

AI HW 1: Search and games

1. Informed search and A*

a) States: $\{(a,b) : a,b \in \{1,2,3,4,5\}\}$. $SO = (1,1)$. $SF = (5,5)$

Operators: North, South, East, West. No preconditions.

Postconditions:

- North $(a,b) \rightarrow (a, b+1)$ if $b < 5$, else (a,b)
- South $(a,b) \rightarrow (a, b-1)$ if $b > 1$, else (a,b)
- East $(a,b) \rightarrow (a+1, b)$ if $a < 5$, else (a,b)
- West $(a,b) \rightarrow (a-1, b)$ if $a > 1$, else (a,b)

Cost associated: Walls = $\{(1,2), (2,2), (3,2), (5,2), (2,4), (3,4), (4,4), (5,4)\}$

If prev-state \notin Walls and next-state \in Walls: cost = 2
Else: cost = 1.

b) i) min branching factors 2 (the corners). max hor. factor: 4.

ii) Avoid cycles: use a graph alg. (remove repeated nodes).

c) Admissible heuristic: $h(a,b) = |5-a| + |5-b| = 10 - (a+b)$

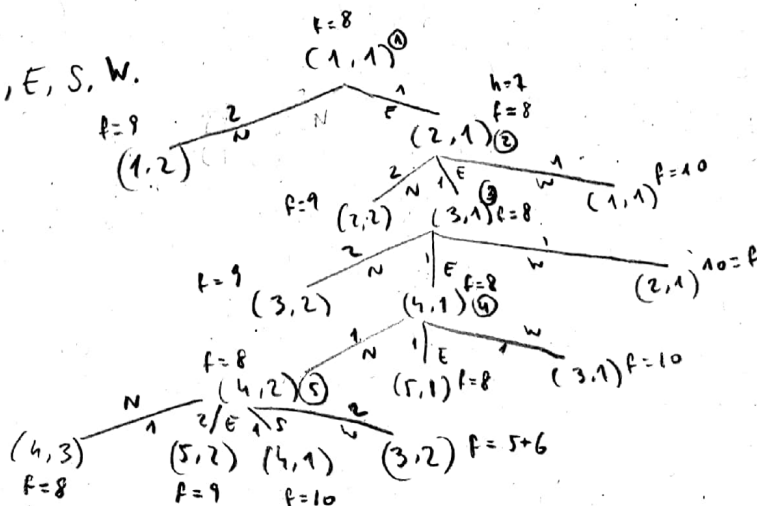
If the cost was 1 for every jump, $h(a,b)$ would be exactly the cost to reach $(5,5)$. As the cost is higher or equal to 1,

$h(a,b)$ is lower or equal to $g^*(a,b)$. Therefore, it's admissible.

d) Yes. Because h' is consistent: If n' is successor of n , then $h(n') = h(n) \pm 1 \geq h(n) - 1$ and $\text{cost}(n \rightarrow n') \in \{1, 2\} \Rightarrow \text{cost}(n \rightarrow n') \geq 1 \Rightarrow h(n') + \text{cost}(n \rightarrow n') \geq h(n) - 1 + 1 = h(n)$

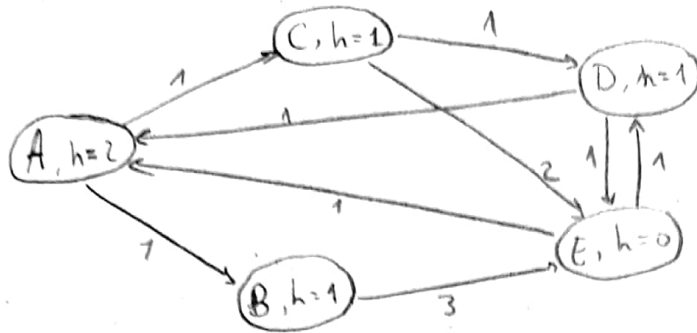
e) Yes.

f) order: N, E, S, W.



