Problema de la Semana (6). Pablo Cuesta Sierra. NIA: 422974

(2) $Q_1(x,y,z) = 2x^2 - 2xz - 6y^2 - 2z^2$

$$= (\sqrt{2}x - 1/\sqrt{2}\, z)^2 - \frac{1}{2}z^2 - 6y^2 - 2z^2$$

$$= (\sqrt{2}\, x - \frac{1}{\sqrt{2}} z)^2 - 6y^2 - \frac{5}{2} z^2$$

$$\begin{cases} x' = \sqrt{2}x - \frac{1}{\sqrt{2}} z \\ y' = \sqrt{6}y \\ z' = \sqrt{\frac{5}{2}} z \end{cases} \Rightarrow \boxed{P = \begin{pmatrix} \sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & \sqrt{6} & 0 \\ 0 & 0 & \sqrt{5/2} \end{pmatrix}}, \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = P \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\beta' = \{\vec{u_1} = (\sqrt{2}, 0, 0), \vec{u_2} = (0, \sqrt{6}, 0), \vec{u_3} = (-1/\sqrt{2}, 0, \sqrt{5/2})\}$$

En esta nueva base,

$$Q_1(x', y', z') = x'^2 - y'^2 - z'^2.$$

$$C = M_{\beta'}(Q_1) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$M_{\beta}(Q_1) = \begin{pmatrix} 2 & 0 & -1 \\ 0 & -6 & 0 \\ -1 & 0 & -2 \end{pmatrix} = P^t \cdot C \cdot P.$$

base inicial

$Q_2(x,y,z,t) = 2xy + 2zt =$

$$= \left(\frac{x+y}{\sqrt{2}}\right)^2 - \left(\frac{x-y}{\sqrt{2}}\right)^2 + \left(\frac{z+t}{\sqrt{2}}\right)^2 - \left(\frac{z-t}{\sqrt{2}}\right)^2$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}. \boxed{P = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix}}$$

En la base $\beta' = \{\vec{u_1} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \vec{u_2} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}, \vec{u_3} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \vec{u_4} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}\}$

$\hookrightarrow Q_2(x', y', z', t') = x'^2 - y'^2 + z'^2 - t'^2.$

$$C = M_{\beta'}(Q_2) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},$$

$$M_{\beta}(Q_2) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = P^t C P$$

base inicial