

# 1. Predicate logic

$$\begin{aligned} a) \cdot \forall p_1, p_2 \quad A(p_1, p_2) &\Rightarrow \neg \exists c_1 \quad C(p_1, c_1) \wedge C(p_2, c_1) \\ &\equiv \forall p_1, p_2 \quad A(p_1, p_2) \Rightarrow \forall c_1 \quad (\neg C(p_1, c_1) \vee \neg C(p_2, c_1)) \\ &\equiv \forall p_1, p_2, c_1 \quad [\neg A(p_1, p_2)] \vee [\neg C(p_1, c_1)] \vee [\neg C(p_2, c_1)] \end{aligned}$$

$$\cdot \forall p_1, p_2 \quad A(p_1, p_2) \Rightarrow A(p_1, p_1)$$

$$\forall p_3 \quad C(p_3, R) \vee C(p_3, G)$$

$$b) \cdot i) A(B_1, B_2), \quad ii) A(B_2, B_3), \quad iii) \neg A(B_1, B_3),$$

$$iv) C(B_2, R)$$

$$c) (A) \neg A(p_1, p_2) \vee \neg (C(p_1, c_1) \vee \neg C(p_2, c_1))$$

$$(B) \neg A(p_3, p_4) \vee A(p_4, p_3)$$

$$(C) C(p_5, R) \vee C(p_5, G)$$

$$d) \text{Goal: } C(B_1, G) \wedge C(B_3, G)$$

$$e) (G) \neg C(B_1, G) \vee \neg C(B_3, G)$$

$$\textcircled{1} A, i) \vdash_{\text{Resol.}} p_1 = B_1, p_2 = B_2, c_1 = R. \neg C(B_2, R)$$

$$\textcircled{2} C, ii) \vdash_{\text{Resol.}} p_5 = B_1. C(B_1, G)$$

$$\textcircled{3} 2, G \vdash_{\text{Resol.}} \neg C(B_3, G)$$

$$\textcircled{4} A, iv, iii) \vdash_{\text{Resol.}} p_1 = B_2, p_2 = B_3, c_1 = R. \neg C(B_3, R)$$

$$\textcircled{5} 4, G \vdash_{\text{Resol.}} p_5 = B_3. C(B_3, G)$$

$$\textcircled{6} 3, 5 \vdash_{\text{Resol.}} \square$$

## 2. Predicate Logic

a) i)  $R(A, B, ABC)$

ii)  $R(B, C, ABC)$

iii)  $\forall w_0, x, y, R(x, y, w_0) \Rightarrow R(y, x, p(y, x, w_0))$

iv)  $\forall w_1, x_1, y_1, z_1$

$$R(x_1, y_1, w_1) \wedge R(y_1, z_1, w_1)$$

$$\Rightarrow R(y_1, x_1, p(y_1, x_1, w_1)) \wedge R(x_1, z_1, p(y_1, x_1, w_1))$$

v)  $\forall w_2, x_2, y_2, z_2$

$$R(x_2, y_2, w_2) \wedge R(y_2, z_2, w_2)$$

$$\Rightarrow R(x_2, z_2, p(z_2, y_2, w_2)) \wedge R(z_2, y_2, p(z_2, y_2, w_2))$$

b) i)  $\neg R(x, y, w_0) \vee R(y, x, p(y, x, w_0))$

1. iv)  $\neg R(x_1, y_1, w_1) \vee \neg R(y_1, z_1, w_1) \vee R(y_1, x_1, p(y_1, x_1, w_1))$

2. iv)  $\neg R(x_3, y_3, w_3) \vee \neg R(y_3, z_3, w_3) \vee R(x_3, z_3, p(y_3, x_3, w_3))$

1. v)  $\neg R(x_2, y_2, w_2) \vee \neg R(y_2, z_2, w_2) \vee R(x_2, z_2, p(z_2, y_2, w_2))$

2. v)  $\neg R(x_4, y_4, w_4) \vee \neg R(y_4, z_4, w_4) \vee R(z_4, y_4, p(z_4, y_4, w_4))$

b) Goal:  $(p(B, C, p(A, B, ABC)) = BCA)$

G1)  $R(B, C, p(B, C, p(A, B, ABC)))$

G2)  $R(C, A, p(B, C, p(A, B, ABC)))$

c) Greens hack:

