Persuasive Advertising for Conformist and Snobbish Goods

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Abstract

I model persuasive advertising for conspicuous goods that can either be made more attractive by greater popularity ("conformist goods") or by greater exclusivity ("snobbish goods"). Consumers are endowed with a latent attribute measuring some aspect of their identity, and a social status implied by this attribute. Consumers wish to signal a high social status, and the function of advertising is to render the use of consumption as a signaling device by linking products with social groups. In a conformist market, I find that advertising increases demand elasticity, inducing firms to converge on low prices, and can be used by a first mover to deter entry and gain monopoly rents. In this setting, advertising promotes a very cutthroat environment in which only one product can survive. In a snobbish market, advertising reduces demand elasticity, dampening price competition and promoting firm entry. In this setting, advertising can act as a public good to firms, increasing all firms' prices and profits. Additionally, it can lead to asymmetric equilibria where a firm appealing to high status consumers advertises more heavily, capturing a greater market share and price. Furthermore, I bring micro-foundation to persuasive advertising, allowing analysis of the channels through which it impacts welfare. Finally, I show that the model can help explain well-documented empirical puzzles in the marketing and empirical industrial organization literatures.

"Advertising is one of the topics in the study of industrial organization for which the traditional assumptions are strained most (especially those with regard to consumer behavior). The advertising of a product has strong psychological and sociological aspects that go beyond optimal inferences about objective quality. For instance, ad agencies constantly try to appeal to consumers' conscious or unconscious desire for social recognition, a trendy lifestyle and the like."

Jean Tirole, *The Theory of Industrial Organization*, pp. 292-293

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I INTRODUCTION

It is estimated that around \$207 billion was spent on advertising in 2018 in the United States alone, roughly \$630 per capita. Despite being a large portion of GDP, the role of advertising is still not well theoretically understood. One aspect of advertising is simply informational, apprising consumers of products and their attributes. However, a significant amount of advertising has little or no informational content, instead attempting to persuade consumers by purportedly manipulating their preferences.[1] Unfortunately, this important phenomenon is difficult to capture in traditional economic models based on the assumption that consumers have fixed preferences.

This model seeks to provide a micro-foundation for the role of advertising as appealing to consumers' desire for social recognition, and associating their consumption with a social identity. The premise of the model is that each consumer is endowed with a latent attribute $x \in [0,1]$ measuring some aspect of her social identity, such as a scale of how New England American or Southern American she is, or a measure of her sophistication. The values and norms governing the community in which consumers reside assign each consumer a social status based on her latent attribute $s(x) : [0,1] \to \Re$. Furthermore, consumers receive a reputational utility from signaling a high social status to a group of non-consuming spectators called "the public."

The function of advertising is to facilitate in this signaling by bringing the public's attention and powers of discrimination to products, so that the public is enabled — and the consumer knows that it is enabled — to infer the social status and latent attributes of consumers from simply observing their consumption choices. In other words, advertising renders the use of consumption as a device for consumers to signal their attribute and the status thereby implied. Thus, advertising affects a consumer's purchase by impacting the strength of the social reputation associated with different products.

But to understand and model this signaling phenomenon, one must recognize that there are two main effects such signaling motives may have on consumer demand: a "conformist effect" (also sometimes called a "bandwagon effect") where market demand for a good increases by others purchasing it; and "snob effect" where market demand for a good decreases by others purchasing it. I analyze the workings and effects of advertising in markets domi-

[1] See, for example, Resnik and Stern (1977), Tom et al. (1984) and Becker and Murphy (1993).

nated by each of these two forms of signaling.

On the one hand, there are "conformist markets" in which goods are made attractive by their popularity. For example, you might be reluctant to frequent Dunkin' Donuts if no one else does, but eager to do so if it is all the rage. On the other hand, there are "snobbish markets" in which goods are made attractive by greater exclusivity. You might drink "artesian water" to seem sophisticated, but as soon as artesian water becomes popular, you will move on to something else.

Economic theory dating back to Leibenstein (1950) has long recognized the potential impact of conformist and snob effects on consumer demand.[2] Surprisingly, however, the theoretical literature has said little about the impact of conformist and snob motives on persuasive advertising. That said, marketing practitioners often distinguish between campaigns built on "bandwagon appeal" (e.g. "America runs on Dunkin"') versus "snob appeal" (e.g. bottled water company Essentia Water's campaign "Overachieving H_2O: someone is going to stand out, it might as well be you").

The specific signaling game adopted is from Corneo and Jeanne (1997)'s study of Veblen effects. I superimpose this signaling game on a classic Hotelling model of two sequentially entering and horizontally differentiated firms.[3] Firms not only choose how much to advertise, but also which consumers to appeal to through their choice of horizontal attributes.

Corneo and Jeanne (1997) show that the instance of snob effects or conformist effects depends, in an identifiable way, on how a community allocates social status on the basis of a consumer's underlying attribute x. Essentially, snob effects or conformist effects are born from one of two like desires: the desire to avoid ostracism and not be considered a low type, or the hope for prestige and being considered a high type. It turns out that if there are *increasing status returns* to a more desired x, then the latter desire outweighs the former, and we get a market characterized by snob effects. By contrast, if there are *decreasing status returns* to a more valued x, then the former desire outweighs the latter, and we get a market characterized by conformist effects.

How does advertising function in these two different markets? By making it more likely the public recognizes the social reputation of a product, advertising amplifies consumers'

[2] Leibenstein (1950) originally coined the terms "bandwagon effects" and "snob effects." A large literature studies the presence of these effects on consumer demand, including, for example: Becker (1991), Bernheim (1994), Karni and Levin (1994), Corneo and Jeanne (1997) and Grilo et al. (2001).

[3] Originally postulated by Veblen (1899), Veblen goods are a type of luxury goods for which the quantity demanded increases in its price.

snobbish or conformist motives, depending on the type of market.

In a snobbish market, by increasing the strength of snob effects, advertising makes firms less willing to cut prices to expand market share, because doing so would imply fewer consumers rush in to buy the product. In other words, advertising reduces the elasticity of demand for snob goods. This dampens price competition and allows firms to converge on higher, supranormal prices.

Furthermore, this promotes firm entry because there are greater profits to be had. In this way, advertising acts as a public good to firms, increasing both firms' profits. Moreover, even if firms locate symmetrically, asymmetries in firms' prices and market shares may result. This is because the firm which does a better job of appealing to high status consumers is considered more prestigious, and uses advertising to extract greater market share at a higher price.

These results speak to many stylized facts. First, in snobbish markets such as that for luxury goods, we often see unusually high prices and an abundance of brands. For example, reusable water bottles have become a millenial status symbol in the last few years, with a proliferation of dozens, if not hundreds, of brands, and prices ranging from $10 to $1,500. As of the writing of this paper, the price on Amazon for a 17 oz water bottle from the most popular brand, S'well, is $30. Furthermore, S'well and its competitors are known for heavily advertising on Instagram and social media.[4] The model shows that this may be due to advertising's influence in reducing demand elasticity.

Additionally, despite goods being physically similar or homogeneous, we often see some firms charging a price premium, advertising heavily and earning greater market share (Bagwell, 2007). Indeed, many decades of empirical research in homogeneous product markets has found price dispersion rather than the "law of one price" to be the norm (Baye et al., 2006; Chioveanu, 2008). This may source from the prestige of a heavily advertised brand targeted at high status consumers, like S'well.

In a conformist market, advertising has the opposite effect. By increasing the strength of consumers' conformist motives, advertising makes firms more willing to cut prices to expand their market share, because doing so results in a greater number of consumers rushing to buy the product. In other words, advertising increases the elasticity of demand. This heightens price competition and induces firms to converge on lower, depressed prices. If the price

[4] See Munro (2018) and Mull (2019).

competition becomes sufficiently severe, then advertising can enable a first-mover to deter the entry of future firms and gain monopoly power. In this setting, advertising promotes a very cutthroat environment in which only one product can survive.

This helps make sense of empirical puzzles observed in conformist markets. We often see a first mover enter a geographic area, build an advertising presence, and dominate it for many years to come (Sutton, 1991). For example: Dunkin Donuts' started in Massachusetts in 1950 and prevails over the northeast United States, appealing to its blue-collar New England identity; Krispy Kreme launched in the North Carolina in 1937 and triumphs in the South, appealing to its wholesome and classy Southern identity; and Tim Hortons was founded by a Canadian hockey player in 1965, with its Canadian customer base often swearing by it with religious zeal.[5]

More rigorously, in a series of papers, Bronnenberg et al. (2007, 2009, 2011) meticulously document the evolution of advertising, prices and firm entry across the United States in the packaged-foods industry, many goods of which might be considered conformist such as beer, soft drinks and yogurt. They find that brands which entered a given geographic area first and built an advertising presence, often over one hundred years ago, are very likely to hold a much stronger, leading presence today. This model provides an economic explanation for this result.

Finally, by giving persuasive advertising a micro-foundation, this model easily lends to welfare analysis, unlike the previous literature. The key insight is that since signaling is a zero-sum game, the total pie of social status available is fixed (Frank, 2005; Heffetz and Frank, 2011). Persuasive advertising can shift which consumers get what portion of that pie, but cannot affect aggregate consumer welfare *directly* since it does not affect the size of the signaling pie. However, advertising can affect consumer welfare *indirectly* through its affect on prices, entry, and on how well consumers' purchases respect their horizontal preferences. For example, in the cases where advertising raises prices, it creates a transfer of welfare from consumers to firms. Furthermore, by limiting entry and inducing status concerns to overpower horizontal preferences, advertising can increase consumers' transportation costs. This knowledge is important for evaluating how policies that limit or tax advertising might impact consumers, firms and market structures.

[5] In the model, the social attribute x a brand appeals to does not have to be a regional identity.

II LITERATURE REVIEW

The theoretical literature on advertising broadly fits into three camps (i) the *informative* view that advertising informs consumers about the existence and attributes of products (ii) the *persuasive* view that advertising in some way shift consumers' tastes and (iii) the *complementary* view that advertising is itself as a good entering consumer utility.[6]

While few would disagree that much advertising is uninformative, the persuasive view has difficulty giving advertising a thorough micro foundation. The most common modeling fix is to assume that persuasive advertising induces an ad hoc change in consumer utility, such as increasing perceived product quality or perceived product differences (Von der Fehr and Stevik, 1998). This change is ad hoc because it does not explain *how* advertising works. Furthermore, it can make it hard to reach generalizable conclusions about advertising's effect on market variables such as prices, market shares, entry decisions, profits, product attributes, etc. Moreover, it makes welfare analysis tricky, as it not clear what standard should be adopted for measuring it. Which preferences should be considered the true preferences, those pre-advertising or post-advertising (Dixit and Norman, 1978)?

The solution of the complementary approach is to treat advertising as itself a good entering consumer utility, and complementary to the main good in question, thus implying a fixed preference (Becker and Murphy, 1993). The solution adopted here has a fixed preference like in complementary models, but without certain implications of such models like creating a consumer budget and demand function for receiving advertisements.

While the literature on persuasive advertising is too large to summarize here, the results for conformist goods speak to arguments developed in Sutton (1991) that advertising can deter market entry. One could interpret the conformist setting as providing a micro-foundation for the Sutton (1991) model to tell a story about why advertising works. Similarly, the results for snobbish goods provide depth to the assumption in models that advertising increases perceived product differences (Von der Fehr and Stevik, 1998; Tremblay and Polasky, 2002), or give a firm greater prestige, by providing a mechanism for how this works.

However, most advertising models that study consumers signaling type focus on the impact of informative rather than persuasive advertising (Campbell et al., 2017; Vikander, 2017). Some exceptions are Krähmer (2006) and Wernerfelt (1990). Krähmer (2006) studies persua-

[6]See Bagwell (2007) for a comprehensive survey of the literature.

sive advertising in the presence of a signaling game where geeky consumers try to emulate cool consumers, thus generating different predictions from those here. Wernerfelt (1990) proposes a model in which each consumer wants to signal her own type, rather than each consumer wanting to signal a high type as in this paper, and advertising is a form of "cheap talk" which helps in selecting among the multiple signaling equilibria. Additionally, there are many models where advertising signals the quality of the firm, rather than the quality of the consumer (these models study informative rather than persuasive advertising).

Furthermore, there is surprisingly little work modeling advertising in settings with conformist or snobbish consumer motives. Buehler and Halbheer (2012) and Sebald and Vikander (2015) are the only models I am aware of to do so. However, both lie outside of a signaling framework. Furthermore, the former maintains a common assumption that persuasive advertising increases the perceived quality of products, and the later studies informative rather than persuasive advertising.

More broadly, this paper relates to industrial organization models studying the effects of a late stage social signaling game on (non-advertising related) firm behavior: Karni and Schmeidler (1990), Karni and Levin (1994), Pesendorfer (1995), Corneo and Jeanne (1997), Bagwell and Bernheim (1996), Kuksov and Xie (2012), Kuksov and Wang (2013) to name a few. As mentioned, the signaling game implemented is adopted from Corneo and Jeanne (1997), which uses it to study the existence of Veblen effects in a monopolist market. Many of these models tend to study games with monopoly rather than oligopoly competition. One hope of this paper is to help demonstrate the possibilities from incorporating late stage signaling games in oligopoly models of competition.

Finally, Grilo et al. (2001) study price competition in a differentiated duopoly market with a reduced form consumption externality of either a conformist or snobbish nature (i.e. one's utility for a good directly increases or decreases in the number of others consuming it). Similar analytic techniques are used in the price subgame of this model. While Grilo et al. (2001) are not motivated by the study of advertising, one could interpret parts of this model as extending Grilo et al. (2001) in various way. First, Grilo et al. (2001) studies a symmetric consumption externality, so that, for example, the utility gains from others going to McDonald's are equivalent to the gains from others going to Burger King. This paper explores the effects of asymmetry in the consumption externality. This arises in the current model because it matters not just *how many* people buy a good, but also *who* buys a good, and the social status they hold. Furthermore, this model endogenizes the consumption externality through the incorporation of a signaling game, thus allowing for welfare analysis, as well as endogenize firms' entry and location decisions.

III A MODEL OF PERSUASIVE ADVERTISING

This section introduces the formal model with sequentially entering firms selling conspicuous and horizontally differentiated goods to status-driven consumers.[7]

The economy is populated by: i) a unit mass of consumers uniformly distributed along some attribute x on the [0,1] interval and ii) a unit mass of non-consuming spectators called the public. The latent attribute x measures some aspect of each consumer's identity, such as the strength of her cultural association with New England America versus Southern America. Furthermore, this attribute helps shape the consumer's demand. As common in models of horizontal differentiation, each consumer x has unit demand with quadratic transportation cost $\tau > 0$ and a bliss point at x.[8] Thus, x both defines the attribute of the consumer and her preferences over brands.

Furthermore, as in Corneo and Jeanne (1997), there is a continuous social status function $s(x) : [0, 1] \to \Re$ that assigns each consumer a social status given her attribute x. The social status function is exogenous, and governed by the norms and values of the community in which consumers reside. Note that the attribute x both defines a consumer's preferences over goods, as well as a consumer's social status. After all, we all face the problem of being endowed with certain attributes which both mold our preferences, as well as give us certain reputation in our relations to society.

As Corneo and Jeanne (1997, p.58) explain:

> Following sociologists, we may define social status as a general claim to deference [e.g., Coleman (1990)]. In economic terms, an individual's status may be called a socially provided private good. Each individual has a certain fixed amount of a special good - say, deference - that he allocates to others according to some social norm... In turn, the norm may be taken as mirroring societal values that characterize the community in which the individuals interact.

Here, those individuals who allocate the good "deference" are the public. At the end

[7] In the real world, firms usually enter sequentially rather than simultaneously. Incorporating sequential entry allows study of the advantages or disadvantages given to the first-mover in such situations. Furthermore, from a technical perspective, sequential entry helps ensure the existence of equilibria in pure strategies in the location and advertising stages (Börgers, 1988).

[8] Quadratic transportation costs help ensure the existence of equilibria at the pricing and location/advertising stages (d'Aspremont et al., 1979; Neven, 1985; Shaked and Sutton, 1987).

of the game, consumers are randomly matched with partners from the public. A member of the public does not know her partner's social status, but tries to infer it. Each consumer receives a reputational utility equal to the inference made by her partner of her social status. We will see exactly how the public's inference is made later. This inference can be thought of as the quantity of deference the public gives the consumer. For example, consumers in New England might receive a certain amount of deference for signaling their New England origins.

There could be other, market based reasons why the public's inference affects consumer utility. For example, a consumer's professional, marriage, mating (Miller, 2011), friendship or leadership opportunities could depend on others' inference of her latent attribute x. I will abstract from such interpretations of a reputational utility.

The ex-post utility u_x a consumer x receives at the end of the game from a given purchase is:

$$u_x = v - \tau(\ell - x)^2 - p + \underbrace{\text{Public's Inference of } s(x)}_{\text{Reputational Utility}}$$

where $v > 0$ is the intrinsic utility of the good, $\ell \in [0, 1]$ is its location and $p \geq 0$ is its price. The only departure from a standard model of horizontal differentiation is the addition of the reputational utility.

The time-line of the game is as follows, summarized in Figure 1. At $t = 0$, the incumbent, firm A, chooses a location $\ell_a \in [0, 1]$. The location choice measures the horizontal attributes of the firm, such as the how well a donut shop appeals to New England American and Southern Americans through the features of its donuts, atmosphere and service. Furthermore, firm A chooses how much to advertise $\lambda_a \in [0, 1]$, paying a convex cost to advertising $\frac{c}{2}\lambda_a^2$ (where $c \geq 0$). The λ_a advertisements are then randomly distributed among members of the public (Grossman and Shapiro, 1984). I normalize firm A's cost of production to zero.

At $t = 1$, firm B observes the location and marketing level of firm A and decides whether to enter the market. Furthermore, if firm B enters the market, then it must decide where to locate $\ell_b \in [0, 1]$ and how much to advertise $\lambda_b \in [0, 1]$, also paying a convex cost to advertising $\frac{c}{2}\lambda_b^2$. The λ_b advertisements are again randomly distributed among the public. Firm B also pays zero cost to production, so that there are no cost advantages to early entry.[9]

Thus, if λ represents the probability that a given member of the public receives an ad-

[9]The assumption that firm B pays zero fixed cost is conservative in that it makes it harder to get a result where firm B is deterred from entering.

vertisement, then $\lambda = \lambda_a + \lambda_b - \lambda_a\lambda_b$. This could be interpreted as a linear advertising production function.[10]

If firm B enters, then at $t = 2$ firms A and B simultaneously choose prices $p_a \geq 0$ and $p_b \geq 0$ as in Bertrand competition. Otherwise, firm A becomes a monopolist and chooses its price free of competition. The assumption that advertising investment take place before pricing decisions is typical in models of persuasive advertising, and motivated by the view of advertising as a long-term investment to generate a brand image, and pricing as a more short-term oriented strategy (Belleflamme and Peitz, 2015, p. 150).

