Persuasive Advertising in Conformist and Snobbish Markets

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Abstract

I model persuasive advertising for conspicuous goods that can either be made more attractive by greater popularity ("conformist markets") or by greater exclusivity ("snobbish markets"). Consumers are endowed with a latent attribute measuring some aspect of their identity, and a social status implied by this attribute. Consumers wish to signal a high social status, and the function of advertising is to render consumption a signaling device by linking products with social groups. In a conformist market, I find that advertising increases demand elasticity, inducing firms to converge on low prices, and can be used by a first mover to deter entry and gain monopoly rents. In this setting, advertising promotes a very cutthroat environment in which only one product can survive. In a snobbish market, advertising reduces demand elasticity, dampening price competition and promoting firm entry. In this setting, advertising can act as a public good to firms, increasing all firms' prices and profits. Additionally, it can lead to asymmetric equilibria where a firm appealing to high status consumers advertises more heavily, capturing a greater market share and price. Furthermore, I bring micro-foundation to persuasive advertising, allowing analysis of the channels through which it impacts welfare. Finally, I show that the model can help explain well-documented empirical puzzles in the marketing and empirical industrial organization literatures.

"Advertising is one of the topics in the study of industrial organization for which the traditional assumptions are strained most (especially those with regard to consumer behavior). The advertising of a product has strong psychological and sociological aspects that go beyond optimal inferences about objective quality. For instance, ad agencies constantly try to appeal to consumers' conscious or unconscious desire for social recognition, a trendy lifestyle and the like."

Jean Tirole, *The Theory of Industrial Organization*, pp. 292-293

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I INTRODUCTION

It is estimated that around \$224 billion was spent on advertising in 2018 in the United States alone, roughly \$685 per capita (Wagner, 2019). Despite being a large portion of GDP, the role of advertising is still not well theoretically understood. One aspect of advertising is simply informational, apprising consumers of products and their attributes. However, a significant amount of advertising has little or no informational content, instead attempting to persuade consumers by purportedly manipulating their preferences.[1] Yet this important phenomenon is difficult to capture in traditional economic models based on the assumption that consumers have fixed preferences.

This model seeks to provide a micro-foundation for the role of advertising as appealing to consumers' desire for social recognition, and associating their consumption with a social identity, without sacrificing the assumption of fixed preferences. The premise of the model is that each consumer is endowed with a latent attribute $x \in [0,1]$ measuring some aspect of her social identity, such as a scale of how Southern or New England she is, or a measure of her sophistication. The values and norms governing the community in which consumers reside assign each consumer a social status based on her latent attribute $s(x) : [0,1] \to \Re$. Furthermore, consumers receive a reputational utility from signaling a high social status to a group of non-consuming spectators called "the public."

The function of advertising is to facilitate this signaling by bringing the public's attention and powers of discrimination to products, so that the public is enabled — and the consumer knows that it is enabled — to infer the latent attributes and social status of consumers from simply observing their consumption choices. In other words, advertising renders consumption a device for consumers to signal their attribute and the status thereby implied. Thus, advertising affects a consumer's purchase by calibrating the social reputation associated with different products.

But to understand and model this signaling phenomenon, one must recognize that there are two main effects such signaling motives may have on consumer demand: a "conformist effect" (sometimes called a "bandwagon effect") where market demand for a good increases by others purchasing it; and "snob effect" where market demand for a good decreases by others purchasing it. I analyze the workings and effects of advertising in markets dominated

[1]See, for example, Resnik and Stern (1977), Tom et al. (1984), Becker and Murphy (1993) and Abernethy and Franke (1996).

by each of these two forms of signaling.

On the one hand, there are "conformist markets" in which goods are made attractive by their popularity. For example, you might be reluctant to frequent Dunkin' Donuts if no one else does, but eager to do so if it is all the rage. On the other hand, there are "snobbish markets" in which goods are made attractive by greater exclusivity. You might drink "artesian water" to seem sophisticated, but as soon as artesian water becomes popular, you will move on to something else.

Economic theory dating back to Leibenstein (1950) has recognized the potential impact of conformist and snob effects on consumer demand.[2] Surprisingly, however, the theoretical literature has said little about the impact of conformist and snobbish motives on persuasive advertising. That said, marketing practitioners often distinguish between campaigns built on "bandwagon appeal" (e.g. "America runs on Dunkin"') versus "snob appeal" (e.g. Essentia Water's campaign "Overachieving H_2O: someone is going to stand out, it might as well be you").[3]

The specific signaling game adopted is from Corneo and Jeanne (1997)'s study of Veblen effects.[4] I superimpose this signaling game on a classic Hotelling model of two sequentially entering and horizontally differentiated firms. Firms not only choose how much to advertise, but also which consumers to appeal to through their choice of horizontal attributes.

Corneo and Jeanne (1997) show that the occurrence of snob effects or conformist effects depends, in an identifiable way, on how a community allocates social status on the basis of a consumer's underlying attribute x. Essentially, snob effects or conformist effects are born from one of two like desires: the desire to avoid ostracism and not be considered a low type, or the hope for prestige and being considered a high type. It turns out that if there are *increasing status returns* to a more desired x, then the latter desire outweighs the former, and we get a market characterized by snob effects. By contrast, if there are *decreasing status returns* to a more valued x, then the former desire outweighs the latter, and we get a market characterized by conformist effects.

[2] Leibenstein (1950) originally coined the terms "bandwagon effects" and "snob effects." A large literature studies the presence of these effects on consumer demand, including: Becker (1991), Bernheim (1994), Karni and Levin (1994), Corneo and Jeanne (1997), Grilo et al. (2001), Amaldoss and Jain (2005a) and Amaldoss and Jain (2005b).

[3] Others are less discreet. For example, in the Fall of 2019 Volvo launched advertisements reading "Follow No One, Update Your Status Symbols."

[4] Originally postulated by Veblen (1899), Veblen goods are a type of luxury goods for which the quantity demanded increases in its price.

How does advertising function in these two different markets? By making it more likely the public recognizes the social reputation of a product, advertising amplifies consumers' snobbish or conformist motives, depending on the type of market.

In a snobbish market, by increasing the strength of snob effects, advertising makes firms less willing to cut prices to expand market share, because fewer new consumers rush in to purchase when they do so. In other words, advertising reduces the elasticity of demand for snob goods. This dampens price competition and allows firms to converge on higher, supranormal prices.

Furthermore, this promotes firm entry because there are greater profits to be had. In this way, advertising acts as a public good to firms, increasing both firms' profits. Moreover, even if firms locate symmetrically, asymmetries in firms' prices and market shares may result. This is because the firm which does a better job of appealing to high status consumers is considered more prestigious, and uses advertising to extract greater market share at a higher price.

These results speak to many stylized facts. First, in snobbish markets such as that for luxury goods, we often see unusually high prices and an abundance of brands. For example, reusable water bottles have become a millennial status symbol in the last few years, with a proliferation of dozens, if not hundreds, of brands, and prices ranging from \$10 to \$1,500. As of this writing, the price on Amazon for a 17 oz water bottle from the most popular brand, S'well, is \$30. Furthermore, S'well and its competitors are known for heavily advertising on Instagram and social media.[5] The model shows that this may be due to advertising's influence in reducing demand elasticity.

Additionally, despite goods being physically similar or homogeneous, we often see some firms charging a price premium, advertising heavily and earning greater market share (Bagwell, 2007). Indeed, many decades of empirical research in homogeneous product markets have found price dispersion rather than the "law of one price" to be the norm (Baye et al., 2006; Chioveanu, 2008). This may source from the prestige of a heavily advertised brand targeted at high status consumers, like S'well.

In a conformist market, advertising has the opposite effect. By increasing the strength of consumers' conformist motives, advertising makes firms more willing to cut prices to expand their market share, because doing so results in a greater number of consumers rushing to

[5]See Munro (2018) and Mull (2019).

buy the product. In short, advertising increases the elasticity of demand. This heightens price competition and induces firms to converge on lower, depressed prices. If the price competition becomes sufficiently severe, then advertising can enable a first-mover to deter the entry of future firms and gain monopoly power. In this setting, advertising promotes a very cutthroat environment in which only one product can survive.

This helps make sense of empirical puzzles observed in conformist markets. We often see a first mover enter a geographic area, build an advertising presence, and dominate it for many years (Sutton, 1991). For example: Dunkin Donuts' started in Massachusetts in 1950 and dominates the northeast United States, appealing to its blue-collar New England identity. Krispy Kreme launched in the North Carolina in 1937, triumphs in the South, appealing to its wholesome and classy Southern identity. And Tim Hortons was founded by a Canadian hockey player in 1965, with its Canadian customer base often swearing by it with religious zeal.[6]

More rigorously, in a series of papers, Bronnenberg et al. (2007, 2009, 2011) meticulously document the evolution of advertising, prices and firm entry across the United States in the packaged-foods industry, many goods of which might be considered conformist such as beer and soft drinks. They find that brands which entered a given geographic area first and built an advertising presence — often over one hundred years ago — are very likely to maintain a stronger, leading presence today. This model provides an economic explanation for this result.

Finally, by giving persuasive advertising a micro-foundation, this model opens a path to welfare analysis, unlike the previous literature. The key insight is that since reputation signaling is a zero-sum game, the total pie of social status available is fixed (Frank, 2005; Heffetz and Frank, 2011). Persuasive advertising can shift which consumers get what portion of that pie, but cannot *directly* affect aggregate consumer welfare since it does not impact the size of the signaling pie. However, advertising can influence consumer welfare *indirectly* through its effect on prices, entry, and on how well consumers' purchases respect their horizontal preferences. For example, in the cases where advertising raises prices, it creates a transfer of welfare from consumers to firms. Furthermore, by limiting entry and inducing status concerns to overpower horizontal preferences, advertising can increase consumers' transportation

[6] In the model, the social attribute x a brand appeals to does not have to be a regional identity. For example, as in the market for reusable water bottles, it could be a measure of sophistication.

costs. This knowledge is important for evaluating how policies that limit or tax advertising might impact consumers, firms and market structures.

II LITERATURE REVIEW

The theoretical literature on advertising broadly fits into three camps (i) the *informative* view that advertising informs consumers about the existence and attributes of products (ii) the *persuasive* view that advertising in some way affects consumer tastes and (iii) the *complementary* view that advertising is itself as a good entering consumer utility.[7]

While few would disagree that much advertising is uninformative, the persuasive view is known to have difficulty giving advertising a micro foundation. It is typically modeled as manipulating a parameter in consumer preferences chosen by the modeler. For example, in the linear city model adopted here, Von der Fehr and Stevik (1998) argues that persuasive advertising may either increase the perceived quality of a good, the perceived differences between goods (transportation costs), or influence the distribution of consumer tastes. Many models begin with the premise that advertising shifts demand up, increasing willingness to pay (Dixit and Norman, 1978). One issue with this approach is that it leaves much freedom to the modeler, making it difficult to reach solid conclusions about advertising's market effects. Moreover, it makes welfare analysis tricky, as it not clear what standard should be adopted for measuring welfare. Which preferences should be considered the true preferences, those pre-advertising or post-advertising (Dixit and Norman, 1978)?

The solution of the complementary approach is to treat advertising as itself a good giving consumer's utility (or disutility), and complementary to the good being sold (Becker and Murphy, 1993). This implies a fixed preference, facilitating market and welfare analysis. The solution adopted here has a fixed preference like in complementary models, but without certain implications of such models like creating a consumer budget and demand function for receiving advertisements.

As mentioned, the approach here is to superimpose a late stage signaling game, where consumers wish to signal their social status to a social contact, and advertising facilitates in that signaling by bringing the social contact's attention to their purchase. Krähmer (2006) also incorporates persuasive advertising in this way. However, Krähmer (2006) studies a model

[7]See Bagwell (2007) for a comprehensive survey of the literature.

with two consumer types, a cool type and a geeky type — where the geeky type wants to mimic the cool type — and firms sell homogenous products. By contrast, this model studies a signaling game with a continuum of consumer types, focusing on the conformist and snob effects signaling may generate, and firms sell horizontally differentiated products.

Despite tying the modeler's hand, this framework can explain many observed empirical regularities and reproduce notable predictions about persuasive advertising's strategic effects. For example, the results support the long-held view that advertising can deter market entry (Braithwaite, 1928; Bain, 1956; Sutton, 1991), providing a process by which demand is affected to the benefit of the first-mover. This literature has pointed out that a full model would explain the mechanism through which advertising influences demand (see, for example, Sutton (1991, pp.312-313)). Furthermore, the model helps in understanding how advertising can build a firm's brand prestige, allowing it to extract a price premium, despite selling a physically similar good (Nichols, 1985; Becker and Murphy, 1993). More generally, questions such as the effect of persuasive advertising on the price elasticity of demand, and whether advertising is combative or mutually beneficial to firms, are often theoretically and empirically ambiguous, and this model suggests that special attention should be paid as to whether advertising takes place in a conformist or snobbish demand setting, as well as the conspicuousness of goods (Comanor and Wilson, 1979; Bagwell, 2007).

Indeed, there is little work modeling advertising in settings with conformist or snobbish demand. Some exceptions are Buehler and Halbheer (2012), which studies persuasive advertising that increases the perceived quality of products, and Sebald and Vikander (2015), which studies informative advertising. Both models lie outside of a signaling framework. There are models that adopt a social signaling game and study informative advertising (Campbell et al., 2017; Vikander, 2017), and a well-developed body of models where advertising helps firms signal type or other private information, rather than consumers signal type (Nelson, 1974; Bagwell and Ramey, 1988, 1990; Albák and Overgaard, 1992; Linnemer, 1998).

More broadly, this paper builds on a growing body of literature studying the effects of consumer signaling type on (non-advertising) firm behavior (Pesendorfer, 1995; Corneo and Jeanne, 1997; Bagwell and Bernheim, 1996; Kuksov and Xie, 2012; Kuksov and Wang, 2013; Yildirim et al., 2016; Liu et al., 2019). As mentioned, the signaling game adopted is introduced in Corneo and Jeanne (1997), which studies the existence of Veblen effects in a monopolist market. I am unaware of any other work studying a social signaling game in the context of a spatial model of competition. As pointed out in Lemma 1, spatial models have a nice single-crossing property that makes them well-suited for superimposing a signaling game, and one hope of this paper is to inspire further exploration of such models.

Along these lines, Grilo et al. (2001) study price competition in a differentiated duopoly market with a reduced form consumption externality of either a conformist or snobbish nature (i.e. a consumer's utility for a good directly increases or decreases in the number of others purchasing it). Similar analytic techniques are used in the price subgame of this model. While Grilo et al. (2001) are not motivated by the study of advertising, one could interpret this model as extending Grilo et al. (2001) along various dimensions. First, Grilo et al. (2001) study a symmetric consumption externality where the utility from others frequenting firm A are equivalent to that of others frequenting firm B. This paper explores asymmetry in the consumption externality, where the utility from others frequenting firm A are not necessarily equal to that of others frequenting firm B. This arises in the current model because it matters not just *how many* people buy a good, but also *who* buys a good, and the social status they hold. The people who occupy firm A may not have the same status as those who occupy firm B. Furthermore, this model endogenizes the consumption externality through the signaling game, thus allowing for welfare analysis, as well as endogenizes firms' entry and location choices.

III A MODEL OF PERSUASIVE ADVERTISING

This section introduces the formal model with sequentially entering firms selling conspicuous and horizontally differentiated goods to status-driven consumers. The economy is populated by i) a unit mass of consumers uniformly distributed along some attribute x on the [0,1] interval and ii) a unit mass of non-consuming spectators called the public. The latent attribute x measures some aspect of each consumer's identity, such as the strength of her cultural association with New England America versus Southern America. Furthermore, this attribute helps shape the consumer's demand. As common in models of horizontal differentiation, each consumer x has unit demand with quadratic transportation cost $\tau > 0$ and a bliss point at x.[8] Thus, x both defines the underlying social attribute of the consumer and her preferences over brands.

[8] Quadratic transportation costs help ensure the existence of equilibria at the pricing and location/advertising stages (d'Aspremont et al., 1979; Neven, 1985; Shaked and Sutton, 1987).

Furthermore, as in Corneo and Jeanne (1997), there is a continuous social status function $s(x) : [0,1] \rightarrow \Re$ that assigns each consumer a social status given her attribute x.[9] The social status function is exogenous, and governed by the norms and values of the community in which consumers reside. Note that the attribute x both defines a consumer's preferences over goods, as well as a consumer's social status. After all, we all face the problem of being endowed with certain attributes which both mold our preferences, as well as give us certain reputation in our relations to society.

As Corneo and Jeanne (1997, p.58) explain:

> Following sociologists, we may define social status as a general claim to deference [e.g., Coleman (1990)]. In economic terms, an individual's status may be called a socially provided private good. Each individual has a certain fixed amount of a special good - say, deference - that he allocates to others according to some social norm... In turn, the norm may be taken as mirroring societal values that characterize the community in which the individuals interact.

As Fershtman and Weiss (1993) notes, Max Weber first introduced this technical definition of status as a claim to "deference" or "esteem" by others (Weber, 1978). Here, the individuals who allocate the good deference are the public. At the end of the game, consumers are randomly matched with partners from the public. A member of the public does not know her partner's social status, but tries to infer it. Each consumer receives a reputational utility equal to the her partner's expectation of her social status (Fershtman and Weiss, 1993; Bernheim, 1994; Ireland, 1994). We will see exactly how the public's inference is made later. This inference can be thought of as the quantity of deference the public gives the consumer. For example, consumers in New England might receive a certain amount of deference for signaling their New England origins.

One could model the game such that the public takes an action, other than allocating deference, that affects consumer utility. For example, a consumer's professional, marriage (Cole et al., 1992; Pesendorfer, 1995), mating (Miller, 2011), friendship or leadership opportunities could depend on others' inference of her latent attribute x. For clarity of exposition, I abstract from such interpretations of the reputational utility.

[9] Corneo and Jeanne (1997) and other papers in the Veblen effects literature model social status as a function of income. This is often motivated by the supposition that income is positively correlated with other desirable underlying qualities such as hard work, intelligence, etc. Indeed, in a laboratory experiment, Clingingsmith and Sheremeta (2018) finds evidence that college students do not allocate status according to a subject's earnings, but rather a subject's academic abilities. Here, I model social status as directly a function of a consumer's underlying social attribute.

The ex-post utility u_x a consumer x receives from a given purchase is:

$$u_x = v - \tau(\ell - x)^2 - p + \underbrace{\text{Public's Expectation of } s(x)}_{\textit{Reputational Utility}}$$

where $v > 0$ is the intrinsic utility of the good, $\ell \in [0, 1]$ is its location and $p \geq 0$ is its price. The only departure from a standard model of horizontal differentiation is the addition of the reputational utility.

The time-line of the game is as follows, summarized in Figure 1. Adopted from Schmalensee (1983), the framework is motivated by the fact that in the real world firms usually enter sequentially rather than simultaneously.[10]

At $t = 0$, the incumbent, firm A, chooses a location $\ell_a \in [0, 1]$. The location choice measures the horizontal attributes of the firm, such as the how well a donut shop appeals to New England American and Southern Americans through the features of its donuts, atmosphere and service. Furthermore, firm A chooses how much to advertise $\lambda_a \in [0, 1]$, paying a convex cost to advertising $\frac{c}{2}\lambda_a^2$ (where $c \geq 0$). λ_a represents the probability that a given member of the public receives an advertisement from firm A (Grossman and Shapiro, 1984). I normalize firm A's cost of production to zero.

At $t = 1$, firm B observes the location and marketing level of firm A and decides whether to enter the market. Furthermore, if firm B enters the market, then it must decide where to locate $\ell_b \in [0, 1]$ and how much to advertise $\lambda_b \in [0, 1]$, also paying a convex cost to advertising $\frac{c}{2}\lambda_b^2$. λ_b represents the probability that a given member of the public receives an advertisement from firm B. The probability a given member of the public receives an advertisement from firm B is independent of that of receiving an advertisement from firm B. Firm B also pays zero cost to production, so that there are no cost advantages to early entry.[11]

Thus, if λ represents the probability that a given member of the public receives an advertisement from either firm, then $\lambda = \lambda_a + \lambda_b - \lambda_a\lambda_b$. This could be interpreted as a linear advertising production function.[12]

[10] Furthermore, from a technical perspective, sequential entry helps ensure the existence of equilibria in pure strategies in the location and advertising stages (Börgers, 1988).

[11] The assumption that firm B pays zero fixed cost is conservative in that it makes it harder to get a result where firm B is deterred from entering.

[12] The model could be extended to accommodate a more general advertising production function $f(\lambda_a, \lambda_b)$ that is increasing in each of its arguments, and determines the probability that a member of the public receives an advertisement given firms' marketing efforts. Furthermore, one could allow for asymmetries in the marketing technologies of firms by letting the first order partial derivatives be unequal $f_1(\lambda_a, \lambda_b) \neq f_2(\lambda_a, \lambda_b)$ for given advertising levels (λ_a, λ_b).

