PHYS364 Report

Cosmological models and observations

An investigation into diverse cosmological frameworks in light of the Euclid and Nancy Grace Roman space telescope missions

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Abstract

In 2023, the European Space Agency (ESA), via SpaceX, launched the Euclid space telescope to study the nature of Dark Energy (DE) by observing Baryonic Acoustic Oscillations (BAOs) and Weak Gravitational Lensing (WL). NASA has scheduled to launch the Nancy Grace Roman space telescope in 2027 to continue this research by measuring the luminosity distance, d_L, to standard candles. We aim to use these observations to analyse cosmological models describing the accelerated Universe expansion. Our best model, Cosmological Constant Cold Dark Matter (ΛCDM), relies on standard assumptions about early-Universe physics. The globular cluster age of the Universe, and its constraints, were used to conclude that a DE density parameter, $\Omega_{\Lambda,0}$, is required to make the theoretical age, $t_0 = 12.8$ Gyr, based on Hubble Space Telescope (HST) Type Ia Supernovae (SNe Ia) measurements, consistent with the observed age, $t_0 = 13.2 \pm 0.4$ Gyr, where the 1-σ range is $0.697 < \Omega_{\Lambda,0} < 0.757$ and the 2-σ range is $0.661 < \Omega_{\Lambda,0} < 0.782$. At $t_\Lambda = 8.4$ Gyr and redshift $z_\Lambda = 0.447$ the Universe became dominated by Λ over matter. We used the constraints on the present matter density parameter, $\Omega_{m,0}$, from galaxy cluster dynamics and globular clusters, obtaining the respective 1-σ and 2-σ limits for the DE barotropic parameter, w_X, $-1.63 < w_X < -0.88$ and $-2.73 < w_X < -0.67$. These limits do not rule out the possible models to be analysed, even thawing quintessence and phantom cosmology. We investigated the Chevalier-Polarski-Linder (CPL) parameterisation model using a χ^2 test, determining the CPL parameters, $\bar{w}_p = -0.985$ and $\bar{w}_a = 0.23$, for simulated BAO angles. The respective 1-σ and 2-σ limits for w_p and w_a are $-0.99 < w_p < -0.98$, $-0.995 < w_p < -0.975$, $0.2 < w_a < 0.26$ and $0.17 < w_a < 0.29$. The ellipses in the (w_p, w_a) plane allowed us to distinguish the model from the Λ, suggesting a dynamical field to describe DE. We used the Figure of Merit (FoM) test to determine the model does not have a time-dependent Equation of state (EoS), albeit observationally distinguishable from ΛCDM. Furthermore, we analysed the fractional deviation in d_L for a test model of DE, Scaling Freezing Quintessence (SFQ), and a modified gravity model, Dvali-Gabadadze-Porrati (DGP) with $\alpha = 1$ against ΛCDM. High aggregate precision measurements from Roman found that the SFQ is indistinguishable from ΛCDM for $0.2 < z < 0.69$, whilst the DGP model was distinguishable over $0.2 < z < 1.7$; hence, SFQ favoured over DGP. Improving these models is the next step at the forefront of cosmology driven forward by ESA and NASA. We suggest that alterations constraining the parameters, where the transition scale factor, $a_t < 0.11$, and transition width, $\tau = 0.33$, for SFQ and $\alpha \leq 1$ for DGP should get tested through these missions.

Contents

1 Introduction

"Why is the Universe's expansion accelerating?" is one of the biggest questions in our understanding of the Universe today. The dominant component of our Universe predicted by Einstein is the cosmological constant. Yet, we know very little about the actual nature of this mysterious parameter. 76% of the estimated energy density for the cosmological constant is DE, an unknown form of energy driving the acceleration of the Universe's expansion. Another 20% of energy density takes the form of Dark Matter, an unknown form of matter that exerts a gravitational attraction, like conventional matter, but does not emit or absorb light. There are several possible candidates for Dark Matter, but a reason to explain one or both is to re-evaluate how we perceive gravity. Doing so could change our view on general relativity (GR).

To attempt to understand the origin of the Universe's accelerating expansion, space agencies utilise advanced telescopes to carry out observations and make the investigation of the dark Universe possible. One such telescope is the Euclid telescope, designed by the ESA. Launched on 1st July 2023, Euclid aims to make a 3D map of the Universe, with time as the third dimension, by observing galaxies out to 10 billion light years to reveal how the Universe has expanded and how structure has formed [1]. Another such telescope is the Nancy Grace Roman Telescope, designed by NASA. Set to be launched in May 2027, one of the primary goals of the Roman mission is to probe the chronology and growth of cosmic structure, with the end goal of measuring the effects of DE. Both of these telescopes use WL and BAOs when probing DE, and BAOs are a significant part of our independent research. Meeting the targets for these missions would contribute significantly to solving the puzzle of the dark Universe and help identify the cosmological constant.

Besides the standard ΛCDM model, other frameworks suggest explanations for the source of DE. The SFQ model incorporates a specific type of quintessence, a canonical scalar field that explains late-time cosmic acceleration. A hypothetical form of DE, quintessence differs from the ΛCDM model in that it is dynamic. The scaling label gets assigned to a type of quintessence where the EoS for the scalar field of DE becomes equivalent to that of the dominant form of matter while freezing, on the other hand, is where the field gradually slows down since the potential is shallow at late times. Another dynamic DE model is the CPL model that uses BAO angles observed by the Euclid space telescope to investigate the nature of DE. It uses a linear dependence for the EoS on the scale factor, a, containing two parameters for the DE EoS for w_X. The fit of the EoS to the low redshift SN Ia data and the high redshift CMB data is concurrently used by all recent cosmological observations, including Planck, to put constraints on DE.

Some cosmologists believe that rather than the cosmological constant of DE and Dark Matter, the accelerated expansion arises from an error in our understanding of GR and gravity. This approach implies a new model that can satisfy this apparent error. DGP, named after the three physicists who proposed the model Dvali, Gabadadze and Porrati, has stated that rather than DE being the cause for the accelerated Universe expansion, it is a breakdown of the standard Friedmann equation. This breakdown suggests a modified Friedmann equation is necessary at certain times of the Universe's expansion to satisfy the assumed error in the ΛCDM model. In this report, we compare these models and comment on the strengths of their respective arguments.

2 Background and theory

2.1 Directly observing the age and matter content of the universe

2.1.1. Determining the age of the universe using globular clusters

The age of the Universe can be estimated by studying globular clusters. They are dense spherical collections of stars born in the same gas cloud shortly after a galaxy is formed; therefore, they are all "chemically homogeneous" [2] and act as "cosmic clocks" [3]. The oldest stellar objects, $t_{\text{oldest stars}}$, provide a lower limit to the age of the Universe. The total age of the Universe, t_U is estimated as the sum of $t_{\text{oldest stars}}$ and the time between the Big Bang (BB) and the formation of the stars, Δ_t,

$$t_U = t_{\text{oldest stars}} + \Delta_t. \tag{1}$$

Estimating the $t_{\text{oldest stars}}$ uses the Main Sequence Turn-Off (MSTO) point on the Hertzsprung-Russell (H-R) diagram where the star's core exhausts Hydrogen burning and starts burning Helium. High-quality photometric data involving the luminosity and colour of each star is collected and transferred into a stellar isochrone. The MSTO point is determined by detecting the deviation from the main sequence. Comparing the observed MSTO against theoretical stellar evolution models infers the age of the stellar population. Many high-redshift galaxies have been found and confirmed at the formation redshift, $z_f > 8$. The Δ_t is estimated as the age of the oldest stars in the Universe, assuming formation at $z_f = 11$ and the maximum redshift at $z_{max} \geq 20$, which depends on cosmology very weakly (despite the "H_0 tension"). The conservative estimate for Δ_t is $\Delta_t \sim 0.5 - 0.1$ Gyr for the redshifts of interest. Identifying the oldest stars as the descendent of the lower-redshift galaxy observed (setting $z_f > 8$) would increase Δ_t by $\sim$0.1 Gyr. Combining the estimations of $t_{\text{oldest stars}}$ and Δ_t provides a method for determining the age of the Universe [3].

A key component in the Universe is dark matter. Independent of the physics regarding the Cosmic Microwave Background (CMB), dark matter content in the late time gets estimated via galaxy clustering. Galaxy clusters contain X-ray-emitting gas. Observations of the X-ray gas allow astrophysicists to calculate the total mass, including the dark matter of the galaxy cluster [4].

Recent technological developments have reduced the error for the magnitude of stars from Gaia parallax measurements. The improved accuracy of the metallicity measurements provides better future comparisons between stellar evolution models. Observing higher-order clustering of galaxies has improved the determinations of the dark matter content, including the Alcock-Paczynski test to voids [5].

Observations for the dark matter content and the late-time age estimate of the Universe will help the ongoing investigations concerning the Hubble tension.

2.1.2. The Hubble tension

The Hubble constant, H_0, is the expansion rate for the Universe, subjective to discrepancies between the early and late-time measurement methods. The H_0 from the cosmic distance ladder in local time disagrees with the value derived from the CMB following the Planck space mission. Recent measurements of H_0 reveal more than a 3-σ tension [6].

The late-time measurement of H_0 is based on the current expansion rate, using the redshifts of distant celestial objects. However, the observations of CMB radiation provide a cosmological model relying on standard assumptions of the early Universe [3].

Direct observational bounds on the age and content of the Universe play a crucial role in understanding the tension. The Friedmann Eqn. (46) relates the age of the Universe, t_0, to H_0. Comparing the local cosmic distance ladder with the CMB observations helps constrain the parameters in the cosmological models. These refinements better our understanding of the amount of DE and dark matter content in the Universe. Incorporating these headways into our cosmological models may resolve the discrepancy in the Hubble tension.

2.1.3. *Determining the age of the universe in the ΛCDM DE model*

The ΛCDM model is our simplest model of DE that considers a flat Universe made of pressureless matter, m, and a cosmological constant, Λ, where the age is specified by,

$$t_0 = \frac{2}{3}\frac{H_0^{-1}}{\sqrt{\Omega_{\Lambda,0}}}\ln\left[\frac{1+\sqrt{\Omega_{\Lambda,0}}}{\sqrt{1-\Omega_{\Lambda,0}}}\right], \tag{2}$$

where $\Omega_{m,0}$ is the present density parameter of pressureless matter (CDM and baryons) and $\Omega_{\Lambda,0}$ is the present density parameter of the cosmological constant, with $\Omega_{m,0} + \Omega_{\Lambda,0} = 1$ for a flat Universe. The general current density parameter is defined by,

$$\Omega_{i,0} = \frac{\rho_{i,0}}{\rho_{c,0}}, \tag{3}$$

for the matter type, i, and the present critical density $\rho_{c,0}$. The full derivation for Eqn. (2) is found in the appendix (A.1).

Finding the age of the Universe, t, at a given redshift tells us the age when an object with redshift, z, is observed. The full derivation can be found in the appendix (A.1), giving,

$$t(z) = \frac{2}{3}\frac{H_0^{-1}}{\sqrt{\Omega_{\Lambda,0}}}\ln\left[\sqrt{\frac{\Omega_{\Lambda,0}+(1+z)^3(1-\Omega_{\Lambda,0})}{(1+z)^3(1-\Omega_{\Lambda,0})}} + \sqrt{\frac{\Omega_{\Lambda,0}}{(1+z)^3(1-\Omega_{\Lambda,0})}}\right]. \tag{4}$$

2.1.4. *Determining the bounds on w_X from the observed age and the matter content*

The observed Age of the Universe, t_0, from globular clusters [3] is,

$$t_0 = 13.2 \pm 0.4\,\mathrm{Gyr}, \tag{5}$$

and the direct observational bound on $\Omega_{m,0}$ from galaxy cluster dynamics provides a 1-σ range,

$$\Omega_{m,0} = 0.30 \pm 0.02. \tag{6}$$

The value of H_0 is directly determined from observation of SN Ia, independent of the DE model, using the HST,

$$H_0 = 73.48\,\mathrm{km\,s^{-1}\,Mpc^{-1}}. \tag{7}$$

The error on H_0 from HST is small and will be neglected. It is larger than the value that indirectly fits the CMB predictions of an assumed DE model (ΛCDM) to the observed CMB temperature fluctuations [7]: the "H_0 problem".

Different types of DE, X, can still satisfy the bounds on Eqn. (5) and Eqn. (6); still from a Λ. The barotropic parameter, w_X is defined by,

$$w_X \equiv \frac{p_X}{\rho_X c^2}, \tag{8}$$

where p_X is the pressure of the DE and c is the speed of light in a vacuum. For a constant w_X, the density of DE varies with scale factor as follows,

$$\rho_X = \rho_{X,0}\left(\frac{a_0}{a}\right)^{3(1+w_X)}. \tag{9}$$

For the ΛCDM model we assume that $\rho_{X,0} = \rho_{\Lambda,0}$.

2.2 Preliminary calculations

The total distance to an object observed at the present time t_0 with redshift is calcuated by considering a photon being emitted by the object at an initial time t_i and integrating it with respect to time until it reaches the observer at t_0. The total physical distance to the object that emits a photon at time t_i is,

$$d(t) = c \int_{t_i}^{t_0} \frac{a(t_0)}{a(t')} dt'. \tag{10}$$

The prime on the t is to distinguish the integration variable from t. The physical distance to the object as a function of redshift, z, is given as

$$d(z) = a_0 \int_{a(z)}^{a_0} \frac{c}{Ha'^2} da'. \tag{11}$$

We will use this integral to study the Modified Gravity model, where H(a') gets substituted with the modified Friedmann equation.

The continuity equation of each independent matter type is,

$$\frac{d\rho_i}{dt} + 3H\left(\rho_i + \frac{p_i}{c^2}\right), \tag{12}$$

where ρ_i and p_i are the density and pressure of the respective matter i. Using Eqn. (12), it is shown how dark energy varies with scale factor a', when w_X is a function of a',

$$\frac{\ln \rho_x(a')}{\ln \rho_{x,0}} = \int_{a_0}^{a'} \frac{-3(1+\omega_x(a''))}{a''} da''. \tag{13}$$

This integral finds the distance as a function of redshift for a dark energy barotropic parameter w_X described by a CPL parameterisation. Furthermore, we will use the integral to compute d_L for a specific DE model. The derivations for the equations above can be found in the appendix (A.1).

2.3 Future space telescope determination of the nature of dark energy

2.3.1. Future Euclid observations of the BAO length

Euclid is a survey mission designed by the ESA to explore the evolution of the dark Universe expansion. It aims to make a 3D map of the Universe, where time is the third dimension, by observing galaxies out to 10 billion light-years. By observing how the Universe evolved over the past 10 billion years, Euclid will reveal how it has expanded and how structure has formed. From this, astronomers can infer the properties of DE, dark matter, and gravity. The spacecraft is approximately 4.7 m tall and 3.7 m in diameter, consisting of the service module and the payload. The service module contains the satellite systems required for function. The payload consists of a 1.2 m aperture telescope with two other observational instruments [1].

The primary aim of Euclid is to understand the origin of the accelerating Universe expansion. It will utilise probes to investigate the nature of DE, Dark Matter, and gravity by tracking their observational signatures on the geometry and the cosmic history of structure formation in the Universe. Euclid will map large-scale structures over a cosmic time covering the last 10 billion years, more than 75% of the age of the Universe. The mission uses two independent cosmological probes - WL and BAO. WL is a technique to map dark matter and measure DE by quantifying the apparent distortions of galaxies, a change in the observed ellipticity caused by mass inhomogeneities along the line of sight. BAOs are wiggle patterns imprinted in the clustering of galaxies that provide a standard ruler to measure the Universe's expansion. The properties of the wiggles are derived from accurate distance measurements of galaxies [1].

WL requires a high image quality on sub-arcsecond scales for the galaxy shape measurements. BAO requires near-infrared (NIR) spectroscopic capabilities to measure accurate redshifts of galaxies out to $z \geq 0.7$. Both probes require high degrees of system stability and the ability to survey vast fractions of the extra-galactic sky. These requirements cannot be met from the surface of Earth, demanding a wide-field-of-view space mission like Euclid.

One of the main scientific objectives of Euclid is to reach a DE FoM > 400, where FoM $= 1/(\Delta \mathrm{w_p} \times \Delta \mathrm{w_a})$, which roughly corresponds to a sensitivity of 1-σ errors on w_p and w_a of 0.02 and 0.1 respectively, meaning to 2% and 10%. These values are then to be used in the CPL parameterisation of $w_X(a)$ with an equation of state of the form

$$w_X(a) \equiv \frac{p_X}{\rho_X c^2} \equiv w_p + \left(\frac{a_p}{a_0} - \frac{a}{a_0}\right) w_a, \tag{14}$$

where w_p is the pivot barotropic parameter, a_p is the pivot scale factor, and w_a is the scale factor dependent barotropic parameter.

The Euclid payload consists of a 1.2 m aperture telescope with two instruments: the visual imager (VIS) and the NIR spectrometer and photometer (NISP). Both instruments share a large, common field of view. The VIS provides high-quality images to perform the galaxy measurements for WL, and the NISP performs imaging photometry to provide NIR photometric measurements for photometric redshifts and carries out slitless spectroscopy to obtain spectroscopic redshifts [1].

The VIS is equipped with 36 Charged-Coupled Devices and measures the shapes of galaxies with a resolution better than 0.2 arcseconds with 0.1 arcsecond pixels in one wide visible band. The NISP contains three NIR bands, employing 16 NIR detectors with 0.3-arcsecond pixels. The spectroscopic channel of the NISP operates in the wavelength range $1.12 < \lambda\,[\mu\mathrm{m}] < 2.0$ at a mean spectral resolution $\frac{\lambda}{\Delta\lambda} \approx 250$, employing 0.3-arcsecond pixels. The VIS and NISP operate in parallel, while the NISP performs the spectroscopy and photometry measurements in sequence [1].

BAOs are paramount for modern cosmology, including the Euclid mission, acting as standard rulers when studying the Universe. They can be considered frozen objects, imprinted on matter, left behind from the pre-decoupling Universe. When the Universe was in its infancy, matter spread in an almost uniform sea of particles, and gravity was trying to change this by pulling large amounts of matter together to form galaxies. The matter heated up as gravity pulled it together, creating an outward pressure that pushed it apart again. As it expanded, it cooled again, and the process repeated. This interaction resulted in an oscillation that created the equivalent of sound waves that spread outward in "wiggles". These wiggles are what we observe today [8]. The oscillations are wiggle patterns imprinted in the clustering of galaxies that provide a standard ruler to measure the expansion of the Universe. The properties of the wiggles are derived from accurate distance measurements of galaxies and require NIR spectroscopic capabilities to measure accurate redshifts of galaxies out to $z \geq 0.7$ [1].

We judge the distance of an object of known length by its angular size. The further away it is, the smaller it appears. The same idea applies in cosmology, with one major complication: space can be curved. This idea is similar to trying to judge the distance of our known object through a smooth lens of unknown curvature. Now, when it appears small, we are no longer sure it is because it is far away. It may be near and appear small because the lens distorts the image. To be useful for cosmology, we need a standard ruler: an object of known size at a single redshift, z, or a population of objects at different redshifts whose size changes in a well-known way (or is constant) with redshift. Ideally, the standard ruler falls into both classes, which can be argued to be the case for the BAO, to a good approximation [9]. The length of this standard ruler is the maximum distance the acoustic waves could travel in the primordial plasma before cooling to the point where it became neutral atoms, which stopped the expansion of the plasma density waves. Today, this is estimated to be 490 million light-years and can be measured by observing the large-scale structure of matter using astronomical surveys [10], similar to Euclid.

A key idea when studying the uses of BAO is the concept of Statistical Standard Rulers (SSR). SSR states that all galaxies get positioned at the intersections of a regular three-dimensional grid of known spacing. Measuring the angular diameter distances as a function of redshift would be simple in this case, and we would be able to find the expansion rate as a function of redshift, measured at a discrete set of redshifts corresponding to the mid-points between galaxies. If we started to randomly insert galaxies into this regular grid, as the number of randomly distributed galaxies increased, the regular grid pattern would rapidly become hard to see by the eye. However, a regular grid distributed throughout space would provide an absolute reference frame that breaks the continuous homogeneity of space down to a discrete subgroup. If we inserted a galaxy at random and, with some fixed probability, put another galaxy at a distance from it, then using the new galaxy as a starting point, we could repeat this process until we had the desired number of galaxies. There is no regular grid of galaxies now, but the length is still preferred in the distribution of the galaxies and forms an SSR [9]. BAOs provide this when studying the Universe.

2.3.2. The observed angular size of the BAO length as a function of redshift and the theoretical d(z) and $\theta_{BAO}(z)$ for the CPL parameterisation

The observed angular size, θ_{BAO}, of the BAO length gets determined by the size of the BAO length at present, l_{BAO}, and the $d(z)$ at present to the objects forming the observed BAO feature.

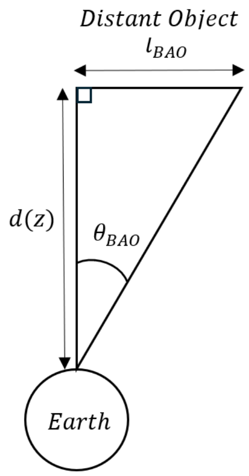

Figure 1: Schematic of the BAO length, l_{BAO}, of an object at a distance, $d(z)$, away from an observer on Earth, procuring an angle, θ_{BAO} at present.

Since $d(z) >> l_{BAO}$, the small-angle approximation for $\sin(\theta_{BAO}) \approx \tan(\theta_{BAO}) \approx \theta_{BAO}$ is applicable,

$$\tan(\theta_{BAO}(z)) \approx \theta_{BAO}(z) = \frac{l_{BAO}}{d(z)}. \tag{15}$$

The physical distance $d(z)$ above is a function of redshift for a dark energy barotropic parameter w_X described by the CPL parameterisation from Eqn (14). To find $d(z)$, Eqn. (13) is solved,

$$\rho_X(a') = \rho_{X,0}\left(\frac{a_0}{a'}\right)^{3(1+w_p+w_a)} \exp\left(3w_a\Big(\frac{a'}{a_0}-1\Big)\right), \tag{16}$$

substituted into Eqn.(11) where,

$$H(a') = \sqrt{\Big(\frac{8\pi G}{3}\Big)(\rho_m+\rho_X)}, \tag{17}$$

and simplified to:

$$d(z) = \frac{c}{a_0^{\frac{1}{2}}}\frac{1}{H_0}\int_{a(z)}^{a_0} \frac{da'}{a'^{\frac{1}{2}}\left(\Omega_{m,0}+\Omega_{X,0}\big(\frac{a_0}{a'}\big)^{3(w_p+w_a)}\exp\left[3w_a\big(\frac{a'}{a_0}-1\big)\right]\right)^{\frac{1}{2}}}. \tag{18}$$

The derivations for the equations above can be found in the appendix (B.1).

2.3.3. The Nancy Grace Roman space telescope mission

NASA is repurposing the Roman telescope for use in the Wide-Field Infrared Survey Telescope (WFIRST) mission scheduled for 2027, with a primary mirror diameter of 1.5 m increased to 2.4 m for finer angular resolution. It features an on-axis telescope, slitless spectroscopy with a grism, Integral Field Unit (IFU) spectrograph for supernova slit spectroscopy, usage of HgCdTe IR detectors with 4k x 4k, 10 µm pixel (H4RG), 18 µm pixel H2RGs, 28.5° inclined geosynchronous orbit and a higher operating temperature than DRM1 [11].

WFIRST is devoted to finding explanations for the crisis in cosmology, concerned with the causation of cosmic acceleration by a new energy component or by the breakdown of GR on cosmological scales. If the cause is a new energy component, is its energy density constant in space and time, or has it evolved over the history of the universe? The mission is interested in measurements regarding the history of cosmic acceleration using three distinct but interlocking methods: WL, BAO, SN and standard candles.

The SN survey uses the wide-field imager for multi-band imaging to discover and monitor more than 2700 (SNe Ia) and measure light curve shapes out to $z = 1.7$ over six months observing time, spread over a 2-year interval. IFU spectroscopy will confirm the Type Ia classification. It will also measure redshifts and spectral diagnostics, performing synthetic photometry to measure luminosity distance. There are three tiers to the SN survey: a shallow survey over 27.44 deg^2 for SNe at $z < 0.4$ using the Y and J bands, a medium and deep survey over 8.96 deg^2 and 5.04 deg^2 for respective SNe at $z < 0.8$ and out to $z = 1.7$ using J and H for the medium and deep tiers.

At every redshift, NIR wavelengths and a stable, space-based imaging platform with sharp PSF reduce the uncertainties associated with dust extinction and photometric calibration. WFIRST-2.4 has a larger collecting area and a sharper PSF relative to DRM1, enabling a survey with more SNe, a more uniform redshift distribution, and reduced systematics afforded by IFU spectroscopy.

IFU observes one object at a time and is more efficient than slitless spectroscopy over the 0.28 deg^2 since the exposure time gets chosen for each SN individually instead of the faintest object in the field driving the observation. Furthermore, IFU observations have substantially lower sky noise per pixel, allowing better isolation of faint SN signals. For each SN, seven IFU spectra on the light curve get used from -10 to +25 days rest-frame days before and after the peak, with a deep spectrum near its peak capable of measuring a highly accurate photometric data point. Identifying SN subtypes and even measuring spectral diagnostics from this can reduce statistical or systematic errors in the survey. After the SN has faded, one reference spectrum gets taken for galaxy subtraction.

Spectral diagnostics helps distinguish intrinsic colour variations from the effects of dust extinction and match high and low-z SNe with similar properties to suppress evolutionary effects in the mix of SNe. The process takes 30 hours to identify and confirm an SN Ia event [11].

2.3.4. Luminosity distance in an expanding universe

The luminosity distance, $d_L(z)$, is the distance at which an object with known absolute luminosity, L, would be in Euclidean space such that the total flux, $\mathcal{F}$, relates according to,

$$d_L = \sqrt{\frac{L}{4\pi\mathcal{F}}}. \tag{19}$$

However, the expansion of the Universe is non-Euclidean, therefore redshift is accounted for leading to the conversion between $d_L(z)$ and physical distance, $d(z)$,

$$d_L(z) = (1+z)d(z). \tag{20}$$

To find $d(z)$, first Eqn.(13) is solved,

$$\rho_x(a') = \rho_{x,0}a^{-3(1+\omega_x)}, \tag{21}$$

followed by a substitution into Eqn.(11), where $w_X = -1$ and,

$$H(a') = \sqrt{\frac{8\pi G}{3}\rho_X(a')}. \tag{22}$$

This is simplified to get,

$$d(z) = \frac{cz}{H_0}. \tag{23}$$

Hence, the luminosity distance becomes,

$$d_L(z) = (1+z)\frac{cz}{H_0}. \tag{24}$$

The derivations for the equations above can be found in the appendix (B.2).

2.3.5. The scaling freezing quintessence model

Quintessence is a canonical scalar field, where $E = 1$, that explains late-time cosmic acceleration from a scalar field DE model. Quintessence has an equation of state EoS where the barotropic parameter, $w_X \geq -1$. The evolution of w_X broadly classifies quintessence into either thawing or freezing models. We focus on the freezing model in this report, where the field gradually slows down because the potential tends to be shallow at late times. A scaling quintessence model is where the EoS for the scalar field of DE becomes equivalent to the EoS for the dominant form of matter [12], scaling as the background fluid [13]. As a result, the system does not enter the phase of cosmic acceleration, corresponding to the exponential potential. Considering the double exponential potential solves this problem,

$$V(\phi) = V_1 \exp\left(-\frac{\lambda_1\phi}{M_{pl}}\right) + V_2 \exp\left(-\frac{\lambda_2\phi}{M_{pl}}\right), \tag{25}$$

where λ_i and $V_i(i = 1, 2)$ are constants and M_{pl} is the Planck mass. The SFQ model behaves according to the scaling quintessence model at early times. If DE behaved as it did at present, it must separate from the scaling solution, transitioning to an EoS similar to the cosmological constant $w_X \approx -1$. When observed, the scaling solution is a case of a tracker along which the $\Omega_\phi = 3(1 + w)/\lambda_2$ is constant. For the matter-dominated era: $w_X = w_m = 0$; therefore, $\Omega_\phi = 3/\lambda_2$. In this context, a tracker is a solution where w_X is almost constant during the matter era, decreasing henceforth. This case belongs to a subclass of freezing models called tracker models [12].

