Experimental Methods in Particle Physics

Homework 2

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1.

If the direction of the Cherenkov photons emitted by a proton with a momentum of 1 GeV/c in a given medium forms an angle of 45 degrees with the direction of the proton, does a pion with the same momentum emit Cherenkov radiation in the same medium, and if so, at what angle?

ANSWER

We know that spatial momentum p can be written as $p = \gamma m c \beta$, where $\gamma = (1 - \beta^2)^{-1/2}$, m is the mass of the given particle, c is the speed of light and $\beta = v/c$ with v as the velocity of the particle. If the momentum of the pion equals that of the proton, hence $p_p \stackrel{!}{=} p_\pi = p = 1$ GeV/c, one can write

$$\beta_{p,\pi} = \frac{p}{\sqrt{p^2 + m_{p,\pi}^2 c^2}} \ . \tag{1}$$

With masses $m_p \cong 0.938$ GeV/c^2 and $m_\pi \cong 0.139$ GeV/c^2 one gets $\beta_p \approx 0.73$ and $\beta_\pi \cong 0.99$. The Cherenkov photon emission angle for the proton is $\vartheta_p = 45°$. For Cherenkov radiation the following is expression is known

$$\cos\vartheta_p = \frac{1}{\sqrt{2}} = \frac{1}{n\beta_p} \ , \tag{2}$$

from which the n refractive index of the medium is $n \cong 1.94$. Knowing n and β_π one can calculate the photon emission angle for the pion:

$$\vartheta_\pi = \arccos\left(\frac{1}{n\beta_\pi}\right) \cong 59^\circ \ . \tag{3}$$

2.

We have a small detector filled with noble gas. Its size is 10×10×10 cm, and the gas pressure is 3% above the atmospheric pressure, and it is at room temperature. Charged particles cross this chamber, ionize the gas, and we measure the amount of ionization for each particle. We would like to determine whether these particles are pions or protons (we know that they can only be either of these). We also measure the momentum p of these particles, which is the same for all particles.

- In approximately which momentum range (for which values of p) can we identify these particles one by one?
- In approximately which momentum range (for which values of p) can we identify these particles statistically (i.e. determine which fraction of them are pions)?

Please do not only give a number, but also some explanation/insight.

ANSWER

Let us assume that the noble gas in the detector is Ar, which is a quite common absorber medium is gaseous ionization detectors. We know that noble gases can be approximated with ideal gas model, hence using the equation of state one can calculate the ϱ mass density of the detector medium:

$$pV = Nk_BT \quad \Longrightarrow \quad \frac{N}{V} = \frac{p}{k_BT} \quad \Longrightarrow \quad \varrho = m_{\mathrm{Ar}}\frac{N}{V} \cong 1.7 \cdot 10^{-3}\ \frac{\mathrm{g}}{\mathrm{cm}^3}\ , \tag{4}$$

where p, V, N, T denotes pressure, spatial volume, particle number and temperature respectively, k_B is the Boltzmann constant and m_{Ar} is the mass of the argon atom.

In order to describe the energy release due to ionisation of different particles in a given detector medium one can use the Bethe–Bloch formula:

$$\left\langle -\frac{dE}{dx}\right\rangle = Kz^2\frac{Z}{A}\frac{1}{\beta^2}\left[\frac{1}{2}\log\left(\frac{2m_ec^2\beta^2\gamma^2W_{\max}}{I^2}\right) - \beta^2\right]\varrho\ , \tag{5}$$

where $K \cong 0.31$ MeV cm^2/mol is a constant[1], z is the charge number of the incident particle ($z = 1$ for both protons and pions), $Z = 18$ is the charge number and $A = 39.95$ g/mol is the so-called atomic mass (or molar mass) of the Ar absorber medium, $\beta = v/c$ – in which v is the velocity of the given particle, while c is the speed of light. In the logarithm $m_e \cong 0.5$ MeV$/c^2$ is the electron mass, $\gamma = (1-\beta^2)^{-1/2}$, $W_{\max}$ is the maximum possible energy transfer to an electron in a single collision and $I \cong Z \cdot 10$ eV $= 180$ eV is the mean excitation energy.

One can make the $W_{\max} \cong 2m_e c^2 \beta^2 \gamma^2$ low energy approximation. Furthermore the mass of the proton is $m_p \cong 938$ MeV$/c^2$, the mass of the pion is $m_\pi \cong 139$ MeV$/c^2$ and using equation (1) one can calculate β for the protons and the pions with given p spatial momentum.

The evaluation of the Bethe-Bloch formula for the protons and the pions at different values of p is shown in Fig. 1.