2. Search with A*

i)



h is monotonic:

Node:

A

B

C

D

E

Successors

B, C

E

D, E

A, E

A, D

$$\rightarrow h(A) = 2 \leq \begin{cases} 1 + h(C) \\ 1 + h(B) \end{cases}$$

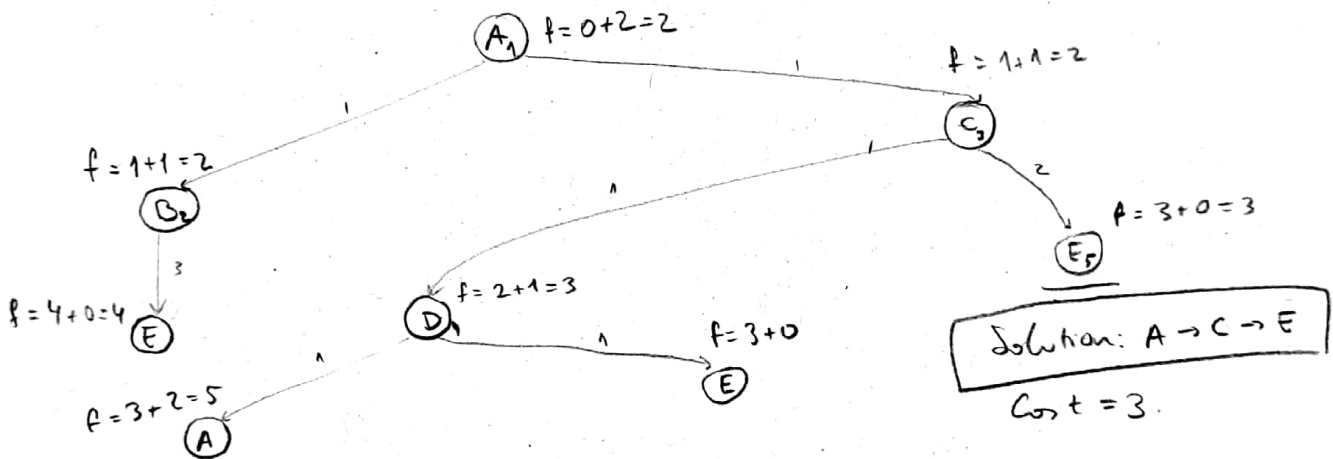
$$\rightarrow h(B) = 1 < \text{cost}(B, E) + h(E) = 3$$

$$\rightarrow h(C) = 1 < \begin{cases} 1 + h(D) \\ 2 + h(E) \end{cases}$$

$$\rightarrow h(D) = 1 \leq \begin{cases} 1 + h(A) \\ 1 + h(E) \end{cases}$$

$$\rightarrow h(E) = 0 < \begin{cases} 1 + h(D) \\ 1 + h(A) \end{cases} \quad \square$$

ii) Beside each node: $f = g + h$ (notation). Subindex: order of expanded nodes.

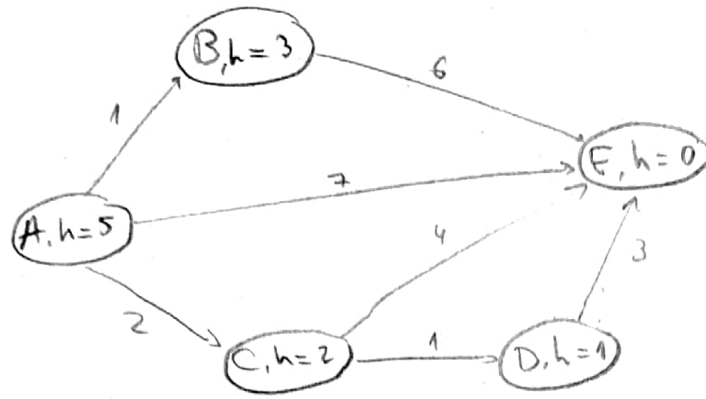


iii) This search has found an optimal path.

The heuristic defined is consistent (i), so the graph search guarantees to find an optimal path.

3. A* Search

i)

