Dijkstra's algorithm (1/2)

- G: Connected, simple, weighted graph with positive weights.
- Initial vertex = a
- Final vertex = z
- Intermediate vertices = $V_1, \ldots. V_n$
- Weights = $w(V_i, V_j)$
- S = "distinguished" set of vertices
- L = Minimum distances

Dijkstra's algorithm (2/2)

procedure Dijkstra (G)

for i := 1 **to** n

    $L(V_i) := \infty$

L(a) := 0

S := Ø

while z NOT in S

begin

    u := vertex with minimum L(u) among those not in S

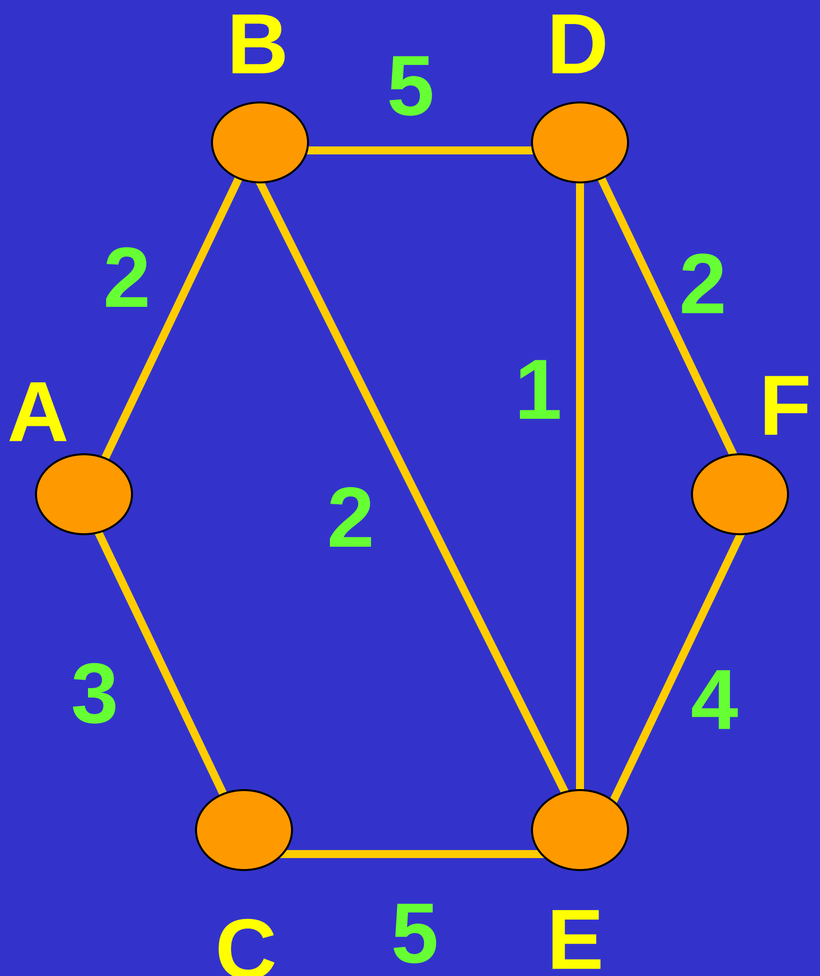
    S := S UNION {u}

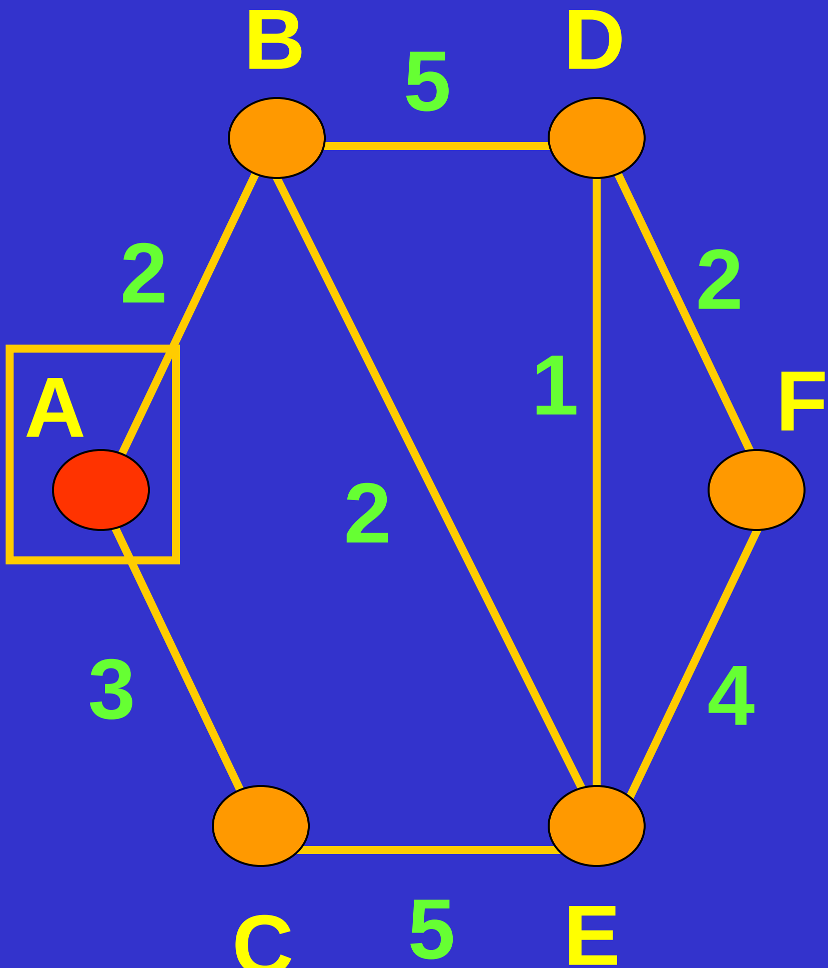
    for all vertices v not S

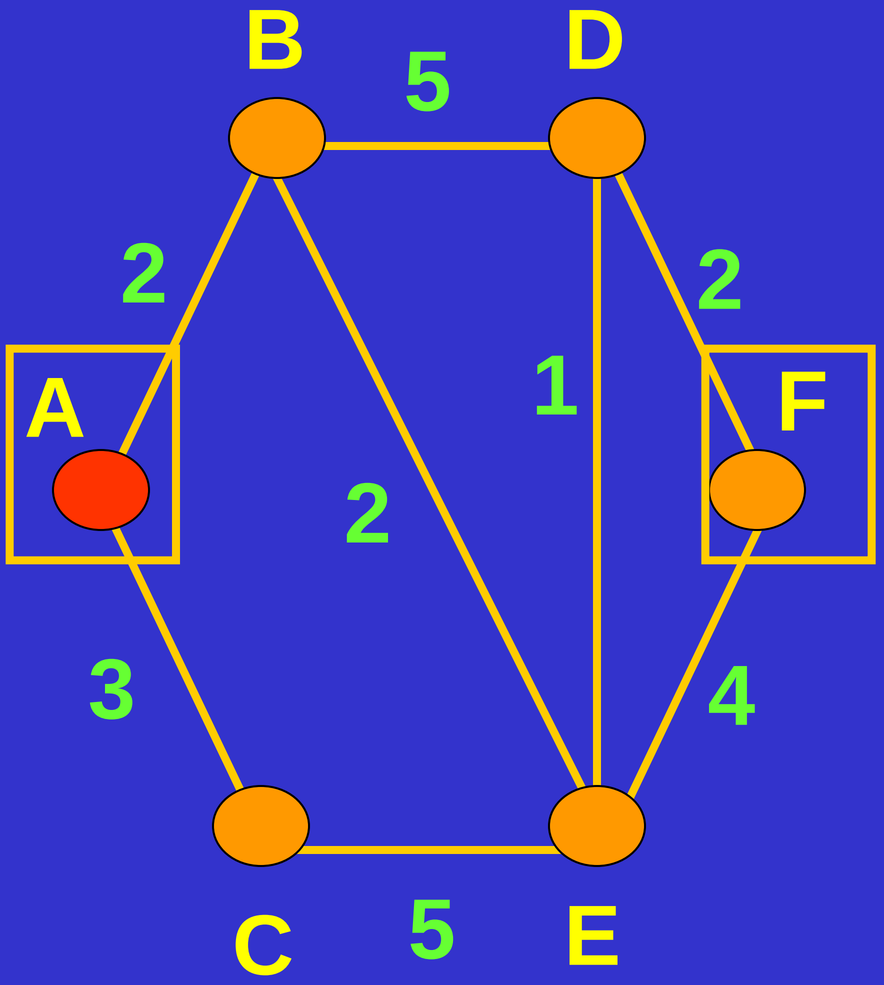
        if L(u) + w(u,v) < L(v) **then** L(v) := L(u) + w(u,v)

    {include in S the vertex with minimum L}

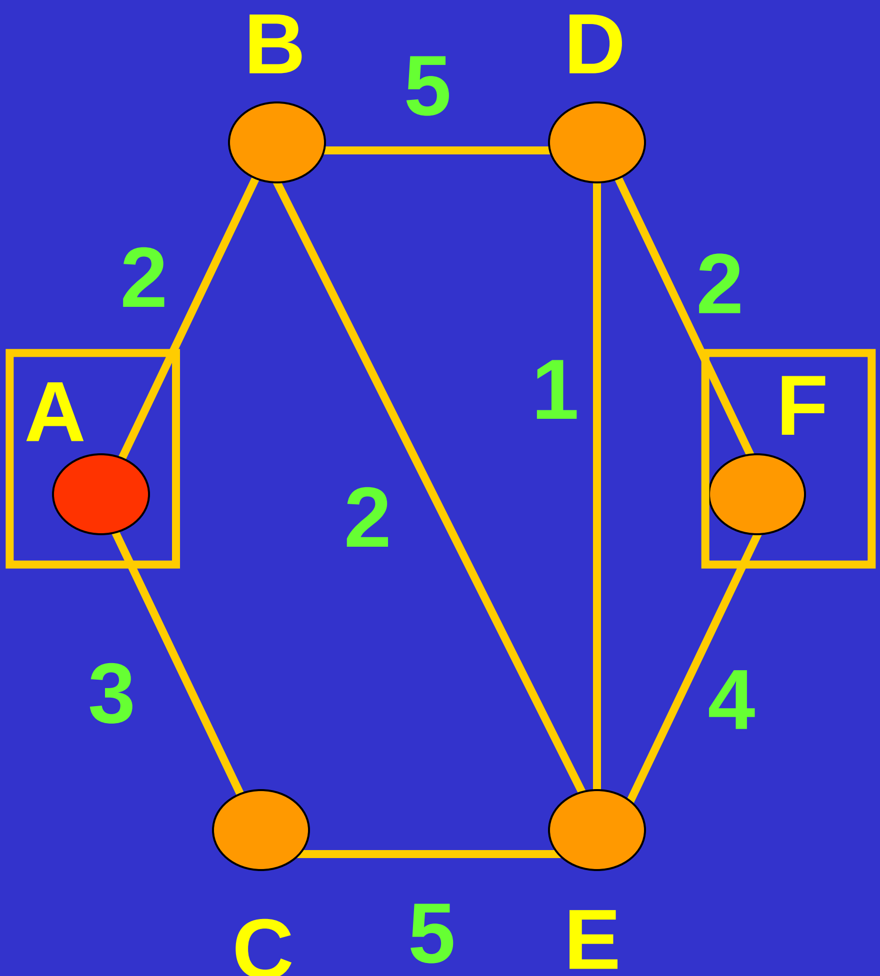
end {L(z) = length of the shortest path between a and z}