A period of cosmic acceleration driven by another exponential potential follows the scaling matter era,

$$V_2 \exp\left(-\frac{\lambda_2\phi}{M_{pl}}\right). \tag{26}$$

The onset of this transition depends on the parameters λ_1, λ_2, and V_2/V_1. The transition redshift is not affected to a large degree by z; therefore, we can set $V2 = V1$ without losing generality. Varying w_X gets accounted for by using the parameterisation,

$$w_X(a) = w_f + \frac{w_{\text{past}} - w_f}{1 + \left(\frac{a}{a_t}\right)^{\frac{1}{\tau}}}, \tag{27}$$

where w_f is the future value (as $a \to \infty$) and w_{past} is the past value of w_X (as $a \to 0$). a_t is the scale factor at the transition where the scaling behaviour ends and τ is the transition width/thickness.

Eqn. (27) represents the time-dependent EoS for the SFQ model. During the period of matter-domination, the scaling solution corresponds to $w_{\text{past}} = 0$ and for $\lambda_2 = 0$ to $w_f = -1$. If multiple fields are present, the scaling matter era is followed by the epoch of cosmic acceleration even when the individual field can not lead to accelerated expansion [12]. Currently, there are no analytical solutions for scaling quintessence models for w_X derived from the scalar field dynamics [13].

2.4 Modified gravity models providing an alternate approach

The DGP model is a gravity model named after the three physicists who proposed it: Dvali, Gabadadze and Porrati. The model proposed a breakdown of the standard Friedmann equation that governs the expansion rate [14], instead of DE being the cause for the acceleration; hence, the Universe only consists of conventional matter and radiation.

The model assumes the existence of a 4+1 dimensional Minkowski space within which ordinary 3+1 Minkowski space is embedded [15]. The Friedmann equation becomes modified when the mean density of the Universe is small enough,

$$H^2 - \frac{H^\alpha}{(r_c/c)^{2-\alpha}} = \frac{8\pi G}{3}\rho_m, \tag{28}$$

where r_c is the cross-over scale (a distance) where GR breaks down and requires an extra dimension to describe the accelerated expansion. It determines the distance above which gravity begins leaking out of the brane. Smaller values of r_c lead to stronger gravity, pushing the prediction away from the data, while larger values bring the DGP model closer to GR [16]. The scale

r_c also sets the distance at which corrections to the usual metric for a gravitating source become important. For example, there are corrections to the Schwarzchild metric, which affect planetary motions. Lunar laser ranging experiments that monitor the moon's perihelion precession with great accuracy and imply a lower bound for r_c that is close to the present cosmological horizon H_0^{-1} [14].

In the original DGP model, it showed $\alpha = 1$, which means that $w = -0.7$ (which is found using $w_{eff} \approx -1 + 0.3\alpha$) [14]. Theory suggests that α is a continuous parameter. However, there is limited supporting physics for this and having $\alpha > 1$ introduces problems into the model.

Problems like the hierarchy and the cosmological constant problems motivate theories with large extra dimensions. These theories modify laws of gravity only at short distances, below the size of the extra dimensions introduced, so all long-distance physics is very close to the standard view. However, theories with infinite-volume extra-dimensions modify the laws of gravity in the far infrared. At early times, gravitational dynamics are very close to the standard ΛCDM model, but at later times get modified. These modifications account for the observed accelerated expansion of the Universe without DE, meaning the modified gravitational dynamics lead to a "self-accelerated" Universe [14].

It is shown that for an arbitrary 4-d brane-localized (y=0) matter source, $\rho M(t)$, $H^2 \pm \frac{H}{r_c} = \frac{8\pi G \rho M}{3}$. The general features of this remain the same for an arbitrary number of dimensions. This idea means the higher-dimensional action is suppressed relative to the 4-d one by an inverse power of the cross-over scale; so, the scaling arguments suggest that the higher-dimensional contribution should scale as a lower power of H^2. This scaling dependence indicates that it is important at late times when H is small, suggesting infinite-volume extra-dimensional theories have the potential to explain cosmic acceleration with DE. It appears that the self-accelerated solutions of the higher-dimensional theory have the same number of new parameters as the simplest model of DE, the cosmological constant, which is replaced by r_c, the cross-over scale [14].

Infinite extra dimensions get used to motivate the modification of the Friedmann equation at late times, assuming that there is a single cross-over scale r_c and that to leading order, the corrections to the Friedmann equation parameterised to a single term, H^α [14].

The present matter density is parameterised by $\Omega_{m,0}$, where,

$$\Omega_{m,0} = \frac{\rho_{m,0}}{\rho_{c,0}}, \tag{29}$$

and the critical density,

$$\rho_{c,0} = \frac{3H_0^2}{8\pi G}. \tag{30}$$

H_0 is the present expansion rate. Using the above equations as well as the Eqn. (28), it is shown that for a general value of α, the cross-over scale would be:

$$\frac{r_c}{c} = [(1 - \Omega_{m,0})^{-1} H_0^{\alpha-2}]^{\frac{1}{2-\alpha}}, \tag{31}$$

which can be simplified for $\alpha = 1$:

$$\frac{r_c}{c} = (1 - \Omega_{m,0})^{-1} H_0^{-1}. \tag{32}$$

The values for the parameters in Eqn. (32) are $c = 2.998 \times 10^8\,\mathrm{m\,s^{-1}}$, $\Omega_{m,0} = 0.308$, and $H_0 = 67.81\,\mathrm{km\,s^{-1}Mpc^{-1}}$. The Eqn. (28), with $\alpha = 1$, is then solved to express $H(a)$ as a function of

H_0, $\Omega_{m,0}$, a_0, and a.

$$H(a) = \frac{H_0(1-\Omega_{m,0}) + H_0\sqrt{(1-\Omega_{m,0})^2 + 4\Omega_{m,0}(a_0/a)^3)}}{2}. \tag{33}$$

The deceleration parameter, q, is defined by,

$$q(z) = \frac{-1}{H^2}\frac{\ddot{a}}{a}, \tag{34}$$

dictating the Universe's acceleration. If $q > 0$, then the expansion decelerates, $\ddot{a} < 0$. A general expression for q, for a given H(a), can be found to be,

$$q(z) = -\left(1 + \frac{a}{H}\frac{dH}{da}\right). \tag{35}$$

Using Eqn. (33) as well as Eqn. (35) an expression for q as a function of a for the $\alpha = 1$ DGP model can be derived to be,

$$q(a) = \frac{6\Omega_{m,0}}{x(a)[1-\Omega_{m,o} + x(a)]}\left(\frac{a_0}{a}\right)^3 - 1, \tag{36}$$

where $x(a)$ is defined as,

$$x(a) = \sqrt{(1-\Omega_{m,0})^2 + 4\Omega_{m,0}\left(\frac{a_0}{a}\right)^3}. \tag{37}$$

In terms of redshift Eqn. (36) and Eqn. (37) provide the respective expressions for $q(z)$ and $x(z)$,

$$q(z) = \frac{6\Omega_{m,0}(1+z)^3}{x(z)\,[(1-\Omega_{m,0}) + x(z)]} - 1; \tag{38}$$

$$x(z) = \sqrt{(1-\Omega_{m,0})^2 + 4\Omega_{m,0}(1+z)^3}. \tag{39}$$

The full derivations for the above Eqn. (31), Eqn. (33), Eqn. (35), Eqn. (36), Eqn. (38), and Eqn. (39) are found in the appendix (C.1).

3 Method

3.1 Directly observing the age and matter content of the universe

3.1.1. The age of the Universe via the cosmological constant.

ΛCDM is a crucial model of the Universe and is our best baseline when deciding if other models are accurate. So, plotting the theoretical age t_0 as a function of $\Omega_{\Lambda,0}$ is a good start to constrain $\Omega_{\Lambda,0}$ based on observations.

Finding these constraints was accomplished in code where the first step is to convert the Hubble constant H_0 from $\mathrm{km\,s^{-1}\,Mpc^{-1}}$ to $\mathrm{Gpc^{-1}}$, done by using the following equation,

$$H_0 = 73.48 \times \frac{1000 \times 86400 \times 365.25 \times 10^9}{10^6 \times 3.086 \times 10^{16}}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}. \tag{40}$$

With this conversion completed, Eqn. (2) is used to calculate t_0 and plot it. At this point, the 1-σ and 2-σ ranges get determined by adding a snippet of code, focusing on the specific points of interest until the correct values to three significant.

For comparison with astrophysics, it is also useful to show the age of the Universe, t, as a function of redshift, z, for the ΛCDM model. We used code to do this, starting with Eqn. (40), to convert H_0 into the correct units where it gets used in Eqn. (4) to calculate t and plotted. In conjunction with the t value code, we looped between two reasonable z values. We printed out a list of numbers for redshift and corresponding age values until we reached the desired precision. The code we used to find these specific values can be found in the appendix (A.2).

3.1.2. Bounds on w_X from the observed age and matter content.

Next, we plotted the values of t_0 as a function of $\Omega_{m,0}$ for given values of w_X as shown in Figure 2 below. We used code to plot a graph starting with Eqn. (40) as usual. Then, by creating a function that employs a numerical trapezium rule integration of 10,000 steps for Eqn. (130), we can calculate values of t_0 corresponding to a set Ω_m and w_X value. By plotting this for the preset values of w_X, all four generated curves can be graphed on the same plot to visually show the effect of w_X on respective models. The 1-σ and 2-σ ranges get plotted as bars on the Ω_m and t_0 axis. To find points of intersection with the vertical bars, we take the corresponding Ω_m values into the function for each w_X value, outputting t_0 values. We increment $\Omega_{m,0}$ for each vertical bar into the function until the t_0 value matches accordingly, shown by Figure 2 below.

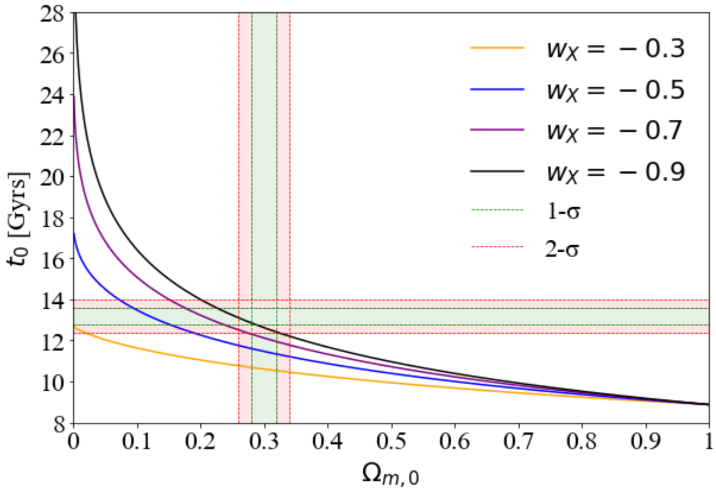

Figure 2: The curves show t_0 over a range of $\Omega_{m,0}$. The barotropic parameter has values w_X = -0.3, -0.5, -0.7, and –0.9, labelled by the respective orange, blue, purple, and black curves. The 1-σ and 2-σ ranges on the observed t_0 and matter content suggest that the w_X requires adjustment.

The problem with using given values means both σ ranges misalign with the curves for each w_X. By changing the values of w_X, a new range of values can be determined that are compatible with the 1-σ and 2-σ ranges.

3.2 Future space telescope determination of the nature of dark energy

3.2.1. Future Euclid observations of the BAO length

In this section, we developed code to determine the w_p and w_a from observed BAO angles. We used a table of simulated Euclid BAO angles that includes observational errors of the magnitude expected from Euclid. Before using this simulated Euclid data, a case with a single unknown, w_p, was considered where $w_a = 0$, a known case, is a good test to see if the code works correctly. The expected results are $\bar{w_p} = -0.936$, with the 1-σ range $w_p = -0.937$ to -0.934 and 2-σ range $w_p = -0.938$ to -0.933. $\chi^2_{min} = 17.73$ and $\frac{\chi^2_{min}}{\nu} = 1.04$.

With this test completed, we moved on to the simulated Euclid data with two unknowns.

Firstly, the values of w_a get incremented in steps of 0.01 (between 0 and 1, giving an accuracy of 2 s.f.), and for each of these, w_p is incremented in steps of 0.001 (between 0 and-1, giving an accuracy of 3 sf). For a unique pair of w_a and w_p, a data set is generated using Eqn. (15), where $d(z)$ is defined in Eqn. (18).

For the integral of dz, we have used an in-built numerical integration library using the trapezium method. This integral has an absolute error in the order of 10^{-10}. This data set has a generation for each value of z, in the range $0.3 < z < 2.0$, by the data provided. However, each of the calculated values will have a dependence on the code accuracy, and so to eliminate we use,

$$\delta\theta_{BAO} = \theta_{BAO}(z) - \theta_{\Lambda CDM}(z). \tag{41}$$

This elimination produces a negligible error in step size, allowing code to have a reasonable runtime. We may now use each set of $\delta\theta_{BAO}$ values in a χ^2 test to calculate the best pair of w_a and w_p values. The χ^2 test functions as follows:

$$\chi^2(w_p, w_a) = \sum_{i=1}^{N} \frac{(\delta\theta_{BAO}(z_i) - \delta\theta_{th}(z_i))^2}{\Delta\theta_i^2}, \tag{42}$$

where $\theta(z_i)$ is the mean value observed at z_i (the given data set), $\theta_{th}(z_i)$ is the theoretical value calculated for a corresponding pair of input (w_p, w_a) values, and $\Delta\theta_i$ is the error (standard deviation) in the value of the observed $\theta(z_i)$ (given as $\Delta\theta_i = 0.00005$).

For each set of w_a and w_p values, a χ^2 value gets generated and appended to a list. By finding the minimum value in this list and its corresponding index, we found the values of w_a and w_p that match the closest with the mean observed data provided. We then calculate the minimum reduced χ-square value using the following formula,

$$\text{Reduced } \chi^2 \text{ value} = \frac{\chi^2_{\min}}{\nu}, \tag{43}$$

where ν is the number of degrees of freedom. In this case we have 2(w_a and w_p) and as $v = N - d$ and we gave 18 points, $v = 18 - 2$.

We then calculate the error in best fit by finding corresponding w_a and w_p values for 1-σ error and 2-σ error using the following formula:

$$\chi^2(w_p, w_a) = \chi^2_{min} + \Delta\chi^2, \tag{44}$$

where, at 1-σ error and 2 free parameters, critical $\Delta\chi^2 = 2.30$ and 2-σ error and at 2 free parameters, critical $\Delta\chi^2 = 6.17$.

This calculation used an iterative loop, testing each value of χ^2 against the minimum χ^2 value shifted from their best-fit values according to one or two σ. Any values in the range have the corresponding indexes appended to a new w_a and w_p list. We then searched this new list, finding the minimum and maximum w_a and w_p for 1-σ and 2-σ. By cleansing this data to include a max and minimum w_p for each value of w_a, we plotted w_p against w_a for 1-σ and 2-σ to generate a scatter plot of two ellipses as shown in Figure 3 below.

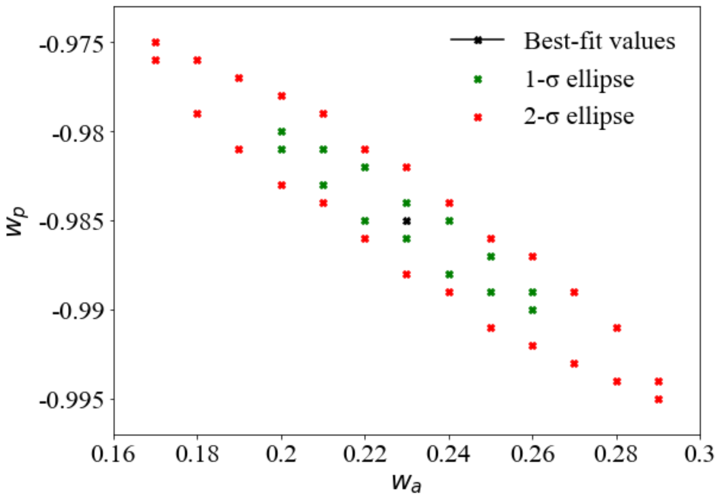

Figure 3: The 1-σ and 2-σ ellipses shown by their respective scatter plots. The 1-σ range is $0.99 < w_p < 0.98$ and the 2-σ range is $0.995 < w_p < 0.975$. The 1-σ range for w_a is $0.2 < w_a < 0.26$ and the 2-σ range is $0.17 < w_a < 0.29$.

Figure 3 shows the general shape of the ellipses, but we added more data points to increase the resolution. By decreasing the step size for both w_a and w_p, we achieve an increased number of corresponding values. We used the same processes seen in the previous code, identifying all w_a and w_p values within the 1-σ and 2-σ limits. These new data points were plotted as scatters on the same plot, reaching a smoother ellipse with a higher degree of accuracy, discussed in Section 4.2.

3.2.2. Calculating luminosity distance and comparing models of the Universe.

Comparing DE models of the Universe determines which one describes it the best. One method is plotting the EoS parameter w_X against redshift for each model for comparison, giving insights into how the models relate. However, a better method where we used the d_L against redshift as a function of the parameter $w_X(a)$ for the models.

To accomplish this, we programmed an array of values of a and loops through it. In this loop, another nested loop is used where the innermost loop integrates Eqn. (13) for each value of a, using a'' for the integration variable and computing ρ_X at a scale factor a''. By taking the value of $\rho_X(a')$ from this integration and substituting it into Eqn. (11), in the middle loop, the physical distance is found. Subsequently, the luminosity distance from Eqn. (20) can be calculated for the given scale factor (and redshift) and saved in another array. Finally, once the scale factor array gets looped through, the d_L can be plotted against redshift.

To test this, the case of a pure cosmological constant Universe with $\Omega_{X,0} = 1$, $w_X = -1$ and $\Omega_{m,0} = 0$ is used. It is a simple model of the Universe where the d_L gets calculated analytically, finishing with Eqn. (24). If the code matches the analytical approach, then the code is confirmed to be working as intended. The test was successful, as seen in Figure 4 below.

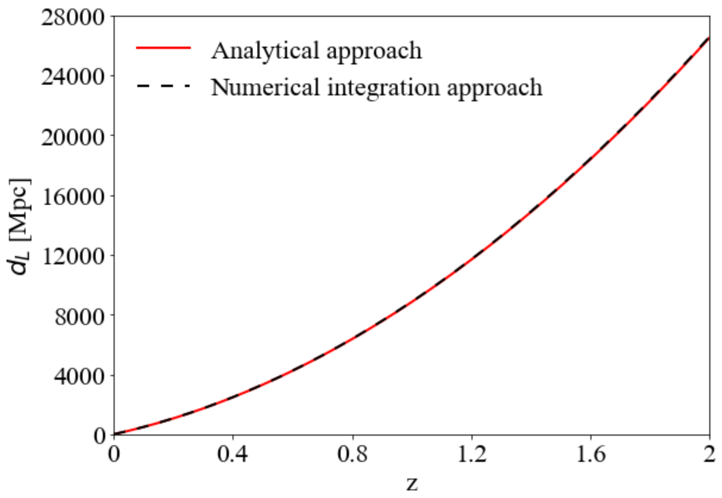

Figure 4: The curves show the test case for calculating the luminosity distance, $d_L(z)$, from $0 < z < 2$, indicated by the approach associated with the red curve and black dashes, of a pure cosmological constant Universe with $\Omega_{X,0} = 1$, $w_X = -1$ and $\Omega_{m,0} = 0$.

With this test completed, we compared ΛCDM and SFQ by setting $w_x(a) = -1$ for the former and using Eqn. (27) with values $a_t = 0.23$, $\tau = 0.33$, $w_{\rm past} = 0$ and $w_f = -1$ for the latter model. A comparison of fractional accuracy to the fractional deviation of the d_L for the DE models from the d_L for the ΛCDM model, by calculating,

$$\left|\frac{\Delta d_L}{d_L}\right| = \left|\frac{d_L(z) - d_{L,\Lambda\rm CDM}(z)}{d_L(z)}\right|, \tag{45}$$

where $d_{L,\Lambda\rm CDM}(z)$ is the luminosity distance for the ΛCDM model.

The curve is then compared with the predicted accuracy of Roman over the range $0.2 < z < 1.7$ where if the fractional deviation is less than 0.36% over the range, then the model is unobservable, but if it is larger, it may be observable.

Lastly, we account for the general SFQ model by finding a region of distinguishability. To do this, a boundary curve comparing a_t to τ is plotted where below the boundary, the deviation isn't detectable and above, it may be observable. The comparisons between the two DE models and the general SFQ model are in Section 4.2.

3.3 Modified gravity models providing an alternate approach

The plot of $q(z)$ for the DGP model uses a function of Eqn. (39) within the function of Eqn. (38) to return values of $q(z)$ over the range $0 < z < 5$, with intervals every $0.01z$, providing a smooth curve. The same was done for the ΛCDM model using a function of Eqn. (228) within the function of Eqn. (227) for comparison of both models regarding their $q(z)$ curves.

The same method used to compare the test SFQ with ΛCDM was to compare the DGP model with the ΛCDM model. First, the luminosity distance can be plotted against redshift using a slightly modified version of the code mentioned before because the modified gravity model does not need to use the innermost loop, so creating a different function that removes this loop will make the code more efficient. We plotted the fractional accuracy for the two models with Eqn. (45), comparing it with the predicted accuracy of Roman again.

We discussed the consequences of generalising the modified Friedmann equation to a value of $\alpha \neq 1$ in Section 2.4. For any given value of α, the model gets compared to the ΛCDM model by plotting luminosity distance against redshift again. We can achieve this via code.

Firstly, all constants can be defined at the start of the code and given variable names. This includes H_0, a_0, $\Omega_{m,0}$ and any other constants that may be used. At this point, the user can choose the value for α.

These values for z get established at this point in an array. We created a sub-routine with z as a parameter, beginning the main bulk of code. Eqn. (93) was used to calculate the values of a, each assigned to an array.

A nested loop is required to calculate the integral for each value of a. The outside loop will go through each value, while the inner loop can calculate the integral shown in Eqn. (141).

Inside the integral mentioned above, $H(a)$ can be calculated with Eqn. (33), inputting all the constants here and then substituting them into the equation. The integration can then take place, going through the values from a_0 to a and adding the final value to an array to use later.

Once the integration is complete and the value saved, the code will run through all values of a, doing the same process until all values of a are tested, with the values for the integration found and saved. Finally, these values get used in conjunction with the array of z values to calculate all values for the luminosity distance with Eqn. (20) and saved in a final array before being plotted.

4 Results

4.1 Directly observing the age and matter content of the universe

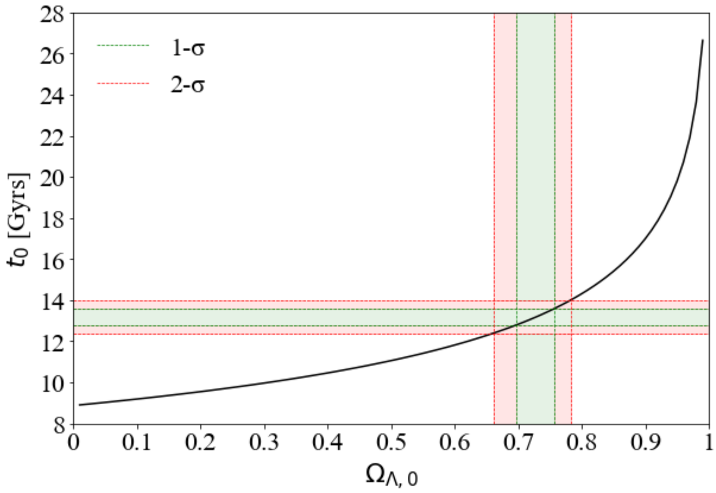

Figure 5: The black curve displays the logarithmic increase in the current age of the Universe, t_0, as the current cosmological energy density parameter, $\Omega_{\Lambda,0}$, becomes more dominant. The green region displays the 1-σ range and the red region displays the 2-σ range for the observed globular cluster age of the Universe $t_0 = 13.2 \pm 0.4\,$Gyr, respectively, where the 1-σ range is $0.697 < \Omega_{\Lambda,0} < 0.757$ and the 2-σ range is $0.661 < \Omega_{\Lambda,0} < 0.782$.

There is a theoretical component of DE consistent with the observed age. Since $\Omega_{\Lambda,0} \neq 0$, established from Figure 17, lies within the t_0 observational bounds from globular clusters [7], it means that a pure matter Universe ($\Omega_{\Lambda,0} = 0, \Omega_{m,0} = 1$) is ruled out. For the case of a pure matter Universe, Eqn. (2) was found to reduce to $t_0 = (2/3)H_o^{-1}$ noted in the appendix (A.1), being consistent with the expected value.

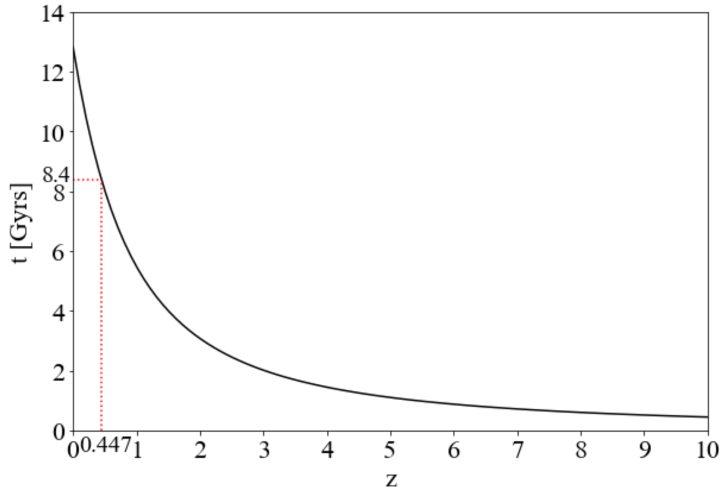

Figure 6: The black curve shows the logarithmic decline in the age of the Universe, t, at higher values of redshift, z, for $\Omega_{\Lambda,0} = 0.7$. The red dotted lines show that at $t_\Lambda = 8.4\,\mathrm{Gyr}$ is when the Universe became dominated by the Λ and the corresponding redshift, $z_\Lambda = 0.447$.

The t_Λ from Figure 6, and its corresponding redshift z_Λ, was determined when the ratio between the ρ_m and ρ_Λ reached unity. ρ_m scales with the inverse square of time, t, and the ρ_Λ remains constant with t. After dividing the ratio through by $\rho_{c,0}$, we obtained $t_\Lambda = t_0\sqrt{\Omega_{m,0}/\Omega_{\Lambda,0}}$ where $t_0 = 12.8\,\mathrm{Gyr}$ was determined using Eqn. (4) and the HST measurements on Type Ia supernovae for H_0. There is a 1-σ discrepancy between the globular cluster age and the SN Ia age determination; hence, there is evidence that the ΛCDM model provides a reliable estimate for t_0, using observational data from SN Ia and galaxy cluster dynamics.

