$\boxed{\text{AALG V1. Exercises}}$ Pablo Cuesta Sierra.

③ ~~ART~~ BO: $p = p * x$

$$ART_{alg\,3}(N) = \sum_{i=2}^{N} 1 = \underline{N-1}.$$

④ a) $ART_{4a}(n) = \sum_{i=1}^{n} 1 = \underline{\underline{n}}$ BO: $\boxed{\text{sum++;}}$

b) BO: $\boxed{\text{sum++;}}$. $ART_{4b}(n) = \sum_{i=1}^{n} \sum_{j=0}^{n-1} 1 = \sum_{i=1}^{n} n = \underline{\underline{n^2}}.$

c) BO: $\boxed{\text{sum++;}}$ $ART_{4c}(n) = \sum_{i=1}^{n} \sum_{j=0}^{n^2-1} 1 = \sum_{i=1}^{n} n^2 = \underline{\underline{n^3}}$

⑤ a) BO: $\boxed{\text{sum++;}}$ $ART_{5a}(n) = \sum_{i=1}^{n} \sum_{j=0}^{i-1} 1 = \sum_{i=1}^{n} i = \underline{\underline{\frac{(n+1)n}{2}}} = \frac{n^2+n}{2}.$

b)

$$ART_{5b}(n) = \sum_{i=1}^{n} \sum_{j=0}^{i^2-1} \sum_{k=0}^{j-1} 1 = \sum_{i=1}^{n} \sum_{j=0}^{i^2-1} j =$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{i^2-1} j = \sum_{i=1}^{n} \frac{i^2(i^2-1)}{2} = \boxed{\frac{1}{2}\left(\sum_{i=1}^{n} (i^4 - i^2)\right)}$$

(5c)

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sum = 0
for (i=1; i≤n; i++)
    for (j=0; j<i²; j++)
        if (j%i == 0)   (*)
            for (K=0; K<j; K++)
                sum++;
```

A, B

(*) $j \% i == 0 \iff j = l \cdot i$ For some $l \in \{0, 1, 2, \dots, i-1\}$ because $j < i^2$

we can take the 0 out, the algorithm won't change.

$$ART_A(n) = \sum_{i=1}^{n} ART_B(i)$$

$$ART_B(i) = \sum_{l=1}^{i-1} \sum_{K=0}^{l \cdot i - 1} 1 = \sum_{l=1}^{i-1} l \cdot i = i \sum_{l=1}^{i-1} l =$$

$$= i \cdot \frac{i(i-1)}{2}$$

$$\Rightarrow \boxed{ART_A(n) = \sum_{i=1}^{n} \frac{i^3 - i^2}{2}} = \frac{1}{24}(n-1)n(n+1)(3n+2).$$

(6) $\max(A,B) \geq A$, $\max(A,B) \geq B$

$\Rightarrow A + B \leq \max(A,B) + \max(A,B) \Rightarrow \frac{A+B}{2} \leq \max(A,B)$.

As $A, B > 0$, $\max(A,B) + \min(A,B) \geq \max(A,B)$

$A + B = \max(A,B) + \min(A,B) \geq \max(A,B)$. ∎