INFERENCE FROM LITERATURE RESEARCH(TENTATIVE DRAFT)

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MASTER'S THESIS ML approaches to the Inverse problem of identifying cracks

Introduction:

Journals:

A. Sensor Concept Based on Piezoelectric PVDF Films for the Structural Health Monitoring of Fatigue Crack Growth by Dennis Bäcker , Andreas Ricoeur and Meinhard Kuna.
B. Piezoelectric sensor for in-situ measurement of stress intensity factors by Dennis Bäcker , C. Haeusler and Meinhard Kuna
C. A PVDF sensor for the in-situ measurement of stress intensity factors during fatigue crack growth by M. Kuna , D. Bäcker

From [A&C] Crack growth influences the life cycle of any structure under a deterministic or a stochastic load(s). Because of complex load collectives and geometries in civil engineering structures, pure calculations may not be effective in ensuring the minimum life cycle of machines and obtaining exact numerical stress analysis. For this reason, regular inspections of highly loaded components for crack growth are critical. [C] new type of crack sensor has been developed, which is particularly suitable for monitoring cracks in plate and shell structures under conditions of linear elastic fracture mechanics.

Cracks are evaluated or detected in two ways:

1. Size and location of cracks.
    a. Interaction principle of high-frequency mechanical waves with the crack.
        i. piezoelectric or magnetostrictive [Kwun et al. (2002)]
        ii. signal of a strain gauge ([Gama and Morikawa (2008)], [Kurosaki et al. (2002)])
2. Stress state quantification at crack tip.
    a. photo elastic methods ([Lu and Chiang (1993)], [Singh and Shuka (1996)])
    b. laser interferometry and the caustic method [Shozu et al. (2002)]
    c. thermo elastic stress methods ([Shiratori et al. (1990)], [Honda et al. (2002)])
    d. optical methods using digital image processing technology ([McNeill et al. (1987)], [Rethore et al. (2005)], [Roux and Hild (2006)]).
    e. Potential difference at crack when high frequent AC applied (skin effect) [Saka et al. (1991)] (rarely used)
    f. Strain gauges at front of crack tip ([Irwin (1957)], [Dally and Sanford (1988)], [Dally and Sanford (1990)], [Putra (2000)]). (Not suitable for long term monitoring)
    g. *piezoelectric polymer films as sensors ([Fujimoto et al. (2003)], [Fujimoto et al. (2003)], [Fujimoto et al. (2004)])* Advantages: compensates for disadvantages of classical resistance strain gauges, are easily applied and are optimally suitable for long-term monitoring.

However, recently developed concepts based on piezoelectric polymers merely aim at the experimental determination of SIFs for a given crack length and do not include crack growth into consideration.[A]

Polyvinylidene fluoride (PVDF) (PIEZOELCTRIC POLYMER MATERIAL):

From [A&B]

- applicable as actuator or sensor
- maximum of polarization in the βÂ-modification(CF2-dipoles aligned ⊥ to molecular chain axis)[Danz and Geiss (1987)]
- piezoelectric and dielectric properties are comparable to ceramic piezoelectrics
- material is orthotropic (due to anisotropy in 1-2-plane by production)
- small elastic modulus and extremely low mass
- application on any curved surface is feasible
- temperature range -70˚C and +90˚C

- strong pyroelectric properties
- possibility for construction of complex sensor arrays accomplishing a high spatial resolution.

Macroscopic piezoelectric properties of the polymeric material are obtained by the polarization process, where in most cases a mechanical extension superimposes an electric field in the thickness direction.[A]

The material constants of PVDF were taken from [Roh et al. (2002)]. Within the framework of a thermodynamically consistent material modelling they are transformed with respect to strain and electric field as independent variables.