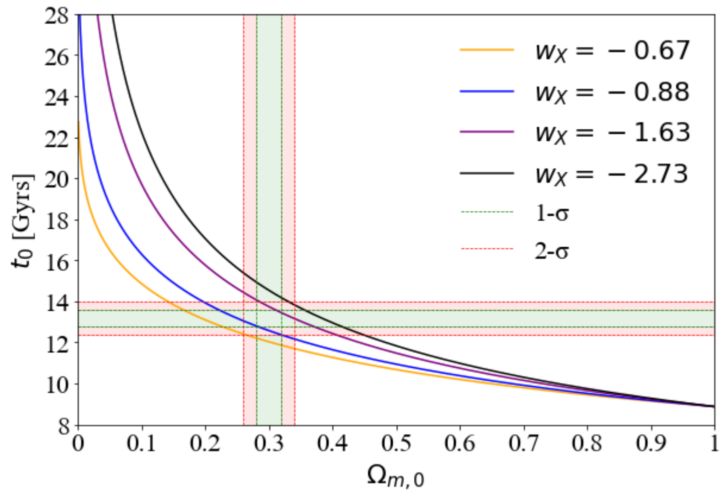

Figure 7: The purple and blue curves intersect the 1-σ observational limits on $t_0 = 13.2 \pm 0.4\,\mathrm{Gyr}$ and $\Omega_{m,0} = 0.30 \pm 0.02$, from galaxy cluster dynamics, providing $-1.63 < w_X < -0.88$ for the DE barotropic parameters. The black and orange curves intersect the 2-σ observational limits providing $-2.73 < w_X < -0.67$.

The curves in Figure 7 use the ΛCDM model of the t_0, derived from the ΛCDM-independent measurement of H_0. However, the 1-σ and 2-σ constraints added to the model are independent of the ΛCDM model, since they are determined from the globular cluster age measurements. The combination provides a compatible range on w_X, refining the ΛCDM model.

4.2 Future space telescope determination of the nature of dark energy

The simulated Euclid data in the CPL model obtained the best-fit values of w_p and w_a, resulting in $\bar{w}_p = -0.985$ and $\bar{w}_a = 0.23$. The values of $\chi^2_{\rm min} = 14.9$ and $\chi^2_{\rm min}/\nu = 0.93 \sim 1$; hence, the best-fit values fit the data well. To test if the model was observationally observable from ΛCDM and has a time-dependent EoS, we plotted 1- and 2-σ ellipses depicted below.

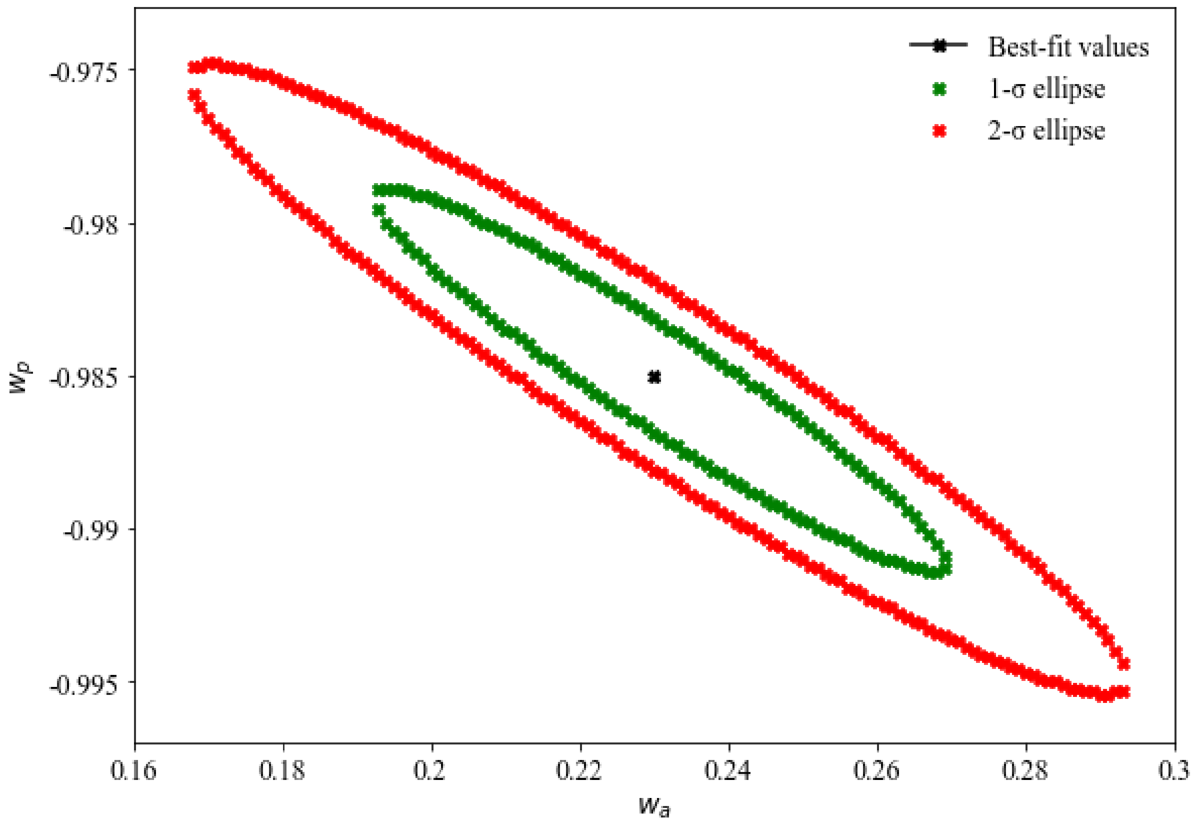

Figure 8: The 1-σ and 2-σ ellipses shown by their respective scatter plots. The 1-σ range for w_p is $-0.99 < w_p < -0.98$ and the 2-σ range is $-0.995 < w_p < -0.975$. The 1-σ range for w_a is $0.2 < w_a < 0.26$ and the 2-σ range is $0.17 < w_a < 0.29$.

Evidence shows that for the 1-σ and 2-σ accuracy, the model is observationally distinguishable from a cosmological constant since $w_X(a) \neq -1$ at any redshift, requiring a dynamical field to describe DE.

For small variations from $w = -1$, the cosmological constant is simpler and favoured because of Occam's razor; for large deviations, the more complex two-parameter becomes favourable. This variation is quantified using a Bayesian evidence calculation, which shows if the data is consistent with a cosmological constant [1].

The 1-σ deviation from the best-fit values is $\Delta w_p = 0.005$ and $\Delta w_a = 0.03$, providing a FoM $= 1/(0.005 \times 0.03) \approx 6700$. This value is > 400, meaning the cosmological constant gets favoured with odds of more than 100:1, considered "decisive" statistical evidence [1]. Therefore, there is evidence the model does not have a time-dependent EoS to a 1-σ accuracy.

The 2-σ deviation from the best-fit values is $\Delta w_p = 0.01$ and $\Delta w_a = 0.06$, providing a FoM $= 1/(0.01 \times 0.06) \approx 1700$, still too large for the sensitivity that Euclid aims to achieve. Hence, it is possible to determine through observations that the model does not have a time-dependent EoS for the 1-σ and 2-σ accuracy, albeit distinguishable from ΛCDM itself.

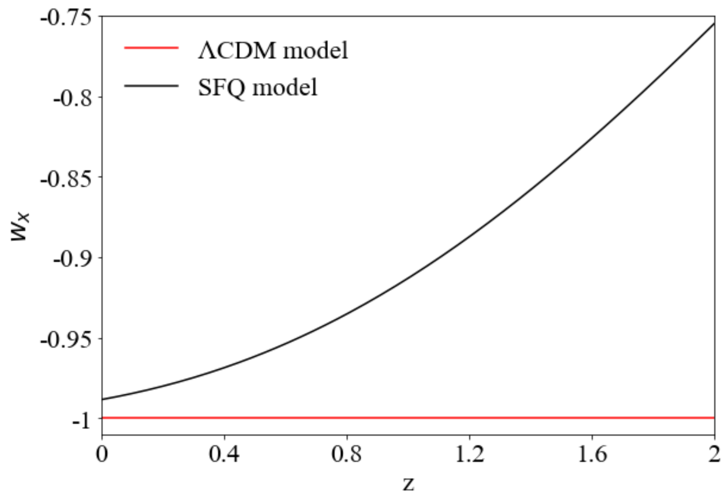

Figure 9: The red line displays the constant value of $w_X = -1$ from $0 < z < 2$ for the ΛCDM model. The black curve displays the decrease in w_X as the Universe is expanding, parameterized by the test model parameters: $a_t = 0.23$, $\tau = 0.33$, $w_{\text{past}} = 0$ and $w_f = -1$ where w_X tends to w_f in the future.

The present barotropic parameter for DE in the test SFQ model is $w_0 = -0.989$, breaking away from the early-time scaling solution for matter as it "freezes", becoming shallow at late times. w_X transitions to an EoS close to a cosmological constant used in the ΛCDM model, helping to explain why the present dark matter and DE densities are similar. As $z \to 2$ on Figure 9, there is evidence of the tracker model since w_X is almost constant with z. The shallowing of the gradient represents the epoch of cosmic acceleration, meaning there is evidence for multiple fields in the test model.

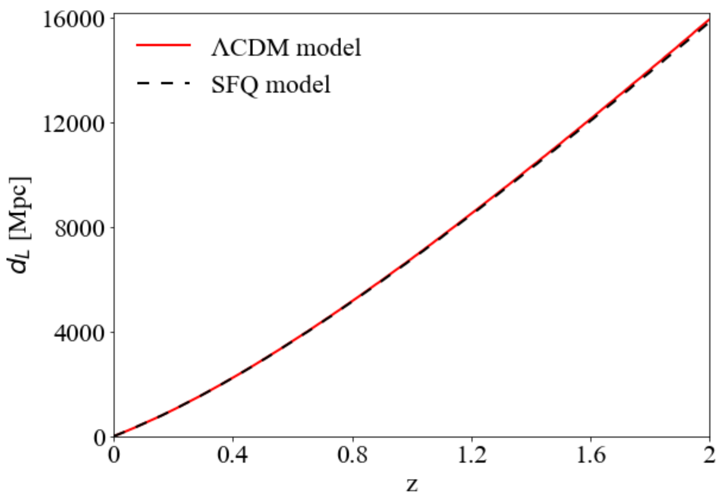

Figure 10: The curves show the $d_L(z)$ for the two DE models: ΛCDM and test SFQ, represented by the respective red line and the black dashes between $0 < z < 2$.

The $d_L(z)$ for the test SFQ model is very close to the corresponding ΛCDM model in the range $0 < z < 2$, seen in Figure 10. There is a minor difference between the models for $z > 1.2$, where the SFQ model is predicting smaller values for $d_L(z)$ at corresponding redshifts, becoming more prominent towards $z = 2$. Therefore, the rise in $w_X \approx 0.25$ from Figure 9 for the test SFQ model has little effect in decreasing the $d_L(z)$ values over the redshift range, as it becomes closer to the scaling solution for matter.

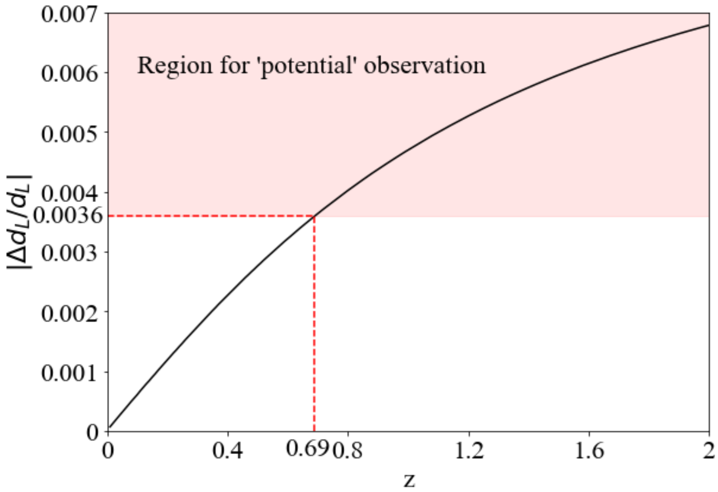

Figure 11: The curve displays the fractional deviation of the $d_L(z)$ for the test SFQ model from $0 < z < 2$. The red dotted line indicates that above 0.36%, outside the 2-σ region for $0.69 > z > 1.70$, the test SFQ model may become observable from the ΛCDM model for the Roman observation of $d_L(z)$.

The high aggregate precision of 0.36% for 2-σ in the Roman measurements of >2700 SN Ia events cannot distinguish the difference between the test SFQ model and the ΛCDM model between $0.2 < z < 0.69$, highlighting the point that $d_L(z)$ for both models are very close to each other, from Figure 10, but becomes potentially observable at higher redshifts. These results provide evidence supporting this test model.

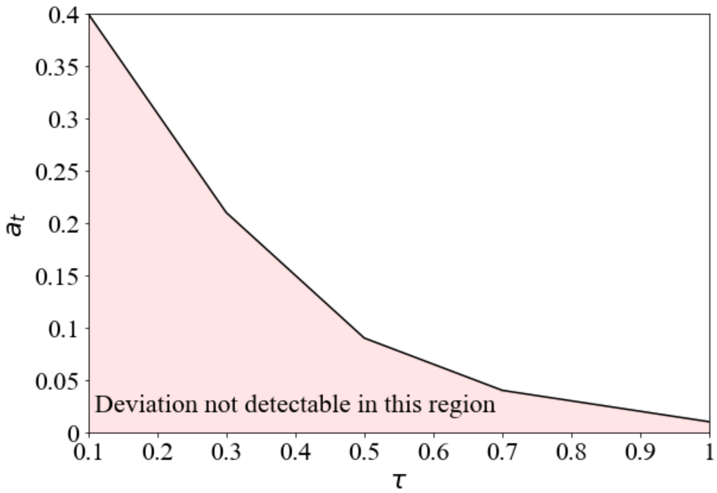

Figure 12: The black line displays the smallest values of a_t for $\tau = 0.1, 0.3, 0.5, 0.7$ and 1 for which the fractional deviation is smaller than the 2-σ Roman accuracy over the whole range $0.2 < z < 1.7$ provides $a_t = 0.40, 0.21, 0.09, 0.04$ and 0.01, respectively. Therefore, the model is indistinguishable in the red region below the curve.

Above the boundary, the Roman telescope can detect the SFQ models from the ΛCDM model in Figure 12. There is an increasing likelihood for detection as the (a_t, τ) value moves further away from the boundary. During early times, there are larger values of a_t [13], so SFQ behaves like matter ($w_X \approx 0$); therefore, transitioning away from the ΛCDM model makes detection possible. Increasing τ causes the test SFQ model to approach the ΛCDM model less rapidly, seen by the shallowing in gradient in Figure 12, making detection more probable. Thus, balancing the two parameters could help to bring the test SFQ model from Figure 10 closer to the ΛCDM model.

4.3 Evaluating the modified gravity DGP model against the ΛCDM model

For the present DGP model where $\alpha = 1$, we used Eqn. (32) to calculate the $r_c = 6\,394\,\text{Mpc}$ for the distance beyond which we apply modified gravity to gravitating sources.

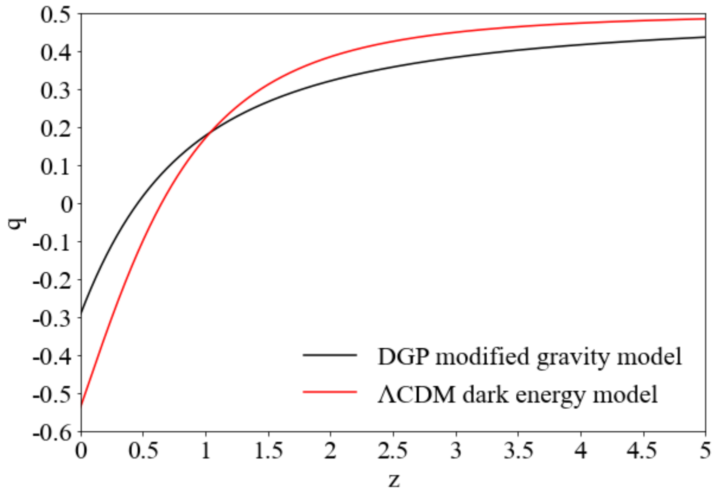

Figure 13: The black and red curves represent their respective models for q, from $0 < z < 5$. At higher values for z, both models show decelerating expansions for the Universe and accelerating expansions at lower values of z. The ΛCDM model accelerates more rapidly than the DGP model since the Universe becomes dominated by DE over matter, whereas DGP accelerates more slowly since gravity is modified.

The deceleration parameter at present, q_0, is seen in Figure 13 for the DGP modified gravity model, which uses the modified Friedmann equation where $\alpha = 1$, and the ΛCDM model that utilises the unmodified Friedmann equation. The values are $q_{0,\text{DGP}} = -0.294$ and $q_{0,\Lambda\text{CDM}} = -0.538$. Both models indicate an accelerating present Universe, with the more prominent acceleration in the ΛCDM model. The curves intersect at $z \approx 1.04$ and $q \approx 0.186$, where the ΛCDM model takes over the acceleration of the DGP model. Using Eqn. (4), we calculated the age of the Universe $t(z = 1.04) = 5.33\,\text{Gyr}$ when this intersection occurs. At this late stage in the Universe, the DGP starts to model the accelerated expansion too less of an effect than DE from the ΛCDM model.

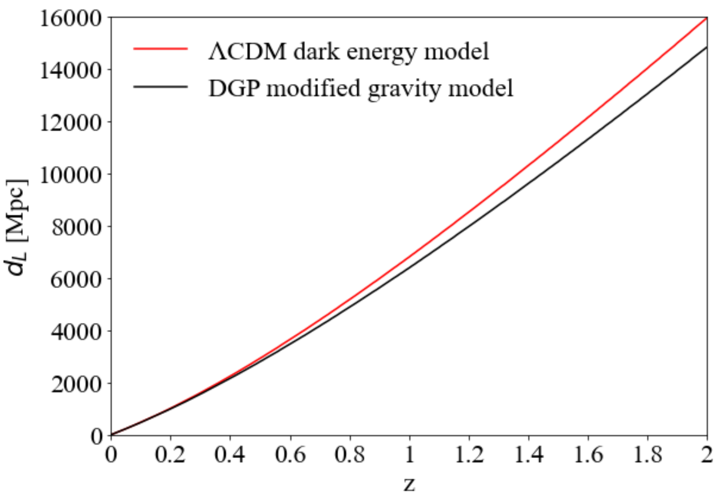

Figure 14: The red and black respective curves show the $d_L(z)$ for the ΛCDM DE model and the DGP modified gravity model where $\alpha = 1$, between $0 < z < 2$. The difference in $d_L(z)$ between the two models follows similar physical regimes at present, before separating towards higher redshifts.

The DGP model assumes a perturbation to the unmodified Friedmann Eqn. (46) that is prominent at $0.6 < z < 2$, moving away from GR and the ΛCDM model. This divergence suggests a smaller value of r_c at higher redshifts, pushing the predictions away from the data and weakening the argument for the DGP model.

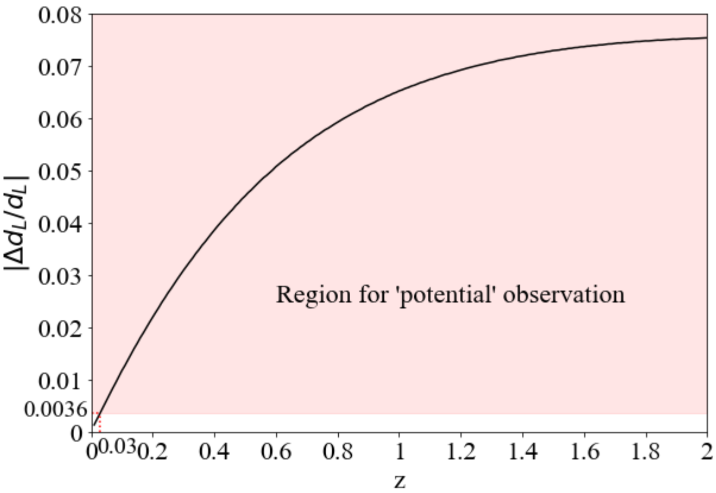

Figure 15: The curve displays the fractional deviation of $d_L(z)$ for the DGP modified gravity model from $0 < z < 2$. The red dotted line indicates that above 0.36%, outside the 2-σ region for $0.2 > z > 1.70$, the $\alpha = 1$ DGP model may become observable from the ΛCDM model for the Roman observation of $d_L(z)$.

The Roman telescope measurements cannot distinguish the difference between the DGP and ΛCDM models for $z < 0.03$, highlighting the point that $d_L(z)$ for both models are not very close to each other from Figure 15. These results suggest a flaw in the DGP model against the observational data.

5 Discussion and further research

5.1 The analysis and future considerations of the SFQ test model against the $\alpha = 1$ DGP model via ΛCDM comparisons

The $d_L(z)$ curve for the test SFQ model from Figure 10 remained closer to the ΛCDM model than the DGP curve from Figure 14, with respective apparent notable differences in $d_L(z)$ for $z > 1.2$ and $z > 0.6$. Both models underpredicted the $d_L(z)$, more prominent for higher redshifts. There is some evidence the test SFQ model explains the epoch of accelerated Universe expansion more effectively than the DGP model, based on our "best" ΛCDM model that uses the Planck CMB best-fit results. Delving further, we analysed the fractional deviation for $d_L(z)$, finding the z-range that Roman could detect the difference between the test and ΛCDM model. The test SFQ model could be observed from $0.69 < z < 1.70$, whereas the DGP model could be throughout the entire 2-σ range of the Roman measurements. Based on the $d_L(z)$ calculations, there is further evidence in support of the SFQ model since the Roman telescope cannot detect the difference between $0.2 < z < 0.69$.

Altering the value for α for the DGP model could bring it closer to the ΛCDM model. Following the condition that α does not interfere with the large-scale structure formation before achieving the present structure from the imprint of early fluctuations left on the CMB, a long period of matter domination is required. This condition means that $w \leq -1/2$. If $w_{\text{eff}} \leq -1/2$ is required, then this leads to $\alpha \leq 1$. From this, we must decrease α to bring the model closer to the ΛCDM model since increasing α would break the condition, bringing the $d_L(z)$ predictions closer to the ΛCDM model [14]. Further work that involves reaching a similar solution to Eqn. (33) for a general value of α may allow us to find a value that best matches the ΛCDM for $d_L(z)$ improving our modified gravity models.

Changing the parameters of a_t and τ in the SFQ model could bring it closer to the ΛCDM. As mentioned in Sec. 4.2 finding, the correct balance between the two parameters would make this possible. Chiba et al. proposed that if the Baryon Oscillation Spectroscopic Survey (BOSS) data gets taken into account in the analysis, the transition redshift is constrained to be at $a_t < 0.23$ (95% CL). For $\lambda_2 = 0$ from Eqn. (25), the transition width is around $\tau \approx 0.33$, while a_t depends on the values of λ_1 [17]. In this case, we can fix $\tau = 0.33$ and find the constraints on a_t. Our test model used $a_t = 0.23$ and $\tau = 0.33$ an "appropriate choice for this analytic expression to fit the numerical solution" [13], undetectable for Roman from ΛCDM between $0.69 < z < 1.70$. Durrive et al. constrained $a_t < 0.11$, via the 1-σ and 2-σ confidence regions in the $(a_t, \Omega_{m,0})$ plane with a 95% CL, meaning that the scaling transition to $w = -1$ occurs very early in the history of the Universe where DE behaves as a cosmological constant. We can take the constraint on $\tau = 0.33$ and vary the a_t to values lower than $a_t < 0.11$ to find the value of a_t that best matches the ΛCDM model for $d_L(z)$. Subsequently, the z region where the Roman telescope cannot distinguish the difference between the two models will determine whether the lower limit of z increases to a value > 0.69, bringing the model closer to ΛCDM.

5.2 Explaining the accelerated expansion of the Universe with alternate models

So far we have considered three models for DE (ΛCDM, CPL, and SFQ) and one for modified gravity (DGP). The DE models have different EoSs, directly influencing the Universe expansion rate from early to late epochs. The cosmological constant drives the expansion in ΛCDM, since matter (and radiation) become less dominant towards the present. Currently, this is our best model explaining Universe expansion with $H_0 = 67.81\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ based on observations of fluctuations in the CMB radiation during the early times of the Universe. However, the "H_0 tension" provides a reason to explore the scope of models that extend the ΛCDM model, since the SNe Ia from HST provides $H_0 = 73.48\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

We have considered a specific case of the scaling quintessence model that freezes over time, which we found more favourable than the modified gravity model. Despite this, considering alternate scaling modes may explain the late-time cosmic acceleration to a better degree. Tsujikawa et al. proposed a thawing model, where the field is nearly frozen by the Hubble friction in the early cosmological epoch. Whilst there is no statistical evidence that the models with $w_0 > -1$ are favoured over the ΛCDM, the thawing models with $-1 < w_0 < -0.7$ are not yet ruled out observationally. Since $w_0 > -1$, the current filed EoS is constrained to be $w_0 < 0.695$ (95% CL) from the joint data analysis of SN Ia, CMB and BAO [12]. Future high-precision observations from Euclid and Roman telescope missions are expected to allow us to distinguish quintessence from ΛCDM.

Euclid also aims to test the phantom DE model, where $w < -1$ at any redshift. There are theoretical problems where taking scalar fields as the phantom field requires the Hamiltonian, $\hat{H}$, to have a negative kinetic term. This $\hat{H}$ leads to vacuum instabilities since it is unbounded from below. Without a negative kinetic term, the theory is not unitary [18]. However, the construction of models prevents this problem. Costa et al. adopt a phantom-like fluid, treating it as a phenomenon, checking its consistency with data and if it conflicts with well-established physical measurements. They ran a Monte Carlo simulation using CMB and BAO datasets for the phantom-like cosmology aided by a non-thermal matter production, finding it alleviates the "H_0 tension" to 3.2-σ, obtaining $H_0 = 69.08 \pm 0.71\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. The Euclid mission will help to constrain the parameters in the phantom DE model, probing a different H_0 with hopes of bringing the value closer to late-time measurements in due course of the Roman mission.

6 Conclusions

We used the HST measurements of SNe Ia in the ΛCDM model, comparing it to the 1-σ and 2-σ globular cluster age bounds of the Universe. As a result, we constrained the density parameters for a cosmological constant using two independent DE observations. We obtained the 1-σ range $0.697 < \Omega_{\Lambda,0} < 0.757$ and the 2-σ range $0.661 < \Omega_{\Lambda,0} < 0.782$. Therefore, a theoretical component of DE is consistent with the observed age, ruling out the case for a pure matter Universe. From the HST standard candle measurements of SNe Ia, the age of the present Universe is $t_0 = 12.8\,\mathrm{Gyr}$ using ΛCDM, within 1-σ of the globular cluster age. There is evidence that the model provides a reliable estimate for t_0. The Universe became dominated by the cosmological constant over matter content during late times, at $t_\Lambda = 8.4\,\mathrm{Gyr}$ and $z_\Lambda = 0.447$. Constraining the limits on w_X was completed using the globular cluster age and the matter density parameter bounds from galaxy cluster dynamics. The respective 1-σ and 2-σ limits for w_X were found to be $-1.63 < w_X < -0.88$ and $-2.73 < w_X < -0.67$. Observations have not ruled out the DE models we would explore, including thawing quintessence, phantom cosmology or even modified gravity models.

