Due: 26 January 2018

NAME: ______________________________

Problem 1. Give a quantitative population with 6 members so that $\mu = 0$ and $\sigma = 1$.

There are several correct answers, one easy one is to make sure that every data point is 1 unit away from 0, and that we have the same number on both sides of 0. That is: $\{-1, -1, -1, 1, 1, 1\}$.

Problem 2. Give a quantitative population with 6 members so that $\mu = 1$ and $\sigma = 0$.

A population with $\sigma = 0$ means that there is no variation, that is, the average distance from the data to the mean is 0. So all the data must be ON the mean. There is exactly one population that satisfies this: $\{1, 1, 1, 1, 1, 1\}$.

Problem 3. A study of the flight times of 6 flights from Dulles to LAX had an median of 263 minutes.

258	257	258	???	272	285

Compute the missing number, please show your work.

We know that the median is the middle value, there are three values below and two values above, to get 263, we need to have a third value above that is the same as the distance from 258 to 263. That is, $x = 268$.

Problem 4. A study of heights of students in a class (rounded up to the nearest inch) is:

$$\{68, 68, 67, 63, 74, 72, 73, 66, 69, 70, 78, 64, 66, 68, 104\}.$$

Compute the following descriptive statistics (include units): median, mode, sample mean, sample variance, the quartiles Q_1, Q_2, and Q_3. Then graph the data as a dot plot, stemplot, and and a boxplot.

First reorder the data set.

$$\{63, 64, 66, 66, 67, 68, 68, 68, 69, 70, 72, 73, 74, 78, 104\}$$

There are 15 data points. Therefore we have that $\ell_{Q_1} = \lceil 15\frac{25}{100} \rceil = 4$,$\ell_{Q_2} = \lceil 15\frac{50}{100} \rceil = 8$, and $\ell_{Q_3} = \lceil 15\frac{75}{100} \rceil = 12$. Therefore our five number summary according to rank order is $63, 66, 68, 73, 104$ in inches. The mode is 68 in. The (sample) mean is calculated by $\bar{x} = \frac{1}{15}\sum x_i = 71\bar{3}$ in. and the sample variance is $S^2 = \frac{1}{14}\sum(x - \bar{x})^2 \approx 97.2381 in^2$.
For the box-plot we need to identify outliers. The IQR is $73 - 66 = 7$. Therefore the lower fence is $66 - 1.5(7) = 55.5$ and the upper fence is $73 + 1.5(7) = 83.5$. Therefore we have an outlier with 104.
Stem-and-leaf of C1 $N = 15$
Leaf Unit = 1.0

6	346678889
7	02348
8	
9	
10	4

Problem 5. Let $A \setminus B$ denote the set of outcomes contained in A that are not in B. Which of the following is another way of writing $A \setminus B$:

$B^c \cup A^c \qquad (A \cap B)^c \cup B \qquad A \cap (B^c) \qquad A \cup (A^c \cap B) \cup A^c.$

Problem 6. Suppose $P(A) = .4$, $P(B) = .3$ and $P(A \cap B) = .3$, compute $P(A \cup B)$.

We have the relation $|A \cap B| + |A \cup B| = |A| + |B|$ and the definition $P(A) = \frac{|A|}{|\Omega|}$.

$$\begin{aligned}
|A \cap B| + |A \cup B| &= |A| + |B| \\
\frac{|A \cap B|}{|\Omega|} + \frac{|A \cup B|}{|\Omega|} &= \frac{|A|}{|\Omega|} + \frac{|B|}{|\Omega|} \\
P(A \cap B) + P(A \cup B) &= P(A) + P(B) \\
.3 + P(A \cup B) &= .4 + .3
\end{aligned}$$

We have $P(A \cup B) = .4$.

Problem 7. Using the previous problem's events and probabilities what can we say about A and B?

If every outcome has positive probability, then $B \subset A$. That is we have $A \cup B = A$ and $A \cap B = B$. When every outcome does not have positive probability, then we could have $B \backslash A \neq \emptyset$ but $P(B \backslash A) = 0$. Hence, there is nothing we can say.

Problem 8. Out of 40 basketball players trying to make a team, 5 will be chosen as starters. There are 26 seniors trying out and the remaining players are juniors. The coach wants 2 seniors and 3 juniors to be starters. In addition the coach wants a senior starter to be the team captain. In how many ways can the coach select the starters with a captain from the 40 players who try out?

There are 26 choices for the captain position or $\binom{26}{1} = 26$. The remaining seniors vie for the last spot or $\binom{25}{1} = 25$. Then the 3 juniors are from a pool of 14. Therefore we have $\binom{14}{3}$ ways to pick the juniors. Hence the number of combinations for the team is $\binom{26}{1}\binom{25}{1}\binom{14}{3} = 26(25)\frac{14(13)12}{3(2)} = 26(25)14(13)2$.

Problem 9. A poker hand consists of five cards. If a hand contains three cards of the same denomination, but the other two are different from the first three and different from each other, we say this hand is a "three-of-a-kind." What is the probability of being dealt a "three-of-a-kind?" Write your answer in terms of binomial coefficients and indicate what each is counting.

To select a poker hand without order mattering we have $\binom{52}{5}$ ways. To select a denomination for the first three cards we have $\binom{13}{1} = 13$ ways. And to select the three suits for those first three cards we have $\binom{4}{3} = 4$ ways. For the next two hards we have to select two different denominations or $\binom{12}{2}$. And for each card we need to select its suit $\binom{4}{1} = 4$.
Thus we have

$$P(\text{“three-of-a-kind”}) = \frac{\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}\binom{4}{1}}{\binom{52}{5}}.$$

Problem 10. Given $P(A|B) = .5$, $P(B|A) = .4$ and $P(B|A^c) = .3$, determine $P(A)$.

$$\begin{aligned}
P(B) &= P(B|A)P(A) + P(B|A^c)P(A^c) \\
&= .4P(A) + .3(1 - P(A)) \\
&= .3 + .1P(A) \\
P(A|B^c)P(B^c) &= P(A \cap B^c) \\
&= P(B^c|A)P(A) \\
&= (1 - P(B|A))P(A) \\
&= .6P(A) \\
P(A) &= P(A|B)P(B) + P(A|B^c)P(B^c) \\
&= .5(.3 + .1P(A)) + .6P(A)) \\
&= .15 + .65P(A) \\
P(A) &= \frac{.15}{.35} = \frac{3}{7}
\end{aligned}$$

Alternatively we have

$$\begin{aligned}
P(A|B) &= \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)} \\
.5 &= \frac{.4P(A)}{.4P(A) + .3(1 - P(A))} \\
.2P(A) + .15 - .15P(A) &= .4P(A) \\
.15 &= .35P(A) \\
P(A) &= \frac{3}{7}
\end{aligned}$$