EQUATIONS:

Separately drafted …. Attached below

SOLUTION OF THE INVERSE PROBLEM:

From [A],

- If the loading situation at the crack tip is to be determined based on these measured potentials the solution of an inverse boundary value problem of the theory of elasticity is required.
- If the crack paths are known, K-factors, T-stress and optionally further terms of the crack solution are the unknown quantities to be determined requiring a system of at least three equations associated with three measuring points.
- However, the crack position with respect to the film is not known and the coordinates of the measuring points (ri , φi) in the crack coordinate system (x, y) thus are not available. Introducing a local coordinate system (x 0 , y 0), the position of any measuring point can be calculated by coordinate transformation to the crack coordinate system (x, y). As the coordinates of the electrodes with respect to the film are known, the fracture quantities are complemented by three more unknowns (x0, y0,β) describing the film position with respect to the crack faces. Thus, three more measuring points and equations need to be considered.
- [B&C] The presented approach allows a simultaneous calculation of stress intensity factors and crack tip position.
- To solve the nonlinear system of equations two different procedures are applied: the principal axis method [Brent (1973)] and the Levenberg-Marquardt method [Moré (1977)].
- Calculations have proven that the influence on the potentials at sensors lying close to the crack path can be neglected.

<u>The principal axis method exhibits some advantages with respect to the Levenberg-Marquardt procedure.</u>

RS – reference solution, PA – principal axis method, LM – Levenberg-Marquardt method, IV – initial values.

[C] An accuracy of 1% is achieved concerning the crack position and the value of stress intensity factors.

[B] Since this information is obtained on-line at a growing crack, the sensor can be used to monitor crack propagation and to predict the further crack path and crack growth rate.

The aim of the thesis comes from the proposed extensions in [B] that is “appropriate choice of the initial values for the solving the inverse problem in the very first measurement”.

Topic Master's Thesis.

Machine Learning Approaches to the Inverse Problem of Identifying Cracks from Electrical Signals in Structural Health Monitoring.

EQUATIONS:-

[A].

Based on independent variables:-

Material equations for an orthotropic piezoelectric sensor.

[A]

1. $\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl} - e_{kij} E_k$.

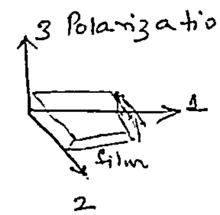
$D_i = e_{ikl}\,\varepsilon_{kl} + k_{ij} E_j$

$C_{ijkl} \rightarrow$ elasticity tensor (Partial derivative of σ_{ij}) w.r.t ε_{ij}

$k_{ij} \rightarrow$ dielectric tensor (derivative of electric displacement D_i)

$e_{kij} \rightarrow$ coupling of mechanical & electrical fields.

2. $\sigma_{33} = C_{31}\varepsilon_{11} + C_{32}\varepsilon_{22} - e_{33}E_3 = 0$



3. $D_3 = e_{31}\varepsilon_{11} + e_{32}\varepsilon_{22} + e_{33}\varepsilon_{33} + k_{33}E_3$.

4. E_3 (Electric field) $= C_1\varepsilon_{11} + C_2\varepsilon_{22} + C_3 D_3$.

C_1, C_2, C_3 (Constants).

$$C_1 = \frac{c_{31}e_{33} - c_{33}e_{31}}{e_{33}^2 + k_{33}c_{33}} \quad (5) \;;\quad C_2 = \frac{c_{32}e_{33} - c_{33}e_{32}}{e_{33}^2 + k_{33}c_{33}} \quad (6) \;;\quad C_3 = \frac{c_{33}}{e_{33}^2 + k_{33}c_{33}} \quad (7).$$

$D_3 = 0$ (No free charges accumulate at film surface).

$\Delta\Phi_{AB} = \Phi_A - \Phi_B$ [Potential difference between lower and upper film surface].

8. $E_3 = -\nabla\Phi = -\dfrac{\Delta\Phi_{AB}}{h}$ $\quad\because$ h = film thickness.

9. Voltage at the film (U_{AB}) $= -h\,(C_1\varepsilon_{11} + C_2\varepsilon_{22})$.

10. $\hookrightarrow$ for transversely isotropic sensor film arrangement, [two orthotropic film are rotated by 90° to each other].

$U_{AB} = -h\,C\,(\varepsilon_{11} + \varepsilon_{22})$. $\quad\because\; C = C_1 = C_2$.

Φ_A, Φ_B, Φ_C (Surfaces of A & B & AB).

