Problema Semana 10. Pablo Cuesta Sierra. NIA: 422974

(2) Mov. helicoidal de recta $x=y=z$, con orientación $(1,1,1)$: ángulo π y desplazamiento $\vec{v}=(3,3,3)$

Función lineal asociada: $\vec{F}$

Construimos una b.o.N con $\vec{u_1} = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}$

$$\vec{u_2} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\\0\end{pmatrix}$$

$$\vec{u_3} = \vec{u_1}\times\vec{u_2} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix} \Big/ \frac{1}{\sqrt{6}} = \frac{1}{\sqrt{6}}\begin{pmatrix}1\\1\\-2\end{pmatrix} \rightarrow \beta = \{\vec{u_1}, \vec{u_2}, \vec{u_3}\}$$

En esta base, $M(\vec{F}, \beta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\pi & -\sin\pi \\ 0 & \sin\pi & \cos\pi \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

$$P = \begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & 0 & -2/\sqrt{6} \end{pmatrix}$$ la matriz del cambio de base.

$$\mathbb{R}^3_e \xrightarrow[P^{-1}=P^t]{Id.} \mathbb{R}^3_\beta \xrightarrow[M(\vec{f},\beta)]{\vec{F}} \mathbb{R}^3_\beta \xrightarrow[P]{Id} \mathbb{R}^3_e$$

$$\rightarrow M = M(\vec{f}, e) = P \cdot M(\vec{f}, \beta) \cdot P^t$$

$$= \begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & 0 & -2/\sqrt{6} \end{pmatrix} \cdot \begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{6} & -1/\sqrt{6} & 2/\sqrt{6} \end{pmatrix}$$

$$= \begin{pmatrix} -1/3 & 2/3 & 2/3 \\ 2/3 & -1/3 & 2/3 \\ 2/3 & 2/3 & -1/3 \end{pmatrix}$$

$$\Rightarrow f\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}3\\3\\3\end{pmatrix} + \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}x'\\y'\\z'\end{pmatrix}$$

$$\begin{cases} x' = 3 - \frac{1}{3}x + \frac{2}{3}y + \frac{2}{3}z \\ y' = 3 + \frac{2}{3}x - \frac{1}{3}y + \frac{2}{3}z \\ z' = 3 + \frac{2}{3}x + \frac{2}{3}y - \frac{1}{3}z \end{cases}$$