B
5
D
2
2
1
A
F
2
3
4
C
5
E


B
5
D
2
2
1
F
A
2
3
4
C
5
E



B
D
5
2
2
1
A
F
2
3
4
C
5
E



$\underline{V_{\underline{0}}}$

A

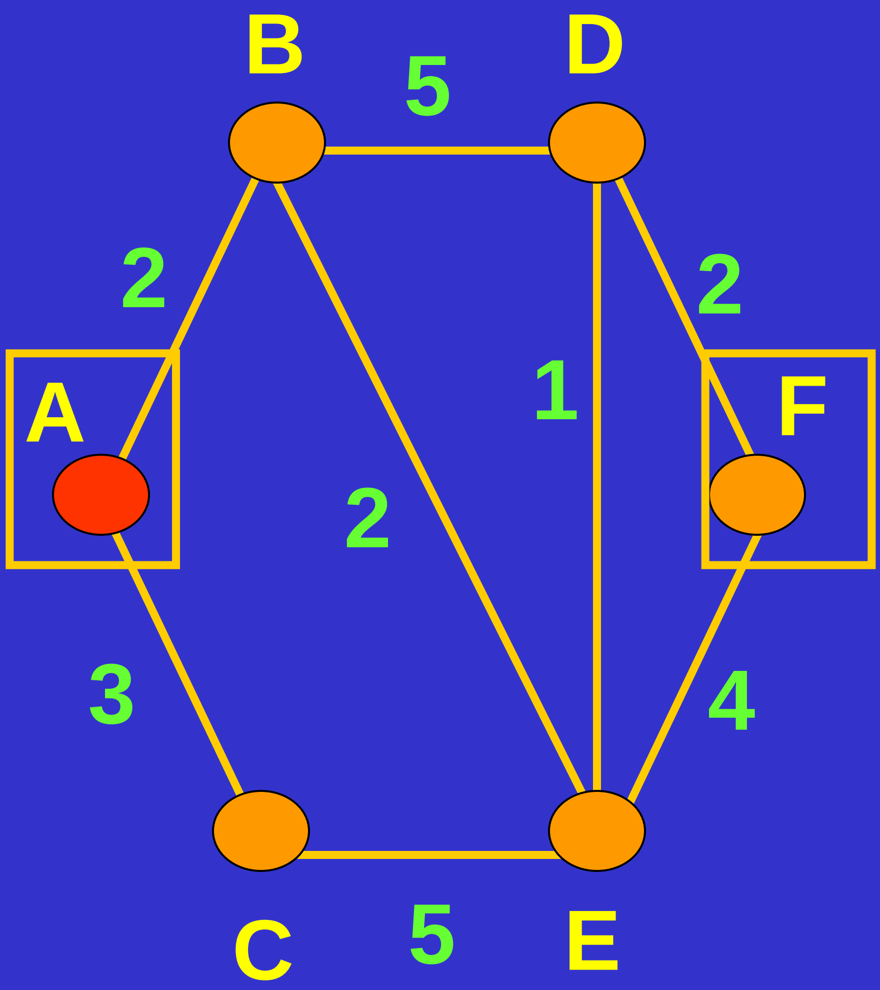
B

C

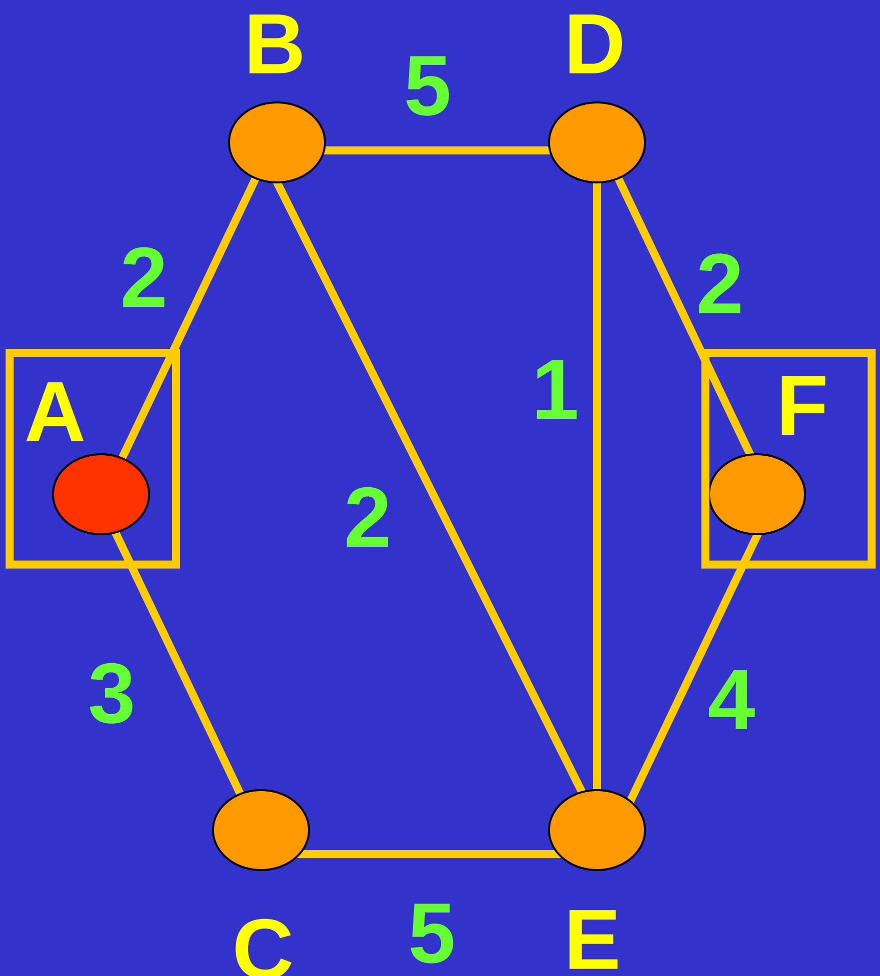
D

E

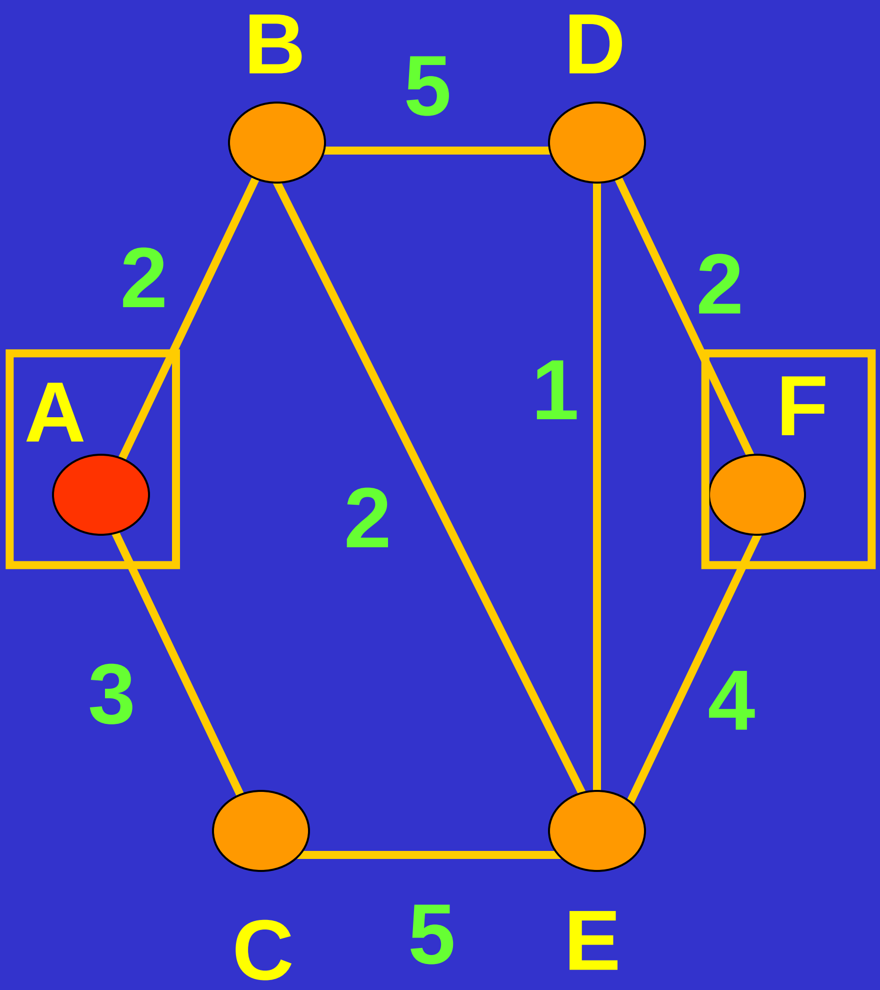
F



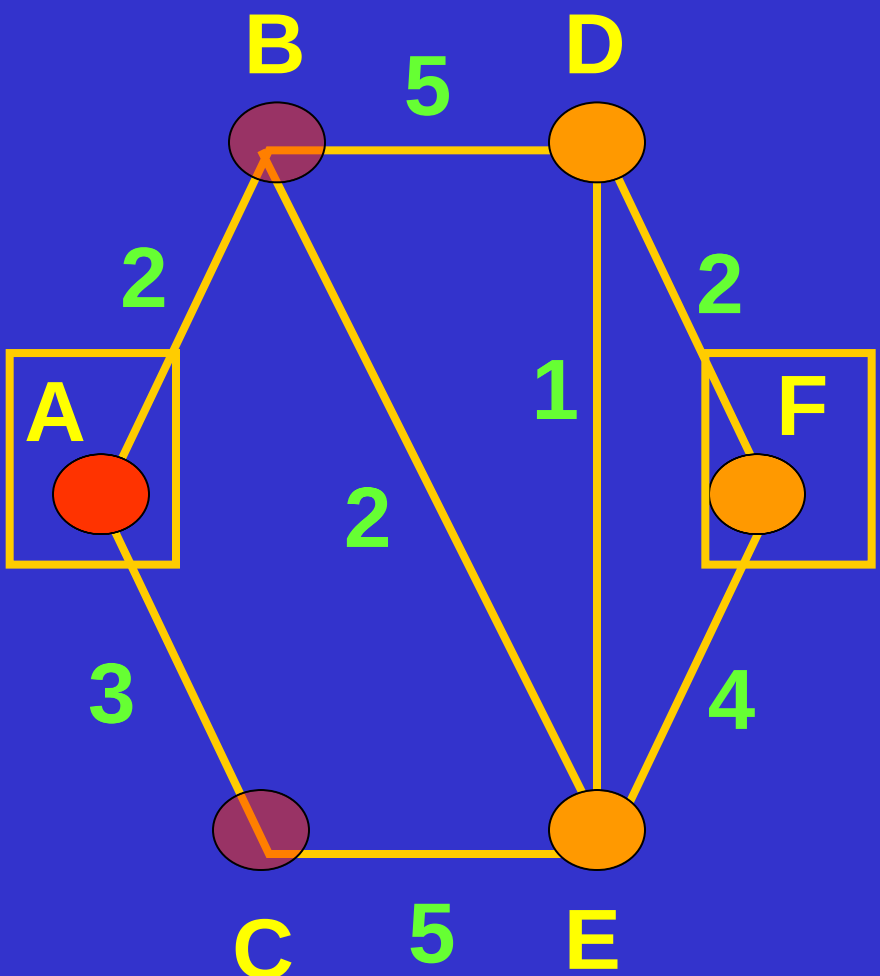
$\underline{V_{\underline{0}}}$	$\underline{L_{\underline{0}}}$
A	(0)
B	∞
C	∞
D	∞
E	∞
F	∞



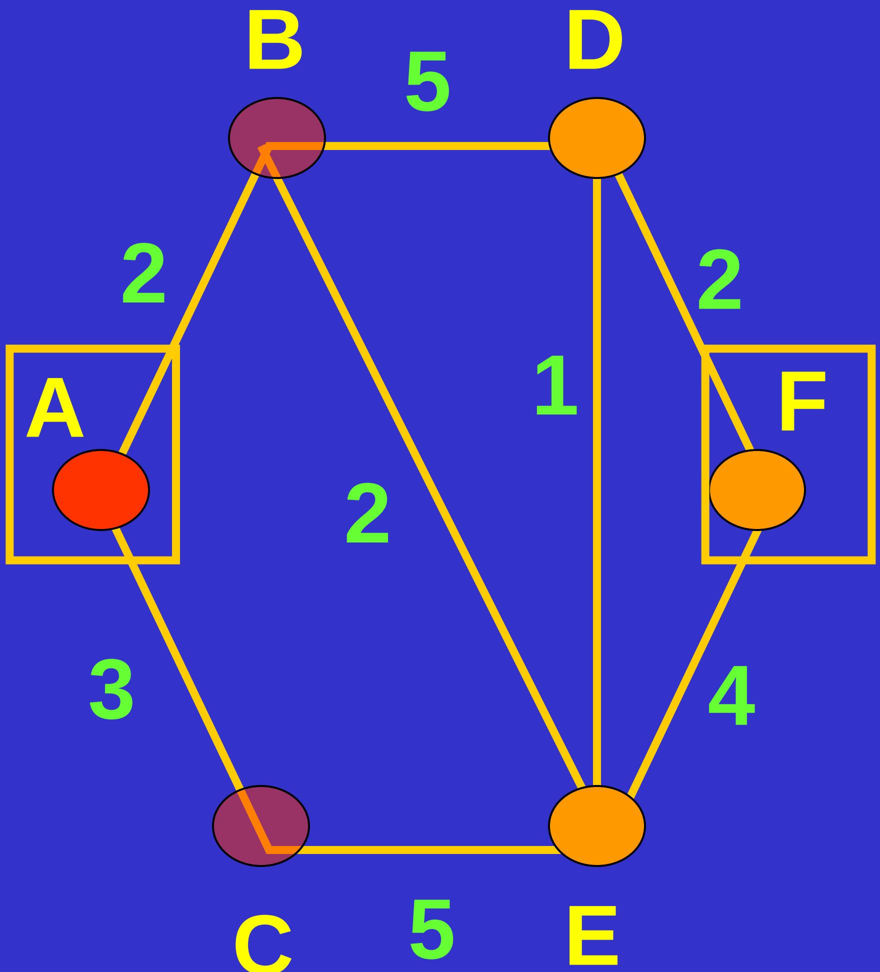
V_1	L_0
A*	(0)
B	∞
C	∞
D	∞
E	∞
F	∞



V_1	L_1	L_0
A*	-	(0)
B		∞
C		∞
D		∞
E		∞
F		∞

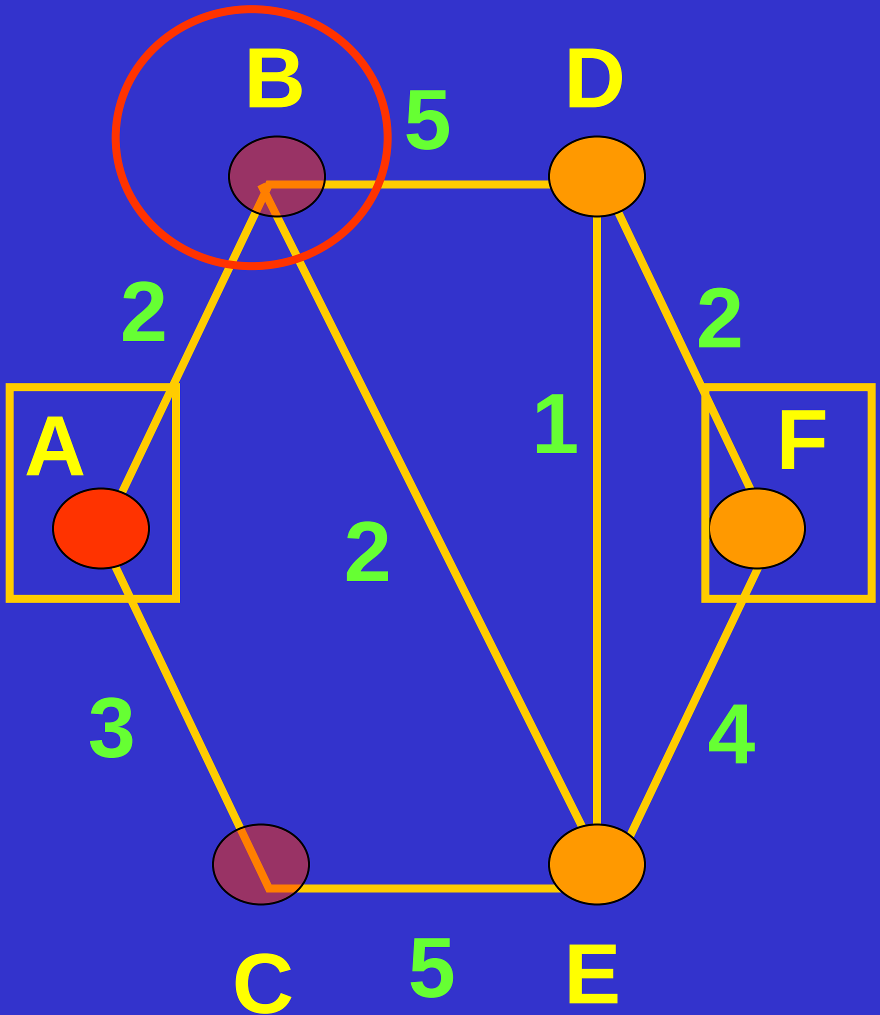


V_1	L_1	L_0
A*	-	(0)
B		∞
C		∞
D		∞
E		∞
F		∞



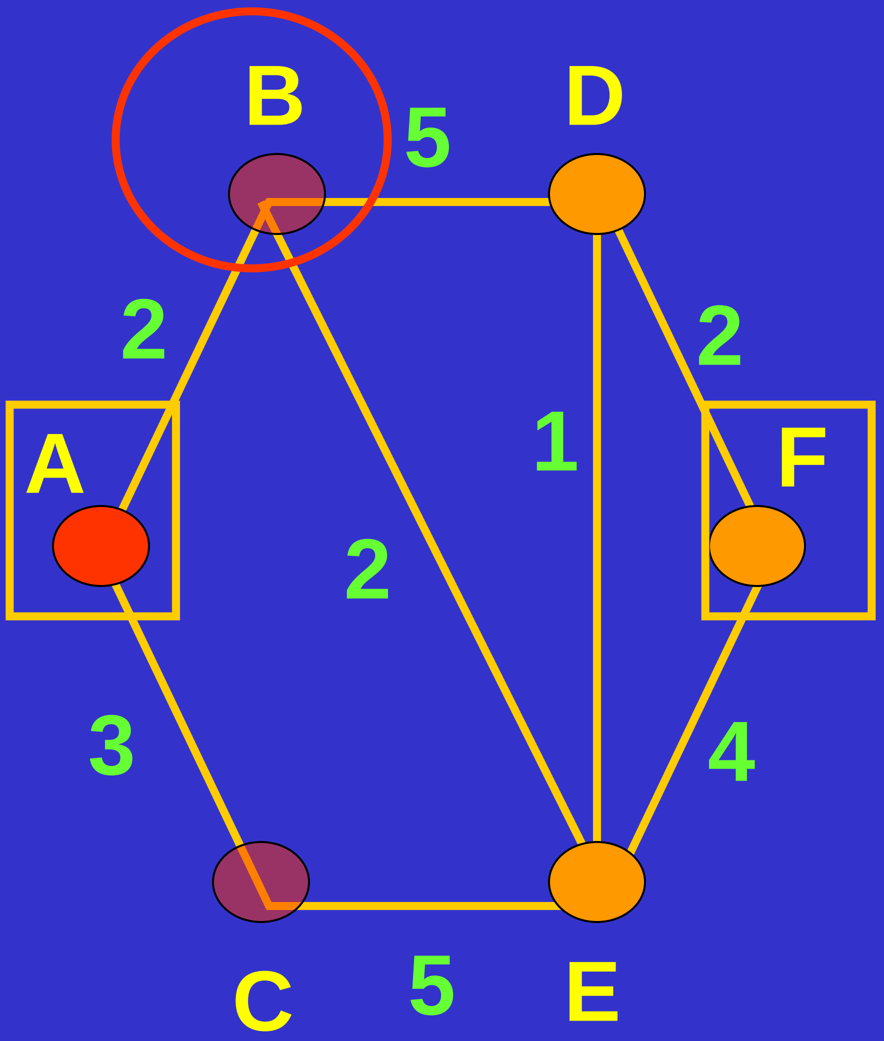
V_1	L_1	L_0
A*	-	(0)
B		∞
C		∞
D		∞
E		∞
F		∞

