

Operational Optimization of an Agricultural Microgrid

Paul Brown
2463461
paul.brown@metu.edu.tr

Abstract—A demonstration agricultural microgrid containing PV, battery energy storage (BSS) and multiple water pumps is presented. A mathematical model of the cost of operating the microgrid is developed, including modeling of battery charging modes and hybrid inverter source selection. The mathematical model is implemented in Python using Pyomo and optimized to plan pumping and water usage. The model may serve as the basis for model predictive control (MPC) or stochastic model predictive control (SMPC).

I. INTRODUCTION

THIS is the introduction. This topic is about something really important. I should probably include a lot of references here. This is the place to show some literature review.

II. DEMONSTRATION SYSTEM

This paper presents an original mathematical formulation of the operational optimization problem for model-predictive control of an agricultural microgrid. The basic problem is to formulate an optimization problem such that the microgrid can be controlled using a model-predictive controller in the EMS. The optimization problem is specifically formulated for a demonstration system to be installed at the Güneşköy farm[1]. A block diagram of the demonstration system is shown in Figure 1.

The existing pump will continue to be fed from the grid. The site electrical load will be fed from a hybrid inverter[2] that will have battery, PV, and grid connections available. A new pump load will be fed from an inverter drive[3] connected to the PV DC bus.

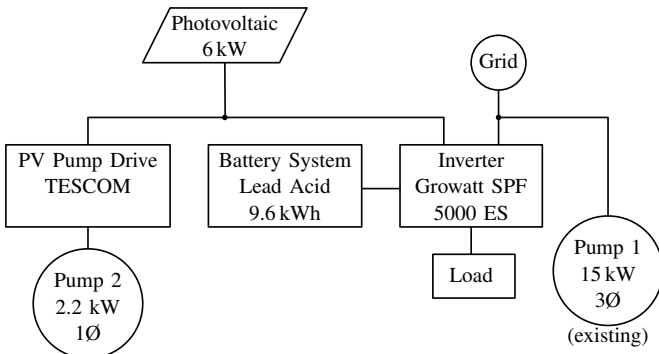


Figure 1. Güneşköy Demonstration System

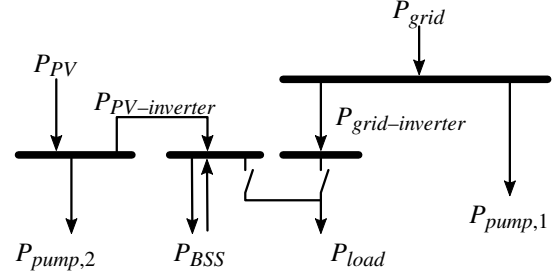


Figure 2. Microgrid power flows

The power flows within the demonstration system are shown in Figure 2.

III. PROBLEM FORMULATION

Table I shows problem variables that are to be determined by the optimization process. Table II shows quantities that are calculated based on the problem equations.

A. Problem variables

The problem variables are shown in Table I. The optimization determines the best times to run Pump 1, when and how much to run Pump 2, and when to utilize water for irrigation.

Table I
PROBLEM VARIABLES

Symbol	Description	Units
$s_{pump1,t}$	Pump 1 on-off selection variable ²	-
$P_{pump2,t}$	Power used for pumping water to Reservoir 2	W
$Q_{use1,t}$	Flow of water used (irrigation or other use) from Reservoir 1	m ³ /h
$Q_{use2,t}$	Flow of water used (irrigation or other use) from Reservoir 2	m ³ /h

¹ Variables subscripted with t have a value for each time period

² Binary variable. 1: Running. 0: Off.

B. Objective function

The objective function includes several components and is shown in Equation 1. The actual cost component is cost of grid power. The cost of battery usage component represents the portion of the replacement cost of the battery system incurred due to the loss of life caused by battery cycling[4]. The other penalty factors encourage the full supply of desired water on

