\Pi_t = (1+\kappa)Y_t - \kappa$$

where $\kappa = \frac{\alpha_p}{1-\alpha_p} > 0$ as before. With this Phillips curve, there always exist two steady states as long as $0 < \delta\beta < 1$ and $\kappa > 0$. We can analytically derive the steady states as in Proposition 2. A takeaway of our analysis is that appropriate price/wage indexation schemes can eliminate expectations trap. These can also be shown to improve the outcome in secular stagnation (as a corollary of paradox of flexibility).

A set of policies that are robust to the kind of stagnation can be framed from our analysis:

Corollary 1. (robust policies) A downwardly rigid price/wage indexation scheme can eliminate expectations trap while also improving welfare under secular stagnation.

C. DSGE Solution and Estimation

C.1. Log-linearized model

This section describes how we obtain the equations of the log-linearized model of Section 4.2. With a slight abuse of notation, we can write the system of equilibrium conditions as follow:

$$\begin{aligned}
1 &= \frac{\beta R}{\pi z} e^{-\tau(c_{t+1}-c_t)+R_t-\pi_{t+1}-z_{t+1}} + \delta c^{\tau} e^{\tau c_t+\delta_t} \\
(1-ve^{v_t}) + v\phi e^{v_t}\left(\pi e^{\pi_t}-\bar{\pi}\right)\pi e^{\pi_t} &= \chi_H c^{\tau} y^{1/\eta} e^{\tau c_t+(1/\eta)y_t} \\
&\quad + v\phi\beta\pi e^{v_t}\left[e^{-\tau(c_{t+1}-c_t)+y_{t+1}-y_t+\pi_{t+1}}\left(\pi e^{\pi_{t+1}}-\bar{\pi}\right)\right] \\
ce^{c_t} &= \frac{y}{g} e^{y_t-g_t}
\end{aligned}$$

Linearization around an arbitrary (R,π) point yields:

$$\begin{aligned}
c_t &= \frac{R\beta}{(R\beta+\pi z\delta c^{\tau})} c_{t+1} - \frac{R\beta}{(R\beta+\pi z\delta c^{\tau})\tau}\left(R_t-\pi_{t+1}-z_{t+1}\right) - \frac{\delta\pi z c^{\tau}}{(R\beta+\pi z\delta c^{\tau})\tau}\delta_t \\
\pi_t &= \beta\frac{(\pi-\bar{\pi})}{2\pi-\bar{\pi}}\left[\tau\left(c_{t+1}-c_t\right)+y_{t+1}-y_t\right] + \beta\pi_{t+1} + \left(\frac{\nu-(1-\beta)\nu\phi\pi(\pi-\bar{\pi})}{\nu\phi\pi(2\pi-\bar{\pi})}\right)\nu_t \\
&\quad + \left(\frac{(1-\nu)y^{\tau+1/\eta}}{\nu\phi\pi(2\pi-\bar{\pi})}\right)\left(\tau c_t + \frac{1}{\eta}y_t\right) \\
c_t &= y_t - g_t
\end{aligned}$$

C.2. Full employment

Around the full employment steady state we have $\pi = \pi^*$ and $R = \pi^* r_0$. In our calibration for the full employment steady state we have $r_0 = \frac{z\left(1-\delta g^{-\tau}\right)}{\beta} = \exp(1/400)$ and $\pi^* = \exp(1/400)$. Moreover, we choose χ_H to normalize the full employment level of output to $y = 1$ and set $\bar{\pi} = \pi^*$.

$$
\begin{aligned}
c_t &= \tau \mathcal{D} c_{t+1} - \mathcal{D}\left(R_t - \pi_{t+1} - z_{t+1}\right) - \mu \delta_t \\
\pi_t &= \beta \pi_{t+1} + \lambda \nu_t + \kappa c_t + \left(\frac{\kappa}{\eta \tau}\right) y_t \\
c_t &= y_t - g_t
\end{aligned}
\tag{C.1}
$$

Where, $\mathcal{D} = \frac{R\beta}{(R\beta + \pi z \delta c^{\tau})}$,$\lambda = \left(\frac{\nu}{\nu \phi \pi^2}\right)$, $\kappa = \tau\left(\frac{1-\nu}{\nu\phi\pi^2}\right)$, and $\mu = \frac{\delta \pi z c^{\tau}}{(R\beta + \pi z \delta c^{\tau})\tau}$ and plugging the log-linearized resourced constraint we obtain the equations in the main text.

