$\boxed{10.2}$ $f:\mathbb{R}^2 \to \mathbb{R}^2$. Matriz de $f$: $F = \begin{pmatrix} 1 & 4 \\ -1/2 & 4 \end{pmatrix}$. Buscamos los autovalores $\lambda$:

(a) $\begin{vmatrix} 1-\lambda & 4 \\ -1/2 & 4-\lambda \end{vmatrix} = 0 \Leftrightarrow \lambda^2 - 5\lambda + (4+2) = 0 \Leftrightarrow \lambda = \dfrac{5 \pm \sqrt{25-24}}{2} = \begin{cases} 3 = \lambda_1 \\ 2 = \lambda_2 \end{cases}$

Con $\lambda_1$: $\begin{pmatrix} -2 & 4 \\ -1/2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ un autovector: $\vec{x}_1 = \begin{pmatrix} 2 \\ +1 \end{pmatrix}$.

Con $\lambda_2$: $\begin{pmatrix} -1 & 4 \\ -1/2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ un autovector: $\vec{x}_2 = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

$\boxed{\beta = \{\vec{x}_1, \vec{x}_2\}}$

$$(\mathbb{R}^2, \beta) \xrightarrow[C]{I} (\mathbb{R}^2, \mathcal{B}) \xrightarrow[F]{f} (\mathbb{R}^2, \mathcal{B}) \xrightarrow[C^{-1}]{I} (\mathbb{R}^2, \beta).$$

$M = C^{-1} \cdot F \cdot C$. $C = \begin{pmatrix} 2 & 4 \\ 1 & 1 \end{pmatrix}$, $C^{-1} = \begin{pmatrix} -1 & +4 \\ +1 & -2 \end{pmatrix}\frac{1}{2}$.

$\Rightarrow M = \frac{1}{2}\begin{pmatrix} -1 & 4 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} 1 & 4 \\ -1/2 & 4 \end{pmatrix}\begin{pmatrix} 2 & 4 \\ 1 & 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -3 & 12 \\ 2 & -4 \end{pmatrix}\begin{pmatrix} 2 & 4 \\ 1 & 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 6 & 0 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$

(b) $M = C^{-1} F C \Rightarrow F = C M C^{-1} \Rightarrow F^{15} = C M^{15} C^{-1} = C \begin{pmatrix} 3^{15} & 0 \\ 0 & 2^{15} \end{pmatrix} C^{-1} =$

$= F^{15} = \begin{pmatrix} 2 & 4 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 3^{15} & 0 \\ 0 & 2^{15} \end{pmatrix} C^{-1} = \begin{pmatrix} 2\cdot 3^{15} & 2^{17} \\ 3^{15} & 2^{15} \end{pmatrix}\begin{pmatrix} -1 & 4 \\ 1 & -2 \end{pmatrix}\frac{1}{2} = \frac{1}{2}\begin{pmatrix} -2\cdot 3^{15} + 2^{17} & 2^3 3^{15} - 2^{18} \\ -3^{15} + 2^{15} & 4\cdot 3^{15} - 2^{16} \end{pmatrix} =$

$= F^{15} = \begin{pmatrix} -3^{15} + 2^{16} & 2^2 \cdot 3^{15} - 2^{17} \\ -\frac{3^{15}}{2} + 2^{14} & 2\cdot 3^{15} - 2^{15} \end{pmatrix}$

$f^{15}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \boxed{F^{15}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 4\cdot 3^{15} - 2^{17} \\ 2\cdot 3^{15} - 2^{15} \end{pmatrix}}$.

H 10-1

$\boxed{10.4}$ 4 (i, iii, v, vi)

(ia) Buscamos autovalores $\lambda$: $f(\vec{x}) = \begin{pmatrix} 0 & 8 \\ -2 & 0 \end{pmatrix} \vec{x}$ $[\mathbb{K} = \mathbb{R}]$

(ib) $\begin{vmatrix} -\lambda & 8 \\ -2 & -\lambda \end{vmatrix} = 0 \Leftrightarrow 0 = \lambda^2 + 16$. No existe $\lambda \in \mathbb{R}$ que

cumple esto, por lo que no existen autovalores.

(ic) Como no hay autovalores, no hay autovectores, por lo que <u>no es diagonizable</u>.

