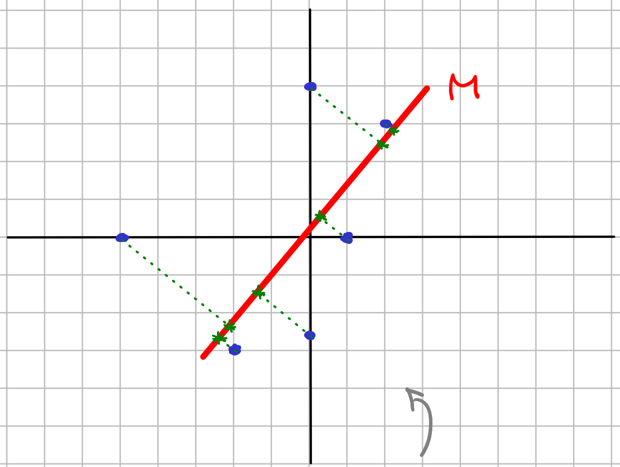
Ejercicio: sean $p_1 = \begin{pmatrix}1\\0\end{pmatrix}$, $p_2 = \begin{pmatrix}-5\\0\end{pmatrix}$, $p_3 = \begin{pmatrix}0\\4\end{pmatrix}$

$p_4 = \begin{pmatrix}-2\\-3\end{pmatrix}$, $p_5 = \begin{pmatrix}0\\-\sqrt{7}\end{pmatrix}$, $p_6 = \begin{pmatrix}2\\3\end{pmatrix}$

1. encontrar la recta por la origen M t.q.

$$\sum_{i=1}^{6} \| p_i - \mathbb{P}_M p_i \|^2 \leq \sum_{i=1}^{6} \| p_i - \mathbb{P}_{M'} p_i \|^2 \quad \forall \text{ recta } M' \text{ por la orig}$$

2. calcular $\sum_{i=1}^{6} \| p_i - \mathbb{P}_M p_i \|^2$



- $\bullet$ = puntos p_i
- $*$ = puntos $\mathbb{P}_M p_i$
- $\vdots$ = $p_i - \mathbb{P}_M p_i$

desde el teorema de Eckhart-Young

$$A = \begin{pmatrix} 1 & -5 & 0 & -2 & 0 & 2 \\ 0 & 0 & 4 & -3 & -\sqrt{7} & 3 \end{pmatrix} = U \Sigma V^t$$

- la matriz de rg $= 1$ $\hookrightarrow$ recta cuyas columnas están más cercanas a las de A es $A_1 = \sigma_1 U^{(1)} \otimes V^{(1)}$

$$\begin{pmatrix} | \\ U^{(1)} \\ | \end{pmatrix} (- V^{(1)} -)$$

los vectores columna de A_1 tienen la dirección de $U^{(1)}$

$\Rightarrow$ $\boxed{\text{la recta } M \text{ dada por el teorema es la que tiene dirección } U^{(1)}}$

- la suma de los cuadrados de las distancias entre los puntos y su proyección sobre M es

$$\sum_{i=1}^{6} \| p_i - \mathbb{P}_M p_i \|^2 = \| A - A_1 \|_F^2 = \sigma_2^2$$

Ejemplo $\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} (v_1 \; v_2 \; v_3) = \begin{pmatrix} u_1 v_1 & u_1 v_2 & u_1 v_3 \\ u_2 v_1 & u_2 v_2 & u_2 v_3 \end{pmatrix} = \begin{pmatrix} | & | & | \\ v_1 U & v_2 U & v_3 U \\ | & | & | \end{pmatrix}$

$\underset{U}{\parallel}$

$\Rightarrow$ para resolver 1. y 2. es suficiente encontrar $U^{(1)}$, σ_2

$$AA^t = U\Sigma V^t V \Sigma^t U^t = U\Sigma\Sigma^t U^t \text{ : diagonalización}$$

$$= \begin{pmatrix} 1 & -5 & 0 & -2 & 0 & 2 \\ 0 & 0 & 4 & -3 & -\sqrt{7} & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -5 & 0 \\ 0 & 4 \\ -2 & -3 \\ 0 & -\sqrt{7} \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 34 & 12 \\ 12 & 41 \end{pmatrix}$$

$$p(\lambda) = (34-\lambda)(41-\lambda) - 144 = \lambda^2 - 75\lambda + \underbrace{34\cdot 41 - 144}_{1250} \Rightarrow \lambda = \begin{cases} 50 \\ 25 \end{cases}$$

$$\Rightarrow \sigma_2 = 5$$

$$AA^t - 50I = \begin{pmatrix} -16 & 12 \\ 12 & -9 \end{pmatrix} \longrightarrow \begin{matrix} 16x = 12y \\ 12x = 9y \end{matrix} \Rightarrow U^{(1)} = \begin{pmatrix} 3/5 \\ 4/5 \end{pmatrix}$$

$$M : y = 4/3\, x$$

$\circ\!\sim\!\circ$

Ejercicio 2 hoja 6

$$A_1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} = U\Sigma V^t, \quad A_1^t A_1 = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = V\Sigma^t\Sigma V^t$$

$$\lambda = \begin{cases} 3 \\ 1 \end{cases}$$

$$\Sigma = \begin{pmatrix} 3 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \Leftarrow$$

$$(A^tA - 3I) = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \rightsquigarrow V^{(1)} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} \Rightarrow V^{(2)} = \begin{pmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

$$AV = U\Sigma V^t V = U\Sigma \quad : \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 2 & 0 \\ 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\Rightarrow U = \left(\begin{array}{cc|c} 2/\sqrt{6} & 0 & \\ 1/\sqrt{6} & 1/\sqrt{2} & U^{(3)} \\ 1/\sqrt{6} & -1/\sqrt{2} & \end{array} \right), \quad U^{(3)} = \frac{1}{\sqrt{3}} \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

compleción de BON

$$\mathbb{P}_{\mathrm{Ran}(A_1)} = \begin{pmatrix} | & | \\ U^{(1)} & U^{(2)} \\ | & | \end{pmatrix} \begin{pmatrix} - & U^{(1)} & - \\ - & U^{(2)} & - \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 2 & 3 & 0 \\ 2 & -3 & 0 \end{pmatrix}, \qquad A_2 A_2^t = \begin{pmatrix} 13 & -5 \\ -5 & 13 \end{pmatrix}, \qquad \lambda = \begin{array}{l} \nearrow 18 \\ \searrow 8 \end{array}$$

$$\left(A_2 A_2^t - 18\, I\right) = \begin{pmatrix} -5 & -5 \\ -5 & -5 \end{pmatrix} \rightsquigarrow \quad U^{(1)} = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}, \quad U^{(2)} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}, \qquad \Sigma = \begin{pmatrix} \sqrt{18} & 0 & 0 \\ 0 & \sqrt{8} & 0 \end{pmatrix}$$

$$A^t U = V \Sigma^t U^t U = V \Sigma^t = \frac{1}{\sqrt{2}} \begin{pmatrix} 2 & 2 \\ 3 & -3 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 4/\sqrt{2} \\ 6/\sqrt{2} & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Rightarrow \quad V = \left(\begin{array}{cc|c} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{array}\right), \qquad \mathbb{P}_{\mathrm{Ker}(A_2)} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$