If firm B enters, then at $t = 2$ firms A and B simultaneously choose prices $p_a \geq 0$ and $p_b \geq 0$ as in Bertrand competition. Otherwise, firm A becomes a monopolist and chooses its price free of competition. The assumption that advertising investment take place before pricing decisions is typical in models of persuasive advertising, and motivated by the view of advertising as a long-term investment to generate a brand image, and pricing as a more short-term oriented strategy (Belleflamme and Peitz, 2015, p. 150).

At $t = 3$, consumers make their purchase decisions. Furthermore, consumers are standard expected utility maximizers, fully informed about the goods in the market, their marketing levels, locations and prices.

At $t = 4$, each consumer is randomly matched with a partner from the public. The public does not know a consumer's underlying identity and social status, but tires to infer it. Each consumers receives a reputational utility equal to the inference made by her partner of her social social status.

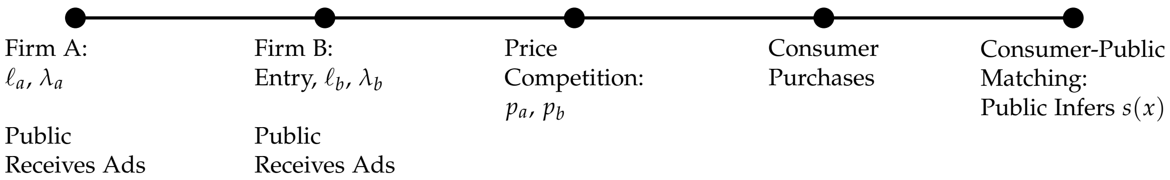


Figure 1: Timeline of Model

Let $\rho(x)$ denote the posterior probability the partner assigns to the consumer being of type x, where $0 \leq \rho(x) \leq 1$ and $\int_0^1 \rho(x)dx = 1$. If the partner receives an advertisement from either firm, then his attention and powers of discrimination is brought to products, and he infers the consumer's social status conditional on the latter's product choice. That is, if we let $o \in \{a, b, \varnothing\}$ denote the chosen option of a given consumer, and Ω the attributes of the products available, then the partner's inference of the consumer's social status is $\rho(x \mid o, \Omega)$. By contrast, if the partner does not receive an advertisement, then he does not pay attention to products and does not condition his inference on the consumer's purchase: $\rho(x)$.

The motivation being that consumers shop at stores, and thus pay full attention to the products available. However, the public does not shop, and thus does not pay full attention to products, and may not be able to readily recognize or distinguish products when they see them. For example, one might not recognize the difference between a S'well water bottle or a Camelbak water bottle unless one receives an advertisement. Advertising does not affect consumer utility directly, but affects consumer utility indirectly by increasing the ability of the public to infer a consumer's social status from her consumption.[13]

Therefore, consumers are senders in a signaling game, making consumption choices based on the trade-off between their horizontal preferences and the signal of social status it conveys to the advertisement receiving public. Let S_o denote a consumer's expected utility from signaling given option o at $t = 3$. The expected utility of consumer x for each choice at $t = 3$ is:

$$U_x(a) = v - \tau(\ell_a - x)^2 - p_a + S_a \tag{1}$$

$$U_x(b) = v - \tau(\ell_b - x)^2 - p_b + S_b \tag{2}$$

$$U_x(\emptyset) = S_\emptyset \tag{3}$$

Furthermore, the signaling value of each choices is:

$$S_a = \lambda \int_0^1 \rho(x \,|\, a, \Omega)\, s(x)\, dx \;+\; (1-\lambda) \int_0^1 \rho(x)\, s(x)\, dx \tag{4}$$

$$S_b = \lambda \int_0^1 \rho(x \,|\, b, \Omega)\, s(x)\, dx \;+\; (1-\lambda) \int_0^1 \rho(x)\, s(x)\, dx \tag{5}$$

$$S_\emptyset = \lambda \int_0^1 \rho(x \,|\, \emptyset, \Omega)\, s(x)\, dx \;+\; (1-\lambda) \int_0^1 \rho(x)\, s(x)\, dx \tag{6}$$

where the first terms are the probability of being matched with a member of the advertisement receiving public multiplied by the perceived status of those choosing said option (found by multiplying the posterior probability a consumer is a given type with that type's social status, and integrating over all possible types), and the second terms are the probability of being matched with a member of the non-advertisement receiving public multiplied by the perceived status of any random consumer. Advertising makes it more likely that a consumer's purchase is recognized for the social status it conveys, and consumers incorporate this into their consumption choices.[14] The signaling game produces a consumption externality where a consumer's utility depends not only on her own consumption choice, but also that of every

[13]This approach accords with evidence that brand advertising is often targeted at people who would not purchase from the given brand, instead aiming to increase the brand's recognizability to serve the signaling needs of its own customer base (Miller, 2011).

[14]There is a second behavioral interpretation of the model following the work of Bénabou and Tirole (2011). Under this interpretation, a consumer has two selves, a present self and a future self. Through a moment of self-reflection, the current self has insight into its own attribute x, and the status implied therein, but the future self may momentarily forget its own attribute (akin to "the public"). Though forgetful, the future self can readily observe its consumption choices, and may try to infer its attribute based on this action. The consumer receives utility based on its future self's inference of its own identity. Thus, the current self is a sender in a signaling game, making consumption choices based on the utility the consumer will get from its future self's inference. As explained in Bénabou and Tirole (2011), the function of advertising is to remind the consumer's future self of the identity associated with products, increasing the salience of the conditional inference, and thus strengthening the use of consumption as signaling device. This alternative interpretation can potentially broaden the application of the model beyond that of conspicuous consumption to that of inconspicuous consumption.

other consumer. It will be shown that the instance of snobbish or conformist effects depends on the shape of the social status function, and additional structure will be imposed on the social status function to highlight such effects.

Equilibrium Definition

Before diving into further detail about the social status function, it's important to define the equilibrium concept. In the consumption subgame (stages $t = 3$ and $t = 4$), the appropriate equilibrium notion is that of a signaling equilibrium where consumers are senders, choosing actions (a, b or $\emptyset$) based on type x and firm decisions prior Ω; while the public are receivers, inferring consumer type x based on information about a consumer's action and the characteristics of firms Ω. I study *pure strategy* Perfect Bayesian Equilibrium (PBE). In a PBE, consumer actions must be optimal given the public's inferences, and the public's inferences must be deducible using Bayes Rule given information available about consumer actions. I restrict attention to PBE satisfying the following reasonable off-equilibrium public beliefs.

Suppose both firms enter the market and $\ell_a < \ell_b$. I assume that if all consumers purchase good a (good b), then a consumer who deviates by purchasing good b (good a) is perceived to be the consumer with greatest benefit from switching: $\rho(x = 1|b, \Omega) = 1$ ($\rho(x = 0|a, \Omega) = 1$). If instead $\ell_b < \ell_a$, then $\rho(x = 0|a, \Omega) = 1$ and $\rho(x = 1|b, \Omega) = 1$.[15] Note that members of the public who do not receive advertisements are unable to distinguish between the choices of consumers. Essentially, they perceive all consumers as choosing a single action. Their inference of a consumer's type is then unconditioned on her action, $\rho(x)$, and hence they form no off-equilibrium beliefs.

Finally, in the larger game I study pure strategy subgame perfect equilibria. Firms maximize profits given demand, as determined in the consumption subgame. All proofs are in the Appendix.

IV SOCIAL STATUS MOTIVATED DEMAND

Working backwards, I first solve for consumer demand, given firm advertising levels, prices and locations. Consumers make their choices based not only on their horizontal preferences

[15] I treat the case of $\ell_a = \ell_b$ at the end of Section IV. I outline analogous off-equilibrium beliefs in the case where firm B does not enter in Section V.

and prices, but also on the status their purchase conveys to the advertisement receiving public. It will be seen how this signaling motive generates conformist and snob effects on demand.

I impose the traditional assumption that v is sufficiently large that all consumers make purchases. Suppose firm B enters the market (I turn to the monopoly case in which firm B does not enter in Section V). The first thing to notice is that if $\ell_a \neq \ell_b$, then any equilibrium in which all consumers make purchases is characterized by a single cut-off $n \in [0, 1]$ such that consumers to the left of n buy the left most good, and consumers to the right of n the right most good.

Lemma 1 (Cut-Off Rule). *If $\ell_a \neq \ell_b$, then in any equilibrium in which all consumers make purchases, there must be a single cut-off $n \in [0, 1]$ such that consumers to the left of n buy the left most good, and consumers to the right of n buy the right most good.*

Consumer demand constitutes a semi-separating equilibrium if $n \in (0, 1)$ and both goods are purchased, and a pooling equilibrium if $n \in \{0, 1\}$ and only one good is purchased. This cut-off rule holds in most spatial models due to a single-crossing property arising from consumers' location dependent transportation costs, and the same applies here. The reason being that all consumers face the same public perception for a given purchase regardless of their type x.[16] Thus, reputation motives add an identical constant to each consumer's utility for a particular option independent of their x, not affecting the single-crossing property.

We can now calculate the signaling value of each good given an arbitrary cut-off n. Without loss of generality, suppose $\ell_a < \ell_b$. The analysis applies equally to the case of $\ell_a > \ell_b$ by switching the a and b terms in what follows. I treat the case of $\ell_a = \ell_b$ at the end of the section.

$$S_a(n) = \frac{\lambda}{n} \int_0^n s(x)dx \, + \, (1 - \lambda) \, E(\, s(x) \,) \tag{7}$$

$$S_b(n) = \frac{\lambda}{1 - n} \int_n^1 s(x)dx \, + \, (1 - \lambda) \, E(\, s(x) \,) \tag{8}$$

where $\int_0^1 \rho(x \,|\, a, \Omega) \, s(x) \, dx = \frac{1}{n} \int_0^n s(x)dx$ and $\int_0^1 \rho(x \,|\, b, \Omega) \, s(x) \, dx = \frac{1}{1-n} \int_n^1 s(x)dx$.

Each consumer must compare the utility of good a and good b. This implies weighing the signaling gain from good a over good b, call it $S_{a/b}(n) \equiv S_a(n) - S_b(n)$, against the difference in their transportation costs and prices.[17] We can calculate the signaling gain from good a,

[16] If the public knew the consumer's type x, then there would be no signaling motives.

[17] Similarly, let $S_{b/a}(n) \equiv S_b(n) - S_a(n)$.

$S_{a/b}(n)$, as follows.

$$S_{a/b}(n) = \lambda \left(\frac{1}{n} \int_0^n s(x)dx - \frac{1}{1-n} \int_n^1 s(x)dx \right) \tag{9}$$

There is a more formal correspondence between the signaling gain from a purchase $S_{a/b}()$ and the social status function.[18] Note that, and this is at the heart of the model, the reputation gain from a purchase $S_{a/b}()$ is a function of the mass of consumers purchasing it n.

Following Leibenstein (1950) and Corneo and Jeanne (1997), I say that demand is characterized by snobbery if the reputation gain from a purchase is enhanced by its rarity and $S'_{a/b}(n) < 0$. Conversely, demand is characterized by conformity if the reputation gain from a purchase is heightened by its popularity and $S'_{a/b}(n) > 0$. In order to highlight the effects of snobbery and conformity on demand, I focus on markets that are either snobbish or conformist for all possible values of n. In other words, cases where the reputation gain function $S_{a/b}(n)$ is monotonic. Furthermore, to make the model tractable, I restrict attention to reputation gain functions $S_{a/b}(n)$ that are linear. The next lemma establishes that $S_{a/b}(n)$ is linear and monotonic if and only if the social status function $s(x)$ is quadratic.

Lemma 2 (Monotonic and Linear Signaling Gain). *$S_{a/b}(n)$ is linear and decreasing if and only if $s(x)$ is quadratic and convex. $S_{a/b}(n)$ is linear and increasing if and only if $s(x)$ is quadratic and concave.*

The quadratic nature of the social status function buys linearity in the signal gain function, and the convexity or concavity in social status generates snobbery or conformity respectively. Considering the result in Lemma 2, I study social status functions with the below functional form.

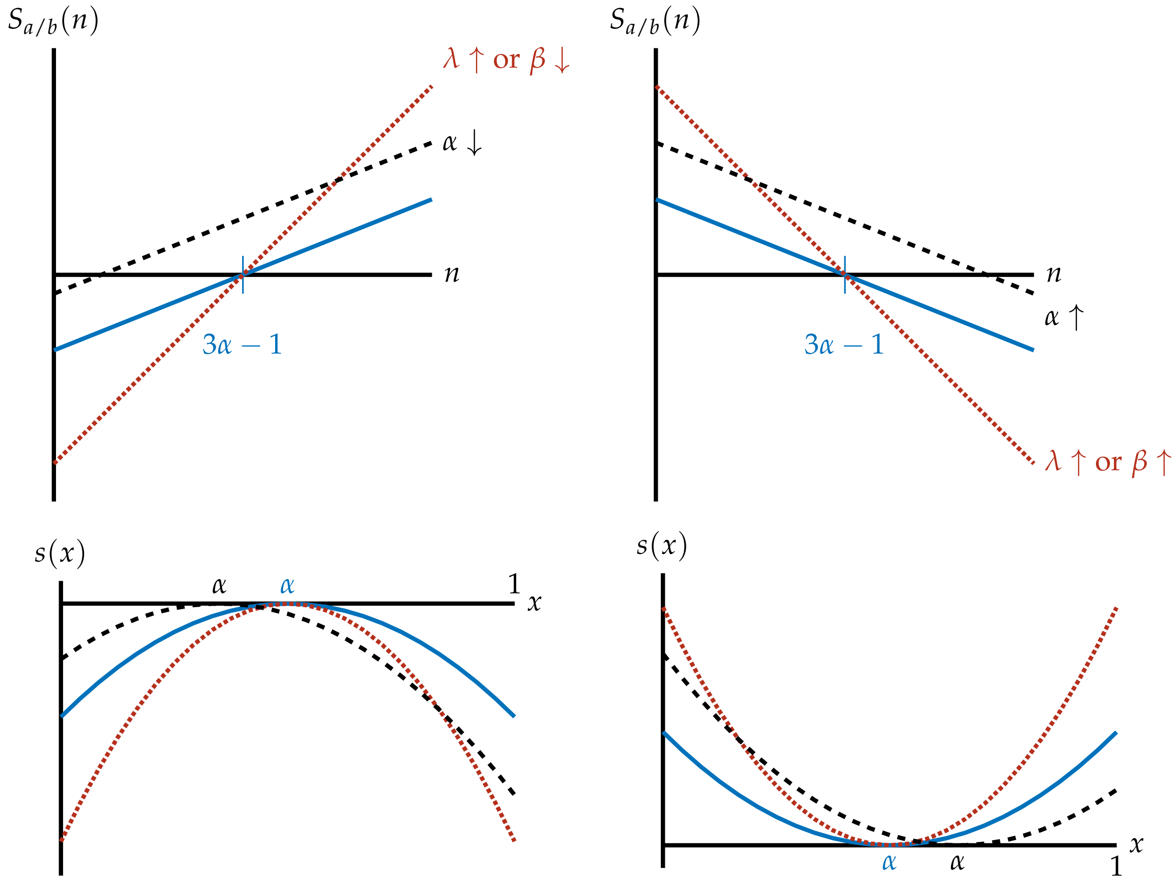
Assumption 1 (Social Status Function). *$s(x)$ is a quadratic function centered at $\alpha \in [0,1]$: $s(x) = \beta(x-\alpha)^2$.*

The sign of β determines whether demand is snobbish or conformist. Calculating $S_{a/b}(n)$ by plugging in the social status function yields:

$$S_{a/b}(n) = -\frac{\lambda\beta}{3}n + \lambda\beta(\alpha - \frac{1}{3}) \tag{10}$$

[18] A continuous and differentiable signaling gain function $S_{a/b}()$ can be rationalized by a social status function of the form $s(x) = (1-2x)S_{a/b}(x) + x(1-x)S'_{a/b}(x) + c$ where c is an arbitrary constant. See equation (11) in Corneo and Jeanne (1997).

As seen in equation (10), if the social status function is convex $\beta > 0$, then demand is snobbish and the reputation gain from a good decreases in the mass of consumers choosing it $\frac{dS_{a/b}(n)}{dn} < 0$. If the social status function is concave $\beta < 0$, then demand is conformist and the reputation gain from a good increases in the mass of consumers choosing it $\frac{dS_{a/b}(n)}{dn} > 0$. Figure 2 shows examples of the social status function for various β and α, mapping it to the reputation gain function generated.



(a) Conformist $\beta < 0$

(b) Snobbish $\beta > 0$

Figure 2: Consumption Externality Implied by Social Status Function
$S_{a/b}(n) = -\frac{\lambda\beta}{3}n + \lambda\beta(\alpha - \frac{1}{3})$

The intuition for the dependence of snob and conformist effects on the concavity or convexity of the social status function is as follows. As Corneo and Jeanne (1997) puts it, whether demand is conformist or snobbish depends on the relative strength of two related desires: the desire to avoid ostracism and not be considered a low type, and the desire to attain prestige and be considered a high type. It turns out that if the latter outweighs the former, then demand turns out snobbish, and if the former outweighs the latter, then demand turns out conformist. Which desire dominates depends on the marginal returns to higher status.

Suppose the social status function is convex on some interval, and for argument's sake, decreasing (again, the snobbery or conformity does not depend on whether social status is increasing or decreasing). Consider some cut-off n in this interval such that those to the left of n buy good a and those to the right of n buy good b. Due to the convexity, it is more costly to lose one's position when one is ranked high than when one is ranked low. Thus, as n shifts right and more consumers buy good a, the expected status of those choosing good a falls more rapidly than that of those choosing good b. This implies that the the signaling gain from good a, $S_{a/b}(n)$, which is the difference between those two, decreases. Essentially, societal norms emphasize a hope of being identified as a high type, and this results in snobbish behavior.

The intuition for the conformist case is just the opposite. Suppose that the social status function is concave, and again for argument's sake, decreasing on some interval. Due to the concavity, it is now more costly to lose one's position when one is ranked low then when one is ranked high. Thus, as n shifts right and more consumers buy good a, the expected status of those buying good a decreases less rapidly than the expected status of those buying good b. This implies that the the signaling gain from good a, $S_{a/b}(n)$, increases as more buy it. Intuitively, societal values instill a fear of being identified as a low type, of being ostracized, and this expresses itself in conformist behavior.

As can be seen in equation (10), the instance of snobbism or conformity is independent of the vertex of the social status function α, and whether the social status function is increasing or decreasing on some interval. However, α also holds economic significance. In the snobbish case, α is interpreted as the least desired value of x. For example, x could be a scale of how sophisticated a consumer is at $x = 0$ to outdoorsy and rugged at $x = 1$.[19] I allow for any $\alpha \in [0, 1]$.[20] α in the middle implies that extremists are more valued than centrists, which may be sensible in many snobbish settings such as the measure of sophistication to ruggedness described. The effect of increasing α is to raise the signaling gain from purchasing the left most good, without affecting the responsiveness of those signaling gain to the good's consumer base, as seen by the shift up of the $S_{a/b}(n)$ line in the right graphs of Figure 2.

[19]Consumers to the left may prefer a water bottle that is more elegant and appealing in business settings, such as the S'well water bottle, and consumers to the right might prefer a water bottle that is more useful in outdoorsy environments, such as a Camelbak water bottle. S'well prides itself on making water bottles that are elegant and impressionable in a board room setting, even though many of its customers are students and others not using it in business settings. Camelbak sells water bottles that are durable, lightweight and well-suited for outdoor activities such as hiking and biking. Camelbak advertises that it supplies the United States military, and that its produced have been used in both the second Gulf war and the War in Afghanistan. However, most Camelbak consumers are probably not using their water bottles in such harsh climates.

[20]In the analysis, I describe the implications of allowing for a broader range of α.

In the conformist case, α is interpreted as the most desired value of x. For example, x could be a measure of how New England is a consumer's identity at $x = 0$ to Southern at $x = 1$. Here, α in the middle implies that centrists are more valued than extremists. Bernheim (1994) argues that α is often in the middle in conformist markets. The effect of lowering α is to raise the signaling gain from purchasing the left most good, without affecting the responsiveness of those signaling gain to the good's consumer base, as seen by the shift up of the $S_{a/b}(n)$ line in the left graphs of Figure 2.

Figure 2 also hints at the effects of advertising on demand in these two settings. An increase in advertising λ increases the magnitude of the slope of $S_{a/b}(n)$, such that the signaling gain from a good becomes more responsive to its consumer base. Intuitively, by increasing the chance that the public recognizes a purchase and the social identity it conveys, advertising increases the strength of conformist or snob effects on demand, depending on the case. We will see how firms can take advantage of this dynamic for strategic purposes when solving the model.