Table II
CALCULATED QUANTITIES

Symbol	Description	Units
$P_{pump1,t}$	Power used by Pump 1 for pumping water to Reservoir 1	W
$Q_{pump1,t}$	Volume of water pumped by Pump 1	m ³
$Q_{pump2,t}$	Volume of water pumped by Pump 2	m ³
$s_{BSS,t}$	Operating mode of BSS ³	-
$P_{BSS,ch,t}$	Power used to charge the BSS	W
$P_{BSS,disch,t}$	Power drawn from discharging the BSS	W
$s_{inv,t}$	Inverter mode ⁴	-
$P_{PV,t}$	Power drawn from the PV array	W
$P_{PV-inverter,t}$	Power flow from the PV bus to the inverter	W
$P_{grid,t}$	Power drawn from the electrical grid or load not served	W
$P_{grid-inverter,t}$	Power drawn from the grid to feed the hybrid inverter	W
$E_{BSS,t}$	Energy stored in the BSS at the end of the period	Wh
$V_{w1,t}$	Volume of water stored in the Reservoir 1 at the end of the period	m ³
$V_{w2,t}$	Volume of water stored in the Reservoir 2 at the end of the period	m ³
$V_{use,d}$	Effectively used volume of water (irrigation or other use) on day d	m ³

¹ Variables subscripted with t have a value for each time period

² Binary variable. 1: Running. 0: Off.

³ Binary variable. 1: Charging. 0: Discharging.

⁴ Binary variable. 1: Inverter fed from utility source. 0: Inverter fed from BSS/PV source.

each day and discourage unnecessary switching of the BSS mode and Pump 1 respectively.

$$\begin{aligned}
 \min & \underbrace{\sum_t C_{grid,t} P_{grid,t} \Delta t}_{\text{Cost of grid power}} + \underbrace{\sum_t C_{BSS} (P_{BSS,ch,t} + P_{BSS,disch,t}) \Delta t}_{\text{Cost of battery usage}} \\
 & + \underbrace{C_{w,short} \sum_d \max((V_{use,desired,d} - V_{use,d}), 0)}_{\text{Penalty for inadequate water}} \\
 & + \underbrace{C_{BSS,switching} \sum_t |s_{BSS,t} - s_{BSS,t-1}|}_{\text{BSS mode-switching penalty}} \\
 & + \underbrace{C_{Pump1,switching} \sum_t |s_{pump1,t} - s_{pump1,t-1}|}_{\text{Pump 1 switching penalty}}
 \end{aligned} \quad (1)$$

C. Constraints

The constraints represent the mathematical representation of the operation and allowable states of the microgrid system. The constraints related to each component of the microgrid are presented in the following subsections.

1) *Power Balance*: There is a power balance constraint equation for each of the four buses shown in Figure 2. Equation 2 enforces balance on the PV bus, Equation 3 enforces balance in the hybrid inverter PV/BSS side, Equation 4 enforces balance in the hybrid inverter grid side, and Equation 5 enforces

Table III
PROBLEM PARAMETERS / DATA

Symbol	Description	Units
$P_{load,t}$	Power drawn by electrical loads	W
$P_{PV,avail,t}$	Power available from PV array (MPP)	W
$P_{pump1,max}$	Operating power of Pump 1	W
$s_{pump1,0}$	Initial state of Pump 1	-
Q_{w1}	Fixed water flow for Pump 1	m ³ /h
$P_{pump2,min}$	Minimum operating power of Pump 2	W
$P_{pump2,max}$	Maximum operating power of Pump 2	W
$Q_{w2}(P_{pump2})$	Function relating pumped water quantity to electrical power for Pump 2	m ³ /h
$P_{BSS,ch,max}$	Bulk charging power for the BSS	W
$P_{BSS,disch,max}$	Maximum power for discharging the BSS	W
K_{BSS}	BSS absorption mode charge rate constant (0 - 1)	-
$E_{BSS,0}$	Initial value of energy stored in the BSS	Wh
$s_{inv,0}$	Initial state of inverter	-
$E_{BSS,max}$	Maximum energy that can be stored in the BSS	Wh
$E_{BSS,lower}$	Inverter threshold to switch from BSS/PV source to utility source	W
$E_{BSS,upper}$	Inverter threshold to switch from utility source to BSS/PV source	W
$V_{w1,0}$	Initial value of water stored in Reservoir 1	m ³
$V_{w1,min}$	Minimum volume of water that can be stored in Reservoir 1	m ³
$V_{w1,max}$	Maximum volume of water that can be stored in Reservoir 1	m ³
$V_{w2,0}$	Initial value of water stored in Reservoir 2	m ³
$V_{w2,min}$	Minimum volume of water that can be stored in Reservoir 2	m ³
$V_{w2,max}$	Maximum volume of water that can be stored in Reservoir 2	m ³
$V_{use,desired,d}$	Desired volume of effectively used water on day d	m ³
D_d	Set of time periods t belonging to day d .	-
$Q_{use1,max}$	Maximum rate of water use from Reservoir 1	m ³ /h
$Q_{use2,max}$	Maximum rate of water use from Reservoir 2	m ³ /h
$C_{grid,t}$	Cost of power from the grid	\$/Wh
C_{BSS}	Cost of storing power in the BSS	\$/Wh
$C_{BSS,switching}$	Penalty for changing BSS charging/discharging mode	\$/ea
$C_{w,short}$	Cost or penalty factor for water that is desired but not used	\$/m ³
η_{BSS}	Efficiency of BSS in charging or discharging	-
$\eta_{w,t}$	Efficiency of water use	-
Δt	Time interval for discretized planning horizon	h