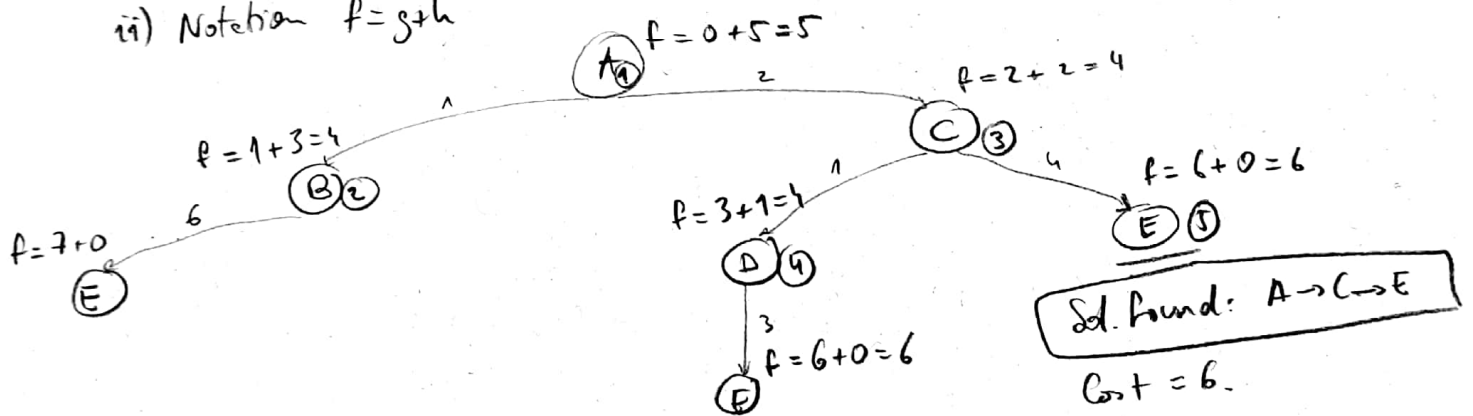


h is not monotonic: $h(A) = 5 > \text{cost}(A, B) + h(B) = 1 + 3 = 4$.

h is admissible: $h(\text{goal}) = h(E) = 0$. Let $M(\text{node}) = \min \text{cost path}(\text{node}, E)$

$$\begin{cases} h(A) = 5 \leq M(A) = 6 \\ h(B) = 3 \leq M(B) = 6 \\ h(C) = 2 \leq M(C) = 4 \\ h(D) = 1 \leq M(D) = 3 \end{cases}$$

ii) Notation $f = g + h$

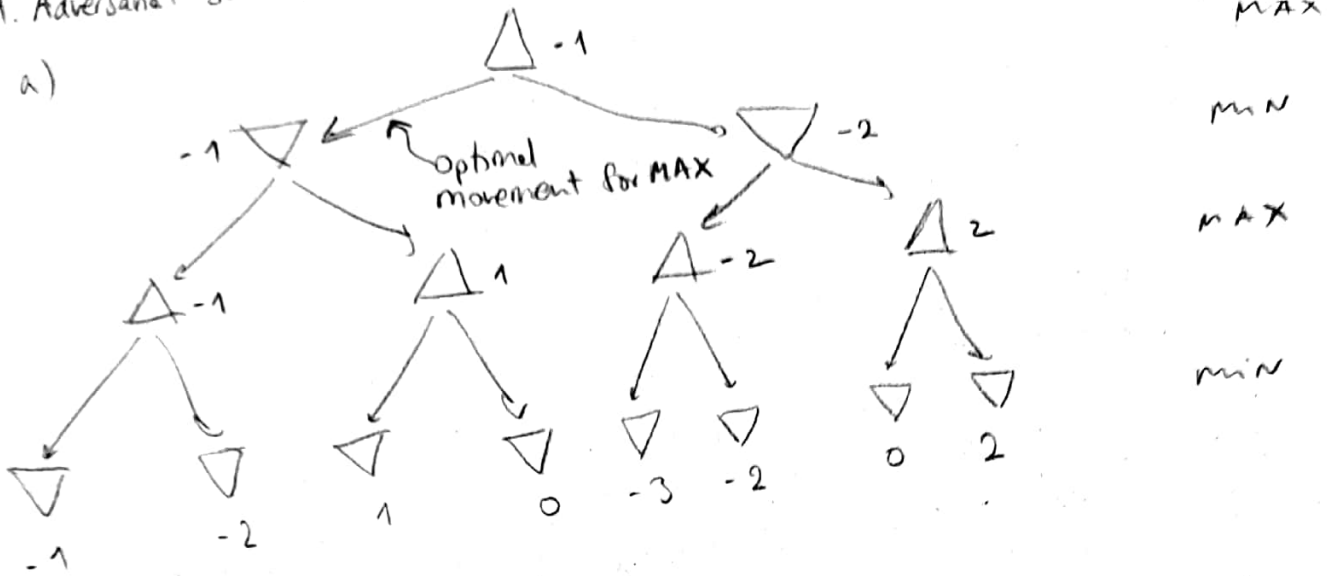


iii) The algorithm has found an optimal path.

