PRINCIPLES OF ECONOMETRICS

5TH EDITION

ANSWERS TO ODD-NUMBERED EXERCISES IN CHAPTER 12

EXERCISE 12.1

(a) Now, $(1-c_1z)(1-c_2z)=1-c_1z-c_2z+c_1c_2z^2=1-(c_1+c_2)z+c_1c_2z^2$. For this expression to be equal to $1-\theta_1z-\theta_2z^2$, we require $c_1+c_2=\theta_1$ and $c_1c_2=-\theta_2$.

(b) We need to show that $\theta_1+\theta_2-1=0$ implies $c_1=1$ or $c_2=1$; and that $c_1=1$ or $c_2=1$ implies $\theta_1+\theta_2-1=0$. From part (a),

$$\begin{aligned}\theta_1+\theta_2-1&=c_1+c_2-c_1c_2-1\\&=c_1(1-c_2)-(1-c_2)\\&=(1-c_2)(c_1-1)\end{aligned}$$

Thus, $\theta_1+\theta_2-1=0$ only if $c_1=1$ or $c_2=1$.

Now suppose $c_1=1$. Then, $c_2=\theta_1-c_1=\theta_1-1$ and $c_2=-\theta_2/c_1=-\theta_2$.

Thus, $-\theta_2=\theta_1-1$, and so $\theta_1+\theta_2-1=0$.

By symmetry the same result is obtained if $c_2=1$.

(c) The AR(2) model will be stationary if $|1/c_1|>1$ and $|1/c_2|>1$. That is, stationarity implies $|c_1|<1$ and $|c_2|<1$. Since $\theta_1+\theta_2-1=(1-c_2)(c_1-1)$, if $|c_1|<1$ and $|c_2|<1$, then $\theta_1+\theta_2-1<0$.

(d) Substituting for γ and a_1, the equation $\Delta y_t=\delta+\gamma y_{t-1}+a_1\Delta y_{t-1}+v_t$ can be written as

$$y_t-y_{t-1}=\delta+(\theta_1+\theta_2-1)y_{t-1}-\theta_2(y_{t-1}-y_{t-2})+v_t$$

$$y_t-y_{t-1}=\delta+\theta_1y_{t-1}+\theta_2y_{t-1}-y_{t-1}-\theta_2y_{t-1}+\theta_2y_{t-2}+v_t$$

Cancelling terms leads to the original model $y_t=\delta+\theta_1y_{t-1}+\theta_2y_{t-2}+v_t$.

The equivalence of the two equations and the results in parts (b) and (c) mean that a test for a unit root (stationarity) can be based on the null and alternative hypotheses $H_0:\gamma=0$ and $H_0:\gamma<0$.

(e) If $\gamma=\theta_1+\theta_2+\cdots+\theta_p-1=0$, then $z=1$ is a solution to the equation

$$1-\theta_1z-\theta_2z^2-\cdots-\theta_pz^p=0$$

Having $z=1$ implies the AR(p) model has a unit root.

(f) We wish to show that

$$\Delta y_t=\alpha+\gamma y_{t-1}+\sum_{s=1}^{p-1}a_s\Delta y_{t-s}+v_t$$

and

$$y_t=\alpha+\theta_1y_{t-1}+\theta_2y_{t-2}+\cdots+\theta_py_{t-p}+v_t$$

are equivalent representations of an AR(p) process if $\gamma=\theta_1+\theta_2+\cdots+\theta_p-1$ and $a_j=-\sum_{r=j}^{p-1}\theta_{r+1}$. Making these substitutions into the first equation, we have

$$y_t - y_{t-1} = \alpha + \left(\theta_1 + \theta_2 + \cdots + \theta_p - 1\right) y_{t-1} + \left(-\theta_2 - \theta_3 - \cdots - \theta_p\right)\left(y_{t-1} - y_{t-2}\right)$$
$$+\left(-\theta_3 - \cdots - \theta_p\right)\left(y_{t-2} - y_{t-3}\right) + \cdots + \left(-\theta_p\right)\left(y_{t-p+1} - y_{t-p}\right) + v_t$$

Collecting coefficients of $y_{t-1}, y_{t-2}, \ldots, y_{t-p}$ leads to the AR(*p*) model

$$y_t = \alpha + \theta_1 y_{t-1} + \theta_2 y_{t-2} + \cdots + \theta_p y_{t-p} + v_t$$

EXERCISE 12.3

(a) Both *W* and *Y* fluctuate around a nonzero mean with no obvious trend upwards or downwards, and so Dickey-Fuller test equations with intercepts and no trend terms were used for these variables. Trend terms were included for *X* and *Z* since they are both trending upwards. A lack of serial correlation in the errors and insignificance of coefficient estimates for the lags of $\Delta W_t, \Delta Y_t, \Delta X_t$ and ΔZ_t would have led to the omission of augmentation terms.