C.3. Permanent Liquidity Trap

When the economy is at a permanent liquidity trap, we have R = 1 and we set $\delta_t = 0$. We denote by $\tilde{x}$ the steady state values corresponding to the liquidity trap

$$
\begin{aligned}
c_t &= \frac{\beta}{(\beta + \tilde{\pi} z \delta c^{\tau})} c_{t+1} - \frac{\beta}{(R\beta + \tilde{\pi} z \delta c^{\tau})\,\tau}\left(-\pi_{t+1} - z_{t+1}\right) \\
\pi_t &= \beta \frac{(\tilde{\pi} - \bar{\pi})}{2\tilde{\pi} - \bar{\pi}}\left[\tau\left(c_{t+1} - c_t\right) + y_{t+1} - y_t\right] + \beta \pi_{t+1} + \left(\frac{\nu - (1-\beta)\nu\phi\tilde{\pi}(\tilde{\pi} - \bar{\pi})}{\nu\phi\tilde{\pi}(2\tilde{\pi} - \bar{\pi})}\right)\nu_t \\
&\quad + \left(\frac{(1-\nu)\tilde{y}^{\tau + 1/\eta}}{\nu\phi\tilde{\pi}(2\tilde{\pi} - \bar{\pi})}\right)\left(\tau c_t + \frac{1}{\eta} y_t\right) \\
c_t &= y_t - g_t
\end{aligned}
$$

Collecting terms and replacing the log-linearized resource constraint we have:

$$
\begin{aligned}
\hat{y}_t &= \tau \tilde{\mathcal{D}}(\hat{y}_{t+1} - \hat{g}_{t+1}) + \tilde{\mathcal{D}}\left(\hat{\pi}_{t+1} + \hat{z}_{t+1}\right) + \hat{g}_t \\
\hat{\pi}_t &= \beta \hat{\pi}_{t+1} + \tilde{\lambda}\nu_t + \tilde{\kappa}(y_t - g_t) + \frac{\tilde{\kappa}}{\eta\tau} y_t + (1+\tau)\tilde{\Gamma}\left(y_{t+1} - y_t\right) - \tau\tilde{\Gamma}(g_{t+1} - g_t)
\end{aligned}
\tag{C.2}
$$

Where $\tilde{\mathcal{D}} = \frac{\beta}{(\beta + \tilde{\pi} z \delta \tilde{c}^{\tau})}$,$\tilde{\lambda} = \left(\frac{\nu - (1-\beta)\nu\phi\tilde{\pi}(\tilde{\pi} - \bar{\pi})}{\nu\phi\tilde{\pi}(2\tilde{\pi} - \bar{\pi})}\right)$, $\tilde{\kappa} = \left(\tau\frac{(1-\nu)\tilde{y}^{\tau+1/\eta}}{\nu\phi\tilde{\pi}(2\tilde{\pi} - \bar{\pi})}\right)$, and $\tilde{\Gamma} = \beta\frac{(\tilde{\pi} - \bar{\pi})}{2\tilde{\pi} - \bar{\pi}}$, we obtain the log-linearized equations presented in the main text.

C.4. MSV solution

This section derives the MSV solution for the analysis in Section 5. We guess that $\hat{y}_t = a_1\hat{z}_t + a_2\hat{g}_t + a_3\hat{v}_t$ and $\hat{\pi}_t = b_1\hat{z}_t + b_2\hat{g}_t + b_3\hat{v}_t$ and solve for the unknown $a's$ and $b's$. Replacing the guess into (C.2), collecting terms and using the method of undetermined coefficients, we obtain the following system of equations:

$$\begin{bmatrix} 0 \\ -\phi_7 - \phi_4\left(\rho_g - 1\right) \\ -\phi_5 \\ -\phi_2\rho_z \\ \phi_1\rho_g - 1 \\ 0 \end{bmatrix} = \begin{bmatrix} (\beta\rho_z - 1) & 0 & 0 & \phi_3(\rho_z - 1) + \phi_6 & 0 & 0 \\ 0 & (\beta\rho_g - 1) & 0 & 0 & \phi_3(\rho_g - 1) + \phi_6 & 0 \\ 0 & 0 & (\beta\rho_v - 1) & 0 & 0 & \phi_3(\rho_v - 1) + \phi_6 \\ \phi_2\rho_z & 0 & 0 & \phi_1\rho_z - 1 & 0 & 0 \\ 0 & \phi_2\rho_g & 0 & 0 & \phi_1\rho_g - 1 & 0 \\ 0 & 0 & \phi_2\rho_v & 0 & 0 & \phi_1\rho_v - 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