Lastly, note that the signaling gain from a good is unaffected by adding a constant to the social status function. Intuitively, if each consumer's social status is increased or decreased by some identical amount, then it does not affect their reputation gain from a specific signal.

Product Locations

In order to understand consumer demand, it is useful to define a couple terms related to product positioning, as this will effect which brands consumers gravitate towards. Following the literature on horizontal differentiation, I say that firm A has a "location advantage" if $\ell_a + \ell_b > 1$ and it is closer to a greater mass of consumers, while firm B has the location advantage if $\ell_a + \ell_b < 1$ and it is closer to a greater mass of consumer. Firms are symmetric if $\ell_a + \ell_b = 1$.

However, and in contrast to the traditional literature on horizontal differentiation, in this model it matters not just *how many* consumers a firm is closer to, but also *who* a firm is closer to. Firms not only wish to appeal to a *large quantity* of consumers, but also to a *high quality* of consumers by winning the patronage of those with high status. This captures an intuitive and real market dynamic, where firms compete to capture the endorsement of celebrities, social media "influencers," and other high types — even-though such high types may constitute a small portion of customers.

I say that the firm on the side with more high types has a "prestige advantage." In the snobbish case where $\beta > 0$, firm A (firm B) has the prestige advantage if $\alpha > \frac{1}{2}$ ($\alpha < \frac{1}{2}$) because then there are higher types on the left than on the right (higher types on the right than on the left). In the conformist case where $\beta < 0$, firm A (firm B) has the prestige advantage if $\alpha < \frac{1}{2}$ ($\alpha > \frac{1}{2}$) because then there are higher types on the right than on the left (on the left than on the right). Firms have a symmetric prestige advantage if $\alpha = \frac{1}{2}$.

However, just because a firm is on the side with more high types does not mean it captures a higher signaling value than the other firm. Firms must also position themselves away from low types to win the better reputation. For example, consider a snobbish market with vertex $\alpha = 0.6$, implying that the left side of the social status function is higher than the right. If $\ell_a = 0.9$ and $\ell_b = 1$, then firm A commands a prestige advantage because it is on the left side. However, it is also more preferred by low types near $x = 0.6$. Indeed, when the market is spit equally between firm locations $n = \frac{\ell_a+\ell_b}{2}$, firm A has the lower signaling value $S_{a/b}(\frac{0.9+1}{2}) < 0$.

Thus, I define the firm with the more "prestigious position" as the firm which holds greater signaling value at an even split of the market between their locations. In other words, firm A has the more prestigious position if $S_{a/b}(\frac{\ell_a+\ell_b}{2}) > 0$, and firm B has the more prestigious position if $S_{a/b}(\frac{\ell_a+\ell_b}{2}) < 0$. Firms have symmetrically prestigious positions if $S_{a/b}(\frac{\ell_a+\ell_b}{2}) = 0$. It can be shown from equation (10) that firm A has the more prestigious position when $\beta > 0$ ($\beta < 0$) and $\ell_a + \ell_b < 6\alpha - 2$ ($\ell_a + \ell_b > 6\alpha - 2$). Furthermore, firm B has the more prestigious position when $\beta > 0$ ($\beta < 0$) and $\ell_a + \ell_b > 6\alpha - 2$ ($\ell_a + \ell_b < 6\alpha - 2$). Firms have symmetrically prestigious positions when $\ell_a + \ell_b = 6\alpha - 2$ or $\beta = 0$. This terminology is summarized in Table 1.

If firms locate symmetrically ($\ell_a + \ell_b = 1$), then a firm has a prestige advantage if and only if it has a more prestigious position. However, if firms locate asymmetrically, then the firm with the prestige advantage may or may not be the firm with the more prestigious position, as in the example described above. That said, there needs to be significant asymmetries in firms locations for a firm with the prestige advantage to not also have the more prestigious position.

	Location Advantage
$\ell_a + \ell_b > 1$	Firm A
$\ell_a + \ell_b < 1$	Firm B
$\ell_a + \ell_b = 1$	Symmetric

	Prestige Advantage
Snobbish: $\beta > 0$ and $\alpha > \frac{1}{2}$	Firm A
Snobbish: $\beta > 0$ and $\alpha < \frac{1}{2}$	Firm B
Conformist: $\beta < 0$ and $\alpha < \frac{1}{2}$	Firm A
Conformist: $\beta < 0$ and $\alpha > \frac{1}{2}$	Firm B
$\beta = 0$ or $\alpha = \frac{1}{2}$	Symmetric

	More Prestigious Position
Snobbish: $\beta > 0$ and $\ell_a + \ell_b < 6\alpha - 2$	Firm A
Snobbish: $\beta > 0$ and $\ell_a + \ell_b > 6\alpha - 2$	Firm B
Conformist: $\beta < 0$ and $\ell_a + \ell_b < 6\alpha - 2$	Firm B
Conformist: $\beta < 0$ and $\ell_a + \ell_b > 6\alpha - 2$	Firm A
$\beta = 0$ or $\ell_a + \ell_b = 6\alpha - 2$	Symmetric

Table 1: Firm with Location Advantage, Prestige Advantage and more Prestigious Position when $\ell_a < \ell_b$

Demand Partitions

Armed with a better understanding of the dynamics of reputation signaling, we can calculate consumer demand. For any cut-off n, the excepted utility of a given consumer x for each good at the purchasing stage is:

$$U_x(a;n) = v - \tau(\ell_a - x)^2 - p_a + S_a(n) \tag{11}$$

$$U_x(b;n) = v - \tau(\ell_b - x)^2 - p_b + S_b(n) \tag{12}$$

where $S_a(n)$ and $S_b(n)$ are as defined in equations (7) and (8). A consumer buys good a if $U_x(a;n) > U_x(b;n)$, and buys good b if $U_x(b;n) > U_x(a;n)$. The equilibrium value of the cut-off n, call it $\hat{n}$, is defined by the consumer x who is just indifferent between buying goods

a and b. Plugging $\hat{n}$ in for x, setting equations (11) and (12) equal and solving yields:

$$\hat{n} = \frac{p_b - p_a + \tau(\ell_b - \ell_a)(\ell_a + \ell_b) + \lambda\beta(\alpha - \frac{1}{3})}{2\tau(\ell_b - \ell_a) + \lambda\frac{\beta}{3}} \tag{13}$$

If $\beta = 0$ and there are no reputation motives, then $\hat{n}$ simplifies to the cut-off in the standard Hotelling model with quadratic transportation costs. Also, note that $\hat{n}$ is a similar cut-off to that in the model of Grilo et al. (2001) when $\alpha = \frac{1}{2}$, and signaling equally affects the market shares of firms A ($\hat{n}$) and B ($1 - \hat{n}$). However, $\alpha \neq \frac{1}{2}$ introduces an asymmetry in how signaling affects the market shares of the two firms.

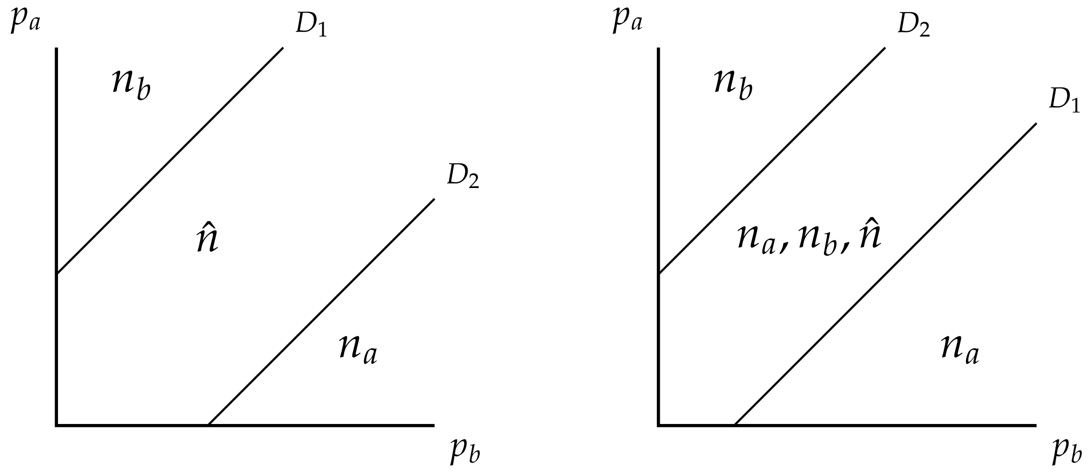
We need to check that $\hat{n}$ is in $[0, 1]$. Whether $\hat{n} \in [0, 1]$ depends on prices and the sign of the denominator. As in Grilo et al. (2001), I say the market is characterized by "Snobbism\Weak Conformity" if $2\tau(\ell_b - \ell_a) > -\lambda\frac{\beta}{3}$ and the denominator is positive. This necessarily holds in a snobbish market, and may hold in a conformist market if firms are sufficiently far and differentiated relative to the degree of advertising and conformity. By contrast, I say the market is characterized by "Strong Conformity" if $2\tau(\ell_b - \ell_a) \leq -\lambda\frac{\beta}{3}$ and the denominator is non-positive.

Definition (Weak and Strong Conformity)**.** *The market is characterized by "weak conformity" if* $\lambda\beta < 0$ *and* $2\tau(\ell_b - \ell_a) > -\lambda\frac{\beta}{3}$*. The market is characterized by "strong conformity" if* $\lambda\beta < 0$ *and* $2\tau(\ell_b - \ell_a) \leq -\lambda\frac{\beta}{3}$.

Suppose the market is characterized by snobbism or weak conformity. Evaluating equation (13), we get a unique semi-separating equilibrium with $\hat{n} \in (0, 1)$ when the difference in firm prices is not too large and lies inside the following range: $\tau(\ell_b - \ell_a)(\ell_a + \ell_b - 2) + \lambda\beta(\alpha - \frac{2}{3}) < p_a - p_b < \tau(\ell_b - \ell_a)(\ell_a + \ell_b) + \lambda\beta(\alpha - \frac{1}{3})$.

If the price difference is large and lies outside this range, then all consumers purchase the cheaper good and we get a pooling equilibrium. Let's call the cut-off n_A when firm A captures the market (i.e. $\hat{n} = 1$), and n_B when firm B captures the market (i.e. $\hat{n} = 0$). We get a unique pooling equilibrium with firm A capturing the market, n_A, when $p_a - p_b \leq \tau(\ell_b - \ell_a)(\ell_a + \ell_b - 2) + \lambda\beta(\alpha - \frac{2}{3})$. Furthermore, we get a unique pooling equilibrium with firm B capturing the market, n_B, when $p_a - p_b \geq \tau(\ell_b - \ell_a)(\ell_a + \ell_b) + \lambda\beta(\alpha - \frac{1}{3})$. Figure 4a maps these demand partitions for any given price vector.

Suppose instead the market is characterized by strong conformity. In this case, there may be multiple equilibria of the consumption subgame, and demand is defined by a correspondence rather than a function. If firm A has a much lower price and $p_a - p_b < \tau(\ell_b - \ell_a)(\ell_a + \ell_b) + \lambda\beta(\alpha - \frac{1}{3})$, then there is a unique pooling equilibrium in which firm A captures the entire market. If firm B has a much lower price and $p_a - p_b > \tau(\ell_b - \ell_a)(\ell_a + \ell_b - 2) + \lambda\beta(\alpha - \frac{2}{3})$, then there is a unique pooling equilibrium in which firm B captures the entire market. However, if $\tau(\ell_b - \ell_a)(\ell_a + \ell_b) + \lambda\beta(\alpha - \frac{1}{3}) \leq p_a - p_b \leq \tau(\ell_b - \ell_a)(\ell_a + \ell_b - 2) + \lambda\beta(\alpha - \frac{2}{3})$, then equilibria with any of the following consumer partitions are possible: n_A, n_B, or $\hat{n} \in (0, 1)$, where $\hat{n}$ is as defined by equation (13). Figure 4 shows a map of demand in (p_b,p_a) space.[21] We will revisit the issue of the multiplicity of demand partitions when we discuss the equilibria at the pricing stage.



(a) Snobbism \Weak Conformity

(b) Strong Conformity

Figure 4: Demand Partitions with Snobbism\Weak Conformity

$$D_1: \; p_a = p_b + \tau(\ell_b - \ell_a)(\ell_a + \ell_b) + \lambda\beta(\alpha - \frac{1}{3})$$
$$D_2: \; p_a = p_b + \tau(\ell_b - \ell_a)(\ell_a + \ell_b - 2) + \lambda\beta(\alpha - \frac{2}{3})$$

The diagonal lines D_1 and D_2 in Figure 4 denote barriers between possible consumer partitions.[22] These diagonal lines will have economic significance in the pricing stage. Intuitively, the firm that commands a larger region of high market share on the (p_b, p_a) plane is likely to be able to use its strategic power to extract a greater price. Furthermore, these diagonal lines are responsive to λ, β, α, τ, ℓ_a and ℓ_b.

In Figure 4 the the diagonal lines are drawn on either side of the 45 degree line, but this

[21] Not shown in Figure 4 is some upper price limits $\overline{p_a}$ and $\overline{p_b}$ above which consumers do not purchase.

[22] In Figure 4a, only partition n_b is possible on the D_1 line, and only partition n_a is possible on the D_2 line. By contrast, in Figure 4b, both partitions n_a and n_b are possible on both the D_1 and D_2 lines, and only partitions n_a and n_b are possible on the D_1 and D_2 lines.

need not always be the case. In the snobbish case, D_1 is always above the 45 degree line and D_2 is below it. As $\lambda\beta$ increases or firms move farther apart, then the diagonal lines separate farther apart. Intuitively, an increase in the degree of snobbery or differentiation implies that it is more likely for consumers to split their demand between the two stores for any given prices. However, as either $\lambda\beta$ decreases or as firms move closer together, the diagonal lines move closer together. Intuitively, an increase in the degree of conformity or a decrease in differentiation implies that it is easier to find prices such that all consumers patron the same store. In the conformist case, the diagonal lines eventually cross as $\lambda\beta$ becomes increasingly negative. The point at which the diagonal lines cross $D_1 = D_2$ is where the market switches from being characterized by weak conformity to strong conformity.

Whether the diagonal lines cross above or below the 45 degree line will have important implications when we solve for prices in a conformist market. If firm A has a more prestigious position and $\ell_a + \ell_b > 6\alpha - 2$, then the diagonal lines cross above the 45 degree line.[23] This implies that there may be a large region in (p_b, p_a) space where all consumers frequent store A. By contrast, if firm B has a more prestigious position and $\ell_a + \ell_b < 6\alpha - 2$, then the diagonal lines cross below the 45 degree line.[24] Thus, there may be a large region in (p_b, p_a) space where all consumers frequent store B. Finally, if $\ell_a + \ell_b = 6\alpha - 2$, then they cross at the 45 degree line.

Tie-Breaking Rule for Undifferentiated Firms

The demand partitions described apply mutatis mutandis when firm B locates to the left $\ell_b < \ell_a$ by flipping the a and b terms. However, if firms choose alike locations $\ell_a = \ell_b$, then the single crossing property of consumer demand fails, and demand can, but does not necessarily have to follow the cut-off rule described in Lemma 1. This issue also arises in the standard Hotelling model.

The Hotelling literature usually accommodates this by assuming, à la Bertrand competition, that homogeneous firms evenly split the market at $n = \frac{1}{2}$ if they charge equivalent prices, and the firm with the lower price captures all demand if they charge different prices. I impose the same tie-breaking rule here in the case of a standard market where $\lambda\beta = 0$.

However, this is not always a possible demand partition in a market characterized by

[23] Note that if $\alpha < \frac{1}{3}$, then the diagonal lines must cross above the 45 degree line.
[24] Note that if $\alpha > \frac{2}{3}$, then the diagonal lines must cross above the 45 degree line.

snobbism or conformity. For example, in a snobbish market, some consumers might still shop at the higher priced firm even if firms are undifferentiated. Furthermore, in a conformist market, consumers prefer to all buy from same firm when alike firms charge alike prices.

To extend the tie-breaking rule to the cases of snobbery and conformity, I assume that demand follows the partition described in the analysis preceding that is most advantageous to firm A. For example, consider snobbish market in which $\alpha \geq \frac{1}{2}$ and the firm to the left has a prestige advantage. I then assume firm A captures the demand described by it being to the left as in equation (13) with $\ell_a = \ell_b$: $\hat{n} = \frac{p_b - p_a + \lambda\beta(\alpha - \frac{1}{3})}{\lambda\frac{\beta}{3}}$. If instead $\alpha < \frac{1}{2}$ and $\lambda\beta > 0$, then firm A captures demand: $\hat{n} = \frac{p_a - p_b + \lambda\beta(\frac{2}{3} - \alpha)}{\lambda\frac{\beta}{3}}$.

In a conformist market, the lack of differentiation between firms implies the market is characterized by strong conformity. If the firm to the left has a prestige advantage and $\alpha \leq \frac{1}{2}$, then I assume demand follows the partition described by the strongly conformist case with firm A to the left as in Figure 4b. If instead the firm to the right has a prestige advantage and $\alpha < \frac{1}{2}$, then I assume that firm A captures the demand described by the strongly conformist case where $\ell_a \geq \ell_b$ (constructed by flipping the a and b terms in Figure 4b).

V PRICING IN CONFORMIST AND SNOBBISH MARKETS

With the above map of consumer demand, we can calculate the equilibrium at the pricing stage, given firm advertising levels and locations. I treat the cases of snobbism\weak conformity and strong conformity separately.

Denote the demand of firm A and firm B by $Q_a(p_a, p_b, \lambda, \ell_a, \ell_b)$ and $Q_b(p_a, p_b, \lambda, \ell_a, \ell_b)$ respectively, as derived in Section IV, so firms maximize $p_a Q_a(p_a, p_b, \lambda, \ell_a, \ell_b)$ and $p_b Q_b(p_a, p_b, \lambda, \ell_a, \ell_b)$.

Snobbism\Weak Conformity

Suppose we are in a market characterized by snobbism\weak conformity. The cut-off $\hat{n}$ is then given by equation (13). A pure strategy price equilibrium exists since demands are linear and decreasing in own prices. Differentiating $p_a Q_a(p_a, p_b, \lambda, \ell_a, \ell_b)$ and $p_b Q_b(p_a, p_b, \lambda, \ell_a, \ell_b)$ with respect to own prices and solving yields:

$$p_a^* = \frac{\tau}{3}(\ell_b - \ell_a)(2 + \ell_a + \ell_b) + \lambda\alpha\frac{\beta}{3} \tag{14}$$

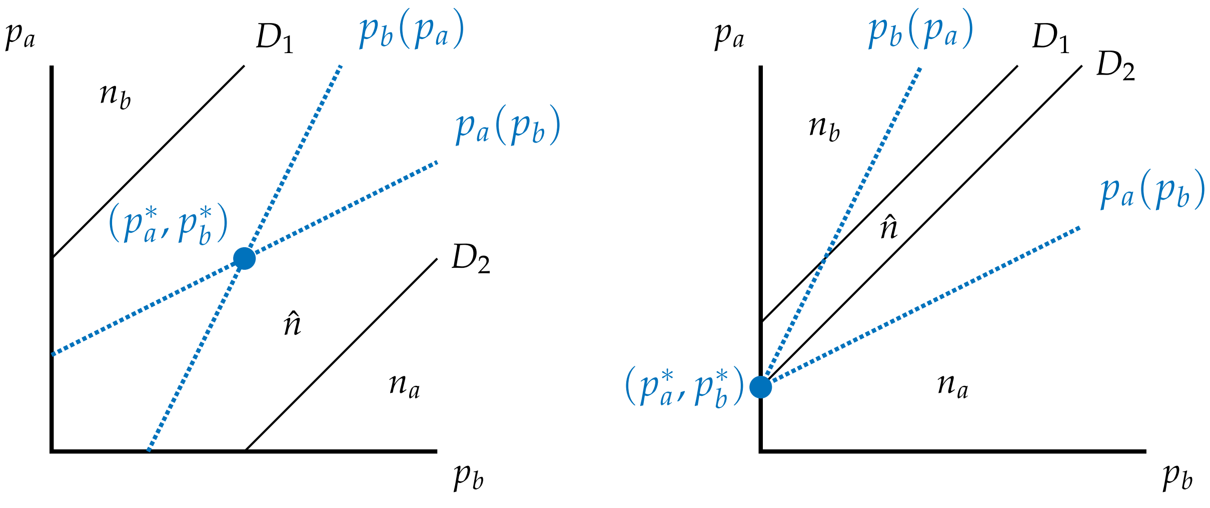
$$p_b^* = \frac{\tau}{3}(\ell_b - \ell_a)(4 - \ell_a - \ell_b) + \lambda(1-\alpha)\frac{\beta}{3} \tag{15}$$

$$n^* = \frac{\frac{\tau}{3}(\ell_b - \ell_a)(2 + \ell_a + \ell_b) + \lambda\alpha\frac{\beta}{3}}{2\tau(\ell_b - \ell_a) + \lambda\frac{\beta}{3}} \tag{16}$$

For this to be a price equilibrium, it remains to be checked that $n^* \in (0,1)$. This holds if and only if:

$$-\lambda\beta < \tau(\ell_b - \ell_a)\min\{\frac{2 + \ell_a + \ell_b}{\alpha}, \frac{4 - \ell_a - \ell_b}{1-\alpha}\} \tag{17}$$

In a snobbish market, equation (17) always holds, so that $n^* \in (0,1)$ and both firms earn positive revenues as described by equations (14) - (15).[25] This equilibrium is depicted graphically in Figure 5a.



