

# Sequential circuits

Computer Fundamentals

Escuela Politécnica Superior. U.A.M

# Index of unit 4

## U4. Sequential circuits

### U4.1. Counters.

U4.1.1. Ring counter.

U4.1.2. Other synchronous counters. Analysis and Synthesis.

### U4.2. Sequential systems. Finite state machines (FSM)

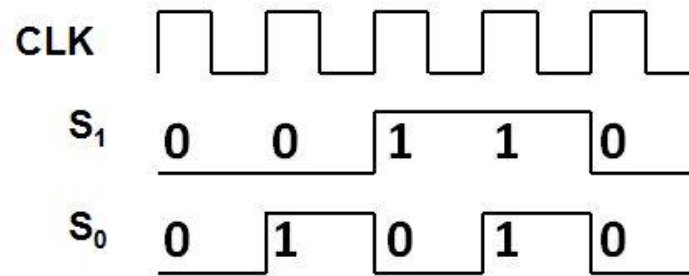
### U4.3. Moore and Mealy circuits. Synthesis of states.

### U4.4. Examples of finite state machines.

# Synchronous counters

**Counter.** Sequential system which changes the value (state) when the clock signal changes.

- ✓ They are used for counting events in the digital systems.  
Ex.: count clock pulses (edges [ ] flip-flops used).

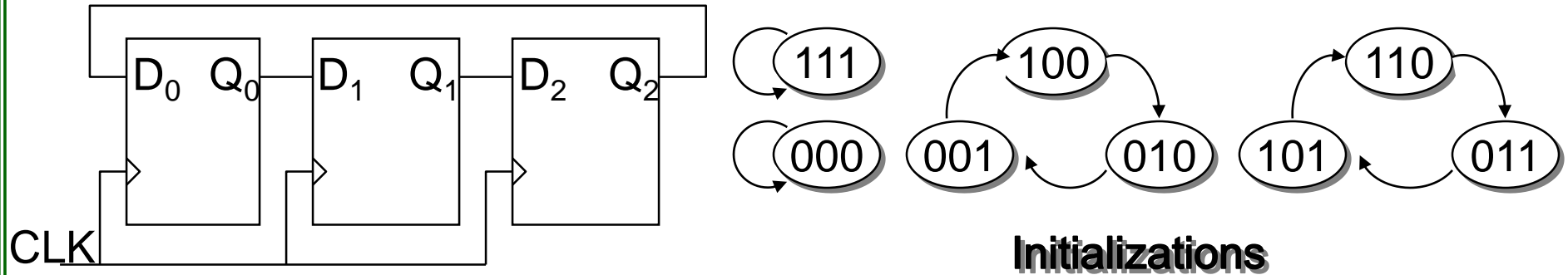


- ✓ Module of a counter is the maximum number of states it can count (for n bits, Module =  $2^n$ ). → Number of bits of the counter