(b) Using a 5% level of significance, we obtain the following Dickey-Fuller test results.

For *W*: since $\tau = -3.23$ is less than the 5% critical value of –2.86, the null hypothesis of nonstationarity is rejected, and we infer that *W* is stationary.

For *Y*: since $\tau = -1.98$ is greater than the 5% critical value of –2.86, the null hypothesis of nonstationarity is not rejected, and we infer that *Y* is not stationary.

For *X*: since $\tau = -3.13$ is greater than the 5% critical value of –3.41, the null hypothesis of nonstationarity is not rejected, and we infer that *X* is not stationary.

For *Z*: since $\tau = -1.87$ is greater than the 5% critical value of –3.41, the null hypothesis of nonstationarity is not rejected, and we infer that *Z* is not stationary.

(c) Using a 5% significance level, we find $\tau = -2.83$ is greater than the critical value of -3.42. Thus, we fail to reject a null hypothesis that the errors are nonstationary, and we conclude *X* and *Z* are not cointegrated.

(d) The value $\tau = -13.76$ is less than the 5% critical value -2.86. Thus, we reject the null hypothesis that ΔZ is nonstationary. Because *Z* is nonstationary, but ΔZ is stationary, we conclude that *Z* is integrated of order 1.

EXERCISE 12.5

(a) For the random walk $y_t = y_{t-1} + v_t$, we have $\hat{y}_{T+1} = 10$ and $\hat{y}_{T+2} = 10$.

(b) For the random walk with drift $y_t = 5 + y_{t-1} + v_t$, we have $\hat{y}_{T+1} = 15$ and $\hat{y}_{T+2} = 20$.

(c) For the random walk $\ln\left(y_t\right) = \ln\left(y_{t-1}\right) + v_t$, we have $\hat{y}_{T+1} = 73.89$ and $\hat{y}_{T+2} = 546$.

(d) For the deterministic trend model $y_t = 10 + 0.1t + v_t$, we have $\hat{y}_{T+1} = 13$ and $\hat{y}_{T+2} = 13.1$.

(e) For the ARDL model $y_t = 6 + 0.6y_{t-1} + 0.3x_t + 0.1x_{t-1} + v_t$, we have $\hat{y}_{T+1} = 14$ and $\hat{y}_{T+2} = 16.4$.

(f) For the error correction model $\Delta y_t = -0.4\left(y_{t-1} - 15 - x_{t-1}\right) + 0.3\Delta x_t + v_t$, we have $\hat{y}_{T+1} = 14$ and $\hat{y}_{T+2} = 16.4$. When $x = 5$, the long run equilibrium value for *y* is $y = 20$.

(g) In the first difference model $\Delta y_t = 0.6\Delta y_{t-1} + 0.3\Delta x_t + 0.1\Delta x_{t-1} + v_t$, we have $\hat{y}_{T+1} = 8.7$ and $\hat{y}_{T+2} = 7.92$

EXERCISE 12.7

Unemployment

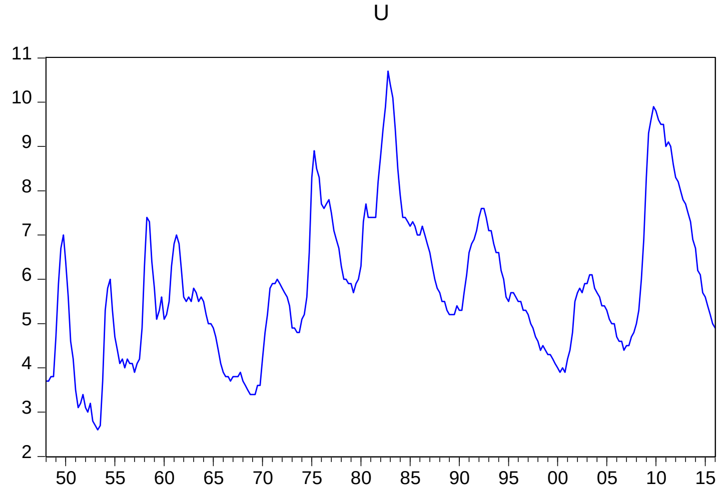
The plot and unit root test results for the unemployment series follow. There is no obvious trend in the series and so a unit root test equation with a constant and no trend is used. Using the Schwarz criterion to choose the number of augmentation terms leads to the inclusion of ΔU_{t-1} in the test equation which becomes