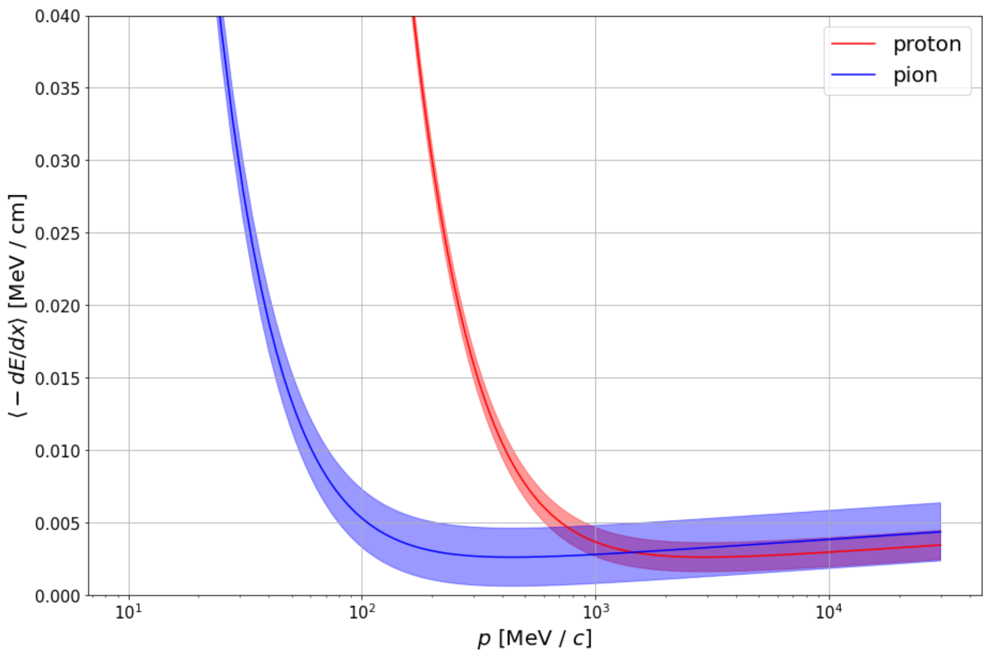

Figure 1: Evaluated Bethe-Bloch formula for protons and pions in Ar detector medium. The error bands are illustrations to represent possible measurement results.

[1] $K = 4\pi N_A r_e^2 m_e c^2$, where $N_A \cong 6 \cdot 10^{23}$ 1/mol is Avogadro's number and r_e is the classical electron radius.

By the analysis of the figure above it becomes clear that for $p > \overline{p} \cong 5 \cdot 10^2$ MeV$/c$ momentum – when the bands start to overlap – the precise identification of particles could become cumbersome, hence it could be performed by statistical means only. However, below $\overline{p}$ particles can be identified one by one.

3.

What is the mean free path of 2 MeV and 2 GeV photon in Lead and in Silicon? How thick material is needed to absorb at least 99.9% of the photons? Use information from the Particle Data Book (`http://pdg.lbl.gov/`). Spell out what assumptions you made, if any.

ANSWER

The density of the Pb and Si absorbers are about 11.34 and 2.33 g/cm^3 respectively. From `https://physics.nist.gov/PhysRefData/XRayMassCoef/tab3.html` and a review of *Passage of particles through matter* from PDG the following μ/ϱ mass attenuation coefficients and $\lambda\varrho$ mass mean free path values were found, from which the λ mean free path can be calculated.

	μ/ϱ [10^{-2} cm^2/g]	λ [cm]
Pb	4.606	1.91
Si	4.480	9.58

Table 1: Found mass attenuation coefficients and calculated mean free path values for 2 MeV photon energy.

	$\lambda\varrho$ [g/cm^2]	λ [cm]
Pb	8	0.71
Si	28	12.02

Table 2: Found mass mean free path and calculated mean free path values for 2 GeV photon energy.

From the Beer–Lambert law one can express the path traveled in the given absorber by the photon as

$$I(x) = I_0 e^{-x/\lambda} \qquad \Longrightarrow \qquad x = -\lambda \log\left(\frac{I(x)}{I_0}\right) . \tag{6}$$

If we want 99.9% of the photons to be absorbed (or in other words: to only 0.1% of the photons survive) we need to $I(x)/I_0 \overset{!}{=} 0.001$. This way one calculate the necessary thickness x of the absorbers.

$E_\gamma = 2$ MeV	x [cm]
Pb	13.19
Si	66.18

$E_\gamma = 2$ GeV	x [cm]
Pb	4.90
Si	83.03

Table 3: Calculated absorber thickness values for Pb and Si absorbers at different photon energies.