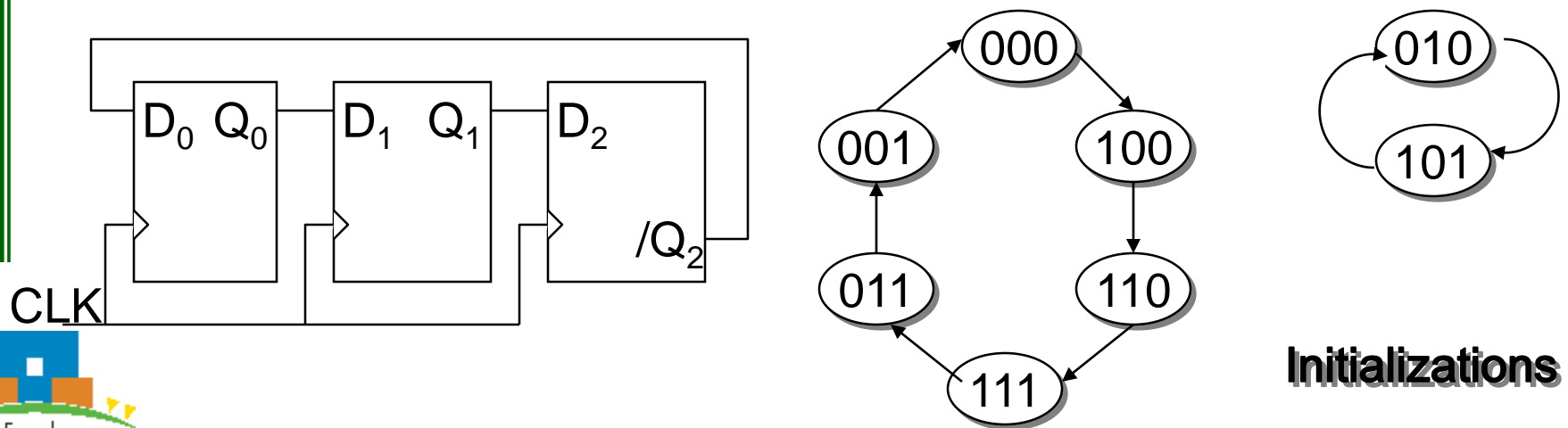
# Synchronous counters. Analysis

## Using shift registers

### a) Ring counter



### b) Johnson counter

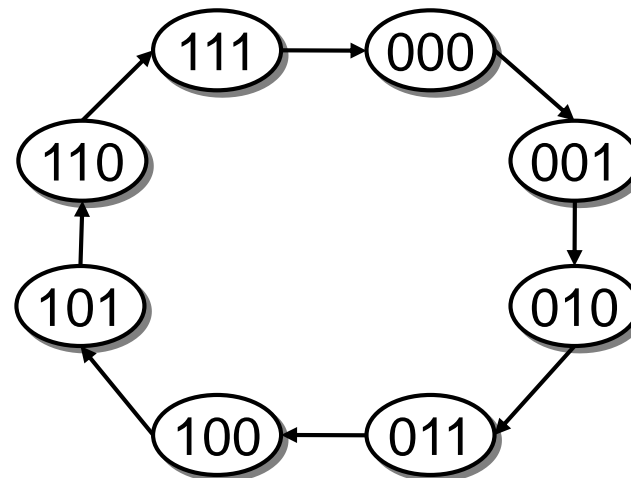
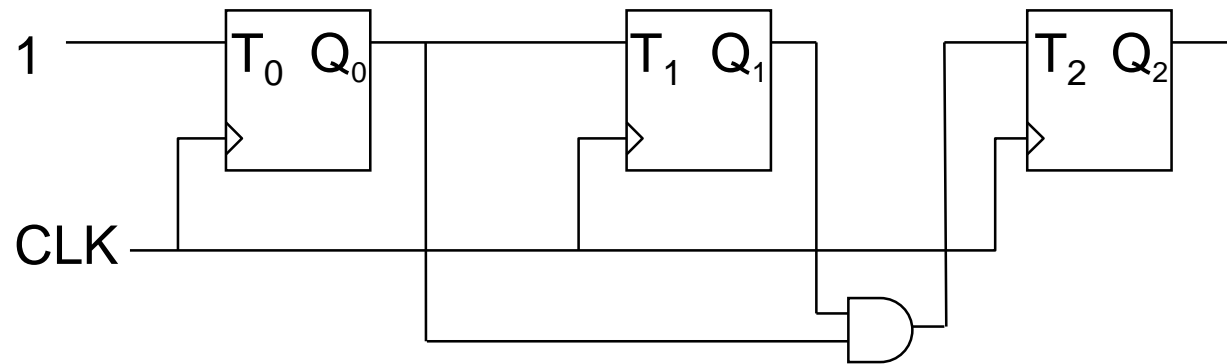


# Synchronous counters. Analysis

## Using shift registers

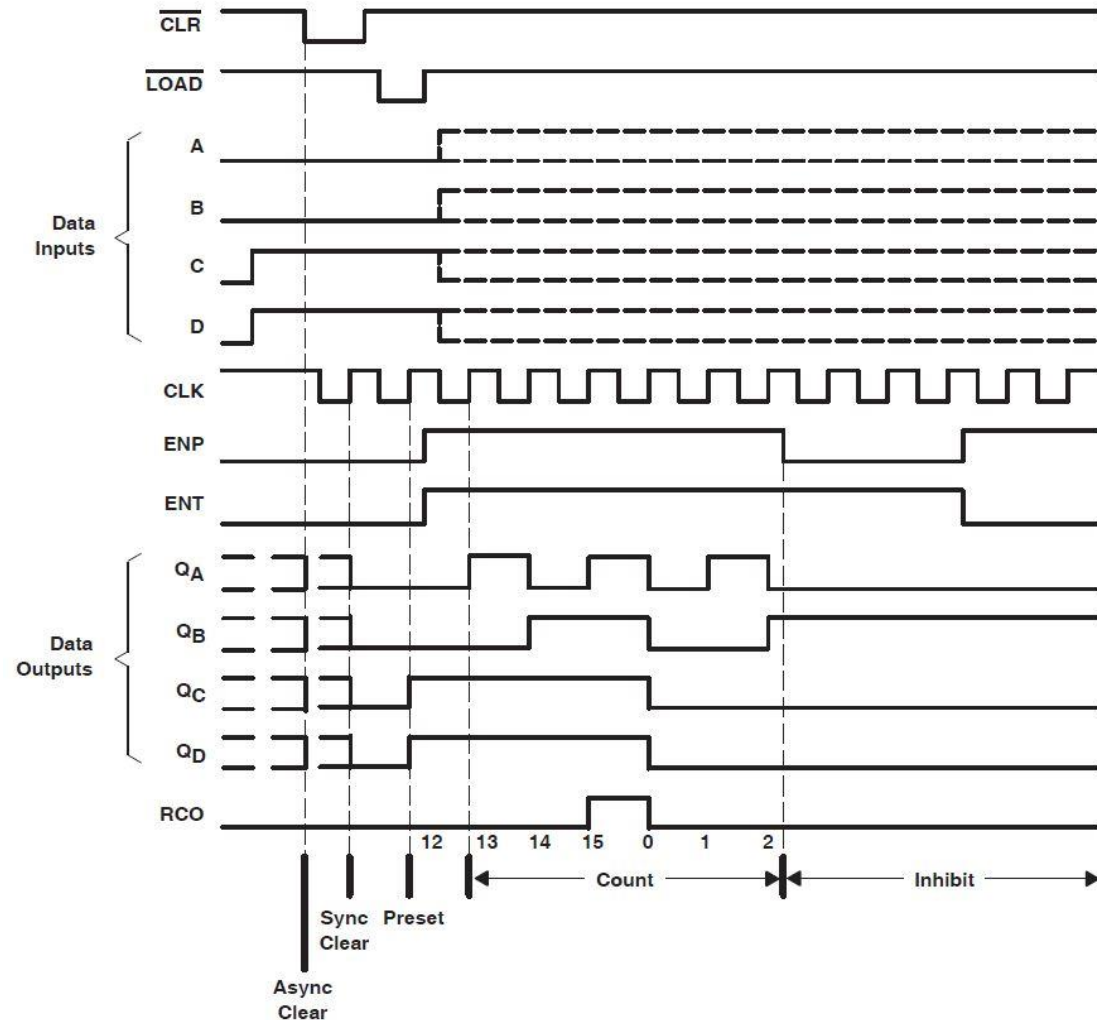
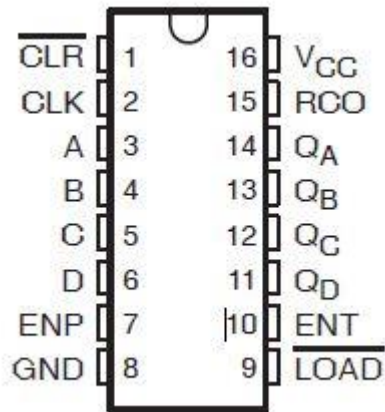
c) Binary counter 3b of maximum module (8 states).