After performing a χ^2 test on simulated BAO data from the concurrent Euclid mission, we obtained the best-fit values for the CPL model to be $\bar{w}_p = -0.985$ and $\bar{w}_a = 0.23$. Our values of $\chi^2_{\min} = 14.9$ and $\chi^2_{\min}/\nu = 0.93 \sim 1$ meant that the best-fit values fitted the data well. The respective 1-σ and 2-σ ellipses in the (w_a, w_p) plane allowed us to clarify that the model is observationally distinguishable from a cosmological constant, requiring a dynamical field to describe DE with the limits of $-0.99 < w_p < -0.98$, $-0.995 < w_p < -0.975$, $0.2 < w_a < 0.26$, and $0.17 < w_a < 0.29$. We used the deviations from these limits, $\Delta w_p = 0.005$, $\Delta w_a = 0.03$, $\Delta w_p = 0.01$, and $\Delta w_a = 0.06$, to determine the FoM for the respective 1-σ and 2-σ accuracy, obtaining values of 6700 and 1700. These values are > 400, the desired FoM for Euclid; therefore, it is possible to determine through observations that the model does not have a time-dependent EoS.

The present barotropic parameter for our SFQ test model is $w_0 = -0.989$, "freezing" from the early-time scaling tracker solution for matter seen as $z \to 2$. The w_X shallowed during late times, transitioning to an EoS close to a cosmological constant used in the ΛCDM model, explaining why the present dark matter and DE densities are similar. This epoch of cosmic acceleration provides evidence for multiple fields in the test model. The curve in the (a_t, τ) plane displayed the smallest values of $a_t = 0.40, 0.21, 0.09, 0.04$ and 0.01 for $\tau = 0.1, 0.3, 0.5, 0.7$ and 1 which the fractional deviation is smaller than the 2-σ Roman accuracy over range $0.2 < z < 1.7$. The region below the curve indicates the parameter values that SFQ is indistinguishable from ΛCDM. The likelihood of detection increases the further the (a_t, τ) value is away from the boundary. This breakdown constrained the parameters in our SFQ model.

Standard length scales, such as $d_L(z)$, were used to analyse the SFQ and DGP models. We found that SFQ remained closer to ΛCDM, whilst the DGP model deviated below the $d_L(z)$ of ΛCDM more prominently at higher-z. Further delving, the fractional deviation for $d_L(z)$ found the z range at which Roman could detect the difference between the test and ΛCDM model at 2-σ. Our test SFQ model was detectable for $0.69 < z < 1.70$, whilst the DGP was detectable over the whole 2-σ range. There is evidence in support of our test SFQ model over the DGP model. The requirement for $\alpha \leq 1$ would bring the DGP model closer to ΛCDM. Modifying the parameters in the SFQ model would bring it closer to ΛCDM, where constraints for $a_t < 0.11$ and $\tau = 0.33$ seem favourable for further testing.

The distance beyond which we apply modified gravity to gravitating sources is $r_c = 6\,394$ Mpc for the present DGP model where $\alpha = 1$. Both DGP and ΛCDM model accelerated expansion of the Universe at present, with values $q_{0,\mathrm{DGP}} = -0.294$ and $q_{0,\Lambda\mathrm{CDM}} = -0.538$; hence, more prominent acceleration in ΛCDM. The curves intersected at $z \approx 1.04$ and $q \approx 0.186$, where ΛCDM takes over the acceleration against DGP. This intersection occurred at $t(z = 1.04) = 5.33\,\mathrm{Gyr}$, representing that DGP starts to model the accelerated expansion weaker than DE from ΛCDM during late times.

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A Directly observing the age and matter content of the universe

A.1 Derivations

Starting from the Friedmann equation where for a flat Universe with the curvature term, $k = 0$ is composed of pressureless matter and the cosmological constant, Λ,

$$H^2 = \frac{8\pi G}{3}\rho_m + \frac{\Lambda c^2}{3}, \tag{46}$$

where H is the Hubble parameter, G is the gravitational constant, and ρ_m is the matter-energy density, was used to derive the Age of the Universe in the ΛCDM DE model. Λ is a constant contribution to the total energy density, ρ, so Eqn. (46) reduces to

$$H^2 = \frac{8\pi G}{3}\Big(\rho_m + \rho_\Lambda\Big). \tag{47}$$

The continuity equation was used to find the scale factor, a, dependence for m and Λ on energy density, ρ_i, being rearranged as follows

$$\dot{\rho}_i + 3(1 + w_i)H\rho_i = 0, \tag{48}$$

$$\frac{d\rho_i}{\rho_i} = -3(1 + w_i)Hdt, \tag{49}$$

where,

$$H = \frac{\dot{a}}{a}. \tag{50}$$

Substitute Eqn. (50) into Eqn. (49),

$$\frac{d\rho_i}{\rho_i} = -3(1 + w_i)\frac{da}{dt}\frac{1}{a}dt. \tag{51}$$

Integrate between ρ_i and $\rho_{i,0}$ after adding a prime to the integration variables,

$$\int_{\rho_{i,0}}^{\rho_i} \frac{d\rho_i'}{\rho_i'} = -3(1 + w_i)\int_{a_0}^{a_i} \frac{da'}{a'}, \tag{52}$$

$$\ln\left(\frac{\rho_i}{\rho_{i,0}}\right) = -3(1+w_i)\ln\left(\frac{a}{a_0}\right), \tag{53}$$

$$\ln\left(\frac{\rho_i}{\rho_{i,0}}\right) = \ln\left[\left(\frac{a}{a_0}\right)^{-3(1+w_i)}\right], \tag{54}$$

$$\frac{\rho_i}{\rho_{i,0}} = \left(\frac{a}{a_0}\right)^{-3(1+w_i)}. \tag{55}$$

After setting the present-day scale factor, a_0, to be to unity, and the barotropic parameter, w_i, for $w_m = 0$ and $w_\Lambda = -1$ the ρ_i dependencies on a_i are,

$$\frac{\rho_m}{\rho_{m,0}} = a^{-3(1+0)}; \tag{56}$$

$$\rho_m = \frac{\rho_{m,0}}{a^3} \tag{57}$$

and,

$$\frac{\rho_\Lambda}{\rho_{\Lambda,0}} = a^{-3(1-1)}, \tag{58}$$

$$\rho_\Lambda = \rho_{\Lambda,0}, \tag{59}$$

respectively. Substitute Eqn. (57) and Eqn. (59) into Eqn. (47),

$$H^2 = \frac{8\pi G}{3}\left(\frac{\rho_{m,0}}{a^3} + \rho_{\Lambda,0}\right). \tag{60}$$

Substitute Eqn. (50) into Eqn. (60),

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\frac{\rho_{m,0}}{a^3} + \rho_{\Lambda,0}\right), \tag{61}$$

$$\frac{\dot{a}}{a} = \left(\frac{8\pi G}{3}\right)^{\frac{1}{2}}\left(\frac{\rho_{m,0}}{a^3} + \rho_\Lambda\right)^{\frac{1}{2}}, \tag{62}$$

$$\frac{da'}{a'(a'^{-3}\rho_{m,0} + \rho_{\Lambda,0})^{\frac{1}{2}}} = \left(\frac{8\pi G}{3}\right)^{\frac{1}{2}} dt. \tag{63}$$

The density parameter, $\Omega_{i,0}$ is defined as

$$\Omega_{i,0} = \frac{\rho_{i,0}}{\rho_{c,0}}, \tag{64}$$

where $\rho_{c,0}$ is the present critical density, from Eqn. (30). Substitute $\rho_{c,0}$ into Eqn. (64),

$$\Omega_{i,0} = \rho_{i,0}\frac{8\pi G}{3H_0^2}, \tag{65}$$

rearranging as follows,

$$\rho_{i,0} = \Omega_{i,0}\frac{3H_0^2}{8\pi G}. \tag{66}$$

Eqn. (66) is then substituted into Eqn. (63) with the corresponding m and Λ notations, and simplified as follows,

$$\frac{da'}{a'(\frac{3}{8\pi G})^{\frac{1}{2}} H_0 (a'^{-3}\Omega_{m,0} + \Omega_{\Lambda,0})^{\frac{1}{2}}} = \Big(\frac{8\pi G}{3}\Big)^{\frac{1}{2}} dt, \tag{67}$$

$$\frac{da'}{(\frac{3}{8\pi G})^{\frac{1}{2}} H_0 (a'^{-1}\Omega_{m,0} + a'^2\Omega_{\Lambda,0})^{\frac{1}{2}}} = \Big(\frac{8\pi G}{3}\Big)^{\frac{1}{2}} dt. \tag{68}$$

Eqn. (127) is then integrated between the following limits,

$$\int_0^a \frac{da'}{(\frac{3}{8\pi G})^{\frac{1}{2}} H_0 (a'^{-1}\Omega_{m,0} + a'^2\Omega_{\Lambda,0})^{\frac{1}{2}}} = \int_0^t \Big(\frac{8\pi G}{3}\Big)^{\frac{1}{2}} dt, \tag{69}$$

where Eqn. (130) uses the assumption that $a(t=0)=0$. Rearranging further,

$$\int_0^a \frac{da'}{(a'^{-1}\Omega_{m,0} + a'^2\Omega_{\Lambda,0})^{1/2}} = H_0 t, \tag{70}$$

$$\int_0^a \frac{da'}{(a'^{-1}\Omega_{m,0})^{\frac{1}{2}} \left(1 + a'^3 \frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}\right)^{1/2}} = H_0 t. \tag{71}$$

Apply the substitution, letting

$$s'^2 = a'^3 \frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}, \tag{72}$$

Differentiating Eqn. (72)

$$2s' ds' = 3a'^2 \frac{\Omega_{\Lambda,0}}{\Omega_{m,0}} da', \tag{73}$$

Rearrange Eqn. (73) for,

$$da' = \frac{2s'}{3a'^2} \frac{\Omega_{\Lambda,0}}{\Omega_{m,0}} ds' \tag{74}$$

Rearrange Eqn. (72) for a'^{-2},

$$a' = s'^{\frac{2}{3}} \left(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\right)^{\frac{1}{3}}, \tag{75}$$

therefore,

$$a'^{-2} = s'^{-\frac{4}{3}} \left(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\right)^{-\frac{2}{3}}. \tag{76}$$

Substitute Eqn. (76) into Eqn. (74),

$$da' = \frac{2s'}{3} \frac{\Omega_{m,0}}{\Omega_{\Lambda,0}} s'^{-\frac{4}{3}} \left(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\right)^{-\frac{2}{3}} ds', \tag{77}$$

$$da' = \frac{2s'^{-\frac{1}{3}}}{3} \left(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\right)^{-\frac{1}{3}} ds'. \tag{78}$$

Also, rearrange Eqn. (72) for $a'^{\frac{1}{2}}$,

$$a'^{\frac{1}{2}} = s'^{\frac{1}{3}} \left(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\right)^{\frac{1}{6}}. \tag{79}$$

Substitute Eqn. (78) and Eqn. (79) into the Eqn. (71), changing the limits accordingly by setting the present-day scale factor, $a_0 = 1$,

$$\int_0^1 \frac{a'^{\frac{1}{2}} da'}{(\Omega_{m,0})^{\frac{1}{2}} \left(1 + a'^3 \frac{\Omega_{\Lambda,0}}{\Lambda_{m,0}}\right)^{\frac{1}{2}}} = H_0 t_0 \tag{80}$$

Adjusting the limits using Eqn. (72),

$$s = 1^{\frac{3}{2}} \left(\frac{\Omega_{\Lambda,0}}{\Lambda_{m,0}}\right)^{\frac{1}{2}}, \tag{81}$$

for the upper limit and the $s = 0$ for the lower limit. Substitute the limits, Eqn. (79), Eqn. (78) and Eqn. (72) into Eqn (94),

$$\int_0^{\sqrt{\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}}} s'^{\frac{1}{3}} \left(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\right)^{\frac{1}{6}} \frac{1}{\Omega_{m,0}^{\frac{1}{2}} (1 + s'^2)^{\frac{1}{2}}} \frac{2s'^{-\frac{1}{3}}}{3} \left(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\right)^{\frac{1}{3}} ds' = H_0 t_0, \tag{82}$$

$$\int_0^{\sqrt{\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}}} \frac{2}{3\Omega_{m,0}^{\frac{1}{2}} (1 + s'^2)^{\frac{1}{2}}} \left(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\right)^{\frac{1}{2}} ds' = H_0 t_0, \tag{83}$$

$$t_0 = \frac{2}{3} \frac{H_0^{-1}}{\sqrt{\Omega_{\Lambda,0}}} \int_0^{\sqrt{\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}}} \frac{ds'}{(1 + s'^2)^{\frac{1}{2}}}. \tag{84}$$

The integrand in Eqn. (100) is a standard integral, computing as,

$$t_0 = \frac{2}{3} \frac{H_0^{-1}}{\sqrt{\Omega_{\Lambda,0}}} \left[\ln\left(\sqrt{s'^2 + 1} + s'\right)\right]_0^{\sqrt{\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}}}, \tag{85}$$

$$t_0 = \frac{2}{3} \frac{H_0^{-1}}{\sqrt{\Omega_{\Lambda,0}}} \ln \left[\frac{\sqrt{\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}} + 1} + \sqrt{\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}}}{1}\right], \tag{86}$$

$$t_0 = \frac{2}{3} \frac{H_0^{-1}}{\sqrt{\Omega_{\Lambda,0}}} \ln \left(\sqrt{\frac{\Omega_{\Lambda,0} + \Omega_{m,0}}{\Omega_{m,0}}} + \sqrt{\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}}\right), \tag{87}$$

where the total energy-density parameter is equal to unity,

$$\Omega_{\Lambda,0} + \Omega_{m,0} = 1. \tag{88}$$

Substitute Eqn. (88) into Eqn. (87),

$$t_0 = \frac{2}{3} \frac{H_0^{-1}}{\sqrt{\Omega_{\Lambda,0}}} \ln \left(\frac{1}{\sqrt{\Omega_{m,0}}} + \sqrt{\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}}\right), \tag{89}$$

$$t_0 = \frac{2}{3} \frac{H_0^{-1}}{\sqrt{\Omega_{\Lambda,0}}} \ln \left(\frac{1 + \Omega_{\Lambda,0}}{\sqrt{\Omega_{m,0}}}\right). \tag{90}$$

Finally, substitute Eqn. (88) into Eqn. (90) for $\Omega_{m,0}$, therefore,

$$t_0 = \frac{2}{3}\frac{H_0^{-1}}{\sqrt{\Omega_{\Lambda,0}}} \ln\left(\frac{1+\Omega_{\Lambda,0}}{\sqrt{1-\Omega_{\Lambda,0}}}\right), \tag{91}$$

is the present Age of the Universe in the ΛCDM DE model.
To determine the age of the universe, t, at a given redshift, z,

$$1 + z = \frac{a_0}{a(t)}, \tag{92}$$

where a_0 is the present scale factor set to unity, reducing to

$$1 + z = \frac{1}{a}. \tag{93}$$

Rearrange Eqn. (71) to the form,

$$t = H_0^{-1} \int_0^a \frac{da'}{(a^{-1}\Omega_{m,0} + a'^2\Omega_{\Lambda,0})^{\frac{1}{2}}}. \tag{94}$$

Rearrange Eqn. (93) for a'^2,

$$a^2 = (1+z)^{-2}, \tag{95}$$

and take the scale factor derivative of Eqn. (93),

$$dz' = -a^{-2}da'. \tag{96}$$

Substitute Eqn. (93) into Eqn. (103), rearranging for da'

$$da' = -(1+z')^{-2}dz'. \tag{97}$$

Substitute Eqn. (93), Eqn. (95) and Eqn. (97) into Eqn. (94) and change limits,

$$t = \int_{z(0)}^{z(a)} \frac{-(1+z')^{-2}dz'}{H_0\big((1+z')\Omega_{m,0} + \Omega_{\Lambda,0}(1+z')^{-2}\big)^{\frac{1}{2}}}, \tag{98}$$

$$t = -H_0^{-1} \int_{z(0)}^{z(a)} \frac{(1+z')^{-2}dz'}{(1+z')^{\frac{1}{2}}\sqrt{\Omega_{m,0}}\Big(1 + (1+z')^{-3}\big(\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}\big)\Big)^{\frac{1}{2}}} \tag{99}$$

$$t = -H_0^{-1} \int_{z(0)}^{z(a)} \frac{(1+z')^{-\frac{5}{2}}dz'}{\sqrt{\Omega_{m,0}}\Big(1 + (1+z')^{-3}\big(\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}\big)\Big)^{\frac{1}{2}}}. \tag{100}$$

Apply a u-substitution, letting

$$u'^2 = (1+z')^{-3}\Big(\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}\Big). \tag{101}$$

Take derivatives of both sides of Eqn. (101),

$$2u'du' = -3(1+z')^{-4}\Big(\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}\Big)dz', \tag{102}$$

rearranging for dz',

$$dz' = -\frac{2}{3}u'(1+z')^4\Big(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\Big)du'. \tag{103}$$

Rearrange Eqn. (101) for $1+z'$ and $(1+z')^4$,

$$(1+z')^3 = u'^{-2}\Big(\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}\Big), \tag{104}$$

$$1+z' = u'^{-\frac{2}{3}}\Big(\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}\Big)^{\frac{1}{3}}, \tag{105}$$

$$(1+z')^4 = u'^{-\frac{8}{3}}\Big(\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}\Big)^{\frac{4}{3}}. \tag{106}$$

Substitute Eqn. (106) into Eqn. (103),

$$dz' = -\frac{2}{3}u'u'^{-\frac{8}{3}}\Big(\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}\Big)^{\frac{4}{3}}\Big(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\Big)du', \tag{107}$$

$$dz' = -\frac{2}{3}u'^{-\frac{5}{3}}\Big(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\Big)^{-\frac{1}{3}}du'. \tag{108}$$

Obtain $(1+z')^{-\frac{5}{2}}$ in terms of u' from Eqn. (105),

$$(1+z')^{-\frac{5}{2}} = u^{\frac{5}{3}}\Big(\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}\Big)^{-\frac{5}{6}}. \tag{109}$$

Substitute Eqn. (108) and Eqn. (109) into Eqn. (100), changing limits in accordance,

$$t = -H_0^{-1}\int_{u(z(0))}^{u(z(a))} u'^{\frac{5}{3}}\Big(\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}\Big)^{-\frac{5}{6}} \times \frac{1}{\sqrt{\Omega_{m,0}}(1+u'^2)^{\frac{1}{2}}} \times -\frac{2}{3}u'^{-\frac{5}{3}}\Big(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\Big)^{-\frac{1}{3}}du', \tag{110}$$

$$t = \frac{2}{3}H_0^{-1}\int_{u(z(0))}^{u(z(a))}\Big(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\Big)\frac{du'}{\sqrt{\Omega_{m,0}}(1+u'^2)^{\frac{1}{2}}}, \tag{111}$$

$$t = \frac{2}{3}\frac{H_0^{-1}}{\sqrt{\Omega_{\Lambda,0}}}\int_{u(z(o))}^{u(z(a))}\frac{du'}{(1+u'^2)^{\frac{1}{2}}}. \tag{112}$$

Find the limits using Eqn. (101) and Eqn. (109),

$$u = (1+z)^{-\frac{3}{2}}\left(\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}\right)^{\frac{1}{2}}. \tag{113}$$

Keep u(z(a)) in terms of z but $z(0) = 1/0-1 \rightarrow \infty$, substituting these values into Eqn. (113) for the respective upper and lower limits,

$$u = (1+z)^{-\frac{3}{2}}\Big(\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}\Big)^{\frac{1}{2}}; u = 0. \tag{114}$$

Substitute the limits in Eqn. (112),

$$t = \frac{2}{3}\frac{H_0^{-1}}{\sqrt{\Omega_{\Lambda,0}}}\int_0^{(1+z)^{-\frac{3}{2}}\left(\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}\right)^{\frac{1}{2}}}\frac{du'}{(1+u'^2)^{\frac{1}{2}}}. \tag{115}$$

Use the same standard integral for Eqn. (115),

$$t = \frac{2}{3}\frac{H_0^{-1}}{\sqrt{\Omega_{\Lambda,0}}}\Big[\ln\Big(\sqrt{u'^2+1}+u'\Big)\Big]_0^{(1+z)^{-\frac{3}{2}}\left(\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}\right)^{\frac{1}{2}}}, \tag{116}$$

$$t = \frac{2}{3}\frac{H_0^{-1}}{\sqrt{\Omega_{\Lambda,0}}} \ln\left[\frac{\sqrt{(1+z)^{-3}\left(\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}\right)+1} + \sqrt{(1+z)^{-3}\left(\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}\right)}}{1}\right], \tag{117}$$

$$t = \frac{2}{3}\frac{H_0^{-1}}{\sqrt{\Omega_{\Lambda,0}}} \ln\left[\sqrt{\frac{\Omega_{\Lambda,0}+(1+z)^3\Omega_{m,0}}{(1+z)^3\Omega_{m,0}}} + \sqrt{\frac{\Omega_{\Lambda,0}}{(1+z}^3\Omega_{m,0}}\right], \tag{118}$$

$$t = \frac{2}{3}\frac{H_0^{-1}}{\sqrt{\Omega_{\Lambda,0}}} \ln\left[\sqrt{\frac{\Omega_{\Lambda,0}+(1+z)^3(1-\Omega_{\Lambda,0})}{(1+z)^3(1-\Omega_{\Lambda,0})}} + \sqrt{\frac{\Omega_{\Lambda,0}}{(1+z)^3(1-\Omega_{\Lambda,0})}}\right]. \tag{119}$$

Check at $z = 0$, i.e. at present where $a = a_0 = 1$, if Eqn. (119) reduces to the formula in Eqn. (2),

$$t(z=0) = \frac{2}{3}\frac{H_0^{-1}}{\sqrt{\Omega_{\Lambda,0}}} \ln\left[\sqrt{\frac{\Omega_{\Lambda,0}+1-\Omega_{\Lambda,0}}{1-\Omega_{\Lambda,0}}} + \sqrt{\frac{\Omega_{\Lambda,0}}{1-\Omega_{\Lambda,0}}}\right], \tag{120}$$

$$t_0 = \frac{2}{3}\frac{H_0^{-1}}{\sqrt{\Omega_{\Lambda,0}}} \ln\left[\frac{1}{\sqrt{1-\Omega_{\Lambda,0}}} + \frac{\sqrt{\Omega_{\Lambda,0}}}{\sqrt{1-\Omega_{\Lambda,0}}}\right], \tag{121}$$

$$t_0 = \frac{2}{3}\frac{H_0^{-1}}{\sqrt{\Omega_{\Lambda,0}}} \ln\left(\frac{1+\Omega_{\Lambda,0}}{\sqrt{1-\Omega_{\Lambda,0}}}\right), \tag{122}$$

as required. This proof confirms Eqn. (119) is correct.

Now, starting from Eqn. (62), we derive an equation of the same form as Eqn. (71) but for the case where ρ_Λ is replaced by $\rho_X(a)$, where the a dependence of $\rho_X(a)$ is determined by w_X. From Eqn. (62),

$$\frac{da}{a(a^{-3}\rho_{m,0}+\rho_{X,0})^{\frac{1}{2}}} = \left(\frac{8\pi G}{3}\right)^{\frac{1}{2}} dt. \tag{123}$$

Then define

$$\rho_\Lambda = \rho_X = \rho_{X,0}a^{-3(1+w_X)} \tag{124}$$

and

$$\Omega_\Lambda = \Omega_{X,0}a^{-3(1+w_X)} \times \frac{3H_0^2}{8\pi G} \tag{125}$$

Subbing these expressions into Eq. (123):

$$\frac{da}{a(\frac{3}{8\pi G})^{\frac{1}{2}}H_0(a^{-3}\Omega_{m,0}+a^{-3(1+w_X)}\Omega_{X,0})^{\frac{1}{2}}} = \left(\frac{8\pi G}{3}\right)^{\frac{1}{2}} dt, \tag{126}$$

Rearranging:

$$\frac{da}{H_0(\frac{3}{8\pi G})^{\frac{1}{2}}(a^{-1}\Omega_{m,0}+a^{-(1+3w_X)}\Omega_{X,0})^{\frac{1}{2}}} = \left(\frac{8\pi G}{3}\right)^{\frac{1}{2}} dt. \tag{127}$$

Taking integrals then gives:

$$\int_0^{a_0} \frac{da'}{H_0(\frac{3}{8\pi G})^{\frac{1}{2}}(a'^{-1}\Omega_{m,0}+a'^{-(1+3w_X)}\Omega_{X,0})^{\frac{1}{2}}} = \int_0^t \left(\frac{8\pi G}{3}\right)^{\frac{1}{2}} dt, \tag{128}$$

Simplifying and integrating the RHS:

$$\int_0^{a_0} \frac{da'}{(a'^{-1}\Omega_{m,0}+a'^{-(1+3w_X)}\Omega_{X,0})^{\frac{1}{2}}} = H_0 t_0, \tag{129}$$

and finally:

$$\int_0^{a} \frac{da'}{(a'^{-1}\Omega_{m,0})^{\frac{1}{2}}(1+a'^{-3w_X}\frac{\Omega_{X,0}}{\Omega_{m,0}})^{\frac{1}{2}}} = H_0 t_0, \tag{130}$$

A.2 Code

All code is in the language: Python.