(a) No Firm Takeover (Snobbism or Weak Conformity)

(b) Firm A Takeover (Weak Conformity)

Figure 5: Price Equilibrium with Snobbism\Weak Conformity

$$p_a(p_b) = \frac{p_b}{2} + \frac{\tau}{2}(\ell_b - \ell_a)(\ell_a + \ell_b) + \frac{\lambda\beta}{2}(\alpha - \frac{1}{3})$$
$$p_b(p_a) = \frac{p_a}{2} + \frac{\tau}{2}(\ell_b - \ell_a)(2 - \ell_a - \ell_b) + \frac{\lambda\beta}{2}(\frac{2}{3} - \alpha)$$

Inspection of the price equilibrium described by equations (14) - (16) reveals interesting insight into the dynamics of snobbish markets. First, both firms' prices are increasing in advertising in a snobbish market. Intuitively, by increasing the strength of consumers' snobbish

[25]Firm A's revenues simplify to $\frac{p_a^{*2}}{2\tau(\ell_b - \ell_a) + \lambda\frac{\beta}{3}}$ and Firm B's revenues to $\frac{p_b^{*2}}{2\tau(\ell_b - \ell_a) + \lambda\frac{\beta}{3}}$.

motives, advertising reduces the elasticity of demand with respect to prices.[26] This is because when firms cut prices, fewer consumers rush in to buy their products, as their signaling gains from the product decrease the more who buy it. This dampens price competition, and induces firms to converge on higher prices. Moreover, advertising by one firm raises the price of the other firm. If the market share effects described below are not too large, then advertising can increase the profits of both firms. Thus, for certain parameters advertising can be non-combative, instead acting as a public good to firms. This is made visually clear in Figure 5a. Advertising shifts some (or all) of the diagonal lines and price response lines out in a way that pushes the price equilibrium in the north east direction.

However, advertising has a greater positive effect on the price of the firm with the prestige advantage. Intuitively, by increasing recognition that one firm has more high types, advertising allows that firm to extract a higher price. In Figure 5a, advertising pushes the price equilibrium out in a direction more favorable to the firm with the prestige advantage. Actually, if we extended the results by allowing for $\alpha \notin [0, 1]$, then advertising would lessen the price of the firm with the prestige disadvantage because it would have to undercut its price to attract customers.

Reputation motives also create interesting effects on firms' market shares. Advertising benefits the market share of the firm with the more prestigious position, at the detriment of the market share of the other. Intuitively, a firm preferred by more high status consumers can use advertising to attract more customers. Even if firms are symmetrically located, then asymmetries arise where the firm with the more prestigious position commands a higher price and earns greater market share. This can explain why we sometimes see physically similar or indistinguishable products with one earning greater market share and charging a price premium. These results are summarized in Proposition 1.

[26] More precisely, advertising reduces a weighted combination of the elasticity of demand of goods a and b with respect to their own and each others' prices that results in higher prices. For example, the elasticity of demand of good a with respect to p_a and p_b are $\epsilon^a_{p_a} = \frac{dn}{dp_a}\frac{p_a}{n} = \frac{-p_a}{p_b - p_a + \tau(\ell_b - \ell_a)(\ell_a + \ell_b) + \lambda\beta(\alpha - \frac{1}{3})}$ and $\epsilon^a_{p_b} = \frac{dn}{dp_b}\frac{p_b}{n} = \frac{p_b}{p_b - p_a + \tau(\ell_b - \ell_a)(\ell_a + \ell_b) + \lambda\beta(\alpha - \frac{1}{3})}$ respectively. Thus, in a snobbish market, the absolute value of elasticity of demand for good a with respect to p_a and p_b decreases in advertising when $\alpha \geq \frac{1}{3}$, and increases in advertising otherwise. Similarly, in a snobbish market, the absolute value of the elasticity of demand for good b with respect to p_a and p_b decreases in advertising when $\alpha \leq \frac{2}{3}$, and increases otherwise. The net result is that advertising effects $\epsilon^a_{p_a}$, $\epsilon^a_{p_b}$, $\epsilon^b_{p_a}$ and $\epsilon^b_{p_b}$ in a way that raises the prices of both goods.

Proposition 1 (Advertising's Effect on Prices and Market Share in a Snobbish Market)**.** *In a snobbish market, advertising by either firm weakly raises the prices of both firms. However, advertising has a greater positive effect on the price of the firm with the prestige advantage. Furthermore, advertising has a positive effect on the market share of the firm with the more prestigious position, and a negative effect on the market share of the firm with the less prestigious position.*

I turn to a weakly conformist market next. If equation (17) holds in a weakly conformist market, then we get the equilibrium described by equations (14) - (16). It is apparent that advertising then has the opposite effect, decreasing firms' prices. The intuition is that by strengthening consumers' conformist motives, advertising increases the elasticity of demand with respect to prices — when firms cut prices, more consumers rush to buy in due to the reputation gains from others doing so. This heightens the price competition and induces firms to converge on lower prices. Here, it is found that advertising has a greater negative effect on the price of the firm with the prestige disadvantage, because it has added need to undercut its price to retain consumers.

Additionally, as in a snobbish market, we see that advertising benefits the market share of the firm with the more prestigious position, at the detriment of the market share of the other. Even if firms are symmetrically located, then the firm with the more prestigious position commands a higher price and earns greater market share. However, there are no parameters for which advertising benefits both firms, as in a snobbish market. These results are summarized in Proposition 2.

Proposition 2 (Advertising's Effect on Prices and Market Share in a Weakly Conformist Market)**.** *Consider a weakly conformist market in which neither firm takes over. Advertising by either firm weakly lowers the prices of both firms. However, advertising has a greater negative effect on the price of the firm with the prestige disadvantage. Furthermore, advertising has a positive effect on the market share of the firm with the more prestigious position, and a negative effect on the market share of the firm with the less prestigious position.*

If price competition is sufficiently severe, then equation (17) may not hold in a conformist market. That is, if product differentiation is sufficiently low relative to the intensity of advertising and conformity, then equation (17) does not hold. In this case, one firm wins all consumer demand. Essentially, conformist motives overpower consumers' transportation costs, and all go to the most popular brand. It turns out that the the firm with the more prestigious positions takes over the market, charging a limit-price. If equation (17) does not hold and firm A has a more prestigious position, then it charges the highest price such that firm B cannot charge a weakly positive price and earn any customers:

$$p_a^* = -\tau(\ell_b - \ell_a)(2 - \ell_a - \ell_b) + \lambda\beta(\alpha - \frac{2}{3}) \tag{18}$$

$$p_b^* = 0 \tag{19}$$

$$n^* = n_a \tag{20}$$

This price equilibrium is depicted in Figure 5b. It occurs when the D_2 and $p_b(p_a)$ lines are sufficiently high that they intersect the $p_a(p_b)$ line on the p_a-axis.

If equation (17) does not hold and firm B has a more prestigious position, then it charges the highest price such that no consumers patron firm A at any $p_a \geq 0$:

$$p_a^* = 0 \tag{21}$$

$$p_b^* = -\tau(\ell_b - \ell_a)(\ell_a + \ell_b) - \lambda\beta(\alpha - \frac{1}{3}) \tag{22}$$

$$n^* = n_b \tag{23}$$

This price equilibrium occurs when the D_1 and $p_a(p_b)$ lines are sufficiently low that they intersect the $p_b(p_a)$ line on the p_b-axis. This single firm dominance is a result we could not get in standard model of horizontal differentiation with zero production costs. Note that if firms are symmetrically located, then equation (17) always holds in a weakly conformist market.

Strong Conformity

Let's now explore equilibrium prices in a market characterized by strong conformity. As shown in Section IV, there may be multiple possible consumer partitions for given prices, and demand is defined by a correspondence rather than a function.

This implies, as in Grilo et al. (2001), there may also be multiple equilibria at the pricing stage. However, many of these price equilibria are unreasonable and rely on unusual off-equilibrium consumer behavior where demand moves highly non-monotonically in prices. It turns out that under a reasonable refinement firms' revenues are uniquely determined in the equilibrium of the pricing stage. The axiom below is predicated on the premise that when there are three possible consumers partitions for a given price vector, firms anticipate consumers settling on the partition most beneficial to the firm with the cheaper product.

Axiom 1 (Cheaper is Better)**.** *If price pair (p_b, p_a) can induce partitions n_a, n_b and $\hat{n} \in (0,1)$ and $p_a \neq p_b$, then consumers settle on the partition giving the highest market share to the firm with the lower price (n_a if $p_a < p_b$ and n_b if $p_a > p_b$).*

I assume Axiom 1 holds throughout the analysis that follows. This imposes a certain degree of monotonicity on the movement of demand with respect to prices, so that when prices move from the north west to the south east of the (p_b, p_a) quadrant in Figure 4b, the anticipated demand partitions generally move from firm B earning greater share to firm A. Under this refinement, firms' equilibrium revenues are uniquely determined in the pricing stage, as described in Proposition 3.[27]

Proposition 3 (Strongly Conformist Price Equilibrium)**.** *Consider a strongly conformist market with $\ell_a \leq \ell_b$. Under Axiom 1, the equilibria at the pricing stage are as follows. If D_1 and D_2 intersect the p_a-axis, then $p_b^* = 0$, $n^* = n_a$ and*

$$p_a^* = \tau(\ell_b - \ell_a)(\ell_a + \ell_b) + \lambda\beta(\alpha - \frac{1}{3}) \tag{24}$$

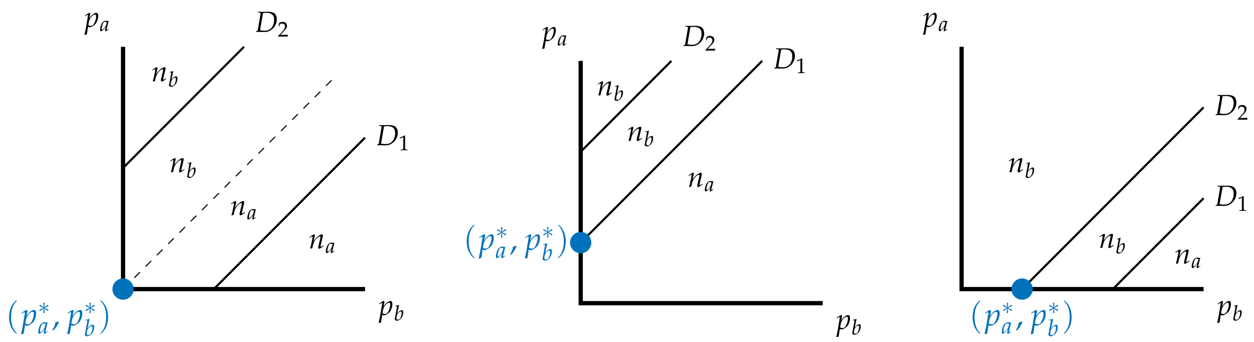
If D_1 and D_2 intersect the p_b-axis, then $p_a^* = 0$, $n^* = n_b$ and

$$p_b^* = \tau(\ell_b - \ell_a)(2 - \ell_a - \ell_b) - \lambda\beta(\alpha - \frac{2}{3}) \tag{25}$$

Otherwise, $p_a^* = p_b^* = 0$, and $n^* \in \{\hat{n}, n_a, n_b\}$.

These equilibria are depicted in Figure 6. The intuition as to why both firms may earn zero revenues is that as conformity grows very large, product differentiation matters comparatively less, and we get an equilibrium resembling Bertrand competition. However, if prestige effects are sufficiently large, then one firm may takeover and earn positive revenues.

[27] One could obtain the same equilibrium described in Proposition 3 by applying the invariance axiom introduced in Grilo et al. (2001) in combination with the following axiom: If price pair (p_b, p_a) can induce partitions n_a, n_b and $\hat{n} \in (0,1)$ and $p_a \neq p_b$, then consumers do *not* settle on the partition giving the highest market share to the firm with the higher price (n_b if $p_a < p_b$ and n_a if $p_a > p_b$). The invariance axiom says that if price pair (p_b, p_a) induces consumer partition n^*, then $(p_b + z, p_a + z)$ induces the same partition n^* where z is an arbitrary constant, $(p_b + z, p_a + z) \geq (0,0)$ and all consumers want to buy. In other words, if consumer utility for each good is added or subtracted by an identical constant, then consumers buy the same goods.



(a) Bertrand Equilibrium **(b)** Firm A Takeover **(c)** Firm B Takeover

Figure 6: Price Equilibria in Strongly Conformist Market

Monopoly Pricing

Let's look at pricing in the monopoly subgame resulting from firm B not entering the market. In congruence with the oligopoly subgame, I maintain the traditional assumption that v is sufficiently large that a monopolist supplies the whole market. This holds for any given locations and advertising levels if $v > \max\{3\tau \pm \beta\}$. I assume this condition is met throughout the analysis that follows.

The calculation of the signaling gains from purchasing good a parallels that in the oligopoly case at the beginning of the section. Here, the signaling gains from good a, call it $S_{a/\emptyset}$, are the perceived status of a consumer who purchases less that of a consumer who picks the outside option of not purchasing (rather than less that of a consumer who purchases good b). Since every consumer purchases in equilibrium, the signaling gains from purchasing are determined by the (advertisement receiving) public's off-equilibrium beliefs about a consumer's identity should she not purchase.[28]

I restrict attention to the following off-equilibrium beliefs. A consumer who does not purchase is believed to be the consumer at the end furthest from the good and, thus benefiting most from foregoing purchase. That is, $\rho(\, x = 1 \,|\, \emptyset, \Omega \,) = 1$ if $\ell_a < \frac{1}{2}$ and $\rho(\, x = 0 \,|\, \emptyset, \Omega \,) = 1$ if $\ell_a > \frac{1}{2}$. If $\ell_a = \frac{1}{2}$, then consumers at both ends benefit equally from not purchasing. Thus, in this case I assume that a consumer who does not purchase is believed to be at the end with the lower social status: $\rho(\, x = 1 \,|\, \emptyset, \Omega \,) = 1$ if $\beta \geq 0$ and $\alpha \geq \frac{1}{2}$, or if $\beta < 0$ and $\alpha \leq \frac{1}{2}$; and $\rho(\, x = 0 \,|\, \emptyset, \Omega \,) = 1$ otherwise.[29] These off-equilibrium beliefs are not only reasonable, but

[28]Just as in the conformist case with takeover, where the signaling gains from the favored good are determined by off-equilibrium beliefs about a consumer's identity should she purchase the less favored good.

[29]This makes firm A's profit function upper semi-continuous at $\ell_a = \frac{1}{2}$, helping ensure the existence of an equilibrium in the larger game where locations are endogenous.

also congruent with the off-equilibrium beliefs assumed in an oligopolistic market.

If a consumer who foregoes purchase is believed to be at end $x = 1$, then:

$$S_{a/\emptyset} = \lambda\beta(\frac{2}{3} - \alpha)$$

If a consumer who does not purchase is believed to be at end $x = 0$, then:

$$S_{a/\emptyset} = \lambda\beta(\alpha - \frac{1}{3})$$

Firm A charges the highest price such that a consumer located at the furthest end is just indifferent to purchasing ($U_x(a) = U_x(\emptyset)$). Firm A's monopoly price (and likewise revenues since $n = n_a$) is thus:

$$p_a^M = \begin{cases} v - \tau(1-\ell_a)^2 + \lambda\beta(\alpha - \frac{2}{3}) & \text{if } \ell_a < \frac{1}{2} \\ & \text{or } \ell_a = \frac{1}{2}, \beta < 0 \text{ and } \alpha \leq \frac{1}{2} \\ & \text{or } \ell_a = \frac{1}{2}, \beta > 0 \text{ and } \alpha \geq \frac{1}{2} \\ v - \tau\ell_a^2 - \lambda\beta(\alpha - \frac{1}{3}) & \text{otherwise} \end{cases} \tag{26}$$

VI PERSUASIVE ADVERTISING EQUILIBRIA

Turning to the main motivation of the paper, I incorporate firm location, advertising and entry choices, characterizing the equilibria of the full game. It will be seen how these market variables are affected by snobbish and conformist demand, and whether such signaling motives can explain the empirical regularities outlined in the introduction.

Firm B must consider whether to enter, ℓ_b and λ_b to maximize profits given ℓ_a and λ_a, and firm A must choose ℓ_a and λ_a to maximize profits given the subsequent influence those choices will have on firm B. As a benchmark of comparison, it is useful to begin with the canonical case of a market without status motives $\beta = 0$.

Proposition 4 (Equilibrium of Standard Market)**.** *If $\beta = 0$, then there exists a unique equilibrium in which firm B enters, firms locate at opposite ends ($\ell_a^* \in \{0, 1\}$, $\ell_b^* = 1 - \ell_a^*$), and do not advertise ($\lambda_a^* = \lambda_b^* = 0$), thus charging equivalent prices ($p_a^* = p_b^* = \tau$) and evenly splitting the market ($n^* = \frac{1}{2}$).*

The intuition is as follows. Firm B enters because there are always positive profits to be had with zero production cost. Firms move as far apart as possible to avoid price competition, "the principal of maximal differentiation." Zero advertising takes place because there are no status motives to capture.

A similar comparison could be drawn by studying a "full information version" of the model, as traditional in the signaling literature, where the public has complete knowledge of each consumer's x and no informational asymmetries persist. If consumer identities are known, then consumers are unable to affect their perceived status through consumption, implying that their choices rely simply on their horizontal preferences and prices. In such a game, firms have no incentive to advertise, yielding the equilibrium described in Proposition 4 above. This signaling free setup does not explain the role of persuasive advertising, asymmetries in the prices and market shares of physically similar goods, and the barriers to entry often faced in heavily advertised markets.

Equilibria of Snobbish Market

Greater insight may be gained in a market characterized by snobbery $\beta > 0$. If firm B enters and, without loss of generality, locates to the right of firm A ($\ell_a < \ell_b$), then the profit functions at the location and advertising stage(s), incorporating the price equilibrium, are given by:

$$\pi_a(\ell_a, \lambda_a; \ell_b, \lambda_b) = \frac{\left(\frac{\tau}{3}(\ell_b - \ell_a)(2 + \ell_a + \ell_b) + \lambda\alpha\frac{\beta}{3} \right)^2}{2\tau(\ell_b - \ell_a) + \lambda\frac{\beta}{3}} - \frac{c}{2}\lambda_a^2 \tag{27}$$

$$\pi_b(\ell_b, \lambda_b; \ell_a, \lambda_a) = \frac{\left(\frac{\tau}{3}(\ell_b - \ell_a)(4 - \ell_a - \ell_b) + \lambda(1-\alpha)\frac{\beta}{3} \right)^2}{2\tau(\ell_b - \ell_a) + \lambda\frac{\beta}{3}} - \frac{c}{2}\lambda_b^2 \tag{28}$$

If firm B locates to the left of firm A, then firm profit functions can be found by flipping the a and b terms in equations (27) - (28) above.[30] Note that the profits functions given in equations (27) - (28) are discontinuous at $\ell_a = \ell_b$ when $\alpha \neq \frac{1}{2}$. Even-though an equilibrium exists at the pricing stage, this discontinuity implies that establishing the existence of an equilibrium at the location and advertising stages is not trivial (Dasgupta and Maskin, 1986; Simon, 1987).[31]

[30] If $\ell_b = \ell_a$ and $\alpha \geq \frac{1}{2}$, then firm A (firm B) earns profits given by equation (27) (by equation (28)); and if $\ell_b = \ell_a$ and $\alpha < \frac{1}{2}$, then firm A (firm B) earns profits given by equation (28) (by equation (27)). See end of section IV.

[31] A large literature studies issues arising from the existence of equilibria at both the pricing and location stages of product differentiation models (see Shaked and Sutton (1987, pp.132-133) for a summary). Many describe the characteristics of an equilibrium, should it exist, without guaranteeing its existence (Prescott and Visscher, 1977; Lane, 1980; Neven, 1987; Shaked and Sutton, 1987).