Where the elements in the right-hand side matrix are: $\phi_1 = \frac{\beta}{(\beta+\pi z\delta c^{\tau})}$, $\phi_2 = \frac{\alpha_1}{\tau}$, $\phi_3 = \beta\frac{(\pi-\bar{\pi})}{2\pi-\bar{\pi}}(1+\tau)$ $\phi_4 = -\beta\frac{(\pi-\pi)}{2\pi-\bar{\pi}}\tau$, $\phi_5 = \left(\frac{\nu-(1-\beta)\nu\phi\pi(\pi-\pi)}{\nu\phi\pi(2\pi-\bar{\pi})}\right)$ $\phi_6 = \left(\frac{(1-\nu)y^{\tau+1/\eta}}{\nu\phi\pi(2\pi-\bar{\pi})}\right)\left(\tau + \frac{1}{\eta}\right)$, $\phi_7 = -\left(\frac{(1-\nu)y^{\tau+1/\eta}}{\nu\phi\pi(2\pi-\bar{\pi})}\right)\tau$.

C.5. Nonlinear solution

For the nonlinear model we use equations (15)-(17) which we repeat here for convenience:

$$\hat{y}_t = \tau\mathbb{E}_t\mathcal{D}\left(\hat{y}_{t+1} - \hat{g}_{t+1}\right) - \mathcal{D}\mathbb{E}_t\left(\hat{R}_t - \hat{\pi}_{t+1} - \hat{z}_{t+1}\right) + \hat{g}_t - \mu\hat{\delta}_t \tag{C.3}$$

$$\hat{\pi}_t = \beta\mathbb{E}_t\hat{\pi}_{t+1} + \lambda\hat{v}_t + \kappa\left(\hat{y}_t - \hat{g}_t\right) + \frac{\kappa}{\eta\tau}\hat{y}_t \tag{C.4}$$

$$\hat{R}_t = \max\left\{-\log(r\pi^*), \psi_1\hat{\pi}_t\right\} \tag{C.5}$$

The max operator in Equation (C.5) and the Markov-switching nature of the solution need to be capture using a non-linear solution. For computation we define the following vector sumarizing the state of the economy $\mathcal{S}^b = (\hat{z}_t, \hat{g}_t, \hat{v}_t, s_t)$ for the expectations trap model and $\mathcal{S}^s = \left(\hat{z}_t, \hat{g}_t, \hat{v}_t, \hat{\delta}_t\right)$ for the secular stagnation model.

Without loss of generality, we illustrate the solution procedure using the expectations trap model in which $\hat{\delta} = 0$. We will approximate the decision rules for output and inflation as flexible functions $\hat{\pi}(\mathcal{S}_t;\Theta)$ and $\hat{y}(\mathcal{S}_t;\Theta)$ that are parameterized by a coefficient vector $\Theta = \left(\theta_\pi, \theta_y\right)'$.

1. Start with a guess for the functional approximation Θ

2. Pick a point $\mathcal{S} \subset [1 - 10\sigma^z, 1 + 10\sigma^z] \times [1 - 10\sigma^g, 1 + 10\sigma^g] \times [1 - 10\sigma^v, 1 + 10\sigma^v] \times \{0, 1\}$

3. Compute the approximated policy functions:

$$\hat{\pi}(\mathcal{S};\Theta) = \begin{cases} f_\pi^1(\mathcal{S};\Theta) & \text{if } s_t = 1 \\ f_\pi^0(\mathcal{S};\Theta) & \text{if } s_t = 0 \end{cases}, \quad \hat{y}(\mathcal{S}_i;\Theta) = \begin{cases} f_y^1(\mathcal{S}_t;\Theta) & \text{if } s_t = 1 \\ f_y^0(\mathcal{S}_t;\Theta) & \text{if } s_t = 0 \end{cases}$$

4. Evaluate D.5 to obtain the implicitly approximated function for $\hat{R}(\mathcal{S};\Theta)$

5. Using the same approximated functions in step 2, compute:

$$\mathcal{R}_y(\mathcal{S};\Theta) = \hat{y}(\mathcal{S}) - \iiiint \left[\tau\mathcal{D}\left(\hat{y}(\mathcal{S}') - \hat{g}'\right) - \mathcal{D}\left(\hat{R}(\mathcal{S}) - \hat{\pi}(\mathcal{S}') - \hat{z}'\right) + \hat{g}\right] dF(\hat{z}')dF(\hat{g}')dF(\hat{v}')dF(s')$$
$$\mathcal{R}_\pi(\mathcal{S};\Theta) = \hat{\pi}(\mathcal{S}) - \iiiint \left[\beta\hat{\pi}(\mathcal{S}') + \lambda\hat{v} + \kappa(\hat{y}(\mathcal{S}) - \hat{g}) + \frac{\kappa}{\eta\tau}\hat{y}(\mathcal{S})\right] dF(\hat{z}')dF(\hat{g}')dF(\hat{v}')dF(s')$$

