Topic Master's Thesis.

Machine Learning Approaches to the Inverse Problem of Identifying Cracks from Electrical Signals in Structural Health Monitoring.

EQUATIONS:-

[A].

Based on independent variables:-

Material equations for an orthotropic piezoelectric sensor.

[A]

1. $\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl} - e_{kij} E_k$.

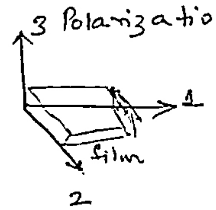
$D_i = e_{ikl}\,\varepsilon_{kl} + K_{ij} E_j$

$C_{ijkl} \rightarrow$ elasticity tensor (Partial derivative of σ_{ij}) w.r.t ε_{ij}

$K_{ij} \rightarrow$ dielectric tensor (derivative of electric displacement D_i)

$e_{kij} \rightarrow$ coupling of mechanical & electrical fields.

2. $\sigma_{33} = C_{31}\varepsilon_{11} + C_{32}\varepsilon_{22} - e_{33}E_3 = 0$



3. $D_3 = e_{31}\varepsilon_{11} + e_{32}\varepsilon_{22} + e_{33}\varepsilon_{33} + K_{33}E_3$.

4. E_3 (Electric field) $= C_1\varepsilon_{11} + C_2\varepsilon_{22} + C_3 D_3$

C_1, C_2, C_3 (Constants).

(5) $C_1 = \dfrac{C_{31}e_{33} - C_{33}e_{31}}{e_{33}^2 + K_{33}C_{33}}$; (6) $C_2 = \dfrac{C_{32}e_{33} - C_{33}e_{32}}{e_{33}^2 + K_{33}C_{33}}$; (7) $C_3 = \dfrac{C_{33}}{e_{33}^2 + K_{33}C_{33}}$.

$D_3 = 0$ (No free charges accumulate at film surface).

$\Delta\Phi_{AB} = \Phi_A - \Phi_B$ [Potential difference between lower and upper film surface].

8. $E_3 = -\nabla\Phi = -\dfrac{\Delta\Phi_{AB}}{h}$ $\because$ h = film thickness.

9. Voltage at the film $(U_{AB}) = -h\,(C_1\varepsilon_{11} + C_2\varepsilon_{22})$.

10. $\hookrightarrow$ for transversely isotropic sensor film arrangement, [two orthotropic film are rotated by 90° to each other].

$U_{AB} = -h\,C\,(\varepsilon_{11} + \varepsilon_{22})$. $\because$ $C = C_1 = C_2$.

Φ_A, Φ_B, Φ_C (Surfaces of A & B & AB).

11. $U_{AC} = -\left(E_3^{F1} h^{F1} + E_3^{F2} h^{F2}\right)$.

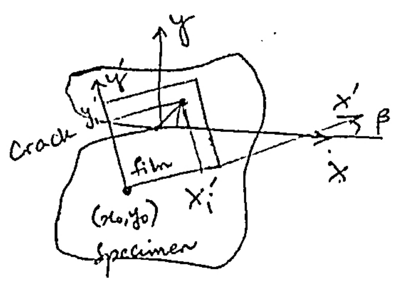
12. if $h = h^{F1} = h^{F2}$ (Both films with same thickness) } $\because U_{AC} = -h C^*(\varepsilon_{11} + \varepsilon_{22})$.

$$C^* = \frac{(c_{31} + c_{32}) e_{33} - (e_{31} + e_{32}) c_{33}}{(e_{33}^2 + K_{33} c_{33})}$$

∴ Due to the adhesive bonding of the film to the specimen (layer representia joint with viscoelastic properties.), discontinuity of the displacement U_1, U_2 at the interface can be taken into account by $g \leq 1$:

13. $\varepsilon_{ij}^F = g\, \varepsilon_{ij}^S \quad (i = j)$ $\because F \to$ Film, $S \to$ structure.

14. $\varepsilon_{11}^F = g\left(\varepsilon_{11}^S \cos^2\beta + \varepsilon_{22}^2 \sin^2\beta + 2\varepsilon_{12}^S \sin\beta \cos\beta\right)$ [∵



(x, y) Crack C.S.
(x', y') film C.S.

∵ Assuming a plane stress state at the surface of the specimen

15. $\varepsilon_{12}^S = \frac{(1 + \nu^S)}{E^S} \sigma_{12}$. ∵ $E^S \to$ elastic modulus
$\nu^S \to$ Poison's ratio.

16. $$U_{AB} = -\frac{hg}{E^S}\left\{\sigma_{11}\left[\cos^2\beta(c_1 - \nu^S c_2) + \sin^2\beta(c_2 - \nu^S c_1)\right] + \sigma_{22}\left[\sin^2\beta(c_1 - \nu^S c_2) + \cos^2\beta(c_2 - \nu^S c_1)\right] + \sigma_{12}(1 + \nu^S)(c_1 - c_2)\sin(2\beta)\right\}.$$

17. $$U_{AC} = -\frac{hg\, C^* (1 - \nu^S)}{E^S}\left\{\sigma_{11} + \sigma_{22}\right\}.$$

[A]

* Asymptotic crack tip near field.