However, note that in any equilibrium, if it exists, firm B enters because there are always positive profits to be had. For example, firm B can locate at any $\ell_b \neq \ell_a$ without advertising $\lambda_b = 0$ and earn positive profits. These profits may be higher than firm B could earn without snobbish demand $\beta = 0$ (note that this sheds light on the abundance of brands often observed in snobbish markets, although only shown here for the case of two firms and zero production cost). Furthermore, I find that firms locate at opposite ends, circumventing issues arising from the discontinuity at $\ell_a = \ell_b$ and facilitating proof of existence.

Proposition 5 (Existence of Equilibrium in Snobbish Market)**.** *If $\beta > 0$ and $\alpha \in [0, 1]$, then there exists an equilibrium of the game.*

The intuition for firms moving to opposite ends is that two forces are at play. The first is the traditional force, described in Proposition 4, of avoiding price competition. However, a second force is introduced. It comes from one firm appealing to more high types, giving it greater prestige, and the ability to extract a higher price. If $\alpha \notin [\frac{1}{3}, \frac{2}{3}]$, then the more prestigious firm may not sufficiently fear price competition to be driven to an end. All else equal, it may extract such a high price that it prefers to drift to the middle to be closer to a greater quantity of consumers and improve its market share. The firm vulnerable to this motive is firm A, as it is the first-mover. However, if firm A drifts to the middle, then firm B would undercut it by advancing to the more prestigious side. In other words, firm B would attain greater prestige. Thus, firm A stays at an end to prevent firm B from winning the high types.

While closed-form solutions are not always admissible (one notable exception is when $\alpha = \frac{1}{2}$), I characterize additional equilibrium properties related to advertising, prices and market share in Proposition 6.

Proposition 6 (Equilibrium of Snobbish Market)**.** *If $\beta > 0$ and $\alpha \in [0, 1]$, then in any equilibrium:*

(1) Firm B enters.

(2) Total advertising is positive.

(3) Firms locate at opposite ends.

(4) The firm with the more prestigious position charges a higher price and earns greater market share.

(5) If $\alpha = \frac{1}{2}$ and firms have symmetric prestige, then firm A invests weakly less in advertising than firm B. Furthermore, the equilibrium has a closed-form solution with $\ell_a^* \in \{0, 1\}$, $\ell_b^* = 1 - \ell_a^*$, $p_a^* = p_b^* = \tau + \lambda^* \frac{\beta}{6}$, $n^* = \frac{1}{2}$ and advertising expenditures:

$$\lambda_a^* = \begin{cases} \frac{\beta - \frac{\beta^2}{6c}}{12c - \frac{\beta^2}{6c}} & \text{if } \beta < 6c \\ 0 & \text{otherwise} \end{cases}$$

$$\lambda_b^* = \max\{(1 - \lambda_a^*)\frac{\beta}{12c}, 1\}$$

Propositions 1 and 6 capture many of the stylized fact observed in snobbish markets: an abundance of brands; inflated prices; and the greater market share, price and marketing efforts of the more prestigious. (1), (3) and (4) are unsurprising given the prior analysis. The intuition for (2) and (5) is as follows. Total advertising is positive because at least one, and possibly both firms have positive marginal revenues from advertising when there are zero advertisements, while the marginal cost is zero. If market prestige is symmetric $\alpha = \frac{1}{2}$, then firms gain equally from advertising. Thus, as the first-mover, firm A invests weakly less in advertising to induce firm B to take on greater advertising expense. However, if firm A has greater prestige, then it may have greater marginal returns from advertising and invest more in advertising.

To give a fuller picture of the equilibria, I provide numerical solutions in Figure 7.[32] The numerical solutions also shed light on how changes in β and α affect market variables, as the envelope theorem cannot be applied to analytically derive these comparative statics due to non-differentiability in firms' best reply functions for certain parameter values. Figure 7 shows firm prices, market shares, advertising levels and profits for various α and β, assuming

[32] With the exception of the right column where $\alpha = 0.5$, which depicts the closed-form solution shown in Proposition 6.

$c = \tau = 2$. In all the equilibria pictured, it is found that firm A locates at the end with the prestige advantage $\ell_a^* = 1$ and firm B locates at the other end $\ell_b^* = 0$.[33] Figure 7 suggests that total advertising expenditures and firm prices increase in β. Except when firms have symmetric prestige $\alpha = \frac{1}{2}$, it is found that firm A invests more in advertising than firm B. Furthermore, as β increases or α moves to an end, the dispersion in firm prices, market shares and profits widens.

[33] This continues to hold unless the cost of advertising is sufficiently high and α is sufficiently close to $\frac{1}{2}$. In this case, firm A may want to locate at the less prestigious end. The intuition being that if the loss of prestige is sufficiently small relative to the cost saving, then firm A may want to locate at the less prestigious end to induce firm B to take on greater advertising expense.

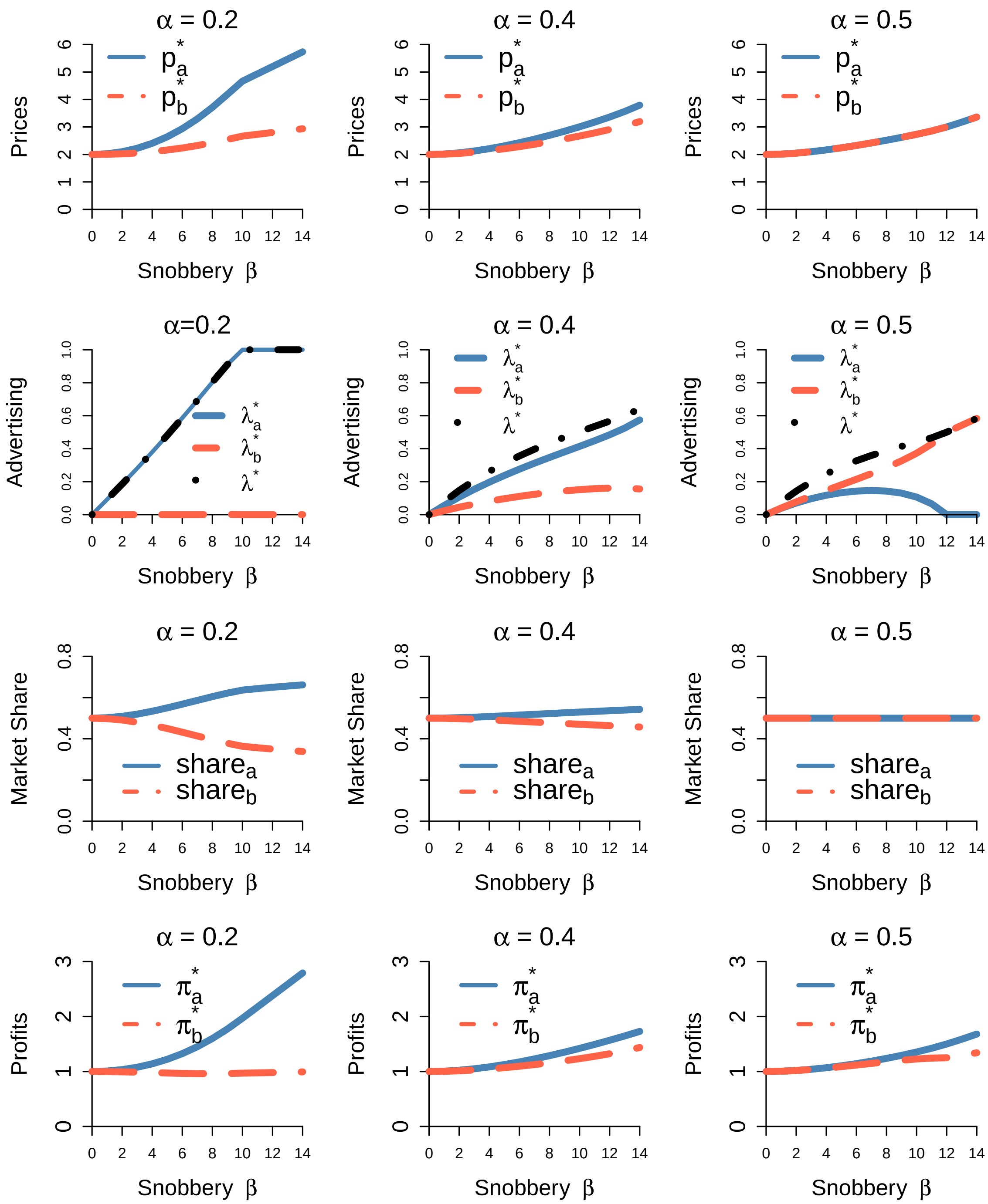


Figure 7: Numerical Solution in Snobbish Market

Snobbery β on the x-axis. In descending rows: prices, advertising, market shares and profits on the y-axis. In columns, from left to right: $\alpha = 0.2$, $\alpha = 0.4$ and $\alpha = 0.5$. The cases of $\alpha = 0.2$ and $\alpha = 0.4$ were calculated using Matlab, while a closed-form analytic solution is given for $\alpha = 0.5$ (see Proposition 6). In all equilibria depicted, $\ell_a^* = 1$ and $\ell_b^* = 0$. Assumes $c = \tau = 2$.

Equilibria of Conformist Market

Turning to the conformist case, let's see if the model can shed light on the barriers to entry often created in such markets by well advertised and branded firms. As before, I first establish the existence of an equilibrium.

Proposition 7 (Existence of Equilibrium in Conformist Market)**.** *If $\beta < 0$ and $\alpha \in [0,1]$, then there exists an equilibrium of the game.*

Firm A can pursue one of two classes of strategies: accommodate firm B's entry, or deterring firm B's entry. Firm A can deter firm B's entry by choosing a prestigious enough position, and advertising heavily enough that firm B could not earn positive profits from entering. I explore equilibria where firm B does not enter if firm B cannot earn positive profits from entry.

In order for an entry deterrence strategy to exist, β needs to be sufficiently negative, creating adequate consumer conformity for firm A to capture. If β is sufficiently negative, then there exists a compact set of strategies $\triangle \subset [0,1]^2$ such that if $(\ell_a, \lambda_a) \in \triangle$, then firm B cannot earn positive profits from entry. Additionally, in order for deterring entry to be profitable, the cost of advertising must be sufficiently small. If these two criteria are met, then firm A chooses $(\ell_a, \lambda_a) \in \triangle$, deterring firm B's entry and seizing monopoly rents.

If firm A chooses to accommodate firm B's entry, then firms locate at opposite ends. Firm A can take the more prestigious end. Firm A only advertises if the positive market share effect of advertising significantly outweighs the negative price effect. This may occur if $\alpha \notin [\frac{1}{3}, \frac{2}{3}]$, but does not occur if $\alpha \in [\frac{1}{3}, \frac{2}{3}]$. Firm B never advertises because, as the firm with the less prestigious position, it can only lose market share and price from advertising. Proposition 8 fully characterizes the equilibria.

Proposition 8 (Equilibrium of Conformist Market)**.** *Suppose $\beta < 0$ and $\alpha \in [0,1]$. If β is sufficiently negative and c is sufficiently small, then firm A chooses some location and positive level of advertising such that deters firm B's entry, allowing firm A to capture monopoly profits. Otherwise, firms locates at opposite ends $\ell_a^* \in \{0,1\}$ and $\ell_b^* = 1 - \ell_a^*$, firm B does not advertise $\lambda_b^* = 0$, and firm A does not advertise $\lambda_a^* = 0$ if $\alpha \in [\frac{1}{3}, \frac{2}{3}]$, and might advertise otherwise $\lambda_a^* \geq 0$ if $\alpha \notin [\frac{1}{3}, \frac{2}{3}]$.*

Interestingly, it may be the case that firm B's entry would make firm A's entry deterring strategy sub optimal. In other words, it may be the case that if firm B happened to enter when firm A chose $(\ell_a, \lambda_a) \in \triangle$, then firm A's strategy would no longer be optimal. However, firm A rationally anticipates that firm B would not enter, thus allowing firm A to gain monopoly power.[34] The intuition resembles that of the chain store paradox (Selten, 1978). In

[34]It is shown in the proof of Proposition 8 that for certain parameters it is can also be the case that firm A would choose $(\ell_a, \lambda_a) \in \triangle$ regardless of whether firm B entered.

the language of the chain store paradox, firm A's location and advertising choice can serve as a commitment to fight by generating harsh conditions of price competition upon firm B's entry.

Another striking aspect of this result is the strength of first mover advantage implied to firm A. For example, there does not exist an equilibrium in which firm B takes over. In order for firm B to take over, firm A would have to act non-rationally or there would have to be an unanticipated shock to the social status function. In other words, firms would have to be out of equilibrium.

The strength of the first mover advantage in the game captures the strength of the first mover advantage observed in conformist markets. Unlike many other models of entry deterrence, this result does not require assumptions about a specific production or cost function implying economies of scale. Indeed, the model assumed zero cost to production. If the model incorporated a fixed cost of production, then firm B's entry would be even less likely.

Figure 8 shows numerical solutions of the value of $\bar{c}$, the highest cost of advertising such that firm A deters entry, for various combinations of β and α. The picture shows an intuitive equilibrium feature, that as β becomes increasingly negative, a higher cost of advertising is needed to prevent firm A from monopolizing.

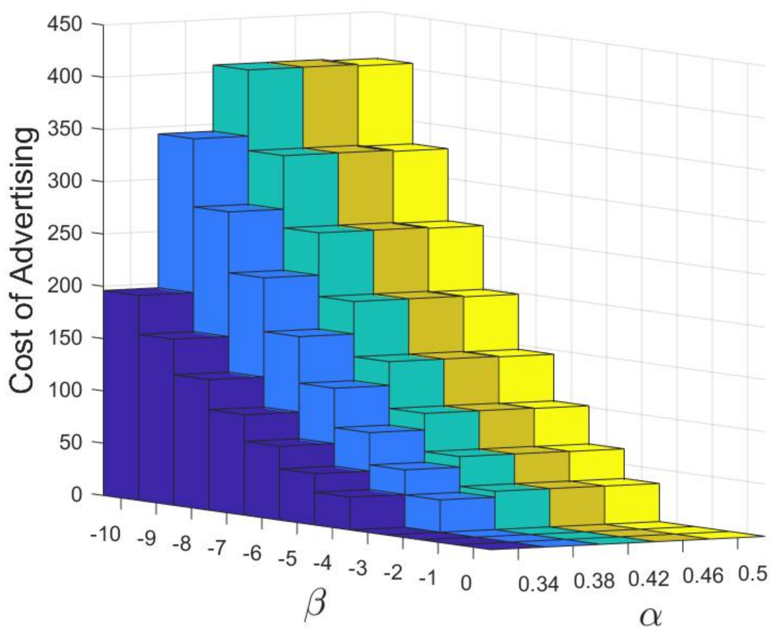


Figure 8: Numerical Solution in Conformist Market
Maximum Cost of Advertising for Firm A to Monopolize ($\bar{c}$)
Calculated using Matlab. Assumes $\tau = 1$ and $v = 13$.

VII WELFARE

With the above equilibria in mind, let's explore the implications for the old question of the welfare consequences of persuasive advertising. As mentioned, since this model has a fixed preference, it easily lends to welfare analysis.

Without loss of generality, suppose both firms enter the market and $\ell_a^* < \ell_b^*$. The equilibrium levels of consumer surplus ("CS"), producer surplus ("PS") and total surplus ("TS" $\equiv$ CS + PS) are given below. Consumer surplus is equivalent to consumers' valuation of the good, minus their money spent and transportation costs incurred, plus their aggregated reputational utility from signaling. Producer surplus is equal to firms' profits. Furthermore, total surplus is equal to producer surplus plus consumer surplus, where money spent by consumers cancels with firm revenues, representing a transfer of welfare from consumers to firms.

$$CS^* = \underbrace{v}_{\text{good value}} - \underbrace{\tau\left(\int_0^{n^*}(x-\ell_a^*)^2dx + \int_{n^*}^1 (x-\ell_b^*)^2dx\right)}_{\text{transport costs}} - \underbrace{(n^*p_a^* + (1-n^*)p_b^*)}_{\text{prices}} + \underbrace{E(s(x))}_{\text{reputational surplus}}$$

$$PS^* = \underbrace{n^*p_a^* + (1-n^*)p_b^*}_{\text{price}} - \underbrace{\left(\frac{c}{2}\lambda_a^{*2} + \frac{c}{2}\lambda_b^{*2}\right)}_{\text{advertising cost}}$$

$$TS^* = \underbrace{v}_{\text{good value}} - \underbrace{\tau\left(\int_0^{n^*}(x-\ell_a^*)^2dx + \int_{n^*}^1 (x-\ell_b^*)^2dx\right)}_{\text{transport costs}} - \underbrace{\left(\frac{c}{2}\lambda_a^{*2} + \frac{c}{2}\lambda_b^{*2}\right)}_{\text{advertising cost}} + \underbrace{E(s(x))}_{\text{reputational surplus}}$$

The consumer reputational surplus is calculated as follows.

$$\begin{aligned}
&n^*\frac{\lambda}{n^*}\int_0^{n^*} s(x)\,dx \;+\; (1-n^*)\frac{\lambda^*}{1-n^*}\int_{n^*}^1 s(x)\,dx \;+\; (1-\lambda^*)\int_0^1 s(x)\,dx \\
&= \int_0^1 s(x)\,dx \\
&= E(s(x))
\end{aligned}$$

The first term on the top line is the mass of consumers who purchase good a (n^*) multiplied by both the fraction of such consumers whose partner receives an advertisement (λ^*), and the status inference of their partners ($\frac{1}{n^*}\int_0^{n^*} s(x)\,dx$). The second term is the same calculation for consumers purchasing good b. The last term is the fraction of the public that does not receive an advertisement ($1-\lambda^*$), multiplied by their inference of a random consumer's status ($\int_0^1 s(x)\,dx$). The main thing to notice is that advertising λ^* does not affect consumer surplus from signaling. This is a result that generalizes to any social status function, firm locations, advertising, prices, or entry decisions.

As mentioned in Section I, the underlying reason is because status signaling is a zero-sum game, so advertising can affect which consumer gets what portion of that reputational utility pie, but cannot affect the overall size of the pie (Frank, 2005; Heffetz and Frank, 2011). In other words, if one consumer's perceived status goes up by a certain amount, then other consumer's perceived status must go down by a proportional amount, as the average perceived status must be held constant. On aggregate, any reputational gains and losses cancel out, and advertising has zero effect on the aggregate consumer surplus from signaling.

Furthermore, since advertising is costly to producers, it negatively affects producer and total surplus. Thus, persuasive advertising is found to be wasteful, and the social planner would select zero persuasive advertising. However, this result should be judged in context, as it could be perturbed by modeling the utility of the public, and allowing for the public's utility to depend in some way on their inference of the consumer's type. For example, this might be a more natural analysis if we modeled consumers and the public as playing a mating or an employee-employer matching game, where there could be higher returns on both sides to certain types of matches.

However, perhaps more interestingly, advertising indirectly affects consumer surplus through other channels. In the cases where advertising increases prices, it decreases consumer surplus and leads to a transfer of welfare from consumers to firms. Furthermore, advertising increases the transportation costs consumers incur in a couple ways. First, given firm locations and entry decisions, advertising creates signaling motives that lead some consumers to purchase products they otherwise would not. In other words, it leads consumers to not fully respect their horizontal preferences. The further n^* is from $\frac{1}{2}$ (the market share cut-off without signaling motives), the greater is this effect. Second, it can limit entry in the conformist case, thus increasing consumers' transportation costs through this mechanism as well.

This helps bring micro foundation to the old and debated sentiment that persuasive advertising can have harmful consequences for consumers and society, by improving our understanding of the channels through which this may take place (Dixit and Norman, 1978). Indeed, while the welfare consequences from the price effects of advertising have been heavily discussed, the welfare consequences from advertising inducing consumers to not fully respect their horizontal preferences and increasing their transportation costs are less often discussed, if at all (Bagwell, 2007). It should be clarified that this paper does not try to address the welfare consequences from other forms of advertising, such as informative advertising.

VIII CONCLUSION

This paper helps give persuasive advertising a micro foundation. The model assumes consumers wish to signal some latent attribute of their identity, and the social status implied by this attribute. The paper focuses on signaling dominated by one of two consumer motives — conformist, motivated by the desire to fit in, and snobbish, motivated by the desire to stand out. The role of persuasive advertising is to facilitate consumers in signaling their social status by rendering consumption a signaling device. The model sheds light on the effects of advertising on firms' prices, entry, locations, market shares and profits, and consumer and societal welfare in these two types of markets.