At $t = 3$, consumers make their purchase decisions. Furthermore, consumers are standard expected utility maximizers, fully informed about the goods in the market, their marketing levels, locations and prices.

At $t = 4$, each consumer is randomly matched with a partner from the public. The public does not know a consumer's underlying identity and social status, but tires to infer it. Each consumers receives a reputational utility equal to the inference made by her partner of her social social status.

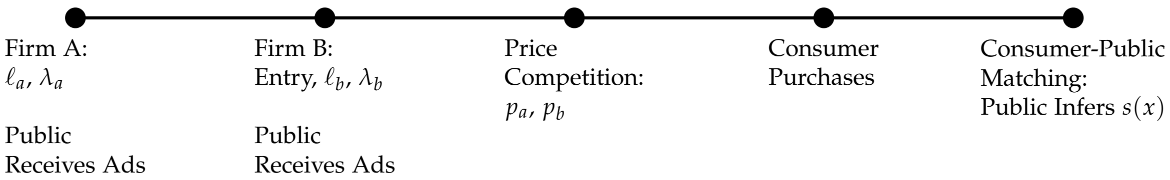


Figure 1: Timeline of Model

Let $\rho(\,x\,)$ denote the posterior probability the partner assigns to the consumer being of type x, where $0 \leq \rho(\,x\,) \leq 1$ and $\int_0^1 \rho(\,x\,)dx = 1$. If the partner receives an advertisement from either firm, then his attention and powers of discrimination is brought to products, and he infers the consumer's social status conditional on the latter's product choice. That is, if we let $o \in \{a, b, \emptyset\}$ denote the chosen option of a given consumer, and Ω the attributes of the products available, then the partner's inference of the consumer's social status is $\rho(\,x \,|\, o, \Omega\,)$. By contrast, if the partner does not receive an advertisement, then he does not pay attention to products and does not condition his inference on the consumer's purchase: $\rho(\,x\,)$.

The motivation being that consumers shop at stores, and thus pay full attention to the

[10]The model could be extended to accommodate a more general advertising production function $f(\lambda_a, \lambda_b)$ that is increasing in each of its arguments, and determines the probability that a member of the public receives an advertisement given firms' marketing efforts. Furthermore, one could allow for asymmetries in the marketing technologies of firms by letting the first order partial derivatives be unequal $f_1(\lambda_a, \lambda_b) \neq f_2(\lambda_a, \lambda_b)$ for given advertising levels (λ_a, λ_b).

products available. However, the public does not shop, and thus does not pay full attention to products, and may not be able to readily recognize or distinguish products when they see them. For example, one might not recognize the difference between an electric Tesla car and a gas car unless one receives an advertisement. Advertising does not affect consumer utility directly, but affects consumer utility indirectly by increasing the ability of the public to infer a consumer's social status from her consumption.[11]

Therefore, consumers are senders in a signaling game, making consumption choices based on the trade-off between their horizontal preferences and the signal of social status it conveys to the advertisement receiving public. Let S_o denote a consumer's expected utility from signaling given option o at $t = 3$. The expected utility of consumer x for each choice at $t = 3$ is:

$$U_x(a) = v - \tau(\ell_a - x)^2 - p_a + S_a \tag{1}$$

$$U_x(b) = v - \tau(\ell_b - x)^2 - p_b + S_b \tag{2}$$

$$U_x(\emptyset) = S_\emptyset \tag{3}$$

Furthermore, the signaling value of each choices is:

$$S_a = \lambda \int_0^1 \rho(x \,|\, a, \Omega)\, s(x)\, dx \; + \; (1-\lambda) \int_0^1 \rho(x)\, s(x)\, dx \tag{4}$$

$$S_b = \lambda \int_0^1 \rho(x \,|\, b, \Omega)\, s(x)\, dx \; + \; (1-\lambda) \int_0^1 \rho(x)\, s(x)\, dx \tag{5}$$

$$S_\emptyset = \lambda \int_0^1 \rho(x \,|\, \emptyset, \Omega)\, s(x)\, dx \; + \; (1-\lambda) \int_0^1 \rho(x)\, s(x)\, dx \tag{6}$$

where the first terms are the probability of being matched with a member of the advertisement receiving public multiplied by the perceived status of those choosing said option (found by multiplying the posterior probability a consumer is a given type with that type's social status, and integrating over all possible types), and the second terms are the probability of being matched with a member of the non-advertisement receiving public multiplied by the perceived status of any random consumer. Advertising makes it more likely that a consumer's purchase is recognized for the social status it conveys, and consumers incorporate this into

[11] This approach accords with evidence that brand advertising is often targeted at people who would not purchase from the given brand, instead aiming to increase the brand's recognizability to serve the signaling needs of its own customer base (Miller, 2011).

their consumption choices.[12]

Social Status Function

The signaling game produces a consumption externality where a consumer's utility depends not only on her own consumption choice, but also that of every other consumer. In reality, the social regard and value a community holds for a given attribute might be complicated and difficult or impossible to empirically measure. I impose a simple functional form on the social status function that is both tractable and nicely captures the intuition of snobbish and conformist markets.

Assumption 1 (Social Status Function). *$s(x)$ is a quadratic function centered at $\alpha \in [0,1]$: $s(x) = \beta(x-\alpha)^2$.*

This is a version of the quadratic social status function introduced in Corneo and Jeanne (1997) in a monopolist market, tailored to suit a horizontally differentiated duopoly market.

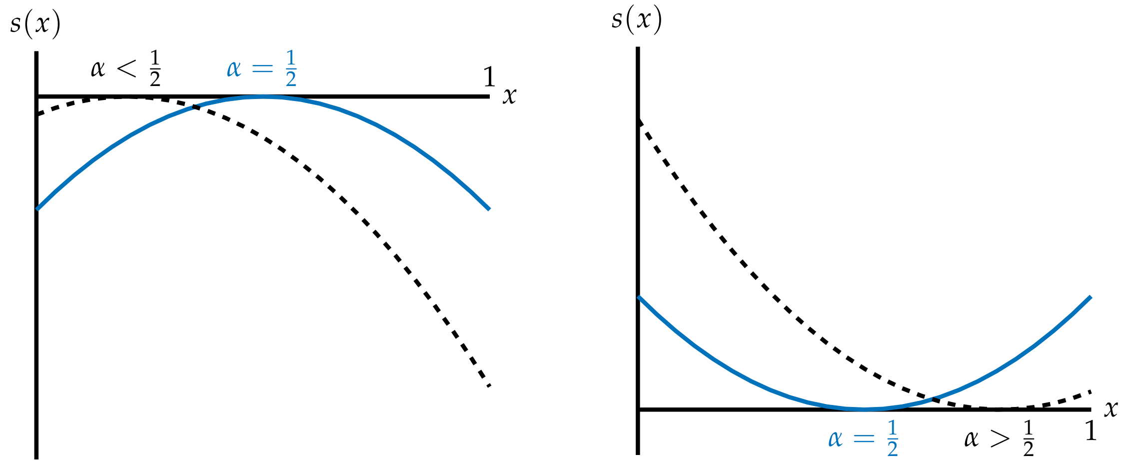
It will be shown in Section IV that the consumption subgame produces snobbish dynamics if there are increasing returns to higher status and $s(x)$ is convex ($\beta > 0$), and conformist dynamics if there are decreasing returns to higher status and $s(x)$ is concave ($\beta < 0$). Furthermore, the model is invariant to the adding of a constant to the social status function, thus it is immaterial that $s(x)$ is weakly negative in the conformist case and weakly positive in the snobbish case.

The vertex of the social status function α also holds economic significance. As will be made clear, if $\alpha \neq \frac{1}{2}$ and the social status function is asymmetric around $x = \frac{1}{2}$, then the brand located on the side with higher types may be considered more prestigious. The distance between α and $\frac{1}{2}$ can be thought of as the amount of prestige in the market. I restrain α to the interval $[0,1]$ since I am primarily interested in the impact of snob and conformist effects, and secondarily interested in the impact of prestige effects. Indeed, in the advertising and

[12] There is a second behavioral interpretation of the model following the work of Bénabou and Tirole (2011). Under this interpretation, a consumer has two selves, a present self and a future self. Through a moment of self-reflection, the current self has insight into its own attribute x, and the status implied therein, but the future self may momentarily forget its own attribute (akin to "the public"). Though forgetful, the future self can readily observe its consumption choices, and may try to infer its attribute based on this action. The consumer receives utility based on its future self's inference of its own identity. Thus, the current self is a sender in a signaling game, making consumption choices based on the utility the consumer will get from its future self's inference. As explained in Bénabou and Tirole (2011), the function of advertising is to remind the consumer's future self of the identity associated with products, increasing the salience of the conditional inference, and thus strengthening the use of consumption as signaling device. This alternative interpretation can potentially broaden the application of the model beyond that of conspicuous consumption to that of inconspicuous consumption.

location stages, I will find that I have to further restrict α to $[\frac{1}{3}, \frac{2}{3}]$ to prove the existence of an equilibrium. However, throughout the analysis I will discuss the implications of a broader range of α.

Figure 2 shows examples of social status functions in a conformist market and a snobbish market. The solid blue lines show social status functions symmetric about $\frac{1}{2}$, and the black dashed lines show social status functions asymmetric about $\frac{1}{2}$.



(a) Conformist $\beta < 0$

(b) Snobbish $\beta > 0$

Figure 2: Social Status Functions

Equilibrium Definition

Before analyzing the game, let's define what we mean by an equilibrium. In the consumption subgame (stages $t = 3$ and $t = 4$), the appropriate equilibrium concept is that of a signaling equilibrium in which consumers are senders, and the public are receivers. I restrict attention to *pure strategy* Perfect Bayesian Equilibrium (PBE) satisfying the following reasonable off-equilibrium public beliefs.

Suppose both firms enter the market and $\ell_a < \ell_b$. I assume that if all consumers purchase good a (good b), then a consumer who deviates by purchasing good b (good a) is perceived to be the consumer who would most benefit from switching: $\rho(x = 1|b, \Omega) = 1$ ($\rho(x = 0|a, \Omega) = 1$). If instead $\ell_b < \ell_a$, then $\rho(x = 0|a, \Omega) = 1$ and $\rho(x = 1|b, \Omega) = 1$. In Section V, I outline analogous off-equilibrium beliefs in the case where firm B does not enter.

Note that members of the public who do not receive advertisements are unable to distinguish between the choices of consumers. Essentially, they perceive all consumers as choosing a single action. Their inference of a consumer's type is then unconditioned on her action,

$\rho(x)$, and hence they form no off-equilibrium beliefs.

Finally, given firms' payoffs as determined by the equilibria of the consumption subgame, I study pure strategy subgame perfect equilibria in the larger game. All proofs are in the Appendix.

IV SOCIAL STATUS MOTIVATED DEMAND

Consumers make their choices based not only on their horizontal preferences and prices, but also on the signal their purchase conveys to the advertisement receiving public. Working backwards, I first solve for the PBE of the consumption subgame, given firm advertising levels, prices and locations. This will give us the demand functions firms face.

I begin with the consumption subgame where firm B enters the market. I turn to the monopoly case in which firm B does not enter in Section V. Furthermore, in the analysis that follows, I make the traditional assumption that v is sufficiently large that all consumers make purchases. The first think to notice is that so long as $\ell_a \neq \ell_b$, then any equilibrium in which all consumers make purchases is characterized by a single cut-off $n \in [0, 1]$ such that consumers to the left of n buy the left most good, and consumers to the right of n the right most good.

Lemma 1 (Cut-Off Rule)**.** *If $\ell_a \neq \ell_b$, then in any equilibrium in which all consumers make purchases, there must be a single cut-off $n \in [0, 1]$ such that consumers to the left of n buy the left most good, and consumers to the right of n buy the right most good.*

We have a semi-separating equilibrium if $n \in (0, 1)$ so that both goods are purchased, and a pooling equilibrium if $n \in \{0, 1\}$ so that only one good is purchased. This cut-off rule holds in most models of product differentiation due to a single-crossing property arising from consumers' location dependent transportation costs, and the same applies here. The reason being that all consumers face the same public perception for a given purchase regardless of their type x.[13] Thus, signaling motives add an identical constant to each consumer's utility for a particular option and does not affect the single-crossing property. This result holds independent of the particular social status function imposed.

We can now calculate the reputational value of both the goods given an arbitrary cut-off n. Without loss of generality, suppose $\ell_a < \ell_b$. The case of $\ell_a > \ell_b$ can be found by switching the

[13] If the public knew the consumer's type x, then there would be no signaling motives.

a and b terms in the analysis that follows. I treat the case of $\ell_a = \ell_b$ in Section IV. Plugging in the social status function to calculate the signaling value of each good for a given n yields:

$$
\begin{aligned}
S_a(n) &= \frac{\lambda}{n}\int_0^n s(x)dx \; + \; (1-\lambda)\, E(\, s(x)\,) \\
&= \lambda\beta\,(\,\frac{n^2}{3} + \alpha^2 - \alpha n\,) \; + \; (1-\lambda)\, E(\, s(x)\,) \qquad (7) \\
S_b(n) &= \frac{\lambda}{1-n}\int_n^1 s(x)dx \; + \; (1-\lambda)\, E(\, s(x)\,) \\
&= \lambda\beta\,(\,\frac{1+n+n^2}{3} + \alpha^2 - \alpha(1+n)\,) \; + \; (1-\lambda)\, E(\, s(x)\,) \qquad (8)
\end{aligned}
$$

where $\int_0^1 \rho(x \,|\, a, \Omega)\, s(x)\, dx = \frac{1}{n}\int_0^n s(x)dx$ and $\int_0^1 \rho(x \,|\, b, \Omega)\, s(x)\, dx = \frac{1}{1-n}\int_n^1 s(x)dx$. We can then obtain the signaling gains to a consumer of choosing good a over good b, call it $S_{a/b}$:[14]

$$S_{a/b}(n) \equiv S_a(n) - S_b(n) = -\frac{\lambda\beta}{3}n + \lambda\beta(\alpha - \frac{1}{3}) \qquad (9)$$

Note that, and this is at the heart of the model, the value of $S_{a/b}()$ (and of $S_{b/a}()$) is a function of the cut-off n. In other words, the reputational gains from a good are a function of the mass of consumers choosing it. Essentially, in its reduced form, the signaling game produces a consumption externality. We said that a market is "conformist" if the social status function is concave and $\beta < 0$ because then $\frac{dS_{a/b}(n)}{dn} > 0$ (and $\frac{dS_{b/a}(n)}{dn} < 0$) and the reputational gains from a good increases in the mass of consumers choosing it. Conversely, we said that a market is "snobbish" if the social status function is convex and $\beta > 0$ because then $\frac{dS_{a/b}(n)}{dn} < 0$ (and $\frac{dS_{b/a}(n)}{dn} > 0$) and the reputational gains from each good decreases in the mass of consumers choosing it.

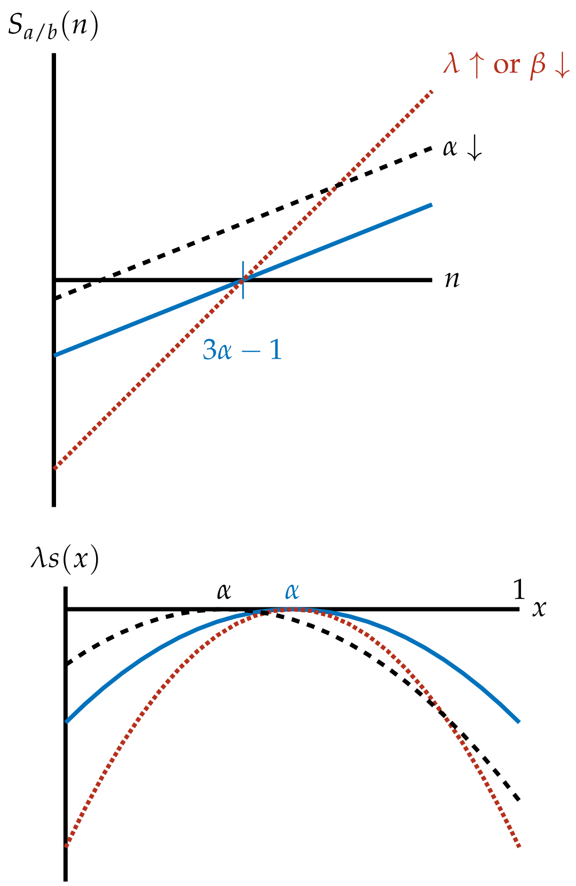
Furthermore, the sign of $\frac{dS_{a/b}(n)}{dn}$ and whether the market is conformist or snobbish is independent of its symmetry α and whether the social status function is increasing or decreasing on a given interval.[15] Instead, α adds a constant to the signaling gains from a purchase, increasing or decreasing the signaling gains in absolute terms, but does not change the responsiveness of those signaling gains to the good's consumer base n or $1-n$. I call the effect of α on demand for a good the "prestige effect."

Figure 3 makes this more clear, showing how $S_{a/b}(n)$ varies based on β, α and λ. As seen

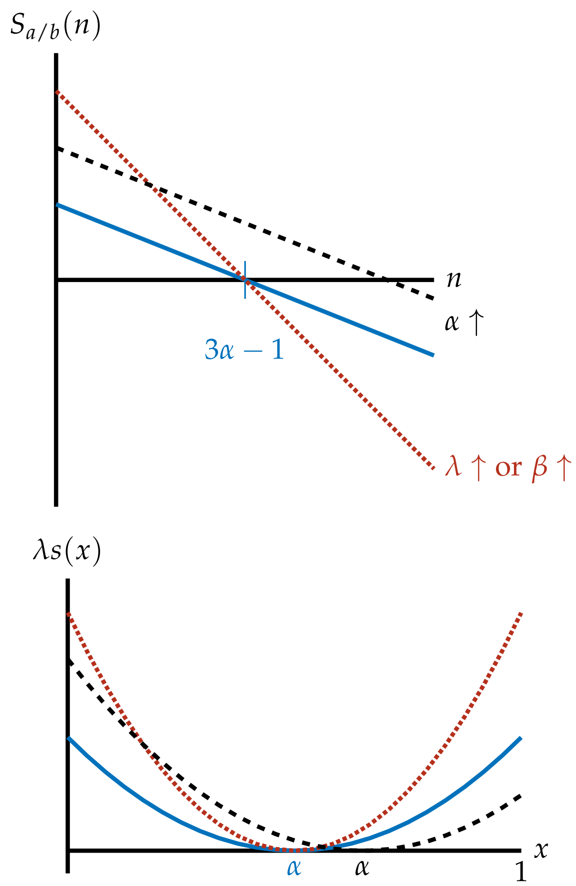
[14] Similarly, $S_{b/a}(n) \equiv S_b(n) - S_a(n) = \frac{\lambda\beta}{3}n - \lambda\beta(\alpha - \frac{1}{3})$.