$$L(v_j) = L(v_{i-1}{}^*) + w(v_{i-1}, v_j)$$



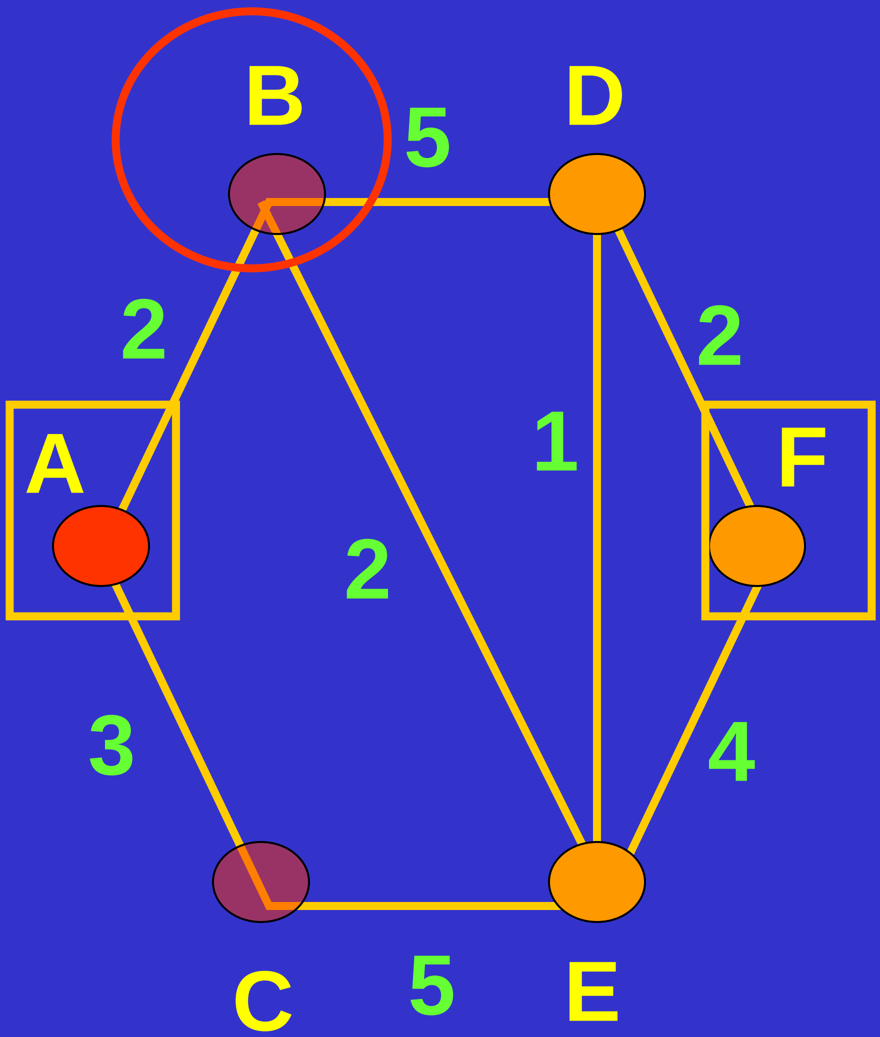
V_1	L_1	L_0
A*	-	(0)
B		∞
C		∞
D		∞
E		∞
F		∞

$$L(v_j) = L(v_{i-1}^*) + w(v_{i-1}, v_j)$$



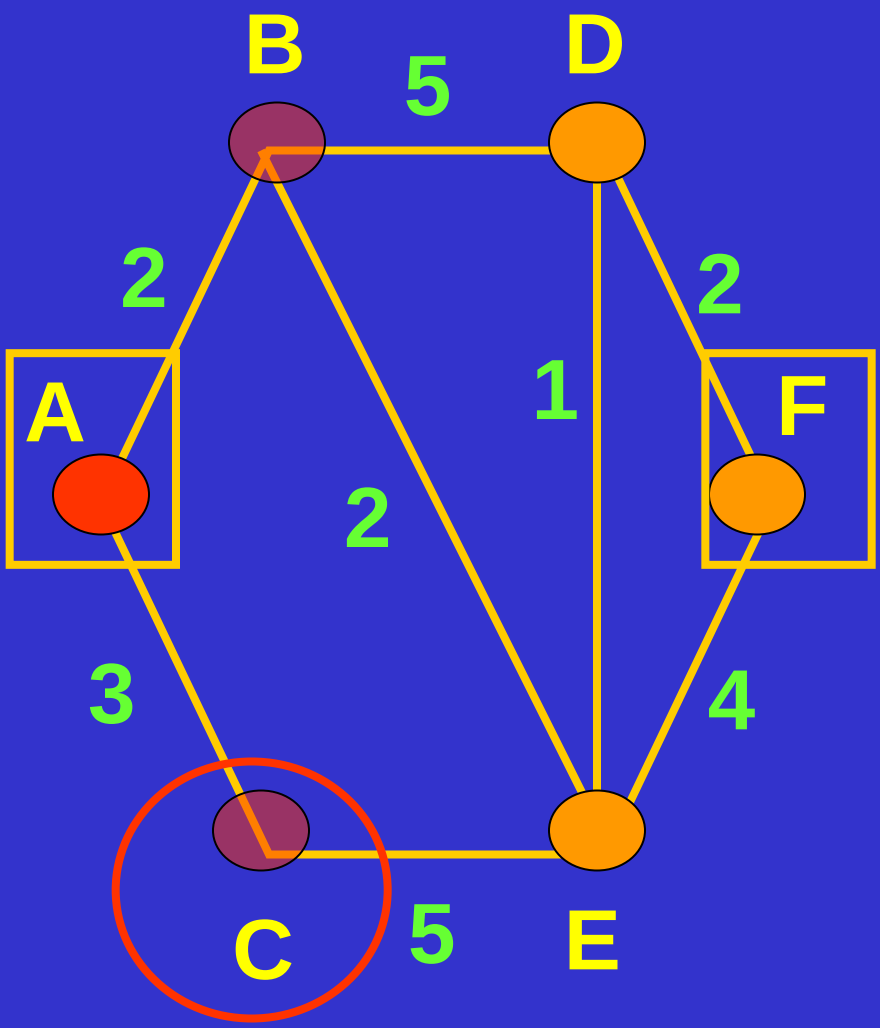
V_1	L_1	L_0
A*	-	(0)
B		∞
C		∞
D		∞
E		∞
F		∞

$$L(B_A) = L(A^*) + w(A^*, B) = 0+2 = 2$$



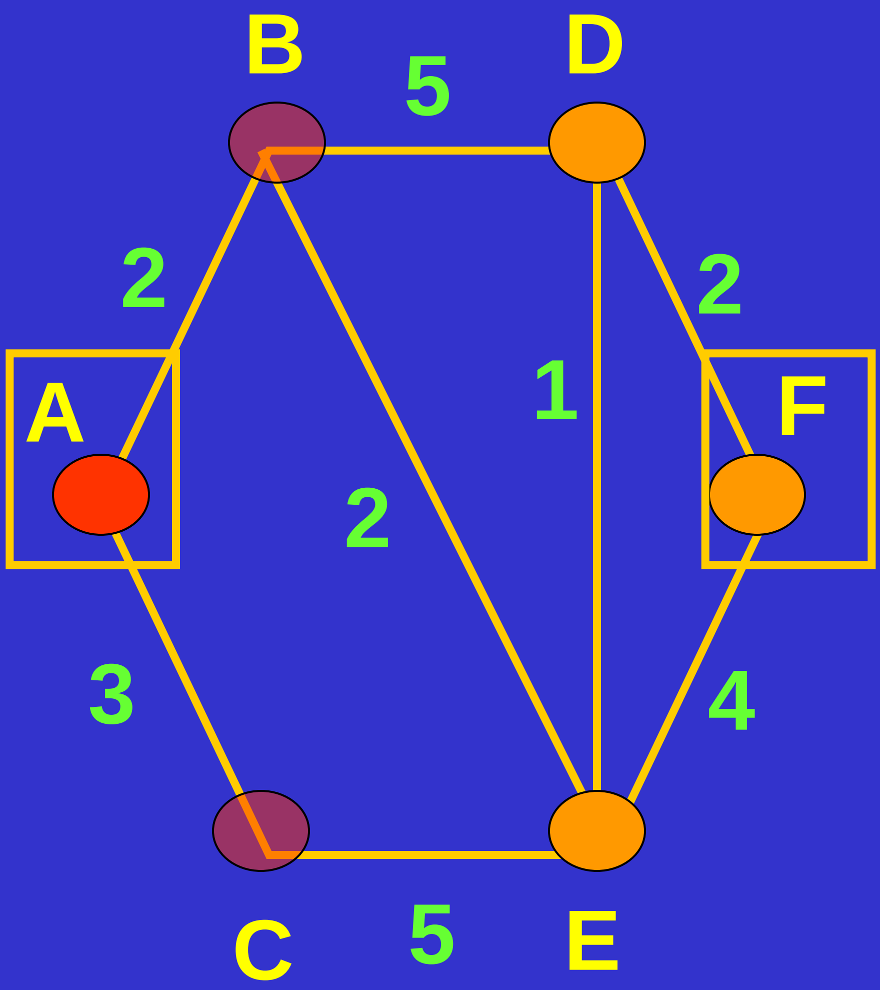
V_1	L_1	L_0
A*	-	(0)
B_A	2	∞
C		∞
D		∞
E		∞
F		∞