In the Linear Elastic Fracture Mechanics (LEFM).

$(r \to 0)$. SIF's. ([Gross & Seelig (2007)], [Kuna (2008)]).

18. $$\sigma_{11} = \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\varphi}{2}\right)\left(1 - \sin\left(\frac{\varphi}{2}\right)\sin\left(\frac{3\varphi}{2}\right)\right) - \frac{K_{II}}{\sqrt{2\pi r}} \sin\left(\frac{\varphi}{2}\right)\left(2 + \cos\left(\frac{\varphi}{2}\right)\cos\left(\frac{3\varphi}{2}\right)\right) + T$$

19. $$\sigma_{22} = \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\varphi}{2}\right)\left(1 + \sin\left(\frac{\varphi}{2}\right)\sin\left(\frac{3\varphi}{2}\right)\right) + \frac{K_{II}}{\sqrt{2\pi r}} \sin\left(\frac{\varphi}{2}\right)\cos\left(\frac{\varphi}{2}\right)\cos\left(\frac{3\varphi}{2}\right).$$

20. $$\sigma_{12} = \frac{K_I}{\sqrt{2\pi r}} \sin\left(\frac{\varphi}{2}\right)\cos\left(\frac{\varphi}{2}\right)\cos\left(\frac{3\varphi}{2}\right) + \frac{K_{II}}{\sqrt{2\pi r}} \cos\left(\frac{\varphi}{2}\right)\left(1 - \sin\left(\frac{\varphi}{2}\right)\sin\left(\frac{3\varphi}{2}\right)\right).$$

$\boxed{-\pi \le \varphi \le \pi}$. $\because$ K_I & K_{II} represent the two-in-plane crack opening modes I and II.

* 21. $$U_{AB} = C_0 \sum_{n=1}^{\infty} r^{\frac{n}{2}-1}\left(a_n \left(M_{11}^{(n)} f_I + M_{22}^{(n)} f_{II} + M_{12}^{(n)} f_{III}\right) + b_n \left(N_{11}^{(n)} f_I + N_{22}^{(n)} f_{II} + N_{12}^{(n)} f_{III}\right)\right),$$

22. $$U_{AC} = C_0^* \left\{ \sum_{n=1}^{\infty} n r^{\frac{n}{2}-1}\left[a_n \cos\alpha_n - b_n \sin\alpha_n\right]\right\}.$$

$$\because \quad C_0^* = -2hg\frac{C^*(1-\nu^S)}{E^S}, \quad \alpha_n = \left(\frac{n}{2}-1\right)\varphi.$$

21 & 22. are electrical potential difference at (r, φ).

The following equations relate to the two coordinate systems:-

23. $x_i = x_i' \cos\beta - y_i' \sin\beta + x_0,$

24. $y_i = x_i' \sin\beta + y_i' \cos\beta + y_0$

25. $r_i = \sqrt{x_i^2 + y_i^2}$

26. $\varphi_i = \arccos \dfrac{x_i}{r_i}$ $\boxed{0 \le \varphi_i \le \pi}$

$\varphi_i = -\arccos \dfrac{x_i}{r_i}$ $\boxed{-\pi \le \varphi_i < 0}$

Near Crack tip Stresses and Strains:-

from [B].

$$\varepsilon_{11}^S = \frac{1}{E^B}\left(\sigma_{11}^S - \nu^S \sigma_{22}^S\right), \quad \varepsilon_{22}^S = \frac{1}{E^B}\left(\sigma_{22}^S - \nu^S \sigma_{11}^S\right), \quad \varepsilon_{12}^S = \frac{(1+\nu^S)}{E^S}\sigma_{12}^S.$$

$S \rightarrow$ stands for specimen.

$$\sigma_{11}^S = \sum_{n=1}^{\infty} r_i^{\frac{n}{2}-1}\left\{a_n M_{11}^{(n)} + b_n N_{11}^{(n)}\right\}, \quad \sigma_{22}^S = \sum_{n=1}^{\infty} r_i^{\frac{n}{2}-1}\left\{a_n M_{22}^{(n)} + b_n N_{22}^{(n)}\right\},$$

$$\sigma_{12}^S = \sum_{n=1}^{\infty} r_i^{\frac{n}{2}-1}\left\{a_n M_{12}^{(n)} + b_n N_{12}^{(n)}\right\}$$

$$* \quad M_{11}^{(n)} = \frac{n}{2}\left\{\left[2 + (-1)^n + \frac{n}{2}\right]\cos\left[\left(\frac{n}{2}-1\right)\varphi_i\right] - \left(\frac{n}{2}-1\right)\cos\left[\left(\frac{n}{2}-3\right)\varphi_i\right]\right\},$$

$$N_{11}^{(n)} = \frac{n}{2}\left\{\left[-2 + (-1)^n - \frac{n}{2}\right]\sin\left[\left(\frac{n}{2}-1\right)\varphi_i\right] + \left(\frac{n}{2}-1\right)\sin\left[\left(\frac{n}{2}-3\right)\varphi_i\right]\right\},$$