[15] To see this another way, consider a social status function of the general quadratic form: $s(x) = a_1x^2 + a_2x + c$. Then, $S_{a/b}(n) = -\lambda(\,\frac{a_2}{2} + \frac{a_1}{3}(n+1)\,)$ and $\frac{dS_{a/b}(n)}{dn} = -\lambda\frac{a_1}{3}$ is only dependent on the concavity or convexity of the function and advertising.

in the figure, if $\beta < 0$ (if $\beta > 0$), then $S_{a/b}(n)$ is an increasing (decreasing) function of n. Moreover, an increase in advertising increases the magnitude of the slope of $S_{a/b}(n)$ so that the signaling gains from a good becomes more responsive to its consumer base. Intuitively, by increasing the chance that the public recognizes a purchase and the social identity it conveys, advertising increases the strength of conformist or snob effects, depending on the case. Furthermore, if $\beta < 0$ (if $\beta > 0$), then a lower vertex α (a higher vertex α) shifts the $S_{a/b}(n)$ line up, so that the signaling gains from buying good a are higher for any given market share due to its greater prestige.



(a) Conformist $\beta < 0$



(b) Snobbish $\beta > 0$

Figure 3: Consumption Externality Implied by Social Status Function
$S_{a/b}(n) = -\frac{\lambda\beta}{3}n + \lambda\beta(\alpha - \frac{1}{3})$

The intuition for the origins of snob and conformist effects is as follows. As Corneo and Jeanne (1997) puts it, whether the market is conformist or snobbish depends on the relative strength of two related desires: the desire to avoid ostracism and not be considered a low type, and the desire to attain prestige and be considered a high type. It turns out that if the latter outweighs the former, then we get a snobbish market, and if the former outweighs the latter, then we get a conformist market. Which desire dominates depends on the marginal returns to higher status.

Suppose the social status function is convex on some interval, and for argument's sake, decreasing (again, the snobbery or conformity does not depend on whether the function is increasing or decreasing). Consider some cut-off n in this interval such that those to the left of n buy good a and those to the right of n buy good b. Due to the convexity, it is more costly to lose one's position when one is ranked high than when one is ranked low. Thus, as n shifts right and more consumers buy good a, the expected status of those choosing good a falls more rapidly than that of those choosing good b. This implies that the the signaling gains from good a, $S_{a/b}(n)$, which is the difference between those two, decreases. Essentially, societal norms emphasize a hope of being identified as a high type, and this results in snobbish behavior.

The intuition for the conformist case is just the opposite. Suppose that the social status function is concave, and again for argument's sake, decreasing on some interval. Due to the concavity, it is now more costly to lose one's position when one is ranked low then when one is ranked high. Thus, as n shifts right and more consumers buy good a, the expected status of those buying good a decreases less rapidly than the expected status of those buying good b. This implies that the the signaling gains from good a, $S_{a/b}(n)$, increases as more buy it. Intuitively, societal values instill a fear of being identified as a low type, of being ostracized, and this expresses itself in conformist behavior.

There is a formal correspondence derivable using the fundamental theorem of calculus between any social status function and the signaling gains it implies.[16] The quadratic nature of the social status function bought us monotonicity and linearity in the consumption externality. If the social status function were linear, then there would be zero consumption externality. If the social status function were a polynomial of degree three or higher, then the consumption externality could oscillate between being conformist and snobbish with the value of n.

Following the literature on horizontal differentiation, I say that firm A has a "location advantage" if $\ell_a + \ell_b > 1$ so that it is closer to a greater mass of consumers, and firm B has the location advantage if $\ell_a + \ell_b < 1$. Firms are symmetric if $\ell_a + \ell_b = 1$.

[16] If $s(x)$ is a social status function integrable on [0,1], then $S_{a/b}(n) = \frac{1}{n}\int_0^n s(x)dx - \frac{1}{1-n}\int_1^n s(x)dx$. Furthermore, a continuous and differentiable signaling gains function $S_{a/b}()$ can be rationalized by a social status function of the form $s(x) = (1-2x)S_{a/b}(x) + x(1-x)S'_{a/b}(x) + c$ where c is an arbitrary constant. See equation (11) in Corneo and Jeanne (1997).

However, and in contrast to the traditional literature on horizontal differentiation, in this model it matters not just *how many* consumers a firm is closer to, but also *who* a firm is closer to. Firms not only wish to appeal to a *large quantity* of consumers, but also to a *high quality* of consumers by winning the patronage of those with high status.

I say that the firm on the side with more high types has a "prestige advantage." In the snobbish case where $\beta > 0$, firm A (firm B) has the prestige advantage if $\alpha > \frac{1}{2}$ ($\alpha < \frac{1}{2}$) because then there are higher types on the left than on the right (higher types on the right than on the left). In the conformist case where $\beta < 0$, firm A (firm B) has the prestige advantage if $\alpha < \frac{1}{2}$ ($\alpha > \frac{1}{2}$) because then there are higher types on the right than on the left (on the left than on the right). Firms have a symmetric prestige advantage if $\alpha = \frac{1}{2}$.

However, just because a firm is on the side with more high types does not mean it captures a higher signaling value than the other firm. Firms must also position themselves away from low types to win the better reputation. For example, consider a snobbish market with vertex $\alpha = 0.6$, implying that the left side of the social status function is higher than the right. If $\ell_a = 0.9$ and $\ell_b = 1$, then firm A commands a prestige advantage because it is on the left side. However, it is also more preferred by low types near $x = 0.6$. Indeed, when the market is spit equally between firm locations $n = \frac{\ell_a+\ell_b}{2}$, firm A has the lower signaling value $S_{a/b}(\frac{0.9+1}{2}) < 0$.

Thus, I define the firm with the more "prestigious position" as the firm which holds greater signaling value at an even split of the market between their locations. In other words, firm A has the more prestigious position if $S_{a/b}(\frac{\ell_a+\ell_b}{2}) > 0$, and firm B has the more prestigious position if $S_{a/b}(\frac{\ell_a+\ell_b}{2}) < 0$. Firms have symmetrically prestigious positions if $S_{a/b}(\frac{\ell_a+\ell_b}{2}) = 0$. It can be shown from equation (9) that firm A has the more prestigious position when $\beta > 0$ ($\beta < 0$) and $\ell_a + \ell_b < 6\alpha - 2$ ($\ell_a + \ell_b > 6\alpha - 2$). Furthermore, firm B has the more prestigious position when $\beta > 0$ ($\beta < 0$) and $\ell_a + \ell_b > 6\alpha - 2$ ($\ell_a + \ell_b < 6\alpha - 2$). Firms have symmetrically prestigious positions when $\ell_a + \ell_b = 6\alpha - 2$ or $\beta = 0$. This terminology is summarized in Table 1.

If firms locate symmetrically ($\ell_a + \ell_b = 1$), then a firm has a prestige advantage if and only if it has a more prestigious position. However, if firms locate asymmetrically, then the firm with the prestige advantage may or may not be the firm with the more prestigious position, as in the example described above.

	Location Advantage
$\ell_a + \ell_b > 1$	Firm A
$\ell_a + \ell_b < 1$	Firm B
$\ell_a + \ell_b = 1$	Symmetric

	Prestige Advantage
Snobbish: $\beta > 0$ and $\alpha > \frac{1}{2}$	Firm A
Snobbish: $\beta > 0$ and $\alpha < \frac{1}{2}$	Firm B
Conformist: $\beta < 0$ and $\alpha < \frac{1}{2}$	Firm A
Conformist: $\beta < 0$ and $\alpha > \frac{1}{2}$	Firm B
$\beta = 0$ or $\alpha = \frac{1}{2}$	Symmetric

	More Prestigious Position
Snobbish: $\beta > 0$ and $\ell_a + \ell_b < 6\alpha - 2$	Firm A
Snobbish: $\beta > 0$ and $\ell_a + \ell_b > 6\alpha - 2$	Firm B
Conformist: $\beta < 0$ and $\ell_a + \ell_b < 6\alpha - 2$	Firm B
Conformist: $\beta < 0$ and $\ell_a + \ell_b > 6\alpha - 2$	Firm A
$\beta = 0$ or $\ell_a + \ell_b = 6\alpha - 2$	Symmetric

Table 1: Firm with Location Advantage, Prestige Advantage and more Prestigious Position when $\ell_a < \ell_b$

Demand Partitions

Armed with a better understanding of the signaling game's dynamics, we can calculate consumer demand. For any cut-off n, the excepted utility of a given consumer x for each good at the purchasing stage is:

$$U_x(a; n) = v - \tau(\ell_a - x)^2 - p_a + S_a(n) \tag{10}$$

$$U_x(b; n) = v - \tau(\ell_b - x)^2 - p_b + S_b(n) \tag{11}$$

where $S_a(n)$ and $S_b(n)$ are as defined in equations (7) and (8). A consumer buys good a if $U_x(a; n) > U_x(b; n)$, and buys good b if $U_x(b; n) > U_x(a; n)$. The equilibrium value of the cut-off n, call it $\hat{n}$, is defined by the consumer x who is just indifferent between buying goods

a and b. Plugging $\hat{n}$ in for x, setting equations (10) and (11) equal and solving yields:

$$\hat{n} = \frac{p_b - p_a + \tau(\ell_b - \ell_a)(\ell_a + \ell_b) + \lambda\beta(\alpha - \frac{1}{3})}{2\tau(\ell_b - \ell_a) + \lambda\frac{\beta}{3}} \tag{12}$$

If $\beta = 0$ and there are no signaling motives, then $\hat{n}$ simplifies to the cut-off in the standard Hotelling model with quadratic transportation costs. Also, note that $\hat{n}$ is a similar cut-off to that in the model of Grilo et al. (2001), except in this model the allowance of $\alpha \neq \frac{1}{2}$ implies an asymmetry in how signaling affects the market shares of firm A ($\hat{n}$) and firm B ($1 - \hat{n}$).

We need to check that $\hat{n}$ is in $[0, 1]$. Whether $\hat{n} \in [0, 1]$ depends on prices and the sign of the denominator. As in Grilo et al. (2001), I say the market is characterized by "Snobbism\Weak Conformity" if $2\tau(\ell_b - \ell_a) > -\lambda\frac{\beta}{3}$ and the denominator is positive. This necessarily holds in a snobbish market, and may hold in a conformist market if firms are sufficiently far and differentiated relative to the degree of advertising and conformity. By contrast, I say the market is characterized by "Strong Conformity" if $2\tau(\ell_b - \ell_a) \leq -\lambda\frac{\beta}{3}$ and the denominator is non-positive.

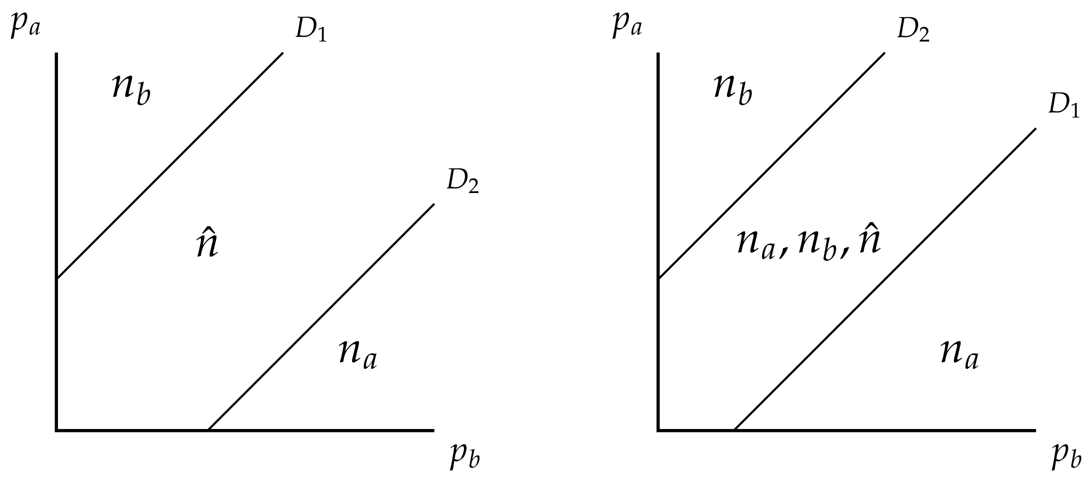
Definition (Weak and Strong Conformity)**.** *The market is characterized by "weak conformity" if* $\lambda\beta < 0$ *and* $2\tau(\ell_b - \ell_a) > -\lambda\frac{\beta}{3}$*. The market is characterized by "strong conformity" if* $\lambda\beta < 0$ *and* $2\tau(\ell_b - \ell_a) \leq -\lambda\frac{\beta}{3}$*.*

Suppose the market is characterized by snobbism or weak conformity. Evaluating equation (12), we get a unique semi-separating equilibrium with $\hat{n} \in (0, 1)$ when the difference in firm prices is not too large and lies inside the following range: $\tau(\ell_b - \ell_a)(\ell_a + \ell_b - 2) + \lambda\beta(\alpha - \frac{2}{3}) < p_a - p_b < \tau(\ell_b - \ell_a)(\ell_a + \ell_b) + \lambda\beta(\alpha - \frac{1}{3})$.

If the price difference is large and lies outside this range, then all consumers purchase the cheaper good and we get a pooling equilibrium. Let's call the cut-off n_A when firm A captures the market (i.e. $\hat{n} = 1$), and n_B when firm B captures the market (i.e. $\hat{n} = 0$). We get a unique pooling equilibrium with firm A capturing the market, n_A, when $p_a - p_b \leq \tau(\ell_b - \ell_a)(\ell_a + \ell_b - 2) + \lambda\beta(\alpha - \frac{2}{3})$. Furthermore, we get a unique pooling equilibrium with firm B capturing the market, n_B, when $p_a - p_b \geq \tau(\ell_b - \ell_a)(\ell_a + \ell_b) + \lambda\beta(\alpha - \frac{1}{3})$. Figure 5a maps these demand partitions for any given price vector.

Suppose instead the market is characterized by strong conformity. In this case, there may be multiple equilibria of the consumption subgame, and demand is defined by a correspondence rather than a function. If firm A has a much lower price and $p_a - p_b < \tau(\ell_b - \ell_a)(\ell_a +$

$\ell_b) + \lambda\beta(\alpha - \frac{1}{3})$, then there is a unique pooling equilibrium in which firm A captures the entire market. If firm B has a much lower price and $p_a - p_b > \tau(\ell_b - \ell_a)(\ell_a + \ell_b - 2) + \lambda\beta(\alpha - \frac{2}{3})$, then there is a unique pooling equilibrium in which firm B captures the entire market. However, if $\tau(\ell_b - \ell_a)(\ell_a + \ell_b) + \lambda\beta(\alpha - \frac{1}{3}) \leq p_a - p_b \leq \tau(\ell_b - \ell_a)(\ell_a + \ell_b - 2) + \lambda\beta(\alpha - \frac{2}{3})$, then equilibria with any of the following consumer partitions are possible: n_A, n_B, or $\hat{n} \in (0,1)$, where $\hat{n}$ is as defined by equation (12). Figure 5 shows a map of demand in (p_b,p_a) space.[17] We will revisit the issue of the multiplicity of demand partitions when we discuss the equilibria at the pricing stage.



(a) Snobbism \Weak Conformity

(b) Strong Conformity

Figure 5: Demand Partitions with Snobbism\Weak Conformity

$$D_1: \; p_a = p_b + \tau(\ell_b - \ell_a)(\ell_a + \ell_b) + \lambda\beta(\alpha - \frac{1}{3})$$
$$D_2: \; p_a = p_b + \tau(\ell_b - \ell_a)(\ell_a + \ell_b - 2) + \lambda\beta(\alpha - \frac{2}{3})$$

The diagonal lines D_1 and D_2 in Figure 5 denote barriers between possible consumer partitions.[18] These diagonal lines will have economic significance in the pricing stage. Intuitively, the firm that commands a larger region of high market share on the (p_b, p_a) plane is likely to be able to use its strategic power to extract a greater price. Furthermore, these diagonal lines are responsive to λ, β, α, τ, ℓ_a and ℓ_b.

In Figure 5 the the diagonal lines are drawn on either side of the 45 degree line, but this need not always be the case. In the snobbish case, D_1 is always above the 45 degree line and

[17]Not shown in Figure 5 is some upper price limits $\overline{p_a}$ and $\overline{p_b}$ above which consumers do not purchase.

[18]In Figure 5a, only partition n_b is possible on the D_1 line, and only partition n_a is possible on the D_2 line. By contrast, in Figure 5b, both partitions n_a and n_b are possible on both the D_1 and D_2 lines, and only partitions n_a and n_b are possible on the D_1 and D_2 lines.

D_2 is below it. As $\lambda\beta$ increases or firms move farther apart, then the diagonal lines separate farther apart. Intuitively, an increase in the degree of snobbery or differentiation implies that it is more likely for consumers to split their demand between the two stores for any given prices. However, as either $\lambda\beta$ decreases or as firms move closer together, the diagonal lines move closer together. Intuitively, an increase in the degree of conformity or a decrease in differentiation implies that it is easier to find prices such that all consumers patron the same store. In the conformist case, the diagonal lines eventually cross as $\lambda\beta$ becomes increasingly negative. The point at which the diagonal lines cross $D_1 = D_2$ is where the market switches from being characterized by weak conformity to strong conformity.

Whether the diagonal lines cross above or below the 45 degree line will have important implications when we solve for prices in a conformist market. If firm A has a more prestigious position and $\ell_a + \ell_b > 6\alpha - 2$, then the diagonal lines cross above the 45 degree line.[19] This implies that there may be a large region in (p_b, p_a) space where all consumers frequent store A. By contrast, if firm B has a more prestigious position and $\ell_a + \ell_b < 6\alpha - 2$, then the diagonal lines cross below the 45 degree line.[20] Thus, there may be a large region in (p_b, p_a) space where all consumers frequent store B. Finally, if $\ell_a + \ell_b = 6\alpha - 2$, then they cross at the 45 degree line.