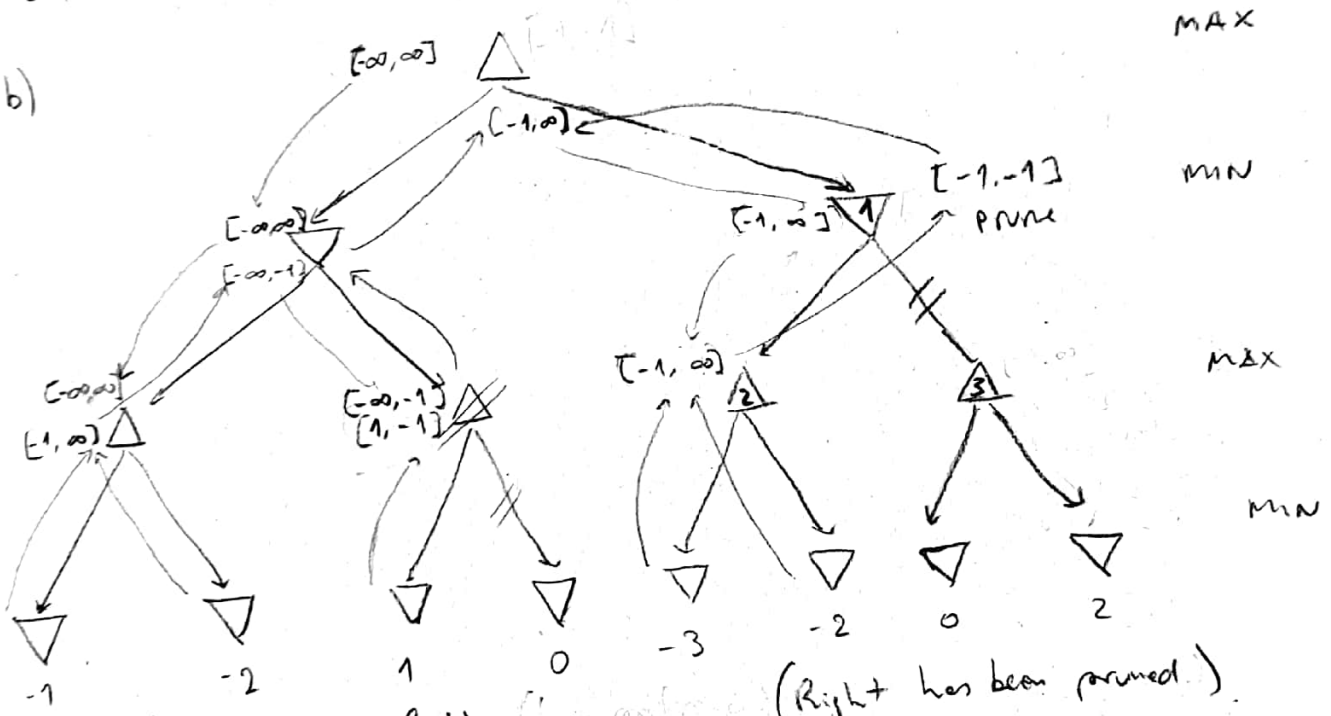
A* does not guarantee, with elimination of repeated states, but a non-monotonic heuristic, to find an optimal path.

4. Adversarial Search.

a)



b)



