Soluciones hoja 2

① ⓐ $\vec{E} = 1{,}798 \cdot 10^{10} \frac{N}{C}\, \hat{u}_x - 1{,}557 \cdot 10^{10} \frac{N}{C}\, \hat{u}_y$

$\vec{F} = q\vec{E} = 53900\, N\, \hat{u}_x - 46700\, \frac{N}{C}\, \hat{u}_y$; $|\vec{E}| = 2{,}378 \cdot 10^{10} \frac{N}{C}$

$\tan \alpha = \frac{|E_y|}{E_x} = \frac{\sqrt{3}}{2} \Rightarrow \alpha = 40{,}89^\circ$; $|\vec{F}| = 71300\, N$

ⓑ a lo largo de la recta que une las cargas, a una distancia de $(\sqrt{2}-1)a \simeq 0{,}41a$ de la carga Q_1

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② $V(x) = \frac{2q}{4\pi\varepsilon_0 r} = \frac{q}{2\pi\varepsilon_0 r} = \frac{q}{2\pi\varepsilon_0 \sqrt{x^2 + \left(\frac{d}{2}\right)^2}}$

$E_x(x) = -\frac{\partial V}{\partial x} = \frac{q}{2\pi\varepsilon_0} \frac{x}{\left(x^2 + \left(\frac{d}{2}\right)^2\right)^{3/2}}$

$E_y(x) = -\frac{\partial V}{\partial y} = 0$

ⓑ $U(x) = e\,V(x)$; $v = \sqrt{\frac{2U(0)}{m}} = 1{,}44 \cdot 10^6\ \frac{m}{s}$

ⓒ $U(x) = -e\,V(x)$

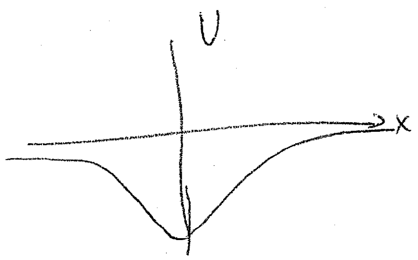
$F(x) = -e\,E(x)$

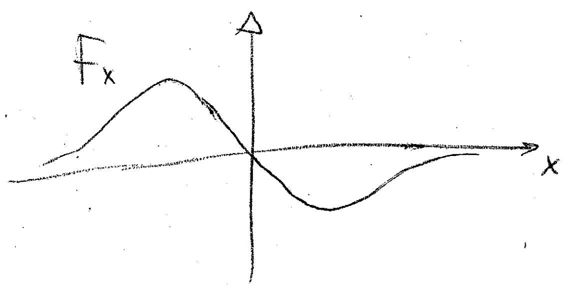
$F(x) = -\frac{qe}{2\pi\varepsilon_0} \frac{x}{\left[x^2 + \left(\frac{d}{2}\right)^2\right]^{3/2}}$

para $x \ll \frac{d}{2} \Rightarrow F(x) \sim -\underbrace{\frac{4qe}{\pi\varepsilon_0 d^3}}_{=K}\, x$

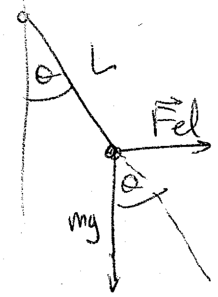
$\omega = \sqrt{\frac{K}{m}} \Rightarrow T = 2\pi\sqrt{\frac{m}{K}}$

$T \simeq 7{,}21 \cdot 10^{-10}\, s$





(3) $\tan\theta = \dfrac{qE}{mg} \Rightarrow q = \dfrac{mg}{E}\tan\theta$



$q = 2{,}83\cdot 10^{-3}\,C = \boxed{\underbrace{1{,}77\cdot 10^{16}}_{N_e}}\,e$

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(4) (A) $\vec{E} = 1{,}26\cdot 10^{5}\,\frac{N}{C}\,\hat{u}_x$

