1. Please convert between number systems -
    a. $(5E4)_{16}$ to Octal
    b. $(299)_8$ to Hexadecimal
    c. $(433)_{10}$ to binary, hexadecimal and Octal
    d. $(25B)_{16}$ to Decimal
    e. $(346)_8$ to Decimal
2. For each pair of expressions, construct truth tables to see if the two compound propositions are logically equivalent:
    a. $p \vee (q \wedge \neg p)$, $p \vee q$
    b. $(\neg p \wedge q) \vee (p \wedge \neg q)$, $(\neg p \wedge \neg q) \vee (p \wedge q)$
3. The following sets have been defined using the | notation. Re-write them by listing some of the elements.
    a. $\{p \mid p$ is a capital city, p is in Europe$\}$
    b. $\{x \mid x = 2n - 5$, x and n are natural numbers$\}$
    c. $\{y \mid 2y^2 = 50$, y is an integer$\}$
    d. $\{z \mid 3z = n^2$, z and n are natural numbers$\}$
4. **U** = {a, b, c, d, e, f, g, h}; P = {c, f}; Q = {a, c, d, e, f, h}; R = {c, d, h}
    a. Draw a Venn diagram, showing these sets with all the elements entered into the appropriate regions. If necessary, redraw the diagram to eliminate any empty regions.
    b. Which of sets P, Q and R are proper subsets of others? Write your answer(s) using the $\subset$ symbol.
    c. P and R are disjoint sets. True or False?
5. Make six copies of the Venn diagram shown alongside, and then shade the areas represented by:
    a. $A' \cup B$
    b. $A \cap B'$
    c. $(A \cap B)'$
    d. $A' \cup B'$
    e. $(A \cup B)'$
    f. $A' \cap B'$
6. Prove the following identities, stating carefully which of the set laws you are using at each stage of the proof.
    a. $B \cup (\varnothing \cap A) = B$
    b. $(A' \cap \mathbf{U})' = A$
    c. $(C \cup A) \cap (B \cup A) = A \cup (B \cap C)$
    d. $(A \cap B) \cup (A \cap B') = A$

e. $(A \cap B) \cup (A \cup B')' = B$

f. $A \cap (A \cup B) = A$

7. $X = \{a, c\}$ and $Y = \{a, b, e, f\}$. Write down the elements of:
   a. $X \times Y$
   b. $Y \times X$
   c. $X^2 (= X \times X)$
   d. What could you say about two sets A and B if $A \times B = B \times A$?
8. Propositions p, q, r and s are defined as follows:
   a. p is "I shall finish my Coursework Assignment"
   b. q is "I shall work for forty hours this week"
   c. r is "I shall pass Maths"
   d. s is "I like Maths"

   Write each sentence in symbols:

   (a) I shall not finish my Coursework Assignment.

   (b) I don't like Maths, but I shall finish my Coursework Assignment.

   (c) If I finish my Coursework Assignment, I shall pass Maths.

   (d) I shall pass Maths only if I work for forty hours this week and finish my Coursework Assignment.

   Write each expression as sensible English sentence:

   (e) $q \vee p$

   (f) $\neg p \Rightarrow \neg r$

9. Draw up truth tables to show that $p \Rightarrow q$, $\neg p \vee q$ and $\neg q \Rightarrow \neg p$ are all logically equivalent.

   The following predicates are defined:

   friend is "… is a friend of mine" = F(x)

   wealthy is "… is wealthy" = W(x)

   clever is "… is clever" = C(x)

   boring is "… is boring" = B(x)

Using the same predicates we just defined, symbolise each of the following.

(a) Some of my friends are clever.

(b) All clever people are boring.

(c) None of my friends is wealthy.

(d) Some of my wealthy friends are clever.

(e) All my clever friends are boring.

(f) All clever people are either boring or wealthy.

10. More set exercise:
    a. If $A = \{1, 2, 3, 4\}$, write down $\mathbf{P}(A)$ by listing its elements. What is the value of $|\ \mathbf{P}(A)\ |$?
    b. If $|\ A\ | = 5$, what is the value of $|\ \mathbf{P}(A)\ |$?
    c. If $|\ A\ | = 10$, what is the value of $|\ \mathbf{P}(A)\ |$?
    d. $\mathbf{U}$ = {natural numbers}; $A = \{2, 4, 6, 8, 10\}$; $B = \{1, 3, 6, 7, 8\}$. State whether each of the following is true or false:
        i. $2 \in A$
        ii. $11 \in B$
        iii. $4 \notin B$
        iv. $A \in \mathbf{U}$
        v. A = {even numbers}
11. Identify each statement as a tautology, a contradiction, or a contingency
    a. $\neg(p \vee \neg q) = \neg p \wedge q$
    b. $(p \vee (\neg p \wedge q)) \rightarrow \neg q$
    c. $(q \vee (\neg p \wedge q)) \rightarrow \neg p$
    d. $(p \rightarrow (q \wedge \neg q)) \rightarrow \neg p$
12. Use mathematical notation to write the following predicates.
    a. Every natural number greater than 0 lies within an interval bounded by powers of 2
    b. There is no largest prime number
    c. Every integer a can be divided by a non-zero integer n to produce an integer quotient and remainder

13. Give the form of each argument. Then prove whether the argument is valid or invalid. For valid arguments, use the rules of inference to prove validity.

I will buy a new car and a new house only if I get a job.
I am not going to get a job.
I will buy a new house.

∴ I will not buy a new car.

- j: I will get a job
- c: I will buy a new car
- h: I will buy a new house

The form of the argument is

$(c \wedge h) \rightarrow j$
$\neg j$
h

$\therefore \neg c$

14. Prove whether the argument is valid or invalid

Every student who missed class or got a detention did not get an A.
Penelope is a student in the class.
Penelope got an A.

Penelope did not get a detention.

- M(x): x missed class
- D(x): x got a detention.
- A(x): x got an A.

$\forall x ((M(x) \vee D(x)) \rightarrow \neg A(x))$
Penelope is a particular student
$A(Penelope)$

$\therefore \neg D(Penelope)$