6. To obtain the correct coefficients we minimize the following objective function:

$$\min_{\Theta} = \sum_{i=1}^{M} \left[\mathcal{R}_y(\mathcal{S}_i;\Theta) + \mathcal{R}_\pi(\mathcal{S}_i;\Theta)\right]^2 \tag{C.6}$$

C.6. Measurement equations

In the data, we measure output and consumption growth in quarter-on-quarter percentages, while we measure inflation annualized percentages. To link the observed series to the model counterparts, we define the following system of measurement equations:

$$\begin{aligned} \Delta y_t^Q &= 100\log(\gamma) + 100\left(\hat{y}_t - \hat{y}_{t-1} + \hat{z}_t\right) \\ \Delta c_t^Q &= 100\log(\gamma) + 100\left(\hat{c}_t - \hat{c}_{t-1} + \hat{z}_t\right) \\ \pi_t^A &= 400\log\left(\tilde{\pi}\right) + 400\hat{\pi}_t \end{aligned}$$

C.7. Prior distributions

Table 3 lists the priors used for estimation of the DSGE model of Section 4, including information on the marginal prior distributions for the estimated parameters. Under the prior, we assume that all estimated parameters are distributed independently which implies that the joint prior distribution can be computed from the product of the marginal distributions.

C.8. Posterior sampler

We can solve the log-linearized system of equations of Section 4 using standard perturbation techniques. As a result, the likelihood function can be evaluated with the Kalman filter. We generate draws from the posterior distribution using the random walk Metropolis algorithm

Table 3: *Prior Distribution of DSGE parameters*

Parameters	Description	Distribution	P(1)	P(2)
ρ_g	Persistence gov. spending shock	$\mathcal{B}$	0.6	0.2
ρ_ν	Persistence markup shock	$\mathcal{B}$	0.6	0.2
ρ_z	Persistence technology. growth shock	$\mathcal{B}$	0.4	0.1
σ_g	Std dev. gov. spending shock	$\mathcal{IG}$	0.004	Inf
σ_ν	Std dev. markup shock	$\mathcal{IG}$	0.004	Inf
σ_z	Std dev. markup shock	$\mathcal{IG}$	0.004	Inf
σ_ζ	Std dev. sunspot shock	$\mathcal{IG}$	0.004	Inf
$corr(\epsilon_z, \epsilon_\zeta)$	Correl. sunspot	$\mathcal{U}$	0	0.5774
$corr(\epsilon_\nu, \epsilon_\zeta)$	Correl. sunspot	$\mathcal{U}$	0	0.5774
$corr(\epsilon_g, \epsilon_\zeta)$	Correl. sunspot	$\mathcal{U}$	0	0.5774

Notes: $\mathcal{G}$ is Gamma distribution; $\mathcal{B}$ is Beta distribution; $\mathcal{IG}$ is Inverse Gamma distribution; and $\mathcal{U}$ is Uniform distribution. P(1) and P(2) are mean and standard deviations for Beta, Gamma, and Uniform distributions. .

(RWM) described in An and Schorfheide (2007). We scale the covariance matrix of the proposal distribution in the RWM algorithm to obtain an acceptance rate of approximately 60%. For posterior inference we generated 50,000 draws from the posterior distribution and discard the first 25,000 draws. Table 4 summarizes key moments of the posterior distribution.

