

Problema de la semana 7. Pablo Cuesta Sierra. NIA: 422974.

$$2) A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}. S = A^t A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}. |S - \lambda I| = (2-\lambda)(1-\lambda) - 1 = \lambda^2 - 3\lambda + 1$$

$$|S - \lambda I| = 0 \Rightarrow \lambda = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}$$

$$E\left(\frac{3+\sqrt{5}}{2}\right) = \ker \begin{pmatrix} \frac{1-\sqrt{5}}{2} & 1 \\ 1 & -\frac{1+\sqrt{5}}{2} \end{pmatrix} = L \left(\vec{q}_1 = \begin{pmatrix} \frac{1+\sqrt{5}}{2} \\ 1 \end{pmatrix} \right) = L \left(\vec{v}_1 = \sqrt{\frac{2}{5+\sqrt{5}}} \begin{pmatrix} \frac{1+\sqrt{5}}{2} \\ 1 \end{pmatrix} \right)$$

$$\|\vec{v}_1\|^2 = \frac{1+5+2\sqrt{5}}{4} + 1 = \frac{5}{2} + \frac{\sqrt{5}}{2}, \|\vec{v}_2\|^2 = \frac{1+5-2\sqrt{5}}{4} + 1 = \frac{5}{2} - \frac{\sqrt{5}}{2}$$

$$E\left(\frac{3-\sqrt{5}}{2}\right) = \ker \begin{pmatrix} \frac{1+\sqrt{5}}{2} & 1 \\ 1 & -\frac{1-\sqrt{5}}{2} \end{pmatrix} = L \left(\vec{q}_2 = \begin{pmatrix} \frac{1-\sqrt{5}}{2} \\ 1 \end{pmatrix} \right) = L \left(\vec{v}_2 = \sqrt{\frac{2}{5-\sqrt{5}}} \begin{pmatrix} \frac{1-\sqrt{5}}{2} \\ 1 \end{pmatrix} \right)$$

$$\rightarrow V = (\vec{v}_1, \vec{v}_2) = \begin{pmatrix} \frac{1+\sqrt{5}}{\sqrt{10+2\sqrt{5}}} & \frac{1-\sqrt{5}}{\sqrt{10-2\sqrt{5}}} \\ \sqrt{\frac{2}{5+\sqrt{5}}} & \sqrt{\frac{2}{5-\sqrt{5}}} \end{pmatrix}$$

$$\vec{u}_1 = \frac{A(\vec{v}_1)}{\sqrt{\frac{3+\sqrt{5}}{2}}} = \frac{\sqrt{2}}{\sqrt{3+\sqrt{5}}} \sqrt{\frac{2}{5+\sqrt{5}}} \begin{pmatrix} \frac{3+\sqrt{5}}{2} \\ \frac{1+\sqrt{5}}{2} \end{pmatrix} = \frac{1}{\sqrt{20+8\sqrt{5}}} \begin{pmatrix} 3+\sqrt{5} \\ 1+\sqrt{5} \end{pmatrix} = \frac{1}{\sqrt{5+2\sqrt{5}}} \begin{pmatrix} \frac{3+\sqrt{5}}{2} \\ \frac{1+\sqrt{5}}{2} \end{pmatrix}$$

$$\vec{u}_2 = \frac{A(\vec{v}_2)}{\sqrt{\frac{3-\sqrt{5}}{2}}} = \frac{\sqrt{2}}{\sqrt{3-\sqrt{5}}} \sqrt{\frac{2}{5-\sqrt{5}}} \begin{pmatrix} \frac{3-\sqrt{5}}{2} \\ \frac{1-\sqrt{5}}{2} \end{pmatrix} = \frac{1}{\sqrt{5-2\sqrt{5}}} \begin{pmatrix} \frac{3-\sqrt{5}}{2} \\ \frac{1-\sqrt{5}}{2} \end{pmatrix}$$

$$\rightarrow U = \begin{pmatrix} \frac{3+\sqrt{5}}{2\sqrt{5+2\sqrt{5}}} & \frac{3-\sqrt{5}}{2\sqrt{5-2\sqrt{5}}} \\ \frac{1+\sqrt{5}}{2\sqrt{5+2\sqrt{5}}} & \frac{1-\sqrt{5}}{2\sqrt{5-2\sqrt{5}}} \end{pmatrix}$$

$$\rightarrow \Sigma = \begin{pmatrix} \sqrt{\frac{3+\sqrt{5}}{2}} & 0 \\ 0 & \sqrt{\frac{3-\sqrt{5}}{2}} \end{pmatrix}$$

$$A = U \Sigma V^t$$