A.2.1. Theoretical age t_0 as a function of $\Omega_{\Lambda,0}$

```
#importing helpful libraries
import numpy as np
import matplotlib
from matplotlib import pyplot as plt
import matplotlib.font_manager
from IPython.core.display import HTML
from matplotlib import patheffects

#gets the font from the font manager.
def make_html(fontname):
    return "<p>{font}: <span style='font-family:{font}; \
    font-size: 24px;'>{font}</p>".format(font=fontname)
code = "\n".join([make_html(font) for font in sorted(
    set([f.name for f in matplotlib.font_manager.fontManager.ttflist]))])
HTML("<div style='column-count: 2;'>{}</div>".format(code))

#calculating H in per Gyr
H = 73.48
H = 1000*H/ (1*10**6)
H = H/(3.086*10**(16))
H = H * 3600 * 24 * 365.25 * (10**9)

#plot formatting
plt.rcParams["figure.figsize"] = [8.6, 6]
plt.rcParams["figure.autolayout"] = True

#calculating the age of the universe
def f(x):
    return (2/(3 *H* (x)**(1/2)))*((np.log((1+(x)**(1/2))/((1-x)**(1/2)))))

#range of y values
omegay = [8,28]
#lists for plotting the 1 and 2 sigma ranges for formatting purposes
omegax1 = [0.661,0.661]
omegax2 = [0.697,0.697]
omegax3 = [0.757,0.757]
omegax4 = [0.782,0.782]
sig1x = [0,1]
sig2x = [0.661,0.697]
sig3x = [0.697,0.757]
sig4x = [0.757, 0.782]
sig1y1 = [13.6,13.6]
sig1y2 = [12.8,12.8]
sig2y1 = [14,14]
sig2y2 = [12.4,12.4]
sig3y3 = [8,8]
sig4y4 = [28,28]
```

```
#array of x values
x=np.linspace(0,1,100)

#plot formatting
fig = plt.figure()
ax = fig.add_subplot(111, autoscale_on=False, xlim=(0, 1), ylim=(8, 28))
custom_font = “Times New Roman”
ax.set_xlabel(r“$Ω_{Λ,0}$”, fontname=custom_font, size=14)
ax.set_ylabel(r“$t_{0}$ [Gyrs]”, fontname=custom_font, size=14)
ax.set_xticks([0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1])
ax.set_yticks([8,10,12,14,16,18,20,22,24,26,28])
for tick in ax.set_xticklabels([0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1]):
    tick.set_fontname(custom_font)
for tick in ax.set_yticklabels([8,10,12,14,16,18,20,22,24,26,28]):
    tick.set_fontname(custom_font)
plt.xticks(fontsize=14)
plt.yticks(fontsize=14)

#graph plotting
plt.plot(x,f(x),color=“black”)
ax.plot(sig1x,sig1y1,color=“green”, linestyle =“dashed”, linewidth=0.7, label=r“1-σ”)
plt.plot(sig1x,sig1y2,color=“green”, linestyle =“dashed”, linewidth=0.7)
ax.plot(sig1x,sig2y1,color=“red”, linestyle =“dashed”, linewidth=0.7, label=r“2-σ”)
plt.plot(sig1x,sig2y2,color=“red”, linestyle =“dashed”, linewidth=0.7)
plt.plot(omegax1,omegay, color=“red”, linestyle=“dashed”, linewidth=0.7)
plt.plot(omegax2,omegay, color=“green”, linestyle=“dashed”, linewidth=0.7)
plt.plot(omegax3,omegay, color=“green”, linestyle=“dashed”, linewidth=0.7)
plt.plot(omegax4,omegay, color=“red”, linestyle=“dashed”, linewidth=0.7)
ax.legend(prop={‘family’:custom_font, ‘size’:14}, loc=0,frameon=False)
plt.fill_between(sig1x,sig1y2,sig1y1, color=‘green’, alpha=.1)
plt.fill_between(sig1x,sig2y2,sig1y2, color=‘red’, alpha=.1)
plt.fill_between(sig1x,sig1y1,sig2y1, color=‘red’, alpha=.1)
plt.fill_between(sig2x, sig3y3, sig2y2, color=‘red’, alpha=.1)
plt.fill_between(sig2x, sig2y1, sig4y4, color=‘red’, alpha=.1)
plt.fill_between(sig3x, sig3y3, sig2y2, color=‘green’, alpha=.1)
plt.fill_between(sig3x, sig2y1, sig4y4, color=‘green’, alpha=.1)
plt.fill_between(sig4x, sig3y3, sig2y2, color=‘red’, alpha=.1)
plt.fill_between(sig4x, sig2y1, sig4y4, color=‘red’, alpha=.1)
plt.plot
plt.show()
```

A.2.2. The age of the Universe t as a function of redshift z

```
#Importing various helpful libraries
import numpy as np
import matplotlib
from matplotlib import pyplot as plt
plt.rcParams[“scatter.marker”] = “x”
import matplotlib.font_manager
from IPython.core.display import HTML
from matplotlib import patheffects
```

```
#gets the font from the font manager
def make_html(fontname):
    return “<p>{font}: <span style=‘font-family:{font}; \
    font-size: 24px;’>{font}</p>”.format(font=fontname)
code = “\n”.join([make_html(font) for font in sorted(
    set([f.name for f in matplotlib.font_manager.fontManager.ttflist]))])
HTML(“<div style=‘column-count: 2;’>{}</div>”.format(code))

#Calculating H in per Gyr
H = 73.48
H = 1000*H/ (1*10**6)
H = H/(3.086*10**(16))
H = H * 3600 * 24 * 365.25 * (10**9)

#function to calculate t0
def f(x):
    return (2/(3*H*(0.7**(1/2))))*np.log(((0.7 + (1-0.7)*(1+x)**3)/((1-0.7)*(1+x)**(3)))**(1/2)
    + ((0.7)/((1-0.7)*(1+x)**3))**(1/2))

#creating an array of values for redshift
z=np.linspace(0,10,100)

#formatting the graph
fig = plt.figure()
ax = fig.add_subplot(111, autoscale_on=False, xlim=(0, 10), ylim=(0, 14))
custom_font = “Times New Roman”
ax.set_xlabel(r“z”, fontname=custom_font, size=14)
ax.set_ylabel(r“t [Gyrs]”, fontname=custom_font, size=14)
ax.set_xticks([0,1,2,3,4,5,6,7,8,9,10])
ax.set_yticks([0,2,4,6,8,10,12,14])
for tick in ax.set_xticklabels([0,1,2,3,4,5,6,7,8,9,10]):
    tick.set_fontname(custom_font)
for tick in ax.set_yticklabels([0,2,4,6,8,10,12,14]):
    tick.set_fontname(custom_font)
plt.xticks(fontsize=14)
plt.yticks(fontsize=14)

#plotting the graph
plt.plot(z,f(z),color=“black”)
plt.plot([0,0.447],[8.4,8.4],color=“red”,linestyle=“dotted”)
plt.plot([0.447,0.447],[0,8.4],color=“red”,linestyle=“dotted”)
ax.legend(prop={‘family’:custom_font, ‘size’:14}, loc=0,frameon=False)
plt.text(0.15,-0.5, “0.447”, fontname=custom_font, fontsize = 14)
plt.text(-0.4,8.3, “8.4”, fontname=custom_font, fontsize = 14)
plt.plot
plt.show()
```

A.2.3. The age of the Universe t_0 as a function of $\Omega_{m,0}$ for specific w_X values.

```
#importing helpful libraries
import numpy as np
import matplotlib
```

```
from matplotlib import pyplot as plt
plt.rcParams[“scatter.marker”] = “x”
import matplotlib.font_manager
from IPython.core.display import HTML
from matplotlib import patheffects

#getting the font
def make_html(fontname):
    return “<p>{font}: <span style=‘font-family:{font}; \
    font-size: 24px;’>{font}</p>”.format(font=fontname)
code = “\n”.join([make_html(font) for font in sorted(
    set([f.name for f in matplotlib.font_manager.fontManager.ttflist]))])
HTML(“<div style=‘column-count: 2;>’{}</div>”.format(code))

#defining constants
xmin = 0
xmax = 1
#calculating H in per Gyr
H = 73.48
H = 1000*H/ (1*10**6)
H = H/(3.086*10**(16))
H = H * 3600 * 24 * 365.25 * (10**9)

#calculation to take place in the integral
def functionx(xvalue,omegam,w):
    y = 1/((((omegam*(xvalue)**-1)**(1/2))*(((1+((1-omegam)/omegam)*((xvalue)**(-3*w))))**(1/2))))
    return y

#the integration function
def integrater(omegam,w):
    #starting integration factors
    interval = (xmax+xmin)/10000
    placeholder = xmin+ interval/2
    total = 0
    #loop this many times, calculating the integral
    for x in range (0,10000):
        total = total + (functionx(placeholder,omegam,w))*interval
        placeholder = placeholder+interval
    #return the value of the integration
    return(total/H)

#creation of empty lists to be appended
listOfOmega = []
ListOfT0w1 = []
ListOfT0w2 = []
ListOfT0w3 = []
ListOfT0w4 = []
#starting value for Omega_m
omegam = 0.001
#finding the values of t0 for different w values.
for h in range(0,1000):
```

```
    listOfOmega.append(omegam)
    ListOfT0w1.append(integrater(omegam,-0.3))
    ListOfT0w2.append(integrater(omegam,-0.5))
    ListOfT0w3.append(integrater(omegam,-0.7))
    ListOfT0w4.append(integrater(omegam,-0.9))
    omegam = omegam+0.001

#plot formatting
fig = plt.figure()
ax = fig.add_subplot(111, autoscale_on=False, xlim=(0, 1), ylim=(8, 28))
custom_font = “Times New Roman”
ax.set_xlabel(r“$Ω_{m,0}$”, fontname=custom_font, size=14)
ax.set_ylabel(r“$t_{0}$ [Gyrs]”, fontname=custom_font, size=14)
ax.set_xticks([0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1])
ax.set_yticks([8,10,12,14,16,18,20,22,24,26,28])
for tick in ax.set_xticklabels([0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1]):
    tick.set_fontname(custom_font)
for tick in ax.set_yticklabels([8,10,12,14,16,18,20,22,24,26,28]):
    tick.set_fontname(custom_font)
plt.xticks(fontsize=14)
plt.yticks(fontsize=14)

#plotting the graph
plt.plot(listOfOmega,ListOfT0w1,color=“orange”,label=r“$w_{X}=-0.3$”)
plt.plot(listOfOmega,ListOfT0w2,color=“blue”,label=r“$w_{X}=-0.5$”)
plt.plot(listOfOmega,ListOfT0w3,color=“purple”,label=r“$w_{X}=-0.7$”)
plt.plot(listOfOmega,ListOfT0w4,color=“black”,label=r“$w_{X}=-0.9$”)
plt.xlabel(r“$Ω_{m,0}$”)
plt.ylabel(r“$t_{0}$ [Gyrs]”)
#range of y values
omegay = [8,32]
#lists for the different omega values
t01 = [8,8]
t02 = [32,32]
omegamrange = [0,1]
omegam1 = [0.26,0.26]
omegam2 = [0.28,0.28]
omegam3 = [0.32,0.32]
omegam4 = [0.34,0.34]
#lists for plotting the 1 and 2 sigma ranges for formatting purposes
sig1x = [0.26,0.28]
sig2x = [0.28,0.32]
sig3x = [0.32,0.34]
sig1y = [12.4,12.4]
sig2y = [12.8,12.8]
sig3y = [13.6,13.6]
sig4y = [14,14]
sig1y1 = [12.4,12.4]
sig1y2 = [12.8,12.8]
sig2y1 = [13.6,13.6]
sig2y2 = [14,14]
```

```
#plotting the omegas and ranges
plt.plot(omegam1,omegay,color=“red”, linestyle =“dashed”, linewidth=0.7)
plt.plot(omegam2,omegay,color=“green”, linestyle =“dashed”, linewidth=0.7, label=r“1-σ”)
plt.plot(omegam3,omegay,color=“green”, linestyle =“dashed”, linewidth=0.7)
plt.plot(omegam4,omegay,color=“red”, linestyle =“dashed”, linewidth=0.7, label=r“2-σ”)
plt.plot(omegamrange,sig1y1,color=“red”, linestyle =“dashed”, linewidth=0.7)
plt.plot(omegamrange,sig1y2,color=“green”, linestyle =“dashed”, linewidth=0.7)
plt.plot(omegamrange,sig2y1,color=“green”, linestyle =“dashed”, linewidth=0.7)
plt.plot(omegamrange,sig2y2,color=“red”, linestyle =“dashed”, linewidth=0.7)
ax.legend(prop={‘family’:custom_font, ‘size’:14}, loc=0,frameon=False)
plt.fill_between(sig1x, t01, sig1y1, color=‘red’, alpha=.1)
plt.fill_between(sig1x, sig4y, t02, color=‘red’, alpha=.1)
plt.fill_between(sig2x, t01, sig1y1, color=‘green’, alpha=.1)
plt.fill_between(sig2x, sig4y, t02, color=‘green’, alpha=.1)
plt.fill_between(sig3x, t01, sig1y1, color=‘red’, alpha=.1)
plt.fill_between(sig3x, sig4y, t02, color=‘red’, alpha=.1)
plt.fill_between(omegamrange, sig1y1, sig1y2, color=‘red’, alpha=.1)
plt.fill_between(omegamrange, sig1y2, sig2y1, color=‘green’, alpha=.1)
plt.fill_between(omegamrange, sig2y1, sig2y2, color=‘red’, alpha=.1)
plt.show()
```

A.2.4. The age of the Universe t_0 as a function of $\Omega_{m,0}$ for varied w_X values.

```
#importing helpful libraries
import numpy as np
import matplotlib
from matplotlib import pyplot as plt
plt.rcParams[“scatter.marker”] = “x”
import matplotlib.font_manager
from IPython.core.display import HTML
from matplotlib import patheffects

#getting the font
def make_html(fontname):
    return “<p>{font}: <span style=‘font-family:{font}; \
    font-size: 24px;’>{font}</p>”.format(font=fontname)
code = “\n”.join([make_html(font) for font in sorted(
    set([f.name for f in matplotlib.font_manager.fontManager.ttflist]))])
HTML(“<div style=‘column-count: 2;’>{}</div>”.format(code))

#defining constants
xmin = 0
xmax = 1
#calculating H in per Gyr
H = 73.48
H = 1000*H/ (1*10**6)
H = H/(3.086*10**(16))
H = H * 3600 * 24 * 365.25 * (10**9)

#calculation to take place in the integral
def functionx(xvalue,omegam,w):
    y = 1/((((omegam*(xvalue)**-1)**(1/2))*(((1+((1-omegam)/omegam)*((xvalue)**(-3*w))))**(1/2))))
```

```
    return y

#the integration function
def integrater(omegam,w):
    #starting integration factors
    interval = (xmax+xmin)/10000
    placeholder = xmin+ interval/2
    total = 0
    #loop this many times, calculating the integral
    for x in range (0,10000):
        total = total + (functionx(placeholder,omegam,w))*interval
        placeholder = placeholder+interval
    return the value of the integration
    return(total/H)

#creation of empty lists to be appended
listOfOmega = []
ListOfT0w5 = []
ListOfT0w6 = []
ListOfT0w7 = []
ListOfT0w8 = []
ListOfT0w9 = []
#starting value for Omega_m
omegam = 0.001
#finding the values of t0 for different w values.
for h in range(0,1000):
    listOfOmega.append(omegam)
    ListOfT0w5.append(integrater(omegam,-0.67))
    ListOfT0w6.append(integrater(omegam,-0.88))
    ListOfT0w7.append(integrater(omegam,-1.63))
    ListOfT0w8.append(integrater(omegam,-2.73))
    ListOfT0w9.append(integrater(omegam,-2.5))
    omegam = omegam+0.001

#plot formatting
fig = plt.figure()
ax = fig.add_subplot(111, autoscale_on=False, xlim=(0, 1), ylim=(8, 28))
custom_font = “Times New Roman”
ax.set_xlabel(r“$Ω_{m,0}$”, fontname=custom_font, size=14)
ax.set_ylabel(r“$t_{0}$ [Gyrs]”, fontname=custom_font, size=14)
ax.set_xticks([0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1])
ax.set_yticks([8,10,12,14,16,18,20,22,24,26,28])
for tick in
ax.set_xticklabels([0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1]):
    tick.set_fontname(custom_font)
for tick in ax.set_yticklabels([8,10,12,14,16,18,20,22,24,26,28]):
    tick.set_fontname(custom_font)
plt.xticks(fontsize=14)
plt.yticks(fontsize=14)

#plotting the graph
```

```
plt.plot(listOfOmega,ListOfT0w5,color="orange",label=r"$w_{X}=-0.67$")
plt.plot(listOfOmega,ListOfT0w6,color="blue",label=r"$w_{X}=-0.88$")
plt.plot(listOfOmega,ListOfT0w7,color="purple",label=r"$w_{X}=-1.63$")
plt.plot(listOfOmega,ListOfT0w8,color="black",label=r"$w_{X}=-2.73$")
plt.plot(listOfOmega,ListOfT0w9,color="blue",label=r"$w_{X}=-1.4$")
plt.xlabel(r"$Ω_{m,0}$")
plt.ylabel(r"$t_{0}$ [Gyrs]")
#range of y values
omegay = [8,32]
#lists for the different omega values
t01 = [8,8]
t02 = [32,32]
omegamrange = [0,1]
omegam1 = [0.26,0.26]
omegam2 = [0.28,0.28]
omegam3 = [0.32,0.32]
omegam4 = [0.34,0.34]
#lists for plotting the 1 and 2 sigma ranges for formatting purposes
sig1x = [0.26,0.28]
sig2x = [0.28,0.32]
sig3x = [0.32,0.34]
sig1y = [12.4,12.4]
sig2y = [12.8,12.8]
sig3y = [13.6,13.6]
sig4y = [14,14]
sig1y1 = [12.4,12.4]
sig1y2 = [12.8,12.8]
sig2y1 = [13.6,13.6]
sig2y2 = [14,14]
#plotting the omegas and ranges
plt.plot(omegam1,omegay,color="red", linestyle ="dashed", linewidth=0.7)
plt.plot(omegam2,omegay,color="green", linestyle ="dashed", linewidth=0.7, label=r"1-σ")
plt.plot(omegam3,omegay,color="green", linestyle ="dashed", linewidth=0.7)
plt.plot(omegam4,omegay,color="red", linestyle ="dashed", linewidth=0.7, label=r"2-σ")
plt.plot(omegamrange,sig1y1,color="red", linestyle ="dashed", linewidth=0.7)
plt.plot(omegamrange,sig1y2,color="green", linestyle ="dashed", linewidth=0.7)
plt.plot(omegamrange,sig2y1,color="green", linestyle ="dashed", linewidth=0.7)
plt.plot(omegamrange,sig2y2,color="red", linestyle ="dashed", linewidth=0.7)
ax.legend(prop={'family':custom_font, 'size':14}, loc=0,frameon=False)
plt.fill_between(sig1x, t01, sig1y1, color='red', alpha=.1)
plt.fill_between(sig1x, sig4y, t02, color='red', alpha=.1)
plt.fill_between(sig2x, t01, sig1y1, color='green', alpha=.1)
plt.fill_between(sig2x, sig4y, t02, color='green', alpha=.1)
plt.fill_between(sig3x, t01, sig1y1, color='red', alpha=.1)
plt.fill_between(sig3x, sig4y, t02, color='red', alpha=.1)
plt.fill_between(omegamrange, sig1y1, sig1y2, color='red', alpha=.1)
plt.fill_between(omegamrange, sig1y2, sig2y1, color='green', alpha=.1)
plt.fill_between(omegamrange, sig2y1, sig2y2, color='red', alpha=.1)
plt.show()
```

A.2.5. Code snippet to find the 1-σ and 2-σ ranges for age as a function of $\Omega_{\Lambda,0}$.

```
f = 0.6
for i in range (1,2000):
    y = (2/(3 *H* (f)**(1/2)))*((np.log((1+(f)**(1/2))/((1-f)**(1/2)))))
    print("x="+str(f)+" y="+str(y))
    f = f + 0.0001
```

A.2.6. Code snippet to find the redshift at cosmological constant domination.

```
f = 0.35
for i in range (1,1000):
    y = (2/(3*H*(0.7**(1/2))))*np.log(((0.7 + (1-0.7)*(1+f)**3)/((1-0.7)*(1+f)**(3)))**(1/2)
    + ((0.7)/((1-0.7)*(1+f)**3))**(1/2))
    print("x="+str(f)+" y="+str(y))
    f = f + 0.0001
```

B Future space telescope determination of the nature of dark energy

B.1 Derivations

The total distance to an object observed at the present time t_0 with redshift can be calculated by considering a photon being emitted by the object, at an initial time t_i and integrating it with respect to time until it reaches the observer at t_0.

The distance the photon travels, to reach the observer, without taking the expansion of the universe can be shown as

$$d(t)_0 = c\int_{t_i}^{t_0} dt'. \tag{131}$$

Taking redshift into account, the distance travelled is

$$d(t) = (1+z)d(t)_0. \tag{132}$$

The relation between redshift and the scale factor can be given by

$$1+z = \frac{\lambda_0}{\lambda} = \frac{a(t_0)}{a(t)}, \tag{133}$$

hence, it can be shown that

$$d(t) = c\int_{t_i}^{t_0} \frac{a(t_0)}{a(t')}dt'. \tag{134}$$

The scale factor, as a function of redshift, where $z = z(t')$ and $a(t_0) = a_0$, can be shown as

$$a(z) = \frac{a_0}{1+z} = \frac{1}{1+z}. \tag{135}$$

Given that the derivative of a with respect to z is

$$\frac{da'}{dz} = \frac{-1}{(1+z)^2} = -a^2, \tag{136}$$

the time derivative of the scale factor can be expressed as

$$\frac{da'}{dt'} = \frac{da'}{dz} \times \frac{dz}{dt'} = \frac{-1}{(1+z)^2}\frac{dz}{dt'} = -a^2\frac{dz}{dt'}. \tag{137}$$

Then, the Hubble parameter would be:

$$H = \frac{\dot{a}}{a} = -a\frac{dz}{dt'}. \tag{138}$$

Rearrange Eqn. (136) for dz, substitute it into the Hubble parameter,

$$H = \frac{1}{a'}\frac{da'}{dt'}, \tag{139}$$

rearrange for dt':

$$dt' = \frac{1}{Ha'}da', \tag{140}$$

and substitute it into the Eqn. (134) to give

$$d(z) = a_0 \int_{a(z)}^{a_0} \frac{c}{Ha'^2} da' \tag{141}$$

The continuity equation for dark energy is given by

$$\frac{d\rho_X}{dt} + 3H(\rho_X + \frac{P_X}{c^2}), \tag{142}$$

where the pressure, P_X is given by

$$P_X = w_X \rho_X c^2. \tag{143}$$

Hence,

$$\dot{\rho}_X + 3H(1 + w_X)\rho_X = 0. \tag{144}$$

Substitute Eqn. (138) to get

$$\dot{\rho}_X = -3(1 + w_X)\left(\frac{\dot{a}}{a}\right)\rho_X. \tag{145}$$

Cancel the time derivative on both sides,

$$d\rho_X = -3(1 + w_X)\left(\frac{1}{a''}\right)\rho_X da'', \tag{146}$$

Bring like terms to each side,

$$\int_0^{a'} \frac{1}{\rho_X} d\rho_X = \int_{a_0}^{a'} -3(1 + w_X)\left(\frac{1}{a''}\right) da'', \tag{147}$$

and integrate to get:

$$\frac{\ln \rho_x(a')}{\ln \rho_{x,0}} = \int_{a_0}^{a'} \frac{-3(1 + w_x(a''))}{a''} da''. \tag{148}$$

The theoretical $d(z)$ and θ_{BAO} for the CPL parameterization from Eqn. (14) was determined using the integral from Eqn. (148) where $a_p = a_0$ so there is no loss of information, substituting the latter for the former,

$$\ln\left(\frac{\rho_X(a')}{\rho_{X,0}}\right) = -\int_{a_0}^{a'} \frac{3\left(1 + w_p + \left(\frac{a_p}{a_0} - \frac{a''}{a_0}\right)w_a\right)}{a''} da'', \tag{149}$$

where a'' denotes another integration variable for the scale factor. Simplifying further,

$$\ln\left(\frac{\rho_X(a')}{\rho_{X,0}}\right) = -\int_{a_0}^{a'} \frac{3\left(1 + w_p + \frac{w_a a_p}{a_0} - \frac{a'' w_a}{a_0}\right)}{a''} da'', \tag{150}$$

$$\ln\left(\frac{\rho_X(a')}{\rho_{X,0}}\right) = -\int_{a_0}^{a'}\left(\frac{3\big(1+w_p+\frac{w_0 a_p}{a_0}\big)}{a''} - \frac{3w_a}{a_0}\right)da'', \tag{151}$$

$$\ln\left(\frac{\rho_X(a')}{\rho_{X,0}}\right) = -\left[3\left(1+w_p+\frac{w_a a_p}{a_0}\right)\ln(a'') - \frac{3w_a}{a_0}a''\right]_{a_0}^{a'}, \tag{152}$$

$$\ln\left(\frac{\rho_X(a')}{\rho_{X,0}}\right) = -\left[\ln\left(a''^{3\left(1+w_p+\frac{w_a a_p}{a_0}\right)}\right) - \frac{3w_a}{a_0}a''\right]_{a_0}^{a'}, \tag{153}$$

$$\ln\left(\frac{\rho_X(a')}{\rho_{X,0}}\right) = -\left[\ln\left(\Big(\frac{a'}{a_0}\Big)^{3\left(1+w_p+\frac{w_a a_p}{a_0}\right)}\right) - \frac{3w_a}{a_0}(a'-a_0)\right], \tag{154}$$

$$\ln\left(\frac{\rho_X(a')}{\rho_{X,0}}\right) = \ln\left(\Big(\frac{a'}{a_0}\Big)^{-3\left(1+w_p+\frac{w_a a_p}{a_0}\right)}\right) + \frac{3w_a}{a_0}(a'-a_0), \tag{155}$$

$$\frac{\rho_X(a')}{\rho_{X,0}} = \left(\frac{a'}{a_0}\right)^{-3\left(1+w_p+\frac{w_a a_p}{a_0}\right)}\exp\left(\frac{3w_a}{a_0}(a'-a_0)\right), \tag{156}$$

$$\rho_X(a') = \rho_{X,0}\left(\frac{a'}{a_0}\right)^{-3\left(1+w_p+\frac{w_a a_p}{a_0}\right)}\exp\left(\frac{3w_a}{a_0}(a'-a_0))\right). \tag{157}$$

Substitute $a_p = a_0$ into Eqn. (157), obtaining

$$\rho_X(a') = \rho_{X,0}\left(\frac{a_0}{a'}\right)^{3(1+w_p+w_a)}\exp\left(3w_a\Big(\frac{a'}{a_0}-1\Big)\right). \tag{158}$$

This is the function for $\rho_X(w_p,\, w_a,\, \rho_{X,0},\, a_0,\, a')$ and gets substituted into Eqn. (141) where,

$$H(a') = \Big(\frac{8\pi G}{3}\Big)^{\frac{1}{2}}(\rho_m+\rho_X)^{\frac{1}{2}}. \tag{159}$$

Substitute Eqn. (159) into Eqn. (141),

$$d(z) = a_0 c\int_{a(z)}^{a_0}\frac{da'}{\big(\frac{8\pi G}{3}\big)^{\frac{1}{2}}(\rho_m+\rho_X)^{\frac{1}{2}}a'^2}. \tag{160}$$

Substitute Eqn. (158) into Eqn. (160),

$$d(z) = a_o c\int_{a(z)}^{a_0}\frac{da'}{\big(\frac{8\pi G}{3}\big)^{\frac{1}{2}}\left(\big(\frac{a_0}{a'}\big)^3\rho_{m,0} + \rho_{X,0}\big(\frac{a_0}{a'}\big)^{3(1+w_p+w_a)}\exp\Big(3w_a\big(\frac{a'}{a_0}-1\big)\Big)\right)^{\frac{1}{2}}a'^2}. \tag{161}$$

Simplifying,

$$d(z) = a_o c\int_{a(z)}^{a_0}\frac{da'}{H_0\left(\big(\frac{a_0}{a'}\big)^3\Omega_{m,0} + \Omega_{X,0}\big(\frac{a_0}{a'}\big)^{3(1+w_p+w_a)}\exp\Big(3w_a\big(\frac{a'}{a_0}-1\big)\Big)\right)^{\frac{1}{2}}a'^2}, \tag{162}$$

$$d(z) = a_o c\int_{a(z)}^{a_0}\frac{da'}{H_0\big(\frac{a_0}{a'}\big)^{\frac{3}{2}}\left(\Omega_{m,0} + \Omega_{X,0}\big(\frac{a_0}{a'}\big)^{3(w_p+w_a)}\exp\Big(3w_a\big(\frac{a'}{a_0}-1\big)\Big)\right)^{\frac{1}{2}}a'^2}, \tag{163}$$

$$d(z) = \frac{c}{a_0^{\frac{1}{2}}} \frac{1}{H_0} \int_{a(z)}^{a_0} \frac{da'}{a^{\frac{1}{2}} \left(\Omega_{m,0} + \Omega_{X,0} \left(\frac{a_0}{a'}\right)^{3(w_p+w_a)} \exp\left(3w_a\left(\frac{a'}{a_0} - 1\right)\right) \right)^{\frac{1}{2}}}, \tag{164}$$

to the form of $d(z)$ used for the χ^2 test of BAO angles.

