Métodos da Segunda Ordem

Em alguns casos, é possível utilizar a seguinte correlação:

$$x^{(p+1)} = x^{(p)} - a \nabla F(x^{(p)}), \; p = 0, 1, \dots$$

ponto mínimo = gradiente menor

Neste caso a velocidade da busca diminui se estamos mais perto de extremo, porque $|\nabla F(x^{(p)})|$ é menor no ponto $x^{(p)}$.

Entretanto, o melhor escolha do comprimento do passo, é possível obter usando o método da descida mais rapida quando o valor de a_p é definido como resultado de minimização da função:

$$\varphi_p(a) = \varphi_p \left[x^{(p)} - a \frac{\nabla F(x^{(p)})}{|\nabla F(x^{(p)})|} \right.$$

ou

$$x^{(p+1)} = x^{(p)} - a \nabla F(x^{(p)}), \quad p = 0, 1, \ldots$$

da uma variavel e é possivel achar

$$a_p = \underset{a \geq 0}{\arg\min}\ \varphi_p(a)$$

para achar comprimento do passo ótimo

Consideramos caso de duas variaveis. Usando a serie de Teilor, é possivel escrever:

até 2ª ordem

ponto α, β

$$F(x_1, x_2) \approx F(\alpha, \beta) + \left[\frac{\partial F(\alpha,\beta)}{\partial x_1} (x_1 - \alpha) + \frac{\partial F(\alpha,\beta)}{\partial x_2} (x_2 - \beta) \right] +$$

ponto que quero estar menos o que estou

$$+ \frac{1}{2} \left[\frac{\partial^2 F(\alpha,\beta)}{\partial x_1^2} (x_1 - \alpha)^2 + 2 \frac{\partial^2 F(\alpha,\beta)}{\partial x_1 \partial x_2} \times (x_1 - \alpha)(x_2 - \beta) + \frac{\partial^2 F(\alpha,\beta)}{\partial x_2^2} (x_2 - \beta)^2 \right] + \ldots$$

objetivo: achar comprimento do passo ótimo comprimento do passo

Temos:

$$x_1 = x_1^{(p+1)}; \quad \alpha = x_1^{(p)};$$

gradiente

$$x_1 - \alpha = x_1^{(p+1)} - x_1^{(p)} = -a \, \frac{\partial F(x_1^{(p)}, x_2^{(p)})}{\partial x_1}$$

$$x_2 = x_2^{(p+1)}; \quad \beta = x_2^{(p)};$$

$$x_2 - \beta = x_2^{(p+1)} - x_2^{(p)} = -a \, \frac{\partial F(x_1^{(p)}, x_2^{(p)})}{\partial x_2}$$

$$F(x_1, x_2) = F(x_1^{(p+1)}, x_2^{(p+1)})$$

$$F(\alpha, \beta) = F(x_1^{(p)}, x_2^{(p)}).$$

Para simplificar, vamos não escrever $x_1^{(p)}$ e $x_2^{(p)}$.

$$F\left(x_1^{(p+1)}, x_2^{(p+1)}\right) \approx F\left(x_1^{(p)}, x_2^{(p)}\right) +$$

parte linear

$$\left[\frac{\partial F}{\partial x_1}\left(-a\frac{\partial F}{\partial x_1}\right) + \frac{\partial F}{\partial x_2}\left(-a\frac{\partial F}{\partial x_2}\right)\right] +$$

parte quadrática

$$+\frac{1}{2}\left[\frac{\partial^2 F}{\partial x_1^2}\left(-a\frac{\partial F}{\partial x_1}\right)^2 + 2\frac{\partial^2 F}{\partial x_1 \partial x_2}\left(-a\frac{\partial F}{\partial x_1}\right)\left(-a\frac{\partial F}{\partial x_2}\right) + \right.$$

$$\left. + \frac{\partial^2 F}{\partial x_2^2}\left(-a\frac{\partial F}{\partial x_2}\right)^2\right] =$$

$$= F\left(x_1^{(p)}, x_2^{(p)}\right) + \left[-a\left(\frac{\partial F}{\partial x_1}\right)^2 - a\left(\frac{\partial F}{\partial x_2}\right)^2\right] +$$

$$+\frac{1}{2}\left[\frac{\partial^2 F}{\partial x_1^2}a^2\left(\frac{\partial F}{\partial x_1}\right)^2 + 2\frac{\partial^2 F}{\partial x_1 \partial x_2}a^2\frac{\partial F}{\partial x_1}\frac{\partial F}{\partial x_2} + \right.$$

$$\left. + \frac{\partial^2 F}{\partial x_2^2}a^2\left(\frac{\partial F}{\partial x_2}\right)^2\right]$$

Agora podemos construir uma derivada.