Suppose  $\neg \text{Goal}: \neg R(B, C, p(C, B, p(B, A, ABC)))$

$\vee \neg R(C, A, p(C, B, p(B, A, ABC)))$

$(G, v) \quad y_2 = A, z_2 = C, x_2 = B, w_2 = p(B, A, ABC)$

Modus tollens

$\vdash$

(1)  $\neg R(B, A, p(B, A, ABC))$

$\vee \neg R(A, C, p(B, A, ABC))$

$((1), iv) \quad y_1 = B, x_1 = A, z_1 = C, w_1 = ABC$

$\vdash_{MT} \neg R(A, B, ABC) \vee \neg R(B, C, ABC) \quad (2)$

(2, i) Resolution  $\neg R(B, C, ABC) \quad (3)$

(3, ii) Resolution  $\square$

### 3. Predicate logic

d) A.  $\text{Factor}(p, q, r) \Leftrightarrow \text{Factor}(p, r, q)$

B.  $\text{Root}(f(x), x)$

C.  $\neg \text{Root}(s, y) \vee \neg \text{Factor}(t, s, v) \rightarrow \text{Root}(t, y)$

D.  $\text{Factor}(\text{product}(p_1, q_1), p_1, q_1)$

e) E.  $\text{Factor}(p, \text{product}(f(2), f(3)), \text{product}(f(2), f(3)))$

f) Goal:  $\text{Root}(p, 2) \wedge \text{Root}(p, 3) \wedge \text{Root}(p, \text{product}(2, 3))$

Introduce:  $G: \neg \text{Root}(p, 2) \vee \neg \text{Root}(p, 3) \vee \neg \text{Root}(p, \text{product}(2, 3))$

①  $x=2, B \text{ Root}(f(2), 2)$

②  $x=3, B \text{ Root}(f(3), 3)$

③  $p_1=f(2), p_2=f(3), \text{Factor}(\text{product}(f(2), f(3)), f(2), f(3))$

④ C.  $s=f(2), y=2, t=\text{product}(f(2), f(3)), v=f(3)$

$\neg \text{Root}(f(2), 2) \vee \neg \text{Factor}(\text{product}(f(2), f(3)), f(2), f(3)) \vee \neg \text{Root}(\text{product}(f(2), f(3)), 2)$

⑤ 4, 3, 1 Resolution:  $\text{Root}(\text{product}(f(2), f(3)), 2)$

⑥ A, 3, Modus tollens:  $\text{Factor}(\text{product}(f(2), f(3)), f(2), f(3))$

⑦ C.  $s=f(3), y=3, t=\text{product}(f(2), f(3)), v=f(2)$

$\neg \text{Root}(f(3), 3) \vee \neg \text{Factor}(\text{product}(f(2), f(3)), f(3), f(2)) \vee \neg \text{Root}(\text{product}(f(2), f(3)), 3)$

⑧ 7, 6, 2 Resolution:  $\text{Root}(\text{product}(f(2), f(3)), 3)$

⑨ E, A: Modus tollens: Factor  $(p, \text{prod}(f(2), f(3)), f(\text{prod}(2, 3)))$

⑩ C.  $s = \text{prod}(f(2), f(3)), y = 3, t = p, v = f(\text{prod}(2, 3))$

$\neg \text{Root}(\text{prod}(f(2), f(3)), 3) \neg \text{Factor}(p, \text{prod}(f(2), f(3)), f(\text{prod}(2, 3)))$   
 $\vee \text{Root}(p, 3)$

⑪ C.  $s = \text{prod}(f(2), f(3)), y = 2, t = p, v = f(\text{prod}(2, 3))$

$\neg \text{Root}(\text{prod}(f(2), f(3)), 2) \neg \text{Factor}(p, \text{prod}(f(2), f(3)), f(\text{prod}(2, 3)))$   
 $\vee \text{Root}(p, 2)$

⑫ 10, 9, 8 Resol.  $\vdash \text{Root}(p, 3)$

⑬ 11, 9, 5, Resol.  $\vdash \text{Root}(p, 2)$

⑭ G; 12, 13 Resol.  $\vdash \neg \text{Root}(p, \text{prod}(2, 3))$

⑮ B.  $x = \text{prod}(2, 3)$   $\text{Root}(f(\text{prod}(2, 3)), \text{prod}(2, 3))$

⑯ C.  $s = f(\text{prod}(2, 3)), y = \text{prod}(2, 3), t = p, v = \text{prod}(f(2), f(3))$

$\neg \text{Root}(f(\text{prod}(2, 3)), \text{prod}(2, 3)) \vee \neg \text{Factor}(p, f(\text{prod}(2, 3)), \text{prod}(f(2), f(3)))$   
 $\vee \text{Root}(p, \text{prod}(2, 3))$

⑰ 16, 15, E. Resolution  $\vdash \text{Root}(p, \text{prod}(2, 3))$

⑱ 17, 14 Resolution  $\vdash \square$