C.9. Impulse responses

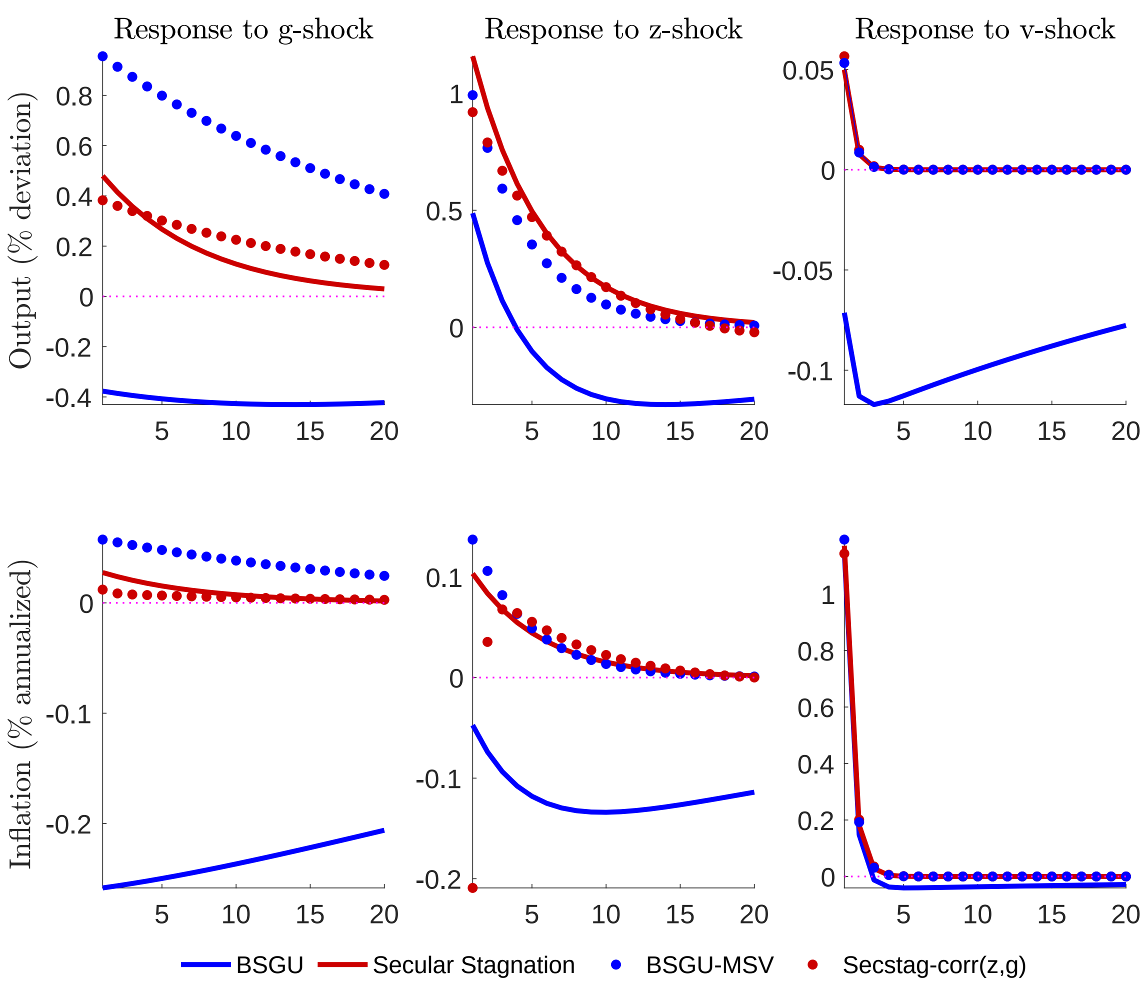
Figure 9 shows impulse responses for the estimated DSGE model of Section 4. For all the model specifications we compute the impulse responses at estimated posterior mean shown in Table 4.

Table 4: *Posterior DSGE estimates*

Parameters	Description	BSGU	Secular Stagnation
ρ_g	Persistence gov. spending shock	0.9605	0.8432
		[0.9273, 0.9949]	[0.7832, 0.9076]
ρ_ν	Persistence markup shock	0.1824	0.1655
		[0.039, 0.3201]	[0.0469, 0.271]
ρ_z	Persistence technology. growth shock	0.604	0.6121
		[0.3503, 0.8636]	[0.3459, 0.9062]
σ_g	Std dev. gov. spending shock	0.0045	0.0045
		[0.0038, 0.0051]	[0.0038, 0.0051]
σ_ν	Std dev. markup shock	0.0172	0.0134
		[0.0142, 0.0202]	[0.011, 0.0156]
σ_z	Std dev. markup shock	0.005	0.0061
		[0.0023, 0.0078]	[0.0024, 0.0098]
σ_ζ	Std dev. sunspot shock	0.0031	-
		[0.0026, 0.0036]	-
$corr(\epsilon_z, \epsilon_\zeta)$	Correl. sunspot & tech. growth shock	0.0025	-
		[-0.167, 0.1759]	-
$corr(\epsilon_\nu, \epsilon_\zeta)$	Correl. sunspot & markup shock	0.9273	-
		[0.8812, 0.9774]	-
$corr(\epsilon_g, \epsilon_\zeta)$	Correl. sunspot & gov.spending shock	-0.2681	-

Notes: The estimation sample is 1998:Q1 - 2012:Q4. For each model we report posterior means and 90% highest posterior density intervals in square brackets. All posterior statstics are based based on the last 25,000 draws from a RWMH algorithm, after discarding the first 25,000 draws. Entries with '-' indicate the parameter is not estimated in that specification.