	$Q_2$	$Q_1$	$Q_0$
$S_0$	0	0	0
$S_1$	0	0	1
$S_2$	0	1	0
$S_3$	0	1	1
$S_4$	1	0	0
$S_5$	1	0	1
$S_6$	1	1	0
$S_7$	1	1	1



# Synchronous counters. Analysis

## d) Binary counter 4b: 74HC163



# Synchronous counters. Synthesis

## ✓ Steps to follow for designing counters of $m$ states.

- The number of states  $m$  determines the minimum number of flip-flops  $n$ ,

$$2^n \geq m$$

- Choose the order in which the states will be run (states diagram).

- Define an initial state.

- Define a solution for possible locking situations? How....

## HARDWARE Synthesis

- Write the transition table between states:

Actual state ( $n$ )  $\rightarrow$  Next state ( $n+1$ ).

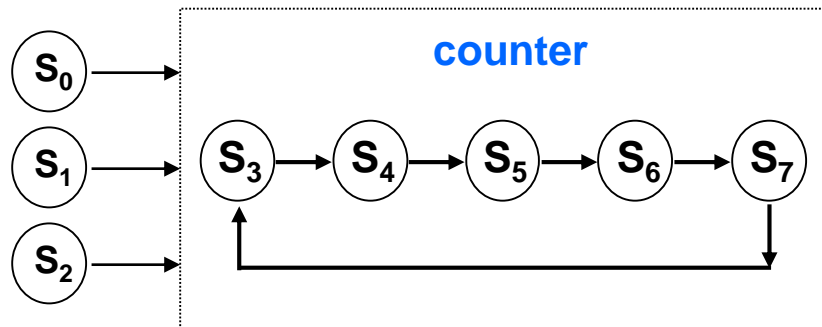
- Choose the type of flip-flop to make the design (**D**, **T**, **JK**).

- Write the State equations  $\{ g(Q_i^n, E_i^n) \}$

# Synchronous counters. Synthesis

## EXAMPLE

- ✓ Counter of 5 states.  $2^3 > 5 > 2^2 \Rightarrow$  (3 flip-flops)
- ✓ Sequential states:  $S_3, S_4, S_5, S_6, S_7$ ;  $Q_2 Q_1 Q_0 = (011, 100, 101, 110, 111)$
- ✓ Add antiblock (Since I want 5 states, I can do): if  $S_0, S_1$  o  $S_2 \Rightarrow S_4$



ACTUAL state			
	$Q_2^n$	$Q_1^n$	$Q_0^n$
$S_0$	0	0	0
$S_1$	0	0	1
$S_2$	0	1	0
$S_3$	0	1	1
$S_4$	1	0	0
$S_5$	1	0	1
$S_6$	1	1	0
$S_7$	1	1	1



# Sequential systems

## Characteristics:

- A sequential circuit is a finite state machine(FSM).
- The change from the current state ( $t = n$ ) to the next state( $t + \Delta t = n + 1$ ) is synchronously produced with the clock signal of the system ( $\uparrow$  or  $\downarrow$ ).
- The system state is defined by the values associated to the outputs ( $Q_i$ ) of the flip-flops conforming the system.
- The transition to the next state ( $Q_i^{n+1}$ ) depends on the actual state ( $Q_i^n$ ) and the inputs applied to the system ( $X_i^n$ ) in the clock edge.
- Regarding the changes in the output:

The outputs ( $S_i$ ) depend only in the state ( $Q_i$ )

**FSM-MOORE:**  $S^n = g(Q^n)$

The outputs ( $S_i$ ) depend on the state ( $Q_i$ ) and on the inputs ( $E_i$ )