$$\Delta U_t = \alpha + \gamma U_{t-1} + a_1 \Delta U_{t-1} + v_t$$

The null and alternative hypotheses are

$$H_0: \gamma = 0 \qquad H_1: \gamma < 0$$

Because the *p*-value = 0.0003, at a 5% significance level we reject $H_0: \gamma = 0$ in favour of $H_1: \gamma < 0$. We conclude that *U* is stationary and of order I(0).



Null Hypothesis: U has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=15)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-4.475914	0.0003
Test critical values:	1% level	-3.454353	
	5% level	-2.872001	
	10% level	-2.572417	

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(U)
Method: Least Squares
Sample (adjusted): 1948Q3 2016Q1
Included observations: 271 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
U(-1)	-0.049269	0.011008	-4.475914	0.0000
D(U(-1))	0.662091	0.045575	14.52759	0.0000
C	0.288522	0.066608	4.331613	0.0000

GDP growth rate

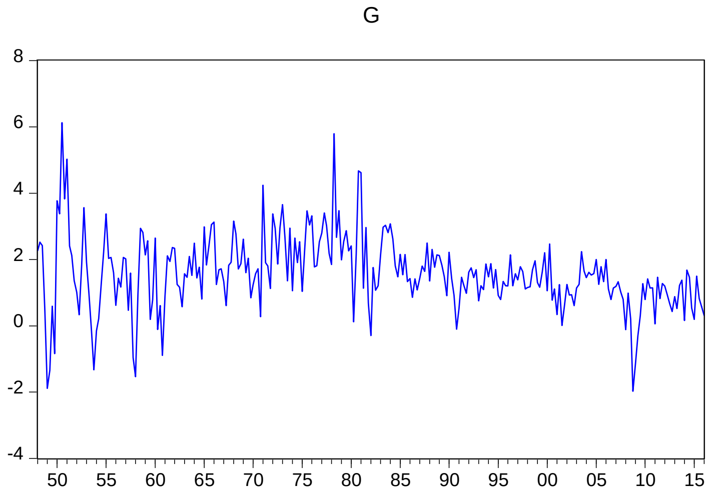
The plot and unit root test results for the GDP growth rate follow. There is no obvious trend in the series and so a unit root test equation with a constant and no trend is used. Using the Schwarz criterion results in no augmentation terms. The test equation is

$$\Delta G_t = \alpha + \gamma G_{t-1} + v_t$$

The null and alternative hypotheses are

$$H_0 : \gamma = 0 \qquad H_1 : \gamma < 0$$

Because the *p*-value = 0.0000, at a 5% significance level we reject $H_0 : \gamma = 0$ in favour of $H_1 : \gamma < 0$. We conclude that G is stationary and of order I(0).



Null Hypothesis: G has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=15)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-9.345206	0.0000
Test critical values:	1% level	-3.454263	
	5% level	-2.871961	
	10% level	-2.572396	

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(G)
Method: Least Squares
Sample (adjusted): 1948Q2 2016Q1
Included observations: 272 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
G(-1)	-0.490444	0.052481	-9.345206	0.0000
C	0.767330	0.101620	7.550965	0.0000

Inflation

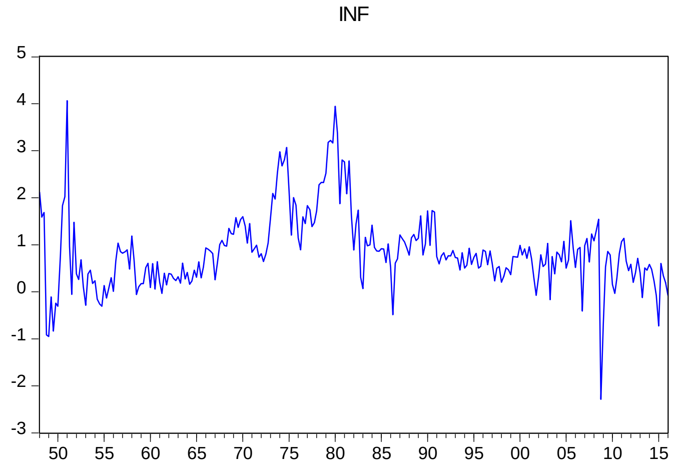
The plot and unit root test results for the inflation series follow. There is no obvious trend in the series and so a unit root test equation with a constant and no trend is used. Using the Schwarz criterion to choose the number of augmentation terms leads to the inclusion of ΔINF_{t-1} and ΔINF_{t-2} in the test equation which becomes

