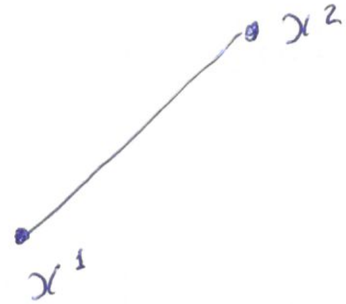
Conjuntos Convexos

Funções Convexas e Côncavas

Segmento:

$x^1 \in R^n$ e $x^2 \in R^n$

$0 \leq K \leq 1$



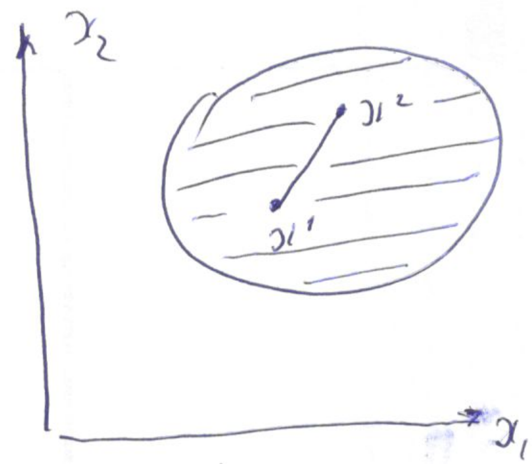
$$\boxed{x = (1-k)x^1 + k x^2}$$

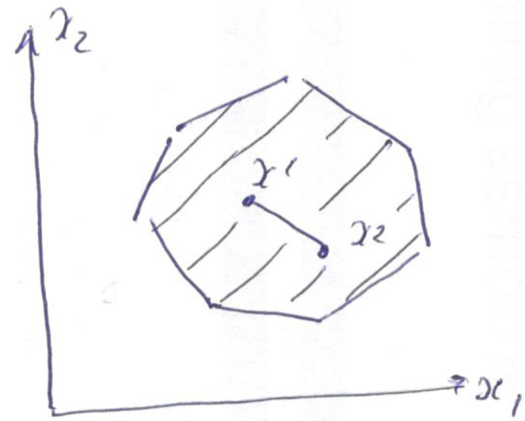
$K = 1 \Rightarrow x^1$

$K = 0 \Rightarrow x^2$

$$x_i = (1-k)x_i^1 + k x_i^2,$$

$i = 1, 2, \ldots, n$





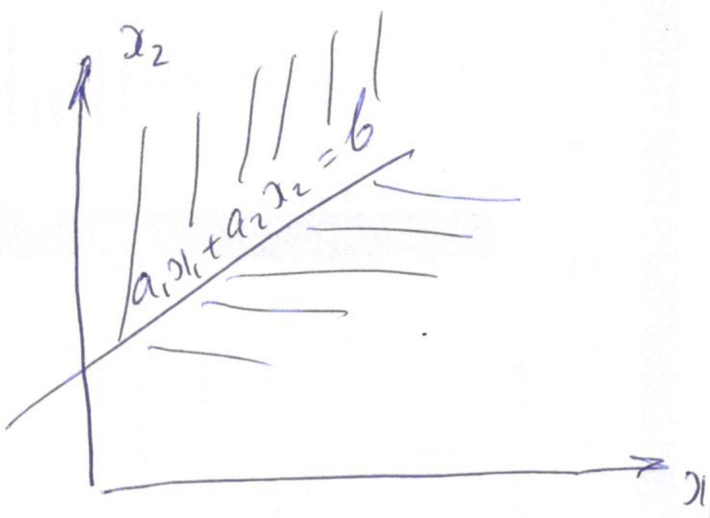
Um hiperplano

$$\sum_{i=1}^{n} a_i x_i = b$$

define dois conjuntos

$$\left.\begin{array}{l} \displaystyle\sum_{i=1}^{n} a_i x_i \geq b \\ \text{e} \quad \displaystyle\sum_{i=1}^{n} a_i x_i \leq b \end{array}\right\} \text{meio-espaços fechados}$$

Se $>$, $<$, então abertos.