Figure 9: *Secular Stagnation vs BSGU*



Notes: Impulse responses are computed at the posterior mean of the estimated parameters. For the BSGU-MSV solution we use the policy functions computed in Section C.4. For the secular stagnation model with cross-shock correlations the estimated posterior mean of the additional parameters are: $\mathcal{M}_s^{corr}$ are $\rho(\varepsilon_z, \varepsilon_g) = -0.7109$, $\rho(\varepsilon_z, \varepsilon_v) = -0.2662$, $\rho(\varepsilon_g, \varepsilon_v) = 0.1978$

C.10. Model simulation and data

Table 5 shows key correlations obtained from model simulated data and compares them with different model specifications.

Table 5: *Key moments in data and models*

	$\mathcal{M}_b$	$\mathcal{M}_s$	$\mathcal{M}_b^{msv}$	$\mathcal{M}_s^{corr}$	**Data**
corr(Δy_t,π_t)	-0.02	0.12	-0.23	-0.10	**-0.15**
$\sigma(\pi_t)$	2.25	1.20	0.82	1.19	**1.15**
$\sigma(\Delta y_t)$	1.12	1.58	1.65	1.25	**1.00**

Notes: The data sample is 1998:Q1 - 2012:Q4. We remove the the periods 1997Q2, and 2008Q4 in which the change in the GDP deflator rose above 4%. The first corresponds to the hike in VAT from 3% to 5% in April of 1997, and the second corresponds to the demand effects on the GDP deflator dues to the sharp decline in oil prices in the second half of 2008.

D. Simple model with downward nominal wage rigiidty

We now derive the results presented in Section 2 with wage-setting frictions and inelastic labor instead of price-rigidities and elastic labor. Our model is a variant of the downward-nominal wage rigidity apparatus introduced by Schmitt-Grohé and Uribe (2017).

Time is discrete and there is no uncertainty. Suppose the representative agent supplies labor $\bar{h} = 1$ inelastically and maximizes the following utility function choosing consumption good C_t and one-period (real) risk-free government bonds b_t:

$$\max_{\{C_t, b_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \theta_t \left[\log C_t\right]$$
$$\theta_0 = 1;$$
$$\theta_{t+1} = \hat{\beta}(\tilde{C}_t)\theta_t \forall t \geq 0$$

where θ_t is an endogenous discount factor (Uzawa, 1968; Epstein and Hynes, 1983; Obstfeld, 1990), C_t is consumption, $\tilde{C}_t$ is average consumption that the household takes as given, and h_t is hours. For tractability, we assume a linear functional form for $\hat{\beta}(\cdot) = \delta_t \beta C_t$, where $0 < \beta < 1$ is a parameter, and $\delta_t > 0$ are exogenous shocks to the discount factor. The household earns wage income $W_t h_t$, interest income on past bond holdings of risk-free government bonds b_{t-1} at gross nominal interest rate R_{t-1}, dividends Φ_t from firms' ownership and makes transfers T_t to the government. Π_t denotes gross inflation rate. The period by period (real) budget constraint faced by the household is given by

$$C_t + b_t = \frac{W_t}{P_t} h_t + \frac{R_{t-1}}{\Pi_t} b_{t-1} + \Phi_t + T_t$$

An interior solution to household optimization yields the Euler equation:

$$1 = \beta(\tilde{C}_t)\mathbb{E}_t \left[\frac{C_t}{C_{t+1}} \frac{R_t}{\Pi_{t+1}} \right]$$

In equilibrium, individual and average per consumption are identical, i.e. $C_t = \tilde{C}_t$. The Euler equation simplifies to:

$$1 = \delta_t \beta C_t \mathbb{E}_t \left[\frac{C_t}{C_{t+1}} \frac{R_t}{\Pi_{t+1}} \right]$$

Consumption goods are produced by competitive firms with labor as the only input using the technology

$$Y_t = F(h_t) = h_t^{\alpha}, \quad \text{where } 0 < \alpha < 1$$

These firms set price of the final good P_t to equate marginal product of labor to the marginal cost.