It is found that advertising in snobbish markets tends to be less combative, increasing prices and accommodating entry. In certain cases, advertising can act as a public good to firms, increasing both firms' profits. This helps make sense of the high number of designer products, as well as the supranormal prices observed in such markets. Furthermore, even when products are similar, the model shows the firm with the more prestigious position can command a greater price and market share through advertising.

In conformist markets, by contrast, the model shows that advertising can be highly combative because it heightens, rather than lessens, price competition. A first mover in a conformist market may use persuasive advertising to deter the entry of future firms and gain monopoly power, even when firms face zero production costs. This is congruent with the sometimes religious zeal of consumers to the brand which was the first mover in their region, and high concentration of market power in conformist markets.

A PROOFS

Proof of Lemma 1

A proof by contradiction is given here. Suppose not. Without loss of generality (WLOG), suppose $\ell_a < \ell_b$. Furthermore, suppose a consumer at some point $x' \in [0,1]$ purchases good b while a consumer at $x'' \in (x', 1]$ purchases good a. Let $S_a \in \Re$ and $S_b \in \Re$ denote the signaling value of choosing goods A and B respectively. For this to be equilibrium behavior, it must be that the expected utility gains to consumer x'' from good a over good b are weakly greater than that of consumer x'.

$$\begin{aligned}
&U_{x''}(a) - U_{x''}(b) \geq U_{x'}(a) - U_{x'}(b)\\
&(\ v - \tau(x'' - \ell_a)^2 - p_a + S_a\ )\ -\ (\ v - \tau(x'' - \ell_b)^2 - p_b + S_b\ )\\
\geq &(\ v - \tau(x' - \ell_a)^2 - p_a + S_a\ )\ -\ (\ v - \tau(x' - \ell_b)^2 - p_b + S_b\ )q
\end{aligned}$$

Canceling like terms, this is equivalent to $-(x'' - \ell_a)^2 + (x'' - \ell_b)^2 \geq -(x' - \ell_a)^2 + (x' - \ell_b)^2$. Further simplifying, this is equivalent to $x'' \leq x'$. This is a contradiction.

Proof of Lemma 2

($\Rightarrow$) Suppose $S_{a/b}(n) = kn + c$. Equation (11) in Corneo and Jeanne (1997) shows that $S_{a/b}(n)$ can be rationalized by any social status function of the form $s(x) = (1-2x)S_{a/b}(x) + x(1-x)S'_{a/b}(x) + d$ where d is an arbitrary constant. This yields $s(x) = (1-2x)(kx+c) + x(1-x)k + d = -4kx^2 + 2x(k-c) + c + k + d$. $s(x)$ is quadratic, $s''(x) = -8k$, and $S'_{a/b}(n) = k$.

($\Leftarrow$) Let $s(x) = k_1x^2 + k_2x + c$. Then, $S_{a/b}(n) = \frac{\lambda}{n}\int_0^n s(x)dx - \frac{\lambda}{n}\int_n^1 s(x)dx = -\frac{k_1}{3}(n+1) - \frac{k_2}{2}$. Furthermore, $s''(x) = 2k_1$ and $S'_{a/b}(n) = -\frac{k_1}{3}$.

Proof of Proposition 1

Suppose $\beta > 0$. Furthermore, WLOG, suppose $\ell_a \leq \ell_b$ and $\alpha \geq \frac{1}{2}$. Then, $\frac{dp_a^*}{d\lambda} = \alpha\frac{\beta}{3} \geq \frac{dp_b^*}{d\lambda} = (1-\alpha)\frac{\beta}{3} \geq 0$. Additionally:

$$\frac{dn^*}{d\lambda} = \frac{[\,2\tau(\ell_b - \ell_a) + \lambda\frac{\beta}{3}\,][\,\alpha\frac{\beta}{3}\,] - [\,\frac{\tau}{3}(\ell_b - \ell_a)(2 + \ell_a + \ell_b) + \lambda\alpha\frac{\beta}{3}\,][\,\frac{\beta}{3}\,]}{[\,2\tau(\ell_b - \ell_a) + \lambda\frac{\beta}{3}\,]^2}$$

The denominator is always positive. Thus, the sign of $\frac{dn^*}{d\lambda}$ is the sign of the numerator. The numerator is positive if $\ell_a + \ell_b < 6\alpha - 2$, zero if $\ell_a + \ell_b = 6\alpha - 2$, and negative if $\ell_a + \ell_b < 6\alpha - 2$. The same technique can be applied to show that $\frac{d(1-n^*)}{d\lambda} > 0$ if $\ell_a + \ell_b > 6\alpha - 2$, $\frac{dn^*}{d\lambda} = 0$ if $\ell_a + \ell_b = 6\alpha - 2$ and $\frac{d(1-n^*)}{d\lambda} < 0$ otherwise.

Proof of Proposition 2

Suppose $\beta < 0$ and neither firm takes over the market. Furthermore, WLOG, suppose $\ell_a < \ell_b$ and $\alpha \leq \frac{1}{2}$. Then, $\frac{dp_b^*}{d\lambda} = (1-\alpha)\frac{\beta}{3} \leq \frac{dp_a^*}{d\lambda} = \alpha\frac{\beta}{3} \leq 0$. The value of $\frac{dn^*}{d\lambda}$ is given in the proof of

Proposition 1. The denominator of $\frac{dn^*}{d\lambda}$ is always positive. Thus, the sign of $\frac{dn^*}{d\lambda}$ is the sign of the numerator. The numerator of $\frac{dn^*}{d\lambda}$ is greater than zero if $\ell_a + \ell_b > 6\alpha - 2$, equals zero if $\ell_a + \ell_b >= \alpha - 2$ and is negative otherwise. The same technique can be applied to show that $\frac{d(1-n^*)}{d\lambda} > 0$ if $\ell_a + \ell_b < 6\alpha - 2$, $\frac{dn^*}{d\lambda} = 0$ if $\ell_a + \ell_b = 6\alpha - 2$ and $\frac{d(1-n^*)}{d\lambda} < 0$ otherwise. Finally, if $\ell_a = \ell_b$ then the market is characterized by strong conformity so the proposition does not apply.

Proof of Proposition 3

Suppose Axiom 1 holds in a strongly conformist market.

First, consider Figure 6a where the D_1 and D_2 lines lie on opposite sides of the 45 degree line. I will show that no strictly positive price vector $(p_a, p_b) > (0, 0)$ can be a price equilibrium. Recall that on the 45 degree line partitions n_a, n_b and $\hat{n}$ are possible, and on the D_1 and D_2 lines only partitions n_a and n_b are possible. Consider a strictly positive price vector to the right of 45 degree line with partition n_a. This cannot be a price equilibrium because firm B can lower its price to some point to the left of the 45 degree line and earn positive revenues. The same applies to any strictly positive price vector to the left of the 45 degree line with partition n_b, because firm A lower its price and earn positive revenues. Additionally, there is no strictly positive price vector on the D_1 line (D_2 line) with partition n_b (n_a) that can be a price equilibrium, because firm A (firm B) can raise its price by an arbitrarily small epsilon and earn positive revues. Similarly, there is no strictly positive price vector on the 45 degree line with consumer partition n_a, n_b or $\hat{n}$ that can be a price equilibrium, because one firm would have an incentive to lower its price by an arbitrarily small epsilon and earn greater revenues. Thus, we are only left with price vectors for which one or both of the firms charge a zero price. Consider a price vector such that $p_b = 0$ and $p_a > 0$. This cannot be a price equilibrium, because firm B could improve its revenues by raising its price by some arbitrarily small epsilon and earning market share n_b. The same holds mutatis mutandis for any price vector such that $p_b > 0$ and $p_a = 0$. We are left to prove that the price vector $(p_a, p_b) = (0, 0)$ can be supported as an equilibrium for any consumer partition $n^* \in \{\hat{n}, n_a, n_b\}$. At such a price vector, both firms earn zero revenues. This zero price vector can be supported as an equilibrium if consumers settle on the following partitions off the equilibrium path: consumers settle on partition n_b at the point on the D_2 line where $p_b = 0$, and consumers settle on partition n_a at the point on the D_1 line where $p_a = 0$. In this case, neither firm can improve its revenues by raising its price. Thus, $(p_a, p_b) = (0, 0)$ is a price equilibrium.

Next, consider the case of Figure 6b where the D_1 and D_2 lines lie above the 45 degree line. There is no price vector on the D_1 line or above it with partition $n = n_b$ that can be supported as a price equilibrium, because firm A could lower its price and earn positive revenues. Furthermore, no price vector on the D_2 line with $n = n_a$ can be supported as a price equilibrium, because firm B can raise its price by epsilon and capture the market. This leaves us with price vectors on the D_1 line with partition n_a, and price vectors below it. There cannot exist a price equilibrium below the D_1 line, because firm A would have an incentive to raise its price. There also cannot be a price equilibrium on the D_1 line with strictly positive prices, because one firm would have an incentive to lower its price and improve its revenues by capturing the market. This leaves only the price vector on the D_1 line such that $p_b = 0$. This can be supported as a price equilibrium if consumers settle on partition n_a, because then neither firm can improve its revenues by changing its price. The same arguments apply mutatis mutandis to the case of Figure 6c where the D_1 and D_2 lie to the right of the 45 degree line.

Proof of Proposition 4

Suppose $\beta = 0$. For convenience, I denote firm strategies as pairs (ℓ_a, λ_a) and (ℓ_b, λ_b). The price strategies firms associate with their location and advertising choices are given by equations (14) - (15). First, let's consider an equilibrium where $\ell_b^* \geq \ell_a^*$. The profit functions of firms at the location and advertising stages are:

$$\pi_a = \frac{\tau}{18}(\ell_b - \ell_a)(2 + \ell_a + \ell_b)^2 - \frac{c}{2}\lambda_a^2$$
$$\pi_b = \frac{\tau}{18}(\ell_b - \ell_a)(4 - \ell_a - \ell_b)^2 - \frac{c}{2}\lambda_b^2$$

From these profit functions, it is apparent that neither firm advertises in any equilibrium since $\frac{d\pi_a(\lambda_a,\ell_a,\lambda_b,\ell_b)}{d\lambda_a} = \frac{d\pi_b(\lambda_a,\ell_a,\lambda_b,\ell_b)}{d\lambda_b} = -c \leq 0$. Next, I examine firm locations. Suppose firm A locates at some $\ell_a \in [0, 1)$. Firm B's strategy $(1, 0)$ strictly dominates any other strategy $(\ell_b, 0)$ such that $\ell_b \geq \ell_a$ since $\frac{d\pi_b(\lambda_a,\ell_a,\lambda_b,\ell_b)}{d\ell_b} > 0$ for all $\ell_b \geq \ell_a$. Similarly, given that firm A anticipates firm B choosing strategy $(1, 0)$, firm A's strategy $(0, 0)$ strictly dominates any other strategy $(\ell_a, 0)$ such that $\ell_a \leq \ell_b$ since $\frac{d\pi_a(\lambda_a,\ell_a,\lambda_b,\ell_b)}{d\ell_a} < 0$ for all $\ell_a \leq 1$. Thus, $(\ell_a, \lambda_a) = (0, 0)$ and $(\ell_b, \lambda_b) = (1, 0)$ is an equilibrium. Furthermore, firms are indifferent between locating at either of opposing ends, because each earns profits $\frac{\tau}{2}$ in either case. Therefore, $(\ell_a, \lambda_a) = (1, 0)$ and $(\ell_b, \lambda_b) = (0, 0)$ is another equilibrium. There are no other equilibria since firms do not advertise in any equilibrium and must locate at opposite ends.

Proof of Proposition 5

Suppose $\beta > 0$ and $\alpha \in [0, 1]$. I will make use of Theorem 1 in Harris (1985) to prove the existence of an equilibrium. This theorem is also proved in Hellwig and Leininger (1987), and applied by Börgers (1988) in a similar game. A specialized version of the theory that is sufficient for present purposes says: Suppose a finite number of players move sequentially and choose actions from compact metric spaces. Furthermore, suppose the set of actions available to each player is independent of the actions made by other players, and each player's payoff is a continuous function of the actions of all players. Such a game has a subgame perfect equilibrium in pure strategies.

Existence would be an immediate implication of of Theorem 1 in Harris (1985) if it were not for the fact that firms' profit functions are discontinuous at $\ell_a = \ell_b$ when $\alpha \neq \frac{1}{2}$. However, existence can be established with a little more work. It is shown below in the proof of part (3) of Proposition 6 that firms must locate at opposite ends in any equilibrium.

We can then reduce the game to two modified subgames in which firms are exogenously located at opposite ends and only choose advertising levels $\lambda_a \in [0, 1]$ and $\lambda_b \in [0, 1]$, and a larger game in which firm A makes a binary choice between the two ends $\ell_a = 0$ and $\ell_a = 1$ given the equilibria of the two advertising subgames. The price strategies firms associate with their location and advertising choices are then given by equations (14) - (15).

WLOG, consider the subgame in which $\ell_a = 0$ and $\ell_b = 1$. Firms' profit functions are then:

$$\pi_a(\lambda_a, \lambda_b) = \frac{(\ \tau + \lambda\alpha\frac{\beta}{3}\ )^2}{2\tau + \lambda\frac{\beta}{3}} - \frac{c}{2}\lambda_a^2 \tag{29}$$

$$\pi_b(\lambda_a, \lambda_b) = \frac{(\ \tau + \lambda(1-\alpha)\frac{\beta}{3}\ )^2}{2\tau + \lambda\frac{\beta}{3}} - \frac{c}{2}\lambda_b^2 \tag{30}$$

Since firms' profit functions are continuous in (λ_a, λ_b), and firms choose sequentially from compact and independent action spaces, Theorem 1 in Harris (1985) guarantees the existence of an equilibrium in this modified subgame. Existence is similarly assured in the modified subgame in which firm locations are $\ell_a = 1$ and $\ell_b = 0$. Since there exists an equilibrium in each of the two modified subgames, there also exists a solution to firm A's binary location choice between the two ends. Therefore, there exists an equilibrium of the overall game.

Proof of Proposition 6

Suppose $\beta > 0$.

(1): Proven in the main text.

(2): I will prove that in any equilibrium at least one firm advertises by contradiction. Suppose neither firm advertises in equilibrium. In this case, Proposition 4 shows that firms must locate at opposite ends. WLOG, consider the case of $\ell_a = 0$ and $\ell_b = 1$. Firms profit functions at the location and advertising stages are then given by equations (29) - (30). For this to be an equilibrium, it must be that $\frac{d\pi_a}{d\lambda_a} \leq 0$ when $(\lambda_a, \lambda_b) = (0, 0)$. The derivative of equation (29) with respect to λ_a when $(\lambda_a, \lambda_b) = (0, 0)$ is:

$$\frac{d\pi_a}{d\ell_a} = \frac{2\alpha\frac{\beta}{3}(\tau)(2\tau) - (\tau)^2\frac{\beta}{3}}{4\tau^2}$$

For this derivative to be weakly negative, it must be that $\alpha \leq \frac{1}{4}$. It must also be that $\frac{d\pi_b}{d\lambda_b} \leq 0$ when $(\lambda_a, \lambda_b) = (0, 0)$. Taking the derivative of equation (30) with respect to λ_b when $(\lambda_a, \lambda_b) = (0, 0)$, I find that α must be greater than or equal to $\frac{3}{4}$ for it to be weakly negative. This is a contradiction, because it cannot be that $\alpha \leq \frac{1}{4}$ and $\alpha \geq \frac{3}{4}$.

(3): **Case 1**. Suppose $\alpha \in [\frac{1}{3}, \frac{2}{3}]$. I will show that firms locate at opposite ends. First, I will show that if $\alpha \leq \frac{2}{3}$, then the firm on the left strictly prefers locating at 0 to any other position to the left. Similarly, I will show that if $\alpha \geq \frac{1}{3}$, then the firm on the right strictly prefers locating at 1 to any other position to the right. This implies that if $\alpha \in [\frac{1}{3}, \frac{2}{3}]$, then firms locate at opposite ends.

WLOG, consider equilibria where firm A locates to the left of firm B, $\ell_a < \ell_b$. In this case, firm A and B's profits at the location and advertising stages are given by equations (27) and (28) respectively. For any given $\alpha \leq \frac{2}{3}$ and λ, we will see that $\ell_a = 0$ strictly dominates any other $\ell_a < \ell_b$. This is easiest to see by checking that $\frac{d\pi_a(\ell_a,\ell_b,\lambda)}{d\ell_a} \leq 0$ at $\ell_a = 0$. Since profits are concave in locations, this is sufficient to show that ℓ_a strictly dominates any other $\ell_a < \ell_b$. The derivative of equation (27) with respect to ℓ_a when $\ell_a = 0$ is:

$$\frac{d\pi_a}{d\ell_a} = \frac{-\frac{4\tau}{3}\left[\,\frac{\tau}{3}\ell_b(2+\ell_b) + \lambda\alpha\frac{\beta}{3}\,\right]\left[\,2\tau\ell_b + \lambda\frac{\beta}{3}\,\right] + 2\tau\left[\,\frac{\tau}{3}\ell_b(2+\ell_b) + \lambda\alpha\frac{\beta}{3}\,\right]^2}{\left[\,2\tau\ell_b + \lambda\frac{\beta}{3}\,\right]^2}$$

The denominator is always positive. Thus, the sign of the derivative is given by the sign of the numerator. Setting the numerator less than or equal to zero and simplifying yields the inequality $4\tau\ell_b + \frac{2}{3}\lambda\beta \geq \tau\ell_b(2+\ell_b) + \lambda\alpha\beta$. It is apparent that i) $4\tau\ell_b > \tau\ell_b(2+\ell_b)$ and ii) $\frac{2}{3}\lambda\beta \geq \lambda\alpha\beta$ when $\alpha \leq \frac{2}{3}$. Thus, if $\alpha \leq \frac{2}{3}$, then firm A must locate at 0 if it locates to the left of firm B. Applying the same procedure, one can show that if $\alpha \geq \frac{1}{3}$, then $\frac{d\pi_b(\ell_a,\ell_b,\lambda)}{d\ell_b} \geq 0$ at $\ell_b = 1$ for any λ. Thus, if $\alpha \geq \frac{1}{3}$, then firm B's most preferred location to the right of firm A is at $\ell_b = 1$. This analysis applies equally to the case of $\ell_b < \ell_a$. Furthermore, note that firm B never locates at $\ell_b = \ell_a$, because its profits are always strictly higher to either the left or right of ℓ_a.

Case 2. Consider $\alpha \in [0,1] \setminus [\frac{1}{3}, \frac{2}{3}]$. I will again show that firms locate at opposite ends. WLOG, suppose $\alpha \in (\frac{2}{3}, 1]$. From case 1 above we know that firm B prefers $\ell_b = 1 > \ell_a$ to any other $\ell_b > \ell_a$. Furthermore, we know that firm B never locates at $\ell_b = \ell_a$. Thus, firm B only needs to compare $\ell_b = 1$ and $\ell_b < \ell_a$.

If firm A prefers $\ell_a = 0 < \ell_b = 1$ to any other $\ell_a < \ell_b = 1$, then it immediately follows that firms locate at opposite ends in equilibrium. However, this need not always be true. Suppose $\frac{d\pi_a}{d\ell_a} > 0$ given $\ell_a = 0$, $\ell_b = 1$ and some $\lambda > 0$ so that this does not hold. In other words, firm A prefers to drift right of $\ell_a = 0$ if $\ell_b = 1$. I will show that then firm B prefers $\ell_b = 0 < \ell_a$ to $\ell_b = 1 > \ell_a$. In other words, firm B moves to the left of firm A if firm A decides to drift right. Firm A then cannot locate at $\ell_a > 0$ and stay to the left of firm B. Thus, firm A must locate at $\ell_a = 0$, or $\ell_a > \ell_b$ in equilibrium.

