$\boxed{1.}$ $f: \mathbb{R}^4 \to \mathbb{R}^4$. $f(\vec{x}) = A\vec{x}$

$$A = \begin{bmatrix} -2 & -1 & 0 & 2 \\ 0 & -2 & 0 & 0 \\ 1 & 0 & -2 & 1 \\ 0 & 0 & 0 & -2 \end{bmatrix}.$$ Polinomio característico de f: $p_f(x)$

$$p_f(x) = |A - xI| = \begin{vmatrix} -2-x & -1 & 0 & 2 \\ 0 & -2-x & 0 & 0 \\ 1 & 0 & -2-x & 1 \\ 0 & 0 & 0 & -2-x \end{vmatrix} =$$

$$= (-2-x)\begin{vmatrix} -2-x & -1 & 0 \\ 0 & -2-x & 0 \\ 1 & 0 & -2-x \end{vmatrix} = (2+x)^2 \begin{vmatrix} -2-x & -1 \\ 0 & -2-x \end{vmatrix} = (2+x)^4.$$

$$\boxed{p_f(x) = (2+x)^4} = x^4 + 2x^3 + 24x^2 + 32x + 16.$$

Sea λ un autovalor de A, entonces $p_f(\lambda) = 0 \Rightarrow \lambda = -2$
(multiplicidad cuatro).

$E_1(-2) = \operatorname{Ker}(A+2I)$:

$$(A+2I) = \begin{bmatrix} 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \to E_1(-2) = \left\langle \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 0 \\ 1 \end{pmatrix} \right\rangle.$$

$E_2(-2) = \operatorname{Ker}(A+2I)^2$.

$$(A+2I)^2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \to E_2(-2) = \left\langle \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \\ 1 \end{pmatrix} \right\rangle$$

$E_3(-2) = \operatorname{Ker}(A+2I)^3$. $(A+2I)^3 = O_{4\times4} \Rightarrow E_3(-2) = \mathbb{R}^4$.

Tomamos $\vec{u_4} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \in E_3(-2) \setminus E_2(-2)$

$E_3(-2)$	u_4	
$E_2(-2)$	u_3	
$E_1(-2)$	u_2	u_1

$$\vec{u_3} = (A-2I)\vec{u_4} = \begin{pmatrix} 2 \\ 0 \\ 1 \\ 0 \end{pmatrix} \in E_2(-2) \setminus E_1(-2)$$

$$\vec{u_2} = (A-2I)\vec{u_3} = \begin{pmatrix} 0 \\ 0 \\ 2 \\ 0 \end{pmatrix} \in E_1(-2)$$

$\vec{u_1} = \begin{pmatrix} -1 \\ 2 \\ 0 \\ 1 \end{pmatrix} \to$ l.i. a $\vec{u_2}$ y perteneciente a $E_1(-2)$

$A(\vec{u_1}) = -2\vec{u_1}$, $A(\vec{u_2}) = -2\vec{u_2}$, $A(\vec{u_3}) = \vec{u_2} - 2\vec{u_3}$

$A(\vec{u_4}) = \vec{u_3} - 2\vec{u_4}$

Base de Jordan: $\boxed{\mathcal{B} = \left\{ \vec{u_1} = \begin{pmatrix} -1 \\ 2 \\ 0 \\ 1 \end{pmatrix}, \vec{u_2} = \begin{pmatrix} 0 \\ 0 \\ 2 \\ 0 \end{pmatrix}, \vec{u_3} = \begin{pmatrix} 2 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \vec{u_4} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}}$

Matriz de f en esta base: $\boxed{J = \begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & -2 \end{pmatrix}}$

$$P = \begin{pmatrix} -1 & 0 & 2 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} \to A = P\,J\,P^{-1}$$