(B) $\vec{E} = \overset{16\,000\,\frac{N}{C}}{\cancel{6{,}\ldots\cdot 10\,\frac{N}{C}}}\,\hat{u}_x - \overset{27\,900\,\frac{N}{C}}{\cancel{3{,}7\ldots\cdot 10^{3}\,\frac{N}{C}}}\,\hat{u}_y$

$|\vec{E}| = 32\,100\,\frac{N}{C}$; $\tan\beta = \dfrac{E_y}{E_x} \Rightarrow \beta = -60{,}17^\circ$

(b) $W_F = -\Delta U$; $W = -W_F$, W: trabajo realizado por la fuerza externa

$W = 1{,}12\cdot 10^{5}\,J$

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(5) $U = -\vec{p}\,\vec{E}$ mínimo cuando $\theta = 0 \Rightarrow \vec{p} \parallel \vec{E}$

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(6) $V(x) = \dfrac{N\cdot\frac{Q}{N}}{4\pi\varepsilon_0 r} = \dfrac{Q}{4\pi\varepsilon_0\sqrt{x^2+R^2}}$

$\vec{E}_x = -\dfrac{\partial V}{\partial x} = \dfrac{Q\;\;x}{4\pi\varepsilon_0\,(x^2+y^2)^{3/2}} = \dfrac{Q}{4\pi\varepsilon_0}\,\dfrac{x}{(x^2+y^2)^{3/2}}$

resultado válido en el caso límite $N\to\infty \Rightarrow$ vale para la circunferencia

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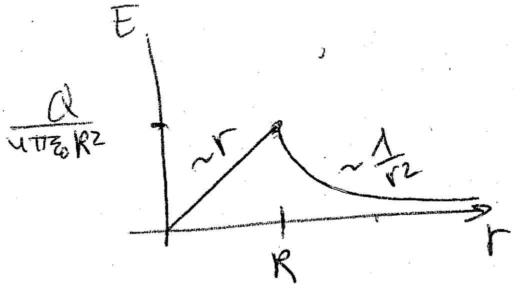
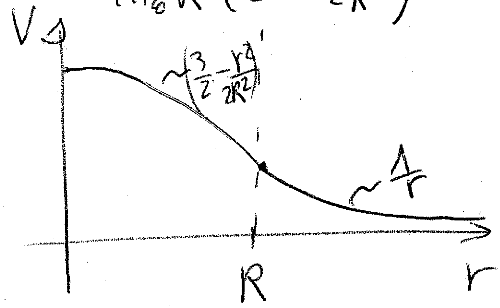
a) $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{u}_r \quad ; \quad V = \frac{Q}{4\pi\varepsilon_0 r}$

b) $r>R \qquad \vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{u}_r \qquad V = \frac{Q}{4\pi\varepsilon_0 r}$

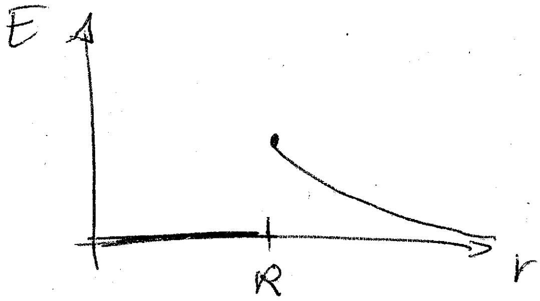
$r<R \qquad \vec{E} = \frac{Q}{4\pi\varepsilon_0}\frac{r}{R^3}\hat{u}_r \qquad V = \frac{Q}{4\pi\varepsilon_0 R}\left(\frac{3}{2} - \frac{r^2}{2R^2}\right)$

