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(2) $L_1 = \begin{cases} x+z+t=1 \\ y-z-t=2 \end{cases}$ $\quad L_2 = \begin{cases} x+y=1 \\ y-z-3t=3 \end{cases}$

$$L_1 = \left\{ \begin{array}{l} x = 1-z-t \\ y = 2+z+t \\ z = z \\ t = t \end{array} \right\} = \underbrace{\begin{pmatrix} 1 \\ 2 \\ 0 \\ 0 \end{pmatrix}}_{a} + \underbrace{\left\langle \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix} \right\rangle}_{\vec{L_1}} = a + \vec{L_1}$$

$$L_2 = \left\{ \begin{array}{l} x = 1-y \\ y = y \\ z = -3+y-3t \\ t = t \end{array} \right\} = \underbrace{\begin{pmatrix} 1 \\ 0 \\ -3 \\ 0 \end{pmatrix}}_{b} + \underbrace{\left\langle \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -3 \\ 1 \end{pmatrix} \right\rangle}_{\vec{L_2}} = b + \vec{L_2}$$

$$d(L_1, L_2) = d(a, L), \quad L = b + (\vec{L_1} + \vec{L_2})$$

$$\vec{L_1} + \vec{L_2} = \left\langle \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -3 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix} \right\rangle$$

$$\begin{pmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & -3 & 1 \\ -1 & 1 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix} \sim \begin{pmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -2 & 0 \end{pmatrix}$$

$$\sim \begin{pmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Rightarrow \vec{L_1} + \vec{L_2} = \left\langle \vec{v_1} = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}, \vec{e_3}, \vec{e_4} \right\rangle$$

$$W = \vec{L_1} + \vec{L_2}, \quad W^{\perp} = \langle \vec{u} \rangle, \quad \vec{u} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix},$$

$$\left. \begin{array}{l} \langle \vec{v_1}, \vec{u} \rangle = a - b = 0 \\ \langle \vec{e_3}, \vec{u} \rangle = c = 0 \\ \langle \vec{e_4}, \vec{u} \rangle = d = 0 \end{array} \right\} |\vec{u}| = 1 \Rightarrow \vec{u} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$d(L_1, L_2) = d(a, L) = \| p_{W^{\perp}}(\vec{ab}) \|$$

$\mathcal{B} = \{\vec{u}, \vec{v_1}, \vec{e_3}, \vec{e_4}\}$ es base de $\mathbb{R}^4$,

sea $\vec{v} \in \mathbb{R}^4$, $\vec{v} = \alpha_1 \vec{u} + \alpha_2 \vec{v_1} + \alpha_3 \vec{e_3} + \alpha_4 \vec{e_4}$

$$P_{W^{\perp}}(\vec{v}) = \alpha_1 \vec{u} \Rightarrow P_{W^{\perp}}(\vec{v}) = \langle \vec{v}, \vec{u} \rangle \vec{u}$$

$$\Rightarrow d(L_1, L_2) = \| p_{W^{\perp}}(\vec{ab}) \| = |\langle \vec{ab}, \vec{u} \rangle| \cdot \overbrace{\|\vec{u}\|}^{(1)} =$$

$$= \left| \left\langle \begin{pmatrix} 0 \\ -2 \\ -3 \\ 0 \end{pmatrix}, \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \\ 0 \end{pmatrix} \right\rangle \right| = \frac{2}{\sqrt{2}} = \sqrt{2}$$