Evaluating the inequality $\frac{d\pi_a}{d\ell_a} > 0$ at $\ell_a = 0$, $\ell_b = 1$ and simplifying yields $4\tau + \frac{2}{3}\lambda\beta \leq 3\tau + \lambda\alpha\beta$. Further simplifying:

$$\tau < \lambda\beta(\alpha - \frac{2}{3}) \tag{31}$$

Inequality (31) must hold for firm A to prefer to drift right of $\ell_a = 0$. Next, I show that if inequality (31) holds, the firm B can make higher revenues at $\ell_b = 0$ than at $\ell_b > \ell_a$. Firm B's price at $\ell_b = 0$ is $p'_b = \frac{\tau}{3}(\ell_a)(2+\ell_a) + \lambda\alpha\frac{\beta}{3}$. Firm B's price at $\ell_b = 1$ (firm B's most preferred location to the right of firm A) is $p''_b = \frac{\tau}{3}(1-\ell_a)(3-\ell_a) + \lambda(1-\alpha)\frac{\beta}{3}$. $p'_b \geq p''_b$ given any $\ell_a > 0$ if $p'_b > p''_b$ when $\ell_a = 0$. Furthermore, $p'_b > p''_b$ when $\ell_a = 0$ if:

$$\tau < \lambda\beta(\frac{2\alpha}{3} - \frac{1}{3}) \tag{32}$$

Note that inequality (31) implies inequality (32). Thus, firm B can earn a higher price at

$\ell_b = 0 < \ell_a$ than at $\ell_b > \ell_a$. Next, I show that firm B can also earn a higher market share at $\ell_b = 0 < \ell_a$ than at $\ell_b > \ell_a$. Firm B's market share at $\ell_b = 0$ is:

$$\frac{\frac{\tau}{3}(\ell_a)(2+\ell_a) + \lambda\alpha\frac{\beta}{3}}{2\tau(\ell_a) + \lambda\frac{\beta}{3}} \tag{33}$$

Firm B's market share at $\ell_b = 1$ is:

$$\frac{\frac{\tau}{3}(1-\ell_a)(3-\ell_a) + \lambda(1-\alpha)\frac{\beta}{3}}{2\tau(1-\ell_a) + \lambda\frac{\beta}{3}} \tag{34}$$

Since the numerators of fractions (33) and (34) are p_b' and p_b'' respectively, the numerator of fraction (33) is greater than the numerator of fraction (34) by the algebra above. Furthermore, the denominator of fraction (33) is less than the denominator of fraction (34) if $\ell_a < \frac{1}{2}$. Thus, if $0 < \ell_a < \frac{1}{2}$, then firm B's revenues must be higher at $\ell_b = 0 < \ell_a$ than at $\ell_b > \ell_a$. It is plain to see that the same holds if $\ell_a \geq \frac{1}{2}$. Since firm B earns higher revenues at $\ell_b = 0 < \ell_a$ than at $\ell_b > \ell_a$ for any given λ, the same holds for firm B's profits.

It is evident that if firm A prefers $0 < \ell_a \leq 1 = \ell_b$ to $\ell_a = 0 < 1 = \ell_b$, then firm B prefers $\ell_b < \ell_a$. Thus, there does not exist an equilibrium in which firm A sets $\ell_a > 0$ and firm B locates at $\ell_b > \ell_a$. Finally, note that if firm A prefers $0 < \ell_a \leq 1 = \ell_b$ to $\ell_a = 0 < 1 = \ell_b$, then firm A prefers $\ell_a < \ell_b$ by the same arguments for which firm B prefers $\ell_b < \ell_a$. Therefore, under such circumstance, firm A must locate at $\ell_a = 0$ and firm B must locate at $\ell_b = 1$.

(4): From (3), we know that firms locate at opposite ends. Without loss of generality, suppose $\alpha > \frac{1}{2}$ and firm B has the less prestigious end ($\ell_b = 1$) in equilibrium. Plugging $\ell_b = 1$ and $\ell_a = 0$ into equations (14) and (15) yields $p_a = \tau + \lambda\alpha\frac{\beta}{3} > \tau + \lambda(1-\alpha)\frac{\beta}{3} = p_b$. Additionally, plugging $\ell_b = 1$ and $\ell_a = 0$ into equation (16) yields $n = \frac{p_a}{2\tau+\lambda\frac{\beta}{3}} > \frac{p_b}{2\tau+\lambda\frac{\beta}{3}} = 1 - n$ where p_a and p_b are as defined above.

(5): Suppose $\alpha = \frac{1}{2}$. (3) shows that firms locate at opposite ends. Furthermore, firm A and firm B's profits functions are symmetric at any $(\ell_a, \ell_b) \in \{(0,1),(1,0)\}$. Thus, firms are indifferent between the two end pairs. Incorporating their location decisions, firm A and B's profits as a function of advertising are:

$$\pi_a(\lambda_a, \lambda_b) = \frac{1}{2}(\tau + \lambda\frac{\beta}{6}) - \frac{c}{2}\lambda_a^2$$

$$\pi_b(\lambda_a, \lambda_b) = \frac{1}{2}(\tau + \lambda\frac{\beta}{6}) - \frac{c}{2}\lambda_b^2$$

where $p_a = p_b = \tau + \lambda\frac{\beta}{6}$ and $n = \frac{1}{2}$. Starting backwards by solving firm B's program yields with respect to λ_b yields:

$$\lambda_b = \max\{(1-\lambda_a)\frac{\beta}{12c}, 1\}$$

Plugging firm B's advertising response function into firm A's program and solving for λ_a

yields:

$$\lambda_a^* = \begin{cases} \frac{\beta - \frac{\beta^2}{6c}}{12c - \frac{\beta^2}{6c}} & \text{if } \beta < 6c \\ 0 & \text{otherwise} \end{cases}$$

It can be checked that $\lambda_a^* \leq \lambda_b^*$.

Proof of Proposition 7

Suppose $\beta < 0$ and $\alpha \in [0,1]$. As shown in the proof of Proposition 8, firm A's strategy at $t = 0$ can be broken down into deterring firm B's entry and choosing an action $(\ell_a, \lambda_a) \in \triangle$ where $\triangle$ is a compact subset of $[0,1]^2$, or choosing an action $(\ell_a, \lambda_a) \in [0,1]^2 \setminus \triangle$ and accommodating firm B's entry. If firm A deters firm B's entry, then it earns monopoly profits. If firm A accommodates firm B's entry, then it earns oligopoly profits as described in the main text.

First, suppose $\triangle$ is empty. Firm B must then enter in any equilibrium. It is shown in the proof of Proposition 8 that firms then locate at opposite ends. Thus, the game can be reduced to firms decisions over advertising given their exogenous locations at either end, with firm A choosing its more preferred end. In this reduced game, firms choose actions sequentially from independent and compact sets $\lambda_a \in [0,1]$ and $\lambda_b \in [0,1]$, and firms profit functions are continuous in their actions. Thus, we can apply Theorem 1 in Harris (1985) to guarantee the existence of an equilibrium in this reduced game. This equilibrium must also be an equilibrium of the original game.

Next, consider the case in which $\triangle$ is non-empty. Firm A must decide between choosing a strategy $(\ell_a, \lambda_a) \in \triangle$ and earning monopoly profits, or choosing $(\ell_a, \lambda_a) \in [0,1]^2 \setminus \triangle$, accommodating firm B's entry and earning oligopoly profits. An equilibrium in which firm A deters entry is given by the solution to:

$$\begin{aligned} \max_{(\ell_a, \lambda_a)} \quad & \pi_a^M(\ell_a, \lambda_a) \\ \text{subject to} \quad & (\ell_a, \lambda_a) \in \triangle \end{aligned}$$

where π_a^M is as defined in equation (26). Since the objective function is upper semi-continuous, and the constraint set is compact, the extreme value theorem guarantees a solution to this program.

If firm A accommodates entry in equilibrium, then once again, the proof of Proposition 8 that firms then locate at opposite ends. Thus, the game can be reduced to firms decisions over advertising given their exogenous locations at either end, with firm A choosing its more preferred end. In this reduced game, firms choose actions sequentially from independent and compact sets $[0,1]$, and firms profit functions are continuous in their action of advertising. Thus, we can apply Theorem 1 in Harris (1985) to guarantee the existence of an equilibrium in this reduced game. This equilibrium must also be an equilibrium of the original game.

Proof of Proposition 8

Suppose $\beta < 0$ and $\alpha \in [0,1]$. One word on notation before proceeding. Since we have already solved for firm pricing decisions in the main text, I often refer to firm A and B's strategy as pairs (ℓ_a, λ_a) and (ℓ_b, λ_b) respectively. Their associated pricing strategies are implied by the analysis in the main text.

Define the compact set $\triangle$ as the set of strategies $(\ell_a, \lambda_a) \in [0,1]^2$ that imply firm B cannot earn positive profits from any $(\ell_b, \lambda_b) \in [0,1]^2$. I consider equilibria where firm B does not enter if firm A chooses $(\ell_a, \lambda_a) \in \triangle$.

Claim 1: If β is sufficiently negative, then there exists a non-empty, compact space $\triangle \subset [0,1]^2$ such that firm B cannot earn positive profits from entry.

Proof of Claim 1. Suppose $\beta < 0$. Let $\triangle \subset [0,1]^2$ be the set of $(\ell_a, \lambda_a) \in [0,1]^2$ satisfying the inequalities below

$$\lambda \geq \frac{-\tau(1-\ell_a)}{\beta} \min\{\frac{3+\ell_a}{\alpha}, \frac{3-\ell_a}{1-\alpha}\} \tag{35}$$

$$\lambda \geq \frac{-\tau\ell_a}{\beta} \min\{\frac{2+\ell_a}{\alpha}, \frac{4-\ell_a}{1-\alpha}\} \tag{36}$$

$$\lambda \geq \frac{-\tau(1-\ell_a)^2}{\beta(\frac{2}{3}-\alpha)} \tag{37}$$

$$\lambda \geq \frac{-\tau\ell_a^2}{\beta(\alpha-\frac{1}{3})} \tag{38}$$

$\triangle$ defines the set of firm A strategies (ℓ_a, λ_a) that prevent firm B from earning positive profits at any $(\ell_b, \lambda_b) \in [0,1]^2$. The significance of the inequalities is as follows. Inequalities (35) and (36) ensure that one of the firms takes over when firm B chooses $(1,0)$ and $(0,0)$ respectively. It can be seen in Figure 6 that if $\ell_a \leq \ell_b$ ($\ell_a > \ell_b$) and some firm takes over when firm B chooses $(1,0)$ ($(0,0)$) so that the distance $D_1 - D_2$ is as large as possible, then some firm takes over for all other $(\ell_b, \lambda_b) \in [0,1]^2$.

Inequalities (37) and (38) ensure that firm B earns non-positive profits when choosing $(1,0)$ and $(0,0)$ respectively. It can be seen in Figure 6 that if $\ell_a \leq \ell_b$ ($\ell_a > \ell_b$) and the D_2 line intercepts the p_a axis when firm B chooses $(1,0)$ (the D_1 line intercepts the p_a axis when firm B chooses $(0,0)$), then firm B does not earn positive profits for any other (ℓ_b, λ_b) in which takeover occurs.

Furthermore, since each of the inequalities forms a compact set in $[0,1]^2$, $\triangle$ must be compact, as the intersection of compact sets is compact. Moreover, each of the inequalities becomes increasingly slack as β becomes increasingly negative. If β is sufficiently close to zero from below, then there are no $(\ell_a, \lambda_a) \in [0,1]^2$ that satisfy all of the inequalities and $\triangle$ is empty. However, as $\beta \to \infty$, then $\triangle \to [0,1]^2$. In other words, if β is sufficiently negative, then $\triangle$ is non-empty. ■

Firm A must decide between a strategy $(\ell_a, \lambda_a) \in \triangle$ and earning monopoly profits, or

$(\ell_a, \lambda_a) \in [0,1]^2 \setminus \triangle$ and earning the oligopoly profits implied by the subgame in which firm B enters.

Note a couple features of $\triangle$. First, if $\triangle$ is non-empty, then $\inf\{\lambda_a | \lambda_a \in \triangle\} > 0$. Second, if $\alpha \leq \frac{1}{3}$ ($\alpha \geq \frac{2}{3}$), then $\triangle = \{(\ell_a, \lambda_a) | \ell_a = 0 \text{ and } \lambda_a \geq \underline{\lambda_a} \text{ for some } \underline{\lambda_a} > 0\}$ ($\triangle = \{(\ell_a, \lambda_a) | \ell_a = 1 \text{ and } \lambda_a \geq \underline{\lambda_a} \text{ for some } \underline{\lambda_a} > 0\}$).

Additionally, let's define a couple more terms here before proceeding. Let the original game as described in the main text be denoted G. It will be useful to also consider a modified game G' in which firm B cannot choose whether to enter, and automatically enters regardless of firm A's strategy (ℓ_a, λ_a). In other words, firm B can take any action $(\ell_b, \lambda_b) \in [0,1]^2$, but firm B cannot chose to not enter. In every other respect besides firm B's entry decision, game G' is defined exactly as game G.

Claim 2: There does not exist an equilibrium of either game G or G' in which firm B earns positive revenues and firm A earns zero revenues.

Proof of Claim 2. Firm A has a cost-less strategy that prevents firm B from capturing all demand at a positive price: $\ell'_a = (\ \min\{\max\{3\alpha - 1, 0\}, 1\},\ 0\ )$ and $\lambda'_a = 0$. If $\ell_b = \ell'_a$, then firm B makes zero revenues. If $\ell_b > \ell'_a$, then $\ell'_a + \ell_b > 6\alpha - 2$ and firm A has the more prestigious position. If $\ell_b < \ell'_a$ then $\ell'_a + \ell_b < 6\alpha - 2$ and firm A again has the more prestigious position. In either case, firm B cannot earn positive revenues in an equilibrium with weak conformity and take over or in an equilibrium with strong conformity. Thus, firm A must earn positive revenues in any equilibrium. ■

Claim 3: Consider game G'. If $(\ell^*_a, \lambda^*_a) \notin \triangle$, then firms locate at opposite ends and $\lambda^*_b = 0$. Additionally, if $\alpha \in [\frac{1}{3}, \frac{2}{3}]$, then $\lambda^*_a = 0$.

Proof of Claim 3. The proof proceeds in steps. Steps 1-5 establish firm B's best-response to firm A and equilibrium strategy of locating at an end and not advertising. Steps 6-7 characterize λ_a and ℓ_a given firm B's best response. This proof only concerns behavior in game G', and assumes $(\ell^*_a, \lambda^*_a) \notin \triangle$.

Step 1: If $\alpha > \frac{1}{3}$ and $\ell^*_b \leq \ell^*_a$, then $\ell^*_b = 0$. *Proof*: Suppose $\alpha > \frac{1}{3}$ and $\ell^*_b \leq \ell^*_a$. As shown in claim 1, ℓ^*_b must be some ℓ_b where neither firm takes over. That is, for a given λ, ℓ^*_b must be some $\ell_b \in [0, \ell^{wc}_b)$ where $\ell^{wc}_b \in [0, \ell_a)$ is the highest $\ell_b < \ell_a$ such that neither firm takes over. It will be demonstrated that given any λ, firm B earns higher profits at $\ell_b = 0$ than any other $\ell_b \in [0, \ell^{wc}_b)$. Using equations (14) - (16), firm B's profits at any $\ell_b \in [0, \ell^{wc}_b]$ are given by:

$$\pi^L_b(\ell_b, \lambda_b) = \frac{(\ \frac{\tau}{3}(\ell_a - \ell_b)(2 + \ell_a + \ell_b) + \lambda\alpha\frac{\beta}{3}\ )^2}{2\tau(\ell_a - \ell_b) + \lambda\frac{\beta}{3}} - \frac{c}{2}\lambda^2_b \tag{39}$$

where L stands for to the left of firm A. It will be show that if $\alpha > \frac{1}{3}$, then π^L_b has no local maximum on $\ell_b \in (0, \ell^{wc}_b)$. It then follows that π^L_b must achieve a maximum at at one of the ends: $\ell_b = 0$ or ℓ^{wc}_b. Furthermore, since firm B cannot earn positive revenues when a firm takes over, π_b^L must be higher at $\ell_b = 0$ than at ℓ_b^{wc}. A necessary condition for an interior local maximum is that $\frac{d\pi_b^L}{d\ell_b} = 0$. The function π_b^L has four first order conditions with respect to ℓ_b:

(A) $\ell_b = -1 - \frac{1}{\tau}\sqrt{\alpha\lambda\beta\tau + \tau^2 + 2\ell_a\tau^2 + \ell_a^2\tau^2}$

(B) $\ell_b = -1 + \frac{1}{\tau}\sqrt{\alpha\lambda\beta\tau + \tau^2 + 2\ell_a\tau^2 + \ell_a^2\tau^2}$

(C) $\ell_b = \frac{(2\lambda\beta - 6\tau + 12\ell_a\tau) - \sqrt{(2\lambda\beta - 6\tau + 12\ell_a\tau)^2 + 36\tau(2\lambda\beta - 3\alpha\beta\lambda + 6\ell_a\tau - 3\ell_a^2\tau)}}{18\tau}$

(D) $\ell_b = \frac{(2\lambda\beta - 6\tau + 12\ell_a\tau) + \sqrt{(2\lambda\beta - 6\tau + 12\ell_a\tau)^2 + 36\tau(2\lambda\beta - 3\alpha\beta\lambda + 6\ell_a\tau - 3\ell_a^2\tau)}}{18\tau}$

(A) is clearly non-positive if real, and thus outside $(0, \ell_b^{wc})$. Furthermore, it can be shown that if $\alpha > \frac{1}{3}$, then (C) is non-positive (if real) and also outside $(0, \ell_b^{wc})$. If the first parenthetical term in (C) is non-positive, $2\lambda\beta - 6\tau + 12\ell_a\tau \leq 0$, then (C) is clearly non-positive because the term in the square root must be non-negative. Suppose instead $2\lambda\beta - 6\tau + 12\ell_a\tau > 0$. Rearranging terms, this is equivalent to $\lambda\beta > 3\tau - 6\ell_a\tau$. If $36\tau(2\lambda\beta - 3\alpha\beta\lambda + 6\ell_a\tau - 3\ell_a^2\tau) > 0$, then (C) is non-positive because the term in the square root is greater than the first parenthetical term on the left. Simplifying, this is equivalent to $\lambda\beta(2 - 3\alpha) > 3\tau\lambda_a(\ell_a - 2)$. If $\alpha \geq \frac{2}{3}$, then this inequality holds because the right hand side is weakly positive and the left hand side is negative. If $\alpha < \frac{2}{3}$, then this condition can be simplified to $\lambda\beta > \frac{3\ell_a\tau(\ell_a-2)}{2-3\alpha}$. Furthermore, note that $3\tau - 6\ell_a\tau \geq \frac{3\ell_a\tau(\ell_a-2)}{2-3\alpha}$ for all $\alpha \in (\frac{1}{3}, \frac{2}{3})$. Thus, if $\alpha > \frac{1}{3}$, then (C) must be non-positive and outside $(0, \ell_b^{wc})$. This leaves only (B) and (D).

Next, I show that either (B) or (D) must be a local maximum outside $(0, \ell_b^{wc})$. There is a clear discontinuity of π_b^L where the denominator equals $\ell_b^{sc} = \ell_a + \frac{\lambda\beta}{6\tau}$. $\ell_b^{sc} > \ell_b^{wc}$ because it is the point where demand becomes characterized by strong conformity. Applying l'Hôpital's rule, note that $\lim_{\ell_b \to \ell_b^{sc+}} \pi_b^L = -\infty$ and $\lim_{\ell_b \to \infty} \pi_b^L = -\infty$. Thus, there must be a local maximum at some $\ell_b > \ell_b^{sc}$. This then implies that can be at most one local extremum in $(0, \ell_b^{wc})$. However, if there exists an extremum in $(0, \ell_b^{wc})$, then it cannot be a local maximum. This is because, again applying l'Hôpital's rule, $\lim_{\ell_b \to \ell_b^{sc-}} \pi_b^L = \infty$. This concludes the proof that if $\alpha > \frac{1}{3}$ and $\ell_b^* \leq \ell_a^*$, then $\ell_b^* = 0$.

Step 2: It follows, mutatits mutandis, that if $\alpha < \frac{2}{3}$ and $\ell_b^* \geq \ell_a^*$, then $\ell_b^* = 1$.

Step 3: $\ell_b^* \in \{0, 1\}$ in any equilibrium. *Proof*: Suppose not. Suppose $\ell_b^* \in (0, 1)$. As demonstrated above, it must then be that $\lambda^* > 0$ and $\alpha \notin (\frac{1}{3}, \frac{2}{3})$, because otherwise firm B would have a dominant strategy in locating at one of the ends. Without loss of generality, suppose $\alpha \geq \frac{2}{3}$ and $\lambda^* > 0$. It must be that $\ell_a^* < \ell_b^*$, otherwise firm B would have chosen $\ell_b = 0$. As demonstrated above, firm A's best location $\ell_a < \ell_b$ is at $\ell_a = 0$. Firm A's price when $\ell_a = 0 < \ell_b < 1$ is $p_a = \frac{\tau}{3}\ell_b(2 + \ell_b) + \lambda\alpha\frac{\beta}{3} < \tau$ and firm A's market share is given by $\frac{\frac{\tau}{3}(\ell_b)(2+\ell_b)+\lambda\alpha\frac{\beta}{3}}{2\tau\ell_b+\lambda\frac{\beta}{3}} < \frac{1}{2}$. However, this cannot be optimal strategic behavior for firm A. If firm A had chosen $\ell_a = 1$ and $\lambda_a = 0$, then firm B's best-response would have been $\ell_b = \lambda_b = 0$. Firm A would have then charged a higher price $p_a = \tau$ while earning greater market share $\frac{1}{2}$. Thus, there cannot exist an equilibrium where $\ell_b^* \in (0, 1)$.