Tie-Breaking Rule for Undifferentiated Firms

The demand partitions described apply mutatis mutandis when firm B locates to the left $\ell_b < \ell_a$ by flipping the a and b terms. However, if firms choose alike locations $\ell_a = \ell_b$, then the single crossing property of consumer demand fails, and demand can, but does not necessarily have to follow the cut-off rule described in Lemma 1. This issue also arises in the standard Hotelling model.

The Hotelling literature usually accommodates this by assuming, à la Bertrand competition, that homogeneous firms evenly split the market at $n = \frac{1}{2}$ if they charge equivalent prices, and the firm with the lower price captures all demand if they charge different prices. I impose the same tie-breaking rule here in the case of a standard market where $\lambda\beta = 0$.

However, this is not always a possible demand partition in a market characterized by snobbism or conformity. For example, in a snobbish market, some consumers might still

[19]Note that if $\alpha < \frac{1}{3}$, then the diagonal lines must cross above the 45 degree line.
[20]Note that if $\alpha > \frac{2}{3}$, then the diagonal lines must cross above the 45 degree line.

shop at the higher priced firm even if firms are undifferentiated. Furthermore, in a conformist market, consumers prefer to all buy from same firm when alike firms charge alike prices.

To extend the tie-breaking rule to the cases of snobbery and conformity, I assume that demand follows the partition described in the analysis preceding that is most advantageous to firm A. For example, consider snobbish market in which $\alpha \geq \frac{1}{2}$ and the firm to the left has a prestige advantage. I then assume firm A captures the demand described by it being to the left as in equation (12) with $\ell_a = \ell_b$: $\hat{n} = \frac{p_b - p_a + \lambda\beta(\alpha - \frac{1}{3})}{\lambda\frac{\beta}{3}}$. If instead $\alpha < \frac{1}{2}$ and $\lambda\beta > 0$, then firm A captures demand: $\hat{n} = \frac{p_a - p_b + \lambda\beta(\frac{2}{3} - \alpha)}{\lambda\frac{\beta}{3}}$.

In a conformist market, the lack of differentiation between firms implies the market is characterized by strong conformity. If the firm to the left has a prestige advantage and $\alpha \leq \frac{1}{2}$, then I assume demand follows the partition described by the strongly conformist case with firm A to the left as in Figure 5b. If instead the firm to the right has a prestige advantage and $\alpha < \frac{1}{2}$, then I assume that firm A captures the demand described by the strongly conformist case where $\ell_a \geq \ell_b$ (constructed by flipping the a and b terms in Figure 5b).

V PRICING IN CONFORMIST AND SNOBBISH MARKETS

With the above map of consumer demand, we can calculate the equilibrium at the pricing stage, given firm advertising levels and locations. I treat the cases of snobbism\weak conformity and strong conformity separately.

Denote the demand of firm A and firm B by $Q_a(p_a, p_b, \lambda, \ell_a, \ell_b)$ and $Q_b(p_a, p_b, \lambda, \ell_a, \ell_b)$ respectively, as derived in Section IV, so firms maximize $p_a Q_a(p_a, p_b, \lambda, \ell_a, \ell_b)$ and $p_b Q_b(p_a, p_b, \lambda, \ell_a, \ell_b)$.

Snobbism\Weak Conformity

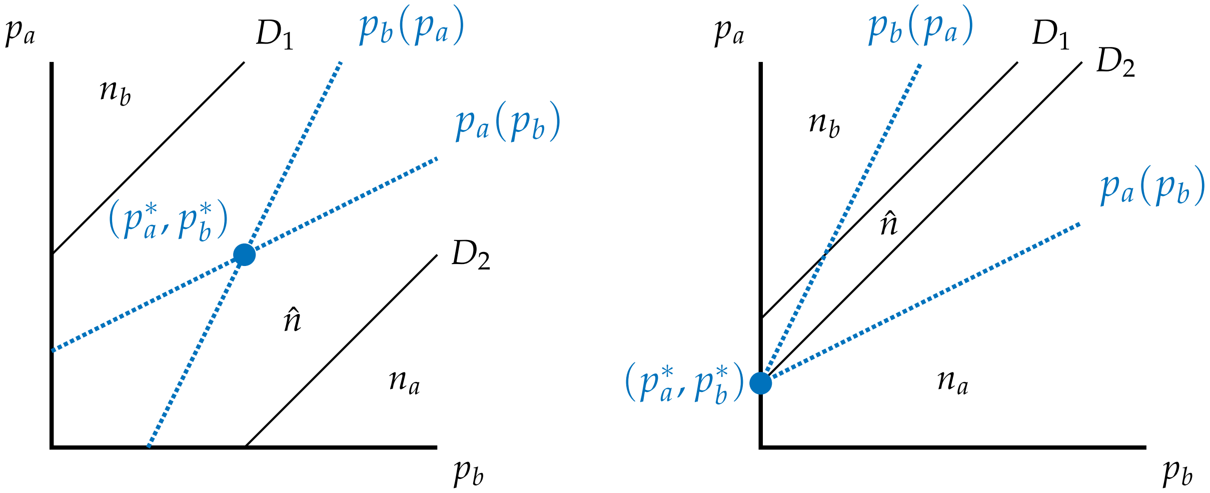
Suppose we are in a market characterized by snobbism\weak conformity. The cut-off $\hat{n}$ is then given by equation (12). A pure strategy price equilibrium exists since demands are linear and decreasing in own prices. Differentiating $p_a Q_a(p_a, p_b, \lambda, \ell_a, \ell_b)$ and $p_b Q_b(p_a, p_b, \lambda, \ell_a, \ell_b)$ with respect to own prices and solving yields:

$$p_a^* = \frac{\tau}{3}(\ell_b - \ell_a)(2 + \ell_a + \ell_b) + \lambda\alpha\frac{\beta}{3} \tag{13}$$
$$p_b^* = \frac{\tau}{3}(\ell_b - \ell_a)(4 - \ell_a - \ell_b) + \lambda(1-\alpha)\frac{\beta}{3} \tag{14}$$
$$n^* = \frac{\frac{\tau}{3}(\ell_b - \ell_a)(2 + \ell_a + \ell_b) + \lambda\alpha\frac{\beta}{3}}{2\tau(\ell_b - \ell_a) + \lambda\frac{\beta}{3}} \tag{15}$$

For this to be a price equilibrium, it remains to be checked that $n^* \in (0,1)$. This holds if and only if:

$$-\lambda\beta < \tau(\ell_b - \ell_a)\min\{\frac{2 + \ell_a + \ell_b}{\alpha}, \frac{4 - \ell_a - \ell_b}{1-\alpha}\} \tag{16}$$

In a snobbish market, equation (16) always holds, so that $n^* \in (0,1)$ and both firms earn positive revenues as described by equations (13) - (14).[21] This equilibrium is depicted graphically in Figure 6a.



(a) No Firm Takeover
(Snobbism or Weak Conformity)

(b) Firm A Takeover
(Weak Conformity)

Figure 6: Price Equilibrium with Snobbism\Weak Conformity

$$p_a(p_b) = \frac{p_b}{2} + \frac{\tau}{2}(\ell_b - \ell_a)(\ell_a + \ell_b) + \frac{\lambda\beta}{2}(\alpha - \frac{1}{3})$$
$$p_b(p_a) = \frac{p_a}{2} + \frac{\tau}{2}(\ell_b - \ell_a)(2 - \ell_a - \ell_b) + \frac{\lambda\beta}{2}(\frac{2}{3} - \alpha)$$

Inspection of the price equilibrium described by equations (13) - (15) reveals interesting insight into the dynamics of snobbish markets. First, both firms' prices are increasing in advertising in a snobbish market. Intuitively, by increasing the strength of consumers' snobbish

[21] Firm A's revenues simplify to $\frac{p_a^{*2}}{2\tau(\ell_b - \ell_a) + \lambda\frac{\beta}{3}}$ and Firm B's revenues to $\frac{p_b^{*2}}{2\tau(\ell_b - \ell_a) + \lambda\frac{\beta}{3}}$.

motives, advertising reduces the elasticity of demand with respect to prices.[22] This is because when firms cut prices, fewer consumers rush in to buy their products, as their signaling gains from the product decrease the more who buy it. This dampens price competition, and induces firms to converge on higher prices. Moreover, advertising by one firm raises the price of the other firm. If the market share effects described below are not too large, then advertising can increase the profits of both firms. Thus, for certain parameters advertising can be non-combative, instead acting as a public good to firms. This is made visually clear in Figure 6a. Advertising shifts some (or all) of the diagonal lines and price response lines out in a way that pushes the price equilibrium in the north east direction.

However, advertising has a greater positive effect on the price of the firm with the prestige advantage. Intuitively, by increasing recognition that one firm has more high types, advertising allows that firm to extract a higher price. In Figure 6a, advertising pushes the price equilibrium out in a direction more favorable to the firm with the prestige advantage. Actually, if we extended the results by allowing for $\alpha \notin [0,1]$, then advertising would lessen the price of the firm with the prestige disadvantage because it would have to undercut its price to attract customers.

Reputation motives also create interesting effects on firms' market shares. Advertising benefits the market share of the firm with the more prestigious position, at the detriment of the market share of the other. Intuitively, a firm preferred by more high status consumers can use advertising to attract more customers. Even if firms are symmetrically located, then asymmetries arise where the firm with the more prestigious position commands a higher price and earns greater market share. This can explain why we sometimes see physically similar or indistinguishable products with one earning greater market share and charging a price premium. These results are summarized in Proposition 1.

[22]More precisely, advertising reduces a weighted combination of the elasticity of demand of goods a and b with respect to their own and each others' prices that results in higher prices. For example, the elasticity of demand of good a with respect to p_a and p_b are $\epsilon^a_{p_a} = \frac{dn}{dp_a}\frac{p_a}{n} = \frac{-p_a}{p_b - p_a + \tau(\ell_b - \ell_a)(\ell_a + \ell_b) + \lambda\beta(\alpha - \frac{1}{3})}$ and $\epsilon^a_{p_b} = \frac{dn}{dp_b}\frac{p_b}{n} = \frac{p_b}{p_b - p_a + \tau(\ell_b - \ell_a)(\ell_a + \ell_b) + \lambda\beta(\alpha - \frac{1}{3})}$ respectively. Thus, in a snobbish market, the absolute value of elasticity of demand for good a with respect to p_a and p_b decreases in advertising when $\alpha \geq \frac{1}{3}$, and increases in advertising otherwise. Similarly, in a snobbish market, the absolute value of the elasticity of demand for good b with respect to p_a and p_b decreases in advertising when $\alpha \leq \frac{2}{3}$, and increases otherwise. The net result is that advertising effects $\epsilon^a_{p_a}$, $\epsilon^a_{p_b}$, $\epsilon^b_{p_a}$ and $\epsilon^b_{p_b}$ in a way that raises the prices of both goods.

Proposition 1 (Advertising's Effect on Prices and Market Share in a Snobbish Market)**.** *In a snobbish market, advertising by either firm weakly raises the prices of both firms. However, advertising has a greater positive effect on the price of the firm with the prestige advantage. Furthermore, advertising has a positive effect on the market share of the firm with the more prestigious position, and a negative effect on the market share of the firm with the less prestigious position.*

I turn to a weakly conformist market next. If equation (16) holds in a weakly conformist market, then we get the equilibrium described by equations (13) - (15). It is apparent that advertising then has the opposite effect, decreasing firms' prices. The intuition is that by strengthening consumers' conformist motives, advertising increases the elasticity of demand with respect to prices — when firms cut prices, more consumers rush to buy in due to the reputation gains from others doing so. This heightens the price competition and induces firms to converge on lower prices. Here, it is found that advertising has a greater negative effect on the price of the firm with the prestige disadvantage, because it has added need to undercut its price to retain consumers.

Additionally, as in a snobbish market, we see that advertising benefits the market share of the firm with the more prestigious position, at the detriment of the market share of the other. Even if firms are symmetrically located, then the firm with the more prestigious position commands a higher price and earns greater market share. However, there are no parameters for which advertising benefits both firms, as in a snobbish market. These results are summarized in Proposition 2.

Proposition 2 (Advertising's Effect on Prices and Market Share in a Weakly Conformist Market)**.** *Consider a weakly conformist market in which neither firm takes over. Advertising by either firm weakly lowers the prices of both firms. However, advertising has a greater negative effect on the price of the firm with the prestige disadvantage. Furthermore, advertising has a positive effect on the market share of the firm with the more prestigious position, and a negative effect on the market share of the firm with the less prestigious position.*

If price competition is sufficiently severe, then equation (16) may not hold in a conformist market. That is, if product differentiation is sufficiently low relative to the intensity of advertising and conformity, then equation (16) does not hold. In this case, one firm wins all consumer demand. Essentially, conformist motives overpower consumers' transportation costs, and all go to the most popular brand. It turns out that the the firm with the more prestigious positions takes over the market, charging a limit-price. If equation (16) does not hold and firm A has a more prestigious position, then it charges the highest price such that firm B cannot charge a weakly positive price and earn any customers:

$$p_a^* = -\tau(\ell_b - \ell_a)(2 - \ell_a - \ell_b) + \lambda\beta(\alpha - \frac{2}{3}) \tag{17}$$

$$p_b^* = 0 \tag{18}$$

$$n^* = n_a \tag{19}$$

This price equilibrium is depicted in Figure 6b. It occurs when the D_2 and $p_b(p_a)$ lines are sufficiently high that they intersect the $p_a(p_b)$ line on the p_a-axis.

If equation (16) does not hold and firm B has a more prestigious position, then it charges the highest price such that no consumers patron firm A at any $p_a \geq 0$:

$$p_a^* = 0 \tag{20}$$

$$p_b^* = -\tau(\ell_b - \ell_a)(\ell_a + \ell_b) - \lambda\beta(\alpha - \frac{1}{3}) \tag{21}$$

$$n^* = n_b \tag{22}$$

This price equilibrium occurs when the D_1 and $p_a(p_b)$ lines are sufficiently low that they intersect the $p_b(p_a)$ line on the p_b-axis. This single firm dominance is a result we could not get in standard model of horizontal differentiation with zero production costs. Note that if firms are symmetrically located, then equation (16) always holds in a weakly conformist market.

Strong Conformity

Let's now explore equilibrium prices in a market characterized by strong conformity. As shown in Section IV, there may be multiple possible consumer partitions for given prices, and demand is defined by a correspondence rather than a function.

This implies, as in Grilo et al. (2001), there may also be multiple equilibria at the pricing stage. However, many of these price equilibria are unreasonable and rely on unusual off-equilibrium consumer behavior where demand moves highly non-monotonically in prices. It turns out that under a reasonable refinement firms' revenues are uniquely determined in the equilibrium of the pricing stage. The axiom below is predicated on the premise that when there are three possible consumers partitions for a given price vector, firms anticipate consumers settling on the partition most beneficial to the firm with the cheaper product.

Axiom 1 (Cheaper is Better)**.** *If price pair (p_b, p_a) can induce partitions n_a, n_b and $\hat{n} \in (0,1)$ and $p_a \neq p_b$, then consumers settle on the partition giving the highest market share to the firm with the lower price (n_a if $p_a < p_b$ and n_b if $p_a > p_b$).*

I assume Axiom 1 holds throughout the analysis that follows. This imposes a certain degree of monotonicity on the movement of demand with respect to prices, so that when prices move from the north west to the south east of the (p_b, p_a) quadrant in Figure 5b, the anticipated demand partitions generally move from firm B earning greater share to firm A. Under this refinement, firms' equilibrium revenues are uniquely determined in the pricing stage, as described in Proposition 3.[23]

Proposition 3 (Strongly Conformist Price Equilibrium)**.** *Consider a strongly conformist market with $\ell_a \leq \ell_b$. Under Axiom 1, the equilibria at the pricing stage are as follows. If D_1 and D_2 intersect the p_a-axis, then $p_b^* = 0$, $n^* = n_a$ and*

$$p_a^* = \tau(\ell_b - \ell_a)(\ell_a + \ell_b) + \lambda\beta(\alpha - \frac{1}{3}) \tag{23}$$

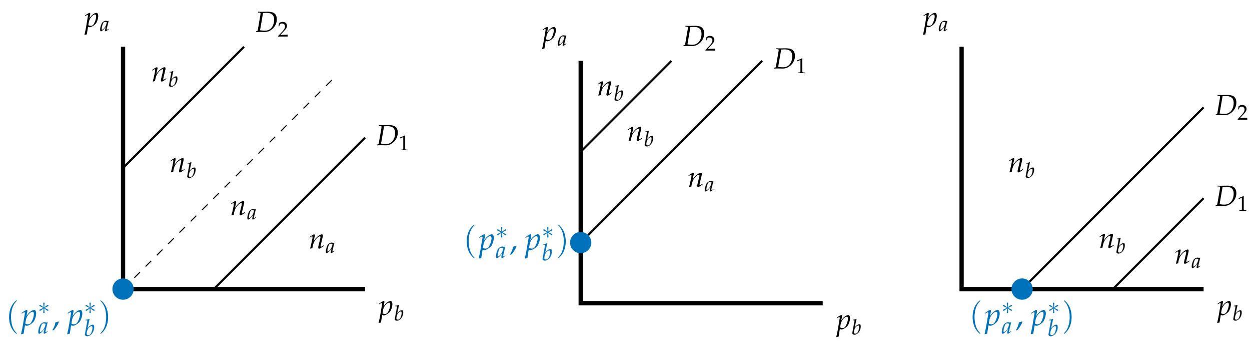
If D_1 and D_2 intersect the p_b-axis, then $p_a^* = 0$, $n^* = n_b$ and

$$p_b^* = \tau(\ell_b - \ell_a)(2 - \ell_a - \ell_b) - \lambda\beta(\alpha - \frac{2}{3}) \tag{24}$$

Otherwise, $p_a^* = p_b^* = 0$, and $n^* \in \{\hat{n}, n_a, n_b\}$.

These equilibria are depicted in Figure 7. The intuition as to why both firms may earn zero revenues is that as conformity grows very large, product differentiation matters comparatively less, and we get an equilibrium resembling Bertrand competition. However, if prestige effects are sufficiently large, then one firm may takeover and earn positive revenues.

[23]One could obtain the same equilibrium described in Proposition 3 by applying the invariance axiom introduced in Grilo et al. (2001) in combination with the following axiom: If price pair (p_b, p_a) can induce partitions n_a, n_b and $\hat{n} \in (0,1)$ and $p_a \neq p_b$, then consumers do *not* settle on the partition giving the highest market share to the firm with the higher price (n_b if $p_a < p_b$ and n_a if $p_a > p_b$). The invariance axiom says that if price pair (p_b, p_a) induces consumer partition n^*, then $(p_b + z, p_a + z)$ induces the same partition n^* where z is an arbitrary constant, $(p_b + z, p_a + z) \geq (0,0)$ and all consumers want to buy. In other words, if consumer utility for each good is added or subtracted by an identical constant, then consumers buy the same goods.