$$\Delta INF_t = \alpha + \gamma INF_{t-1} + a_1 \Delta INF_{t-1} + a_2 \Delta INF_{t-2} + v_t$$

The null and alternative hypotheses are

$$H_0 : \gamma = 0 \qquad H_1 : \gamma < 0$$

Because the *p*-value = 0.0017, at a 5% significance level we reject $H_0 : \gamma = 0$ in favour of $H_1 : \gamma < 0$. We conclude that *INF* is stationary and of order I(0).



Null Hypothesis: INF has a unit root
Exogenous: Constant
Lag Length: 2 (Automatic - based on SIC, maxlag=15)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.986120	0.0017
Test critical values:	1% level	-3.454443	
	5% level	-2.872041	
	10% level	-2.572439	

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(INF)
Method: Least Squares
Sample (adjusted): 1948Q4 2016Q1
Included observations: 270 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
INF(-1)	-0.182792	0.045857	-3.986120	0.0001
D(INF(-1))	-0.219250	0.063123	-3.473391	0.0006
D(INF(-2))	-0.228658	0.059539	-3.840499	0.0002
C	0.147452	0.051447	2.866115	0.0045

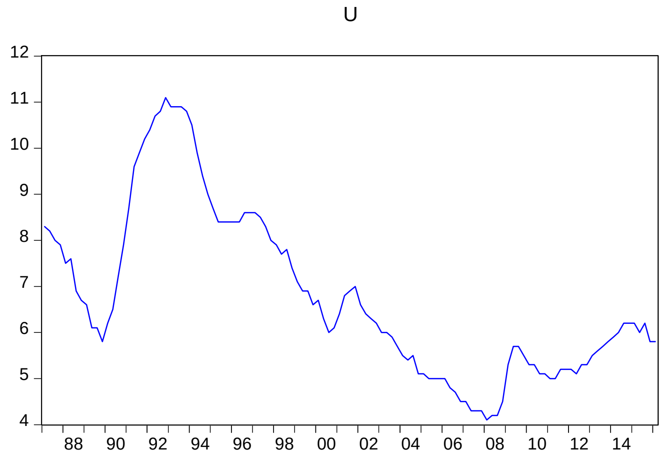
EXERCISE 12.9

Unemployment

The plot and unit root test results for the unemployment series follow. There is an obvious downward trend in the series from 1992 to 2008, but a less obvious trend over the complete sample period. Because 1992-2008 makes up a large part of the sample period, we include both a constant and a trend in the unit root test equation. Using the Schwarz criterion to choose the number of augmentation terms, the test equation becomes

$$\Delta U_t = \alpha + \lambda t + \gamma U_{t-1} + a_1 \Delta U_{t-1} + a_2 \Delta U_{t-2} + v_t$$

The null and alternative hypotheses are $H_0 : \gamma = 0$ and $H_1 : \gamma < 0$. The null hypothesis is not rejected at a 5% significance level, the *p*-value being 0.1009. Because $H_0 : \gamma = 0$ cannot be rejected, we treat U as nonstationary.



Null Hypothesis: U has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 2 (Automatic - based on SIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.146056	0.1009
Test critical values:	1% level	-4.040532	
	5% level	-3.449716	
	10% level	-3.150127	

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(U)
Method: Least Squares
Sample (adjusted): 1987Q4 2016Q1
Included observations: 114 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
U(-1)	-0.049051	0.015591	-3.146056	0.0021
D(U(-1))	0.347451	0.086482	4.017604	0.0001
D(U(-2))	0.405974	0.089387	4.541772	0.0000
C	0.447330	0.148134	3.019761	0.0032
@TREND("1987Q1")	-0.002011	0.000860	-2.339011	0.0212

Concluding U is nonstationary means a unit root test on its first difference is needed to assess its order of integration. The following output from this unit root test reveals a p-value of 0.0087, implying the first difference is stationary and hence U is of order I(1).

