Ej. 1 Patient: $S_1 S_2$. D = disease ...

(?)

a) Priors: $P(D) = P(\neg D) = 1/2$.

b) Max. likelyhood:

$$P(S_1 S_2 \mid D) = 1/4 \qquad \Rightarrow \underline{H_{ML} = D}.$$
$$P(S_1 S_2 \mid \neg D) = 0$$

c) MAP

$$P(D \mid S_1 S_2) = 1 \qquad \Rightarrow \underline{H_{MAP} = D}.$$
$$P(\neg D \mid S_1 S_2) = 0$$

d) NB assumes S_1 and S_2 independent.

e) $P(S_1 S_2 \mid D) \simeq P(S_1 \mid D)\, P(S_2 \mid D) = 1/4 \cdot 2/4 = 1/8$.

$P(S_1 S_2 \mid \neg D) \simeq \underbrace{3/4}_{P(S_1 \mid \neg D)} \cdot \underbrace{1/4}_{P(S_2 \mid \neg D)} = 3/16$

$$\Rightarrow P(D \mid S_1 S_2) = \frac{P(S_1 S_2 \mid D)\, P(D)}{P(S_1 S_2)} \simeq \frac{1/8 \cdot 1/2}{\text{Norm}} = \frac{1/16}{0,25} = \frac{2}{5}$$

$$P(\neg D \mid S_1 S_2) = \frac{P(S_1 S_2 \mid \neg D)\, P(\neg D)}{\text{Norm}} \simeq \frac{3/16 \cdot 1/2}{\text{Norm}} = \frac{3/32}{0,25} = \frac{3}{5}.$$

$\Rightarrow H_{\text{naive Bayes MAP}} = \neg D$.

Hw 3 1

Ej. 2. $P(D) = 10^{-4}$. T = test positive. $\neg T$: test negative

- $P(T \mid D) = 0,99$.
- $P(\neg T \mid \neg D) = 0,95$. $\Rightarrow P(T \mid \neg D) = 0,05$

a) $\cancel{P(D) = P(T \wedge D) + P(\neg T \wedge D)}$
$\cancel{= P(T \mid D)}$

$$P(D \mid T) = \frac{P(T \mid D) \cdot P(D)}{P(T)} =$$

$$= \frac{P(T \mid D) P(D)}{P(T \mid D) P(D) + P(T \mid \neg D) P(\neg D)}$$

$$= \frac{0,99 \cdot 10^{-4}}{0,99 \cdot 10^{-4} + 0,05 \cdot 9999 \, 10^{-4}} \cong 0,001976$$

F: fever of $> 41°$. C: dry cough.

- $P(F \mid D) = 0,95$
- $P(F \mid \neg D) = 0,001$
- $P(C \mid D) = 0,6$
- $P(C \mid \neg D) = 0,01$
- $P(C \mid F D) = 0,9$
- $P(C \mid F \neg D) = 0,4$

$$P(D \mid TFC) = \frac{P(TFC \mid D) P(D)}{\cancel{P(TFC)} \text{ Norm}} \overset{NB}{\approx} \frac{0,95 \cdot 0,99 \cdot 0,6 \cdot 10^{-4}}{\text{Norm}} = \frac{5,643 \cdot 10^{-5}}{\text{Norm}} \approx 0,9912$$

$$P(\neg D \mid TFC) = \frac{P(TFC \mid \neg D) P(D)}{\text{Norm}} \approx \frac{0,05 \cdot 0,001 \cdot 0,01 \cdot \frac{9999}{10^4}}{\text{Norm}} = \frac{5 \cdot 10^{-7}}{\text{Norm}}$$

$8,78 \cdot 10^{-3}$

$H_{\text{Naive Bayes}} = D$

$\Rightarrow$

HW 2

d)

$$\begin{tikzcd}
 & D \arrow[dl] \arrow[d] \arrow[dr] & \\
T & F \arrow[r] & C
\end{tikzcd}$$

e)

- $P(TFC \mid D) = P(T \mid D) \cdot P(FC \mid D) =$

$= P(T \mid D) \cdot P(C \mid FD) \cdot P(F \mid D)$

$= 0,99 \cdot 0,9 \cdot 0,95 = 0,84645$

- $P(TFC \mid \neg D) = P(T \mid \neg D) \cdot P(C \mid F \neg D) \cdot P(F \mid \neg D) =$

$= 0,05 \cdot 0,4 \cdot 0,001 = 0,00038 = 3,2 \cdot 10^{-5}$

$$P(D \mid TFC) = \frac{P(TFC \mid D)\, P(D)}{Norm} = \frac{8,4645 \cdot 10^{-6}}{Norm} \simeq 0,02616\ldots$$

$$P(\neg D \mid TFC) = \frac{P(TFC \mid \neg D)\, P(\neg D)}{Norm} = \frac{1,9998 \cdot 10^{-5}}{Norm} \simeq 0,7383$$

$\boxed{\neg D} = H_{MAP}$

f) Son diferentes. Porque realmente las variables no son independientes.