(a) Bertrand Equilibrium **(b)** Firm A Takeover **(c)** Firm B Takeover

Figure 7: Price Equilibria in Strongly Conformist Market

Monopoly Pricing

Let's look at pricing in the monopoly subgame resulting from firm B not entering the market. In congruence with the oligopoly subgame, I maintain the traditional assumption that v is sufficiently large that a monopolist supplies the whole market. This holds for any given locations and advertising levels if $v > \max\{3\tau \pm \beta\}$. I assume this condition is met throughout the analysis that follows.

The calculation of the signaling gains from purchasing good a parallels that in the oligopoly case at the beginning of the section. Here, the signaling gains from good a, call it $S_{a/\emptyset}$, are the perceived status of a consumer who purchases less that of a consumer who picks the outside option of not purchasing (rather than less that of a consumer who purchases good b). Since every consumer purchases in equilibrium, the signaling gains from purchasing are determined by the (advertisement receiving) public's off-equilibrium beliefs about a consumer's identity should she not purchase.[24]

I restrict attention to the following off-equilibrium beliefs. A consumer who does not purchase is believed to be the consumer at the end furthest from the good and, thus benefiting most from foregoing purchase. That is, $\rho(\, x = 1 \,|\, \emptyset, \Omega\,) = 1$ if $\ell_a < \frac{1}{2}$ and $\rho(\, x = 0 \,|\, \emptyset, \Omega\,) = 1$ if $\ell_a > \frac{1}{2}$. If $\ell_a = \frac{1}{2}$, then consumers at both ends benefit equally from not purchasing. Thus, in this case I assume that a consumer who does not purchase is believed to be at the end with the lower social status: $\rho(\, x = 1 \,|\, \emptyset, \Omega\,) = 1$ if $\beta \geq 0$ and $\alpha \geq \frac{1}{2}$, or if $\beta < 0$ and $\alpha \leq \frac{1}{2}$; and $\rho(\, x = 0 \,|\, \emptyset, \Omega\,) = 1$ otherwise.[25] These off-equilibrium beliefs are not only reasonable, but

[24]Just as in the conformist case with takeover, where the signaling gains from the favored good are determined by off-equilibrium beliefs about a consumer's identity should she purchase the less favored good.

[25]This makes firm A's profit function upper semi-continuous at $\ell_a = \frac{1}{2}$, helping ensure the existence of an equilibrium in the larger game where locations are endogenous.

also congruent with the off-equilibrium beliefs assumed in an oligopolistic market.

If a consumer who foregoes purchase is believed to be at end $x = 1$, then:

$$S_{a/\emptyset} = \lambda\beta(\frac{2}{3} - \alpha)$$

If a consumer who does not purchase is believed to be at end $x = 0$, then:

$$S_{a/\emptyset} = \lambda\beta(\alpha - \frac{1}{3})$$

Firm A charges the highest price such that a consumer located at the furthest end is just indifferent to purchasing ($U_x(a) = U_x(\emptyset)$). Firm A's monopoly price (and likewise revenues since $n = n_a$) is thus:

$$p_a^M = \begin{cases} v - \tau(1-\ell_a)^2 + \lambda\beta(\alpha - \frac{2}{3}) & \text{if } \ell_a < \frac{1}{2} \\ & \text{or } \ell_a = \frac{1}{2}, \beta < 0 \text{ and } \alpha \leq \frac{1}{2} \\ & \text{or } \ell_a = \frac{1}{2}, \beta > 0 \text{ and } \alpha \geq \frac{1}{2} \\ v - \tau\ell_a^2 - \lambda\beta(\alpha - \frac{1}{3}) & \text{otherwise} \end{cases} \tag{25}$$

VI EQUILIBRIA OF FULL MODEL OF PERSUASIVE ADVERTISING

This section turns to the main motivation of the paper, characterizing equilibria of the full game in snobbish and conformist markets. We will see how the entry, advertising, location, pricing and market shares of firms are effected in the two types of markets, and how such signaling motives can explain the empirical regularities outlined in the introduction.

First, as a benchmark of comparison, it is useful to characterize the equilibrium in the canonical case of a market without signaling motives and $\beta = 0$.

Proposition 4 (Equilibrium of Standard Market)**.** *If $\beta = 0$, then there exists a unique equilibrium in which firm B enters, firms locate at opposite ends $\ell_a^* \in \{0, 1\}$ and $\ell_b^* = 1 - \ell_a^*$, and do not advertise $\lambda_a^* = \lambda_b^* = 0$, thus charging equivalent prices $p_a^* = p_b^* = \tau$ and evenly splitting the market $n^* = \frac{1}{2}$.*

A similar benchmark of comparison could be made by instead studying a "full information version" of the model, as common in the study of signaling games, where the public has full knowledge of each consumer's attribute x and no information asymmetries exist. If consumers' attributes were known, then consumers would be unable to affect their perceived social status through their purchases. Consumers would then make purchases only according to their horizontal preferences, and firms would have no incentive to advertise, thus generating the same equilibrium as in Proposition 4. This signaling free setup does not explain the role of persuasive advertising, asymmetries in the prices and market shares of physically similar goods, and the barriers to entry often faced in heavily advertised markets.

Equilibria of Snobbish Market

Perhaps we can gain more insight in a market characterized by snobbery $\beta > 0$. If firms B enters and, without loss of generality, locates to the right of firm A ($\ell_a < \ell_b$), then the implied profit functions of firms at the location and advertising stages, taking into account their later pricing decisions, are:

$$\pi_a(\ell_a, \lambda_a; \ell_b, \lambda_b) = \frac{\left(\frac{\tau}{3}(\ell_b - \ell_a)(2 + \ell_a + \ell_b) + \lambda\alpha\frac{\beta}{3} \right)^2}{2\tau(\ell_b - \ell_a) + \lambda\frac{\beta}{3}} - \frac{c}{2}\lambda_a^2 \tag{26}$$

$$\pi_b(\ell_b, \lambda_b; \ell_a, \lambda_a) = \frac{\left(\frac{\tau}{3}(\ell_b - \ell_a)(4 - \ell_a - \ell_b) + \lambda(1-\alpha)\frac{\beta}{3} \right)^2}{2\tau(\ell_b - \ell_a) + \lambda\frac{\beta}{3}} - \frac{c}{2}\lambda_b^2 \tag{27}$$

If firm B locates to the left of firm A, then firms' profit functions are found by flipping the a and b terms in equations (26) - (27).[26]

The first thing to notice is that in any equilibrium, if it exists, firm B must enter because it can always earn positive profits. For example, firm B can locate at any $\ell_b \neq \ell_a$ without advertising $\lambda_b = 0$ and earn positive profits. While this is only shown here for the case of two firms, this sheds light on the abundance of brands often seen in snobbish markets.

However, before characterizing the equilibria, let's establish their existence. Even though equilibria exist at the demand and pricing stages, this does not guarantee the existence of an equilibrium at the earlier stages of the game. There is a large literature dealing with issues related to the existence of equilibria at both the pricing and location stages of product differentiation models.[27] The approach of many is to describe the characteristics of an equilibrium, should it exist, without guaranteeing its existence.[28]

Indeed, the profit functions given in equations (26) - (27) reveal a complication with estab-

[26]If $\ell_b = \ell_a$ and $\alpha \geq \frac{1}{2}$, then firm A (firm B) earns profits given by equation (26) (by equation (27)); and if $\ell_b = \ell_a$ and $\alpha < \frac{1}{2}$, then firm A (firm B) earns profits given by equation (27) (by equation (26)).

[27]Summarized in Shaked and Sutton (1987, p.132-133).

[28]For example, see Prescott and Visscher (1977); Lane (1980); Neven (1987) and Shaked and Sutton (1987).

lishing the existence of an equilibrium. This complication is brought about because of the fact that the profit functions have a jump discontinuity at $\ell_a = \ell_b$ if $\alpha \neq \frac{1}{2}$. It is well known that it is difficult to prove the existence of an equilibrium when payoff functions are discontinuous (Dasgupta and Maskin, 1986; Simon, 1987).

Nonetheless, if the prestige effect from a firm being preferred by more high types is not too large, then it can be shown that firms do not locate at the discontinuity, instead locating at opposite ends. Thus, we are able to guarantee the existence of an equilibrium in this case. This necessarily holds if $\alpha \in [\frac{1}{3}, \frac{2}{3}]$, and may hold outside this range if β is not too large.

Proposition 5 (Existence of Equilibrium in Snobbish Market)**.** *If $\beta > 0$ and $\alpha \in [\frac{1}{3}, \frac{2}{3}]$, then there exists an equilibrium of the game.*

The intuition for $\alpha \in [\frac{1}{3}, \frac{2}{3}]$ inducing firms to locate at the ends is that two forces are at play. The first is the traditional motivation for firms to avoid price discrimination by locating as far apart as possible ("the principal of maximal differentiation"). The second force comes from one firm having greater prestige, and thus being able to extract a higher price. This firm may be able to so easily extract a higher price that it does not fear price competition and wants to drift to the middle to be closer to more consumers and earn greater market share. If α is sufficiently close to 0 or 1 and β is sufficiently high, then one firm can have strong enough prestige advantage that the second effect comes to dominate, and locating at an end is no long strictly dominant. In this case, the discontinuity in firms' profit functions comes into play and creates complications in proving the equilibrium's existence.

Whether an equilibrium exists or not, Proposition 6 describes its properties, should it exist, for any $\alpha \in [0, 1]$.

Proposition 6 (Equilibrium of Snobbish Market)**.** *If $\beta > 0$, then in any equilibrium:*

(1) Firm B enters.

(2) Total advertising is positive.

(3) The firm with the less prestigious position locates at an end. Furthermore, if $\alpha \in [\frac{1}{3}, \frac{2}{3}]$, then both firms locate at opposite ends.

(4) The firm with the more prestigious position charges a higher price and earns greater market share.

(5) If $\alpha = \frac{1}{2}$ and firms have symmetric prestige, then firm A invests weakly less in advertising than firm B. Furthermore, the equilibrium has a closed-form solution with $\ell_a^* \in \{0, 1\}$, $\ell_b^* = 1 - \ell_a^*$, $p_a^* = p_b^* = \tau + \lambda^* \frac{\beta}{6}$, $n^* = \frac{1}{2}$ and advertising expenditures:

$$\lambda_a^* = \begin{cases} \frac{\beta - \frac{\beta^2}{6c}}{12c - \frac{\beta^2}{6c}} & \textit{if } \beta < 6c \\ 0 & \textit{otherwise} \end{cases}$$

$$\lambda_b^* = \max\{(1 - \lambda_a^*)\frac{\beta}{12c}, 1\}$$

Proposition 6 captures many of the stylized facts we see in snobbish markets: the abundance of brands; inflated prices; and the greater market share, price and marketing efforts of the more prestigious.

It is also of interest how market variables are effected by the degree of snobbery (β) and prestige in social status. However, due to the potential non-differentiability in firms' best reply functions, analytic results for these comparative statics are unavailable using the envelope theorem. It stands to reason that as either the degree of snobbery or prestige increases, then so does the advertising expenditures of the more prestigious firm as it holds greater marginal returns to advertising. This would imply its price and market share also increases. As the degree of snobbery increases, the advertising expenditure of the less prestigious firm may or may not increase, depending on whether the positive price effects outweigh the negative market share effects. Furthermore, as the degree of prestige increases, the advertising expenditures of the less prestigious firm should decrease due to lower returns to advertising. The effect of snobbery and prestige on the overall level of advertising λ is ambiguous.

To shed light on these comparative statics, as well as give a graphical intuition for the equilibria, I provide numerical solutions in Figure 8.[29] Figure 8 shows firms' prices, advertising levels and profits for various levels of α and β, assuming $c = \tau = 2$. In all the equilibria pictured, it is found that firm A locates at the end with the prestige advantage $\ell_a^* = 1$ and firm B locates at the other end $\ell_b^* = 0$.[30] These results support the conjectures above. Moreover, Figure 8 suggests a first mover advantage to firm A, as it earns a weakly greater price, market share and profit in every equilibrium found.

[29] To be more precise, the right column with $\alpha = 0.5$ is found by a closed-form solution shown in Proposition 6. When $\alpha \neq \frac{1}{2}$, closed form solutions are unavailable or intractable and instead numerical solutions are shown.

[30] Analytically, I find that unless the cost of advertising is really high and α is very close to $\frac{1}{2}$, then firm A locates at a more prestigious position. The intuition being that if the loss of prestige is sufficiently small relative to the cost savings, then firm A may want to locate at a less prestigious position to induce firm B to take on greater advertising expenditures.

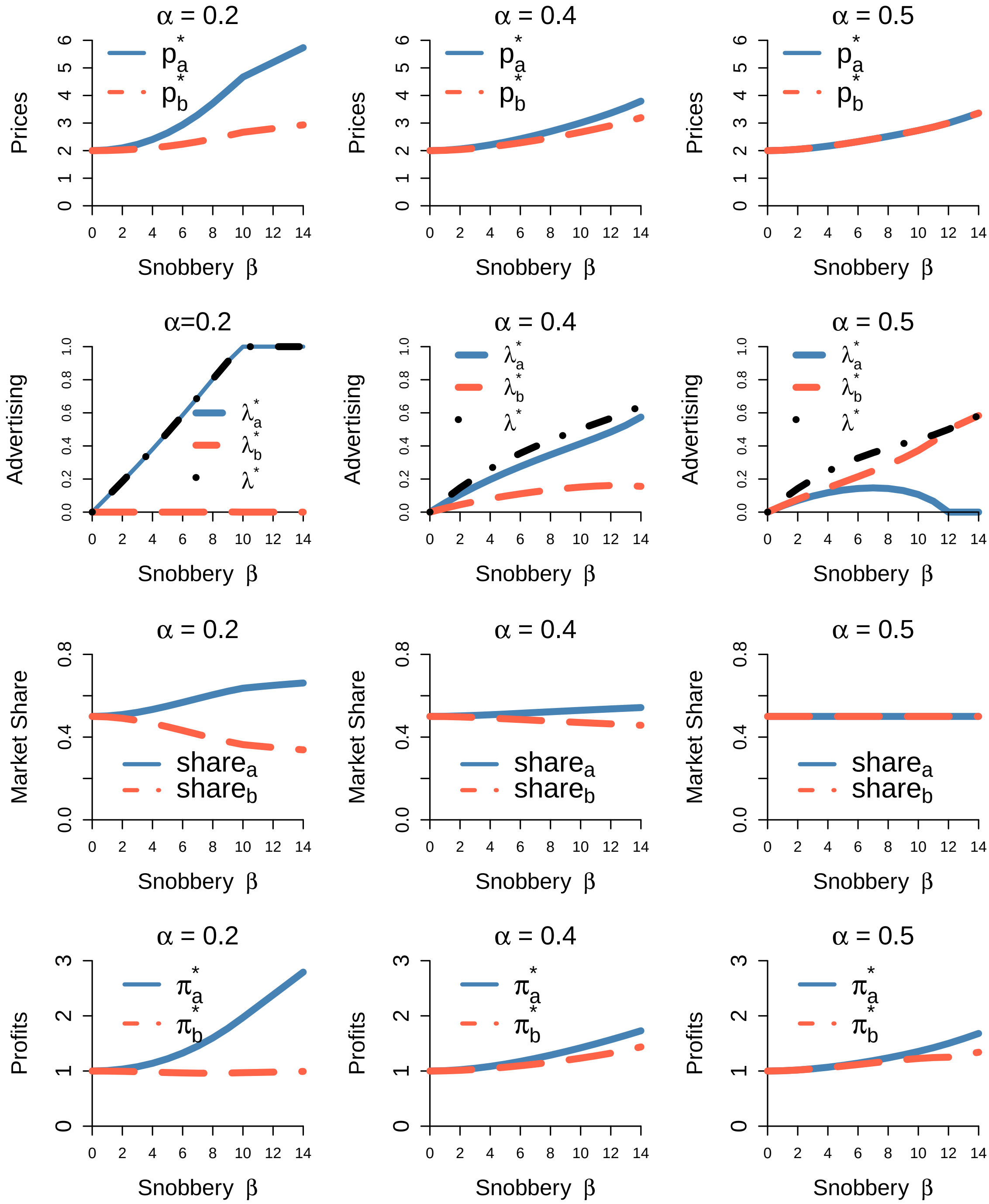


Figure 8: Numerical Solution in Snobbish Market

Snobbery β on the x-axis. In descending rows: prices, advertising, market shares and profits on the y-axis. In columns, from left to right: $\alpha = 0.2$, $\alpha = 0.4$ and $\alpha = 0.5$. The cases of $\alpha = 0.2$ and $\alpha = 0.4$ were calculated using Matlab, while a closed-form analytic solution is given for $\alpha = 0.5$ (see Proposition 6). In all equilibria depicted, $\ell_a^* = 1$ and $\ell_b^* = 0$. Assumes $c = \tau = 2$.

Equilibria of Conformist Market

Turning to the conformist case, let's see if the model can shed light on the barriers to entry often created by well advertised and branded firms in such markets.

As in a snobbish market, the existence of an equilibrium is guaranteed if $\alpha \in [\frac{1}{3}, \frac{2}{3}]$. However, the reason guaranteeing the existence of an equilibrium is somewhat different. In the conformist case, firms' profit functions are continuous at $\ell_a = \ell_b$ if $\alpha \in [\frac{1}{3}, \frac{2}{3}]$, thus ensuring the existence of an equilibrium (Harris, 1985).[31] This follows from the following facts. When firms locations are sufficiently close and their differentiation sufficiently low, then the market is characterized by strong conformity. If $\alpha \in [\frac{1}{3}, \frac{2}{3}]$, then prestige effects are small enough that both make zero profits when arbitrarily close, rather than the firm on one side capturing the entire market. This can be seen graphically in Figure 7 where $\ell_a = \ell_b$ implies the D_1 and D_2 lines lie on either side of the 45 degree line. If on the other hand $\alpha \notin [\frac{1}{3}, \frac{2}{3}]$, then prestige effects are so overpowering that firms' profit functions are discontinuous at $\ell_a = \ell_b$.