$$M_{22}^{(n)} = \frac{n}{2}\left\{\left[2 - (-1)^n - \frac{n}{2}\right]\cos\left[\left(\frac{n}{2}-1\right)\varphi_i\right] + \left(\frac{n}{2}-1\right)\cos\left[\left(\frac{n}{2}-3\right)\varphi_i\right]\right\},$$

$$N_{22}^{(n)} = \frac{n}{2}\left\{\left[-2 - (-1)^n + \frac{n}{2}\right]\sin\left[\left(\frac{n}{2}-1\right)\varphi_i\right] - \left(\frac{n}{2}-1\right)\sin\left[\left(\frac{n}{2}-3\right)\varphi_i\right]\right\},$$

$$M_{12}^{(n)} = \frac{n}{2}\left\{\left(\frac{n}{2}-1\right)\sin\left[\left(\frac{n}{2}-3\right)\varphi_i\right] - \left[\frac{n}{2} + (-1)^n\right]\sin\left[\left(\frac{n}{2}-1\right)\varphi_i\right]\right\},$$

$$N_{12}^{(n)} = \frac{n}{2}\left\{\left(\frac{n}{2}-1\right)\cos\left[\left(\frac{n}{2}-3\right)\varphi_i\right. - \left[\frac{n}{2} - (-1)^n\right]\cos\left[\left(\frac{n}{2}-1\right)\varphi_i\right]\right\}.$$

Conversion of strains from crack tip coordinate system to the local film coordinate system:-

$$\varepsilon_{11}^F = \varepsilon_{11}^S \cos^2\beta + \varepsilon_{22}^S \sin^2\beta + \varepsilon_{12}^S \sin(2\beta).$$

$$\varepsilon_{22}^F = \varepsilon_{11}^S \sin^2\beta + \varepsilon_{22}^S \cos^2\beta - \varepsilon_{12}^S \sin(2\beta).$$

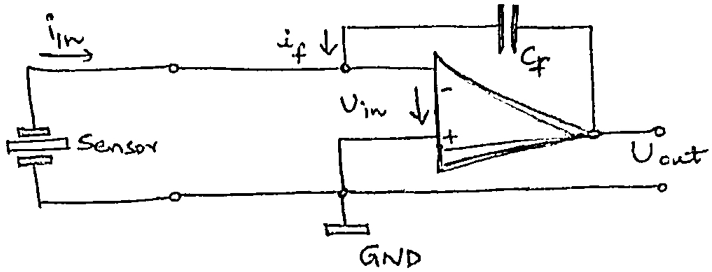
$$\varepsilon_{12}^F = -\left(\varepsilon_{11}^S - \varepsilon_{22}^S\right)\sin\beta\cos\beta + \varepsilon_{12}^S\left(\cos^2\beta - \sin^2\beta\right).$$

[B].

Inverse Problem

$i_{in} \rightarrow$ current

Output Voltage, $U_{out} = \frac{1}{C_f}\int_0^t i_{in}\,dt = \frac{Q_{in}}{C_f}$ or $U_{out} = -\frac{1}{C_f}\int_0^t i_f\,dt = -\frac{Q_f}{C_f}$.



IP.

$$\sigma_{33}^F = C_{31}\varepsilon_{11}^F + C_{32}\varepsilon_{22}^F + C_{33}\varepsilon_{33}^F = 0$$

$$D_3 = e_{31}\varepsilon_{11}^F + e_{32}\varepsilon_{22}^F + e_{33}\varepsilon_{33}^F.$$

$Q_{in} = D_3 A$. $\because$ A (small electrode area).

$$\because U_{out} = -\frac{A}{C_f}\left(\varepsilon_{11}^F\left(e_{31} - e_{33}\frac{C_{31}}{C_{33}}\right) + \varepsilon_{22}^F\left(e_{32} - e_{33}\frac{C_{32}}{C_{33}}\right)\right)$$

$\because$ Relationship between measured output voltages and the load at the crack tip.

$$U_{out} = C_0\left\{\sum_{n=1}^{\infty} r_i^{\frac{n}{2}-1}\left[a_n\left(M_{11}^{(n)}f_I + M_{22}^{(n)}f_{II} + M_{12}^{(n)}f_{III}\right) + b_n\left(N_{11}^{(n)}f_I + N_{22}^{(n)}f_{II} + N_{12}^{(n)}f_{III}\right)\right]\right\}$$

$$f_I = \cos^2\beta\,(C_{11} - \nu^S C_{22}) + \sin^2\beta\,(C_{22} - \nu^S C_{11}),$$

$$f_{II} = \sin^2\beta\,(C_{11} - \nu^S C_{22}) + \cos^2\beta\,(C_{22} - \nu^S C_{11}),$$

$$f_{III} = (1+\nu^S)(C_{11} - C_{22})\sin(2\beta),.$$

$$C_0 = -\frac{A}{E^S C_f},\quad C_{11} = e_{31} - \frac{C_{31}}{C_{33}}e_{33},\quad C_{22} = e_{32} - \frac{C_{32}}{C_{33}}e_{33}.$$