

PROBLEM SET 1: Propositional Logic EDyL 2019-2020

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[Due date: 2019/10/01, 09:00]

[Solutions (in class): 2019/10/01]

NOTE: Make sure to include explanations in your answers. An exercise whose solution is correct but does not include an explanation can be considered incomplete and not receive full credit.

EXERCISE 1. Consider the knowledge base

$$\{ B \Leftrightarrow [A \Leftrightarrow (\neg B \wedge A \wedge \neg C)], C \Leftrightarrow (A \vee B) \}$$

- (i) Write down the truth table of the knowledge base and indicate which interpretations are models.

Based on this truth table, give an answer to the following questions. To get full credit, explain the reasoning behind your answers.

- (ii) Is the knowledge base, UNSAT, SAT, but not a tautology, or a tautology.
- (iii) Is the well-formed formula A a logical consequence of the knowledge base $\{B \Leftrightarrow [A \Leftrightarrow (\neg B \wedge A \wedge \neg C)], C \Leftrightarrow (A \vee B)\}$?
- (iv) Is the well-formed formula $\neg A$ a logical consequence of the knowledge base $\{B \Leftrightarrow [A \Leftrightarrow (\neg B \wedge A \wedge \neg C)], C \Leftrightarrow (A \vee B)\}$?
- (v) Is the well-formed formula B a logical consequence of the knowledge base $\{B \Leftrightarrow [A \Leftrightarrow (\neg B \wedge A \wedge \neg C)], C \Leftrightarrow (A \vee B)\}$?
- (vi) Is the well-formed formula $\neg B$ a logical consequence of the knowledge base $\{B \Leftrightarrow [A \Leftrightarrow (\neg B \wedge A \wedge \neg C)], C \Leftrightarrow (A \vee B)\}$?
- (vii) Is the well-formed formula C a logical consequence of the knowledge base $\{B \Leftrightarrow [A \Leftrightarrow (\neg B \wedge A \wedge \neg C)], C \Leftrightarrow (A \vee B)\}$?
- (viii) Is the well-formed formula $\neg C$ a logical consequence of the knowledge base $\{B \Leftrightarrow [A \Leftrightarrow (\neg B \wedge A \wedge \neg C)], C \Leftrightarrow (A \vee B)\}$?

EXERCISE 2.

(i) Let w_1 , w_2 , and w be WFFs, which simultaneously fulfill

$\{w_1, w_2, \neg w\}$ is UNSAT,

$\{w_1, w_2, w\}$ is SAT.

Indicate which of the following statements are correct, which are incorrect, and for which it is not possible to determine whether they are correct or incorrect with the information available. Explain why.

(a) $\{w_1, w_2\} \models w$

(b) $\{w_1, w_2\} \models \neg w$

(c) w is SAT

(d) $w_1 \wedge w_2$ is SAT

(e) $w_1 \wedge w_2 \wedge w$ is a tautology

(ii) Transform into conjunctive normal form the knowledge base

$\Delta = \{A \vee B, (A \Rightarrow B) \vee (A \Leftrightarrow \neg C), \neg C \Leftrightarrow (\neg A \vee \neg B)\}$.

Determine, using resolution on clauses, whether this knowledge base has as logical consequence C or $\neg C$.

EXERCISE 3. [Adapted from

<https://demonstrations.wolfram.com/DeafIslandPuzzleGenerator/>]

A space traveler lands on a planet inhabited by members of two different species: the Akritians and the Denobulans. They are both humanoids that are indistinguishable from their appearance. There is strong mistrust and a history of betrayal between these two species. This has led to a peculiarity in their communication patterns: on the one hand, when two individuals of the same species speak to each other they always tell the truth. On the other hand, when two individuals of different species speak to each other they always lie. The space traveler comes across three individuals engaged in pairwise conversations:

A to B: You are Akritian if and only if C is Akritian

B to C: Either you are Denobulan or A is Akritian (or both)

C to A: B and you are Akritians

The space traveler needs to determine which of the creatures is from which species.

Use the following atoms to formalize the problem

Atoms	Denotation
A	"A is Akritian" $\neg A$: "A is Denobulan"
B	"B is Akritian" $\neg B$: "B is Denobulan"
C	"C is Akritian" $\neg C$: "C is Denobulan"

- (i) Formalize the knowledge base as WFFs in propositional logic (use as many rows as needed)

Sentence in natural language	Well-formed formulas (WFF)

- (ii) Transform the knowledge base to conjunctive normal form, indicating at each step the equivalence rule used

- (iii) Using direct resolution only (no natural, semiformal or case-based reasoning is allowed) determine to which species does each of the humanoids belong.

EXERCISE 4. [adapted from *Alice in Puzzleland* by R. Smullyan]

The creatures that appear in this story are either sane or mad. We know that the beliefs of sane creatures are always true and that beliefs of mad creatures are always false.

Let me introduce you the three creatures of our story: The Lobster, the Gryphon and the Mock Turtle. The Lobster once expressed the belief that the Gryphon believes that he (the Gryphon) is the only sane one of the three. The Mock Turtle believes that either the Lobster or the Gryphon, or both are sane.

Using only direct inference in propositional logic (no truth tables, case-based, natural, or semi-natural reasoning are allowed), what can be deduced about whether each of these three creatures is mad or sane?

- a. Specify the atoms that are needed to solve the problem

Atoms	Symbol	Denotation
	L	"The lobster is sane" = "The lobster's beliefs are true"
	G	"The gryphon is sane" = "The gryphon's beliefs are true"
	M	"The mock turtle is sane" = "The mock turtle's beliefs are true"

- b. Write down the well-formed formulas in propositional logic that make up the knowledge base (as many as needed)

Knowledge base		Wff	Meaning

- c. Transform the wff's in the knowledge base into conjunctive normal form (CNF)
- d. Apply resolution to derive new clauses and give an interpretation of the resulting clauses, including clauses with more than one literal.
- e. What can we say if we also know that either the Lobster or the Mock Turtle, but not both, is sane?