Proposition 7 (Existence of Equilibrium in Conformist Market)**.** *If $\beta < 0$ and $\alpha \in [\frac{1}{3}, \frac{2}{3}]$, then there exists an equilibrium of the game.*

Since I am primarily interested in the effects of conformity, and secondarily interested in the effects of prestige, I restrict attention to the case of $\alpha \in [\frac{1}{3}, \frac{2}{3}]$. However, I later discuss the implications for α outside this range.

Firm A can pursue one of two classes of strategies: accommodate firm B's entry, or deter firm B's entry. Firm A can deter firm B's entry by choosing a prestigious enough position, and advertising heavily enough that firm B could not earn positive profits from entering. I assume firm B does not enter if doing so would garner non positive profits.

In order for an entry deterrence strategy to exist, β needs to be sufficiently negative, creating adequate consumer conformity for firm A to capture. If β is sufficiently negative, then there exists a compact set of strategies $\triangle \subset [0,1]^2$ such that if $(\ell_a, \lambda_a) \in \triangle$, then firm B cannot earn positive profits from entry. Additionally, in order for deterring entry to be profitable, the cost of advertising must be sufficiently small. If these two criteria are met, then firm A chooses $(\ell_a, \lambda_a) \in \triangle$, deterring firm B's entry and seizing monopoly rents. Proposition 8 characterizes the equilibria.

[31] It is also true, similar to the snobbish case, that if $\alpha \in [\frac{1}{3}, \frac{2}{3}]$, then firms locate at the ends in any equilibrium in which firm B enter. However, this is not necessary to guarantee the equilibrium's existence.

Proposition 8 (Equilibrium of Conformist Market). *Suppose $\alpha \in [\frac{1}{3}, \frac{2}{3}]$ and $\beta < 0$. If β is sufficiently negative and c is sufficiently small, then firm A chooses some location and positive level of advertising such that firm B does not enter, allowing firm A to capture monopoly profits. Otherwise, neither firm advertises, and firms locates at opposing ends, splitting the market and earning $\frac{\tau}{2}$ profits (as in the equilibrium of a standard market with $\beta = 0$).*

Interestingly, it may be the case that firm B's entry would make firm A's entry deterring strategy sub optimal. In other words, it may be the case that if firm B happened to enter when firm A chose $(\ell_a, \lambda_a) \in \triangle$, then firm A's strategy would no longer be optimal. However, firm A rationally anticipates that firm B would not enter, thus allowing firm A to gain monopoly power.[32] The intuition resembles that of the chain store paradox (Selten, 1978). In the language of the chain store paradox, firm A's location and advertising choice can serve as a commitment to fight by generating harsh conditions of price competition upon firm B's entry.

Another striking aspect of this result is the strength of first mover advantage implied to firm A. For example, there does not exist an equilibrium in which firm B takes over. In order for firm B to take over, firm A would have to act non-rationally or there would have to be an unanticipated shock to the social status function. In other words, firms would have to be out of equilibrium.

The strength of the first mover advantage in the game captures the strength of the first mover advantage observed in conformist markets. Unlike many other models of entry deterrence, this result does not require assumptions about a specific production or cost function implying economies of scale. Indeed, the model assumed zero cost to production. If the model incorporated a fixed cost of production, then firm B's entry would be even less likely.

Figure 9 shows numerical solutions for the value of $\bar{c}$ for various configurations of β and α. As can be seen, if β is very negative, then a fairly high cost of advertising may be needed to prevent firm A from taking over.

[32] It is shown in the proof of Proposition 8 that for certain parameters it is can also be the case that firm A would choose $(\ell_a, \lambda_a) \in \triangle$ regardless of whether firm B entered.

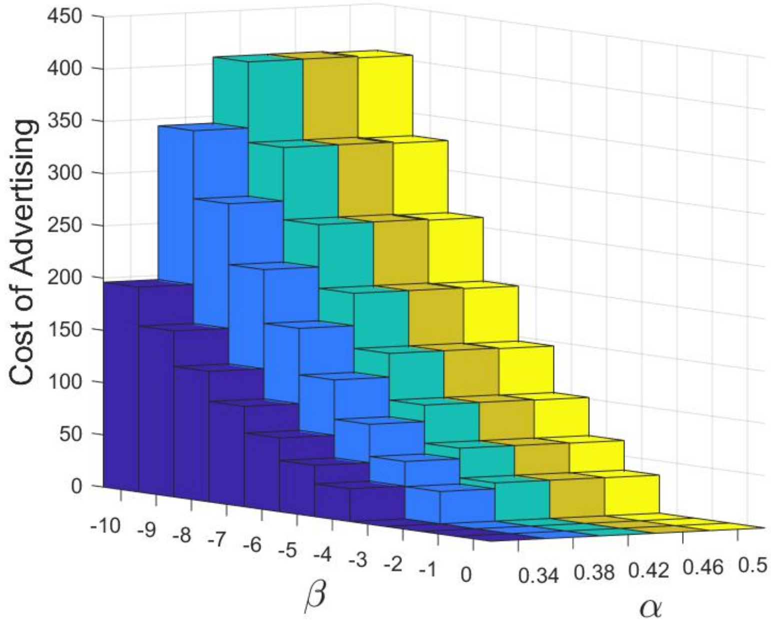


Figure 9: Numerical Solution in Conformist Market
Maximum Cost of Advertising for Firm A to Monopolize ($\bar{c}$)
Calculated using Matlab. Assumes $\tau = 1$ and $v = 13$.

Finally, let's discuss how these results would be affected by $\alpha \notin [\frac{1}{3}, \frac{2}{3}]$. As mentioned, the discontinuity in firms' profit functions prevents the use of traditional mathematical tools to guarantee the existence of an equilibrium. Nonetheless, we can discuss it's characteristics, should it exist. The proof of Proposition 8 establishes the following facts:

i) If β is sufficiently negative, then $\triangle$ is non-empty and firm A has an entry deterring strategy. In fact, it's entry deterring strategy is to locate at the end with the prestige advantage and advertise some positive amount: if $\alpha < \frac{1}{3}$, then $\triangle = \{[0, \lambda_a] | \lambda_a \geq \underline{\lambda_a}$ where $\underline{\lambda_a} \in (0, 1]\}$ and if $\alpha > \frac{2}{3}$, then $\triangle = \{[1, \lambda_a] | \lambda_a \geq \underline{\lambda_a}$ where $\underline{\lambda_a} \in (0, 1]\}$.

ii) Firm B never takes over the market, and firm A always earns positive profits.

Unfortunately, it's difficult to fully characterize the equilibria of the subgame in which firm B enters. This is because, as in the snobbish case, it may be that the firm with the prestige advantage wants to drift to the middle and advertise a positive amount. In this case, the discontinuity in firms' profit functions comes into play. Since we cannot give a closed-form solution for firm A's profits in the subgame where firm B enters, we cannot prove that firm A's gains from monopolization over firm B's entry increase *monotonically* as the cost of advertising decreases. This prevents proof of a *unique* cut-off $\bar{c}$ such that if the cost of advertising is above it, then firm B enters, and if the cost of advertising is below it, firm A monopolizes. However, it can be shown that if β is sufficiently negative and c is very low, then firm A would deter firm B's entry as its monopoly profits would be higher than *for any* price equilibrium outcome in which firm B entered.

VII WELFARE

With the above equilibria in mind, let's explore the implications for the old question of the welfare consequences of persuasive advertising. As mentioned, since this model has a fixed preference, it easily lends to welfare analysis.

Without loss of generality, suppose both firms enter the market and $\ell_a^* < \ell_b^*$. The equilibrium levels of consumer surplus ("CS"), producer surplus ("PS") and total surplus ("TS" $\equiv$ CS + PS) are given below. Consumer surplus is equivalent to consumers' valuation of the good, minus their money spent and transportation costs incurred, plus their aggregated reputational utility from signaling. Producer surplus is equal to firms' profits. Furthermore, total surplus is equal to producer surplus plus consumer surplus, where money spent by consumers cancels with firm revenues, representing a transfer of welfare from consumers to firms.

$$CS^* = \underbrace{v}_{\text{good value}} \underbrace{-\tau(\int_0^{n^*}(x-\ell_a^*)^2dx + \int_{n^*}^1(x-\ell_b^*)^2dx)}_{\text{transport costs}} - \underbrace{(n^*p_a^* + (1-n^*)p_b^*)}_{\text{prices}} + \underbrace{E(s(x))}_{\text{reputational surplus}}$$

$$PS^* = \underbrace{n^*p_a^* + (1-n^*)p_b^*}_{\text{price}} - \underbrace{(\frac{c}{2}\lambda_a^{*2} + \frac{c}{2}\lambda_b^{*2})}_{\text{advertising cost}}$$

$$TS^* = \underbrace{v}_{\text{good value}} \underbrace{-\tau(\int_0^{n^*}(x-\ell_a^*)^2dx + \int_{n^*}^1(x-\ell_b^*)^2dx)}_{\text{transport costs}} - \underbrace{(\frac{c}{2}\lambda_a^{*2} + \frac{c}{2}\lambda_b^{*2})}_{\text{advertising cost}} + \underbrace{E(s(x))}_{\text{reputational surplus}}$$

The consumer reputational surplus is calculated as follows.

$$\begin{aligned}
& n^*\frac{\lambda}{n^*}\int_0^{n^*} s(x)\,dx \;+\; (1-n^*)\frac{\lambda^*}{1-n^*}\int_{n^*}^1 s(x)\,dx \;+\; (1-\lambda^*)\int_0^1 s(x)\,dx \\
&= \int_0^1 s(x)\,dx \\
&= E(s(x))
\end{aligned}$$

The first term on the top line is the mass of consumers who purchase good a (n^*) multiplied by both the fraction of such consumers whose partner receives an advertisement (λ^*), and the status inference of their partners ($\frac{1}{n^*}\int_0^{n^*} s(x)\,dx$). The second term is the same calculation for consumers purchasing good b. The last term is the fraction of the public that does not receive an advertisement ($1-\lambda^*$), multiplied by their inference of a random consumer's status ($\int_0^1 s(x)\,dx$). The main thing to notice is that advertising λ^* does not affect consumer surplus from signaling. This is a result that generalizes to any social status function, firm locations, advertising, prices, or entry decisions.

As mentioned in Section I, the underlying reason is because signaling is a zero-sum game, so advertising can affect which consumer gets what portion of that reputational utility pie, but cannot affect the overall size of the pie (Frank, 2005; Heffetz and Frank, 2011). In other words, if one consumer's perceived status goes up by a certain amount, then other consumer's perceived status must go down by a proportional amount, as the average perceived status must be held constant. On aggregate, any reputational gains and losses cancel out, and advertising has zero effect on the aggregate consumer surplus from signaling.

Furthermore, since advertising is costly to producers, it negatively affects producer and total surplus. Thus, persuasive advertising is found to be wasteful, and the social planner would select zero persuasive advertising. However, this result should be judged in context, as it could be perturbed by modeling the utility of the public, and allowing for the public's utility to depend in some way on their inference of the consumer's type. For example, this might be a more natural analysis if we modeled consumers and the public as playing a mating or an employee-employer matching game, where there could be higher returns on both sides to certain types of matches.

However, perhaps more interestingly, advertising indirectly affects consumer surplus through other channels. In the cases where advertising increases prices, it decreases consumer surplus and leads to a transfer of welfare from consumers to firms. Furthermore, advertising increases the transportation costs consumers incur in a couple ways. First, given firm locations and entry decisions, advertising creates signaling motives that lead some consumers to purchase products they otherwise would not. In other words, it leads consumers to not fully respect their horizontal preferences. The further n^* is from $\frac{1}{2}$ (the market share cut-off without signaling motives), the greater is this effect. Second, it can limit entry in the conformist case, thus increasing consumers' transportation costs through this mechanism as well.

This helps bring micro foundation to the old and debated sentiment that persuasive advertising can have harmful consequences for consumers and society, by improving our understanding of the channels through which this may take place (Dixit and Norman, 1978). Indeed, while the welfare consequences from the price effects of advertising have been heavily discussed, the welfare consequences from advertising inducing consumers to not fully respect their horizontal preferences and increasing their transportation costs are less often discussed, if at all (Bagwell, 2007). It should be clarified that this paper does not try to address the welfare consequences from other forms of advertising, such as informative advertising.

VIII CONCLUSION

This paper helps give persuasive advertising a micro foundation. The model assumes consumers wish to signal some latent attribute of their identity, and the social status implied by this attribute. The paper focuses on signaling dominated by one of two consumer motives — conformist, motivated by the desire to fit in, and snobbish, motivated by the desire to stand out. The role of persuasive advertising is to facilitate consumers in signaling their social status by rendering the use of conspicuous consumption as a signaling device. The model sheds light on the effects of advertising on firms' prices, entry, locations, market shares and profits, and consumer and societal welfare in these two types of markets.

It is found that advertising in snobbish markets tends to be less combative, increasing prices and accommodating entry. In certain cases, advertising can act as a public good to firms, increasing both firms' profits. This helps make sense of the high number of designer products, as well as the supranormal prices observed in such markets. Furthermore, even when products are similar, the model shows the firm with the more prestigious position can command a greater price and market share through advertising.

In conformist markets, by contrast, the model shows that advertising can be highly combative because it heightens, rather than lessens, price competition. A first mover in a conformist market may use persuasive advertising to deter the entry of future firms and gain monopoly power, even when firms face zero production costs. This is congruent with the sometimes religious zeal of consumers to the brand which was the first mover in their region, and high concentration of market power in conformist markets.

A PROOFS

Proof of Lemma 1

A proof by contradiction is given here. Suppose not. Without loss of generality (WLOG), suppose $\ell_a < \ell_b$. Furthermore, suppose a consumer at some point $x' \in [0,1]$ purchases good B while a consumer at $x'' \in (x', 1]$ purchases good a. Let $S_a \in \Re$ and $S_b \in \Re$ denote the signaling value of choosing goods A and B respectively. For this to be equilibrium behavior, it must be that the expected utility gains to consumer x'' from good a over good b are weakly greater than that of consumer x'.

$$\begin{aligned}
&U_{x''}(a) - U_{x''}(b) \geq U_{x'}(a) - U_{x'}(b) \\
&(\ v - \tau(x'' - \ell_a)^2 - p_a + S_a\ )\ -\ (\ v - \tau(x'' - \ell_b)^2 - p_b + S_b\ ) \\
\geq &(\ v - \tau(x' - \ell_a)^2 - p_a + S_a\ )\ -\ (\ v - \tau(x' - \ell_b)^2 - p_b + S_b\ )q
\end{aligned}$$

Canceling like terms, this is equivalent to $-(x'' - \ell_a)^2 + (x'' - \ell_b)^2 \geq -(x' - \ell_a)^2 + (x' - \ell_b)^2$. Further simplifying, this is equivalent to $x'' \leq x'$. This is a contradiction.

Proof of Proposition 1

Suppose $\beta > 0$. Furthermore, WLOG, suppose $\ell_a \leq \ell_b$ and $\alpha \geq \frac{1}{2}$. Then, $\frac{dp_a^*}{d\lambda} = \alpha\frac{\beta}{3} \geq \frac{dp_b^*}{d\lambda} = (1-\alpha)\frac{\beta}{3} \geq 0$. Additionally:

$$\frac{dn^*}{d\lambda} = \frac{[\,2\tau(\ell_b - \ell_a) + \lambda\frac{\beta}{3}\,][\,\alpha\frac{\beta}{3}\,] - [\,\frac{\tau}{3}(\ell_b - \ell_a)(2 + \ell_a + \ell_b) + \lambda\alpha\frac{\beta}{3}\,][\,\frac{\beta}{3}\,]}{[\,2\tau(\ell_b - \ell_a) + \lambda\frac{\beta}{3}\,]^2}$$

The denominator is always positive. Thus, the sign of $\frac{dn^*}{d\lambda}$ is the sign of the numerator. The numerator is positive if $\ell_a + \ell_b < 6\alpha - 2$, zero if $\ell_a + \ell_b = 6\alpha - 2$, and negative if $\ell_a + \ell_b < 6\alpha - 2$. The same technique can be applied to show that $\frac{d(1-n^*)}{d\lambda} > 0$ if $\ell_a + \ell_b > 6\alpha - 2$, $\frac{dn^*}{d\lambda} = 0$ if $\ell_a + \ell_b = 6\alpha - 2$ and $\frac{d(1-n^*)}{d\lambda} < 0$ otherwise.

Proof of Proposition 2

Suppose $\beta < 0$ and neither firm takes over the market. Furthermore, WLOG, suppose $\ell_a < \ell_b$ and $\alpha \leq \frac{1}{2}$. Then, $\frac{dp_b^*}{d\lambda} = (1-\alpha)\frac{\beta}{3} \leq \frac{dp_a^*}{d\lambda} = \alpha\frac{\beta}{3} \leq 0$. The value of $\frac{dn^*}{d\lambda}$ is given in the proof of Proposition 1. The denominator of $\frac{dn^*}{d\lambda}$ is always positive. Thus, the sign of $\frac{dn^*}{d\lambda}$ is the sign of the numerator. The numerator of $\frac{dn^*}{d\lambda}$ is greater than zero if $\ell_a + \ell_b > 6\alpha - 2$, equals zero if $\ell_a + \ell_b >= \alpha - 2$ and is negative otherwise. The same technique can be applied to show that $\frac{d(1-n^*)}{d\lambda} > 0$ if $\ell_a + \ell_b < 6\alpha - 2$, $\frac{dn^*}{d\lambda} = 0$ if $\ell_a + \ell_b = 6\alpha - 2$ and $\frac{d(1-n^*)}{d\lambda} < 0$ otherwise. Finally, if $\ell_a = \ell_b$ then the market is characterized by strong conformity so the proposition does not apply.

Proof of Proposition 3

Suppose Axiom 1 holds in a strongly conformist market.