$$L(B_A) = L(A^*) + w(A^*, B)$$

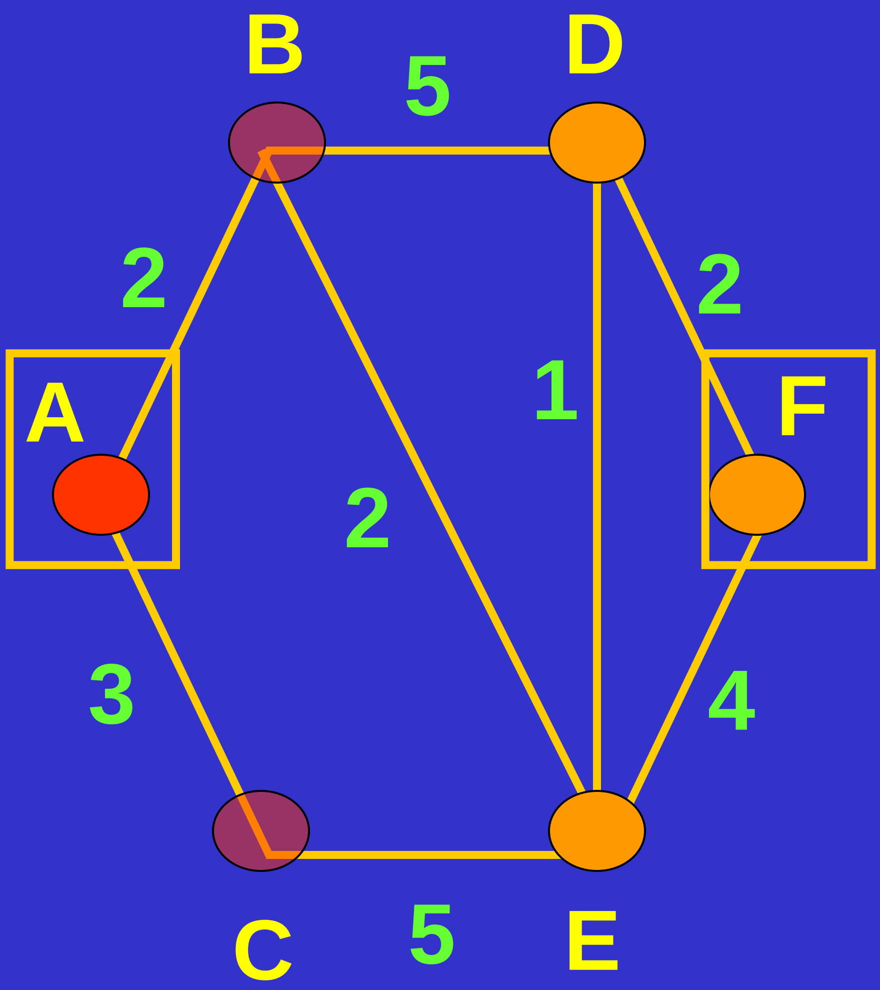


V_1	L_1	L_0
A^*	-	(0)
B_A	2	∞
C_A	3	∞
D		∞
E		∞
F		∞

$$L(C_A) = L(A^*) + w(A^*, C) = 0+3 = 3$$

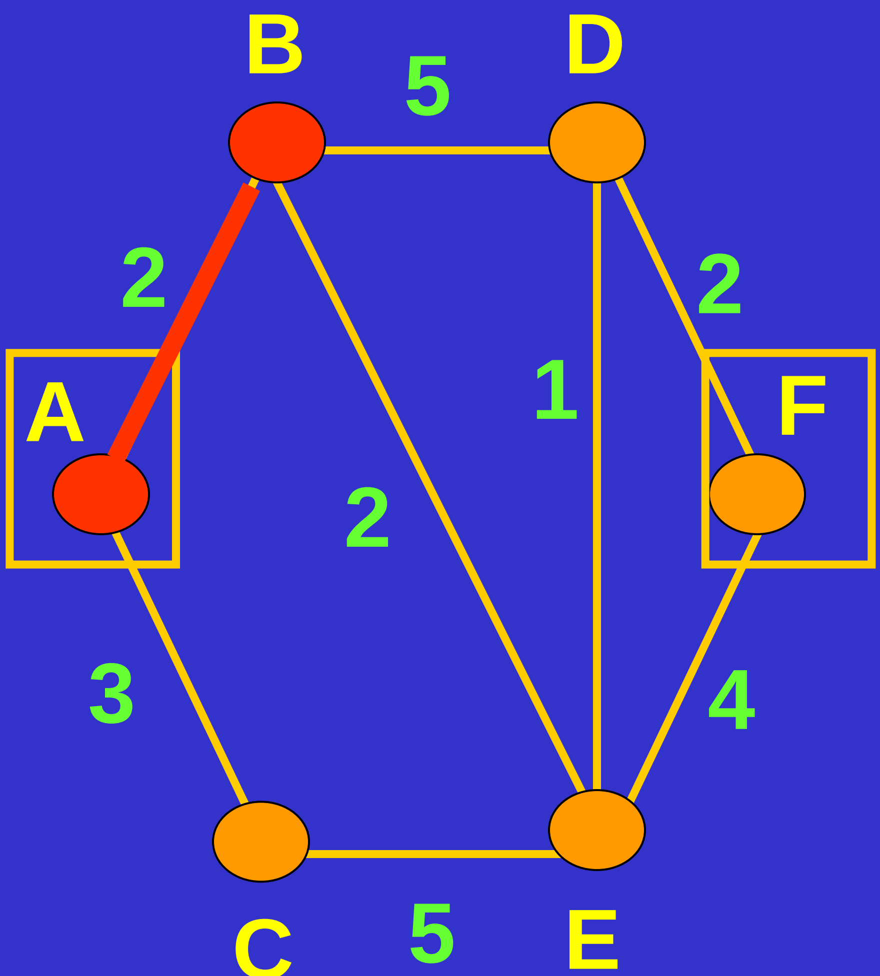


V_1	L_1	L_0
A*	-	(0)
B_A	2	∞
C_A	3	∞
D	∞	∞
E	∞	∞
F	∞	∞



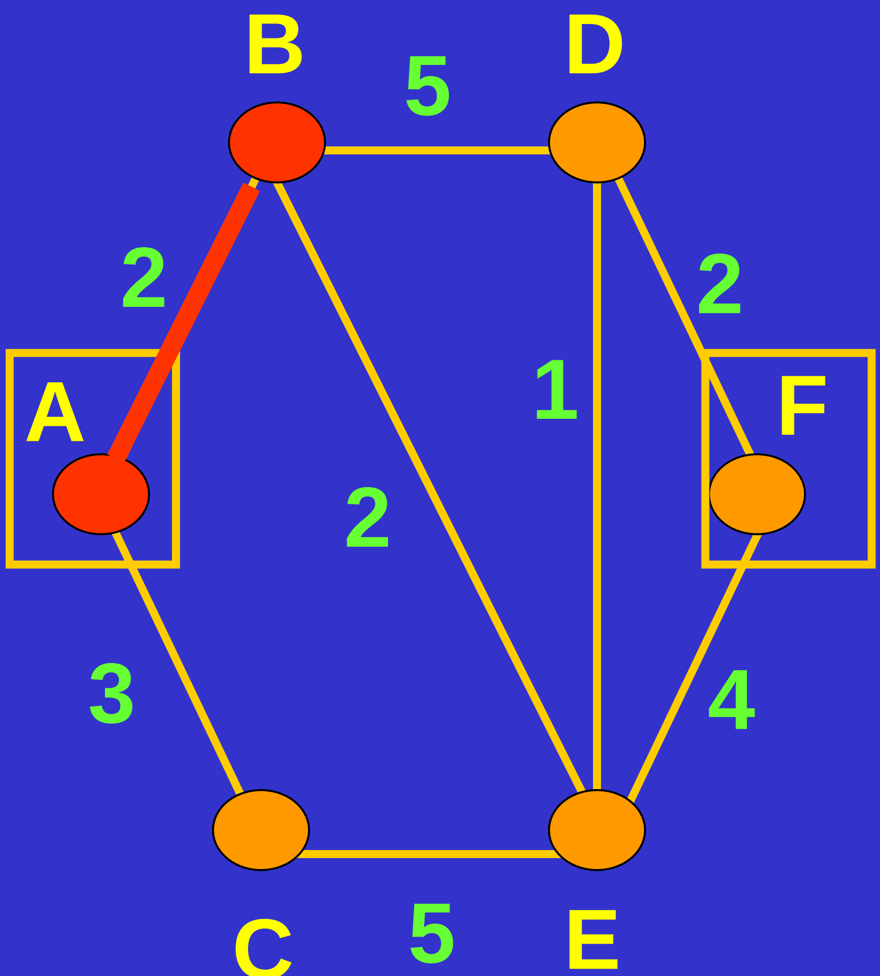
V_1	L_1	L_0
A*	-	(0)
B_A	2	∞
C_A	3	∞
D	∞	∞
E	∞	∞
F	∞	∞

L(B) < L(C)

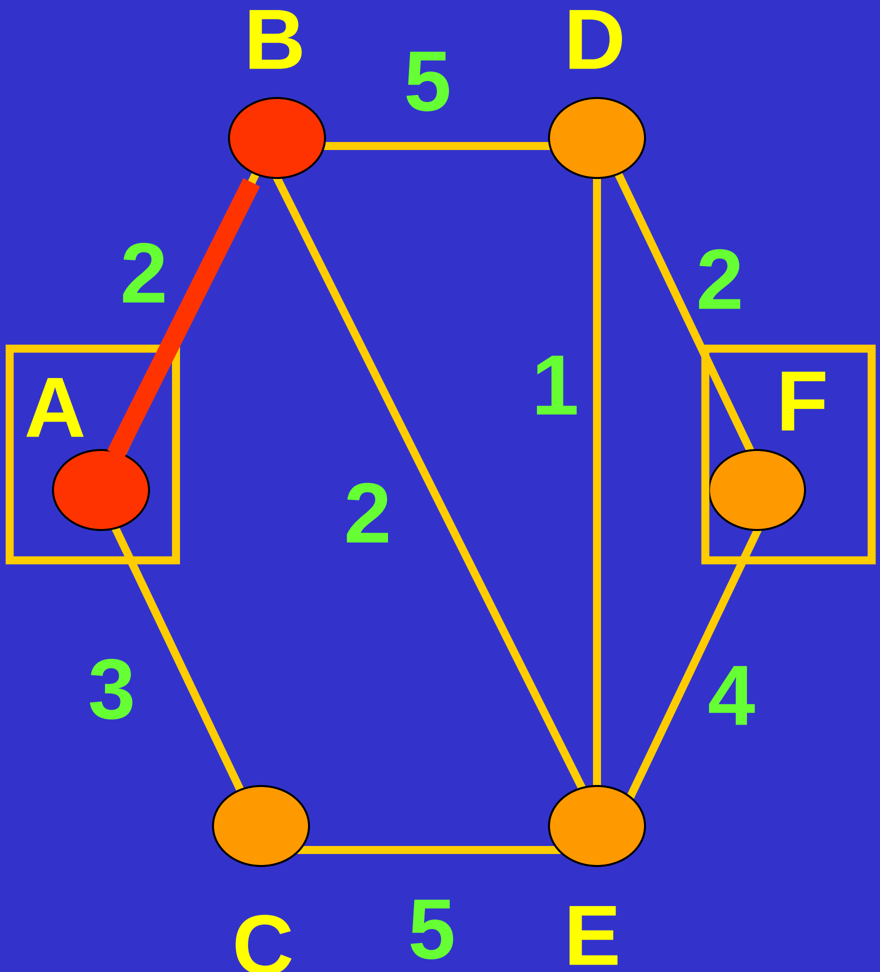


$\underline{V_1}$	$\underline{L_1}$	$\underline{L_0}$
A*	-	(0)
B_A	2	∞
C_A	3	∞
D	∞	∞
E	∞	∞
F	∞	∞

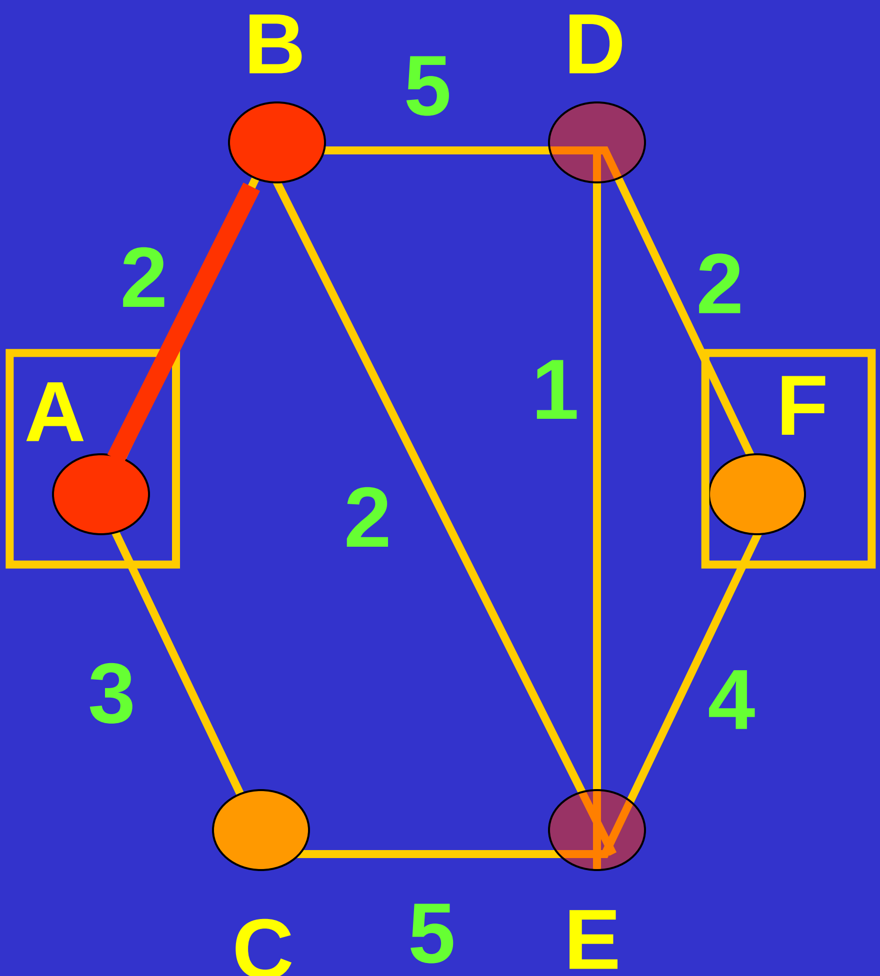
L(B) < L(C)



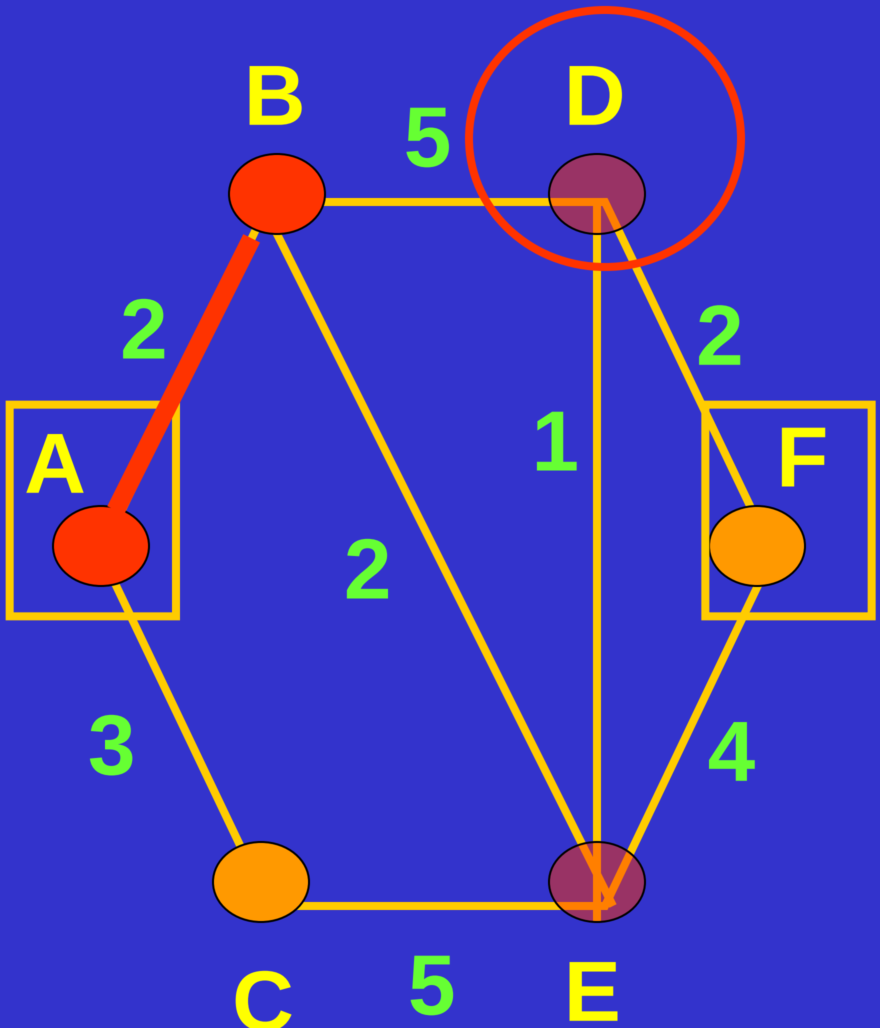
V_2	L_1
A*	-
B_A*	(2)
C_A	3
D	∞
E	∞
F	∞



$\underline{V_2}$	$\underline{L_2}$	$\underline{L_1}$
A*	-	-
B_A*	-	(2)
C_A	3	3
D		∞
E		∞
F		∞

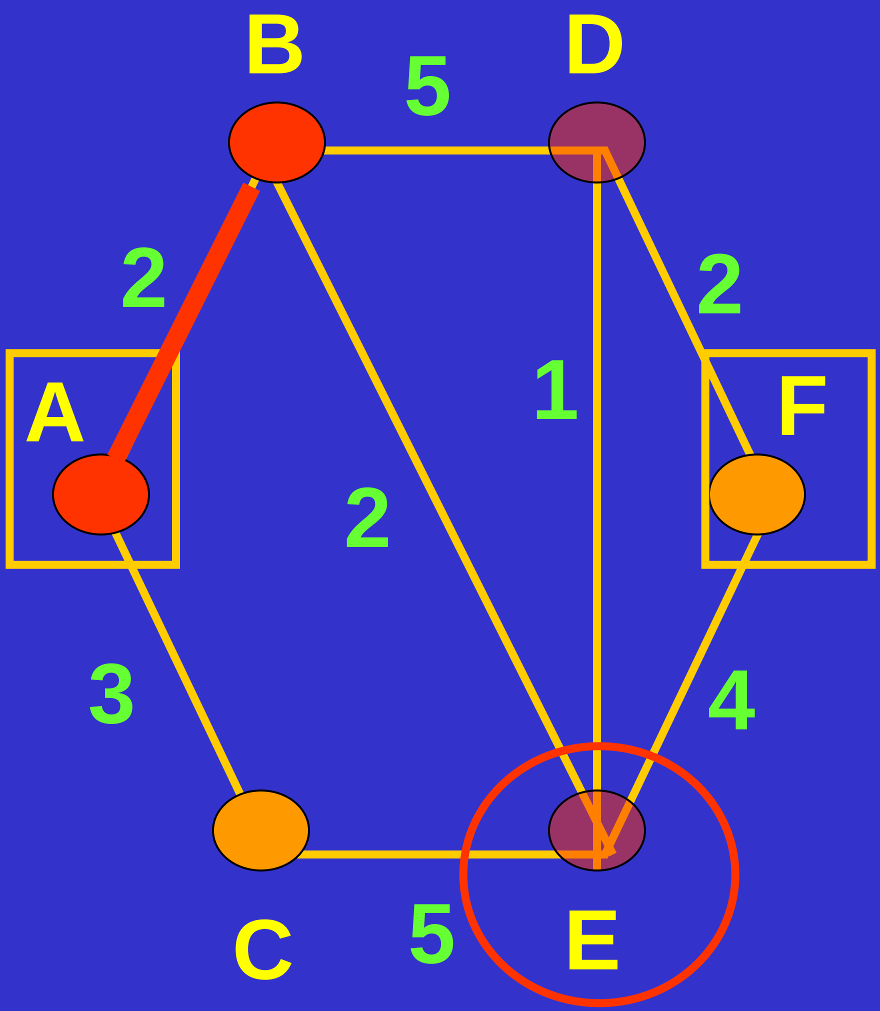


V_2	L_2	L_1
A*	-	-
B_A*	-	(2)
C_A	3	3
D		∞
E		∞
F		∞



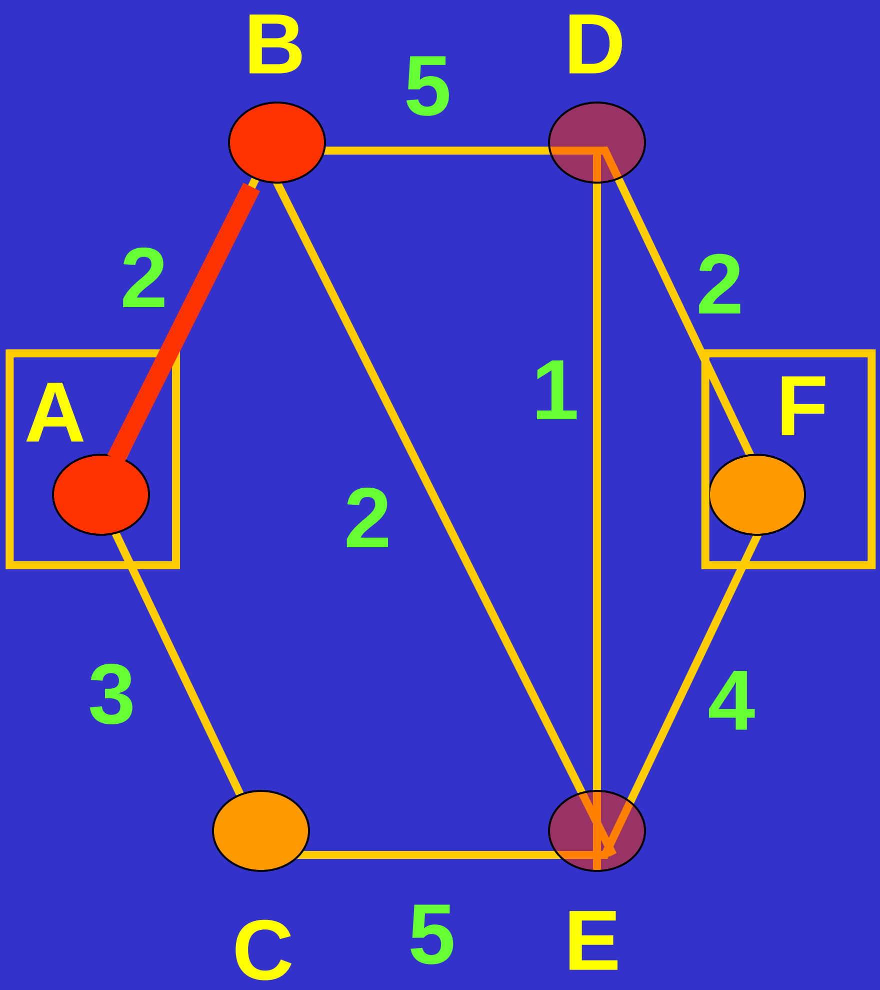
V_2	L_2	L_1
A*	-	-
B_A*	-	(2)
C_A	3	3
D_B	7	∞
E		∞
F		∞