$$F'(h_t) = \frac{W_t}{P_t}$$

We introduce a very stylized form of downward nominal wage rigidity (following Schmitt-Grohé and Uribe 2017):

$$W_t \geq (1 - \kappa + \kappa(1 - u_t)^{\alpha}) W_{t-1} \equiv \tilde{\gamma}(u_t) W_{t-1}$$

where $\kappa > 0$, and $u_t \equiv 1 - \frac{h_t}{\bar{h}}$ is involuntary unemployment. This downward rigidity implies that employment cannot exceed the total labor supply in the economy i.e. $h_t \leq 1$. We further assume that the following slackness condition holds:

$$(\bar{h} - h_t)(W_t - \tilde{\gamma}(u_t) W_{t-1}) = 0$$

We close the model by assuming a government that balances budget,

$$b_t + T_t = \frac{R_{t-1}}{\Pi_t} b_{t-1}$$

and a monetary authority that sets nominal interest rate on the net zero supply of nominal risk-free one-period bonds using the following Taylor rule

$$R_t = \max\{1, (1+r^*)\Pi_t^{\phi_\pi}\}$$

where $(1+r^*) \equiv \frac{1}{\delta_t \beta}$ is the natural interest rate, and $\phi_\pi > 1$.[24] The zero lower bound (ZLB) constraint on the short-term nominal interest rate introduces an additional nonlinearity in the policy rule. Finally, we assume that the resource constraints hold in the aggregate:

$$C_t = Y_t, \quad \text{and} \quad b_t = 0.$$

D.1. Equilibrium

Let $w_t \equiv \frac{W_t}{P_t}$ denote the real wage. The competitive equilibrium is given by the sequence of seven endogenous processes $\{C_t, Y_t, R_t, \Pi_t, h_t, w_t, u_t\}$ that satisfy the conditions (D.1) - (D.7) for a given exogenous sequence of process $\{\delta_t\}_{t=0}^{\infty}$ and the initial condition w_{-1}:

$$1 = \delta_t \beta C_t \mathbb{E}_t \left[\frac{C_t}{C_{t+1}} \frac{R_t}{\Pi_{t+1}} \right] \tag{D.1}$$

$$Y_t = h_t^{\alpha}. \tag{D.2}$$

$$\alpha h_t^{\alpha-1} = w_t \tag{D.3}$$

$$h_t \le \bar{h}, \quad w_t \ge (1-\kappa+\kappa h_t^{\alpha}) \frac{w_{t-1}}{\Pi_t}, \quad (\bar{h}-h_t)\left(w_t - (1-\kappa+\kappa h_t^{\alpha}) \frac{w_{t-1}}{\Pi_t} \right) = 0 \tag{D.4}$$

$$u_t = 1 - \frac{h_t}{\bar{h}} \tag{D.5}$$

$$R_t = \max\{1, (1+r^*)\Pi_t^{\phi_\pi}\}\} \tag{D.6}$$

$$Y_t = C_t \tag{D.7}$$

where the exogenous sequence of natural interest rate is given by $1 + r_t^* \equiv \frac{1}{\delta_t \beta}$.

D.2. Non-stochastic steady state

In the steady state, we can simplify the system of equations to an aggregate demand block and an aggregate supply block.

Aggregate Demand (AD) is a relation between output and inflation and is derived by combining the Euler equation and the Taylor rule. Mathematically, the AD curve is given

[24] The natural interest rate is defined as the real interest rate on one-period government bonds that would prevail in the absence of nominal rigidities.

by

$$Y_{AD} = \frac{1}{\beta\delta} \begin{cases} \frac{1}{(1+r^*)\Pi^{\phi\pi - 1}}, & \text{if R } > 1, \\ \Pi, & \text{if R } = 1 \end{cases} \tag{D.8}$$

When ZLB is not binding, the AD curve has a strictly negative slope, and it becomes linear and upward sloping when the nominal interest rate is constrained by the ZLB. The kink in the aggregate demand curve occurs at the inflation rate at which monetary policy is constrained by the ZLB: $\Pi_{kink} = \left(\frac{1}{(1+r^*)}\right)^{\frac{1}{\phi\pi}}$. When $1 + r^* > 1$, the kink in the AD curve occurs at an inflation rate below 1. For the natural interest rate to be positive, the patience parameter must be low enough i.e. $\delta < \frac{1}{\beta}$. The dashed red line in panel a) of Figure 10 plots the aggregate demand curve with a positive natural rate.