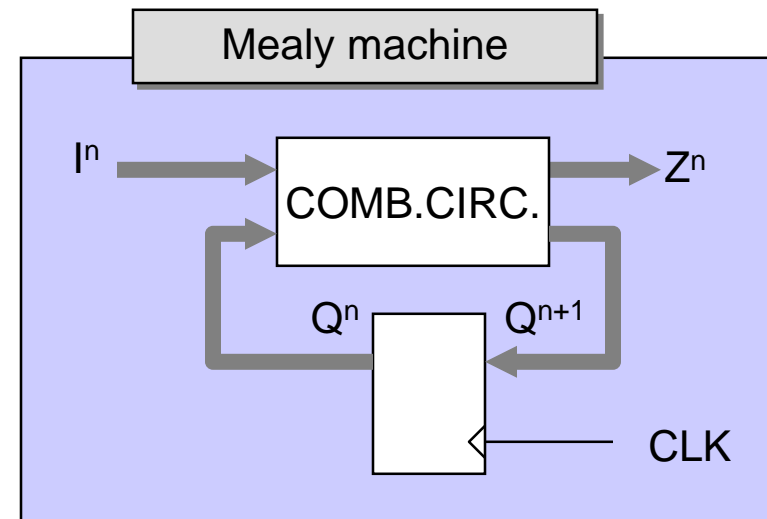
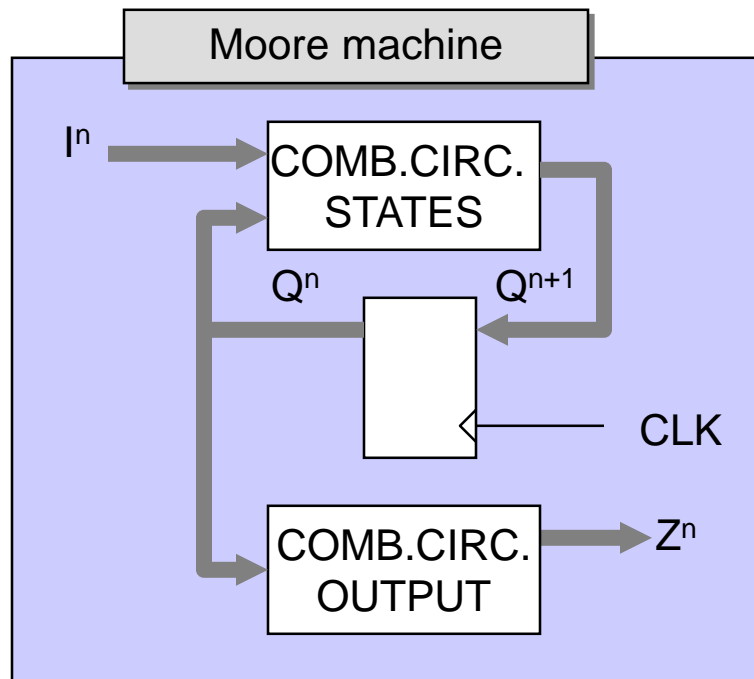
**FSM-MEALY:**  $S^n = g(Q^n, E^n)$

# Sequential systems

## STATE DIAGRAMS

- ✓ Represents the functionality of a system in a structured way

SYNTHESIS: State diagrams  $\Rightarrow$  Circuit



The type of FSM is determined by the combinational circuits it contains

# Sequential systems. Implementation

A Identification of inputs and outputs.