11. $U_{AC} = -\left(E_3^{F1} h^{F1} + E_3^{F2} h^{F2}\right).$

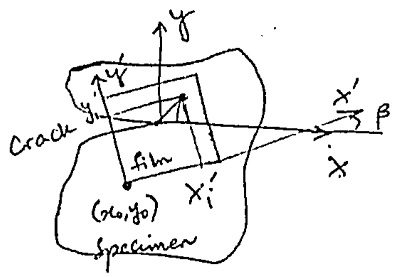
12. if $h = h^{F1} = h^{F2}$ (Both films with same thickness) } $\because U_{AC} = -h C^*(\varepsilon_{11} + \varepsilon_{22}).$

$$C^* = \frac{(c_{31} + c_{32}) e_{33} - (e_{31} + e_{32}) c_{33}}{(e_{33}^2 + K_{33} c_{33})}$$

$\therefore$ Due to the adhesive bonding of the film to the Specimen (layer representing a joint with viscoelastic properties.), discontinuity of the displacements U_1, U_2 at the interface can be taken into account by $g \leq 1$:

13. $\varepsilon_{ij}^F = g\, \varepsilon_{ij}^S \quad (i = j)$ $\because$ f → Film, S → structure.

14. $\varepsilon_{11}^F = g\left(\varepsilon_{11}^S \cos^2\beta + \varepsilon_{22}^2 \sin^2\beta + 2\varepsilon_{12}^S \sin\beta \cos\beta\right)$ [$\because$



(x, y) Crack C.S.
(x', y') film C.S.

$\because$ Assuming a plane stress state at the surface of the specimen

15. $\varepsilon_{12}^S = \dfrac{(1 + \nu^S)}{E^S} \sigma_{12}.$ $\because$ E^S → elastic modulus, ν^S → Poisson's ratio.

16. $$U_{AB} = -\frac{hg}{E^S}\left\{ \sigma_{11}\left[\cos^2\beta (c_1 - \nu^S c_2) + \sin^2\beta (c_2 - \nu^S c_1)\right] + \sigma_{22}\left[\sin^2\beta (c_1 - \nu^S c_2) + \cos^2\beta (c_2 - \nu^S c_1)\right] + \sigma_{12}(1 + \nu^S)(c_1 - c_2)\sin(2\beta) \right\}.$$

17. $$U_{AC} = -\frac{hg\, C^* (1 - \nu^S)}{E^S} \left\{ \sigma_{11} + \sigma_{22} \right\}.$$

[A]

* Asymptotic crack tip near field.

In the Linear Elastic Fracture Mechanics (LEFM).

$(r \to 0)$. SIF's. ([Gross & Seelig (2007)], [Kuna (2008)]).

18. $$\sigma_{11} = \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\varphi}{2}\right)\left(1 - \sin\left(\frac{\varphi}{2}\right)\sin\left(\frac{3\varphi}{2}\right)\right) - \frac{K_{II}}{\sqrt{2\pi r}} \sin\left(\frac{\varphi}{2}\right)\left(2 + \cos\left(\frac{\varphi}{2}\right)\cos\left(\frac{3\varphi}{2}\right)\right) + T$$

19. $$\sigma_{22} = \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\varphi}{2}\right)\left(1 + \sin\left(\frac{\varphi}{2}\right)\sin\left(\frac{3\varphi}{2}\right)\right) + \frac{K_{II}}{\sqrt{2\pi r}} \sin\left(\frac{\varphi}{2}\right)\cos\left(\frac{\varphi}{2}\right)\cos\left(\frac{3\varphi}{2}\right).$$

20. $$\sigma_{12} = \frac{K_I}{\sqrt{2\pi r}} \sin\left(\frac{\varphi}{2}\right)\cos\left(\frac{\varphi}{2}\right)\cos\left(\frac{3\varphi}{2}\right) + \frac{K_{II}}{\sqrt{2\pi r}} \cos\left(\frac{\varphi}{2}\right)\left(1 - \sin\left(\frac{\varphi}{2}\right)\sin\left(\frac{3\varphi}{2}\right)\right).$$

$\boxed{-\pi \le \varphi \le \pi}$. $\because$ K_I & K_{II} represent the two-in-plane crack opening modes I and II.