Null Hypothesis: D(U) has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.534644	0.0087
Test critical values:	1% level	-3.488585	
	5% level	-2.886959	
	10% level	-2.580402	

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(U,2)
Method: Least Squares
Sample (adjusted): 1987Q4 2016Q1
Included observations: 114 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(U(-1))	-0.302109	0.085471	-3.534644	0.0006
D(U(-1),2)	-0.344275	0.090219	-3.816005	0.0002
C	-0.005512	0.020624	-0.267255	0.7898

Inflation

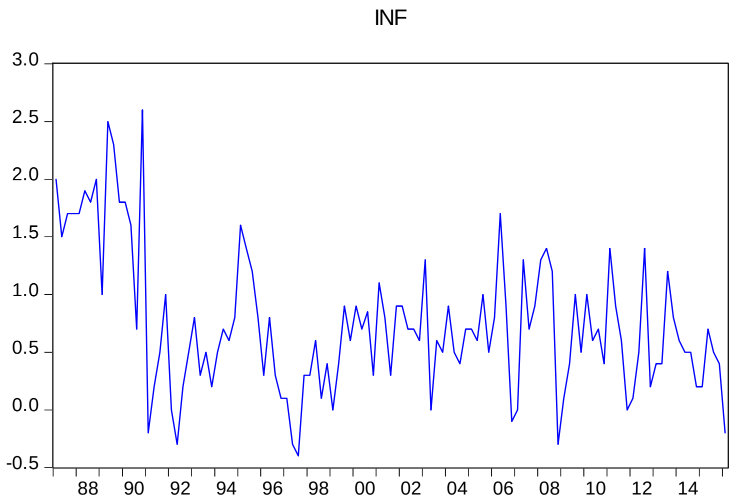
The plot and unit root test results for the inflation series follow. Although there is sharp drop in inflation around 1990, there is no obvious trend in the series and so a unit root test equation with a constant and no trend is used. The Schwarz criterion does not suggest inclusion of augmentation terms, and so the test equation becomes

$$\Delta INF_t = \alpha + \gamma INF_{t-1} + v_t$$

The null and alternative hypotheses are

$$H_0 : \gamma = 0 \qquad H_1 : \gamma < 0$$

Because the *p*-value = 0.0000, at a 5% significance level we reject $H_0 : \gamma = 0$ in favour of $H_1 : \gamma < 0$. We conclude that *INF* is stationary and of order I(0).



Null Hypothesis: INF has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-6.221769	0.0000
Test critical values:	1% level	-3.487550	
	5% level	-2.886509	
	10% level	-2.580163	

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(INF)
Method: Least Squares
Sample (adjusted): 1987Q2 2016Q1
Included observations: 116 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
INF(-1)	-0.498510	0.080123	-6.221769	0.0000
C	0.354272	0.077133	4.593006	0.0000