B State diagrams

C Checking and diagram reduction

D Determining of the number and type of flip-flops

E Assigning the states

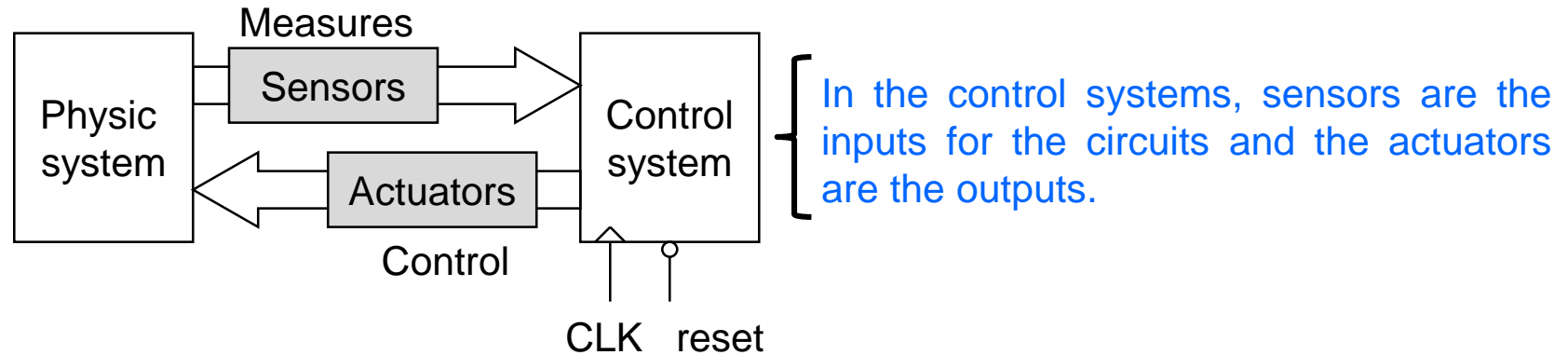
F Truth Table

G Minimization of logic functions

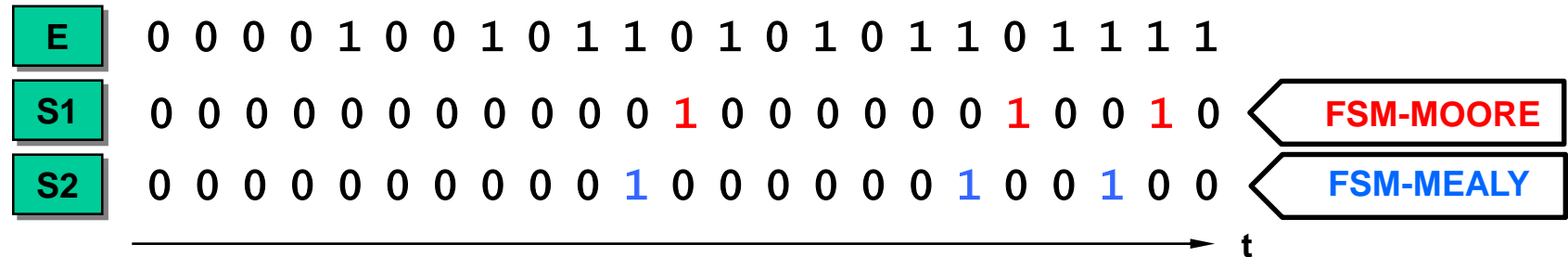
H Circuit design

There can be changes  
as a function of the  
type of implementation

# Sequential systems. Implementation



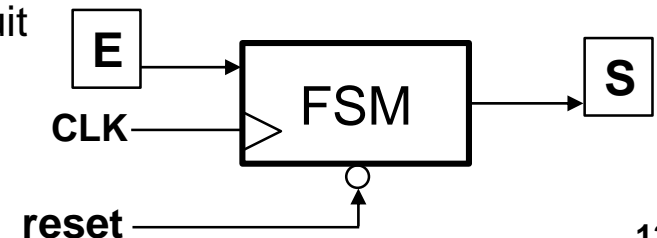
**Example\_1:** Detect the sequences overlap of value "1011" synchronous with a clock.



A

Identification of inputs and outputs

- Determine the input and output signals of the designed circuit
- The clock and the reset must always appear, and they are not considered as inputs



# Sequential systems. Implementation

**Example\_1:** Detect the sequence "1011" accepting overlap .

E	0	0	0	0	1	0	0	1	0	1	1	0	1	0	1	1	0	1	1	1	1		
S1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	1	0	
S2	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	1	0	0

t →

FSM-MOORE

FSM-MEALY

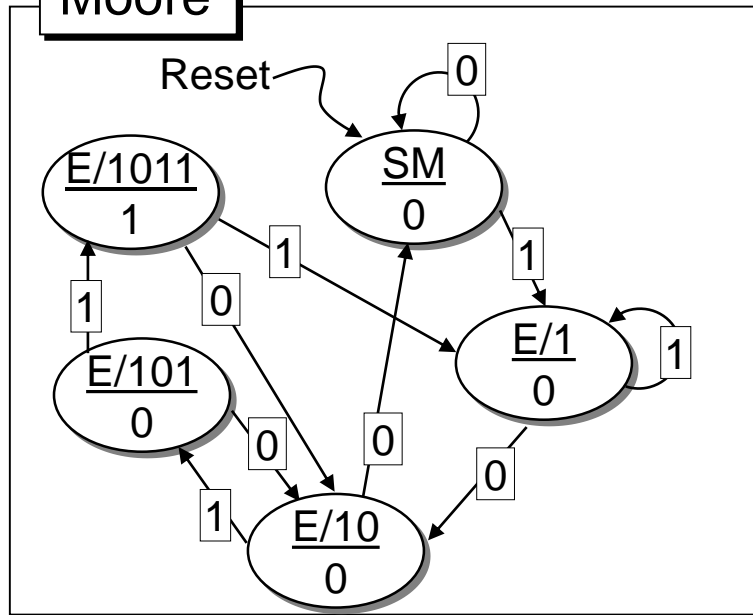
FSM-MOORE

FSM-MEALY

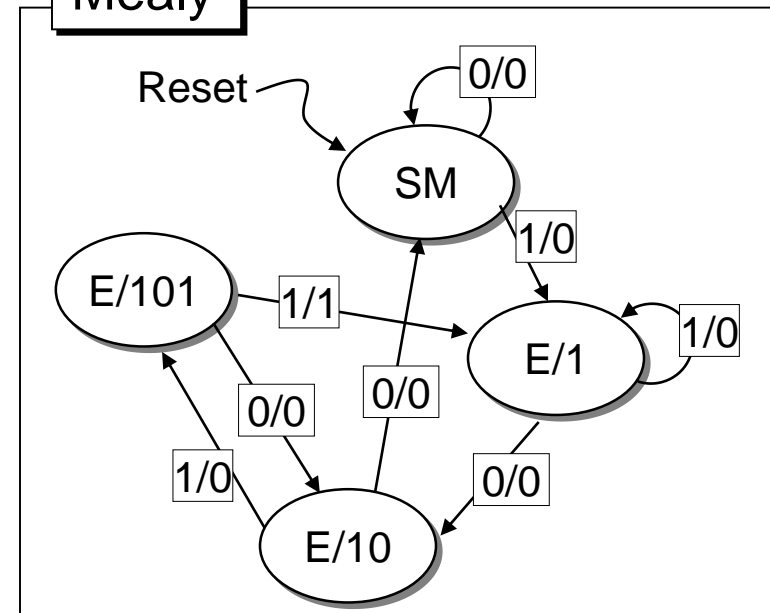
B

States diagram (NOT AUTOMATABLE (there is no systematic method))