First, consider Figure 7a where the D_1 and D_2 lines lie on opposite sides of the 45 degree line. I will show that no strictly positive price vector $(p_a, p_b) > (0, 0)$ can be a price equilibrium. Recall that on the 45 degree line partitions n_a, n_b and $\hat{n}$ are possible, and on the D_1 and D_2 lines only partitions n_a and n_b are possible. Consider a strictly positive price vector to the right of 45 degree line with partition n_a. This cannot be a price equilibrium because firm B can lower its price to some point to the left of the 45 degree line and earn positive revenues. The same applies to any strictly positive price vector to the left of the 45 degree line with partition n_b, because firm A lower its price and earn positive revenues. Additionally, there is no strictly positive price vector on the D_1 line (D_2 line) with partition n_b (n_a) that can be a price equilibrium, because firm A (firm B) can raise its price by an arbitrarily small epsilon and earn positive revues. Similarly, there is no strictly positive price vector on the 45 degree line with consumer partition n_a, n_b or $\hat{n}$ that can be a price equilibrium, because one firm would have an incentive to lower its price by an arbitrarily small epsilon and earn greater revenues. Thus, we are only left with price vectors for which one or both of the firms charge a zero price. Consider a price vector such that $p_b = 0$ and $p_a > 0$. This cannot be a price equilibrium, because firm B could improve its revenues by raising its price by some arbitrarily small epsilon and earning market share n_b. The same holds mutatis mutandis for any price vector such that $p_b > 0$ and $p_a = 0$. We are left to prove that the price vector $(p_a, p_b) = (0, 0)$ can be supported as an equilibrium for any consumer partition $n^* \in \{\hat{n}, n_a, n_b\}$. At such a price vector, both firms earn zero revenues. This zero price vector can be supported as an equilibrium if consumers settle on the following partitions off the equilibrium path: consumers settle on partition n_b at the point on the D_2 line where $p_b = 0$, and consumers settle on partition n_a at the point on the D_1 line where $p_a = 0$. In this case, neither firm can improve its revenues by raising its price. Thus, $(p_a, p_b) = (0, 0)$ is a price equilibrium.

Next, consider the case of Figure 7b where the D_1 and D_2 lines lie above the 45 degree line. There is no price vector on the D_1 line or above it with partition $n = n_b$ that can be supported as a price equilibrium, because firm A could lower its price and earn positive revenues. Furthermore, no price vector on the D_2 line with $n = n_a$ can be supported as a price equilibrium, because firm B can raise its price by epsilon and capture the market. This leaves us with price vectors on the D_1 line with partition n_a, and price vectors below it. There cannot exist a price equilibrium below the D_1 line, because firm A would have an incentive to raise its price. There also cannot be a price equilibrium on the D_1 line with strictly positive prices, because one firm would have an incentive to lower its price and improve its revenues by capturing the market. This leaves only the price vector on the D_1 line such that $p_b = 0$. This can be supported as a price equilibrium if consumers settle on partition n_a, because then neither firm can improve its revenues by changing its price. The same arguments apply mutatis mutandis to the case of Figure 7c where the D_1 and D_2 lie to the right of the 45 degree line.

Proof of Proposition 4

Suppose $\beta = 0$. For convenience, I denote firm strategies as pairs (ℓ_a, λ_a) and (ℓ_b, λ_b). The price strategies firms associate with their location and advertising choices are given by equations (13) - (14). First, let's consider an equilibrium where $\ell_b^* \geq \ell_a^*$. The profit functions of firms at the location and advertising stages are:

$$\pi_a = \frac{\tau}{18}(\ell_b - \ell_a)(2 + \ell_a + \ell_b)^2 - \frac{c}{2}\lambda_a^2$$
$$\pi_b = \frac{\tau}{18}(\ell_b - \ell_a)(4 - \ell_a - \ell_b)^2 - \frac{c}{2}\lambda_b^2$$

From these profit functions, it is apparent that neither firm advertises in any equilibrium since $\frac{d\pi_a(\lambda_a,\ell_a,\lambda_b,\ell_b)}{d\lambda_a} = \frac{d\pi_b(\lambda_a,\ell_a,\lambda_b,\ell_b)}{d\lambda_b} = -c \leq 0$. Next, I examine firm locations. Suppose firm A locates at some $\ell_a \in [0,1)$. Firm B's strategy $(1,0)$ strictly dominates any other strategy $(\ell_b, 0)$ such that $\ell_b \geq \ell_a$ since $\frac{d\pi_b(\lambda_a,\ell_a,\lambda_b,\ell_b)}{d\ell_b} > 0$ for all $\ell_b \geq \ell_a$. Similarly, given that firm A anticipates firm B choosing strategy $(1,0)$, firm A's strategy $(0,0)$ strictly dominates any other strategy $(\ell_a, 0)$ such that $\ell_a \leq \ell_b$ since $\frac{d\pi_a(\lambda_a,\ell_a,\lambda_b,\ell_b)}{d\ell_a} < 0$ for all $\ell_a \leq 1$. Thus, $(\ell_a, \lambda_a) = (0,0)$ and $(\ell_b, \lambda_b) = (1,0)$ is an equilibrium. Furthermore, firms are indifferent between locating at either of opposing ends, because each earns profits $\frac{\tau}{2}$ in either case. Therefore, $(\ell_a, \lambda_a) = (1,0)$ and $(\ell_b, \lambda_b) = (0,0)$ is another equilibrium. There are no other equilibria since firms do not advertise in any equilibrium and must locate at opposite ends.

Proof of Proposition 5

Suppose $\beta > 0$ and $\alpha \in [\frac{1}{3}, \frac{2}{3}]$. I will make use of Theorem 1 in Harris (1985) to prove the existence of an equilibrium in this case. This theorem is also proved in Hellwig and Leininger (1987), and applied by Börgers (1988) in a similar game. A specialized version of the theory that is sufficient for present purposes says: Suppose a finite number of players move sequentially and choose actions from compact metric spaces. Furthermore, suppose the set of actions available to each player is independent of the actions made by other players, and each player's payoff is a continuous function of the actions of all players. Such a game has a subgame perfect equilibrium in pure strategies.

Existence would be an immediate implication of of Theorem 1 in Harris (1985) if it were not for the fact that firms' profit functions are discontinuous at $\ell_a = \ell_b$ when $\alpha \neq \frac{1}{2}$. However, existence can be established with a little more work. It is shown that in Proposition 6 that if $\alpha \in [\frac{1}{3}, \frac{2}{3}]$, then firms have strictly dominant strategies in locating at opposite ends. We can then reduce the game to two modified subgames in which firms are exogenously located at opposite ends and only choose advertising levels $\lambda_a \in [0,1]$ and $\lambda_b \in [0,1]$, and a larger game in which firm A makes a binary choice between the two ends $\ell_a = 0$ and $\ell_a = 1$ given the equilibria of the two advertising subgames. The price strategies firms associate with their location and advertising choices are then given by equations (13) - (14).

WLOG, consider the subgame in which $\ell_a = 0$ and $\ell_b = 1$. Firms' profit functions are then:

$$\pi_a(\lambda_a, \lambda_b) = \frac{(\ \tau + \lambda\alpha\frac{\beta}{3}\ )^2}{2\tau + \lambda\frac{\beta}{3}} - \frac{c}{2}\lambda_a^2 \tag{28}$$

$$\pi_b(\lambda_a, \lambda_b) = \frac{(\ \tau + \lambda(1-\alpha)\frac{\beta}{3}\ )^2}{2\tau + \lambda\frac{\beta}{3}} - \frac{c}{2}\lambda_b^2 \tag{29}$$

Since firms' profit functions are continuous in (λ_a, λ_b), and firms choose sequentially from compact and independent action spaces, Theorem 1 in Harris (1985) guarantees the existence of an equilibrium in this modified subgame. Existence is similarly assured in the modified subgame in which firm locations are $\ell_a = 1$ and $\ell_b = 0$. Since there exists an equilibrium in each of the two modified subgames, there also exists a solution to firm A's binary location choice between the two ends. Therefore, there exists an equilibrium of the overall game.

Proof of Proposition 6

Suppose $\beta > 0$.

(1): Proven in the main text.

(2): I will prove that in any equilibrium at least one firm advertises by contradiction. Suppose neither firm advertises in equilibrium. In this case, Proposition 4 shows that firms must locate at opposite ends. WLOG, consider the case of $\ell_a = 0$ and $\ell_b = 1$. Firms profit functions at the location and advertising stages are then given by equations (28) - (29). For this to be an equilibrium, it must be that $\frac{d\pi_a}{d\lambda_a} \leq 0$ when $(\lambda_a, \lambda_b) = (0,0)$. The derivative of equation (28) with respect to λ_a when $(\lambda_a, \lambda_b) = (0,0)$ is:

$$\frac{d\pi_a}{d\ell_a} = \frac{2\alpha\frac{\beta}{3}(\tau)(2\tau) - (\tau)^2\frac{\beta}{3}}{4\tau^2}$$

For this derivative to be weakly negative, it must be that $\alpha \leq \frac{1}{4}$. It must also be that $\frac{d\pi_b}{d\lambda_b} \leq 0$ when $(\lambda_a, \lambda_b) = (0,0)$. Taking the derivative of equation (29) with respect to λ_b when $(\lambda_a, \lambda_b) = (0,0)$, I find that α must be greater than or equal to $\frac{3}{4}$ for it to be weakly negative. This is a contradiction, because it cannot be that $\alpha \leq \frac{1}{4}$ and $\alpha \geq \frac{3}{4}$.

(3): I will show that if $\alpha \leq \frac{2}{3}$, then the firm on the left in equilibrium must locate at 0. Similarly, I will show that if $\alpha \geq \frac{1}{3}$, then the firm on the right in equilibrium must locate at 1. This implies that if $\alpha \in [\frac{1}{3}, \frac{2}{3}]$, then firms locate at opposite ends. Furthermore, since the firm on the left has the less prestigious position if $\alpha \leq \frac{1}{3}$, and the firm on the right has the less prestigious position if $\alpha \geq \frac{2}{3}$, this also implies that the firm with the less prestigious position locates at an end for any $\alpha \in [0,1]$.

WLOG, consider equilibria where firm A locates to the left of firm B, $\ell_a < \ell_b$. In this case, firm A and B's profits at the location and advertising stages are given by equations (26) and (27) respectively. I will show that for any given any $\alpha \leq \frac{2}{3}$ and λ, $\ell_a = 0$ strictly dominates any other $\ell_a < \ell_b$. I do this by showing that $\frac{d\pi_a(\ell_a,\ell_b,\lambda)}{d\ell_a} \leq 0$ at $\ell_a = 0$. Since profits are concave in locations, this is sufficient to show that ℓ_a strictly dominates any other $\ell_a < \ell_b$. The derivative of equation (26) with respect to ℓ_a when $\ell_a = 0$ is:

$$\frac{d\pi_a}{d\ell_a} = \frac{-\frac{4\tau}{3}\ [\ \frac{\tau}{3}\ell_b(2+\ell_b) + \lambda\alpha\frac{\beta}{3}\ ]\ [\ 2\tau\ell_b + \lambda\frac{\beta}{3}\ ] + 2\tau\ [\ \frac{\tau}{3}\ell_b(2+\ell_b) + \lambda\alpha\frac{\beta}{3}\ ]^2}{[\ 2\tau\ell_b + \lambda\frac{\beta}{3}\ ]^2}$$

The denominator is always positive. Thus, the sign of the derivative is given by the sign of the numerator. Setting the numerator less than or equal to zero and simplifying yields the inequality $4\tau\ell_b + \frac{2}{3}\lambda\beta \geq \tau\ell_b(2+\ell_b) + \lambda\alpha\beta$. It is apparent that i) $4\tau\ell_b > \tau\ell_b(2+\ell_b)$ and ii) $\frac{2}{3}\lambda\beta \geq \lambda\alpha\beta$ when $\alpha \leq \frac{2}{3}$. Thus, if $\alpha \leq \frac{2}{3}$, then firm A must locate at 0 if it locates to the left of firm B. Applying the same procedure, one can show that if $\alpha \geq \frac{1}{3}$, then $\frac{d\pi_b(\ell_a,\ell_b,\lambda)}{d\ell_b} \geq 0$ at $\ell_b = 1$ for any λ. Thus, if $\alpha \geq \frac{1}{3}$, then firm B's most preferred location to the right of firm A is at $\ell_b = 1$. This analysis applies equally to the case of $\ell_b < \ell_a$. Finally, note that firm B never locates at $\ell_b = \ell_a$, because its profits are always strictly higher to either the left or right of ℓ_a.

(4): From (3), we know that the firm with the less prestiguous position locates at an end. Without loss of generality, suppose $\alpha > \frac{1}{2}$ and firm B has the less prestigious position in equilibrium. It must then be that firm B locates at $\ell_b = 1$. Plugging $\ell_b = 1$ into equations (13) and (14), this implies that $p_a = \frac{\tau}{3}(1-\ell_a)(3+\ell_a) + \lambda\alpha\frac{\beta}{3} > \frac{\tau}{3}(1-\ell_a)(3-\ell_a) + \lambda(1-\alpha)\frac{\beta}{3} = p_b$. Additionally, plugging $\ell_b = 1$ into equation (15), we find $n = \frac{p_a}{2\tau(1-\ell_a)+\lambda\frac{\beta}{3}} > \frac{p_b}{2\tau(1-\ell_a)+\lambda\frac{\beta}{3}} = 1-n$ where p_a and p_b are as defined above.

(5): If $\alpha = \frac{1}{2}$, then (3) shows that firms locate at opposite ends. Furthermore, firm A and firm B's profits functions are symmetric at any $(\ell_a, \ell_b) \in \{(0,1),(1,0)\}$. Thus, firms are indifferent between the two end pairs. Incorporating their location decisions, firm A and B's profits as a function of advertising are:

$$\pi_a(\lambda_a, \lambda_b) = \frac{1}{2}(\tau + \lambda\frac{\beta}{6}) - \frac{c}{2}\lambda_a^2$$

$$\pi_b(\lambda_a, \lambda_b) = \frac{1}{2}(\tau + \lambda\frac{\beta}{6}) - \frac{c}{2}\lambda_b^2$$

where $p_a = p_b = \tau + \lambda\frac{\beta}{6}$ and $n = \frac{1}{2}$. Starting backwards by solving firm B's program yields with respect to λ_b yields:

$$\lambda_b = \max\{(1-\lambda_a)\frac{\beta}{12c}, 1\}$$

Plugging firm B's advertising response function into firm A's program and solving for λ_a yields:

$$\lambda_a^* = \begin{cases} \frac{\beta - \frac{\beta^2}{6c}}{12c - \frac{\beta^2}{6c}} & \text{if } \beta < 6c \\ 0 & \text{otherwise} \end{cases}$$

It can be checked that $\lambda_a^* \leq \lambda_b^*$.

Proof of Proposition 7

Suppose $\beta < 0$ and $\alpha \in [\frac{1}{3}, \frac{2}{3}]$. As shown in the proof of Proposition 8, firm A's strategy at $t = 0$ can be broken down into deterring firm B's entry and choosing an action $(\ell_a, \lambda_a) \in \triangle$ where $\triangle$ is a compact subset of $[0,1]^2$, or choosing an action $(\ell_a, \lambda_a) \in [0,1]^2 \setminus \triangle$ and accommodating firm B's entry. If firm A deters firm B's entry, then it earns monopoly profits. If firm A accommodates firm B's entry, then it earns oligopoly profits as described in the main text. $\triangle$ is non-empty if β is sufficiently negative.

First, suppose $\triangle$ is empty. Firm B must then enter in any equilibrium, implying firms profit functions are defined by the pricing equilibrium in the oligopoly case. Furthermore, firms' action spaces can be reduced to that over independent and compact sets $[0,1]^2$. If firms' profit functions are continuous, then since firms choose sequentially from compact and independent action spaces, we can apply Theorem 1 in Harris (1985) to guarantee the existence of an equilibrium. Note that if $\alpha \in [\frac{1}{3}, \frac{2}{3}]$, then firms' profits functions are continuous at $\ell_a = \ell_b$. As can be seen graphically in Figure 7, both firms' profits are 0 when ℓ_a and ℓ_b are within an arbitrary small neighborhood of each other. Firms' profit functions are then everywhere continuous in their locations and advertising levels, thus guaranteeing the existence of an equilibrium.

Next, consider the case in which $\triangle$ is non-empty. At $t = 0$, firm A must decide between choosing a strategy $(\ell_a, \lambda_a) \in \triangle$ and earning monopoly profits, or choosing $(\ell_a, \lambda_a) \in [0,1]^2 \setminus \triangle$ and earning the oligopoly profits implied by the subgame in which firm B enters.

We can prove the existence of an equilibrium in each of the two subgames. First, the existence of an equilibrium in the subgame in which firm A chooses a strategy $(\ell_a, \lambda_a) \in \triangle$ is shown in the proof of Proposition 8.

Next, we can consider a modified version of the second subgame in which firm B enters, and firm A chooses from any (ℓ_a, λ_a) in $[0,1]^2$ rather than (ℓ_a, λ_a) in the open set $[0,1]^2 \setminus \triangle$. Since firms' strategy spaces are compact in this modified subgame, we can apply Theorem 1 in Harris (1985) to guarantee the existence of an equilibrium it. Let (ℓ_a', λ_a') denote firm A's strategy in an equilibrium of this modified subgame.

Suppose $(\ell_a', \lambda_a') \in \triangle$. In this case, there must be an equilibrium of the original game in which firm B does not enter, because firm A would choose a strategy that prevents it from making positive profits regardless of firm B's entry decision, and, furthermore, firm A would make larger profits if firm B did not enter for any given (ℓ_a', λ_a'). This last result holds because firm A can commands a higher price for any given market share when it is a monopolist, as it is free from price competition.

Suppose instead $(\ell_a', \lambda_a') \in [0,1]^2 \setminus \triangle$. Then, there must again exist an equilibrium of the original larger game, as the game can be reduced to the binary decision of firm A over choosing between $(\ell_a, \lambda_a) \in \triangle$ and (ℓ_a', λ_a').

Proof of Proposition 8

I present the proof of the proposition here, utilizing lemmas that are proved in the subsections below. One word on notation before proceeding. Since we have already solved for firm pricing decisions in the main text, I refer to firm A and B's strategy as pairs (ℓ_a, λ_a) and (ℓ_b, λ_b) respectively. Their associated pricing strategies are implied by the analysis in the main text.

Suppose $\beta < 0$ and $\alpha \in [\frac{1}{3}, \frac{2}{3}]$. First, Lemma 2 establishes the existence of a strategy space for firm A, $\triangle \subset [0,1]^2$, in which firm A deters firm B's entry.

Lemma 2. *If β is sufficiently negative, and firm B does not enter when doing so implies non-positive profits, then there exists a non-empty, compact space $\triangle \subset [0,1]^2$ such that firm B does not enter the market if firm A chooses any strategy $(\ell_a, \lambda_a) \in \triangle$.*

Given a sufficiently negative β, firm A's decision can be broken down in that of choosing $(\ell_a, \lambda_a) \in \triangle$ and deterring firm B's entry, or $(\ell_a, \lambda_a) \in [0,1]^2 \setminus \triangle$ and accommodating firm B's entry. An equilibrium in which firm A does not accommodate firm B's entry is given by the solution to:

$$\begin{aligned} \max_{(\ell_a, \lambda_a)} \quad & \pi_a^M(\ell_a, \lambda_a) \\ \text{subject to} \quad & (\ell_a, \lambda_a) \in \triangle \end{aligned}$$

where π_a^M is as defined in equation (25). Since the objective function is upper semi-continuous, and the constraint set is compact, the extreme value theorem guarantees a solution to this program when $\triangle$ is non-empty. Next, let's solve for firm A's profits in an equilibrium in which it accommodates firm B's entry.