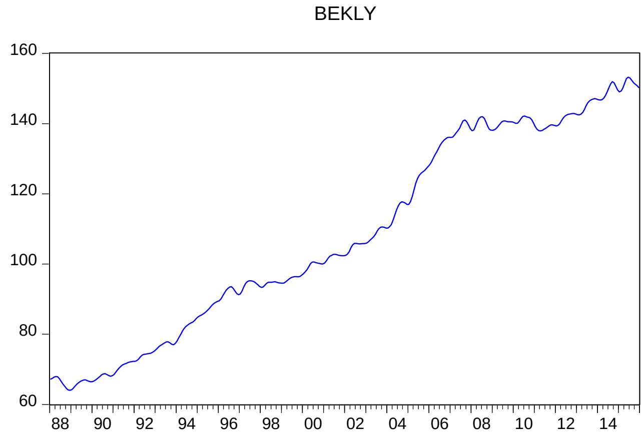
EXERCISE 12.11

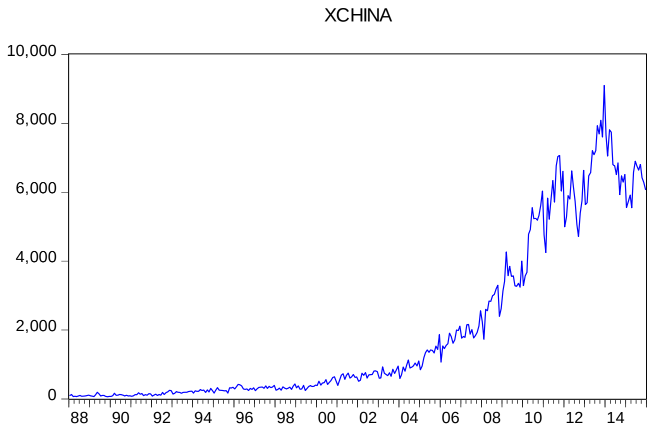
(a) The results from estimating the equation $XCHINA_t = \beta_1 + \beta_2 BEKLY_t + e_t$ follow. They suggest a very strong relationship between house prices in Beckley, West Virginia, and the value of Australian exports to China. However, since there is no reason why these two variables should be related, there is a need to ask whether the relationship is spurious.

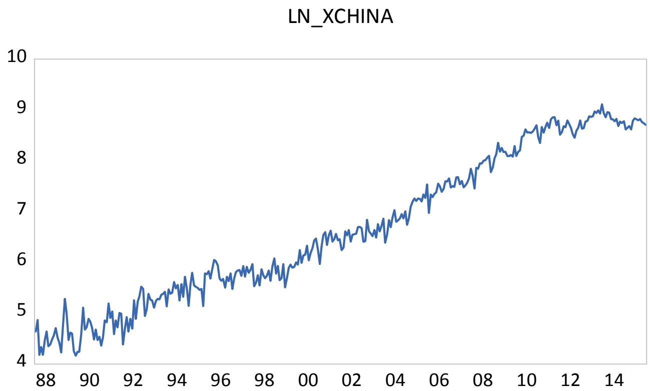
Dependent Variable: XCHINA
Method: Least Squares
Sample: 1988M01 2015M12
Included observations: 336

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-5689.274	283.5430	-20.06494	0.0000
BEKLY	70.23848	2.526920	27.79608	0.0000
R-squared	0.698181	Mean dependent var		1934.580

(b) The plots of the three series given below show an increasing trend, with *BEKLY* and $\ln(XCHINA)$ increasing approximately linearly, and *XCHINA* increasing at an increasing rate. They suggest that the strong relationship estimated in part (a) is attributable to the upward trends of the two variables and the omission of a trend from the equation.







(c) Estimates for the equation $\ln(XCHINA_t) = \beta_1 + \delta t + \beta_2 BEKLY_t + e_t$ follow. Since $\ln(XCHINA)$ increases linearly, and *XCHINA* increases at an increasing rate, the linear trend on the right-hand side of the equation is likely to be better at capturing the trend in $\ln(XCHINA)$ than that in *XCHINA*.

Despite controlling for the trends by including a trend in the equation, house prices in Beckley still seem to have a significant impact on Australian exports to China. We need to investigate further why the equation might be spurious.

Dependent Variable: LN_XCHINA
Method: Least Squares
Sample: 1988M01 2015M12
Included observations: 336

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	3.142563	0.175741	17.88176	0.0000
@TREND	0.009814	0.000849	11.55742	0.0000
BEKLY	0.017080	0.002895	5.899484	0.0000
R-squared	0.973163	Mean dependent var		6.640334

(d) The results from unit root tests on $\ln(XCHINA)$ and *BEKLY* follow. Given the trending behaviour of both variables, a constant and a trend term have been included in the test equations. Two augmentation terms were selected by the Schwarz criterion for the $\ln(XCHINA)$ equation and nine were selected for the *BEKLY* equation. The test *p*-value of 0.0012 for $\ln(XCHINA)$ suggests this series is trend stationary. However, for *BEKLY*, the test *p*-value is 0.7332. A null hypothesis of a unit root cannot be rejected. Thus, we can treat *BEKLY* as nonstationary which means the relationship seemingly uncovered in part (c) is likely to be spurious.