Moore



Mealy



# Sequential systems. Implementation

**Example\_1:** Detect the sequence “1011” (overlap allowed) synchronous with a clock.

D Determine the number and type of flip-flops

**N states  $\Rightarrow$  n flip-flops such that  $2^n \geq N$**

Moore

5 states  $\rightarrow$  3 flip-flops

Mealy

4 states  $\rightarrow$  2 flip-flops

E Assign the states

Moore

STATE	Binary value		
	$Q_2$	$Q_1$	$Q_0$
SM	0	0	0
E/1	0	0	1
E/10	0	1	0
E/101	1	0	0
E/1011	1	1	1

Mealy

STATE	Binary value	
	$Q_1$	$Q_0$
SM	0	0
E/1	0	1
E/10	1	0
E/101	1	1

# Sequential systems. Implementation

**Example\_1:** Detect the sequence “1011” (overlap allowed) synchronous with a clock.

F Truth tables: State and output (Moore)

Actual State			
$Q_2^n$	$Q_1^n$	$Q_0^n$	$E^n$
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
1	0	0	0
1	0	0	1
1	1	1	0
1	1	1	1

Next state		
$Q_2^{n+1}$	$Q_1^{n+1}$	$Q_0^{n+1}$
0	0	0
0	0	1
0	1	0
0	0	1
0	0	0
1	0	0
0	1	0
1	1	1
0	1	0
0	0	1

Output
$S_1$
0
0
0
0
0
0
0
0
1
1

State Eq.(3)

Output Eq.

# Sequential systems. Implementation

**Example\_1:** Detect the sequence “1011” (overlap allowed) synchronous with a clock.

F Truth table: State and output (Mealy)

Actual state		
$Q_1^n$	$Q_0^n$	$E^n$
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

Next state	
$Q_1^{n+1}$	$Q_0^{n+1}$
0	0
0	1
1	0
0	1
0	0
1	1
1	0
0	1

Output
$S_2$
0
0
0
0
0
0
0
1

State Eq.(2)

Output Eq.



# Sequential systems. Implementation

**Example\_1:** Detect the sequence “1011” (overlap allowed) synchronous with a clock.

**G** Minimization of logic functions

**H** Circuit designs

- The state equations depends on the type of flip-flops selected. In each case the transition equations which must be applied are:

$Q_i^n \rightarrow Q_i^{n+1}$
0 $\rightarrow$ 0
0 $\rightarrow$ 1
1 $\rightarrow$ 0
1 $\rightarrow$ 1

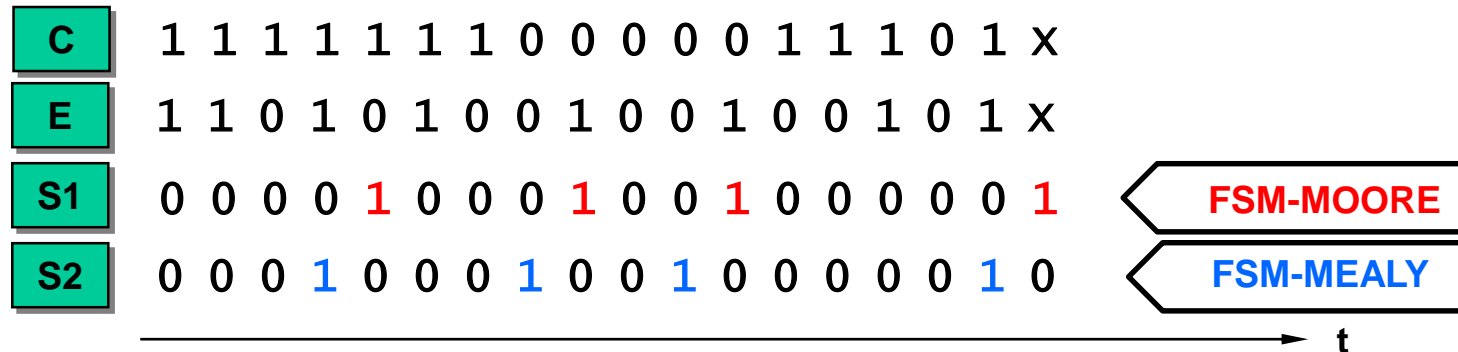
$D_i^{n+1}$
0
1
0
1

$T_i^{n+1}$
0
1
1
0

$J_i^{n+1}$	$K_i^{n+1}$
0	X
1	X
X	1
X	0

# Other Moore or Mealy circuits

**Example\_2:** Detect a complete sequence without overlapping of three consecutive bits as a function of a control signal C (if C = 0 sequence 100, if C = 1 sequence 101). If in the middle of the search for a valid sequence, the control signal is changed, the circuit must respond on the next clock edge to the new signal.



# Other Moore or Mealy circuits

**Example\_3:** You are asked to control a beverage vending machine. The inputs to the circuit are signals that come from the input of coins. The outputs are signals that must be sent to dispense the drink (only) and activate the return of the change when necessary. The price of the drink is € 1.5, the system only accepts coins of 50 cents, € 1 and € 2, coins that are detected one by one independently and in a single cycle of the system. Design the system using Moore and Mealy approaches.

