2. Function growth.

7)

(μs) \ $N=$	10	10^2 100	10^3 1000	10^4 10000	10^5 100000	10^6 1000000
(1) $10000N$	10^5	10^6	10^7	10^8	10^9	10^{10}
(2) $1000N$	10^4	10^5	10^6	10^7	10^8	10^9
(3) $\log_{10} N$	1	2	3	4	5	6
(4) $100N^2$	10^4	10^6	10^8	10^{10}	10^{12}	10^{14}
(5) $10N^3$	10^4	10^7	10^{10}	10^{13}	10^{16}	10^{19}
(6) $10^{-3}\cdot 10^{N/10}$	10^{-2}	10^7	10^{97}	10^{997}	10^{9997}	10^{99997}

$$f(N_1) = 1\text{min} = 1\text{min}\cdot\frac{60\text{ sec.}}{1\text{min}}\cdot\frac{10^6\mu s}{1\text{sec}} = 60\cdot 10^6 \mu s = 6\cdot 10^7 \mu s$$

(1) $N_1 = \frac{60\cdot 10^6}{10^4} = 60\cdot 10^2 = 6\cdot 10^3$

(2) $N_1 = 6\cdot 10^7\cdot 10^{-3} = 6\cdot 10^4$

(3) $N_1 = 10^{6\cdot 10^7}$

(4) $N_1 = \sqrt{6\cdot 10^7\cdot 10^{-2}} = \sqrt{6\cdot 10^5} = \sqrt{60}\cdot 10^2$

(5) $N_1 = \sqrt[3]{6\cdot 10^7\cdot 10^{-1}} = \sqrt[3]{6}\cdot 10^2$

(6) $10^{-3}\cdot 10^{N/10} = 6\cdot 10^7 \Rightarrow 10^{N/10} = 6\cdot 10^{10}$

$\Rightarrow \log_{10}\left(\frac{N}{10}\right) = \log_{10}(6\cdot 10^{10}) \Rightarrow \log_{10}(N_1) - 1 = \log_{10}(6) + 10$

$\Rightarrow N_1 = 10^{11}\cdot 10^{\log_{10} 6} = 10^{17}.$

8) $\frac{2}{n} \le 37 \le \sqrt{n} \le n \le \cancel{\ } n\log(\log n) \le n\log n \le n^2 \le n^2\log n \le n^3 \le 2^{n/2} \le 2^n$

$n\log^2 n \le n^{1.5} \le$

10) $W_A(N) = O(1000N\log_{10}(N))$, $W_B(N) = O(N^2)$

For problems where $N \le 1000$, B is best (for small problems)

" " $N \ge 10000$, A is best

11) $\lim_{n\to\infty}\frac{\log n}{n^a} \underset{a>0}{\overset{(L.H)}{=}} \lim_{n\to\infty}\frac{1/n}{a\cdot n^{a-1}} = \lim_{n\to\infty}\frac{1}{a n^a} = 0 \Rightarrow \log n = o(n^a)\ \forall a>0.$

14) $T_1 \approx f$, $T_2 \approx f$. (a) F. $\lim_{n\to\infty}\frac{T_1(n)+T_2(n)}{f(n)} = 2 \neq 1$.

(b) True, (c) True.

15) (a) True (b) $T_1(n) = n+1$, $T_2(n) = n-1$, $f(n) = n$.

$$\lim_{n\to\infty}\frac{T_1^2 - T_2^2}{f} = \lim_{n\to\infty}\frac{n^2+2n+1-n^2+2n-1}{n} = \lim_{n\to\infty}\frac{4n}{n} = 4 \neq 0 \to \text{False.}$$

(c) $\lim_{n\to\infty}\frac{T_1(n)}{T_2(n)} = \lim_{n\to\infty}\frac{T_1(n)}{f}\cdot\frac{f}{T_2(n)} = 1$ (True)

16) (a) True, (b) False: $T_1(n) = 2n$, $T_2(n) = 3n$, $f(n) = 4n \Rightarrow T_1(n) - T_2(n) = -n \neq o(4n)$

(c) ~~True $\exists N, L$ tq. $N_1(n) \le L f(n)$~~ False. $T_1(n) = n$, $T_2(n) = 1$, $f(n) = 2n$

$\forall n > 1$, $T_1(n) \le f(n)$, $T_2(n) \le f(n)$

$\frac{T_1(n)}{T_2(n)} = n$, $\nexists N, L$ tq $\forall n \ge N$ $n \le L\cdot 1$.

d) ~~$T_1(n) \nleq L_1 f(n)$, $T_2(n) \le L_2 f(n)$ $\Rightarrow T_1(n)T_2(n) \le L_1 L_2 f(n)$~~ False (*)

17) $T_1(n) = \Theta(f(n))$, $T_2(n) = \Theta(f(n))$

(a) True , (b) False $t_1(n) = 2n$, $T_2(n) = n$, $f(n) = 3n$.

(c) $\frac{T_1(n)}{T_2(n)} = \Theta(1)$ (true)

$$\lim_{n\to\infty} \frac{T_1(n)}{f(n)} = L_1 \neq 0 \text{ , } \lim_{n\to\infty} \frac{T_2(n)}{f(n)} = L_2 \neq 0$$

$$\rightarrow \lim_{n\to\infty} \frac{\frac{T_1(n)}{T_2(n)}}{1} = \lim_{n\to\infty} \frac{t_1(n)}{f(n)} \cdot \frac{f(n)}{T_2(n)} = \frac{L_1}{L_2} \neq 0 .$$

(d) $T_1(N) = \Theta(T_2(n))$ (True)

20) $\forall f, c$, $f(cn) \overset{(a)}{=} O(f(n))$. $f(n+c) \overset{(b)}{=} O(f(n))$.

Yes. (Polynomial ~~for~~ sure , ~~exp too~~)

$f(n) = n^k \Rightarrow f(cn) = c^k n^k \leq (c^k+1) n^k = (c^k+1) f(n)$

(a) $f(n) = a^n \Rightarrow f(cn) = a^{cn}$

$f(n) = 2^n \Rightarrow f(2n) = 2^{2n} \neq O(2^n)$ b.c. $2^n = o(2^{2n})$

(b) True

21) $S(N) = \sum_1^N \frac{1}{i^{1/3}} \Rightarrow \sum_1^N f(i)$, $f(x) = \frac{1}{x^{1/3}}$

$$\rightarrow \int_0^N x^{-1/3}\,dx \leq S(N) \leq \int_1^{N+1} x^{-1/3}\,dx \Rightarrow \frac{3}{2}x^{2/3}\Big|_0^N \leq S(N) \leq \frac{3}{2}x^{2/3}\Big|_1^{N+1}$$

$$\Rightarrow \frac{3}{2} N^{2/3} \leq S(N) \leq \frac{3}{2}(N+1)^{2/3} - \frac{3}{2}.$$

$$\Rightarrow 1 \leq \frac{S(N)}{\frac{3}{2}N^{2/3}} \leq \frac{(N+1)^{2/3} - 1}{N^{2/3}}$$

$$\lim_{N\to\infty} 1 = \lim_{N\to\infty} \frac{(N+1)^{2/3} - 1}{N^{2/3}} = 1 \Rightarrow \lim_{N\to\infty} \frac{S(N)}{\frac{3}{2}N^{2/3}} = 1$$

$$\Rightarrow \boxed{S(N) \sim \frac{3}{2} N^{2/3}.}$$

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