Null Hypothesis: LN_XCHINA has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 2 (Automatic - based on SIC, maxlag=16)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-4.598655	0.0012
Test critical values:	1% level	-3.985857	
	5% level	-3.423377	
	10% level	-3.134639	

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LN_XCHINA)
Method: Least Squares
Sample (adjusted): 1988M04 2015M12
Included observations: 333 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LN_XCHINA(-1)	-0.201075	0.043725	-4.598655	0.0000
D(LN_XCHINA(-1))	-0.291388	0.058020	-5.022170	0.0000
D(LN_XCHINA(-2))	-0.132585	0.053697	-2.469122	0.0141
C	0.853237	0.181488	4.701355	0.0000
@TREND("1988M01")	0.002968	0.000656	4.525557	0.0000

Null Hypothesis: BEKLY has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 9 (Automatic - based on SIC, maxlag=16)

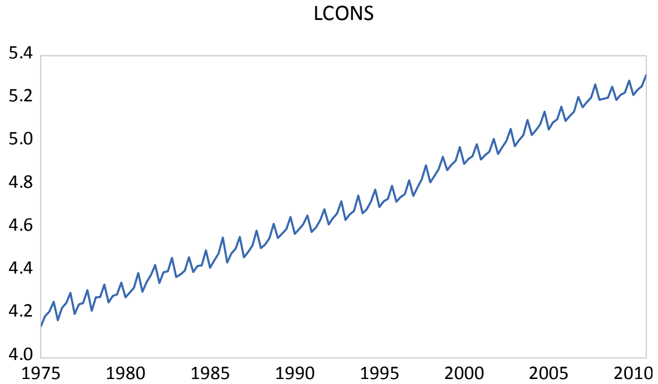
		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.735750	0.7332
Test critical values:	1% level	-3.986459	
	5% level	-3.423669	
	10% level	-3.134812	

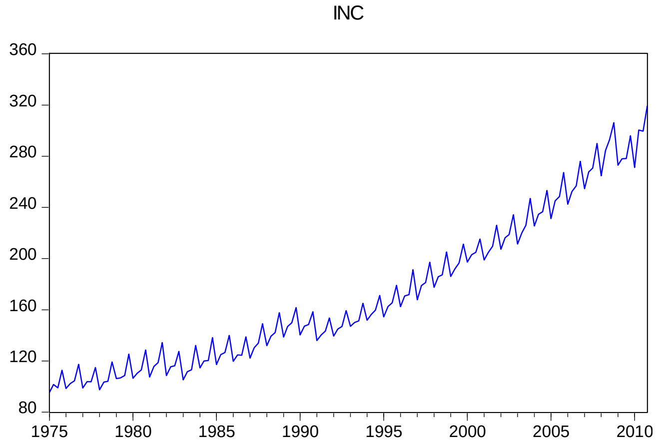
Augmented Dickey-Fuller Test Equation
Dependent Variable: D(BEKLY)
Method: Least Squares
Sample (adjusted): 1988M11 2015M12
Included observations: 326 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
BEKLY(-1)	-0.002159	0.001244	-1.735750	0.0836
D(BEKLY(-1))	2.418710	0.055517	43.56677	0.0000
D(BEKLY(-2))	-3.139904	0.144513	-21.72743	0.0000
D(BEKLY(-3))	2.944103	0.221902	13.26757	0.0000
D(BEKLY(-4))	-2.414760	0.266480	-9.061707	0.0000
D(BEKLY(-5))	1.852850	0.281823	6.574505	0.0000
D(BEKLY(-6))	-1.393284	0.268806	-5.183240	0.0000
D(BEKLY(-7))	0.993406	0.225135	4.412488	0.0000
D(BEKLY(-8))	-0.579190	0.147175	-3.935387	0.0001
D(BEKLY(-9))	0.247610	0.056634	4.372101	0.0000
C	0.156847	0.073282	2.140304	0.0331
@TREND("1988M01")	0.000569	0.000368	1.546132	0.1231

EXERCISE 12.13

(a) The plots of *LCONS* and *INC* displayed below exhibit increasing trends as well as a strong seasonal component.





(b) Estimates for the trend equations $LCONS_t = a_1 + a_2 t + u_{1t}$ and $INC_t = c_1 + c_2 t + u_{2t}$ follow. The residuals which represent the detrended series were stored as *LCONS_T* and *INC_T*.