(iii.a) $h: \mathbb{R}^3 \to \mathbb{R}^3$. $M(h) = \begin{pmatrix} 1 & 1 & 3 \\ 0 & -2 & 0 \\ 5 & -1 & 3 \end{pmatrix}$. Sea $\lambda \in \mathbb{R} = \mathbb{K}$ un autovalor

$\Rightarrow |M(h) - \lambda I| = \begin{vmatrix} 1-\lambda & 1 & 3 \\ 0 & -2-\lambda & 0 \\ 5 & -1 & 3-\lambda \end{vmatrix} = (-2-\lambda)(3 + \lambda^2 - 4\lambda - 18) = 0$

$\Rightarrow \lambda = -2$ ó $\lambda^2 - 4\lambda - 12 = 0 \Rightarrow \lambda_1 = -2$ ó $\lambda = \frac{4 \pm \sqrt{16+48}}{2} = \begin{cases} 6 = \lambda_2 \\ -2 = \lambda_1 \end{cases}$

Autovalores de h: $\lambda_1 = 2$, $\lambda_2 = 6$,

$\to \lambda_1 = -2 \Rightarrow \begin{pmatrix} 3 & 1 & 3 \\ 0 & 0 & 0 \\ 5 & -1 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \sim \left(\begin{array}{ccc|c} 3 & 1 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -a \\ 0 \\ a \end{pmatrix} = a \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ Autovector: $\vec{x}_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$.

$\to \lambda_2 = 6 \Rightarrow \begin{pmatrix} -5 & 1 & 3 \\ 0 & -8 & 0 \\ 5 & -1 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \to \begin{pmatrix} x \\ y \\ z \end{pmatrix} = a \begin{pmatrix} 3 \\ 0 \\ 5 \end{pmatrix}$, $\vec{x}_2 = \begin{pmatrix} 3 \\ 0 \\ 5 \end{pmatrix}$

(iii.b) Cada autovalor nos da un solo autovector, por lo que solamente podemos encontrar 2 autovectores que sean l.i. Por lo que

(iii.c) <u>No es diagonalizable</u>.

(v.a) $g : \mathbb{C}^3 \to \mathbb{C}^3$. $g(\vec{z}) = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 3 & 6 \\ 1 & -1 & 0 \end{bmatrix} \vec{z}$.

$\lambda \in \mathbb{C}$ es autovalor $\iff \begin{vmatrix} 1-\lambda & 1 & 1 \\ -1 & 3-\lambda & 6 \\ 1 & -1 & -\lambda \end{vmatrix} = 0$

$\iff -\lambda \begin{vmatrix} 1-\lambda & 1 \\ -1 & 3-\lambda \end{vmatrix} + \begin{vmatrix} 1-\lambda & 1 \\ -1 & 6 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 3-\lambda & 6 \end{vmatrix} = 0$

$\iff -\lambda(3 + \lambda^2 - 4\lambda + 1) + (6 - 6\lambda + 1) + (6 - 3 + \lambda) = 0$

$\iff -\lambda^3 + 4\lambda^2 - 4\lambda - 6\lambda + 7 + \lambda + 3 = 0$

$\iff -\lambda^3 + 4\lambda^2 - 9\lambda + 10 = 0$ (una sol $\lambda = 2$.)

$\iff \lambda = 2$, ó $+\lambda^2 - 2\lambda + 5 = 0$

$$\begin{array}{c|cccc} & -1 & 4 & -9 & 10 \\ 2 & & -2 & 4 & -10 \\ \hline & -1 & 2 & -5 & \boxed{0} \end{array}$$

$\iff \lambda = 2$ ó $\lambda = \dfrac{2 \pm \sqrt{4-20}}{2} = 1 \pm 2i$

Autovalores de $g$: $\{\lambda_1 = 2;\ \lambda_2 = 1+2i,\ \lambda_3 = 1-2i.\}$.

$\lambda_1 = 2$: $\begin{pmatrix} -1 & 1 & 1 \\ -1 & 1 & 6 \\ 1 & -1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \sim \left(\begin{array}{ccc|c} -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ +1 \\ 0 \end{pmatrix} \alpha \to \vec{x}_1 = \begin{pmatrix} 1 \\ +1 \\ 0 \end{pmatrix}$ autovector.