**Example\_4:** There are two processors that share the same memory area but the memory only accepts the access from one at a time. The processors can request access to the memory by the  $X_i$  lines ( $i = 1,2$ ). Upon a request of access by any of the processors, the FSM to design, activates one of the  $Z_i$  lines ( $i = 1,2$ ), to indicate which processor grants the permission use. If the memory is busy, no new requests are answered, until it is free. If the memory is free, the request of the processor that demands it is accepted. In the case of a simultaneous request, the FSM applies the following priority protocol, the highest priority is for the processor that has not used the memory for a longer time. It is requested to design the Moore diagram for this control FSM.

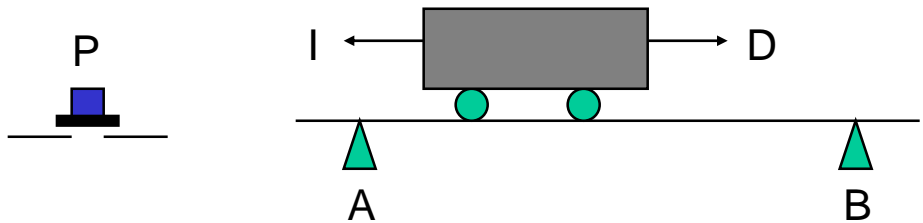
# Other Moore or Mealy circuits

**Example\_5:** In order to control the movement of a material transport trolley between two points A and B, two presence sensors are available at those points and one Push-button. The motor is controlled by two signals (D and I) to move to the left and right. . The operation of the system must be as follows:

- ☞ Initially, the trolley is stopped at position A.
- ☞ When the Push button (P) is pressed, the trolley will start to move through position B without stopping. Once arrived at B, the trolley starts moving in the opposite direction, without stopping, until reaching the starting point A. During the movement, the value of P is not taken into account by the system.
- ☞ If the P button is not pressed at the point A, the trolley will stop. If the P button is pressed when reaching A, a new movement will be started.

Determine:

- Mealy states diagram
- Truth table
- Minimized state equations



1000

1000 JOURNAL OF CLIMATE


$$D_1 = Q_1 Q_0 X + Q_1 Q_0 S X + Q_1 Q_0 S X$$
$$Z = Q_1 Q_0$$

$$Z = Q_1Q_0$$

## Example\_3:

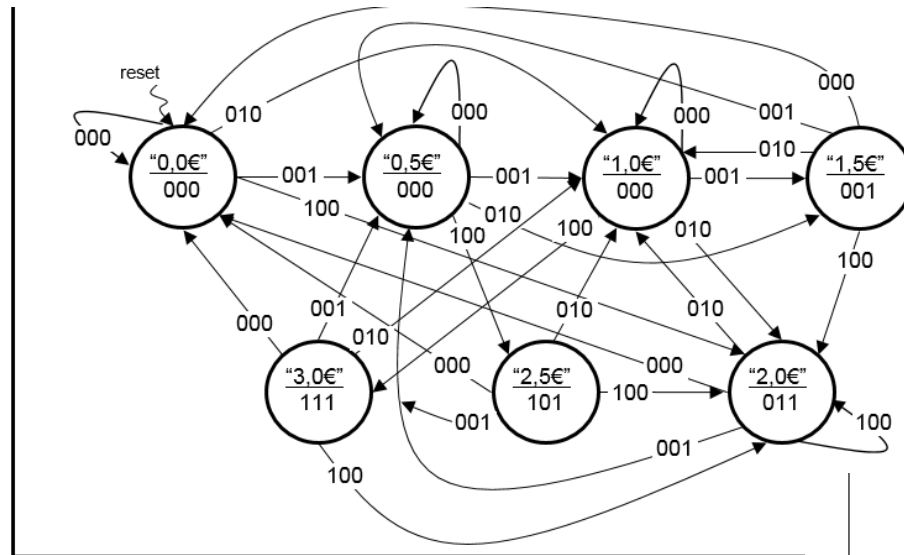
**Solution:**

Assign the inputs  $X_2, X_1, X_0$  is equal to introduce a coin of 2€, 1€ and 0.5€ respectively.

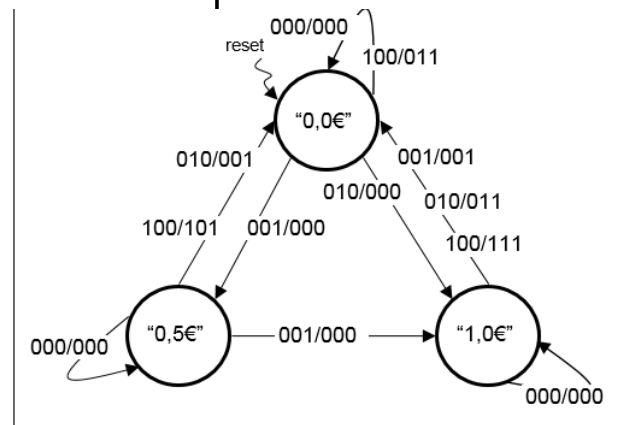
Any combination which implies receive more than one coin at a time is not considered.

Output assignment  $Z_2, Z_1, Z_0$  is equal to give back 1€, 0.5€ and dispense drink respectively.