$$L(D_B) = L(B_A^*) + w(B_A^*, D) = 2+5 = 7$$

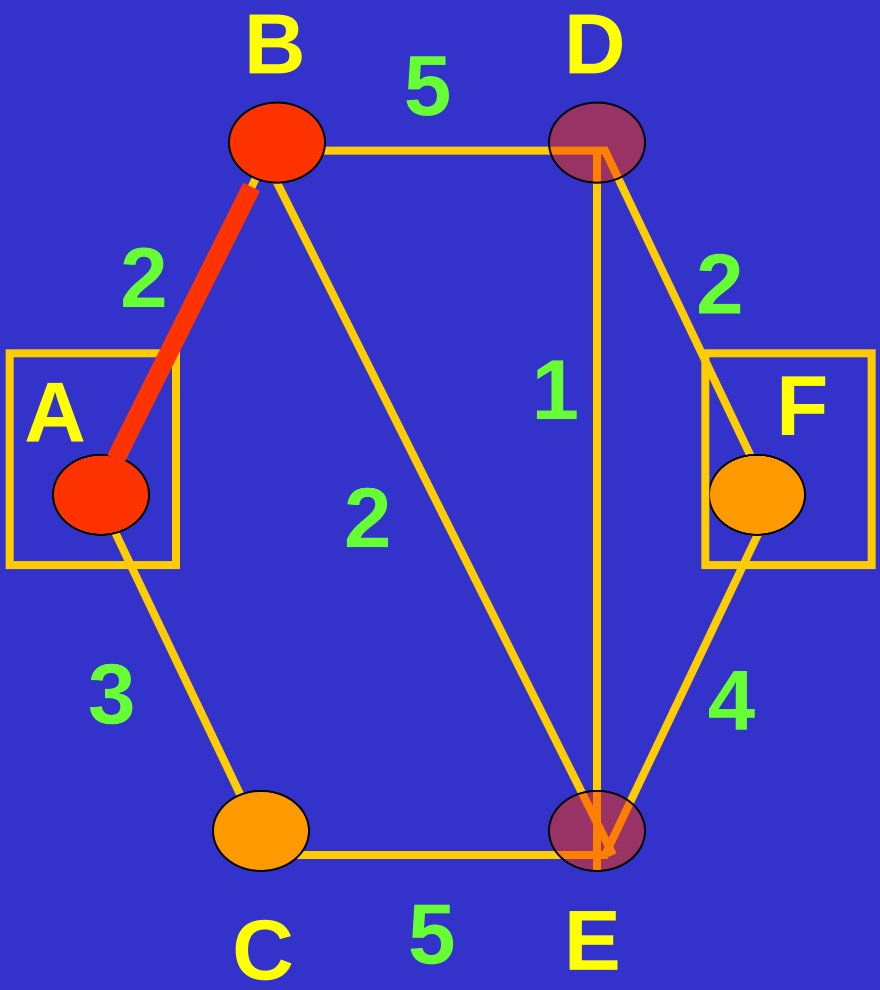


V_2	L_2	L_1
A^*	-	-
B_A^*	-	(2)
C_A	3	3
D_B	7	∞
E		∞
F		∞

$$L(E_B) = L(B_A^*) + w(B_A^*, E) = 2+2 = 4$$

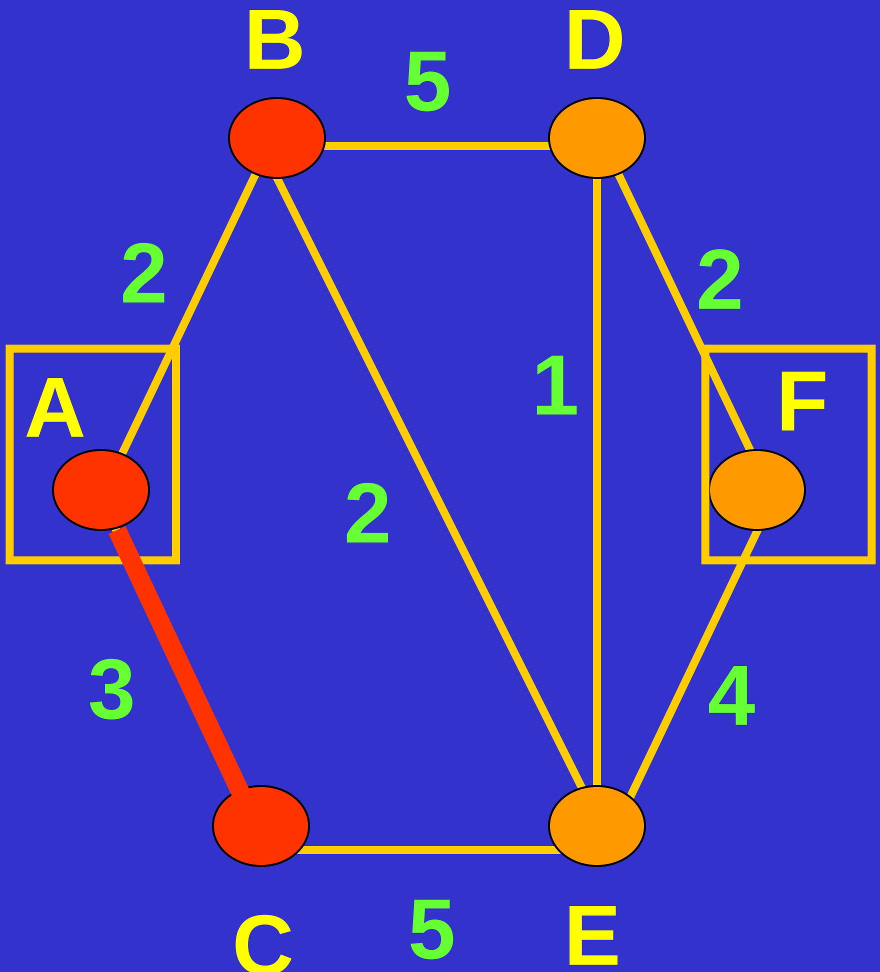


V_2	L_2	L_1
A*	-	-
B_A*	-	(2)
C_A	3	3
D_B	7	∞
E_B	4	∞
F	∞	∞

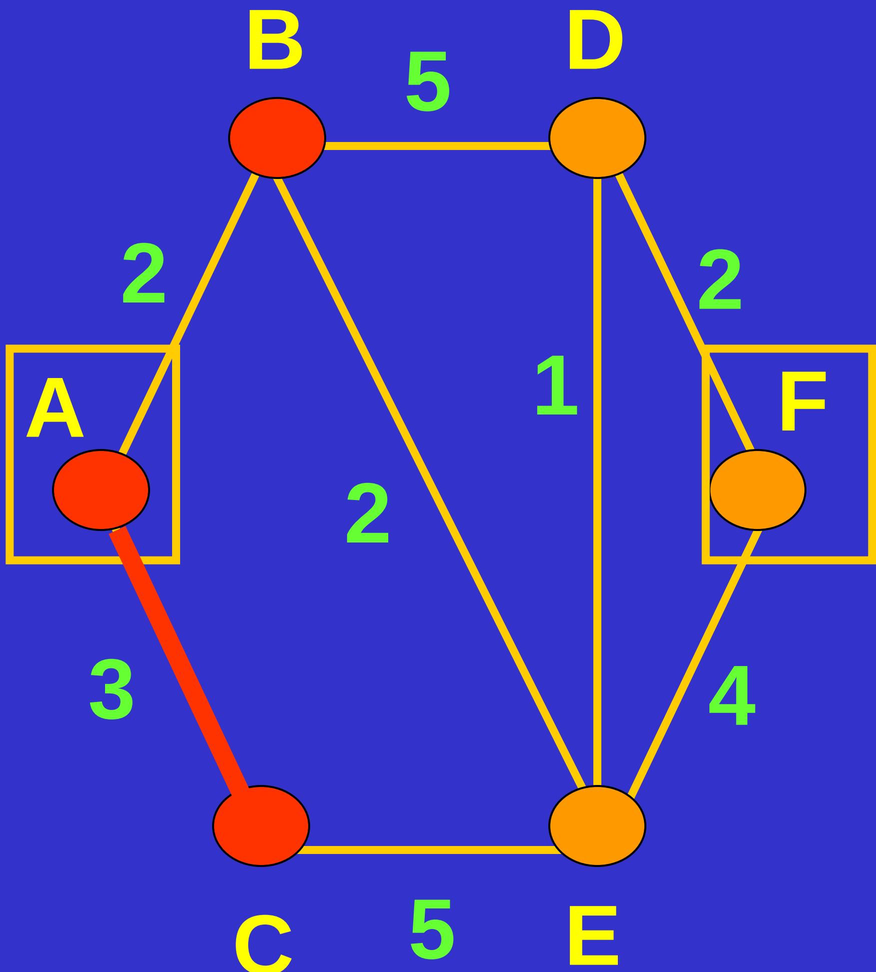


$\underline{V_2}$	$\underline{L_2}$	$\underline{L_1}$
A*	-	-
B_A*	-	(2)
C_A	3	3
D_B	7	∞
E_B	4	∞
F	∞	∞

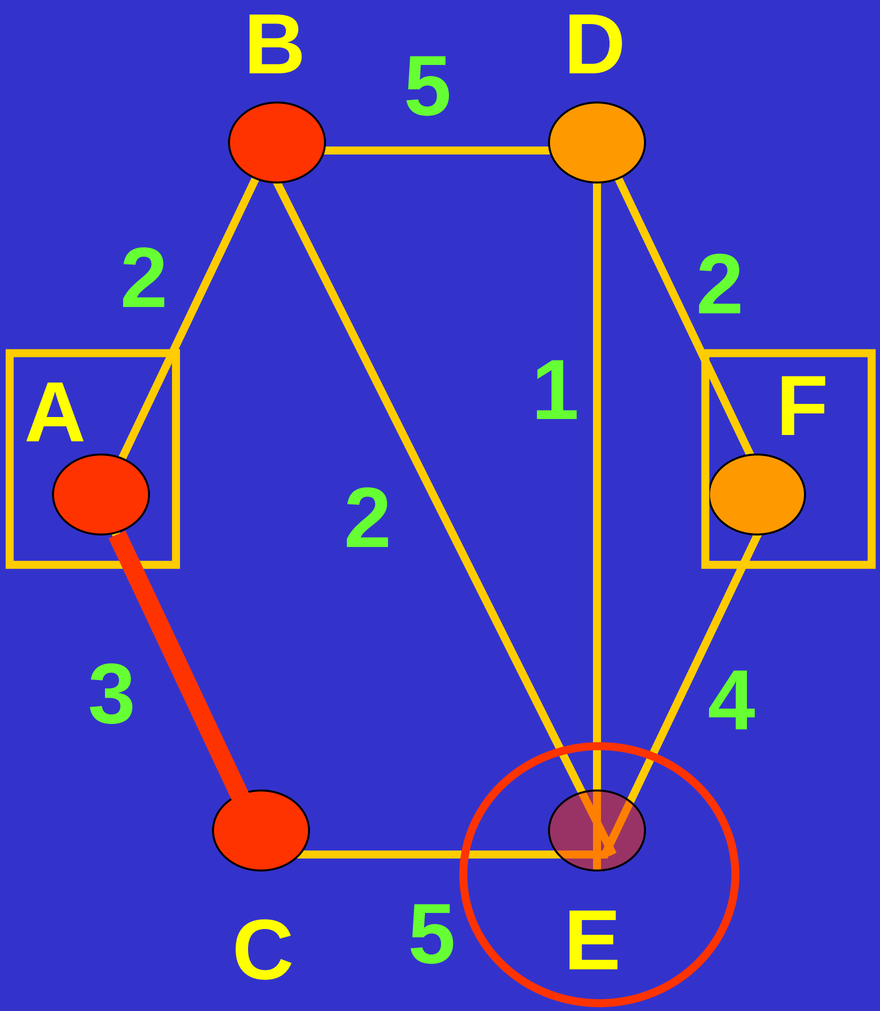
L(C) < L(E) < L(D)



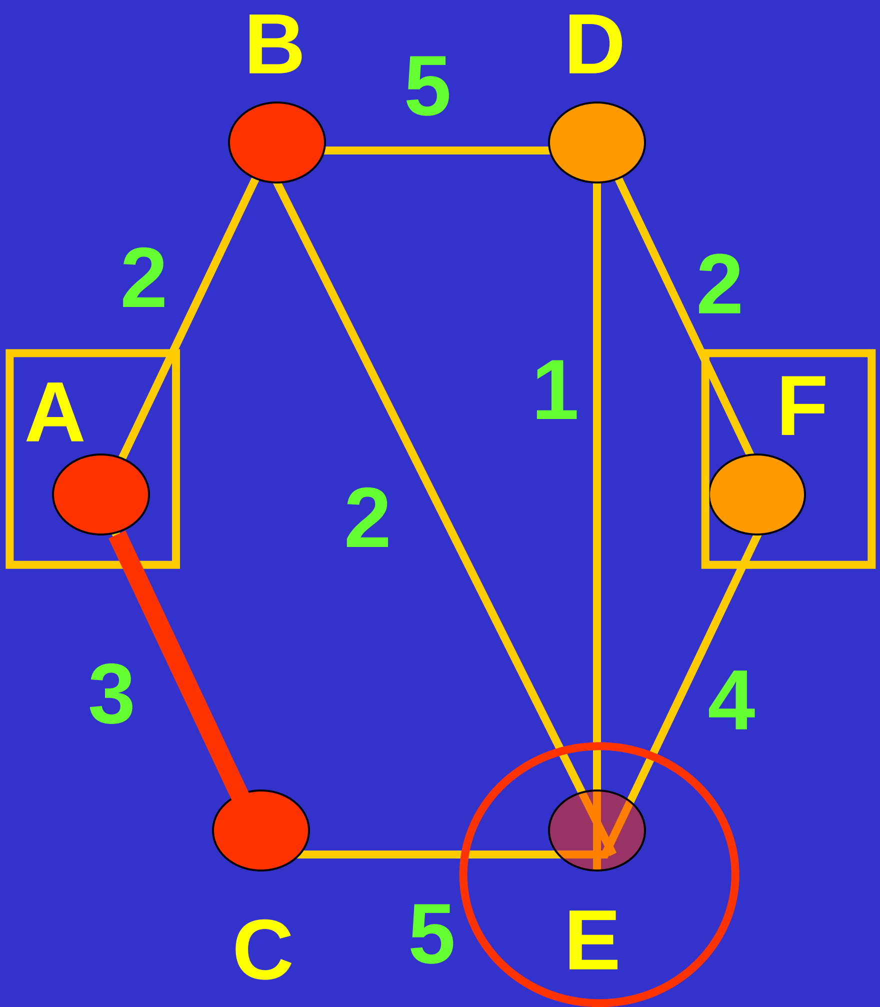
$\underline{V_3}$	$\underline{L_2}$
A*	-
B_A*	-
C_A*	(3)
D_B	7
E_B	4
F	∞



