PROBLEM SET 2: Predicate logic.

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EXERCISE 1.

Considering the ontology:

Constants: Thelma, Louise (people)

Variables: p (people), x (objects)

Predicates:

Name	Arity	Description (including the type of arguments)
Likes	2	Likes(p,x) evaluates to "True" if and only if p likes the type of food x.
Cooked	1	Cooked(x) evaluates to "True" if and only if x is cooked (not raw).
Food	1	Food(x) evaluates to "True" if and only if x is a type of food.
Fish	1	Fish(x) evaluates to "True" if and only if x is a type of fish.

Now we write in predicate logic the following statements:

I. "There are some types of food that Thelma dislikes but Louise likes":

$$\exists x[Food(x) \wedge \neg Likes(Thelma, x) \wedge Likes(Louise, x)]$$

II. "Louise dislikes all types of food that Thelma likes (and possibly others)":

$$\forall x[(Food(x) \wedge Likes(Thelma, x)) \implies \neg Likes(Louise, x)]$$

III. "Thelma likes all types of food except raw fish (which she dislikes)":

$$\forall x\{Food(x) \implies [Likes(Thelma, x) \iff (\neg Fish(x) \vee Cooked(x))\}$$

EXERCISE 2.

Ontology:

	Symbol	Interpretation
Constant	0	Integer value of 0
Variables	x, y, m	Integers
Predicates	$Equal(x, y)$	True if and only if $x = y$
	$NeutralSum(x)$	True if and only if x is the neutral element of the sum
Functions	$Sum(x, y)$	The result of $x + y$
	$Negative(x)$	$-x$

NOTE: subtracting one element is the same as adding its negative value, therefore $(x - y)$ would be $Sum(x, Negative(y))$.

Formulate the following statements as WFFs in predicate logic.

a) "The result of subtracting two integers is zero if and only if these integers are equal":

$$\forall x, y[Equal(Sum(x, Negative(y)), 0) \iff Equal(x, y)]$$

b) "An integer number is the neutral element with respect to the sum if and only if when subtracted from any integer the result is equal to this second integer":

$$\forall x\{NeutralSum(x) \iff \forall y[Equal(Sum(y, Negative(x)), y)]\}$$

c) "For any integer there is another integer such that the result of subtracting the second integer from the first one yields the neutral element":

$$\forall x \exists y NeutralSum(Sum(x, Negative(y)))$$

d) "The neutral element with respect to the sum is unique and equal to zero":

$$\exists x(NeutralSum(x) \iff Equal(x, 0))$$

EXERCISE 3.

Let us consider a series of elections in which only three voters can cast votes according to their preferences.

- Variables: x, y, z, ... (candidates)
  p, q, r, ... (voters)
- Predicates: - P^3 "Prefers": P(p,x,y) evaluates to True if voter p prefers candidate x to candidate y, False otherwise.
  - B^2 "Beats": B(x,y) evaluates to True if candidate x beats candidate y in a two-candidate election, False otherwise.

Translate the following knowledge base into well-formed formulas in predicate logic. The predicate "Equal" (E^2) can be used if needed.

(a) "Predicate "Beats" is antisymmetric: If a candidate beats another one, the second one does not beat the first one":

$$\forall x, y \ \{[\neg E(x,y)] \Longrightarrow [B(x,y) \iff \neg B(y,x)]\}$$

(b) "Predicate "Prefers" is transitive: If a voter prefers a candidate to another one, and also prefers this second candidate to a third one, she prefers the first candidate to the third one."

$$\forall x, y, z, p \ \{[\neg E(x,y) \wedge \neg E(x,z) \wedge \neg E(y,z)] \Longrightarrow [[P(p,x,y) \wedge P(p,y,z)] \Longrightarrow P(p,x,z)]\}$$

(c) "In a two-candidate election, if at least two different voters prefer one candidate to another one, the first one beats the second one."

$$\forall x, y \{\exists p, q [P(p,x,y) \wedge P(q,x,y) \wedge \neg E(p,q)] \Longrightarrow B(x,y)\}$$

(d) "Predicate "Beats" is not transitive."

$$\exists x, y, z \ \{[B(x,y) \wedge B(y,z)] \wedge \neg[B(x,z)]\}$$

(In this case, we do not contemplate the possibility of the two candidates x and z having the exact same amount of votes, so either x beats z or vice versa.)