Problema de la Semana 9. Pablo Cuesta Sierra. NIA: 422974

(2). La familia de planos: $2\lambda x + (\lambda+1)y - 3(\lambda-1)z + 2\lambda - 4 = 0$ en $\mathbb{A}^3(\mathbb{R})$

(a) Dem. que estos planos tienen una recta en común.

Sea $\pi_\lambda = \{2\lambda x + (\lambda+1)y - 3(\lambda-1)z + 2\lambda - 4 = 0\}$

Dados $\alpha \neq \beta \in \mathbb{R}$, tomemos π_α, π_β en $\mathbb{A}^3(\mathbb{R})$

$$\pi_\alpha \cap \pi_\beta = \left\{\begin{array}{l} 2\alpha x + (\alpha+1)y - 3(\alpha-1)z + 2\alpha - 4 = 0 \\ 2\beta x + (\beta+1)y - 3(\beta-1)z + 2\beta - 4 = 0 \end{array}\right\}$$

$$= \left\{\begin{array}{l} 2(\alpha-\beta)x + (\alpha-\beta)y - 3(\alpha-\beta)z + 2(\alpha-\beta) = 0 \\ 2\beta x + (\beta+1)y - 3(\beta-1)z + 2\beta - 4 = 0 \end{array}\right\}$$

$$= \left\{\begin{array}{l} 2x + y - 3z + 2 = 0 \\ 2\beta x + (\beta+1)y - 3(\beta-1)z = 4 - 2\beta \end{array}\right\} = \left\{\begin{array}{l} 2x + y - 3z + 2 = 0 \\ 0 + y + 3z = 4 \end{array}\right\}$$

$$\Rightarrow \left\{\begin{array}{l} y = 4 - 3t \\ z = t \\ x = \frac{1}{2}(-2 - y + 3t) = -1 + \frac{1}{2}(-4 + 3t + 3t) \end{array}\right\} = \pi_\alpha \cap \pi_\beta$$

$$\Rightarrow \pi_\alpha \cap \pi_\beta = \left\{\begin{array}{l} x = -3 + 3t \\ y = 4 - 3t \\ z = t \end{array} \middle| \; t \in \mathbb{R} \right\} \quad \forall \alpha \neq \beta \in \mathbb{R}.$$

b) Determina los planos de la familia que pasan por $(1, -1, 2)$

Si $(1,-1,2) \in \pi_\lambda \Leftrightarrow 2\lambda - \lambda - 1 - 6\lambda + 6 + 2\lambda - 4 = 0 \Leftrightarrow -3\lambda + 1 = 0 \Rightarrow \boxed{\lambda = \frac{1}{3}}$

$$\boxed{(1,-1,2) \in \pi_\lambda \Leftrightarrow \lambda = \frac{1}{3}}$$

Escaneado con CamScanner

c) Determine cuáles de los planos de esta familia son paralelos a:

$$L = \begin{cases} x + 3z = 1 \\ y - 5z = -2 \end{cases} = \left\{ \begin{array}{l} x = 1 - 3t \\ y = -2 + 5t \\ z = t \end{array} \;\middle|\; t \in \mathbb{R} \right\} = \underbrace{\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}}_{a} + \underbrace{\left\langle \begin{pmatrix} -3 \\ 5 \\ 1 \end{pmatrix} \right\rangle}_{V_1}$$

$$\pi_\lambda = \{ 2\lambda x + (\lambda+1) y - 3(\lambda - 1) z + 2\lambda - 4 = 0 \}$$

$$= p + W_\lambda \;, \quad p = \begin{pmatrix} -3 \\ 4 \\ 0 \end{pmatrix}$$, que está en la recta de la intersección de todos los planos de la familia

$$W_\lambda = \{ 2\lambda x + (\lambda + 1) y - 3(\lambda - 1) z = 0 \}$$

$$\boxed{L \parallel \pi_\lambda} \iff V_1 \subset W_\lambda \iff \begin{pmatrix} -3 \\ 5 \\ 1 \end{pmatrix} \in W_\lambda$$, ya que $V_1 = \left\langle \begin{pmatrix} -3 \\ 5 \\ 1 \end{pmatrix} \right\rangle$

$$\iff -6\lambda + (\lambda+1)\cdot 5 - 3(\lambda - 1) = 0$$

$$\iff -6\lambda + 5\lambda + 5 - 3\lambda + 3 = 0$$

$$\iff -4\lambda = -8$$

$$\iff \boxed{\lambda = 2}$$