B.2 Analytical approach derivation

To start, Eqn. (148) is used, a_0 is set to 1 and for ease of notation, $\omega_x(a'')$ is set to ω_x. Rearranging this equation gives,

$$\rho_x(a') = \rho_{x,0} e^{\int_1^{a'} \frac{-3(1+\omega_x)}{a''} da''} \tag{165}$$

Taking the negative from the integral, the bounds get flipped,

$$\rho_x(a') = \rho_{x,0} e^{\int_{a'}^1 \frac{3(1+\omega_x)}{a''} da''} \tag{166}$$

Completing the integral,

$$\int_{a'}^1 \frac{3(1+\omega_x)}{a''} da'' = [3(1+\omega_x) \ln a'']_{a'}^1 \tag{167}$$

$$\int_{a'}^1 \frac{3(1+\omega_x)}{a''} da'' = -3(1+\omega_x) \ln a' \tag{168}$$

$$\int_{a'}^1 \frac{3(1+\omega_x)}{a''} da'' = \ln a'^{-3(1+\omega_x)} \tag{169}$$

Substituting this back into Eqn. (166),

$$\rho_x(a') = \rho_{x,0} e^{\ln a'^{-3(1+\omega_x)}} \tag{170}$$

$$\rho_x(a') = \rho_{x,0} a^{-3(1+\omega_x)} \tag{171}$$

Stopping here, Eqn. (60) is used where $\rho_m = 0$ and $\rho_m = \rho_x$ becoming,

$$H = \sqrt{\frac{8\pi G}{3} \rho_x(a')} \tag{172}$$

This equation can then be substituted into Eqn. (141) becoming,

$$d(z) = \int_a^1 \frac{c}{\sqrt{\frac{8\pi G \rho_x(a')}{3}}} \frac{1}{a'^2} da' \tag{173}$$

Taking the constants outside the integral gives,

$$d(z) = c \left(\frac{3}{8\pi G \rho_{x,0}} \right)^{\frac{1}{2}} \int_a^1 \frac{1}{a'^2} \frac{1}{(a'^{-3(1+\omega_x)})^{\frac{1}{2}}} da' \tag{174}$$

$$d(z) = c \left(\frac{3}{8\pi G \rho_{x,0}} \right)^{\frac{1}{2}} \int_a^1 \frac{1}{a'^{2-\frac{3}{2}(1+\omega_x)}} da' \tag{175}$$

Completing the integral gives,

$$d(z) = c \left(\frac{3}{8\pi G \rho_{x,0}} \right)^{\frac{1}{2}} \frac{-2(a^{\frac{3\omega_x}{2}} a^{\frac{-1}{2}} - 1)}{3\omega_x + 1} \tag{176}$$

Setting $\omega_x = -1$ gives,

$$d(z) = c\left(\frac{3}{8\pi G\rho_{x,0}}\right)^{\frac{1}{2}} \frac{-2(a^{\frac{-3}{2}}a^{\frac{-1}{2}} - 1)}{-3+1} \tag{177}$$

$$d(z) = c\left(\frac{3}{8\pi G\rho_{x,0}}\right)^{\frac{1}{2}} \left(\frac{1-a}{a}\right) \tag{178}$$

The rearranged equation for critical density is,

$$H_0 = \sqrt{\frac{8\pi G\rho_{x,0}}{3}} \tag{179}$$

Substituting this into Eqn. (178) becomes,

$$d(z) = \frac{c}{H_0}\left(\frac{1-a}{a}\right) \tag{180}$$

Substituting Eqn. (93) into this gives,

$$d(z) = \frac{cz}{H_0} \tag{181}$$

Then converting to luminosity distance becomes,

$$d_L(z) = (1+z)\frac{cz}{H_0} \tag{182}$$

B.3 χ^2 Code

All code is in the language: Python.

B.3.1. Code to complete the χ^2 test.

```
#importing helpful libraries
import math
from scipy import integrate
import matplotlib
from matplotlib import pyplot as plt

#the provided test data
#euclidTestList =[0.000933,0.000835,0.000804,0.000705,0.000674,0.000592,0.000630,0.000594,0.000505,
#0.000479,0.000459,0.000476,0.000463,0.000382,0.000410,0.000376,0.000323,0.000176]

#Gamma Group Unique data
euclidTestList = [0.000469,0.000476,0.000528,0.000496,0.000518,0.000479,0.000552,0.000544,0.000478,
0.000472,0.000466,0.000496,0.000494,0.000421,0.000456,0.000429,0.000380,0.000237]

#integrates function for given z, wa and wp values
def in2grator(z,wa,wp):
    #defines lbao in parsec
    lbao = 147.6*(10**6)
    #defines H and sets conistent units
    H = 67.81
    H = 1000*H/ (10**6)
    H = H*3.2407792896664*10**-17
```

```
    H = H * 3600 * 24 * 365.25 * (10**9)
    #defines c as doccument describes
    c = 2.998 *10**8
    #defines density m and x
    omegam = 0.308
    omegax = 0.692
    #defines a0 as 1, and az as 1/(a+z)
    a0 = 1
    az = 1/(1+z)
    #defines the function dz before the integration
    function = lambda a: (((c/(H *( (a0)**0.5))) * ((math.sqrt(a)) * (omegam + omegax*((a0/a)**
    (3*(wp + wa)))* (math.e)**(3*wa*((a/a0)-1)) )**0.5)**(-1)))
    # using integrate.quad() method for a trapezium numerical integral of the function dz
    #with an absolute error of e-10
    dz = integrate.quad(function, az, a0,epsabs=1e-10)
    #returns the value lbao/dz , yielding the calculated theta
    return(lbao/dz[0])

# defines an empty list for each chi value, wp value and wa value
chivalues = []
wpvalues = []
wavalues = []

#outside loop iterating 100 wa values between 0 and 1 (number of iterations for required 3sf)
for y in range(0,100):
    #prints the value of y/100 to represent progress of program completion, purely cosmetic
    print(y/100)
    #middle loop iterating 1000 wp values between 0 and -1
    #(number of iterations for required 4sf)
    for x in range(0,1000):
        # sets the initial value of z to 0.3, corresponding to the data given
        counterz = 0.3
        #sets corresponding wa and wp values for current itteration
        wpValue = -(x/1000)
        waValue = y/100
        #the next line should be unhashed for test data only as wa is 0 and
        #not varied for this one variable process
        #waValue = 0
        #sets initial chi value as 0
        chivalue = 0
        #iterates loop 18 times to vary z value between 0.3 and 2 by change of 0.2,
        #corresponding with the data
        for bruh in range(0,18):
            #thetaTH defined as difference between calculated data and thetaTHCDM
            thetaTH = ((in2grator(counterz,waValue,wpValue) - in2grator(counterz,0,-1)))
            #chivalue is the sum of the differences between the test data and thetaTH squared,
            #over uncertainty squared
            chivalue = chivalue + ((euclidTestList[bruh]-thetaTH)**2)/((0.00005)**2)
            #incriments the z value up by 0.1
            counterz = counterz + 0.1
        #adds the current chi,wp and wa values to the repective lists
```

```
        chivalues.append(chivalue)
        wpvalues.append(wpValue)
        wavalues.append(waValue)

# defines minChiValue as the minimum value in the list of chi values
minChiValue = min(chivalues)

#locates index of this best chi value in the list
bestIndex = chivalues.index(minChiValue)

#uses this index to return the corresponding chi value, wa value, and wp value
print("bestchi = "+str(chivalues[bestIndex]))
print("bestwp = "+str(wpvalues[bestIndex]))
print("bestwa = "+str(wavalues[bestIndex]))

#these functions are repeated with different conditions to occur, if test data is used, it is one
variable
#and so set v1 (varable 1) test to true
#v1Test = True
v1Test = False

#a function to find every wp, wa pair within one sigma for 1 variable
if v1Test == True:
    oneSigChiWpList = []
    for oneSig in range(0, len(chivalues)):
        if chivalues[oneSig] <(chivalues[bestIndex]+1.01):
            oneSigChiWpList.append(wpvalues[oneSig])
    minOneSig = min(oneSigChiWpList)
    correspondingIndex = oneSigChiWpList.index(minOneSig)
    print("min one sig wp = " + str(oneSigChiWpList[correspondingIndex]))
    maxOneSig = max(oneSigChiWpList)
    correspondingIndex = oneSigChiWpList.index(maxOneSig)
    print("max one sig wp = " +str(oneSigChiWpList[correspondingIndex]))

#a function to find every wp, wa pair within two sigma for 1 variable
if v1Test == True:
    oneSigChiWpList = []
    for oneSig in range(0, len(chivalues)):
        if chivalues[oneSig] <(chivalues[bestIndex]+4.0):
            oneSigChiWpList.append(wpvalues[oneSig])
    minOneSig = min(oneSigChiWpList)
    correspondingIndex = oneSigChiWpList.index(minOneSig)
    print("min two sig wp = " +str(oneSigChiWpList[correspondingIndex]))
    maxOneSig = max(oneSigChiWpList)
    correspondingIndex = oneSigChiWpList.index(maxOneSig)
    print("max two sig wp = " +str(oneSigChiWpList[correspondingIndex]))

if v1Test == True:
    print("chi/v = "+ str(chivalues[bestIndex]/(18-1)))

#if the gamma group data is used, set v2 (variable two) test to true
```

```
v2Test = True
#v2Test = False

#a function to find every wp, wa pair within one sigma for 2 variable
if v2Test == True:
    #creates empty lists for new refined wp and wa values
    oneSigChiWpList = []
    oneSigChiWaList = []
    #repeats loop for how many points in original wa list
    for oneSig in range(0, len(chivalues)):
        #if the corresponding chi value is within the assigned 1 sigma range, adds the wa and
        #wp values to the respective lists
        if chivalues[oneSig] <(chivalues[bestIndex]+2.3):
            oneSigChiWpList.append(wpvalues[oneSig])
            oneSigChiWaList.append(wavalues[oneSig])
    #finds the minimum wp value in this list and corresponding index
    minOneSig = min(oneSigChiWpList)
    correspondingIndex = oneSigChiWpList.index(minOneSig)
    #outputs the minimum wa and wp value
    print("min one sig wp = " + str(oneSigChiWpList[correspondingIndex]))
    print("min one sig wa = " + str(oneSigChiWaList[correspondingIndex]))
    #finds the maximum wp value in this list and corresponding index
    maxOneSig = max(oneSigChiWpList)
    correspondingIndex = oneSigChiWpList.index(maxOneSig)
    #outputs the minimum wa and wp value
    print("max one sig wp = " +str(oneSigChiWpList[correspondingIndex]))
    print("min one sig wa = " + str(oneSigChiWaList[correspondingIndex]))

#assigns the refined range lists respectively
semifinalonesigwa =oneSigChiWaList
semifinalonesigwp =oneSigChiWpList

#a function to find every wp, wa pair within two sigma for 2 variable
if v2Test == True:
    #creates empty lists for new refined wp and wa values
    oneSigChiWpList = []
    oneSigChiWaList = []
    #repeats loop for how many points in original wa list
    for oneSig in range(0, len(chivalues)):
        #if the corresponding chi value is within the assigned 2 sigma range, adds the wa and
        #wp values to the respective lists
        if chivalues[oneSig] <(chivalues[bestIndex]+6.17):
            oneSigChiWpList.append(wpvalues[oneSig])
            oneSigChiWaList.append(wavalues[oneSig])
    #finds the minimum wp value in this list and corresponding index
    minOneSig = min(oneSigChiWpList)
    correspondingIndex = oneSigChiWpList.index(minOneSig)
    #outputs the minimum wa and wp value
    print("min two sig wp = " +str(oneSigChiWpList[correspondingIndex]))
    print("min two sig wa = " +str(oneSigChiWaList[correspondingIndex]))
    #finds the maximum wp value in this list and corresponding index
```

```
    maxOneSig = max(oneSigChiWpList)
    correspondingIndex = oneSigChiWpList.index(maxOneSig)
    #outputs the minimum wa and wp value
    print("max two sig wp = " +str(oneSigChiWpList[correspondingIndex]))
    print("max two sig wa = " +str(oneSigChiWaList[correspondingIndex]))

#prints the chi/v value for two variable
if v2Test == True:
    print("chi/v = "+ str(chivalues[bestIndex]/(18-2)))

#assigns the refined range lists respectively
semifinaltwosigwa =oneSigChiWaList
semifinaltwosigwp =oneSigChiWpList

#reassigns the lists to new names in order to process in next function
wavalues =semifinalonesigwa
wpvalues = semifinalonesigwp

# finds minimum and maximum values in wa
minwavalue = min(wavalues)
maxwavalue = max(wavalues)

#calculates how many iterations needed for following loop
itnum = round(abs(minwavalue-maxwavalue)*100)+1
wavalcurrent = minwavalue

#creates new lists for final values
newwavalues = []
newwpvalues = []

# loop repeats calculated number of times
for h in range(0,itnum):
    # determines every correpsonding index of wa is in list
    indexs = [i for i, e in enumerate(wavalues) if e == wavalcurrent]
    # makes a new temporary list
    templist = []
    #repeats loop for every found corresponding index
    for x in range(0,len(indexs)):
        templist.append(wpvalues[indexs[x]])
    #calculates min and max wp for a given wa as only want to plot outside points of elipse,
    #appends these to new list
    tempmin = min(templist)
    tempmax = max(templist)
    newwavalues.append(wavalcurrent)
    newwpvalues.append(tempmin)
    newwavalues.append(wavalcurrent)
    newwpvalues.append(tempmax)
    #incriments wa value
    wavalcurrent = round( (wavalcurrent + 0.01)*100)/100

#assigns final wa and wp values
```

```
finalwavaluesonesig = newwavalues
finalwpvaluesonesig = newwpvalues

#assigns the refined range lists respectively
wavalues =semifinaltwosigwa
wpvalues = semifinaltwosigwp

# finds minimum and maximum values in wa
minwavalue = min(wavalues)
maxwavalue = max(wavalues)

#calculates how many iterations needed for following loop
itnum = round(abs(minwavalue-maxwavalue)*100)+1
wavalcurrent = minwavalue

# creates new lists for final values
newwavalues = []
newwpvalues = []

# loop repeats calculated number of times
for h in range(0,itnum):
    # determines every correpsonding index of wa is in list
    indexs = [i for i, e in enumerate(wavalues) if e == wavalcurrent]
    # makes a new temporary list
    templist = []
    #repeats loop for every found corresponding index
    for x in range(0,len(indexs)):
        templist.append(wpvalues[indexs[x]])
    #calculates min and max wp for a given wa as only want to plot outside points of elipse,
    #appends these to new list
    tempmin = min(templist)
    tempmax = max(templist)
    newwavalues.append(wavalcurrent)
    newwpvalues.append(tempmin)
    newwavalues.append(wavalcurrent)
    newwpvalues.append(tempmax)
    #incriments wa value
    wavalcurrent = round( (wavalcurrent + 0.01)*100)/100

#assigns final wa and wp values
finalwavaluestwosig = newwavalues
finalwpvaluestwosig = newwpvalues

#plots wa and wp for 1 and 2 sigma as scatters on the same plot
plt.scatter(finalwavaluesonesig,finalwpvaluesonesig)
plt.scatter(finalwavaluestwosig,finalwpvaluestwosig)
plt.show()
```

B.3.2. Code to plot the results of the χ^2 test.

```
#importing helpful libraries
import math
```

```
from matplotlib import pyplot as plt
import numpy as np
import matplotlib.font_manager
from IPython.core.display import HTML
from matplotlib import patheffects

#getting the font
def make_html(fontname):
    return “<p>{font}: <span style=‘font-family:{font}; \
    font-size: 24px;’>{font}</p>”.format(font=fontname)
code = “\n”.join([make_html(font) for font in sorted(
    set([f.name for f in matplotlib.font_manager.fontManager.ttflist]))])
HTML(“<div style=‘column-count: 2;’>{}</div>”.format(code))

#plot formatting
plt.rcParams[“figure.figsize”] = [8.6, 6]
plt.rcParams[“figure.autolayout”] = True

#lists of values for the 1-sigma and 2-sigma ranges obtained from the chi-squared test
finalwavaluesonesig =[0.193, 0.193, 0.194, 0.194, 0.195, 0.195, 0.196, 0.196, 0.197, 0.197, 0.198,
0.198, 0.199, 0.199, 0.2, 0.2, 0.201, 0.201, 0.202, 0.202, 0.203, 0.203, 0.204, 0.204, 0.205, 0.205,
0.206, 0.206, 0.207, 0.207, 0.208, 0.208, 0.209, 0.209, 0.21, 0.21, 0.211, 0.211, 0.212, 0.212, 0.213,
0.213, 0.214, 0.214, 0.215, 0.215, 0.216, 0.216, 0.217, 0.217, 0.218, 0.218, 0.219, 0.219, 0.22, 0.22,
0.221, 0.221, 0.222, 0.222, 0.223, 0.223, 0.224, 0.224, 0.225, 0.225, 0.226, 0.226, 0.227, 0.227,
0.228, 0.228, 0.229, 0.229, 0.23, 0.23, 0.231, 0.231, 0.232, 0.232, 0.233, 0.233, 0.234, 0.234, 0.235,
0.235, 0.236, 0.236, 0.237, 0.237, 0.238, 0.238, 0.239, 0.239, 0.24, 0.24, 0.241, 0.241, 0.242, 0.242,
0.243, 0.243, 0.244, 0.244, 0.245, 0.245, 0.246, 0.246, 0.247, 0.247, 0.248, 0.248, 0.249, 0.249,
0.25, 0.25, 0.251, 0.251, 0.252, 0.252, 0.253, 0.253, 0.254, 0.254, 0.255, 0.255, 0.256, 0.256, 0.257,
0.257, 0.258, 0.258, 0.259, 0.259, 0.26, 0.26, 0.261, 0.261, 0.262, 0.262, 0.263, 0.263, 0.264, 0.264,
0.265, 0.265, 0.266, 0.266, 0.267, 0.267, 0.268, 0.268, 0.269, 0.269]
finalwpvaluesonesig =[-0.9796, -0.9789, -0.98, -0.9789, -0.9803, -0.9789, -0.9805, -0.9789, -0.9808,
-0.979, -0.981, -0.9791, -0.9812, -0.9791, -0.9815, -0.9792, -0.9817, -0.9793, -0.9819, -0.9794,
-0.9821, -0.9795, -0.9823, -0.9796, -0.9825, -0.9797, -0.9827, -0.9799, -0.9829, -0.98, -0.9831, -
0.9801, -0.9833, -0.9802, -0.9835, -0.9803, -0.9836, -0.9805, -0.9838, -0.9806, -0.984, -0.9807,
-0.9842, -0.9808, -0.9844, -0.981, -0.9845, -0.9811, -0.9847, -0.9812, -0.9849, -0.9814, -0.9851,
-0.9815, -0.9852, -0.9817, -0.9854, -0.9818, -0.9856, -0.9819, -0.9857, -0.9821, -0.9859, -0.9822,
-0.9861, -0.9824, -0.9862, -0.9825, -0.9864, -0.9827, -0.9865, -0.9828, -0.9867, -0.983, -0.9869,
-0.9831, -0.987, -0.9833, -0.9872, -0.9835, -0.9873, -0.9836, -0.9875, -0.9838, -0.9876, -0.9839,
-0.9878, -0.9841, -0.9879, -0.9843, -0.9881, -0.9844, -0.9882, -0.9846, -0.9884, -0.9848, -0.9885,
-0.9849, -0.9886, -0.9851, -0.9888, -0.9853, -0.9889, -0.9854, -0.9891, -0.9856, -0.9892, -0.9858,
-0.9893, -0.986, -0.9895, -0.9861, -0.9896, -0.9863, -0.9897, -0.9865, -0.9898, -0.9867, -0.99, -
0.9869, -0.9901, -0.9871, -0.9902, -0.9873, -0.9903, -0.9875, -0.9904, -0.9877, -0.9906, -0.9879,
-0.9907, -0.9881, -0.9908, -0.9883, -0.9909, -0.9885, -0.991, -0.9887, -0.991, -0.9889, -0.9911, -
0.9891, -0.9912, -0.9894, -0.9913, -0.9896, -0.9913, -0.9899, -0.9914, -0.9902, -0.9914, -0.9905,
-0.9913, -0.9909]
finalwavaluestwosig=[0.168, 0.168, 0.169, 0.169, 0.17, 0.17, 0.171, 0.171, 0.172, 0.172, 0.173,
0.173, 0.174, 0.174, 0.175, 0.175, 0.176, 0.176, 0.177, 0.177, 0.178, 0.178, 0.179, 0.179, 0.18, 0.18,
0.181, 0.181, 0.182, 0.182, 0.183, 0.183, 0.184, 0.184, 0.185, 0.185, 0.186, 0.186, 0.187, 0.187,
0.188, 0.188, 0.189, 0.189, 0.19, 0.19, 0.191, 0.191, 0.192, 0.192, 0.193, 0.193, 0.194, 0.194, 0.195,
0.195, 0.196, 0.196, 0.197, 0.197, 0.198, 0.198, 0.199, 0.199, 0.2, 0.2, 0.201, 0.201, 0.202, 0.202,
0.203, 0.203, 0.204, 0.204, 0.205, 0.205, 0.206, 0.206, 0.207, 0.207, 0.208, 0.208, 0.209, 0.209,
```

```
0.21, 0.21, 0.211, 0.211, 0.212, 0.212, 0.213, 0.213, 0.214, 0.214, 0.215, 0.215, 0.216, 0.216, 0.217,
0.217, 0.218, 0.218, 0.219, 0.219, 0.22, 0.22, 0.221, 0.221, 0.222, 0.222, 0.223, 0.223, 0.224, 0.224,
0.225, 0.225, 0.226, 0.226, 0.227, 0.227, 0.228, 0.228, 0.229, 0.229, 0.23, 0.23, 0.231, 0.231, 0.232,
0.232, 0.233, 0.233, 0.234, 0.234, 0.235, 0.235, 0.236, 0.236, 0.237, 0.237, 0.238, 0.238, 0.239,
0.239, 0.24, 0.24, 0.241, 0.241, 0.242, 0.242, 0.243, 0.243, 0.244, 0.244, 0.245, 0.245, 0.246, 0.246,
0.247, 0.247, 0.248, 0.248, 0.249, 0.249, 0.25, 0.25, 0.251, 0.251, 0.252, 0.252, 0.253, 0.253, 0.254,
0.254, 0.255, 0.255, 0.256, 0.256, 0.257, 0.257, 0.258, 0.258, 0.259, 0.259, 0.26, 0.26, 0.261, 0.261,
0.262, 0.262, 0.263, 0.263, 0.264, 0.264, 0.265, 0.265, 0.266, 0.266, 0.267, 0.267, 0.268, 0.268,
0.269, 0.269, 0.27, 0.27, 0.271, 0.271, 0.272, 0.272, 0.273, 0.273, 0.274, 0.274, 0.275, 0.275, 0.276,
0.276, 0.277, 0.277, 0.278, 0.278, 0.279, 0.279, 0.28, 0.28, 0.281, 0.281, 0.282, 0.282, 0.283, 0.283,
0.284, 0.284, 0.285, 0.285, 0.286, 0.286, 0.287, 0.287, 0.288, 0.288, 0.289, 0.289, 0.29, 0.29, 0.291,
0.291, 0.292, 0.292, 0.293, 0.293]
finalwpvaluestwosig=[-0.9758, -0.9749, -0.9762, -0.9749, -0.9766, -0.9748, -0.9769, -0.9748, -
0.9771, -0.9749, -0.9774, -0.9749, -0.9777, -0.975, -0.9779, -0.975, -0.9782, -0.9751, -0.9784, -
0.9752, -0.9786, -0.9752, -0.9789, -0.9753, -0.9791, -0.9754, -0.9793, -0.9755, -0.9795, -0.9756,
-0.9797, -0.9757, -0.9799, -0.9758, -0.9801, -0.9759, -0.9803, -0.976, -0.9806, -0.9761, -0.9808,
-0.9762, -0.981, -0.9763, -0.9811, -0.9764, -0.9813, -0.9766, -0.9815, -0.9767, -0.9817, -0.9768,
-0.9819, -0.9769, -0.9821, -0.977, -0.9823, -0.9772, -0.9825, -0.9773, -0.9827, -0.9774, -0.9829,
-0.9775, -0.983, -0.9777, -0.9832, -0.9778, -0.9834, -0.9779, -0.9836, -0.978, -0.9838, -0.9782, -
0.9839, -0.9783, -0.9841, -0.9784, -0.9843, -0.9786, -0.9845, -0.9787, -0.9846, -0.9788, -0.9848,
-0.979, -0.985, -0.9791, -0.9851, -0.9793, -0.9853, -0.9794, -0.9855, -0.9795, -0.9857, -0.9797, -
0.9858, -0.9798, -0.986, -0.98, -0.9862, -0.9801, -0.9863, -0.9803, -0.9865, -0.9804, -0.9866, -0.9806,
-0.9868, -0.9807, -0.987, -0.9809, -0.9871, -0.981, -0.9873, -0.9812, -0.9875, -0.9813, -0.9876, -
0.9815, -0.9878, -0.9816, -0.9879, -0.9818, -0.9881, -0.9819, -0.9882, -0.9821, -0.9884, -0.9822,
-0.9885, -0.9824, -0.9887, -0.9826, -0.9889, -0.9827, -0.989, -0.9829, -0.9892, -0.983, -0.9893,
-0.9832, -0.9895, -0.9834, -0.9896, -0.9835, -0.9898, -0.9837, -0.9899, -0.9838, -0.99, -0.984, -
0.9902, -0.9842, -0.9903, -0.9843, -0.9905, -0.9845, -0.9906, -0.9847, -0.9908, -0.9848, -0.9909,
-0.985, -0.991, -0.9852, -0.9912, -0.9854, -0.9913, -0.9855, -0.9915, -0.9857, -0.9916, -0.9859, -
0.9917, -0.9861, -0.9919, -0.9862, -0.992, -0.9864, -0.9921, -0.9866, -0.9923, -0.9868, -0.9924,
-0.987, -0.9925, -0.9871, -0.9926, -0.9873, -0.9928, -0.9875, -0.9929, -0.9877, -0.993, -0.9879,
-0.9931, -0.9881, -0.9933, -0.9883, -0.9934, -0.9884, -0.9935, -0.9886, -0.9936, -0.9888, -0.9937, -
0.989, -0.9939, -0.9892, -0.994, -0.9894, -0.9941, -0.9896, -0.9942, -0.9898, -0.9943, -0.99, -0.9944,
-0.9902, -0.9945, -0.9905, -0.9946, -0.9907, -0.9947, -0.9909, -0.9948, -0.9911, -0.9949, -0.9913,
-0.995, -0.9916, -0.995, -0.9918, -0.9951, -0.992, -0.9952, -0.9923, -0.9952, -0.9925, -0.9953, -
0.9928, -0.9953, -0.993, -0.9954, -0.9933, -0.9954, -0.9936, -0.9953, -0.994, -0.9953, -0.9944]

#plot formatting
fig = plt.figure()
ax = fig.add_subplot(111, autoscale_on=False, xlim=(0.16, 0.30), ylim=(-0.997, -0.973))
custom_font = "Times New Roman"
ax.set_xlabel(r"$w_a$", fontname=custom_font, size=14)
ax.set_ylabel(r"$w_p$", fontname=custom_font, size=14)
ax.set_xticks([0.16,0.18,0.20,0.22,0.24,0.26,0.28,0.3])
ax.set_yticks([-0.995,-0.990,-0.985,-0.980,-0.975])
for tick in ax.set_xticklabels([0.16,0.18,0.20,0.22,0.24,0.26,0.28,0.3]):
    tick.set_fontname(custom_font)
tick in ax.set_yticklabels([-0.995,-0.990,-0.985,-0.980,-0.975]):
    tick.set_fontname(custom_font)
plt.xticks(fontsize=14)
plt.yticks(fontsize=14)
```

```
#plotting the graph
plt.scatter(finalwavaluesonesig,finalwpvaluesonesig, marker=“X”, color=“green”, label=r“1-σ ellipse”)
plt.scatter(finalwavaluestwosig,finalwpvaluestwosig, marker=“X”, color=“red”, label=r“2-σ ellipse”)
plt.plot(0.23,-0.985, marker=“X”, color=“black”, label=r“Best-fit values”)
ax.legend(prop={‘family’:custom_font, ‘size’:14}, loc=0,frameon=False)
plt.plot()
plt.show()
```

B.4 Luminosity Distance Code

All code is in the language: Python.