¹ Variables subscripted with t have a value for each time period

balance on the grid-side bus.

$$P_{PV,t} - P_{pump2,t} - P_{PV-inverter,t} = 0 \quad (2)$$

$$P_{PV-inverter,t} + P_{BSS,disch,t} - P_{BSS,ch,t} - (1 - s_{inv,t}) P_{load,t} = 0 \quad (3)$$

$$P_{grid-inverter,t} = s_{inv,t} P_{load,t} \quad (4)$$

$$P_{grid,t} - P_{grid-inverter,t} - P_{pump1,t} = 0 \quad (5)$$

The direction of power flow from the PV bus to the inverter and from the grid to the inverter must be constrained to be positive.

$$0 \leq P_{PV-inverter,t} \quad (6)$$

$$0 \leq P_{grid-inverter,t} \quad (7)$$

2) *Photovoltaic*: It is assumed that the microgrid has the ability to track the photovoltaic array maximum power point (MPP) regardless of the operation of either or both of the hybrid

inverter and the drive for Pump 2. It is also assumed that the control system has the ability to know what the maximum available PV power is, even if the system is not operating at the MPP. The full available output of the PV system may not be used if the load is less than the available PV power.

$$0 \leq P_{PV,t} \leq P_{PV,avail,t} \quad (8)$$

3) *Pumps*: Pump 1 is operated in a simple on-off fashion at a fixed power level.

$$P_{pump1,t} = s_{pump1,t} P_{pump1,max} \quad (9)$$

$$Q_{pump1,t} = s_{pump1,t} Q_{w1} \quad (10)$$

It is assumed that Pump 2 may be operated at a specified range-limited setpoint chosen by the controller. The pumping power $P_{pump2,t}$ is a semi-continuous variable, being continuous between a minimum and a maximum or else 0.

$$P_{pump2,t} = 0 \vee P_{pump2,min} \leq P_{pump2,t} \leq P_{pump2,max} \quad (11)$$

$$Q_{pump2,t} = Q_{w2} (P_{pump2,t}) \quad (12)$$

Equation 13 is a placeholder for the currently unknown relationship between pump flow and power. For real-time control, the EMS controller could infer this relationship by observing the operation of the pump. Until then, a simple linear efficiency coefficient η_{pump2} is used to characterize the power-flow relationship of Pump 2.

$$Q_{w2} (P_{pump2,t}) = \frac{P_{pump2,t} \eta_{pump2}}{h\rho g} \quad (13)$$

4) *Hybrid Inverter and BSS*: The hybrid inverter selected for the demonstration system at Güneşköy has limited capability to receive external control. It is assumed that the hybrid inverter will be configured with a priority order for supplying load such that the load will be supplied from PV if available, supplemented by power from the BSS. If PV is not sufficient and the BSS charge level is too low, then the utility grid source will be used. It is assumed that the hybrid inverter will charge the BSS only from PV and not from the utility grid source.

It is planned for the BSS to consist of lead-acid batteries. The hybrid inverter is responsible for charging the lead-acid battery bank. The inverter uses a four-stage charging cycle:

- 1) **Bulk charging**. Charges at a settable maximum charging current. The power to the battery is nearly constant and is approximated in this formulation as a constant power $P_{BSS,ch,max}$.
- 2) **Absorption charging**. Once the voltage reaches set maximum charging voltage, the voltage is held for a duration of 10 times the time spent in bulk charging mode. The power to the battery declines exponentially with time as the BSS voltage approaches the charging voltage and the SOC approaches 100%.
- 3) **Float charging**. Once the absorption charge timer completes, the charger switches to float charging mode in which a fixed voltage is held and minimal charging current is output except to compensate the small battery internal discharge or small load discharging of the battery.
- 4) **Equalization charging**. Equalization charging is only applicable to flooded lead acid batteries and not to sealed

lead acid batteries. In this mode, the battery is temporarily overcharged in order to reduce sulfation on the battery plates.