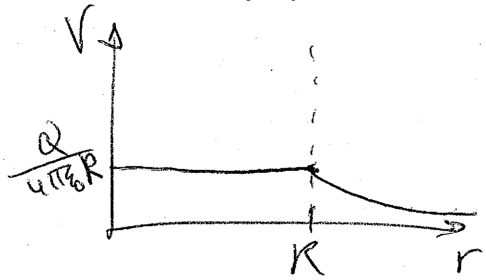




c) $r>R \qquad \vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{u}_r \qquad V = \frac{Q}{4\pi\varepsilon_0 r}$

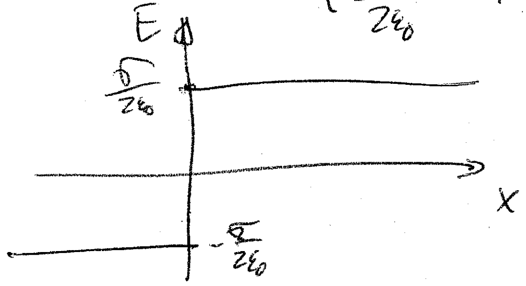
$r<R \qquad \vec{E} = 0 \qquad V = \text{cte} = \frac{Q}{4\pi\varepsilon_0 R}$

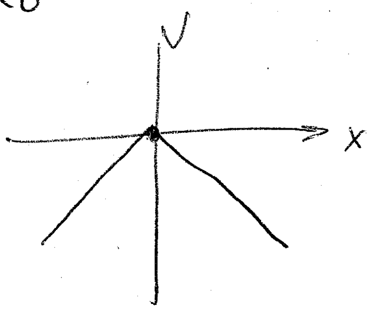




d) $E = \begin{cases} \frac{\sigma}{2\varepsilon_0}\hat{u}_x & x>0 \\ -\frac{\sigma}{2\varepsilon_0}\hat{u}_x & x<0 \end{cases}$

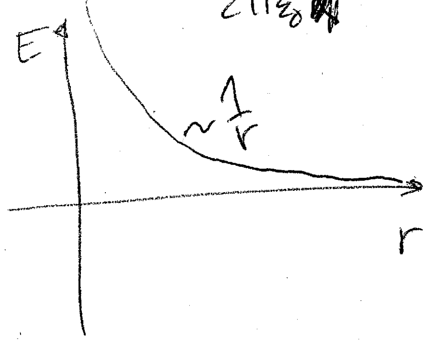
$V = \begin{cases} -\frac{\sigma}{2\varepsilon_0}x + \text{cte} & x>0 \\ \frac{\sigma}{2\varepsilon_0}x + \text{cte} & x<0 \end{cases} = -\frac{\sigma}{2\varepsilon_0}|x| + \underset{0}{\underset{\parallel}{\text{cte}}}$

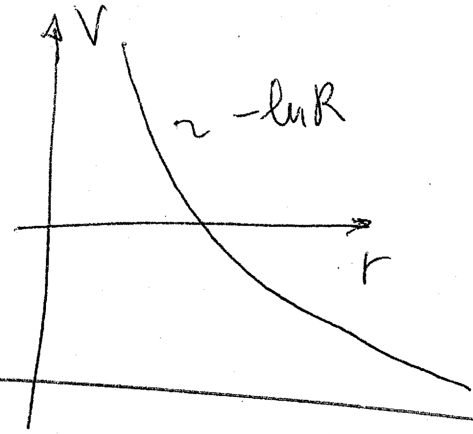




(e) $$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 R}\,\hat{u}_R$$

$$V = -\frac{\lambda}{2\pi\varepsilon_0}\ln R + cte$$





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(8) $$\Delta V = \frac{Q_1 + |Q_2|}{2\varepsilon_0 A}\,d \;;\quad v = 2{,}64\cdot 10^7\ \frac{m}{s}$$

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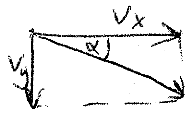
(9)

(a) $y = -6{,}4$ mm

(b) 

$\tan\alpha = 0{,}32$; $\alpha = 17{,}7°$

(c) $Y = -3{,}84$ cm ; $Y_{tot} = -4{,}48$ cm