$$\frac{\partial F\left(x_1^{(p+1)}, x_2^{(p+1)}\right)}{\partial a} = -\left(\frac{\partial F}{\partial x_1}\right)^2 - \left(\frac{\partial F}{\partial x_2}\right)^2 +$$

$$+ \frac{\partial^2 F}{\partial x_1^2}\left(\frac{\partial F}{\partial x_1}\right)^2 a + 2\,\frac{\partial^2 F}{\partial x_1 \partial x_2}\, a\, \frac{\partial F}{\partial x_1}\frac{\partial F}{\partial x_2} +$$

$$+ \frac{\partial^2 F}{\partial x_2^2}\left(\frac{\partial F}{\partial x_2}\right)^2 a = 0.$$

$$a\left[\frac{\partial^2 F}{\partial x_1^2}\left(\frac{\partial F}{\partial x_1}\right)^2 + 2\,\frac{\partial^2 F}{\partial x_1 \partial x_2}\,\frac{\partial F}{\partial x_1}\,\frac{\partial F}{\partial x_2} +\right.$$

$$\left. + \frac{\partial^2 F}{\partial x_2^2}\left(\frac{\partial F}{\partial x_2}\right)^2\right] = \left(\frac{\partial F}{\partial x_1}\right)^2 + \left(\frac{\partial F}{\partial x_2}\right)^2$$

$$a_p = \frac{\left(\frac{\partial F}{\partial x_1}\right)^2 + \left(\frac{\partial F}{\partial x_2}\right)^2}{\frac{\partial^2 F}{\partial x_1^2}\left(\frac{\partial F}{\partial x_1}\right)^2 + 2\,\frac{\partial^2 F}{\partial x_1 \partial x_2}\,\frac{\partial F}{\partial x_1}\,\frac{\partial F}{\partial x_2} + \frac{\partial^2 F}{\partial x_2^2}\left(\frac{\partial F}{\partial x_2}\right)^2}$$

De tal forma, necessitem fer:

$$\frac{\partial F(x_1^{(p)}, x_2^{(p)})}{\partial x_1} \qquad \frac{\partial F(x_1^{(p)}, x_2^{(p)})}{\partial x_2}$$

$$\left(\frac{\partial F(x_1^{(p)}, x_2^{(p)})}{\partial x_1}\right)^2 \qquad \left(\frac{\partial F(x_1^{(p)}, x_2^{(p)})}{\partial x_2}\right)^2$$

$$\frac{\partial^2 F(x_1^{(p)}, x_2^{(p)})}{\partial x_1^2} \qquad \frac{\partial^2 F(x_1^{(p)}, x_2^{(p)})}{\partial x_2^2}$$

$$\frac{\partial^2 F(x_1^{(p)}, x_2^{(p)})}{\partial x_1 \partial x_2}$$

$$\frac{\partial F(x_2, x_3)}{\partial x_2} = \frac{1}{110^2}\left[-53 + 2x_2 + 0{,}6\, x_3\right]$$

$$\frac{\partial (x_2, x_3)}{\partial x_3} = \frac{1}{110^2}\left[-40 + 0{,}6\, x_2 + 2{,}8\, x_3\right]$$

$$\frac{\partial^2 F(x_2, x_3)}{\partial x_2^2} = \frac{2}{110^2} \qquad \frac{\partial^2 F(x_2, x_3)}{\partial x_3^2} = \frac{2{,}8}{110^2}$$

$$\frac{\partial^2 F(x_2, x_3)}{\partial x_2 \, \partial x_3} = \frac{0,6}{110^2}$$

$p=0$: $$\frac{\partial F(x_2^{(0)}, x_3^{(0)})}{\partial x_2} = \frac{1}{110^2}[-53] = -0,000438$$

$$\frac{\partial F(x_2^{(0)}, x_3^{(0)})}{\partial x_3} = \frac{1}{110^2}[-40] = -0,00331$$

$$a_0 = \frac{(-0,00438)^2 + (-0,00331)^2}{\frac{2}{110^2}(-0,00438)^2 + \frac{0,6}{110^2}(-0,00438)(-0,00331) + \frac{2,8}{110^2}(-0,00331)^2} =$$

$$= 4537,5$$

$$x_2^{(0)} = 0 - 4537,5(-0,00438) = 19,87$$

$$x_3^{(0)} = 0 - 4537,5(-0,00331) = 15,02$$

$$\frac{\partial F(x_2^{(1)}, x_3^{(1)})}{\partial x_2} = \frac{1}{110^2}\left[-53 + 2 \cdot 19{,}87 + 0{,}6 \cdot 15{,}02\right] = -0{,}00035$$

$$\frac{\partial F(x_2^{(1)}, x_3^{(1)})}{\partial x_3} = \frac{1}{110^2}\left[-40 + 0{,}6 \cdot 19{,}87 + 2{,}8 \cdot 15{,}02\right] = 0{,}00116$$