For the purposes of a mathematical model of the hybrid inverter's battery charging system for this optimization problem, only the bulk charging and absorption charging modes are represented. The other charging modes are neglected since most of the energy transfer to the BSS is completed these modes.

Equation 14 forces the charger state to charge if power is available. Equation 15 enforces the limit for absorption charging mode of the hybrid inverter, limits charging power to available power from the PV after meeting pumping and load power, and only allows charging when $s_{BSS,t}$ is in charging mode. Equation 16 only allows discharging when $s_{BSS,t}$ is in discharging mode.

$$s_{BSS,t} = (P_{PV,avail,t} - P_{pump2,t} - (1 - s_{inv,t})P_{load,t} \geq 0) \quad (14)$$

$$P_{BSS,ch,t} = \min \left(\underbrace{K_{BSS} \frac{E_{BSS,max} - E_{BSS,t-1}}{\eta_{BSS} \Delta t}}_{\text{Absorption mode charging limit}}, \underbrace{P_{PV,avail,t} - P_{pump2,t} - (1 - s_{inv,t})P_{load,t}}_{\text{Unused available PV power}}, \underbrace{s_{BSS,t} P_{BSS,ch,max}}_{\text{Enforce charger mode}} \right) \quad (15)$$

$$0 \leq P_{BSS,disch,t} \leq (1 - s_{BSS,t}) P_{BSS,disch,max} \quad (16)$$

The constant K_{BSS} takes a value between 0 and 1 and determines this switchover point from bulk charging mode to absorption charging and the rate of decrease in the charging power in absorption charging mode. A K_{BSS} value of 1 indicates that the charger remains in bulk charging mode until the BSS is fully charged. The value of K_{BSS} can be related to the time constant of the exponential decay of charging power in absorption mode, τ_{BSS} as shown in Equation 17. This relationship can be used to calculate K_{BSS} or to convert a known K_{BSS} from one modeling time interval Δt to another.

$$K_{BSS} = 1 - e^{-\Delta t / \tau_{BSS}} \quad (17)$$

According to the the hybrid inverter user manual, when in "SBU priority" mode, the hybrid inverter switches from PV/battery source to utility source when the battery goes below a minimum voltage level and switches back to the battery when the battery rises above a minimum voltage level. Equation 18 represents this logic to determine the connection of the inverter in time period t based on the connection during the previous period and the BSS energy level at the end of the previous period. In this way of modeling, the mode is switched only at discrete time intervals, so the model will show the battery BSS charge level will going a little above and below the set thresholds rather than switching mid-period as the actual hybrid inverter will do.

$$s_{inv,t} = (s_{inv,t-1} \text{ and } (E_{BSS,t-1} \leq E_{BSS,upper})) \text{ or } (E_{BSS,t-1} \leq E_{BSS,lower}) \quad (18)$$

Equation 19 couples the battery system energy balance from one period to the next. It includes a factor for conversion losses on energy input and energy output. An alternative formulation would be to make the efficiency factor be “round trip” and only include it on one of charging or discharging power rather than both.

$$E_{BSS,t} = E_{BSS,t-1} + P_{BSS,ch,t} \eta_{BSS} \Delta t - \frac{P_{BSS,disch,t} \Delta t}{\eta_{BSS}} \quad (19)$$

5) *Water Flow*: Equation 20 and Equation 21 couple the water level in Reservoir 1 and Reservoir 2 from one period to the next. It does not include water losses at this time, but water losses could be incorporated into this equation in the future.

$$V_{w1,t} = V_{w1,t-1} + Q_{pump1,t} \Delta t - Q_{use1,t} \Delta t \quad (20)$$

$$V_{w2,t} = V_{w2,t-1} + Q_{pump2,t} \Delta t - Q_{use2,t} \Delta t \quad (21)$$

Equation 22 sums the effective irrigation water across periods in each day, taking into consideration the varying efficiency of irrigation in different periods. It is assumed that water use between the two reservoirs is interchangeable.

$$V_{use,d} = \sum_{t \in D_d} \eta_{w,t} (Q_{use1,t} + Q_{use2,t}) \Delta t \quad (22)$$

Equations 23 through 26 are the limits on feasible values of the water flow and reservoir level values.

$$0 \leq Q_{use1,t} \leq Q_{use1,max} \quad (23)$$

$$0 \leq Q_{use2,t} \leq Q_{use2,max} \quad (24)$$

$$V_{w1,min} \leq V_{w1,t} \leq V_{w1,max} \quad (25)$$

$$V_{w2,min} \leq V_{w2,t} \leq V_{w2,max} \quad (26)$$

IV. DATA VALUES

Rather than providing data values for all the problem parameters shown in Table III, due to space limitations, a more general description of the parameters selection is given.