Lemma 3. *Suppose $\beta < 0$ and $\alpha \in [\frac{1}{3}, \frac{2}{3}]$. In any equilibrium in which $(\ell_a^*, \lambda_a^*) \in [0,1]^2 \setminus \triangle$, both firms locate at opposite ends and do not advertise, thus earning profits $\frac{\tau}{2}$.*

This implies that if $\triangle$ is empty, then the equilibrium is as described in Lemma 3. If instead $\triangle$ is non-empty, then firm A then has to decide whether to deter firm B's entry. Firm A deters firm B's entry if its profits from doing so are higher than its profits from not deterring firm B's entry, $\frac{\tau}{2}$.

Firm A's profits from not deterring firm B's entry, $\frac{\tau}{2}$ are independent of c. However, by the envelope theorem, we know that firm A's profits from deterring firm B's entry are monotonically decreasing in c. Furthermore, the next lemma establishes that at $c = 0$ firm A's profits from deterring firm B's entry are greater than that from accommodating firm B's entry.

Lemma 4. *Suppose $c = 0$ and $\triangle$ is non-empty. Firm A's optimal profits from deterring firm B's entry are greater than its highest profits from accommodating firm B's entry, $\frac{\tau}{2}$.*

Moreover, using the envelope theorem, we see that as $c \to \infty$, firm A's profits from deterring firm B's entry approach $-\infty$ as it must always advertise above some lower bound. Thus, there must exist some unique $\underline{c} > 0$ such that if $c \leq \underline{c}$, then firm A deters firm B's entry, and if $c > \underline{c}$, then firm A accommodates firm B's entry.

Proof of Lemma 2

Definition. *Suppose $\beta < 0$. Let $\triangle \subset [0,1]^2$ be the set of $(\ell_a, \lambda_a) \in [0,1]^2$ satisfying the inequalities below*

$$\lambda \geq \frac{-\tau(1-\ell_a)}{\beta} \min\{\frac{3+\ell_a}{\alpha}, \frac{3-\ell_a}{1-\alpha}\} \tag{30}$$

$$\lambda \geq \frac{-\tau\ell_a}{\beta} \min\{\frac{2+\ell_a}{\alpha}, \frac{4-\ell_a}{1-\alpha}\} \tag{31}$$

$$\lambda \geq \frac{-\tau(1-\ell_a)^2}{\beta(\frac{2}{3}-\alpha)} \tag{32}$$

$$\lambda \geq \frac{-\tau\ell_a^2}{\beta(\alpha-\frac{1}{3})} \tag{33}$$

$\triangle$ defines the set of firm A strategies (ℓ_a, λ_a) that prevent firm B from earning positive profits at any $(\ell_b, \lambda_b) \in [0,1]^2$. The significance of the inequalities is as follows. Inequalities (30) and (31) ensure that one of the firms takes over when firm B chooses $(1,0)$ and $(0,0)$ respectively. It can be seen in Figure 7 that if $\ell_a \leq \ell_b$ ($\ell_a > \ell_b$) and some firm takes over when firm B chooses $(1,0)$ ($(0,0)$) so that the distance $D_1 - D_2$ is as large as possible, then some firm takes over for all other $(\ell_b, \lambda_b) \in [0,1]^2$.

Inequalities (32) and (33) ensure that firm B earns non-positive profits when choosing $(1,0)$ and $(0,0)$ respectively. It can be seen in Figure 7 that if $\ell_a \leq \ell_b$ ($\ell_a > \ell_b$) and the D_2 line intercepts the p_a axis when firm B chooses $(1,0)$ (the D_1 line intercepts the p_a axis when firm B chooses $(0,0)$), then firm B does not earn positive profits for any other (ℓ_b, λ_b) in which takeover occurs.

Furthermore, since each of the inequalities forms a compact set in $[0,1]^2$, $\triangle$ must be compact, as the intersection of compact sets is compact. Moreover, each of the inequalities becomes increasingly slack as β becomes increasingly negative. If β is sufficiently close to zero from below, then there are no $(\ell_a, \lambda_a) \in [0,1]^2$ that satisfy all of the inequalities and $\triangle$ is empty. However, as $\beta \to \infty$, then $\triangle \to [0,1]^2$. In other words, if β is sufficiently negative, then $\triangle$ is non-empty.

Proof of Lemma 3

Suppose $\beta < 0$ and $\alpha \in [\frac{1}{3}, \frac{2}{3}]$. Consider an equilibrium in which $(\ell_a^*, \lambda_a^*) \in [0,1]^2 \setminus \triangle$. In any such equilibrium, firm B enters the market because it can earn positive profits from entering. We want to show that firms locate at opposite ends and do not advertise, thus each earning profits $\frac{\tau}{2}$.

We know that in such an equilibrium both firms earn positive profits. Firm B must earn positive profits, because otherwise it would not have entered. Furthermore, firm A must earn positive profits, as it has a zero-cost strategy available that guarantees firm B does not take over. For example, if firm A chooses $(3\alpha-1, 0)$ (for the general case of $\alpha \in [0,1]$, firm A could choose $(\,\min\{\max\{3\alpha-1,0\},1\},\,0\,)$), then firm B does not have a strategy (ℓ_b, λ_b) available in which it takes over the market. If $\ell_b \leq \ell_a^*$, then $\ell_a^* + \ell_b \leq 6\alpha - 2$ and firm B does not take over. If $\ell_b > \ell_a^*$, then $\ell_a^* + \ell_b > 6\alpha - 2$ and firm B again does not take over. Thus, both firms must earn positive profits in any equilibrium in which firm B enters.

Furthermore, Claim 1 shows that in such an equilibrium firm B must locate at an end.

Claim 1. *In any equilibrium in which $\beta < 0$, $\alpha \in [\frac{1}{3}, \frac{2}{3}]$ and both firms earn positive profits, firm B locates at an end.*

Proof of Claim 1. Supppose $\beta < 0$, $\alpha \in [\frac{1}{3}, \frac{2}{3}]$ and both firms earn positive profits in equilibrium. Without loss of generality, suppose $\ell_a^* \in (0, 1]$, and firm B considers $\ell_b \leq \ell_a^*$. I will show that for any given level of advertising λ, $\ell_b = 0$ strictly dominates any other ℓ_b such that $\ell_b \leq \ell_a^*$ and $\ell_b \neq 0$.

Holding $\lambda\beta < 0$ fixed, let's consider firm B's location decision. We need only consider ℓ_b such that both firms earn positive profits, because it cannot be a best response for firm B to take over. Both firms earn positive revenues for any $\ell_b \in [0, \ell_b^{wc})$ where $\ell_b^{wc} \in [0, \ell_a)$ is defined by setting inequality (16) to an equality constraint. In other words, ℓ_b^{wc} is defined implicitly by $-\lambda\beta = \tau(\ell_a - \ell_b^{wc}) \min\{ \frac{2+\ell_a+\ell_b^{wc}}{\alpha}, \frac{4-\ell_a-\ell_b^{wc}}{1-\alpha} \}$. Firm B's profits at any $\ell_b \in [0, \ell_b^{wc})$ are given by:

$$\pi_b(\ell_b) = \frac{(\frac{\tau}{3}(\ell_a - \ell_b)(2 + \ell_a + \ell_b) + \lambda\alpha\frac{\beta}{3})^2}{2\tau(\ell_a - \ell_b) + \lambda\frac{\beta}{3}} - \frac{c}{2}\lambda_b^2 \tag{34}$$

as shown in equations (13) - (15). Firm B's profits for $\ell_b >\in [\ell_b^w c, \ell_a]$ are not given by equation (34), and can be found in the main text. I will show that there exists no local max in the interior of the considered domain $\ell_b \in (0, \ell_b^{wc})$. Thus, the function defined in equation (34) must either achieve a global maximum at $\ell_b = 0$ or have no global maximum in the domain $[0, \ell_b^{wc})$. Furthermore, since firm B prefers to not take over and its actual profits are continuous at ℓ_b^{wc} (see proof of Proposition 7), firm B's profits at $\ell_b = 0$ must be greater than at ℓ_b^{wc}. This then implies that $\ell_b = 0$ is a global max of firm B's profits over $\ell_b \in [0, \ell_b^{wc})$.

Equation (34) has four first order conditions:

(A) $\ell_b = -1 - \frac{1}{\tau}\sqrt{\alpha\lambda\beta\tau + \tau^2 + 2\ell_a\tau^2 + \ell_a^2\tau^2}$

(B) $\ell_b = -1 + \frac{1}{\tau}\sqrt{\alpha\lambda\beta\tau + \tau^2 + 2\ell_a\tau^2 + \ell_a^2\tau^2}$

(C) $\ell_b = \frac{(2\lambda\beta - 6\tau + 12\ell_a\tau) - \sqrt{(2\lambda\beta - 6\tau + 12\ell_a\tau)^2 + 36\tau(2\lambda\beta - 3\alpha\beta\lambda + 6\ell_a\tau - 3\ell_a^2\tau)}}{18\tau}$

(D) $\ell_b = \frac{(2\lambda\beta - 6\tau + 12\ell_a\tau) + \sqrt{(2\lambda\beta - 6\tau + 12\ell_a\tau)^2 + 36\tau(2\lambda\beta - 3\alpha\beta\lambda + 6\ell_a\tau - 3\ell_a^2\tau)}}{18\tau}$

I will one by one eliminate each first order condition as a contender for a local maximum in $(0, \ell_b^{wc})$. First, (A) is clearly negative (if real), and thus outside the considered domain.

Furthermore, I will show that if $\alpha \in [\frac{1}{3}, \frac{2}{3}]$, then (C) must be non-positive (if real) and thus outside the considered domain. If the first parenthetical term $2\lambda\beta - 6\tau + 12\ell_a\tau \leq 0$, then (C) is clearly non-positive (if real) because the term in the square root must be non-negative. Suppose instead that $2\lambda\beta - 6\tau + 12\ell_a\tau > 0$. If we can show that $36\tau(2\lambda\beta - 3\alpha\beta\lambda + 6\ell_a\tau - 3\ell_a^2\tau) > 0$, then it follows that (C) is non-positive because the term in the square root must be bigger than $2\lambda\beta - 6\tau + 12\ell_a\tau$. Rearranging terms, $2\lambda\beta - 6\tau + 12\ell_a\tau > 0$ is equivalent to $\lambda\beta > 3\tau - 6\ell_a\tau$ and $36\tau(2\lambda\beta - 3\alpha\beta\lambda + 6\ell_a\tau - 3\ell_a^2\tau) > 0$ is equivalent to $\lambda\beta > \frac{3\ell_a\tau(\ell_a-2)}{2-3\alpha}$.

Furthermore, $3\tau - 6\ell_a\tau \geq \frac{3\ell_a\tau(\ell_a-2)}{2-3\alpha}$ for all $\alpha \in [\frac{1}{3}, \frac{2}{3}]$, $\ell_a \in [0,1]$ and $\tau \geq 0$. Thus, (C) must be non-positive (if real) and outside the considered domain.

Since (A) and (C) are outside the considered domain, we need only consider the two remaining critical points (B) and (D), which may or may not be within the considered domain.

There is a clear discontinuity of equation (34) at $\ell_b^{sc} = \ell_a + \frac{\lambda\beta}{6\tau} \geq \ell_b^{wc}$ where the denominator equals zero. By applying L'Hôpital's rule, we see that $\lim_{\ell_b \to \ell_b^{sc+}} \pi_b = -\infty$ and $\lim_{\ell_b \to \infty} \pi_b = -\infty$ where π_b is defined by equation (34). Thus, one of the two remaining first order conditions must be a local max to the right of ℓ_b^{sc}, and outside the considered domain.

We are then left with one potential critical point in $(0, \ell_b^{wc})$, corresponding to the last remaining first order condition (either (B) or (D)). However, I will show that if this first order condition point is in $(0, \ell_b^{wc})$, then it cannot be a local max. One way to do this would be analytically using the second order conditions. However, this is quite tricky given the complexity of the derivatives and critical points, and ambiguity about whether (B) or (D) is the relevant critical point. An easier path is to again apply L'Hôpital's rule and see that $\lim_{\ell_b \to \ell_b^{sc-}} \pi_b = \infty$ where π_b is defined by equation (34). Thus, if there is a first order condition in $[0, \ell_b^{sc}]$, which $[0, \ell_b^{wc})$ is contained within, then it must be either a saddle point or a local min.

Thus, in any equilibrium in which neither firm takes over $\ell_b^* \leq \ell_a^*$, $\ell_b^* = 0$. Furthermore, mutatis mutandis, in any equilibrium in which neither firm takes over and $\ell_b^* > \ell_a^*$, $\ell_b^* = 1$. ■

Next, let's look at the location decision of firm A. Since firm A also does not take over the market in equilibrium, and firm A has the same profit function as firm B, firm A's first-best is likewise to locate at an end. However, firm A cannot necessarily choose its first-best location, because it must also prevent firm B from taking over.

I will show that if firm A locates at the end with the prestige advantage, then firm B does not take over. First, I show that if firms locate at opposite ends, then neither firm advertises.

Claim 2. *Suppose $\beta < 0$ and $\alpha \in [\frac{1}{3}, \frac{2}{3}]$. Neither firm advertises in any equilibrium in which firms locate at opposite ends and both earn positive profits.*

Proof of Claim 2. WLOG, suppose $\alpha \leq \frac{1}{2}$, $\ell_a = 0$ and $\ell_b = 1$. Both firms earn positive profits when $\lambda \in [0, \frac{-3\tau}{\beta(1-\alpha)})$. The right bound of this domain can be found by setting inequality (16) to an equality constraint. Plugging in firms locations, firm A's revenues as a function of λ over the aforementioned domain are $\frac{(\tau+\lambda\frac{\alpha\beta}{3})^2}{2\tau+\lambda\frac{\beta}{3}}$. The second derivative of firm A's revenues with respect to λ is positive in the considered domain. Thus firm A's revenues are convex in λ in this region.

Furthermore, firm A's revenues are higher at $\lambda = 0$ then at the right closure of the relevant domain, $\lambda = \frac{-3\tau}{\beta(1-\alpha)}$. Firm A's revenues at $\lambda = 0$ are $\frac{\tau}{2}$. Firm A's revenues at $\lambda = \frac{-3\tau}{\beta(1-\alpha)}$ are $\frac{\tau(1-2\alpha)}{1-\alpha)}$, and $\frac{\tau}{2} \geq \frac{\tau(1-2\alpha)}{(1-\alpha)}$ if $\alpha \geq \frac{1}{3}$. Since firm A's revenues are highest at $\lambda = 0$, and firm A's costs are strictly increasing in λ_a, firm A's profits are maximized at $\lambda_a = 0$.

Moreover, firm B's revenues as a function of advertising over the aforementioned domain are $\frac{(\tau+\lambda\frac{(1-\alpha)\beta}{3})^2}{2\tau+\lambda\frac{\beta}{3}}$. The first derivative of firm B's revenues with respect to λ are negative for any $\alpha \leq \frac{1}{2}$. Thus, firm B also does not advertise. ■

Now we can show that firm B does not take over when firm A locates at the end with the prestige advantage and does not advertise.

Claim 3. *Suppose $\beta < 0$. If firm A locates at the end with the prestige advantage and does not advertise, then firm B does not take over.*

Proof of Claim 3. Suppose $\beta < 0$. WLOG, suppose $\alpha \le \frac{1}{2}$ and firm A chooses $(0, 0)$. If firm B were to take over, its highest revenues would be when the D_1 line and D_2 lines are equal, as seen in Figures 6 and 7. This occurs when $\ell_b = \frac{-\lambda\beta}{6}$. Plugging this into firm B's profits from takeover, firm B's optimal takeover strategy can be found by solving:

$$\begin{aligned} \max_{\lambda_b} \quad & \tau\left(\frac{-\lambda_b\beta}{6\tau}\right)\left(2 + \frac{\lambda_b\beta}{6\tau}\right) - \lambda_b\beta\left(\alpha - \frac{2}{3}\right) - \frac{c}{2}\lambda_b^2 \\ & \text{subject to } \lambda_b \in [0, 1] \end{aligned}$$

This program is concave in λ_b, and has solution: $\lambda_b = \frac{6\tau\beta(1-3\alpha)}{\beta^2+18c\tau} \in (0, 1)$ and $\ell_b = \frac{\beta^2(3\alpha-1)}{\beta^2+18c\tau} \in [0, 1)$. Furthermore, by the envelope theorem, firm B's optimized profits from takeover are strictly decreasing in c. Thus, if firm B does not takeover at $c = 0$, then it does not takeover at any $c > 0$.

Indeed, firm B's profits at $c = 0$ are $\tau(1 - 3\alpha)^2$. Firm B's profits from strategy $(1, 0)$ are $\frac{\tau}{2}$, and $\frac{\tau}{2} \ge \tau(1 - 3\alpha)^2$ for any $\alpha \in [\frac{1}{3} - \frac{1}{3\sqrt{2}}, \frac{1}{3} + \frac{1}{3\sqrt{2}}]$, which has a right bound greater than $\frac{1}{2}$. Furthermore, if $\alpha \le \frac{1}{3}$ and firm A chooses $(0, 0)$, then there does not exist a (ℓ_b, λ_b) where firm B takes over. Thus, firm B does not take over. ∎

Proof of Lemma 4

Suppose $\beta < 0$, $c = 0$ and $\triangle$ is non-empty. Without loss of generality, suppose $\alpha \le \frac{1}{2}$. It can be seen analytically by inequalities (30) - (32) that there must then exist some $(\ell_a', \lambda_a') \in \triangle$ with $\ell_a' \le \frac{1}{2}$. Firm A's monopoly profits at (ℓ_a', λ_a') are:

$$\pi_a^M(\ell_a', \lambda_a') = v - \tau(1 - \ell_a')^2 + \lambda_a'\beta\left(\alpha - \frac{2}{3}\right)$$

as shown in equation (25). Since $\alpha \le \frac{1}{2}$ and $v > 3\tau$ (see Section V), $\pi_a^M(\ell_a', \lambda_a') > \frac{\tau}{2}$. Thus, firm A earns greater profits than it would if it accommodates firm B's entry. Finally, since firm A's profits at its optimal entry deterring strategy must be weakly greater than that at (ℓ_a', λ_a'), it follows that firm A prefers to deter firm B's entry.

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