Verificamos, por exemplo,

$$\sum_{i=1}^{n} a_i x_i \leq b$$

é convexo?

Escolhemos dois pontos arbitrarios que pertencem ao

$$\sum_{i=1}^{n} a_i x_i \leq b$$

Então

$$\sum_{i=1}^{n} a_i x_i^1 \leq b$$

e

$$\sum_{i=1}^{n} a_i x_i^2 \leq b \, .$$

$$\sum_{i=1}^{n} a_i x_i^1 \leq b \quad \times (1-k)$$

$$\sum_{i=1}^{n} a_i x_i^2 \leq b \quad \times k$$

$$(1-k) \sum_{i=1}^{n} a_i x_i^1 + k \sum_{i=1}^{n} a_i x_i^2 \leq (1-k) b + k b = b$$

$$\sum_{i=1}^{n} a_i \left[(1-k) x_i^1 + k x_i^2 \right] \leq b$$

Teorema

Uma interseção de conjuntos convexos é, também, convexo.

Temos igualdades

$$\sum_{i=1}^{n} a_{ji} x_i = b_j, \quad j=1,\dots,J$$

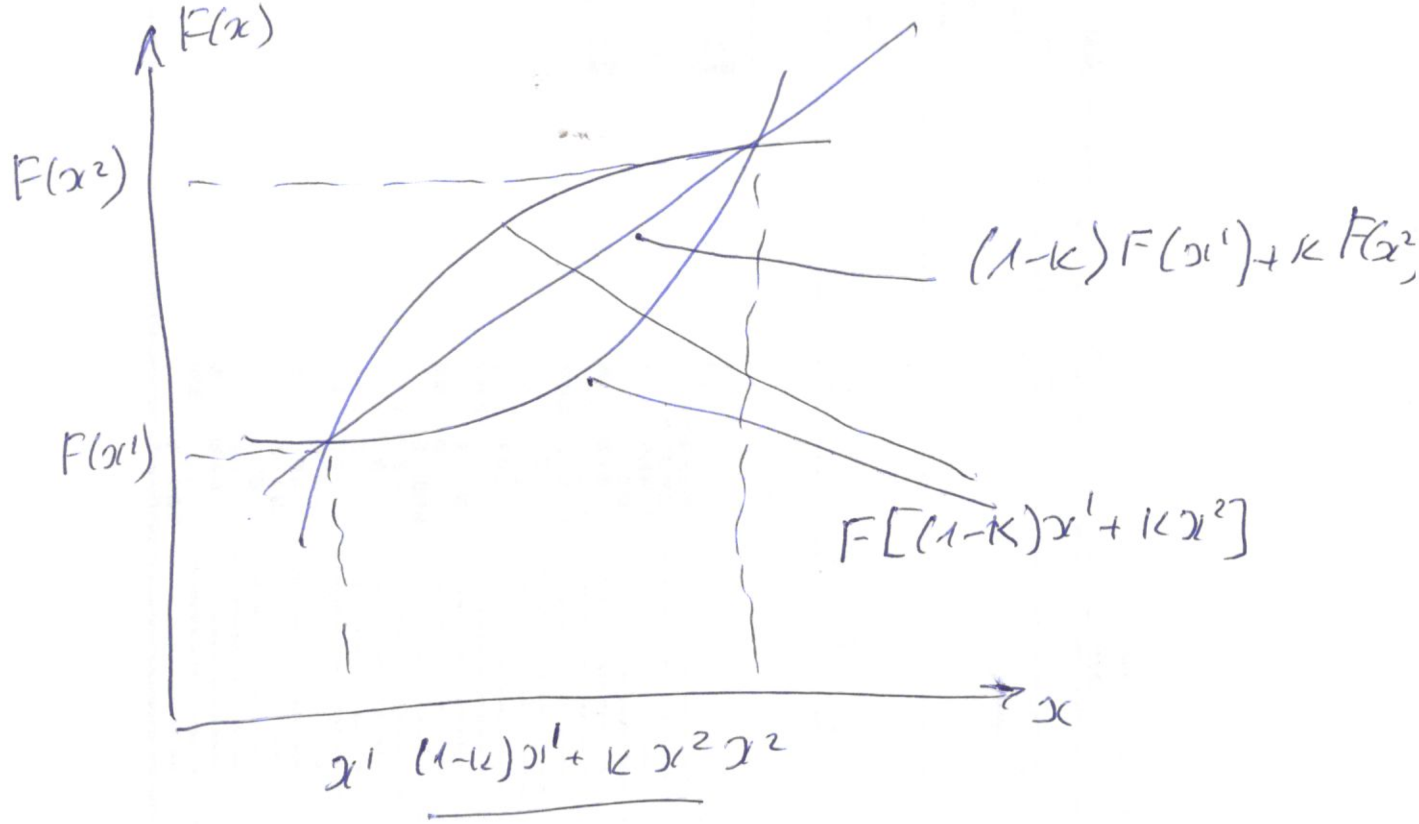
e desigualdades

$$\sum_{i=1}^{n} a_{ki} x_i \ge b_k, \quad k=1,\dots,K$$

$$\sum_{i=1}^{n} a_{li} x_i \le b_l, \quad l=1,\dots,L$$

é conjunto convexo ou vazio.

Funções Convexas e Concavas

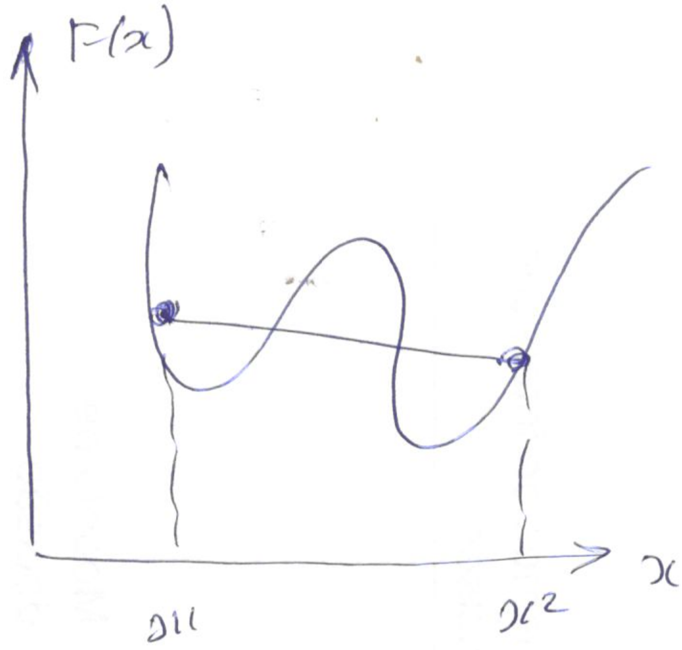


Uma FO chama-se convexa no segmento $[x^1, x^2]$ se

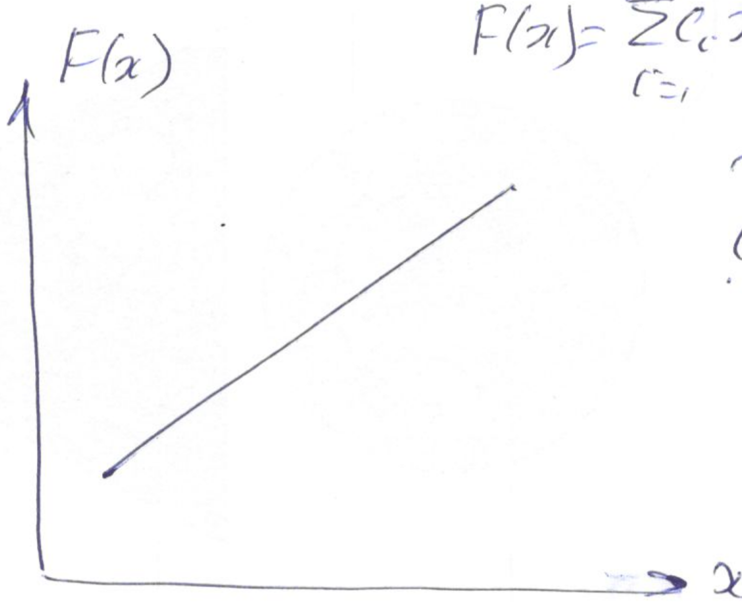
$$F[(1-k)x^1 + kx^2] \leq (1-k)F(x^1) + kF(x^2)$$