Grid energy costs $C_{grid,t}$ were set for the daytime, peak, and nighttime rates and daily periods that Güneşköy was billed at in 2020, not including fees for power factor, etc.

Water usage efficiency $\eta_{w,t}$ was set using different values for each hour of the day, with the highest efficiency (1.0) during nighttime hours and the lowest efficiency (0.5) during early afternoon.

Random values were generated for the load and the desired water use. Load was drawn from a uniform distribution from 0 to 1 kW. Daily desired water use was drawn from a normal distribution with a mean of 70 m³ and a standard deviation of 30 m³.

Pump, battery, and PV parameters were selected to represent the demonstration system shown in Figure 1. The hybrid inverter source selection was configured to stay on the PV/BSS source until the BSS charge dropped below 30% and then switch to the grid to charge until it reached a charge level of 95%. Hourly averages of the recorded output power of the rooftop PV array on the METU EEE Department machinery building was scaled to the rating of the demonstration system and used for the available PV power.

The optimization period was set for a length of 72 h with a time step Δt of 1 h used. The time period of 72 h (3 days) was used so that the optimizer wouldn't use up all the stored water and energy to meet the needs of the first day, neglecting its benefit for future days. Initial values for E_{BSS} , V_{w1} , V_{w2} , and s_{inv} were set arbitrarily for the first period optimized, and for subsequent periods, the time period being optimized was slid forward by 24 h and the value for the 24th period of the previous period was used as the initial value for the current period. The results shown in section VI are from the fourth in this series of sliding optimization runs.

V. IMPLEMENTATION

The problem was modeled in the Python programming language using the Pyomo library[5, 6] and solved using the COIN-OR CBC[7] open-source linear mixed-integer program (MIP) solver. In order to use a MIP solver, auxiliary binary variables were introduced to handle non-linear functions such as max, min, absolute value, \geq , \leq , and logic functions and and or. The formulation of the non-linear functions was based on [8] and original work by the author. Semi-continuous variables were modeled using the approach described in [9].

The problem, solved for a time period of 72 h with an interval Δt of 1 h, contained a total of 566 continuous and 1008 binary variables. CBC converges near to a solution in less than 30 seconds.

Initialization to a feasible point was performed using a single forward-pass heuristic of greedy pumping with Pump 2 if PV was available, then meeting loads with PV and BSS energy, discharging BSS to meet loads if necessarily and energy was available. Initial Pump 1 pumping and water use was set by iteratively solving a linear problem to try to add pumping one hour at a time until the desired water usage was met. The linear problem was implemented in Pyomo and solved using GLPK[10].

VI. RESULTS

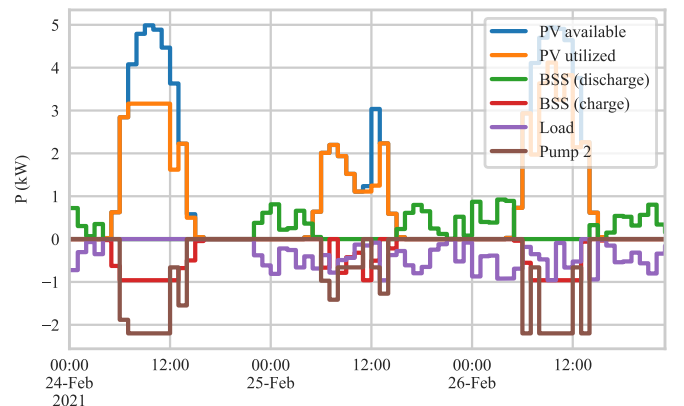


Figure 3. PV-Side Power

VII. CONCLUSION

This paper has presented a detailed mathematical model for an agricultural microgrid suitable for optimization of the operation of the pumps and water usage. The model was implemented in open-source software and results have been presented.

The model is not intended to stand alone but serves as one component of a microgrid energy management system (EMS). One possible application of this model is for use in model-predictive control (MPC) of the microgrid. In such a scheme, the system operation is optimized over some time horizon, but only the first step is used as the control output. The time horizon is then slid forward one step and the optimization is repeated.

