1 Zadania na liczenie pochodnych

Policz pochodną:

1. $f(x) = \ln\bigl(x + \sqrt{x^2+1}\bigr) \quad\Longrightarrow\quad f'(x) = \dfrac{1}{\sqrt{x^2+1}}.$

2. $f(x) = \ln\sqrt{\dfrac{1+x}{1-x}} = \frac{1}{2}\ln\dfrac{1+x}{1-x} \quad\Longrightarrow\quad f'(x) = \dfrac{1}{1-x^2}.$

3. $f(x) = \dfrac{x}{a^2\sqrt{x^2+a^2}} = \dfrac{1}{a^2}x(x^2+a^2)^{-1/2} \quad\Longrightarrow\quad f'(x) = \dfrac{1}{(x^2+a^2)^{3/2}}.$

4. $f(x) = \sqrt[3]{2e^x - 2^x + 1} = (2e^x - 2^x + 1)^{1/3} \quad\Longrightarrow\quad f'(x) = \dfrac{2e^x - 2^x\ln 2}{3(2e^x - 2^x + 1)^{2/3}}.$

5. $f(x) = \sqrt{xe^x + x} = (xe^x + x)^{1/2} \quad\Longrightarrow\quad f'(x) = \dfrac{e^x(1+x)+1}{2\sqrt{xe^x + x}}.$

6. $f(x) = \frac{1}{2}\sin(x^2) \quad\Longrightarrow\quad f'(x) = x\cos(x^2).$

7. $f(x) = \ln(\arcsin(5x)) \quad\Longrightarrow\quad f'(x) = \dfrac{5}{\arcsin(5x)\sqrt{1-25x^2}}.$

8. $f(x) = \dfrac{1}{(\ln x)^2} = (\ln x)^{-2} \quad\Longrightarrow\quad f'(x) = -\dfrac{2}{x(\ln x)^3}.$

9. $f(x) = \ln\bigl(\ln(\ln x)\bigr) \quad\Longrightarrow\quad f'(x) = \dfrac{1}{x\ln x\ln(\ln x)}.$

10. $f(x) = \frac{1}{2}\ln\tan\bigl(\frac{x}{2}\bigr) \quad\Longrightarrow\quad f'(x) = \dfrac{1}{4}\cdot\dfrac{\sec^2\bigl(\frac{x}{2}\bigr)}{\tan\bigl(\frac{x}{2}\bigr)} = \dfrac{1}{4\sin\bigl(\frac{x}{2}\bigr)\cos\bigl(\frac{x}{2}\bigr)} = \dfrac{1}{2\sin x}.$

11. $f(x) = x^x \quad (x>0) \quad\Longrightarrow\quad f'(x) = x^x(\ln x + 1).$

2 Zadania z pochodnych

Policz pochodne następujących funkcji

1. $f(x) = \frac{\pi}{x} + \ln 2$
2. $f(x) = x^2 \sqrt[3]{x^2}$
3. $f(x) = \frac{1+2x}{3+4x}$
4. $f(x) = \frac{\sin(x)+\cos(x)}{\sin(x)-\cos(x)}$
5. $f(x) = \frac{e^x}{x^2}$
6. $f(x) = e^x \arcsin x$
7. $f(x) = x \sinh x$
8. $f(x) = \sqrt{1-x^2}$
9. $f(x) = \frac{1}{3\cos^3 x} - \frac{1}{\cos x}$
10. $f(x) = \frac{1}{\operatorname{arctg}(x)}$
11. $f(x) = \arcsin \frac{1}{x^2}$
12. $f(x) = \ln(1-x^2)$
13. $f(x) = \operatorname{arctg}(\ln(x)) - \ln(\operatorname{arctg}(x))$
14. $f(x) = \frac{4}{3}\sqrt{\frac{x-1}{x+2}}$
15. $f(x) = \sin^2(x^3)$
16. $f(x) = e^{\sin^2(x)}$
17. $f(x) = \ln(\cos \frac{x-1}{x})$