Aggregate Supply (AS): Because of the assumptions of downward nominal wage rigidity and capacity constraints on production, the AS curve features a kink at full employment level of output and gross inflation rate equal to one. When inflation rate is less than one, the downward wage rigidity constraint becomes binding. As a result, inflation cannot fall to completely adjust any demand deficiency and firms layoff workers. The aggregate supply curve can be summarized by:

$$Y_{AS} \leq 1, \quad \Pi \geq (1-\kappa) + \kappa Y_{AS}, \quad (Y_{AS} - 1)\left(1 - (1 - \kappa + \kappa Y_{AS})\frac{1}{\Pi}\right) = 0 \tag{D.9}$$

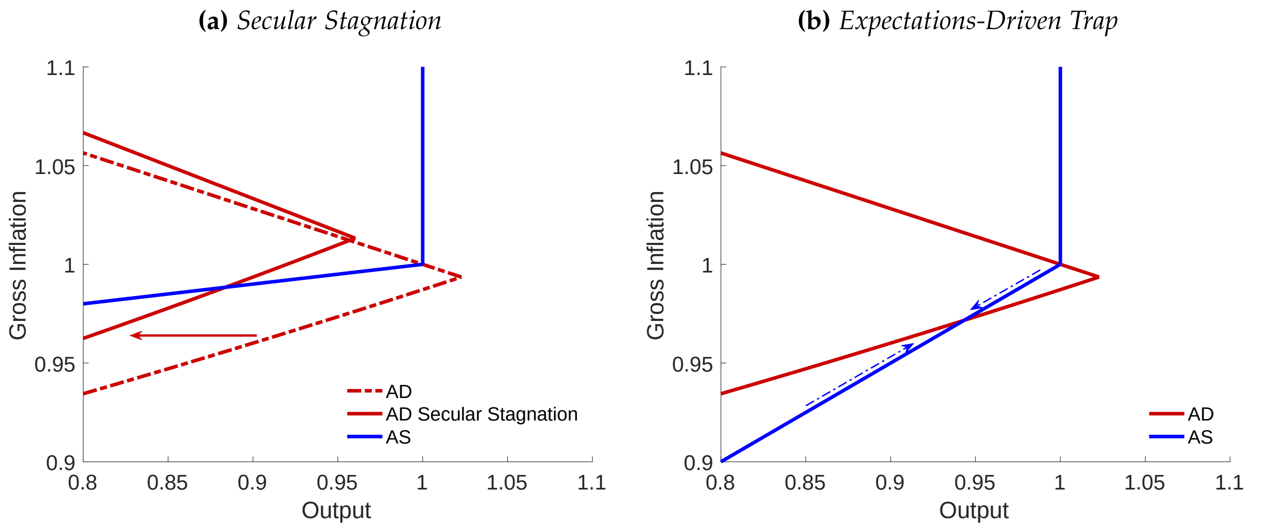
When $h = \bar{h} \equiv 1$, $\Pi \geq 1$. The AS curve is a vertical line at full employment. For $h < 1$, $\Pi = (1 - \kappa + \kappa h^{\alpha})$. The AS curve is linear and upward sloping with slope=κ for $y < 1$. The kink in the AS curve occurs at the coordinate $Y = 1, \Pi = 1$. Because of this assumed linear aggregate supply curve under deflation, the degree of nominal rigidity κ also determines the lower bound on inflation $(1 - \kappa)$. The solid blue line in both panels of Figure 10 plots the aggregate supply curve.

Note that the equilibrium conditions are similar to those presented in Section 2, with the major exception being that the steady state AS graph is vertical at $h = 1$ due to the upper bound on labor endowment. One can derive similar results as in Section 2. We only note the following proposition to complete proofs for statements regarding robust minimum wage policy in Section 7.2.1.

Proposition 5. (Minimum wage policy): Let $0 < \delta < \frac{1}{\beta}$. A minimum income policy that installs a lower bound $1 - \kappa$ on nominal wage growth can preclude the expectations trap.

Proof. The downward sloping portion of AD curve goes through $Y = 1$, $\Pi = 1$, and so

Figure 10: *Steady-State Representation*



does the vertical portion of the AS curve. When $0 < \delta < \frac{1}{\beta}$, $1 + r^* > 1$. The kink in the AD curve occurs at inflation rate below 1. Thus, there always exists an intersection between the AS and the AD at $Y = 1$ and $\Pi = 1$. To show that there does not exist another equilibrium, note that, AD is linear and upward sloping when ZLB is binding and AS is also linear and upward sloping for gross inflation below 1. Furthermore for $\Pi_{kink} \leq \Pi < 1$, $Y_{AD} > 1 > Y_{AS}$. And for $\Pi = 1 - \kappa$, $Y_{AS} = 0 < Y_{AD}$. Thus there does not exist a steady state with zero nominal interest rate.

$\square$