Step 4: $\lambda_b^* = 0$ in any equilibrium. *Proof*: Suppose not. Suppose $\lambda_b^* > 0$ in some equilibrium. WLOG, suppose $\ell_b^* = 0$. Since p_b is always decreasing in λ, it must be that firm B has the more prestigious position and its market share is increasing in λ. Thus, it must be that $\alpha < \frac{1}{2}$. It is demonstrated above that if $\ell_b = 0$ and $\alpha < \frac{1}{2}$, then firm A's profits are maximized when $\ell_a = 1$. If $\ell_a = 1$ and $\ell_b = 0$, firm A's price is given by $p_a = \tau + \lambda(1-\alpha)\frac{\beta}{3} < \tau$ and market share by $\frac{\tau+\lambda(1-\alpha)\frac{\beta}{3}}{2\tau+\lambda\frac{\beta}{3}} < \frac{1}{2}$. However, this cannot be optimal strategic behavior for firm A. If firm A had chosen $\ell_a = \lambda_a = 0$, then firm B's best-response would have been $\ell_b = 1$ and $\lambda_b = 0$. Firm A would have then charged a higher price $p_a = \tau$ while earning greater market share $\frac{1}{2}$. Thus, there cannot exist an equilibrium where $\lambda_b^* > 0$.

Step 5: If firm A locates at the end with the prestige advantage and does not advertise $\lambda_a = 0$, then firm B's best response is to locate at the opposite end and not advertise: $\ell_b(\ell_a, \lambda_a) = 1 - \ell_a$ and $\lambda_b(\ell_a, \lambda_a) = 0$. *Proof*: WLOG, suppose $\alpha \leq \frac{1}{2}$, $\ell_a = \lambda_a = 0$. It has already been shown that if firm B prefers not to take over, then firm B's best reply is $\ell_b = 1$ and $\lambda_b = 0$. It will next be shown that firm B prefers not to take over. If $\alpha \leq \frac{1}{3}$, then firm A always has the more prestigious position and firm B cannot take over. If $\alpha \in (\frac{1}{3}, \frac{1}{2}]$, then there may exist some combination of $\ell_b \in (0,1)$ and $\lambda_b > 0$ where firm B takes over.

Suppose $\alpha \in (\frac{1}{3}, \frac{1}{2}]$. If firm B took over, its revenues would be maximized when the D_1 line and D_2 lines are equal, as seen in Figures 5 and 6. This occurs when $\ell_b = \frac{-\lambda\beta}{6\tau}$ and demand is at the border of strong conformity and weak conformity. Plugging $\ell_b = \frac{-\lambda\beta}{6\tau}$ into firm B's profits from takeover, firm B's optimal takeover strategy can be found by solving:

$$\begin{aligned} \max_{\lambda_b} \quad & \tau\left(\frac{-\lambda_b\beta}{6\tau}\right)\left(2 + \frac{\lambda_b\beta}{6\tau}\right) - \lambda_b\beta\left(\alpha - \frac{2}{3}\right) - \frac{c}{2}\lambda_b^2 \\ & \text{subject to } \lambda_b \in [0,1] \end{aligned}$$

This program is concave in λ_b, and has solution: $\lambda_b = \frac{6\tau\beta(1-3\alpha)}{\beta^2+18c\tau} \in (0,1)$ and $\ell_b = \frac{\beta^2(3\alpha-1)}{\beta^2+18c\tau} \in [0,1)$. By the envelope theorem, firm B's optimized profits from takeover are strictly decreasing in c. Thus, if firm B does not takeover at $c = 0$, then it does not takeover at any $c > 0$.

Indeed, firm B's profits at $c = 0$ are $\tau(1-3\alpha)^2$. Firm B's profits from $\ell_b = 1$ and $\lambda_b = 0$ are $\frac{\tau}{2}$, and $\frac{\tau}{2} \geq \tau(1-3\alpha)^2$ for any $\alpha \in [\frac{1}{3} - \frac{1}{3\sqrt{2}}, \frac{1}{3} + \frac{1}{3\sqrt{2}}]$. Furthermore, $\frac{1}{3} - \frac{1}{3\sqrt{2}} < \frac{1}{3}$ and $\frac{1}{3} + \frac{1}{3\sqrt{2}} > \frac{1}{2}$. Thus, firm B's best response is $\ell_b = 1$ and $\lambda_b = 0$.

Step 6: Given firm B's best-reply $\ell_b(\ell_a, \lambda_a)$ and $\lambda_b(\ell_a, \lambda_a)$, firm A cannot do better than by locating at the end with the prestige advantage and not advertising $\lambda_a = 0$ if $\alpha \in (\frac{1}{3}, \frac{2}{3})$. *Proof*: WLOG, suppose $\alpha \geq \frac{1}{2}$. It has been shown that if firm A locates at the end with the prestige advantage $\ell_a = 1$, then firm B chooses $\ell_b = \lambda_b = 0$. Next, I show that this strategy maximizes firm A's profits given firm B's reaction. I divide into two cases based on the value of α. First, suppose $\alpha \in [\frac{1}{2}, \frac{2}{3})$.

If $\alpha \in [\frac{1}{2}, \frac{2}{3})$, then given any λ and $\ell_b = 0$, it has been demonstrated that firm A's profits are maximized when $\ell_a = 1$. In other words, fixing firm B's actions at $\ell_b = 0$ and some λ_b, $\ell_a = 1$ maximizes firm A's profits. However , in order to show that firm A cannot improve its profits at some other $\ell_a \neq 1$, it needs to also be shown that firm A cannot induce firm B to a strategy more profitable to firm A by choosing some $\ell_a \neq 1$. First, notice if firm A chooses $\ell_a < 1$, then firm B will still choose $\ell_b = 0$ if firm B remains to the left of firm A. Furthermore, firm A's profits are higher to the right of firm B than to the left, as demonstrated above. Thus, firm A cannot influence firm B's location in its favor. Next, notice that if firm B found it profitable to advertise $\lambda_b > 0$, then this advertising must hurt firm A because advertising can benefit at most one firm. This implies that firm A cannot improve its profits by choosing some ℓ_a that induces $\lambda_b > 0$.

Next, it also needs to be shown that firm A's profits are maximized by $\lambda_a = 0$ when $\alpha \in [\frac{1}{2}, \frac{2}{3})$. If we fix $\ell_a = 1$ and $\ell_b = \lambda_a = 0$, then firm A's revenues as a function of its advertising is:

$$\frac{(\tau + \lambda_a \frac{(1-\alpha)\beta}{3})^2}{2\tau + \lambda_a \frac{\beta}{3}}$$

The highest λ_a firm A can set without taking over is $\lambda_a = \frac{-3\tau}{\beta\alpha}$. The second derivative of firm A's revenues with respect to λ_a is positive in the considered domain: $\lambda_a \in [0, \frac{-3\tau}{\beta\alpha})$. Thus firm A's revenues are convex in λ on this domain and have no interior maximum. Furthermore, firm A's revenues are higher at $\lambda = 0$ then at the right closure of the relevant domain, $\lambda_a = \frac{-3\tau}{\beta\alpha}$. Firm A's revenues at $\lambda_a = 0$ are $\frac{\tau}{2}$. Firm A's revenues at $\lambda_a = \frac{-3\tau}{\beta\alpha}$ are $\frac{\tau(2\alpha-1)}{\alpha}$, and $\frac{\tau}{2} \geq \frac{\tau(2\alpha-1)}{\alpha}$ if $\alpha \leq \frac{2}{3}$. Since firm A's revenues are highest at $\lambda_a = 0$, and firm A's costs are strictly increasing in λ_a, firm A's profits are maximized at $\lambda_a = 0$.

I now move to the second case, where $\alpha \geq \frac{2}{3}$. Here, it needs to be shown that firm A cannot do better than locating at $\ell_a = 1$, given the reaction of firm B. It has already been demonstrated that the optimal strategy of firm B such that $\ell_b < \ell_a$ is $\ell_b = \lambda_b = 0$. However, it might be that given $\ell_b = \lambda_b = 0$, firm A could do better at some strategy where $\ell_a < 1$. I will show that if firm A prefers some $\ell_a < 1$

Suppose that given $\ell_b = \lambda_b = 0$ and some λ_a, firm A earns higher profits at some $\ell_a < 1$ than at $\ell_a = 1$. I will show that if firm A chooses $\ell_a < 1$, then firm B will move to the other side and choose $\ell_b > \ell_a$ rather than $\ell_b = 0$. In other words, there cannot be an equilibrium where $\ell_b = 0 < \ell_a < 1$. Furthermore, firm A would prefer $\ell_b = \lambda_b = 0$ and $\ell_a = 1$ to $\ell_a < \ell_b$. Thus, firm A chooses $\ell_a = 0$.

Using equations (14) - (16), firm A's profits to the right of $\ell_b = 0$ are given by:

$$\pi_a^R(\ell_a, \lambda_a) = \frac{(\frac{\tau}{3}\ell_a(4-\ell_a) + \lambda(1-\alpha)\frac{\beta}{3})^2}{2\tau\ell_a + \lambda\frac{\beta}{3}} - \frac{c}{2}\lambda_a^2 \tag{40}$$

whee R stands for to the right. It can be shown that if π_a^R has a local maximum at some $\ell_a < 1$, then $\frac{d\pi^R}{d\ell_a} < 0$ at $\ell_a = 1$. To see this, let $\ell_a^{sc} = -\lambda\frac{\beta}{6\tau}$ denote the value of ℓ_a such that the denominator equals 0 and demand becomes characterized by strong conformity. From the analysis before, we know that since $\alpha > \frac{2}{3}$, π_a^R has at most two extrema in $[\ell_a^{sc}, 1]$. Suppose that there is a local maxmimum in $[\ell_a^{sc}, 1]$. Applying l'Hôpital's rule, note that $\lim_{\ell_a \to \ell_a^{sc+}} \pi_a^R = \infty$. Thus, if π_a^R has a local maximum in $[\ell_a^{sc}, 1]$, then it also has a local minimum to the left of it in $[\ell_a^{sc}, 1]$. Since there are at most two local extrema in $[\ell_a^{sc}, 1]$, this implies that $\frac{d\pi^R}{d\ell_a} < 0$ at $\ell_a = 1$.

Setting the derivative of firm A's profits less than zero $\frac{d\pi^R}{d\ell_a} < 0$ and simplifying, this is equivalent to $\tau < \lambda\beta(\frac{1}{3} - \alpha)$. In what follows, suppose $\tau < \lambda\beta(\frac{1}{3} - \alpha)$ and firm A chooses some $\ell'_a < 1$. I will show that firm B's best response is some $\ell_b > \ell'_a$. If firm B chooses $\ell_b \leq \ell_a$, then (as demonstrated) the best it can do is $\ell_b = 0$. At $\ell_b = 0$, $p^L_b = \frac{\tau}{3}\ell'_a(2 + \ell'_a) + \lambda\alpha\frac{\beta}{3}$ and firm B earns some market share less than 1. If firm B chooses an $\ell^R_b > \ell_a$ and sufficiently close to ℓ_a such that $2(\ell^R_b - \ell_a) < -\lambda\frac{\beta}{3}$, then demand is characterized by strong conformity. Furthermore, since $\alpha \geq \frac{2}{3}$, firm B earns all the market share at a positive price $p^R_b = \tau(\ell^R_b - \ell_a)(2 - \ell_a - \ell^R_b) - \lambda\beta(\alpha - \frac{2}{3})$. If ℓ^R_b is arbitrarily close to ℓ_a, then $p^R_b \approx -\lambda\beta(\alpha - \frac{2}{3})$. I will show that $p^R_b > p^L_b$. Since firm B earns greater market share at ℓ^R_b than $\ell_b = 0$, this is sufficient to show that firm B's best-response is some $\ell_b > \ell'_a$. The inequality $p^R_b > p^L_b$ is equivalent to $-\lambda\beta(\alpha - \frac{2}{3}) > \frac{\tau}{3}\ell'_a(2 + \ell'_a) + \lambda\alpha\frac{\beta}{3}$. This can be simplified to $\tau < \frac{6\lambda\beta(\frac{1}{3}-\alpha)}{\ell'_a(2+\ell'_a)}$. This inequality must hold, because $\tau < \lambda\beta(\frac{1}{3} - \alpha) < \frac{6\lambda\beta(\frac{1}{3}-\alpha)}{\ell'_a(2+\ell'_a)}$. In other words, there does not exist an equilibrium where $\ell_b = 0 < \ell_a < 1$.

Step 7: It follows from the above arguments that there is an equilibrium where firm A locates at the end with the prestige advantage, $\ell^*_b = 1 - \ell^*_a$, $\lambda^*_b = 0$ and $\lambda^* = 0$ if $\alpha \in (\frac{1}{3}, \frac{2}{3})$. In any other equilibrium, firm A must earn the same amount of profits as in the aforementioned equilibrium. This can occur if neither firm advertises. In this case, there may also be an equilibrium where firm A locates at the end with the prestige disadvantage, firm B locates at the opposite end, and neither firm advertises. ■

Claim 4: If $(\ell^*_a, \lambda^*_a) \in \triangle$ in game G', then $(\ell^*_a, \lambda^*_a) \in \triangle$ in game G.

Proof Claim 4. Suppose $(\ell^*_a, \lambda^*_a) \in \triangle$ in game G'. It will be shown that given the best reply of firm B to $(\ell_a, \lambda_a) \in \triangle$ in the two games, firm A's profits are higher in game G than game G'. WLOG, suppose $\alpha \leq \frac{1}{2}$. Firm B's best response to $(\ell_a, \lambda_a) \in \triangle$ in game G' is any (ℓ_b, λ_b) such that $\lambda_b = 0$ since firm B cannot earn positive revenues and minimizes profit loss. By contrast, in game G, it is assumed that firm B does not enter if $(\ell_a, \lambda_a) \in \triangle$.

If $(\ell^*_a, \lambda^*_a) \in \triangle$ in game G', then firm A must earn positive revenues and take over the market. This implies that at the pricing stage firm A sets a price low enough such that the marginal consumer at $x = 1$ is just indifferent to not purchasing: $p^M_a = v - \tau(1 - \ell_a)^2 + \lambda\beta(\alpha - \frac{2}{3})$. In game G', firm A must charge a positive price and take over the market, otherwise it would not have chosen $(\ell_a, \lambda_a) \in \triangle$. Suppose firm B chooses $\ell'_b \geq \ell_a$. In this case, firm A must charge a price low enough such that the marginal consumer at $x = 1$ purchase good a over good b when $p_b = 0$: $p'_a \leq \tau(1 - \ell'_b)^2 - \tau(1 - \ell_a)^2 + \lambda\beta(\alpha - \frac{2}{3})$. Since $v > \tau$ (this follows from our assumption that v is sufficiently large that all consumers make purchases in the price equilibrium for any ℓ_a, ℓ_b and λ), it follows that $p'_a < p^M_a$.

Suppose firm B chooses $\ell_b < \ell_a$. In this case, firm A must charge a price low enough such that the marginal consumer at $x = 0$ purchase good a over good b when $p_b = 0$: $p''_a \leq \tau(\ell''^2_b - \ell^2_a) - \lambda\beta(\alpha - \frac{1}{3})$. Compare the upper bounds of p'_a and p''_a. Note that $\lambda\beta(\alpha - \frac{2}{3}) \geq -\lambda\beta(\alpha - \frac{1}{3})$ for any $\alpha \leq \frac{1}{2}$. Furthermore, note that $\tau(1 - \ell'_b)^2 - \tau(1 - \ell_a)^2 \geq \tau(\ell''^2_b - \ell^2_a)$ since $\ell''_b < \ell_a \leq \ell'_b$. Thus, the upper bound of p''_a is less than or equal to that of p'_a, and $p''_a < p^M_a$.

Thus, given any equilibrium of game G' with $(\ell_a^*, \lambda_a^*) \in \triangle$, firm A charges a strictly higher price for that $(\ell_a^*, \lambda_a^*) \in \triangle$ in game G. Since firm A has identical market shares and cost in both scenarios, firm A's profits are strictly greater in game G than game G'. It follows that if $(\ell_a^*, \lambda_a^*) \in \triangle$ in game G', then $(\ell_a^*, \lambda_a^*) \in \triangle$ in game G. ■

Claim 5: Consider game G. There exists some $\bar{c} > 0$, such that if $c \leq \bar{c}$, then $(\ell_a^*, \lambda_a^*) \in \triangle$ and firm B does not enter, and if $c > \bar{c}$, then $(\ell_a^*, \lambda_a^*) \notin \triangle$ and firm B enters, yielding the equilibrium described in claim 2.

Proof. If $\triangle$ is empty, then the proof follows trivially by setting $\overline{c'} < 0$. Suppose $\triangle$ is non-empty.

First, I show that if $c = 0$, then given firm B's best-response firm A's profits from deterring firm B's entry $(\ell_a, \lambda_a) \in \triangle$ are greater than that from not deterring firm B's entry $(\ell_a, \lambda_a) \notin \triangle$. WLOG, suppose $\alpha \leq \frac{1}{2}$. It can be seen analytically by inequalities (35) - (37) that there must then exist some $(\ell_a', \lambda_a') \in \triangle$ with $\ell_a' \leq \frac{1}{2}$. Firm A's monopoly price at (ℓ_a', λ_a') is:

$$p^M(\ell_a', \lambda_a') = v - \tau(1 - \ell_a')^2 + \lambda_a' \beta(\alpha - \frac{2}{3})$$

as shown in equation (26). If firm A chooses $(\ell_a, \lambda_a) \notin \triangle$, then the best it can do is a strategy where $\ell_a = 0$, inducing $\ell_b = 1$ and $\lambda_b = 0$. It's price is then given by $p_a^E = \tau + \lambda_a \alpha \frac{\beta}{3}$. If $\alpha > \frac{1}{3}$, then $\lambda_a = 0$, and if $\alpha < \frac{1}{3}$, then λ_a could be positive but must be less than λ_a (otherwise $(\ell_a, \lambda_a) \in \triangle$). Since $v > \tau$, $p_a^M > p_a^E$. Furthermore, since firm A's market share is greater given strategy (ℓ_a', λ_a'), and firm A's advertising cost is 0 in both cases, firm A's profits are higher given strategy (ℓ_a', λ_a'). This concludes the proof that when $c = 0$ and $\triangle$ is non-empty, firm A deters firm B's entry.

Next, I complete the proof by showing the existence of a unique cut-off $\bar{c}$.

Firm A's maximum profits from deterring firm B's entry are strictly decreasing in c. This follows from the fact that both firm A's monopoly revenues and $\triangle$ are independent of c, while firm A's cost of advertising is strictly increasing in c for any $\lambda > 0$. Furthermore, as $c \to \infty$, these maximum profits from entry deterrence approach $-\infty$.

Let's start with the case where $\alpha \in (\frac{1}{3}, \frac{2}{3})$. If c is low enough that firm A would choose $(\ell_a, \lambda_a) \in \triangle$ even when firm B enters, then $(\ell_a^*, \lambda_a^*) \in \triangle$ (see Claim 3). If c is greater than this amount, then firm A's maximum profits from accommodating entry $\frac{\tau}{2}$ are independent of c. It immediately follows that there must exist some unique $\bar{c}$ such that $(\ell_a^*, \lambda_a^*) \in \triangle$ if $c \leq \bar{c}$, and $(\ell_a^*, \lambda_a^*) \notin \triangle$ otherwise.

Suppose $\alpha \notin (\frac{1}{3}, \frac{2}{3})$. WLOG, suppose $\alpha \leq \frac{1}{3}$. If deterring firm B's entry, firm A chooses $\ell_a = 0$ and solves:

$$\max_{\lambda_a} \quad v - \tau + \lambda_a \beta(\alpha - \frac{2}{3}) - \frac{c}{2}\lambda_a^2$$
$$\text{subject to } (0, \lambda_a) \in \triangle$$

If firm A accommodates firm B's entry, then firm B chooses $\ell_b = 1$ and $\lambda_b = 0$, and firm

A chooses $\ell_a = 0$ and solves:

$$\max_{\lambda_a} \quad \frac{(\tau + \lambda_a \alpha \frac{\beta}{3})^2}{2\tau + \lambda_a \frac{\beta}{3}} - \frac{c}{2}\lambda_a^2$$

Let Π_a^M and Π_a^E denote firm A's optimized profits from deterring and not deterring entry respectively. By the envelope theorem, $\frac{d\Pi_a^M}{dc} \geq \frac{d\Pi_a^E}{dc}$ for all c. Thus, there must exist some unique $\bar{c}$ such that $(\ell_a^*, \lambda_a^*) \in \triangle$ if $c \leq \bar{c}$, and $(\ell_a^*, \lambda_a^*) \notin \triangle$ otherwise.

■

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