Ejercicio: sea $n \geq 0$, $a < b$

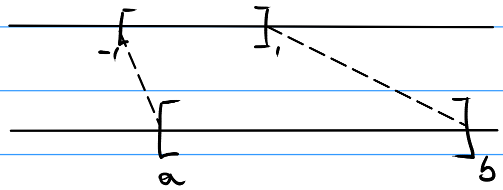
1) escribir los nodos de Chebyshev en $[a,b]$

2) demostrar que, si $f \in C^{n+1}([a,b])$ y $p \in P_n$ es el polinomio interp. por los nodos de C.

$$\Rightarrow \quad |f(x) - p(x)| \leq \max_{[a,b]} |f^{(n+1)}| \cdot \frac{1}{2^n (n+1)!} \left(\frac{b-a}{2}\right)^{n+1} \quad \forall x \in [a,b]$$

1) nodos de Chebyshev en $[-1,1]$

$$t_k = \cos\left(\frac{2k+1}{n+1} \frac{\pi}{2}\right), \quad k \in \{0, .., n\}$$



$x(t) = \frac{b-a}{2} t + \frac{b+a}{2}$ es t.q.

$x(-1) = a$, $x(1) = b$

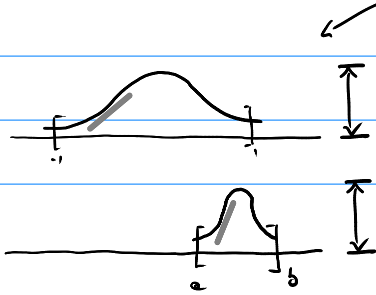
$$\Rightarrow \quad x_k = \frac{b-a}{2} t_k + \frac{b+a}{2}$$

2) sabemos que

- $\max_{[a,b]} |f - p_n| \leq \max_{[a,b]} |f^{(n+1)}| \cdot \frac{1}{(n+1)!} \max_{[a,b]} |\pi_{n+1}|$

- con nodos de Cheby en $[-1,1]$

$$\max_{[-1,1]} |f - p_n| \leq \underbrace{\max_{[-1,1]} |f^{(n+1)}|} \cdot \frac{1}{(n+1)!} \underbrace{\overbrace{\max_{[-1,1]} |\tilde{T}_{n+1}|}^{= 1/2^n}}$$



reescalamiento: influye sobre la magnitud de las derivadas, no de la función si $f:[a,b]\to\mathbb{R}$ y decimos

$\tilde{f}:[-1,1]\to\mathbb{R}$ la función $\tilde{f}(t)=f\left(\frac{b-a}{2}t+\frac{b+a}{2}\right)$

$$\frac{d}{dt}\tilde{f}(t)=\frac{b-a}{2}f'\left(\frac{b-a}{2}t+\frac{b+a}{2}\right)$$

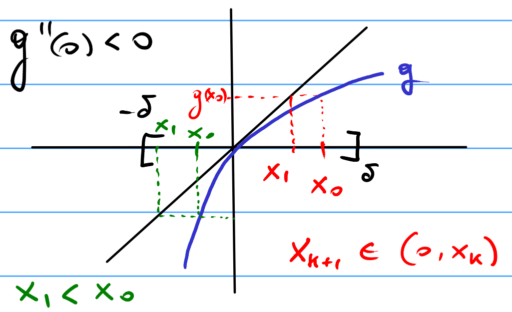
$$\frac{d^k}{dt^k}\tilde{f}(t)=\left(\frac{b-a}{2}\right)^k f^{(k)}\left(\frac{b-a}{2}t+\frac{b+a}{2}\right)$$

$$\Rightarrow \max_{[-1,1]}|\tilde{f}^{(k)}|=\left(\frac{b-a}{2}\right)^k\max_{[a,b]}|f^{(k)}|$$