V_3	L_3	L_2
A*	-	-
B_A*	-	-
C_A*	-	(3)
D_B	7	7
E_B	4	4
F		∞

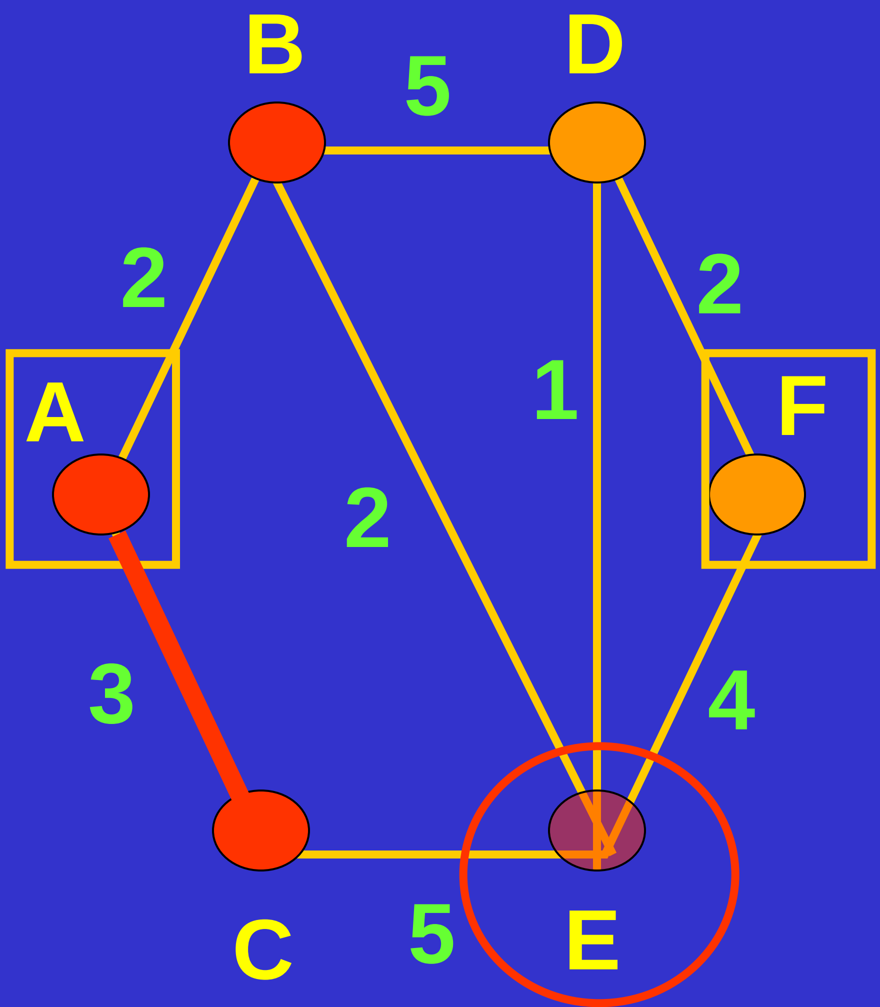


V_4	L_3	L_2
A*	-	-
B_A*	-	-
C_A*	-	(3)
D_B	7	7
E_B	4	4
F		∞



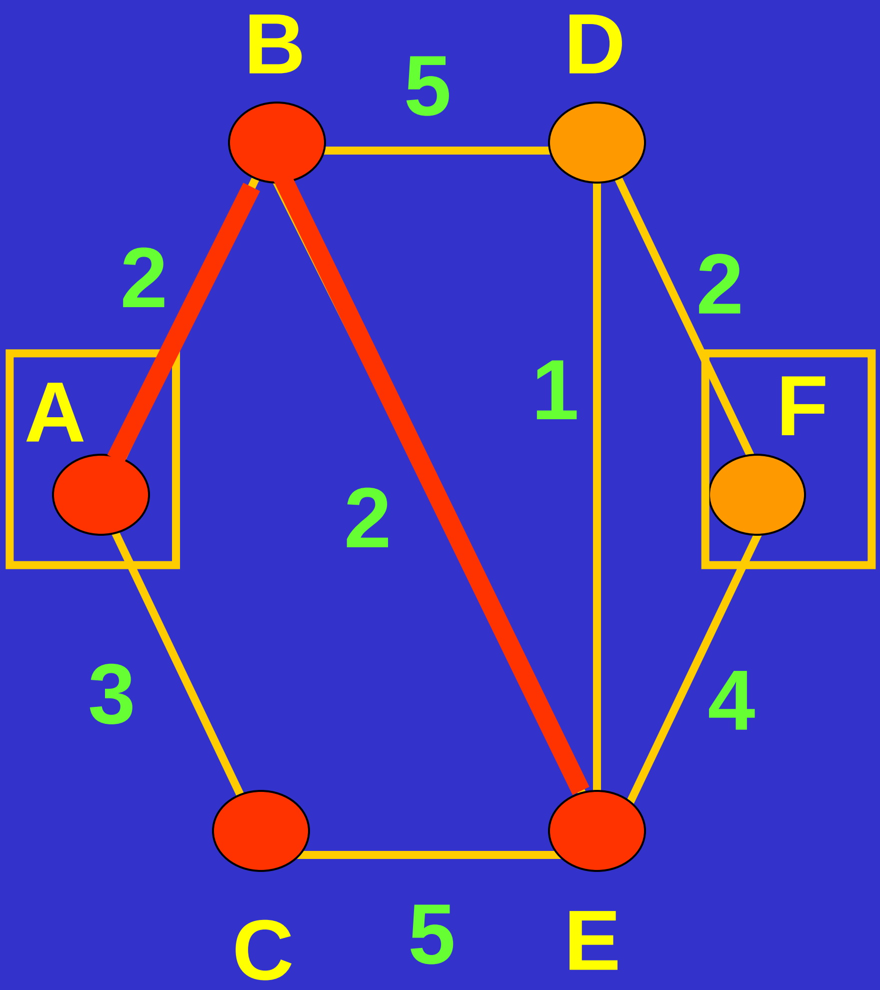
$\underline{V_4}$	$\underline{L_3}$	$\underline{L_2}$
A*	-	-
B_A*	-	-
C_A*	-	(3)
D_B	7	7
E_B	4	4
F		∞

$$L(E_C) = L(C_A^*) + w(C_A^*, E) = 3+5 = 8$$

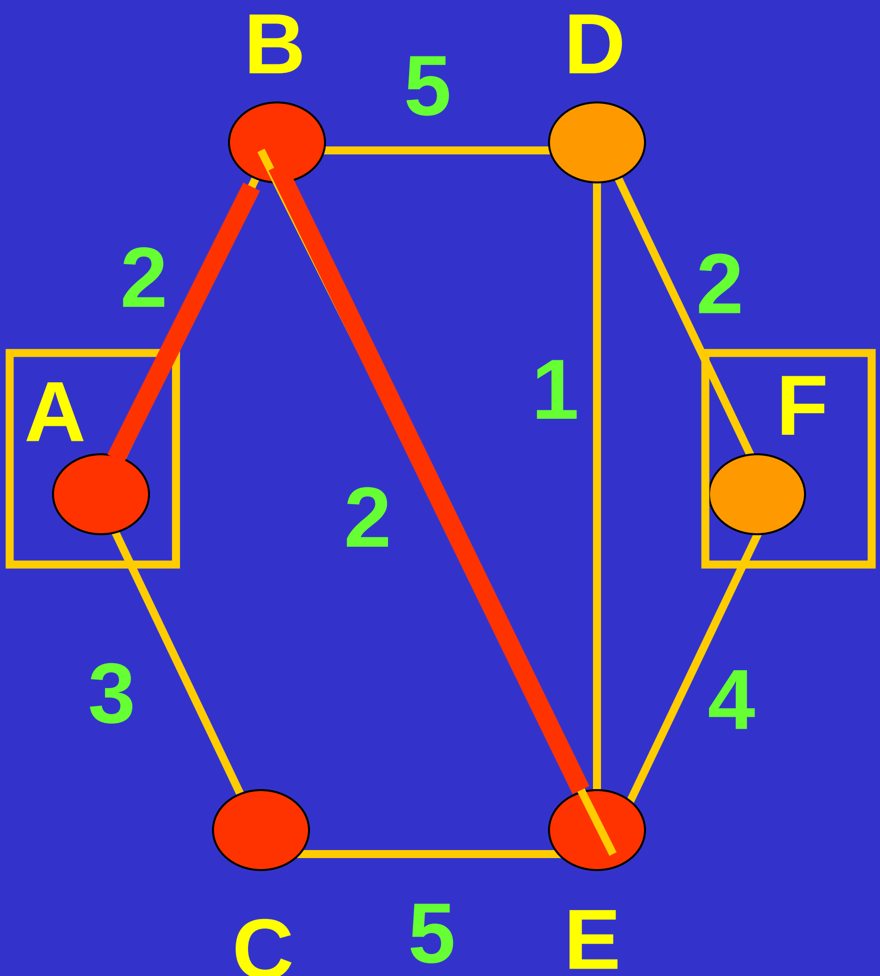


V_4	L_3	L_2
A*	-	-
B_A*	-	-
C_A*	-	(3)
D_B	7	7
E_B	4	4
F		∞

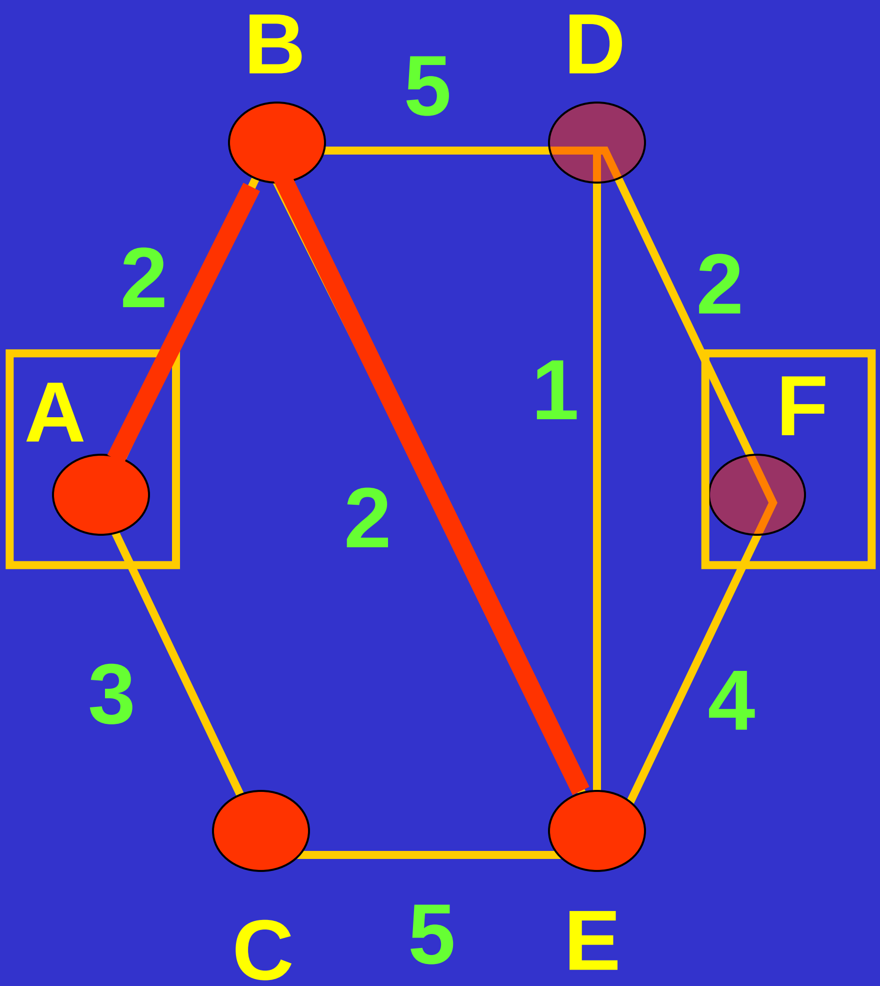
$$L(E_C) = L(C_A^*) + w(C_A^*, E) = 3+5 = 8 > 4 \ldots$$



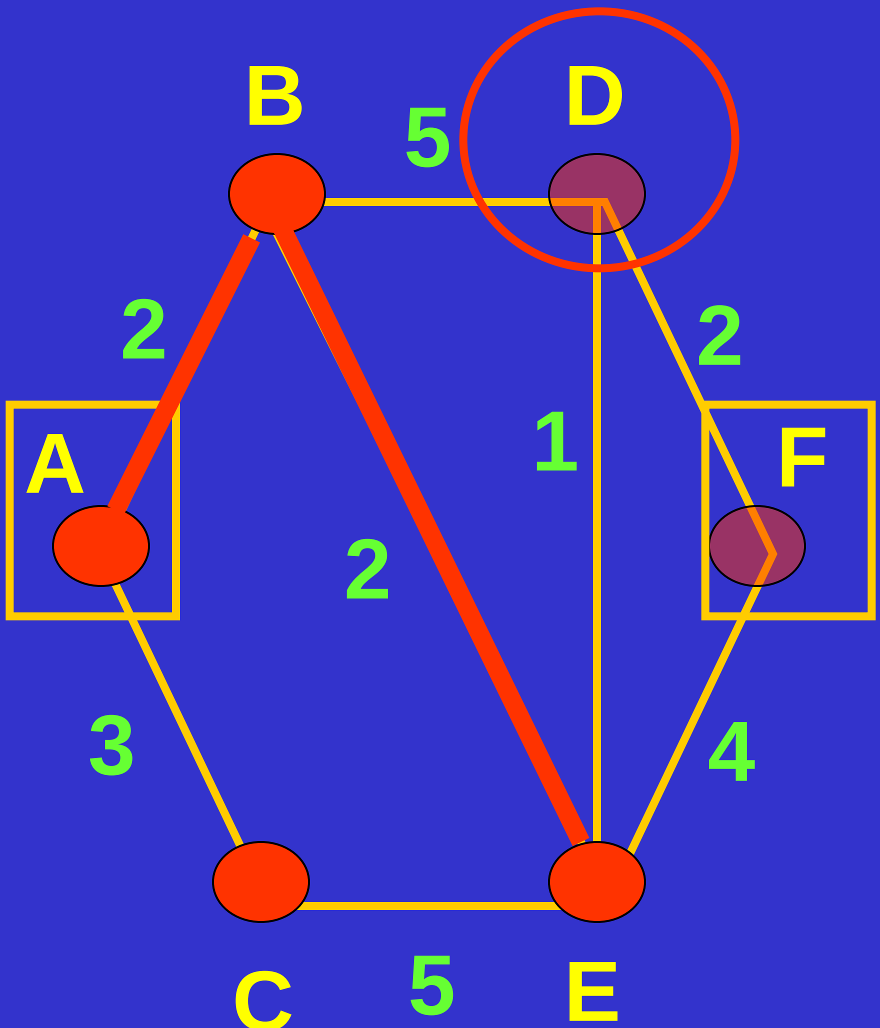
V_4	L_3
A*	-
B_A*	-
C_A*	-
D_B	7
E_B*	(4)
F	∞