* 21. $$U_{AB} = C_0 \sum_{n=1}^{\infty} r^{\frac{n}{2}-1}\left(a_n \left(M_{11}^{(n)} f_I + M_{22}^{(n)} f_{II} + M_{12}^{(n)} f_{III}\right) + b_n \left(N_{11}^{(n)} f_I + N_{22}^{(n)} f_{II} + N_{12}^{(n)} f_{III}\right)\right),$$

22. $$U_{AC} = C_0^{*} \left\{ \sum_{n=1}^{\infty} n r^{\frac{n}{2}-1}\left[a_n \cos\alpha_n - b_n \sin\alpha_n\right]\right\}.$$

$$\because\ C_0^{*} = -\frac{2hg\, C^{*}(1-\nu^{s})}{E^{s}}, \quad \alpha_n = \left(\frac{n}{2}-1\right)\varphi.$$

21 & 22. are electrical potential difference at (r, φ).

The following equations relate to the two coordinate systems:-

23. $x_i = x_i' \cos\beta - y_i' \sin\beta + x_0,$

24. $y_i = x_i' \sin\beta + y_i' \cos\beta + y_0$

25. $r_i = \sqrt{x_i^2 + y_i^2}$

26. $\varphi_i = \arccos \frac{x_i}{r_i}$ $\boxed{0 \le \varphi_i \le \pi}$

$\varphi_i = -\arccos \frac{x_i}{r_i}$ $\boxed{-\pi \le \varphi_i < 0}$

Near Crack tip Stresses and Strains:-

From [B].

$$\varepsilon_{11}^S = \frac{1}{E^B}\left(\sigma_{11}^S - \nu^S \sigma_{22}^S\right), \quad \varepsilon_{22}^S = \frac{1}{E^B}\left(\sigma_{22}^S - \nu^S \sigma_{11}^S\right), \quad \varepsilon_{12}^S = \frac{(1+\nu^S)}{E^S}\sigma_{12}^S.$$

S → stands for specimen.

$$\sigma_{11}^S = \sum_{n=1}^{\infty} r_i^{\frac{n}{2}-1}\left\{a_n M_{11}^{(n)} + b_n N_{11}^{(n)}\right\}, \quad \sigma_{22}^S = \sum_{n=1}^{\infty} r_i^{\frac{n}{2}-1}\left\{a_n M_{22}^{(n)} + b_n N_{22}^{(n)}\right\},$$

$$\sigma_{12}^S = \sum_{n=1}^{\infty} r_i^{\frac{n}{2}-1}\left\{a_n M_{12}^{(n)} + b_n N_{12}^{(n)}\right\}$$

$$* \quad M_{11}^{(n)} = \frac{n}{2}\left\{\left[2 + (-1)^n + \frac{n}{2}\right]\cos\left[\left(\frac{n}{2}-1\right)\varphi_i\right] - \left(\frac{n}{2}-1\right)\cos\left[\left(\frac{n}{2}-3\right)\varphi_i\right]\right\},$$

$$N_{11}^{(n)} = \frac{n}{2}\left\{\left[-2 + (-1)^n - \frac{n}{2}\right]\sin\left[\left(\frac{n}{2}-1\right)\varphi_i\right] + \left(\frac{n}{2}-1\right)\sin\left[\left(\frac{n}{2}-3\right)\varphi_i\right]\right\},$$

$$M_{22}^{(n)} = \frac{n}{2}\left\{\left[2 - (-1)^n - \frac{n}{2}\right]\cos\left[\left(\frac{n}{2}-1\right)\varphi_i\right] + \left(\frac{n}{2}-1\right)\cos\left[\left(\frac{n}{2}-3\right)\varphi_i\right]\right\},$$

$$N_{22}^{(n)} = \frac{n}{2}\left\{\left[-2 - (-1)^n + \frac{n}{2}\right]\sin\left[\left(\frac{n}{2}-1\right)\varphi_i\right] - \left(\frac{n}{2}-1\right)\sin\left[\left(\frac{n}{2}-3\right)\varphi_i\right]\right\},$$

$$M_{12}^{(n)} = \frac{n}{2}\left\{\left(\frac{n}{2}-1\right)\sin\left[\left(\frac{n}{2}-3\right)\varphi_i\right] - \left[\frac{n}{2} + (-1)^n\right]\sin\left[\left(\frac{n}{2}-1\right)\varphi_i\right]\right\},$$

$$N_{12}^{(n)} = \frac{n}{2}\left\{\left(\frac{n}{2}-1\right)\cos\left[\left(\frac{n}{2}-3\right)\varphi_i\right] - \left[\frac{n}{2} - (-1)^n\right]\cos\left[\left(\frac{n}{2}-1\right)\varphi_i\right]\right\}.$$