Uma FO chama-se concava no segmento $[x^1, x^2]$ se

$$F[(1-k)x^1 + kx^2] \geq (1-k)F(x^1) + kF(x^2)$$



$$F(x) = \sum_{i=1}^{n} c_i \cdot x_i$$



?

Para identificar o caráter da FO (convexa, côncava ou não convexa e não côncava).

Para isso, é possível usar uma matriz das derivadas parciais da segunda ordem (matriz de Hesse (Hessiano).

$$H = \nabla^2 F(x) = \begin{bmatrix} \frac{\partial^2 F(x)}{\partial x_1^2} & \frac{\partial^2 F(x)}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 F(x)}{\partial x_1 \partial x_n} \\ \frac{\partial^2 F(x)}{\partial x_2 \partial x_1} & \frac{\partial^2 F(x)}{\partial x_2^2} & \cdots & \frac{\partial^2 F(x)}{\partial x_2 \partial x_n} \\ & \cdots & & \\ \frac{\partial^2 F(x)}{\partial x_n \partial x_1} & \frac{\partial^2 F(x)}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 F(x)}{\partial x_n^2} \end{bmatrix}$$

Minores principais:

$$\Delta_1 = h_1$$

$$\Delta_2 = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

$$\Delta_3 = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$$

convexa concava

Δ_1	>0	<0
Δ_2	>0	>0
Δ_3	>0	<0
$\vdots$		$\vdots$

$$\Delta P = F(Q_{c1}, Q_{c2}) = \frac{1}{10^3 V^2}\left[(Q_1+Q_2-Q_{c1}-Q_{c2})^2 R_{0-1} + (Q_1-Q_{c1})^2 R_{T1} + (Q_2-Q_{c2})^2 (R_{1-2}+R_{T2})\right] + \operatorname{tg}\delta\,(Q_{c1}+Q_{c2})$$

$Q_1 = 400\,\text{kVar}$ $\quad R_{0-1} = 0{,}225\,\Omega$ $\quad R_{T1} = 1{,}22\,\Omega$

$Q_2 = 300\,\text{kVar}$ $\quad R_{1-2} = 0{,}315\,\Omega$ $\quad R_{T2} = 2{,}12\,\Omega$

$$F(Q_{c1}, Q_{c2}) = 5{,}246 - 0{,}00841\,Q_{c1} - 0{,}01326\,Q_{c2} + 0{,}45\cdot10^{-5}\,Q_{c1}Q_{c2} + 1{,}445\cdot10^{-5}\,Q_{c1}^2 + 2{,}660\cdot10^{-5}\,Q_{c2}^2$$

$$H = \begin{bmatrix} 2,89 \cdot 10^{-5} & 0,45 \cdot 10^{-5} \\ 0,45 \cdot 10^{-5} & 5,32 \cdot 10^{-5} \end{bmatrix}$$

$$\Delta_1 = 2,89 \cdot 10^{-5}$$

$$\Delta_2 = \begin{vmatrix} 2,89 \cdot 10^{-5} & 0,45 \cdot 10^{-5} \\ 0,45 \cdot 10^{-5} & 5,32 \cdot 10^{-5} \end{vmatrix} = 15,17 \cdot 10^{-5}$$