1 Funkje trygonometryczne

$\sin(x)$, $\cos(x)$

$\sin^2(x) + \cos^2(x) = 1$

$\sin(x+y) = \sin x \cos y + \sin y \cos x$ (=s)

$\cos(x+y) = \cos x \cos y - \sin y \sin x$

$0 = \sin(x-x) = \sin x \cos(-x) + \sin(-x)\cos x$ $\quad / \cdot \sin(-x)$

$1 = \cos(x-x) = \cos x \cos(-x) - \sin(-x)\sin x$ $\quad / \cdot \cos(-x)$

$0 = \sin x \cos(-x)\sin(-x) + \sin^2(-x)\cos x$

$\cos(-x) = \cos x \cos^2(-x) - \sin(-x)\sin(-x)\cos(-x)$

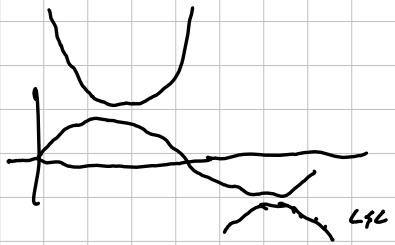
$\cos(-x) = \cos x$ miejsca zerowe

Granice nie istnieją

$\sin(x + \pi/2) = \sin x \cos \pi/2 + \cos x \sin \pi/2 = \cos x$

$\operatorname{tg} x = \frac{\sin x}{\cos x}$, $\operatorname{ctg} x = \frac{\cos x}{\sin x}$

$\sec x = \frac{1}{\cos x}$ $\quad \operatorname{cosec} = \frac{1}{\sin x}$



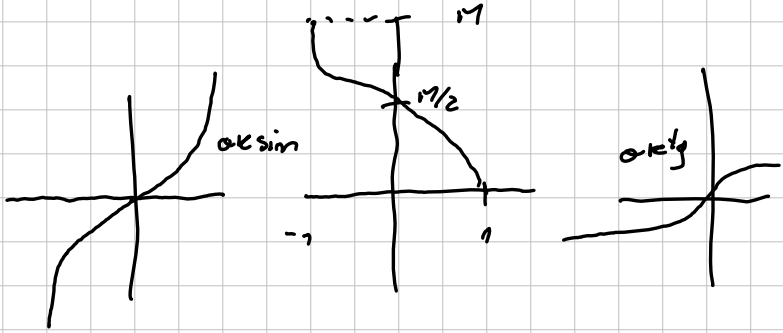
$\operatorname{tg} 2x = \frac{2\operatorname{tg} x}{1-\operatorname{tg}^2 x}$, $\sin 2x = \frac{2\operatorname{tg} x}{1+\operatorname{tg}^2 x}$, $\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \frac{1-\operatorname{tg}^2 x}{1+\operatorname{tg}^2 x}$

$$\sin 2x = 2\sin x \cos x = 2\sin x \cos x \cdot \frac{\cos x}{\cos x} = 2\operatorname{tg} x \cdot \cos^2 x$$

$$1 + \operatorname{tg}^2 x = 1 + \frac{\sin^2 y}{\cos^2 x} = \frac{1}{\cos^2 x}$$

Funkcje odwrotne

arcsin
arccos
arctg



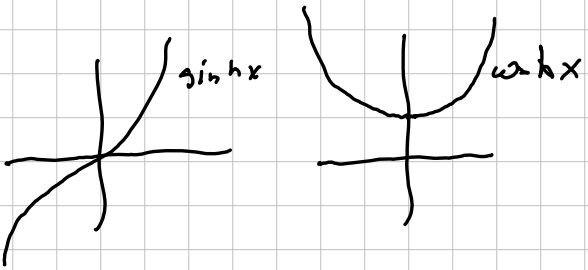
Funkcje hiperboliczne

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\operatorname{tgh} x = \frac{\sinh x}{\cosh x}$$

$$\operatorname{ctgh} x = \frac{\cosh x}{\sinh x}$$



$$\sinh(x+y) = \sinh x \cosh y + \sinh y \cosh x$$

$$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$$

$$\cosh^2(x) - \sinh^2(x) = 1$$

Funkcje area

$$\operatorname{arsinh}(x) = \ln\left(x + \sqrt{x^2+1}\right) \qquad \operatorname{artgh}(x) = \frac{1}{2}\ln\frac{1+x}{1-x}$$

$$\operatorname{arcosh}(x) = \ln\left(x + \sqrt{x^2-1}\right)$$

Funkcje wymierne

$f(x) = 1/x$

Dziedzina, granice

$$\frac{(x-2)(x+1)}{(x-1)(x-3)} = \frac{x^2 - x - 2}{x^2 - 4x + 3}$$

$$\frac{(x-1)(x-2)}{(x-1)(x-3)} = \frac{x^2 - 3x + 2}{x^2 - 4x + 3}$$

$$\frac{x-1}{x^2+2x+5}$$

$$\frac{(x-1)(x-2)(x-3)}{x^2+2x+5} = \frac{(x^2-3x+2)(x-3)}{x^2+2x+5} = \frac{x^3 - 3x^2 + 2x - 3x^2 + 9x - 6}{}$$

$$\frac{x^3 - 6x^2 + 11x - 6}{x^2 + 2x + 5}$$

1. Rozwiąż równanie

$$\sin\left(\arccos\left(\sqrt{x}\right)\right) = \frac{1}{2}$$

Wskazówka:
pokaż że $\sin(\arccos x) = \sqrt{1-x^2}$

2. Pokaż, że

$$\operatorname{arctg}(x) + \operatorname{arctg}(y) = \operatorname{arctg}\left(\frac{x+y}{1-xy}\right)$$