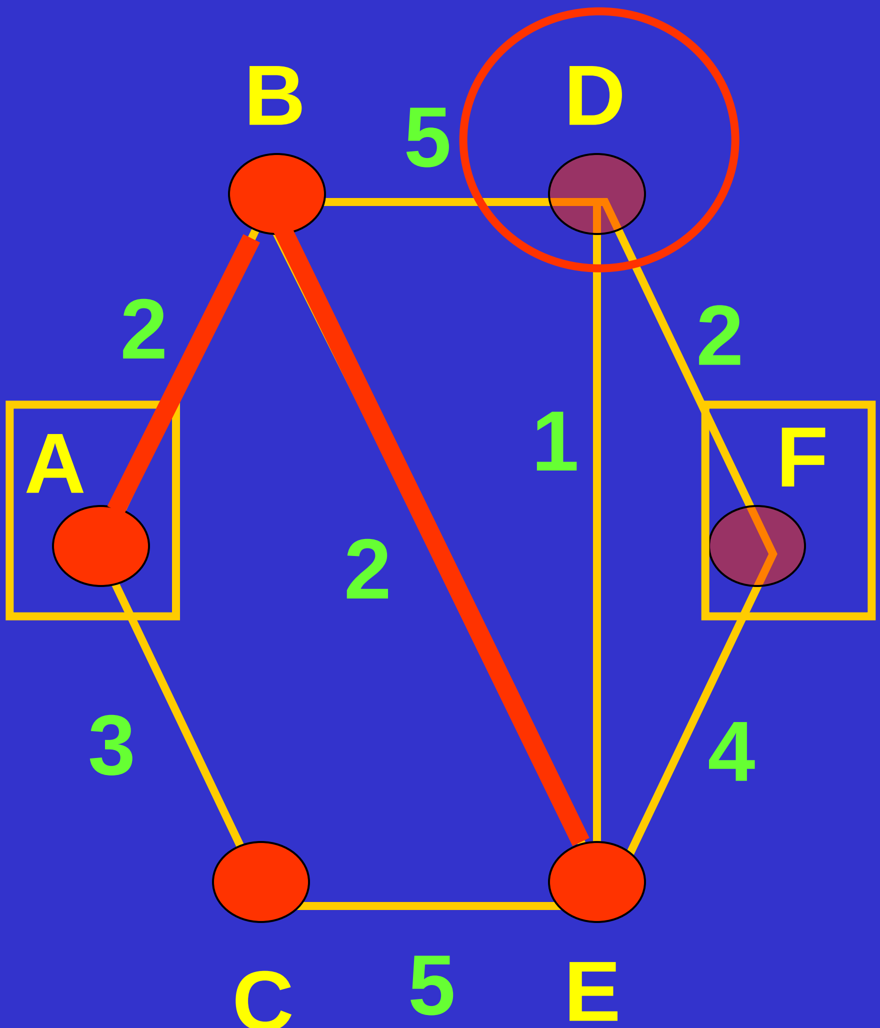
V_4	L_4	L_3
A*	-	-
B_A*	-	-
C_A*	-	-
D_B	7	7
E_B*	-	(4)
F		∞



$\underline{V_4}$	$\underline{L_4}$	$\underline{L_3}$
A*	-	-
B_A*	-	-
C_A*	-	-
D_B	7	7
E_B*	-	(4)
F		∞

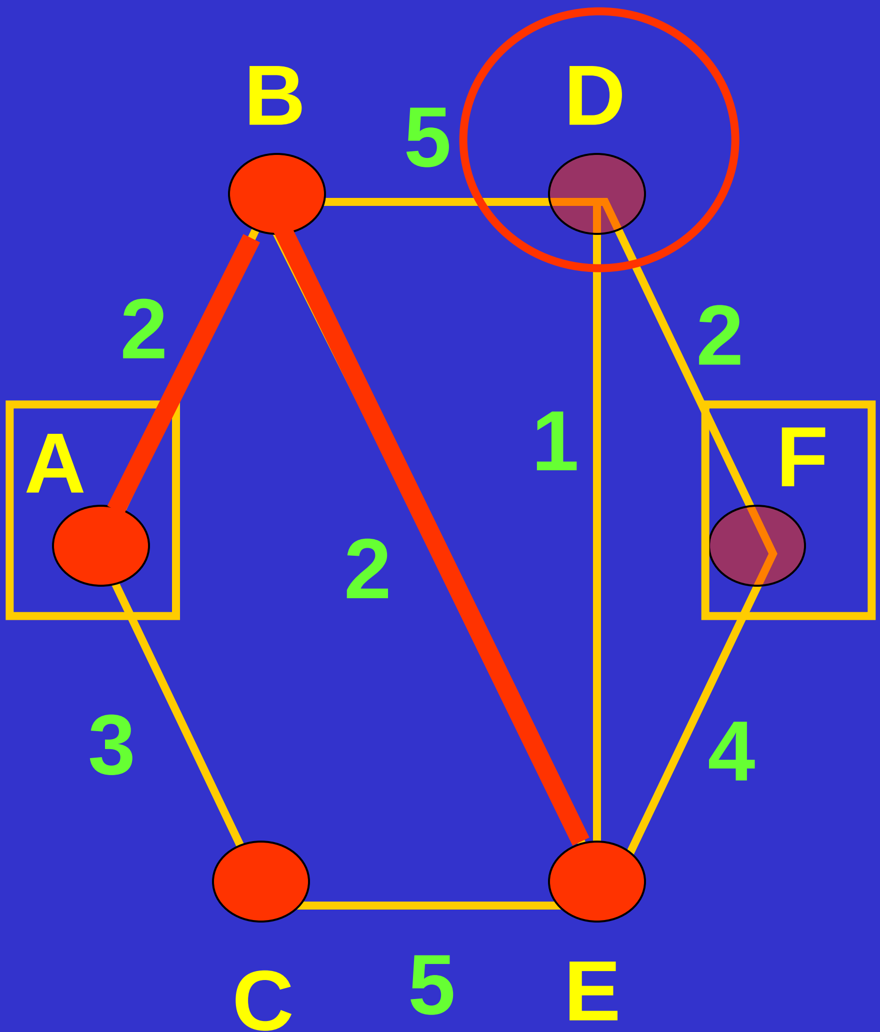


V_4	L_4	L_3
A*	-	-
B_A*	-	-
C_A*	-	-
D_B	7	7
E_B*	-	(4)
F		∞



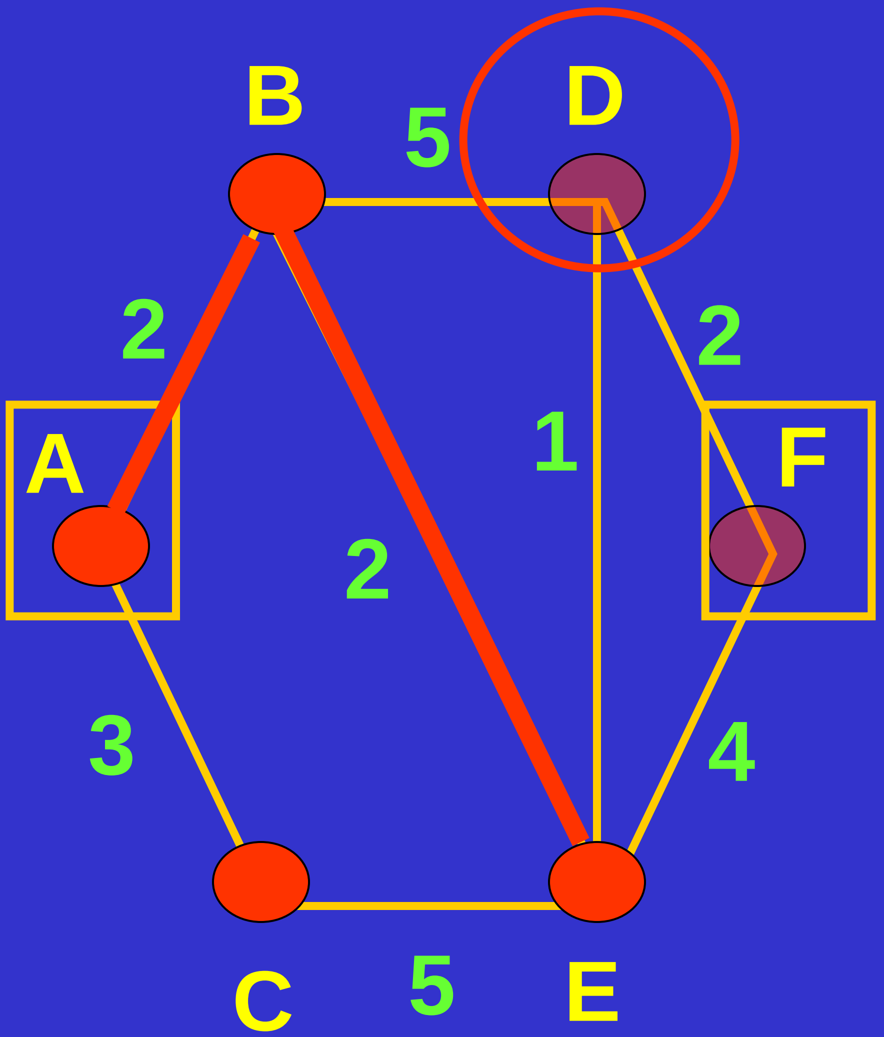
V_5	L_4	L_3
A^*	-	-
B_A^*	-	-
C_A^*	-	-
D_B	7	7
E_B^*	-	(4)
F		∞

$$L(D_E) = L(E_B^*) + w(E_B^*, D) = 4+1 = 5$$



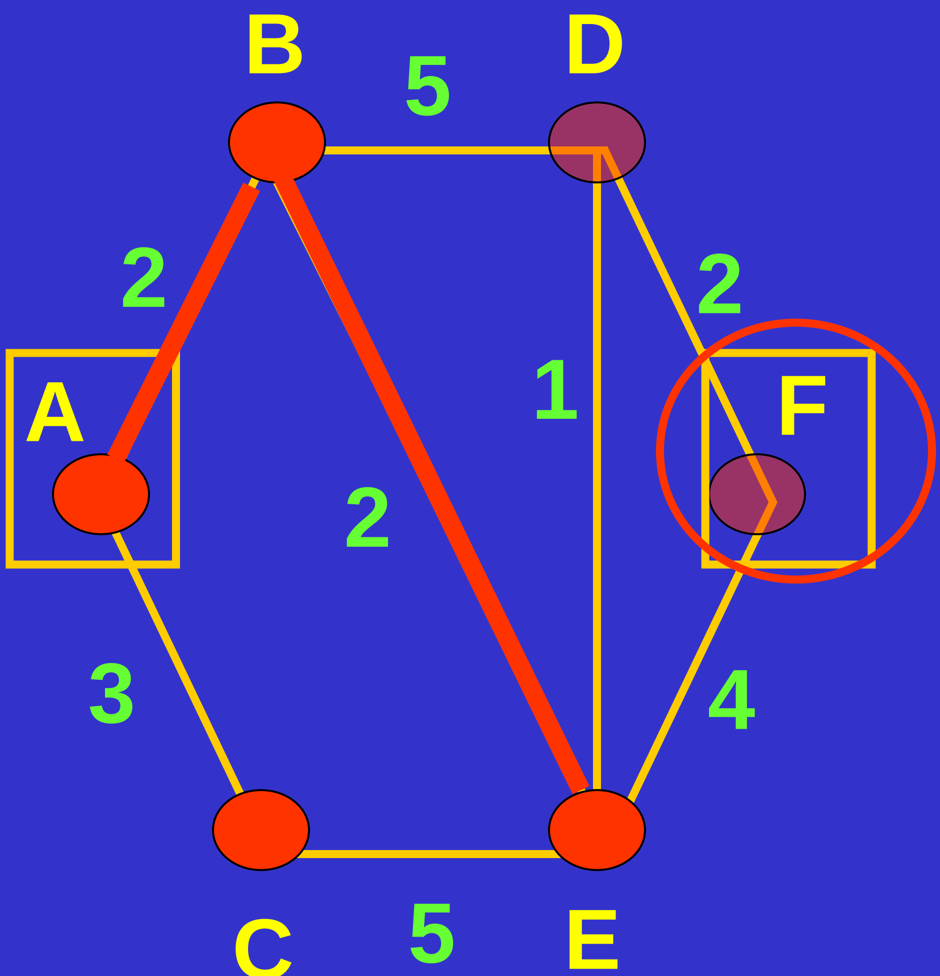
V_5	L_4	L_3
A*	-	-
B_A*	-	-
C_A*	-	-
D_B	7	7
E_B*	-	(4)
F		∞

$$L(D_E) = L(E_B^*) + w(E_B^*, D) = 4+1 = 5 < 7$$



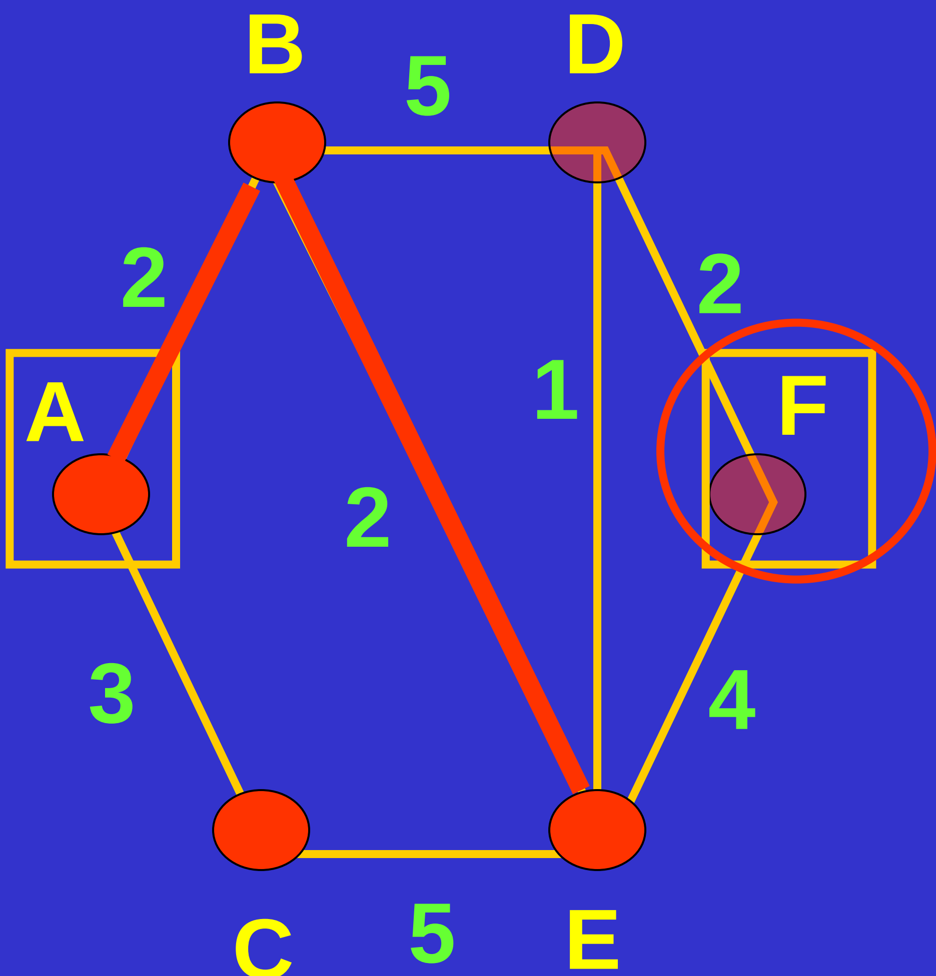
$\underline{V_5}$	$\underline{L_4}$	$\underline{L_3}$
A*	-	-
B_A*	-	-
C_A*	-	-
D_E	5	7
E_B*	-	(4)
F		∞