$\lambda_2 = 1+2i \to \begin{pmatrix} -2i & 1 & 1 \\ -1 & 2-2i & 6 \\ 1 & -1 & -1-2i \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \sim \left(\begin{array}{ccc|c} 2i & -1 & -1 & 0 \\ -1 & 2-2i & 6 & 0 \\ 0 & 1-2i & 5-2i & 0 \end{array}\right)$

$\sim \left(\begin{array}{ccc|c} 2i & -1 & -1 & 0 \\ 0 & 3+4i & -1+12i & 0 \\ 0 & 1-2i & 5-2i & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2i & -1 & -1 & 0 \\ 0 & 3+4i & -1+12i & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$(2-2i)(2i) = 4i+4$ $\qquad \begin{vmatrix} 3+4i & -1+12i \\ 1-2i & 5-2i \end{vmatrix} = 0$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \alpha \frac{1}{2i}\left(1 + \left(\frac{1-12i}{3+4i}\right)\right) \\ \alpha \left(\frac{1-12i}{3+4i}\right) \\ \alpha \end{pmatrix} \underset{*}{=} \alpha \begin{pmatrix} -4+2i \\ -9-8i \\ 5 \end{pmatrix} \quad \vec{x}_2 = \begin{pmatrix} -4+2i \\ -9-8i \\ 5 \end{pmatrix}$

$*$ $\begin{cases} \dfrac{1-12i}{3+4i} = \dfrac{(1-12i)(3-4i)}{9+16} = \dfrac{-45-40i}{25} = \dfrac{-9-8i}{5} \\[2ex] \dfrac{1}{2i} = \dfrac{2i}{2i \cdot 2i} = \dfrac{-i}{2} \to \dfrac{1}{2i}\left(1 + \left(\dfrac{1-12i}{3+4i}\right)\right) = \dfrac{+i}{\cancel{2}}\left(\dfrac{+2+4i}{5}\right) = \dfrac{-4+2i}{5} \end{cases}$

$$\lambda_3 = 1 \cdot 2i \to \left(\begin{array}{ccc|c} 2i & 1 & 1 & 0 \\ -1 & 2+2i & 6 & 0 \\ 1 & -1 & -1+2i & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2i & 1 & 1 & 0 \\ 0 & (-3+4i) & (1+12i) & 0 \\ 0 & 1+2i & 5+2i & 0 \end{array}\right)$$

$$\det\begin{pmatrix} -3+4i & 1+12i \\ 1+2i & 5+2i \end{pmatrix} = 0$$

$$\sim \left(\begin{array}{ccc|c} 2i & 1 & 1 & 0 \\ 0 & 1+2i & 5+2i & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \alpha \frac{1}{2i}\left(-1 + \frac{5+2i}{1+2i}\right) \\ -\alpha\left(\frac{5+2i}{1+2i}\right) \\ \alpha \end{pmatrix} = \alpha \begin{pmatrix} \frac{-4-2i}{5} \\ \frac{-9+8i}{5} \\ 1 \end{pmatrix}$$

Autovectores

(v.c) $\vec{x}_3 = \begin{pmatrix} -4-2i \\ -9+8i \\ 5 \end{pmatrix}$.

$$\boxed{\beta = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -4+2i \\ -9-8i \\ 5 \end{pmatrix}, \begin{pmatrix} -4-2i \\ -9+8i \\ 5 \end{pmatrix} \right\}} = \{\vec{x_1}, \vec{x_2}, \vec{x_3}\}$$

(v.b) Es diagonalizable, porque con $\beta$, la matriz queda diagonal.

$$\begin{pmatrix} 1 & -4+2i & -4-2i \\ 1 & -9-8i & -9+8i \\ 0 & 5 & 5 \end{pmatrix} \sim \begin{pmatrix} 1 & -4 & -4-2i \\ 1 & -9 & -9+8i \\ 0 & 5 & 5 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & -4 & -4-2i \\ 0 & 5 & 5-16i \\ 0 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & -4 & -4-2i \\ 0 & 5 & 5-16i \\ 0 & 0 & -16i \end{pmatrix} (\text{l. i}) \to \beta \text{ base.}$$

(vi.a) $f: \mathbb{Q}^2 \to \mathbb{Q}^2$; $f(\vec{q}) = \begin{pmatrix} 2 & 1 \\ 2 & 0 \end{pmatrix} \vec{q}$.

$\lambda$ es autovalor $\Leftrightarrow \det\begin{pmatrix} 2-\lambda & 1 \\ 2 & -\lambda \end{pmatrix} = 0 \Leftrightarrow -2\lambda + \lambda^2 - 2 = 0$

$\Leftrightarrow \lambda = \frac{2 \pm \sqrt{4+8}}{2} \Rightarrow \lambda \notin \mathbb{Q}$.

(vi.b) No ~~podemos~~ existen autovalores en $\mathbb{Q} \to$ No es diagonizable.

M10-3