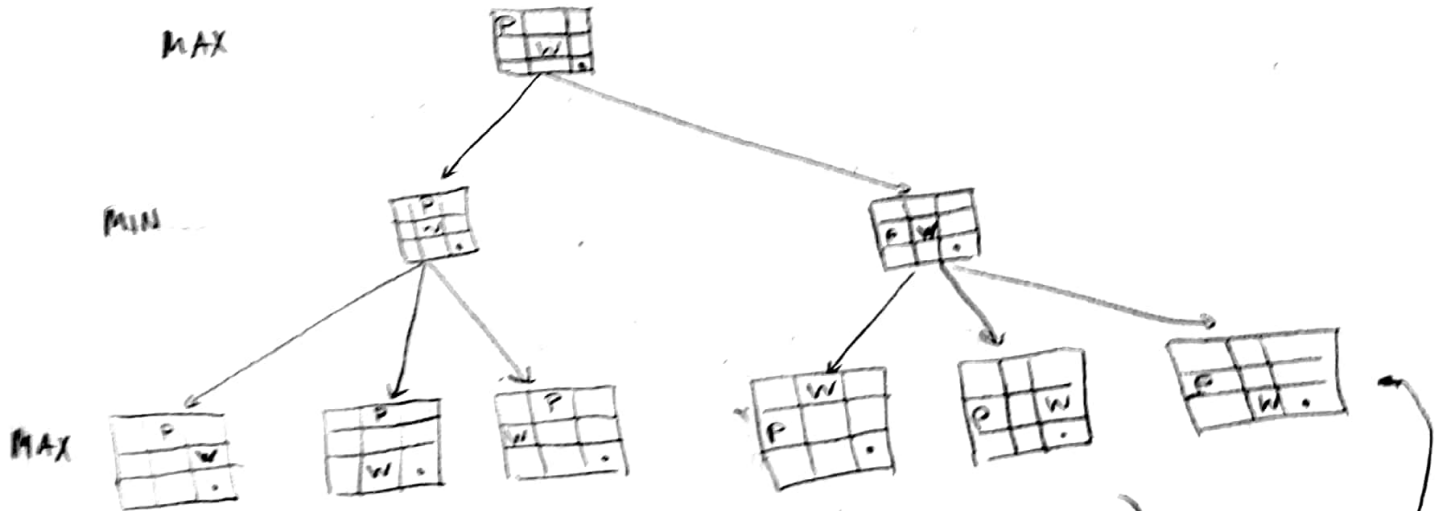
c) $A=B=1, C=D=0$

This way, node 2 would have: $[1, \infty] \rightarrow$ node 1 $[-1, 1]$
 \rightarrow node 3 $[-1, 1] \rightarrow$ node 3 $[-0, 1] \rightarrow$ node 1 $[-1, 0]$

i.e. no pruning

5 Adversarial search

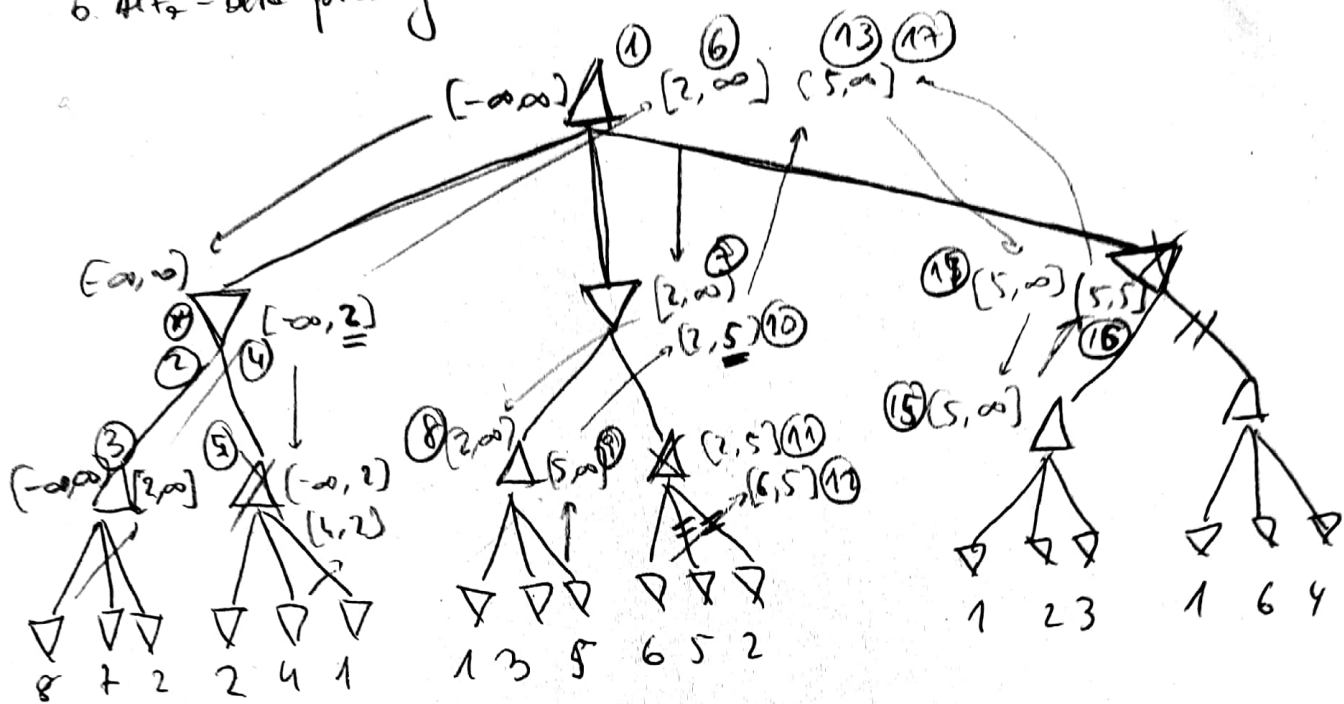
a) $P \equiv \text{Pacman}$, $W \equiv \text{Wall}$, $\cdot \equiv \text{dot}$. Order: $\uparrow, \rightarrow, \downarrow, \leftarrow$



c) Pacman has eaten 0 dots in every state shown in a),
 So every node has a value of 0. d) The value would be 1. Pacman here can always choose a move to eat the dot, whatever the W does.



6. Minimax - beta pruning



a) Minimax value at root: 5

b) Optimal play: left *

c) Left to right

d) MAX: Δ , MIN: ∇

e) Doesn't look like a 0-sum game

f) Minimax does not guarantee the optimal strategy, because it relies on the assumption that MIN plays optimally.