$$L(D_E) = L(E_B*) + w(E_B*, D) = 4+1 = 5 < 7$$



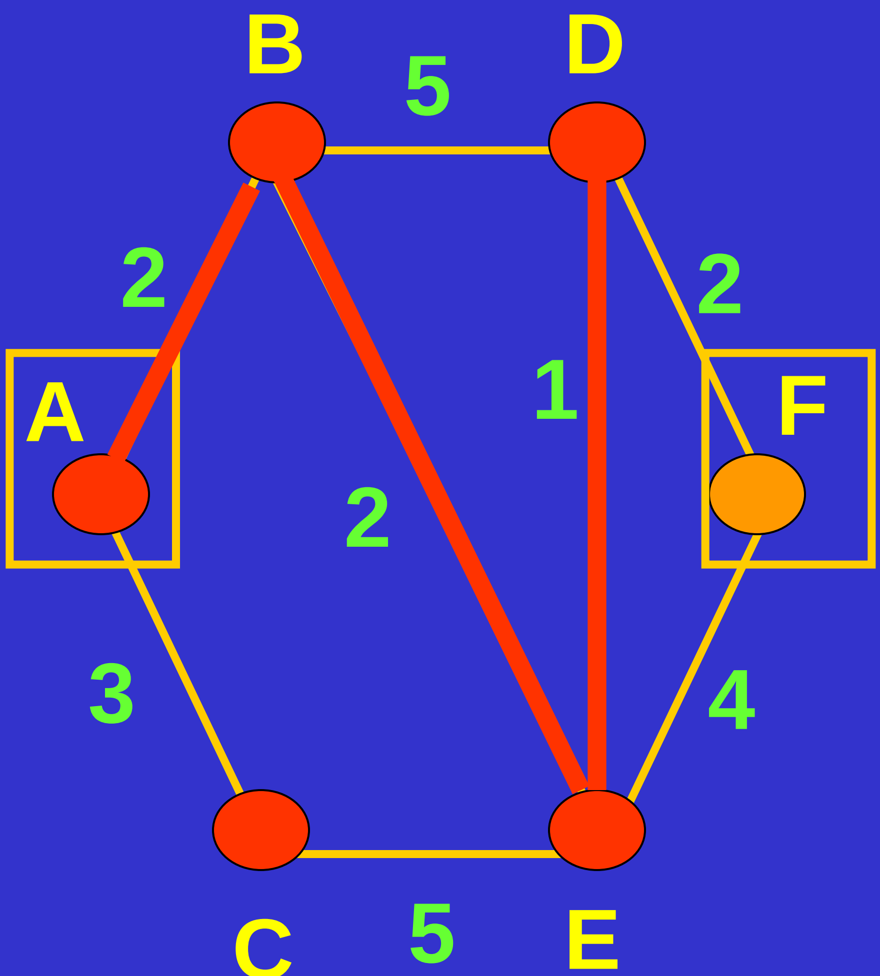
$\underline{V_5}$	$\underline{L_4}$	$\underline{L_3}$
A*	-	-
B_A*	-	-
C_A*	-	-
D_E	5	7
E_B*	-	(4)
F_E	8	∞

$$L(F_E) = L(E_B^*) + w(E_B^*, F) = 4+4 = 8$$

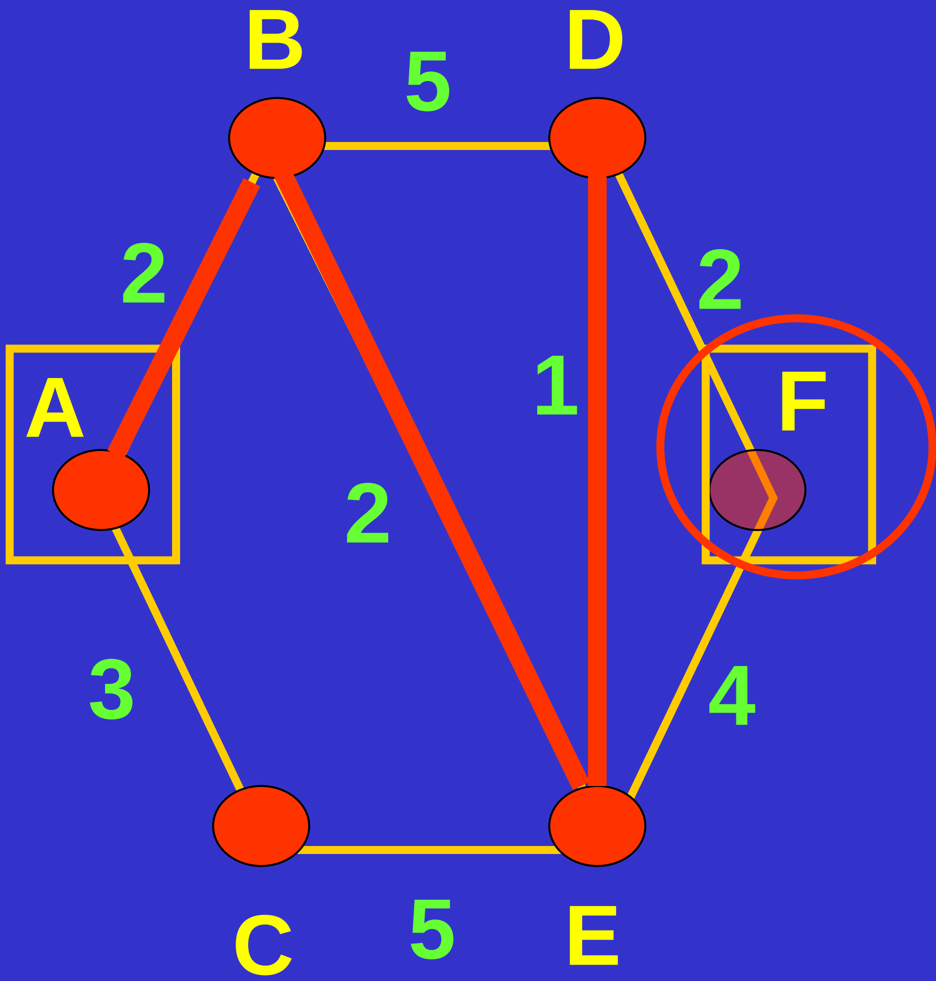


V_5	L_4	L_3
A*	-	-
B_A*	-	-
C_A*	-	-
D_E	5	7
E_B*	-	(4)
F_E	8	∞

$$L(F_E) = L(E_B^*) + w(E_B^*, F) = 4+4 = 8$$

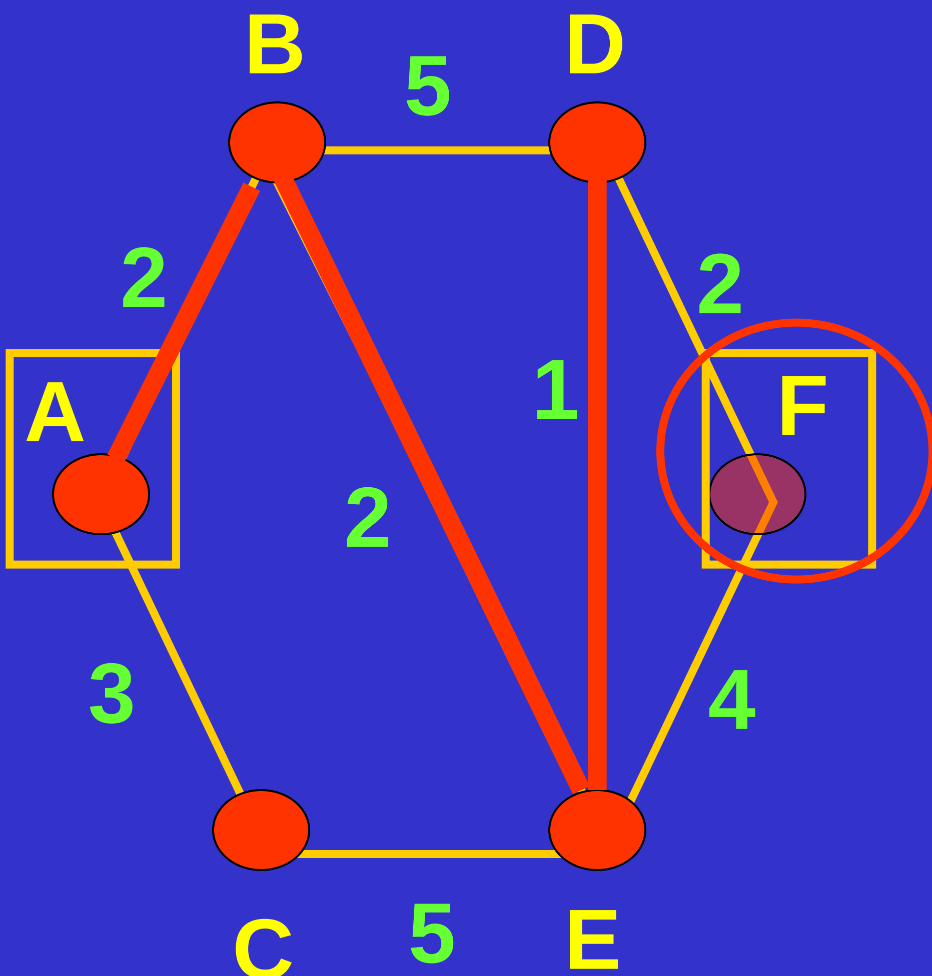


V_5	L_4
A*	-
B_A*	-
C_A*	-
D_E *	(5)
E_B*	-
F_E	8



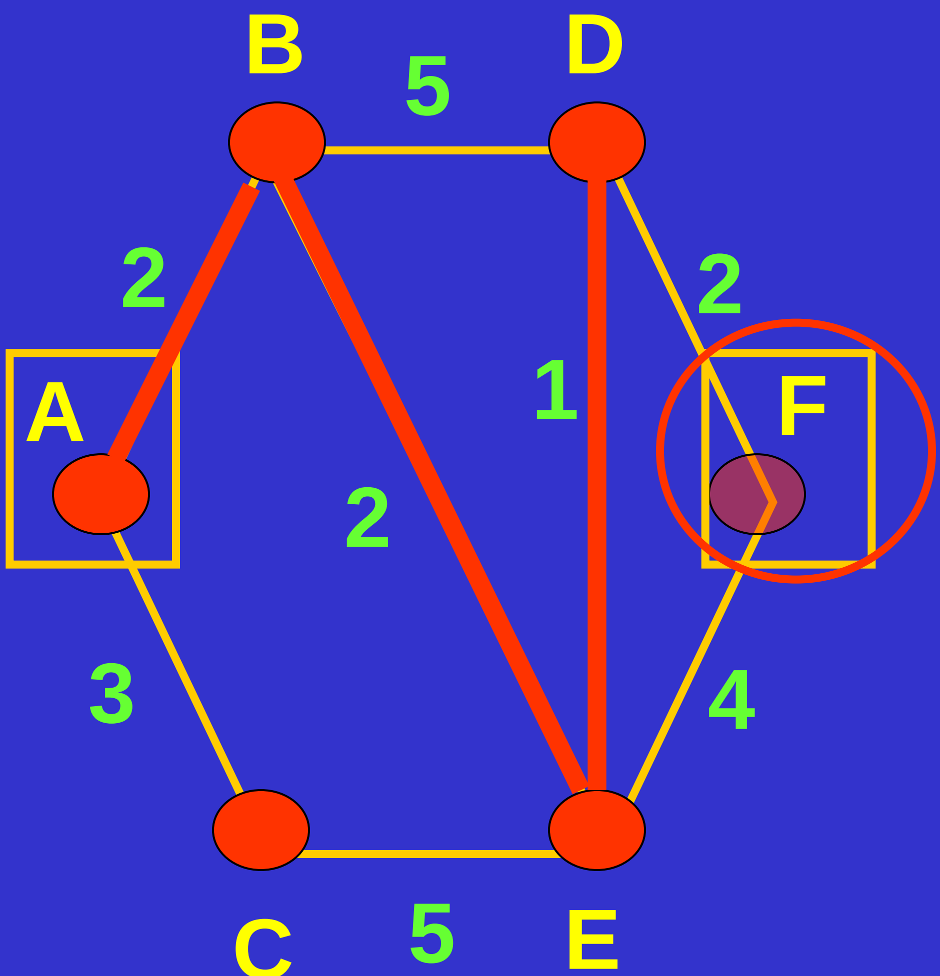
V_6	L_5	L_4
A^*	-	-
B_A^*	-	-
C_A^*	-	-
D_E^*	-	(5)
E_B^*	-	-
F_E	8	8

$$L(F_D) = L(D_E^*) + w(D_E^*, F) = 5+2 = 7$$



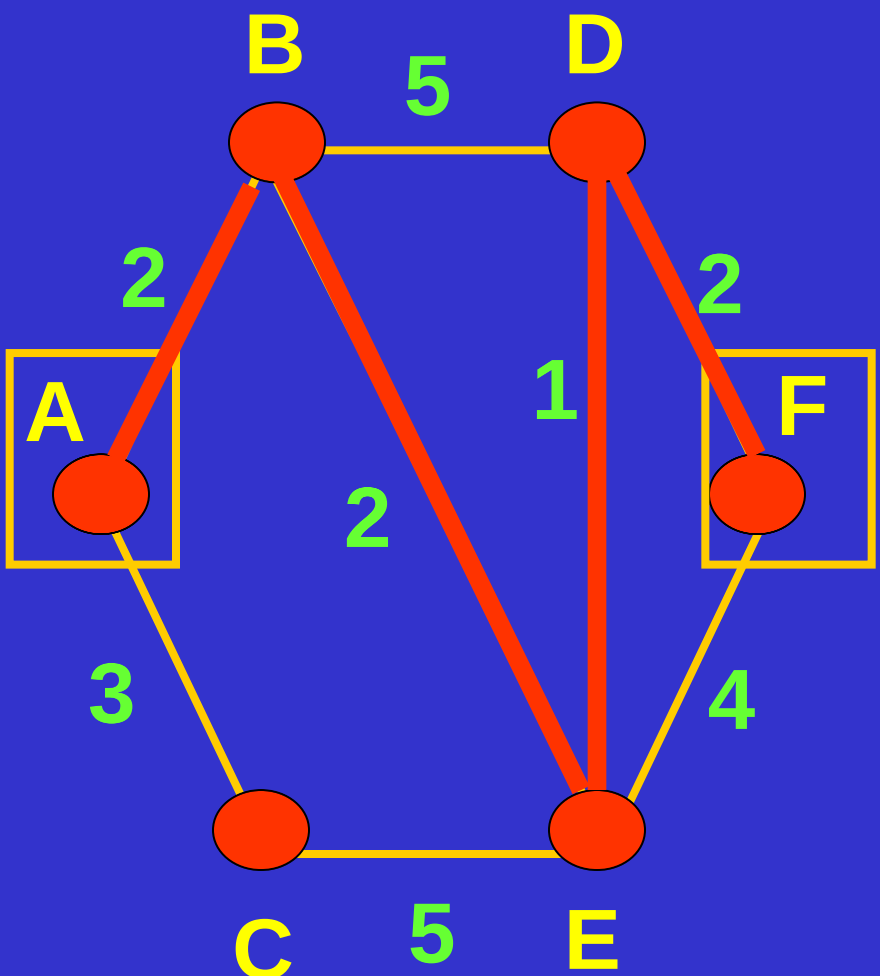
$\underline{V_6}$	$\underline{L_5}$	$\underline{L_4}$
A*	-	-
B_A*	-	-
C_A*	-	-
D_E *	-	(5)
E_B*	-	-
F_E	8	8

$$L(F_D) = L(D_E^*) + w(D_E^*, F) = 5+2 = 7 < 8 \ldots$$



V_6	L_5	L_4
A^*	-	-
$B_A{}^*$	-	-
$C_A{}^*$	-	-
$D_E{}^*$	-	(5)
$E_B{}^*$	-	-
F_D	7	8

$L(F_D) = L(D_E{}^*) + w(D_E{}^*, F) = 5+2 = 7 < 8$ ...



$\underline{V_6}$	$\underline{L_5}$
A*	-
B_A*	-
C_A*	-
D_E*	-
E_B*	-
F_D*	(7)