$\cdot\sim\cdot$

Ej. 14 hoja 7 : $g\in\mathcal{C}^2(\mathbb{R})$, $g(0)=0$, $g'(0)=1$

a) estudiar la convergencia de la iteración $x_{k+1}=g(x_k)$ si $g''(0)\neq 0$



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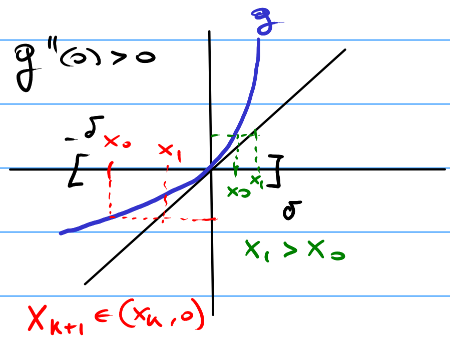


$$x_{k+1}=g(x_k)=\overset{0}{g(0)}+\overset{1}{g'(0)}\,x_k+\frac{1}{2}g''(\xi_k)\,x_k^2$$

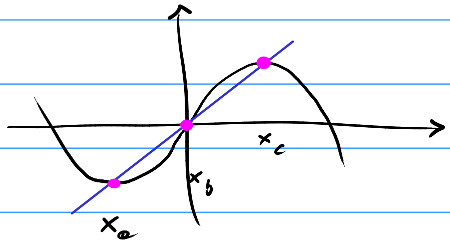
↳ entre 0 y x_k

$$\boxed{x_{k+1}=x_k\left(1+\frac{1}{2}g''(\xi_k)\,x_k\right)}$$

$g''(0)<0$ $\begin{cases}\cdot\ \text{si } x_k\in(0,\delta) \text{ y } g''(0)<0 \Rightarrow x_{k+1}\in(0,x_k)\\ \cdot\ \text{si } x_k\in(-\delta,0) \text{ y } g''(0)<0 \Rightarrow x_{k+1}<x_k\end{cases}$

Ejercicio 6 hoja 7

$g(x) = \frac{\pi}{2} \sin x$



x_b repulsor

x_a, x_c atract.

$g\left(\pi/2\right) = \pi/2$

Ejercicio 5, hoja 7

$g(x) = \frac{x}{2} - x^3 = x\left(1/2 - x^2\right)$

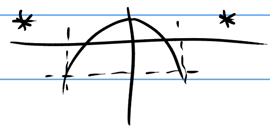
$g(x) = x$ — $x = 0$

— $x \neq 0$ $\quad 1/2 - x^2 = 1$, $x^2 = -1/2$

$g(\beta) = -\beta$ — $\beta = 0$

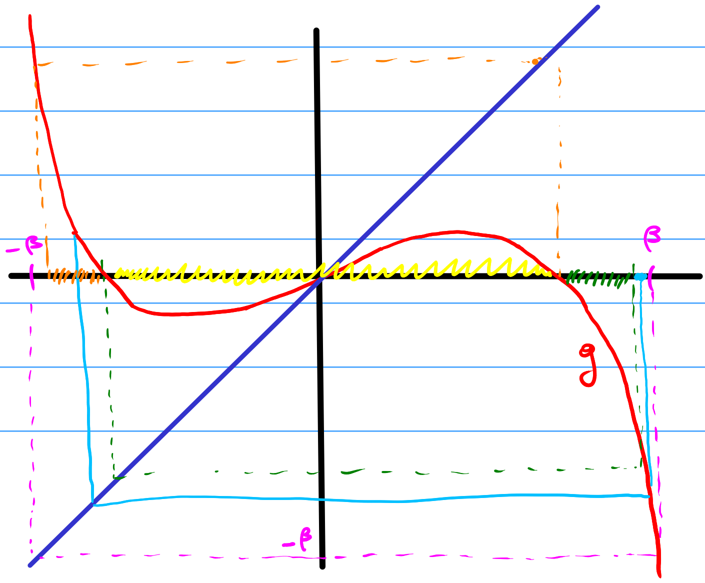
— $\beta \neq 0$ $\quad 1/2 - \beta^2 = -1$, $\beta = \pm\sqrt{3/2}$

$g'(x) = \frac{1}{2} - 3x^2$,



$g' = -1$

$1/2 - 3x^2 = -1 \qquad 3x^2 = 3/2 \qquad x = \pm 1/\sqrt{2}$



$x_0 = \beta \qquad x_1 = -\beta \qquad x_2 = \beta$