In this formulation, a single value represents all the parameters at each time step, including available PV power, loads, desired water volume, all of which are dependent on random processes. One possible extension of this model would be to make it stochastic so that multiple possible future scenarios are represented. Unlike in standard model-predictive control (MPC), where the output of the optimization problem is the control input (feed-forward), in stochastic model predictive control (SMPC), the output of the optimization problem is the optimal control law to be applied in the future time periods[11]. Thus, the system is able to respond to stochastic events dynamically without having to re-run the MPC problem to obtain new control outputs. A microgrid control system incorporating SMPC has been presented in [12].

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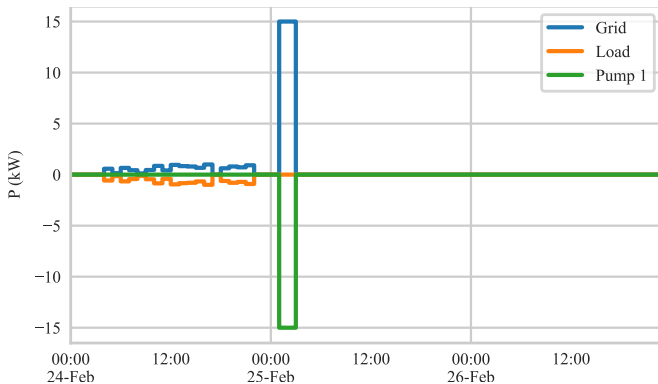


Figure 4. Grid-Side Power

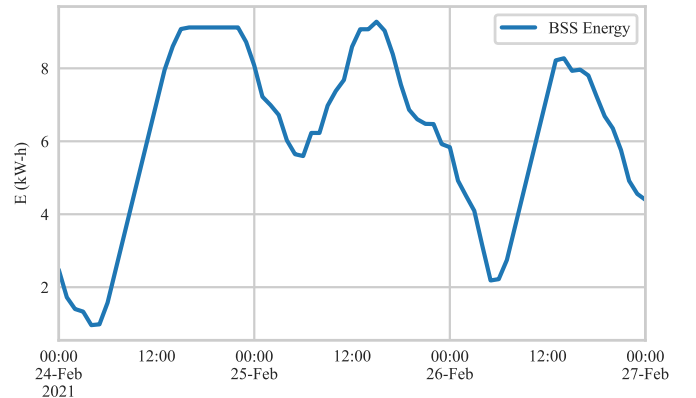


Figure 5. BSS Energy Stored

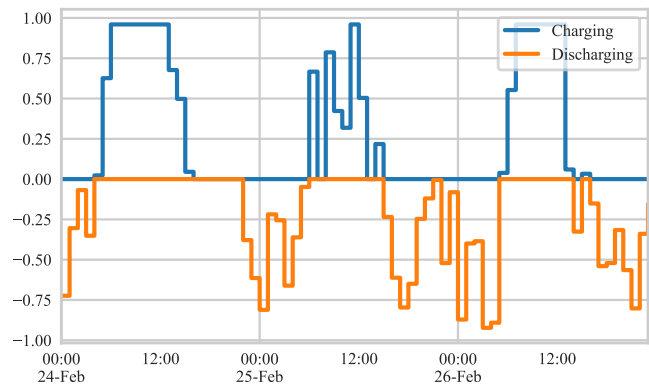


Figure 6. BSS Power (Charging & Discharging)

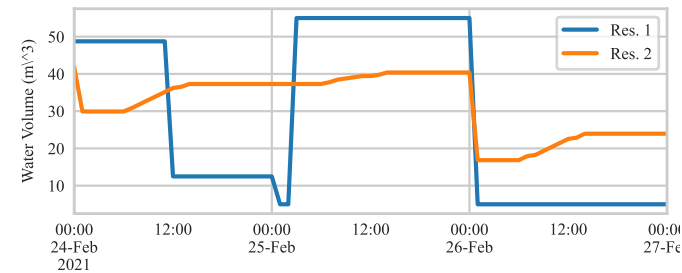


Figure 7. Water Stored

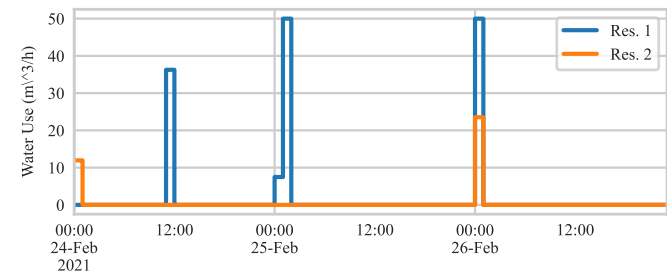


Figure 8. Water Used