B.4.1. Calculating luminosity distance and testing against the analytical approach.

```
#importing helpful libraries
import math
import numpy as np
from matplotlib import pyplot as plt

#plot formatting
plt.rcParams[“figure.figsize”] = [8.6, 6]
plt.rcParams[“figure.autolayout”] = True

#defining constants
c = 2.998*(10**8)
H_0 = 67.81*(10**3)
G = 6.67/(10**11)
Omega_X = 1
Omega_m = 0
w_X = -1
p_c = (3*(H_0**2))/(math.pi*8*G)
p_X = Omega_X*p_c
#p_m doesn’t need to be calculated considering Omega_m = 0

#calculating the distance analytically
def d(z):
    return c*z/H_0

#calculating the luminosity distance numerically
#a_0 is 1, so the value of 1 has been used directly
def f(z):
    #scale factor “a” defined
    a = 1/(1+z)
    #empty array defined now to hold d(z).
    d = np.array([])
    #loop through each value of a
    for aPrime in a:
        #starting integration values
        dz = 0.0
        ddz = 0.0
        da = 0.001
```

```
        #integration to find distance
        while aPrime <1:
            #starting integration values
            rho = 0.0
            drho = 0.0
            aPrime2 = aPrime
            dda = 0.001
            #integration to find ln(rho_x(a'))/ln(rho_x,0)
            while aPrime2 <1:
                drho = ((3*(1+w_X))/aPrime2) * dda
                rho = rho + drho
                aPrime2 = aPrime2 + dda
            #calculating rho_x(a') from the integral
            p_a = p_X*(math.e**rho)
            #calculating H(a'). Only p_a matters here because omega_m = 0.
            H = ((8*math.pi*p_a*G)/3)**0.5
            ddz = (c/(H*(aPrime**2)))*da
            dz = dz + ddz
            aPrime = aPrime + da
        #adding the distances to an array
        d = np.append(d, dz)
    #calculating the luminosity distance and adding to array
    dl = np.add(1, z)*d
    return dl

#creating array of z values
z = np.linspace(0, 2, 201)

#calculating luminosity distance analytically
d_L = np.add(1, z)*d(z)

#plot formatting
fig = plt.figure()
ax = fig.add_subplot(111, autoscale_on=False, xlim=(0, 2), ylim=(0, 28000))
custom_font = "Times New Roman"
ax.set_xlabel(r"z", fontname=custom_font, size=14)
ax.set_ylabel(r"$d_L$ [Mpc]", fontname=custom_font, size=14)
ax.set_xticks([0,0.4,0.8,1.2,1.6,2])
ax.set_yticks([0,4000,8000,12000,16000,20000,24000,28000])
for tick in ax.set_xticklabels([0,0.4,0.8,1.2,1.6,2]):
    tick.set_fontname(custom_font)
for tick in ax.set_yticklabels([0,4000,8000,12000,16000,20000,24000,28000]):
    tick.set_fontname(custom_font)
plt.xticks(fontsize=14)
plt.yticks(fontsize=14)

#graph plotting
plt.plot(z, d_L, color='red',linewidth=2, label="Analytical approach")
plt.plot(z,f(z),color="black", linestyle=(0,(5,5)), linewidth=2,
    label="Numerical integration approach")
plt.legend(prop={'family':custom_font, 'size':14}, loc=0,frameon=False)
```

```
plt.show()
```

B.4.2. Plotting w_X as a function of z for ΛCDM and SFQ models.

```
#importing helpful libraries
import math
from matplotlib import pyplot as plt
import numpy as np
import matplotlib.font_manager
from IPython.core.display import HTML
from matplotlib import patheffects

#gets the font from the font manager
def make_html(fontname):
    return “<p>{font}: <span style=‘font-family:font; \
    font-size: 24px;’>{font}</p>”.format(font=fontname)
code = “\n”.join([make_html(font) for font in sorted(
    set([f.name for f in matplotlib.font_manager.fontManager.ttflist]))])
HTML(“<div style=‘column-count: 2;’>{}</div>”.format(code))

#plot formatting
plt.rcParams[“figure.figsize”] = [8.6, 6]
plt.rcParams[“figure.autolayout”] = True

#defining constants
a_t = 0.23
tau = 0.33
w_p = 0
w_f = -1

#calculating w for the SFQ model
def f(z):
    return (w_f+((w_p-w_f)/(1+((1/(1+z))/a_t)**(1/tau))))

#plot formatting
fig = plt.figure()
ax = fig.add_subplot(111, autoscale_on=False, xlim=(0, 2), ylim=(-1.01, -0.75))
custom_font = “Times New Roman”
ax.set_xlabel(r“z”, fontname=custom_font, size=14)
ax.set_ylabel(r“$w_x$”, fontname=custom_font, size=14)
ax.set_xticks([0,0.4,0.8,1.2,1.6,2])
ax.set_yticks([-1,-0.95,-0.9,-0.85,-0.8,-0.75])
for tick in ax.set_xticklabels([0,0.4,0.8,1.2,1.6,2]):
    tick.set_fontname(custom_font)
for tick in ax.set_yticklabels([-1,-0.95,-0.9,-0.85,-0.8,-0.75]):
    tick.set_fontname(custom_font)
plt.xticks(fontsize=14)
plt.yticks(fontsize=14)

#plotting the graph
z = np.linspace(0, 2, 201)
plt.plot([0, 2],[-1,-1],color=“red”, label=“ΛCDM model”)
```

```
plt.plot(z,f(z),color="black", label="SFQ model")
ax.legend(prop={'family':custom_font, 'size':14}, loc=0,frameon=False)
plt.plot()
plt.show()
```

B.4.3. Calculating luminosity distance for the SFQ and ΛCDM models.

```
#importing helpful libraries
import math
from matplotlib import pyplot as plt
import numpy as np
import matplotlib.font_manager
from IPython.core.display import HTML
from matplotlib import patheffects

#getting the font
def make_html(fontname):
    return ">{font}: <span style='font-family:{font}; \
    font-size: 24px;'>{font}</p>".format(font=fontname)
code = "\n".join([make_html(font) for font in sorted(
    set([f.name for f in matplotlib.font_manager.fontManager.ttflist]))])
HTML("<div style='column-count: 2;'>{}</div>".format(code))

#plot formatting
plt.rcParams["figure.figsize"] = [8.6, 6]
plt.rcParams["figure.autolayout"] = True

#defining constants
Omega_X = 0.692
Omega_m = 0.308
c = 2.998*(10**8)
H_0 = 67.81*(10**3)
G = 6.67/(10**11)
a_t = 0.23
tau = 0.33
w_p = 0
w_f = -1
p_c = (3*(H_0**2))/(math.pi*8*G)
p_X = Omega_X*p_c
p_m = Omega_m*p_c

#function to calculate the luminosity distance for either model.
def f(z, SFQ):
    #setting base barotropic parameter
    w_X = -1
    #scale factor "a" defined
    a = 1/(1+z)
    #empty array defined now to hold d(z).
    d = np.array([])
    #loop through each value of a
    for aPrime in a:
        #starting integration values
```

```
        dz = 0.0
        ddz = 0.0
        da = 0.001
        #integral to find distance
        while aPrime <1:
            #starting integration values
            rho = 0.0
            drho = 0.0
            aPrime2 = aPrime
            dda = 0.001
            #integral to find ln(rho_x(a'))/ln(rho_x,0)
            while aPrime2 <1:
                #if the SFQ model is being used, the barotropic parameter needs to be changed.
                if SFQ:
                    w_X = (w_f+((w_p-w_f)/(1+((aPrime2/a_t)**(1/tau)))))
                drho = ((3*(1+w_X))/aPrime2) * dda
                rho = rho + drho
                aPrime2 = aPrime2 + dda
            #calculating rho_x(a') from the integral
            p_a = p_X*(math.e**rho)
            #calculating rho_m(a')
            p_b = p_m*((1/aPrime)**3)
            #calculating H(a')
            H = ((8*math.pi*(p_a+p_b)*G)/3)**0.5
            ddz = (c/(H*(aPrime**2)))*da
            dz = dz + ddz
            aPrime = aPrime + da
        #adding the distances to array
        d = np.append(d, dz)
    #calculating the luminosity distance and adding to array
    dl = np.add(1, z)*d
    return dl

#plot formatting
fig = plt.figure()
ax = fig.add_subplot(111, autoscale_on=False, xlim=(0, 2), ylim=(0, 16200))
custom_font = "Times New Roman"
ax.set_xlabel(r"z", fontname=custom_font, size=14)
ax.set_ylabel(r"$d_L$ [Mpc]", fontname=custom_font, size=14)
ax.set_xticks([0,0.4,0.8,1.2,1.6,2])
ax.set_yticks([0,4000,8000,12000,16000])
for tick in ax.set_xticklabels([0,0.4,0.8,1.2,1.6,2]):
    tick.set_fontname(custom_font)
for tick in ax.set_yticklabels([0,4000,8000,12000,16000]):
    tick.set_fontname(custom_font)
plt.xticks(fontsize=14)
plt.yticks(fontsize=14)

#graph plotting
z = np.linspace(0, 2, 201)
plt.plot(z,f(z, False),color="red", linewidth=2, label="ΛCDM model")
```

```
plt.plot(z, f(z, True), color=“black”,linestyle=(0,(5,5)), linewidth=2, label=“SFQ model”)
ax.legend(prop={‘family’:custom_font, ‘size’:14}, loc=0,frameon=False)
plt.plot()
plt.show()
```

B.4.4. Calculating fractional deviation in luminosity distance for the SFQ and ΛCDM models.

```
#importing helpful libraries
import math
from matplotlib import pyplot as plt
from itertools import count
import numpy as np
import matplotlib.font_manager
from IPython.core.display import HTML
from matplotlib import patheffects

#getting the font
def make_html(fontname):
    return “<p>{font}: <span style=‘font-family:{font}; \
    font-size: 24px;’>{font}</p>”.format(font=fontname)
code = “\n”.join([make_html(font) for font in sorted(
    set([f.name for f in matplotlib.font_manager.fontManager.ttflist]))])
HTML(“<div style=‘column-count: 2;’>{}</div>”.format(code))

#graph formatting
plt.rcParams[“figure.figsize”] = [8.6, 6]
plt.rcParams[“figure.autolayout”] = True

#defining constants
Omega_X = 0.692
Omega_m = 0.308
c = 2.998*(10**8)
H_0 = 67.81*(10**3)
G = 6.67/(10**11)
a_t = 0.23
tau = 0.33
w_p = 0
w_f = -1
p_c = (3*(H_0**2))/(math.pi*8*G)
p_X = Omega_X*p_c
p_m = Omega_m*p_c

#function to calculate the luminosity distance for either model.
def f(z, SFQ):
    #setting base barotropic parameter
    w_X = -1
    #scale factor “a” defined
    a = 1/(1+z)
    #empty array defined now to hold d(z).
    d = np.array([])
    #loop through each value of a
```

```
    for aPrime in a:
        #starting integration values
        dz = 0.0
        ddz = 0.0
        da = 0.001
        #integral to find distance
        while aPrime <1:
            #starting integration values
            rho = 0.0
            drho = 0.0
            aPrime2 = aPrime
            dda = 0.001
            #integral to find ln(rho_x(a'))/ln(rho_x,0)
            while aPrime2 <1:
                #if the SFQ model is being used, the barotropic parameter needs to be changed.
                if SFQ:
                    w_X = (w_f+((w_p-w_f)/(1+((aPrime2/a_t)**(1/tau)))))
                drho = ((3*(1+w_X))/aPrime2) * dda
                rho = rho + drho
                aPrime2 = aPrime2 + dda
            #calculating rho_x(a') from the integral
            p_a = p_X*(math.e**rho)
            #calculating rho_m(a')
            p_b = p_m*((1/aPrime)**3)
            #calculating H(a')
            H = ((8*math.pi*(p_a+p_b)*G)/3)**0.5
            ddz = (c/(H*(aPrime**2)))*da
            dz = dz + ddz
            aPrime = aPrime + da
        #adding the distances to array
        d = np.append(d, dz)
    #calculating the luminosity distance and adding to array
    dl = np.add(1, z)*d
    return dl

#array of redshift values
z = np.linspace(0, 2, 201)

#various lists for formatting purposes
sigmafrac = [0.0036,0.0036]
sigmaz = [0.69,0.69]
yrange = [0,0.0036]
xrange = [0,0.69]
xtotalrange = [0,2]
ysigmamax = [1,1]

#plot formatting
fig = plt.figure()
ax = fig.add_subplot(111, autoscale_on=False, xlim=(0, 2), ylim=(0, 0.007))
custom_font = "Times New Roman"
ax.set_xlabel(r"z", fontname=custom_font, size=14)
```

```
ax.set_ylabel(r“$—Δd_{L}/d_{L}—$”, fontname=custom_font, size=14)
ax.set_xticks([0,0.4,0.8,1.2,1.6,2])
ax.set_yticks([0,0.001,0.002,0.003,0.004,0.005,0.006,0.007])
for tick in ax.set_xticklabels([0,0.4,0.8,1.2,1.6,2]):
    tick.set_fontname(custom_font)
for tick in ax.set_yticklabels([0,0.001,0.002,0.003,0.004,0.005,0.006,0.007]):
    tick.set_fontname(custom_font)
plt.xticks(fontsize=14)
plt.yticks(fontsize=14)

#Calculating the fractional deviation
CDM = f(z, False)
SFQ = f(z, True)
frac = abs((SFQ-CDM)/SFQ)

#plotting the graph
plt.plot(z, frac, color=“black”)
plt.plot(xrange, sigmafrac, linestyle=“dashed”,color=“red”)
plt.plot(sigmaz, yrange, linestyle=“dashed”,color=“red”)
plt.fill_between(xtotalrange, ysigmamax, sigmafrac,color=“red”,alpha=.1)
plt.text(0.1,0.0051, “Region for ‘potential’ observation”, fontname=custom_font, fontsize = 14)
plt.text(-0.15,0.0035, “0.0036”, fontname=custom_font, fontsize = 14)
plt.text(0.64,-0.0003, “0.69”, fontname=custom_font, fontsize = 14)
plt.plot()
plt.show()
```

B.4.5. Plotting a_t as a function of τ for the SFQ model and determining the region which definitely cannot be detected.

```
#importing helpful libraries
from matplotlib import pyplot as plt
import numpy as np
import matplotlib.font_manager
from IPython.core.display import HTML
from matplotlib import patheffects

#getting the font
def make_html(fontname):
    return “<p>{font}: <span style=‘font-family:{font}; \
    font-size: 24px;’>{font}</p>”.format(font=fontname)
code = “\n”.join([make_html(font) for font in sorted(
    set([f.name for f in matplotlib.font_manager.fontManager.ttflist]))])
HTML(“<div style=‘column-count: 2;’>{}</div>”.format(code))

#plot formatting
plt.rcParams[“figure.figsize”] = [8.6, 6]
plt.rcParams[“figure.autolayout”] = True

#list of values for the transition width
tau = [0.1, 0.3, 0.5, 0.7, 1.0]
#list of values for the scale factor at the transition
a = [0.4, 0.21, 0.09, 0.04, 0.01]
```

```
#plot formatting
fig = plt.figure()
ax = fig.add_subplot(111, autoscale_on=False, xlim=(.1, 1), ylim=(0, 0.4))
custom_font = "Times New Roman"
ax.set_xlabel(r"$\tau$", fontname=custom_font, size=14)
ax.set_ylabel(r"$a_t$", fontname=custom_font, size=14)
ax.set_xticks([0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1])
ax.set_yticks([0,0.05,0.10,0.15,0.20,0.25,0.30,0.35,0.40])
for tick in ax.set_xticklabels([0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1]):
    tick.set_fontname(custom_font)
for tick in ax.set_yticklabels([0,0.05,0.10,0.15,0.20,0.25,0.30,0.35,0.40]):
    tick.set_fontname(custom_font)
plt.xticks(fontsize=14)
plt.yticks(fontsize=14)

#plotting the graph
plt.plot(tau, a, color="black")
plt.fill_between(tau, a, color='red',alpha=.1)
plt.text(0.15,0.045, "Deviation not detectable in this region", fontname=custom_font, fontsize
= 14)
plt.plot()
plt.show()
```

B.4.6. Code snippet to find the redshift value that may be observed.

```
z = np.linspace(0.68, 0.71, 100)
for index,item in enumerate(frac):
    print("i="+str(index)+" f="+str(item))
for index,item in enumerate(z):
    if round(frac[index], 4) == 0.0036:
        print("i="+str(index)+" f="+str(item))
```

B.4.7. Code snippet to find the a_t values for the respective τ values.

```
z = np.linspace(1.6, 1.7, 21)
#These two values can be changed for each value of tau
tau = 0.1
a_t = 0.45
observable = True
while observable:
    CDM = f(z, False)
    SFQ = f(z, True)
    frac = abs((SFQ-CDM)/SFQ)
    if frac[-1] <0.0036:
        observable = False
    else:
        a_t = round(a_t-0.01, 3)
print(a_t)
print(frac[-1])
```

C Modified gravity models providing an alternate approach

C.1 Derivations

The Friedmann equation for a modified gravity model is given by

$$H^2 - \frac{H^\alpha}{(r_c/c)^{2-\alpha}} = \frac{8\pi G}{3}\rho_m \tag{183}$$

where H is the Hubble parameter, G is the gravitational constant, c is the speed of light, α is the model-dependent parameter, and r_c is the cross-over scale. The present matter density is given by,

$$\Omega_{m,0} = \frac{\rho_{m,0}}{\rho_{c,0}}, \tag{184}$$

where the critical density, by definition is,

$$\rho_{c,0} = \frac{3H_0^2}{8\pi G}. \tag{185}$$

The cross-over scale equation, Eqn. (31), is derived now. First, rearrange Eqn. (183)to show the cross-over scale in terms of the H parameter

$$\left(\frac{r_c}{c}\right)^{2-\alpha} = \frac{H^\alpha}{H^2 - \frac{8\pi G}{3}\rho_m}. \tag{186}$$

Rearranging Eqn. (185) next would give,

$$\frac{8\pi G}{3} = \frac{H_0^2}{\rho_{c,0}}, \tag{187}$$

which can then be substituted into Eqn. (186) to give,

$$\left(\frac{r_c}{c}\right)^{2-\alpha} = \frac{H^\alpha}{H^2 - \frac{H_0^2}{\rho_{c,o}}\rho_m}. \tag{188}$$

At time $t = 0, H = H_0$. So, at t = 0 the Eqn. (188) would become,

$$\left(\frac{r_c}{c}\right)^{2-\alpha} = \frac{H_0^\alpha}{H_0^2 - \frac{H_0^2}{\rho_{c,o}}\rho_{m,0}}, \tag{189}$$

and the H_0 in the numerator would cancel with the one in the denominator to give,

$$\left(\frac{r_c}{c}\right)^{2-\alpha} = \frac{H_0^\alpha}{H_0^2(1 - \frac{\rho_{m,0}}{\rho_{c,o}})}. \tag{190}$$

Substituting Eqn. (184) in, the r_c equation would finally come to:

$$r_c/c = [H_0^{\alpha-2}(1 - \Omega_{m,0})^{-1}]^{\frac{1}{2-\alpha}}. \tag{191}$$

For a model where $\alpha = 1$, the cross-over scale equation becomes:

$$r_c/c = H_0^{-1}(1 - \Omega_{m,0})^{-1}. \tag{192}$$

Next, the modified friedmann equation is solved to express $H(a)$ as a function of H_0, $\Omega_{m,0}$, a_0 and a. The modified Friedmann equation for $\alpha = 1$ is given by

$$H^2 - \frac{H}{(r_c/c)} = \frac{8\pi G}{3}\rho_m. \tag{193}$$

Insert Eqn.(192) in,

$$H^2 - HH_0(1-\Omega_{m,0}) = \frac{8\pi G}{3}\rho_m, \tag{194}$$

followed by Eqn. (187),

$$H^2 - HH_0(1-\Omega_{m,0}) = \frac{H_0^2}{\rho_{c,0}}\rho_m, \tag{195}$$

and lastly substitute Eqn. (185) in and bring all terms to one side,

$$H^2 - HH_0(1-\Omega_{m,0}) - H_0^2\frac{\Omega_{m,0}}{\rho_{m,0}}\rho_m = 0. \tag{196}$$

Matter density can be shown as a function of the scale factor, a,

$$\rho_m = \rho_{m,0}\left(\frac{a_0}{a}\right)^3. \tag{197}$$

Substituting this into Eqn. (196) would give

$$H^2 - HH_0(1-\Omega_{m,0}) - H_0^2\Omega_{m,0}\left(\frac{a_0}{a}\right)^3 = 0. \tag{198}$$

As can be seen, this is a quadratic equation. Using the quadratic formula, the solutions are given as:

$$H(a) = \frac{H_0(1-\Omega_{m,0}) \pm H_0\sqrt{(1-\Omega_{m,0})^2 + 4\Omega_{m,0}\left(\frac{a_0}{a}\right)^3}}{2}. \tag{199}$$

We used $a = a_0$ in Eqn. (199) to determine whether the sign is positive or negative. At $a = a_0$, $H(a_0) = H_0$. Hence, it was found that this only happens when the sign is positive. So, the final expression for $H(a)$ is given as,

$$H(a) = \frac{H_0(1-\Omega_{m,0}) + H_0\sqrt{(1-\Omega_{m,0})^2 + 4\Omega_{m,0}\left(\frac{a_0}{a}\right)^3}}{2}. \tag{200}$$

The deceleration parameter, q, is defined by

$$q(z) = \frac{-1}{H^2}\frac{\ddot{a}}{a} \tag{201}$$

The deceleration parameter dictates the universe's expansion acceleration. The expression for $q(z)$ as a function of a, H, and dH/da is derived below. The hubble parameter as a function of the scale factor, a, is given as

$$H = \frac{\dot{a}}{a}. \tag{202}$$

The expression for the derivative of H with respect to can be found by obtaining

$$\frac{dH}{da} = \frac{dH}{dt} \times \frac{dt}{da}. \tag{203}$$

By using the quotient rule on Eqn. (202), it can be found that

$$\frac{dH}{dt} = \frac{\ddot{a}}{a} - H^2, \tag{204}$$

therefore,

$$\frac{dH}{da} = \left(\frac{\ddot{a}}{a} - H^2\right) \times \frac{1}{\dot{a}}. \tag{205}$$

Rearrange this expression,

$$\dot{a}\frac{dH}{da} + H^2 = \frac{\ddot{a}}{a}, \tag{206}$$

substitute it back into Eqn. (201),

$$q = \frac{-1}{H^2}\left(\dot{a}\frac{dH}{da} + H^2\right), \tag{207}$$

and simplify,

$$q = -\left(1 + \frac{a}{H}\frac{dH}{da}\right). \tag{208}$$

Using the equations for q, Eqn. (208), and H, Eqn. (200), an expression for q as a function of a for the $\alpha = 1$ DGP model can be derived, as shown below. Derive Eqn. (200) with respect to the scale factor a,

$$\frac{dH}{da} = \frac{-3H_0\Omega_{m,0}a_0^3}{\sqrt{(1-\Omega_{m,0})^2 + 4\Omega_{m,0}\left(\frac{a_0}{a}\right)^3}}. \tag{209}$$

Let x(a) be:

$$x(a) = \sqrt{(1-\Omega_{m,0})^2 + 4\Omega_{m,0}\left(\frac{a_0}{a}\right)^3}. \tag{210}$$

Substitute Eqn. (209) and Eqn. (200), into the expression for q, from Eqn. (208), to get q as a function of a:

$$q(a) = \frac{6\Omega_{m,0}}{x(a)[(1-\Omega_{m,0}) + x(a)]}\left(\frac{a_0}{a}\right)^3 - 1. \tag{211}$$

The scale factor a is given as:

$$a = \frac{a_0}{1+z} = \frac{1}{1+z} \tag{212}$$

Substituting this into Eqn. (211) and Eqn. (210), the respective expression for q(z) and x(z) are:

$$q(z) = \frac{6\Omega_{m,0}(1+z)^3}{x(z)\,[(1-\Omega_{m,0}) + x(z)]} - 1; \tag{213}$$

$$x(z) = \sqrt{(1-\Omega_{m,0})^2 + 4\Omega_{m,0}(1+z)^3}. \tag{214}$$

The $q(z)$ for the ΛCDM model uses the Eqn. (35) and the Friedmann Eqn. (47) in terms of $H(a)$, where

$$H^2(a) = \frac{8\pi G}{3}\left(\rho_{m,0}\left(\frac{a_0}{a}\right)^3 + \rho_{\Lambda,0}\right). \tag{215}$$

Substituting the present critical density Eqn. (30) into Eqn. (215), obtaining

$$H^2(a) = H_0^2\left(\frac{\rho_{m,0}}{\rho_{c,0}}\left(\frac{a_0}{a}\right)^3 + \frac{\rho_{\Lambda,0}}{\rho_{c,0}}\right). \tag{216}$$

Using the energy-density parameter from Eqn. (3), substituting and simplifying

$$H^2(a) = H_0^2\left(\Omega_{m,0}\left(\frac{a_0}{a}\right)^3 + \Omega_{\Lambda,0}\right), \tag{217}$$

$$H(a) = H_0\left(\Omega_{m,0}\left(\frac{a_0}{a}\right)^3 + \Omega_{\Lambda,0}\right)^{\frac{1}{2}}. \tag{218}$$

Take the scale factor derivative of Eqn. (218),

$$\frac{dH}{da} = \frac{d}{da}\left(H_0\left(\Omega_{m,0}\left(\frac{a_0}{a}\right)^3 + \Omega_{\Lambda,0}\right)^{\frac{1}{2}}\right), \tag{219}$$

$$\frac{dH}{da} = \frac{H_0}{2}\left(\Omega_{m,0}\left(\frac{a_0}{a}\right)^3 + \Omega_{\Lambda,0}\right)^{-\frac{1}{2}} \times -\frac{3\Omega_{m,0}a_0^3}{a^4}, \tag{220}$$

$$\frac{dH}{da} = -\frac{3H_0\Omega_{m,0}a_0^3}{2a^4}\left(\Omega_{m,0}\left(\frac{a_0}{a}\right)^3 + \Omega_{\Lambda,0}\right)^{-\frac{1}{2}}. \tag{221}$$

Let $x(a)$ equal from Eqn. (221),

$$x(a) = \left(\Omega_{m,0}\left(\frac{a_0}{a}\right)^3 + \Omega_{\Lambda,0}\right)^{-\frac{1}{2}}. \tag{222}$$

Substitute Eqn. (221) into Eqn. (35),

$$q = -1\left(1 + \frac{1}{H} \times -\frac{3H_0\Omega_{m,0}}{2}\left(\frac{a_0}{a}\right)^3 x(a)\right), \tag{223}$$

$$q = -1\left(1 - \frac{3\Omega_{m,0}}{2}\left(\frac{a_0}{a}\right)^3 x^2(a)\right). \tag{224}$$

Writing Eqn. (222) in terms of z from Eqn. (92),

$$x(z) = \left(\Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0}\right)^{-\frac{1}{2}}. \tag{225}$$

Writing Eqn. (224) in terms of z from Eqn. (92) and simplifying,

$$q(z) = -1\left(1 - \frac{3\Omega_{m,0}}{2}(1+z)^3 x^2(z)\right), \tag{226}$$

therefore,

$$q(z) = \frac{3\Omega_{m,0}(1+z)^3 x^2(z)}{2} - 1, \tag{227}$$

where,

$$x(z) = \left(\Omega_{m,0}(1+z)^3 + 1 - \Omega_{m,0}\right)^{-\frac{1}{2}}. \tag{228}$$

This formula was used for plotting Figure 13, comparing the DGP model to ΛCDM.

C.2 Code

All code was done in the language: Python.