Conversion of strains from crack tip coordinate system to the local film coordinate system:-

$$\varepsilon_{11}^F = \varepsilon_{11}^S \cos^2\beta + \varepsilon_{22}^S \sin^2\beta + \varepsilon_{12}^S \sin(2\beta).$$

$$\varepsilon_{22}^F = \varepsilon_{11}^S \sin^2\beta + \varepsilon_{22}^S \cos^2\beta - \varepsilon_{12}^S \sin(2\beta).$$

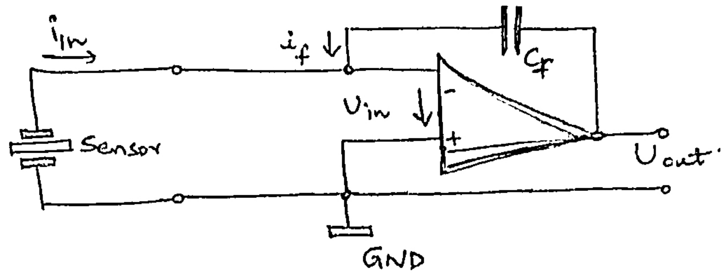
$$\varepsilon_{12}^F = -(\varepsilon_{11}^S - \varepsilon_{22}^S)\sin\beta\cos\beta + \varepsilon_{12}^S(\cos^2\beta - \sin^2\beta).$$

[B].

Inverse Problem

$i_{in} \rightarrow$ current

Output Voltage, $U_{out} = \frac{1}{C_f}\int_0^t i_{in}\,dt = \frac{Q_{in}}{C_f}$ or $U_{out} = -\frac{1}{C_f}\int_0^t i_f\,dt = -\frac{Q_f}{C_f}$.



IP.

$$\sigma_{33}^F = C_{31}\varepsilon_{11}^F + C_{32}\varepsilon_{22}^F + C_{33}\varepsilon_{33}^F = 0$$

$$D_3 = e_{31}\varepsilon_{11}^F + e_{32}\varepsilon_{22}^F + e_{33}\varepsilon_{33}^F.$$

$Q_{in} = D_3 A$. $\because$ A (small electrode area).

$$\because U_{out} = -\frac{A}{C_f}\left(\varepsilon_{11}^F\left(e_{31} - e_{33}\frac{C_{31}}{C_{33}}\right) + \varepsilon_{22}^F\left(e_{32} - e_{33}\frac{C_{32}}{C_{33}}\right)\right)$$

$\because$ Relationship between measured output voltages and the load at the crack tip.

$$U_{out} = C_0\left\{\sum_{n=1}^{\infty} r_i^{\frac{n}{2}-1}\left[a_n\left(M_{11}^{(n)} f_I + M_{22}^{(n)} f_{II} + M_{12}^{(n)} f_{III}\right) + b_n\left(N_{11}^{(n)} f_I + N_{22}^{(n)} f_{II} + N_{12}^{(n)} f_{III}\right)\right]\right\}$$

$$f_I = \cos^2\beta\,(C_{11} - \nu^S C_{22}) + \sin^2\beta\,(C_{22} - \nu^S C_{11}),$$

$$f_{II} = \sin^2\beta\,(C_{11} - \nu^S C_{22}) + \cos^2\beta\,(C_{22} - \nu^S C_{11}),$$

$$f_{III} = (1+\nu^S)(C_{11} - C_{22})\sin(2\beta),.$$

$$C_0 = -\frac{A}{E^S C_f}, \quad C_{11} = e_{31} - \frac{C_{31}}{C_{33}} e_{33}, \quad C_{22} = e_{32} - \frac{C_{32}}{C_{33}} e_{33}.$$