### Moore Design (7 states)



### Mealy design (3 states)



**Solution:**

Assign the states:

$Q_1 Q_0 = 00$  Receives 0 €

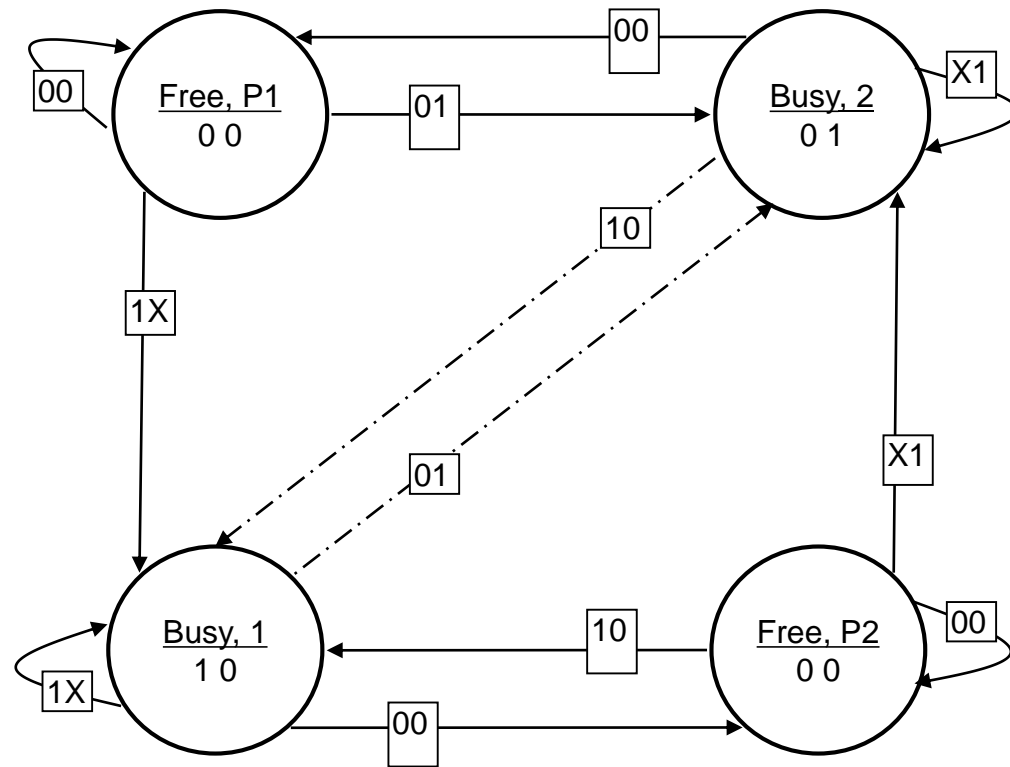
$Q_1 Q_0 = 01$  Receives 0.5 €

$Q_1 Q_0 = 10$  Receives 1 €

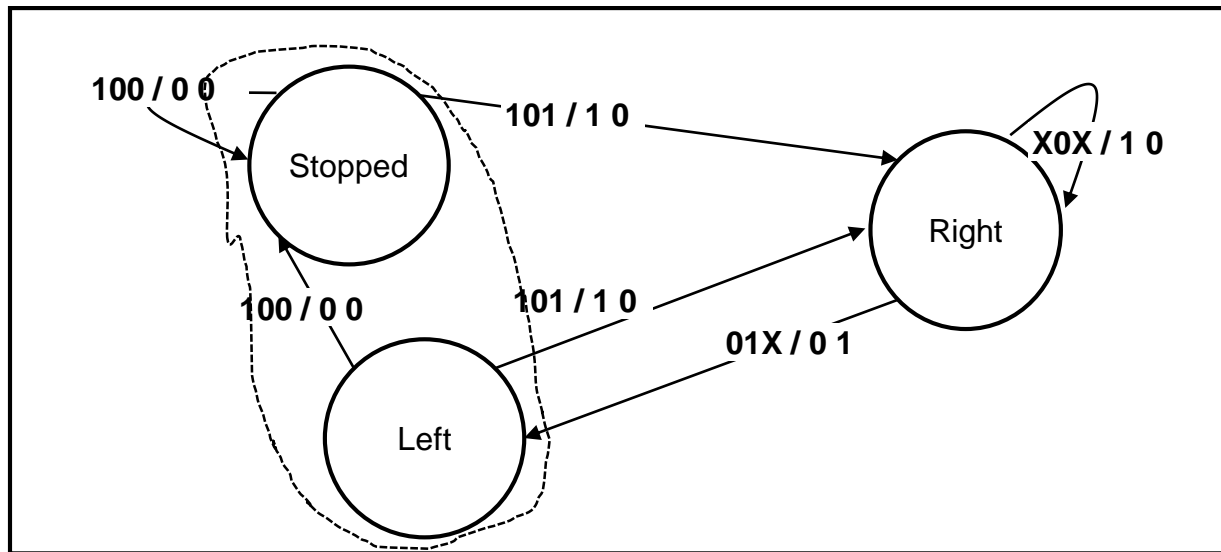
**Output Equation:**  $Z_0 = X_2 + Q_0 X_1 + Q_1 X_1 + Q_1 X_0$ ;  $Z_1 = Q_1 X_1 + \overline{Q_0} X_2$ ;  $Z_2 = Q_1 X_2 + Q_0 X_2$

**State Equation:**  $D_0 = \overline{Q_1} \overline{Q_0} X_0 + Q_0 \overline{X_2} \overline{X_1} X_0$ ;  $D_1 = Q_0 X_0 + \overline{Q_1} \overline{Q_0} X_1 + Q_1 \overline{X_2} \overline{X_1} X_0$

## Example\_4:



## Example\_5:



**ABP / DI**

If left  $\equiv$  Stopped