C.2.1. $q(z)$ for the DGP and ΛCDM model

```
#importing helpful libraries
import math
from matplotlib import pyplot as plt
import numpy as np

#gets the font from the font manager
import matplotlib.font_manager
from IPython.core.display import HTML
from matplotlib import patheffects
def make_html(fontname):
    return “<p>{font}: <span style=‘font-family:{font}; \
    font-size: 24px;’>{font}</p>”.format(font=fontname)
code = “\n”.join([make_html(font) for font in sorted(set([f.name for f in
matplotlib.font_manager.fontManager.ttflist]))])
HTML(“<div style=‘column-count: 2;’>{}</div>”.format(code))

#plot formatting
plt.rcParams[“figure.figsize”] = [8.6, 6]
plt.rcParams[“figure.autolayout”] = True

#defining constants
omegamo = 0.308
#Calculating x(z) for the DGP model
def w(z):
    return ((1-omegamo)**2+4*omegamo*(1+z)**3)**(1/2)

#Calculating q(z) for the DGP model, using the function above within
def q(z):
    return (6*omegamo*(1+z)**3)/(w(z)*((1-omegamo)+w(z)))-1

#Sets the range between 0 <z <5, with a 0.001 iteration interval
z = np.linspace(0, 5, 5001)

#Prints the q value at present for DGP
print(“DGP q(z=0)=”,q(0))

#Calculating x(z) for the ΛCDM model
def f(z):
    return (omegamo*(1+z)**3+(1-omegamo))**(-1/2)

#Calculating q(z) for the ΛCDM model, using the function above within
def g(z):
    return ((3*omegamo*(1+z)**3*(f(z))**2)/2) - 1

#Prints the q value at present for ΛCDM
print(“ΛCDM q(z=0)=”,g(0))
```

```
#Prints the closest intersect point for the two curves and the z-value
print("z=",z[1040],"DGP q(z=1.04)=",q(z)[1040],"ΛCDM q(z=1.04)=",g(z)[1040])

#plot formatting
fig = plt.figure()
ax = fig.add_subplot(111, autoscale_on=False, xlim=(0, 5), ylim=(0, 0.5))
custom_font = "Times New Roman"
ax.set_xlabel(r"z", fontname=custom_font, size=14)
ax.set_ylabel(r"q", fontname=custom_font, size=14)
ax.set_xticks([0,0.5,1,1.5,2,2.5,3,3.5,4,4.5,5])
ax.set_yticks([-0.6,-0.5,-0.4,-0.3,-0.2,-0.1,0,0.1,0.2,0.3,0.4,0.5])
for tick in ax.set_xticklabels([0,0.5,1,1.5,2,2.5,3,3.5,4,4.5,5]):
    tick.set_fontname(custom_font)
for tick in ax.set_yticklabels([-0.6,-0.5,-0.4,-0.3,-0.2,-0.1,0,0.1,0.2,0.3,0.4,0.5]):
    tick.set_fontname(custom_font)
plt.xticks(fontsize=14)
plt.yticks(fontsize=14)

#plotting the graph
plt.plot(z,q(z),color="black", label="DGP modified gravity model")
plt.plot(z,g(z),color="red", label="ΛCDM dark energy model")
ax.legend(prop={'family':custom_font, 'size':14}, loc=0,frameon=False)
plt.plot()
plt.show()
```

C.2.2. Calculating luminosity distance for the modified gravity and ΛCDM models.

```
#importing helpful libraries
import math
from matplotlib import pyplot as plt
import numpy as np
import matplotlib.font_manager
from IPython.core.display import HTML
from matplotlib import patheffects

#getting the font
def make_html(fontname):
    return "<p>{font}: <span style='font-family:{font}; \
    font-size: 24px;'>{font}</p>".format(font=fontname)
code = "\n".join([make_html(font) for font in sorted(
    set([f.name for f in matplotlib.font_manager.fontManager.ttflist]))])
HTML("<div style='column-count: 2;'>{}</div>".format(code))

#plot formatting
plt.rcParams["figure.figsize"] = [8.6, 6]
plt.rcParams["figure.autolayout"] = True

#defining constants
Omega_X = 0.692
Omega_m = 0.308
c = 2.998*(10**8)
H_0 = 67.81*(10**3)
```

```
G = 6.67/(10**11)
p_c = (3*(H_0**2))/(math.pi*8*G)
p_X = Omega_X*p_c
p_m = Omega_m*p_c

#function to find the luminosity distance for the lambdaCDM model
def lambdaCDM(z):
    #setting base barotropic parameter
    w_X = -1
    #scale factor "a" defined
    a = 1/(1+z)
    #empty array defined now to hold d(z).
    d = np.array([])
    #loop through each value of a
    for aPrime in a:
        #starting integration values
        dz = 0.0
        ddz = 0.0
        da = 0.001
        #integral to find distance
        while aPrime <1:
            #starting integration values
            rho = 0.0
            drho = 0.0
            aPrime2 = aPrime
            dda = 0.001
            #integral to find ln(rho_x(a'))/ln(rho_x,0)
            while aPrime2 <1:
                drho = ((3*(1+w_X))/aPrime2) * dda
                rho = rho + drho
                aPrime2 = aPrime2 + dda
            #calculating rho_x(a') from the integral
            p_a = p_X*(math.e**rho)
            #calculating rho_m(a')
            p_b = p_m*((1/aPrime)**3)
            #calculating H(a')
            H = ((8*math.pi*(p_a+p_b)*G)/3)**0.5
            ddz = (c/(H*(aPrime**2)))*da
            dz = dz + ddz
            aPrime = aPrime + da
        #adding the distances to the array
        d = np.append(d, dz)
    #calculating luminosity distance and adding to array
    dl = np.add(1, z)*d
    return dl

#function to calculate the luminosity distance for the modified gravity model.
def gravity(z):
    #scale factor "a" defined
    a = 1/(1+z)
    #empty array defined now to hold d(z).
```

```
    d = np.array([])
    #loop through each value of a
    for aPrime in a:
        #starting integration values
        dz = 0.0
        ddz = 0.0
        da = 0.001
        #integral to find distance
        while aPrime <1:
            #calculating H(a')
            H = (H_0*(1-Omega_m)+H_0*((((1-Omega_m)**2)+(4*Omega_m*
            ((1/aPrime)**3)))**0.5))/2
            ddz = (c/(H*(aPrime**2)))*da
            dz = dz + ddz
            aPrime = aPrime + da
        #adding the distances to the array
        d = np.append(d, dz)
    #calculating luminosity distance and adding to array
    dl = np.add(1, z)*d
    return dl

#range of redshift values
z = np.linspace(0, 2, 201)

#plot formatting
fig = plt.figure()
ax = fig.add_subplot(111, autoscale_on=False, xlim=(0, 2), ylim=(0, 16000))
custom_font = "Times New Roman"
ax.set_xlabel(r"z", fontname=custom_font, size=14)
ax.set_ylabel(r"$d_{L}$ [Mpc]", fontname=custom_font, size=14)
ax.set_xticks([0,0.2,0.4,0.6,0.8,1,1.2,1.4,1.6,1.8,2])
ax.set_yticks([0,2000,4000,6000,8000,10000,12000,14000,16000])
for tick in ax.set_xticklabels([0,0.2,0.4,0.6,0.8,1,1.2,1.4,1.6,1.8,2]):
    tick.set_fontname(custom_font)
for tick in ax.set_yticklabels([0,2000,4000,6000,8000,10000,12000,14000,16000]):
    tick.set_fontname(custom_font)
plt.xticks(fontsize=14)
plt.yticks(fontsize=14)

#plotting the graph
plt.plot(z, lambdaCDM(z),color="red", label='ΛCDM dark energy model')
plt.plot(z, gravity(z), color="black", label='DGP modified gravity model')
ax.legend(prop={'family':custom_font, 'size':14}, loc=0,frameon=False)
plt.plot()
plt.show()
```

C.2.3. Calculating fractional deviation in luminosity distance for the modified gravity and ΛCDM models.

```
#importing helpful library
import math
from matplotlib import pyplot as plt
```

```
import numpy as np
import matplotlib.font_manager
from IPython.core.display import HTML
from matplotlib import patheffects

#getting the font
def make_html(fontname):
    return “<p>{font}: <span style=‘font-family:{font}; \
    font-size: 24px;’>{font}</p>”.format(font=fontname)
code = “\n”.join([make_html(font) for font in sorted(
set([f.name for f in matplotlib.font_manager.fontManager.ttflist]))])
HTML(“<div style=‘column-count: 2;’>{}</div>”.format(code))

#plot formatting
plt.rcParams[“figure.figsize”] = [8.6, 6]
plt.rcParams[“figure.autolayout”] = True

#defining constants
Omega_X = 0.692
Omega_m = 0.308
c = 2.998*(10**8)
H_0 = 67.81*(10**3)
G = 6.67/(10**11)
p_c = (3*(H_0**2))/(math.pi*8*G)
p_X = Omega_X*p_c
p_m = Omega_m*p_c

#function to find the luminosity distance for the lambdaCDM model
def lambdaCDM(z):
    #setting base barotropic parameter
    w_X = -1
    #scale factor “a” defined
    a = 1/(1+z)
    #empty array defined now to hold d(z).
    d = np.array([])
    #loop through each value of a
    for aPrime in a:
        #starting integration values
        dz = 0.0
        ddz = 0.0
        da = 0.001
        #integral to find distance
        while aPrime <1:
            #starting integration values
            rho = 0.0
            drho = 0.0
            aPrime2 = aPrime
            dda = 0.001
            #integral to find ln(rho_x(a’))/ln(rho_x,0)
            while aPrime2 <1:
                drho = ((3*(1+w_X))/aPrime2) * dda
```

```
                rho = rho + drho
                aPrime2 = aPrime2 + dda
            #calculating rho_x(a’) from the integral
            p_a = p_X*(math.e**rho)
            #calculating rho_m(a’)
            p_b = p_m*((1/aPrime)**3)
            #calculating H(a’)
            H = ((8*math.pi*(p_a+p_b)*G)/3)**0.5
            ddz = (c/(H*(aPrime**2)))*da
            dz = dz + ddz
            aPrime = aPrime + da
        #adding the distances to the array
        d = np.append(d, dz)
    #calculating luminosity distance and adding to array
    dl = np.add(1, z)*d
    return dl

#function to calculate the luminosity distance for the modified gravity model.
def gravity(z):
    #scale factor “a” defined
    a = 1/(1+z)
    #empty array defined now to hold d(z).
    d = np.array([])
    #loop through each value of a
    for aPrime in a:
        #starting integration values
        dz = 0.0
        ddz = 0.0
        da = 0.001
        #integral to find distance
        while aPrime <1:
            #calculating H(a’)
            H = (H_0*(1-Omega_m)+H_0*((((1-Omega_m)**2)+(4*Omega_m*
            ((1/aPrime)**3)))**0.5))/2
            ddz = (c/(H*(aPrime**2)))*da
            dz = dz + ddz
            aPrime = aPrime + da
        #adding the distances to the array
        d = np.append(d, dz)
    #calculating luminosity distance and adding to array
    dl = np.add(1, z)*d
    return dl

#range of redshift values
z = np.linspace(0, 2, 201)

#calculating the fractional deviation
grav = gravity(z)
CDM = lambdaCDM(z)
frac = abs((grav-CDM)/grav)
```

```
#lists for the observable region for formatting
sigmafrac = [0.0036,0.0036]
xtotalrange = [0,2]
ysigmamax = [0.08,0.08]

#plot formatting
fig = plt.figure()
ax = fig.add_subplot(111, autoscale_on=False, xlim=(0, 2), ylim=(0, 0.08))
custom_font = “Times New Roman”
ax.set_xlabel(r“z”, fontname=custom_font, size=14)
ax.set_ylabel(r“$—Δd_{L}/d_{L}—$”, fontname=custom_font, size=14)
ax.set_xticks([0,0.2,0.4,0.6,0.8,1,1.2,1.4,1.6,1.8,2])
ax.set_yticks([0,0.01,0.02,0.03,0.04,0.05,0.06,0.07,0.08])
for tick in ax.set_xticklabels([0,0.2,0.4,0.6,0.8,1,1.2,1.4,1.6,1.8,2]):
    tick.set_fontname(custom_font)
for tick in ax.set_yticklabels([0,0.01,0.02,0.03,0.04,0.05,0.06,0.07,0.08]):
    tick.set_fontname(custom_font)
plt.xticks(fontsize=14)
plt.yticks(fontsize=14)

#plotting the graph
plt.plot(z, frac, color=“black”)
plt.plot([0,0.03],[0.0036,0.0036], color=“red”, linestyle=“dotted”)
plt.plot([0.03, 0.03],[0, 0.0036], color=“red”, linestyle=“dotted”)
plt.fill_between(xtotalrange, ysigmamax, sigmafrac,color=“red”,alpha=.1)
plt.text(0.75,0.025, “Region for ‘potential’ observation”, fontname=custom_font, fontsize = 14)
plt.text(-0.15,0.0032, “0.0036”, fontname=custom_font, fontsize = 14)
plt.text(0.02,-0.003, “0.03”, fontname=custom_font, fontsize = 14)
plt.plot()
plt.show()
```

C.2.4. Code snippet to find the redshift value that may be observed.

```
z = np.linspace(0, 0.04, 100)
for index,item in enumerate(frac):
    print(“i=”+str(index)+“ f=”+str(item))
for index,item in enumerate(z):
    print(“i=”+str(index)+“ f=”+str(item))
```

D Agendas

D.1 Week 1

Gamma Collaboration Meeting

13:00 Tuesday 30th January, Library A11.

AGENDA

Apologies for absence

No note of any absences.

Minutes of previous meeting

No previous meeting.

Matters arising
No previous minutes or actions to confirm.

Matters for discussion
1. Choose a coordinator.
2. We read through the project so that we knew what we were doing.
3. Assign major roles with responsibilities.
   - Literature Searches and Background Physics
   - Theory Calculations
   - Computing
   - Figure Generation and Result Presentation
   - Report Collation, Writing and Editing
4. Assign any sub-roles that may arise.
5. Create a research plan.
6. Create teams/chat to share files.

Any other business
No other business.

D.2 Week 2

Gamma Collaboration Meeting

14:00 Tuesday 6th February, Library B7.

AGENDA

Apologies for absence
No note of any absences.

Minutes of the previous meeting
Read through previous minutes. They are complete.

Matters arising
1. Check on task progression.

Matters for discussion
1. Following on from the matters arising, what tasks have been completed?
2. Review content for completed tasks.
3. What tasks are currently being done?
4. Help with tasks that are being struggled on currently.
5. What tasks still need to be assigned.
6. Comparison with Gantt Chart.
7. Report planning.

Any other business
No other business.

D.3 Week 3

Gamma Collaboration Meeting

14:00 Tuesday 13th February, Library B10.

AGENDA

Apologies for absence
No note of any absences.

Minutes of the previous meeting
Read through previous minutes. They are complete.

Matters arising
1. Check on task progression.

Matters for discussion
1. Following on from the matters arising, what tasks have been completed?
2. Review content for completed tasks.
3. What tasks are currently being done?
4. Help with tasks that are being struggled on currently.
5. What tasks still need to be assigned.
6. Comparison with Gantt Chart.

Any other business
No other business.

D.4 Week 4

Gamma Collaboration Meeting

14:00 Tuesday 20th February, Library B10.

AGENDA

Apologies for absence
No note of any absences.

Minutes of the previous meeting
Read through previous minutes. They are complete.

Matters arising
1. Check on task progression.

Matters for discussion
1. Following on from the matters arising, what tasks have been completed?
2. Review content for completed tasks.
3. What tasks are currently being done?
4. Help with tasks that are being struggled on currently.
5. What tasks still need to be assigned.
6. Comparison with Gantt Chart.

Any other business
No other business.

D.5 Week 5

Gamma Collaboration Meeting

13:00 Wednesday 28th February, Library B10.

AGENDA

Apologies for absence
Owen will be absent.

Minutes of the previous meeting
Read through previous minutes. They are complete.

Matters arising
1. Check on task progression.

Matters for discussion
1. Review what tasks have been done.
2. Make sure everyone is happy with the few remaining tasks. Owen – Chi-Squared, Paresh – the very first task again, Joseph – The very last task, Dan – graph cleanup, Will – starting the report.
3. Go over all the completed work thus far making sure everyone understands what has been done.
4. Any clean-up that needs to be done will also be discussed.
5. Make sure everyone has access to the Overleaf project.

Any other business
No other business.

D.6 Week 6

Gamma Collaboration Meeting

11:00 Tuesday 5th March, Library B11.

AGENDA

Apologies for absence
No absences.

Minutes of the previous meeting
Read through previous minutes. They are complete.

Matters arising
1. Check on task progression.

Matters for discussion
1. Checked that everyone had finished their tasks.
2. Go through the entire project, everyone explaining what they have done.
3. Start planning the written report.

Any other business
No other business.

E Minutes

E.1 Week 1

Gamma Collaboration Meeting

13:00 Tuesday 30th January, Library A11.

Minutes

Attendance (X/P)
Owen Applegarth - P
Dan Astbury - P
Joseph Ashworth - P
Will Crick - P
Paresh Dokka - P

Apologies for absence
No in-advance absences to note of.

Minutes of the previous meeting
No previous meeting.

Matters arising
No action has been taken as of yet.

Matters Discussed
1. Choosing a coordinator. Dan is the coordinator, Joseph is the administrator.
2. Reading through the project document.
   - Dan has already started tasks 1a, 2.
3. Our timetables were discussed, this may decide what roles people play.
4. Create individual logbooks – We already have logbooks.
5. Going through the project, assigning roles/tasks. We have decided not to assign specific roles, but, individual tasks instead.
6. We went through half the project assigning the tasks while leaving the second half for later planning.
7. Those with less to do will help others who have more and maybe struggling.
8. Creating the presentation plan.
9. Teams were created.
10. Meeting for next week planned.

Any other business
1. Install libraries for coding
2. Start tasks that have already been given.

E.2 Week 2

Gamma Collaboration Meeting

14:00 Tuesday 6th February, Library B7.

Minutes

Attendance (X/P)
Owen Applegarth - P
Dan Astbury - P
Joseph Ashworth - P
Will Crick - P
Paresh Dokka - P

Apologies for absence
No in-advance absences to note of.

Minutes of the previous meeting
Reviewed previous minutes. They are complete.

Matters arising
1. Good progress has been made. Most tasks assigned have been completed, a couple longer ones remain.

Matters Discussed
1. Discussed part 1. b) with the derivation and talking about how to plot it. Still not entirely sure yet.
2. Reviewed which tasks were completed and the content coming from those tasks in the project document.
3. For tasks currently being worked on, we need to figure out how to plot the point talked about in point 1. We may need to ask for help with this one.
4. Assigned the 2 large coding tasks to 2 pairs to do together.
5. We assigned a few extra tasks so everyone had enough to do, but some tasks remain to be assigned.
6. We are happy with the progress that has been made. So far we are mostly ahead of schedule, although we have a couple of large tasks this week.
7. We decided the report doesn't need to be planned yet.

Any other business
No other business.

E.3 Week 3

Gamma Collaboration Meeting

14:00 Tuesday 13th February, Library B10.

Minutes

Attendance (X/P)

Owen Applegarth - P
Dan Astbury - P
Joseph Ashworth - P
Will Crick - P
Paresh Dokka - P

Apologies for absence

No in-advance absences to note of.

Minutes of the previous meeting

Reviewed previous minutes. They are complete.

Matters arising

1. Good progress has been made. Most tasks assigned have been completed, but some in part 3 remain to be allocated.

Matters Discussed

1. Discussed the work that has been completed. Everyone has made good progress.
2. Discussed the work currently taking place. Will is doing research, Joseph and Owen are currently programming, and Dan is making graphs and finishing some tasks we accidentally skipped over. Paresh has been assigned some additional tasks.
3. Joseph's and Will's tasks align so we will work together for the meantime and Paresh will continue with research.
4. Everyone is happy with what has been completed and what they need to do.
5. Comparison with the Gantt chart shows we are making excellent progress.
6. We will assign the few remaining tasks next time.

Any other business

1. The meeting room has been booked for next week.

E.4 Week 4

Gamma Collaboration Meeting

14:00 Tuesday 20th February, Library B10.

Minutes

Attendance (X/P)

Owen Applegarth - P
Dan Astbury - P
Joseph Ashworth - P
Will Crick - P
Paresh Dokka - X

Apologies for absence

No in-advance absences to note of.

Minutes of the previous meeting

Reviewed previous minutes. They are complete.

Matters arising

1. Good progress has been made. Most tasks assigned have been completed, but some in part 3 remain to be allocated.

Matters Discussed

1. Discussed the work that has been completed. Everyone has made good progress.
2. Everyone is happy with the content of what has been completed.
3. Joseph has completed his current tasks and can continue to the next lot. Will is continuing to do research. Paresh is continuing a derivation and then needs to upload it to Teams. Owen is continuing the chi-squared code. Dan has been making the graphs/plots and will start writing the report/setting it up.
4. Everyone is happy with their current tasks and what needs to be done next.
5. Will and Paresh will potentially work together to complete a few tasks in the future.
6. We still have a couple of tasks to be allocated, but they rely on work yet to be done, so they will be allocated once the tasks beforehand have been completed. We will most likely be allocated to Joseph or Owen.
7. When all tasks have been completed, all the content will be reviewed so everyone understands what has been done.

Any other business

1. The meeting room has been booked for next week.
2. We need to place the agendas and minutes onto the Teams chat for everyone to see.

E.5 Week 5

Gamma Collaboration Meeting

12:00 Wednesday 28th February, Library B10.

Minutes

Attendance (X/P)
Owen Applegarth – X
Dan Astbury - P
Joseph Ashworth - P
Will Crick - P
Paresh Dokka - P

Apologies for absence
Owen is absent and that was known.

Minutes of the previous meeting
Reviewed previous minutes. They are complete.

Matters arising
1. Good progress has been made. Almost all tasks have been completed. Very few remain by this point.

Matters Discussed
1. The tasks that were supposed to be done last week have been completed. Everyone is happy with the progress made. Everything that has been done makes sense.
2. Owen – continues the chi-squared, Paresh – continues the research, Joseph and Will – the generalisation of modified gravity, and Dan – graph cleanup.
3. Owen is not here so we can't review the whole project so that we all understand. This will be conducted next week.
4. We have all logged into the Overleaf group project allowing us to work on it concurrently. We will need to make sure Owen has done this next week too.
5. If people finish their tasks next week, some clean-up in code/writing can take place and report writing can ensue.
6. To do this Friday – finish the remaining tasks. Clean up the code and make sure the writing and derivations are legible. Place work in the correct files for organisation and ease of use.

Any other business
1. The meeting room has been booked for next week.

E.6 Week 6

Gamma Collaboration Meeting

11:00 Tuesday 5th March, Library B11.

Minutes

Attendance (X/P)

Owen Applegarth – P
Dan Astbury - P
Joseph Ashworth - P
Will Crick - P
Paresh Dokka - P

Apologies for absence

No Absences

Minutes of the previous meeting

Reviewed previous minutes. They are complete.

Matters arising

1. Good progress has been made. Essentially all tasks have been completed. Owen just needs to do 1 more graph.

Matters Discussed

1. We are going through the entire project explaining what we have done.
2. The age of the Universe was determined by globular clusters, by getting the age of the cluster + time between the Big Bang and the cluster formed. It is estimated that the youngest stars formed. The age of the cluster is from the main sequence turn-off method. Dark matter content – estimated using galaxy clustering, with x-ray gas to determine the internal matter in a cluster.
3. Derivation for the age of the Universe starts from the Friedmann and continuity equations Just following the derivation explaining what is going on. Also, we went through the age of the universe as a function of redshift.
4. Explain the next couple of graphs with the code. A lot of the code is the same, and it has been commented on, so we will mostly skip through the code unless something major needs to be explained.
5. Explaining the main numerical integration code for the t0 against matter universe.
6. Explaining the chi-squared code.
7. Explaining how we got to the point where the Universe became dominated by the cosmological constant.
8. Everyone explains their part of task 1b. The derivation is similar to the one before. Code and plot are also similar so not much needs to be explained, but we are reviewing it nonetheless.
9. Going through the part 2 preliminary derivations.
10. Explaining the Euclid tasks and BAO angles. The research, derivations and luminosity distance were discussed.
11. Explaining the Roman telescope and its missions.
12. Explaining the SFQ and the comparison with LamdbaCDM.
13. Explaining part 3 – modified gravity. The concept, derivations and graphs etc.
14. Discussed generalising to $\alpha \neq 1$.

Any other business

1. No more meetings from here on out.

F Research Plan

Dark Energy Research Plan

Gamma Collaboration

D Astbury, J Ashworth, O Applegarth, W Crick, P Dokka

Gantt Chart

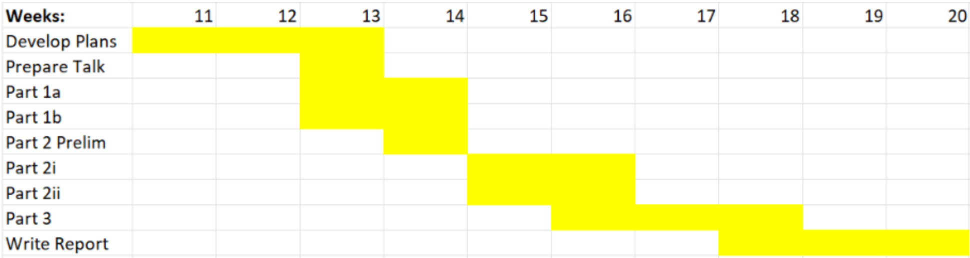

Figure 16: Gantt chart showing a rough rundown of which tasks need to be done in which time periods.

Reactive Task Allocation

During each stage of this project, there are collaborators with higher and lower workloads across specific periods. The higher workloads will take on a persistent task designed to span the period, whereas the lower workloads will finish partway through this set time and use the remaining resources to assist the longer tasks when required. This stratagem allows adaptation of resources and time management. We believe two people focused on one task in the later stages is more efficient than each person alone.

Table of Individual Tasks

Owen	Dan	Joseph	Will	Paresh
Part I (a) 3.)	Part I (a) 2.)	Admin: Write agenda and minutes each week. -Collate the data (potentially start putting it into a report style) - explicitly state the highlighted parts. -Derive BAO angle. -Luminosity Distance - Code parts in the second half of the project.	Part I (b) 1.)	Part I (a) 1.)
Plot the theoretical age t_0 as a function of Ω_Λ Plot derivation of t0 against z Calculate value when universe becomes dominated by the cosmological constant. PLot $\Omega_{m,0}$ for $w_x = -0.3, -0.5, -0.7, -0.9$. On the same graph, plot the 1-σ and 2-σ observational limits on t_0 and $\Omega_{m,0}$.	Derivation for the Age of the Universe, t_0, and the Age of the Universe at a redshift, z.		Numerically integrate Equation (2) for the case where ρ_Λ is replaced by $\rho_X(a)$.	Brief explanation of using globular clusters to estimate age of universe.
	Part II (i) 4) **		Part II (i) 1.) & 2.)	Part II
	Derivation for the energy density, ρ_X, and the physical distance, $d(z)$.		Description of the Euclid mission. Explain the physics of BAO and how the BAO length is obtained. Assist others with larger tasks if needed.	Preliminary measurements: derive the theory. **

Figure 17: Table showing the rundown of tasks assigned to each member for the beginning of the project.

Remaining Tasks

We have allocated tasks until the Chi-squared test at the end of part II (i) and decided the workload for each collaborator is sufficient until the next meeting. We endeavour to complete each task to the best of our ability, cooperating to check and resolve any issues in the meeting or working together during research sessions. We will assess current workloads in the upcoming meeting, distributing the remaining tasks upon agreement. We aim to discuss the collation process for the report in preparation for the report writing sessions.