En NB la probabilidad de tener la enfermedad es mucho mayor porque no tiene en cuenta que la probabilidad de tener tos seca ~~teniendo a la vez fiebre~~ sabiendo que tengo fiebre es mayor que si no tenemos información sobre la fiebre.

Ex. 3

$$\begin{cases} P(S) = 0{,}75 \\ P(\neg S) = 0{,}25 \end{cases} \qquad \begin{cases} \text{flavor} = \text{"S"} \equiv S \\ \text{flavor} = \text{"A"} \equiv \neg S \end{cases}$$

$$\begin{cases} P(C \mid S) = 0{,}7 \\ P(R \mid S) = 0{,}7 \end{cases} \qquad \begin{cases} \text{shape} = \text{"C"} \equiv C \\ \text{shape} = \text{"O"} \equiv \neg C \end{cases} \qquad \begin{cases} \text{wrapper} = \text{"R"} \equiv R \\ \text{wrapper} = \text{"B"} \equiv \neg R \end{cases}$$

$$\begin{cases} P(\neg C \mid \neg S) = 0{,}9 \\ P(\neg R \mid \neg S) = 0{,}9 \end{cases}$$

---

a)

$$\begin{tikzcd} W \arrow[dr] & & S \arrow[dl] \\ & F & \end{tikzcd}$$

(i) $P(W, Sh, F) = P(W)\,P(Sh)\,P(F \mid W, Sh)$

$$\begin{tikzcd} W \arrow[r] \arrow[dr] & Sh \arrow[d] \\ & F \end{tikzcd}$$

(ii) $P(W, Sh, F) = P(F \mid Sh, W)\,P(Sh \mid W)\,P(W)$

$$\begin{tikzcd} & F \arrow[dl] \arrow[dr] & \\ W & & Sh \end{tikzcd}$$

(iii) $P(W, Sh, F) = P(F)\,P(W \mid F)\,P(Sh \mid F)$

b) $P(W, Sh, F) = \underbrace{P(W \mid Sh, F)}_{P(W \mid F)}\,P(Sh \mid F)\,P(F)$

By the way the info is given, it could be understood that $P(W \mid Sh, F) = P(W \mid F)$ i.e. $\boxed{\text{net iii}}$

d) $\boxed{P(R) = P(R, S) + P(R, \neg S) = P(R \mid S)P(S) + P(R \mid \neg S)P(\neg S) = 0{,}7 \cdot 0{,}75 + 0{,}1 \cdot 0{,}25 = 0{,}55}$

e) $\boxed{P(S \mid R) = \dfrac{P(R \mid S)\,P(S)}{P(R)} = \dfrac{0{,}7 \cdot 0{,}75}{0{,}55} = \dfrac{21}{20}} \simeq 0{,}9545$

f) $\boxed{P(C, R)} = P(C R \cap S) + P(C R \cap \neg S) =$

$= \underbrace{P(CR \mid S)}_{P(C \mid S)P(R \mid S)} P(S) + \underbrace{P(CR \mid \neg S)}_{P(C \mid \neg S)P(R \mid \neg S)} P(\neg S) = 0{,}7 \cdot 0{,}7 \cdot 0{,}75 + 0{,}1 \cdot 0{,}1 \cdot 0{,}25 =$

$\boxed{= 0{,}37}$

HW 3-3

---

j) $\boxed{P(S \mid CR) = \frac{P(CR \mid S)\,P(S)}{P(CR)}} = \frac{0{,}49 \cdot 0{,}75}{0{,}37} = \boxed{\frac{147}{148}} \approx \underline{\overline{0{,}99324}}$

$P(CR \mid S) \overset{\circledast}{=} P(C \mid S)\,P(R \mid S) = 0{,}7^2 = 0{,}49$

Hw 3 - 4