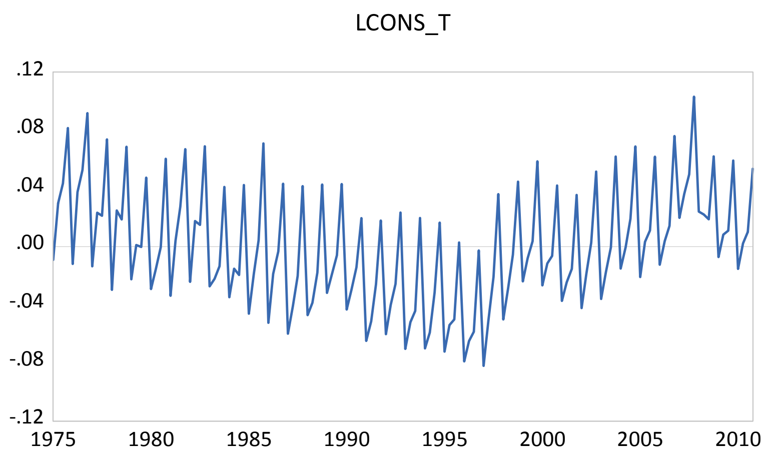
Dependent Variable: LCONS
Method: Least Squares
Sample: 1975Q1 2010Q4
Included observations: 144

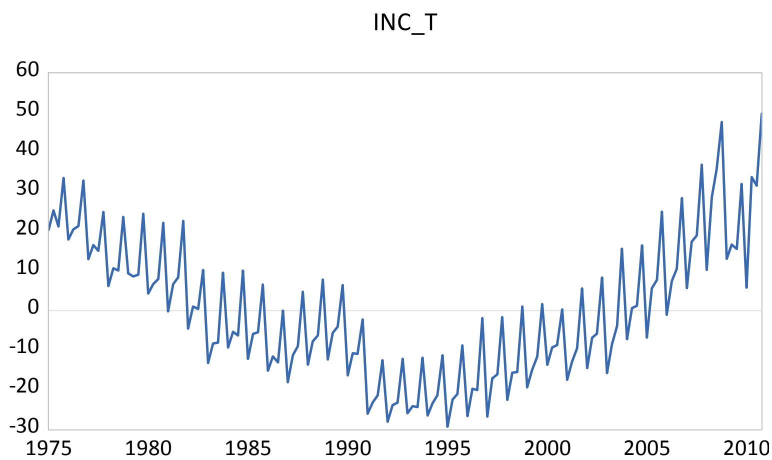
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	4.154279	0.006725	617.7642	0.0000
@TREND	0.007716	8.13E-05	94.89534	0.0000
R-squared	0.984476	Mean dependent var		4.705963

Dependent Variable: INC
Method: Least Squares
Sample: 1975Q1 2010Q4
Included observations: 144

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	74.95214	2.894387	25.89568	0.0000
@TREND	1.359009	0.034996	38.83283	0.0000
R-squared	0.913939	Mean dependent var		172.1213

(c) The plots of *LCONS_T* and *INC_T* are displayed below. The strong seasonal component continues to be evident. Also, there is a predominance of negative values in the middle of the sample, and positive values at the ends of the sample, particularly for *INC_T*. This outcome suggests a linear trend may have been inadequate. Perhaps you would like to check out the implications of trying a quadratic trend.





(d) Estimates for the equations $LCONS_t = \pi_0 t + \pi_1 D_{1t} + \pi_2 D_{2t} + \pi_3 D_{3t} + \pi_4 D_{4t} + u_t$ and $INC_t = \pi_0 t + \pi_1 D_{1t} + \pi_2 D_{2t} + \pi_3 D_{3t} + \pi_4 D_{4t} + u_t$ follow. The residuals which represent the "seasonally-adjusted detrended" series were stored as *LCONS_STAR* and *INC_STAR*.

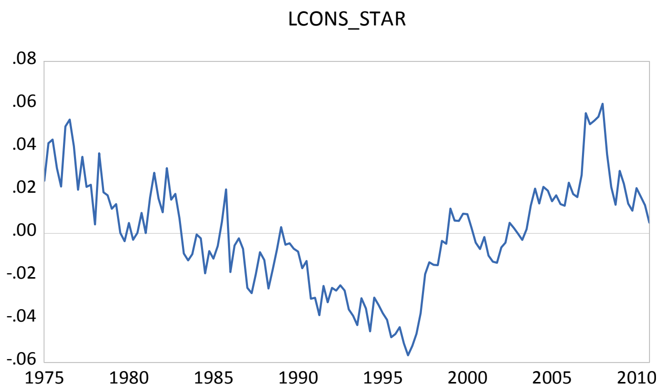
Dependent Variable: LCONS
Method: Least Squares
Sample: 1975Q1 2010Q4
Included observations: 144

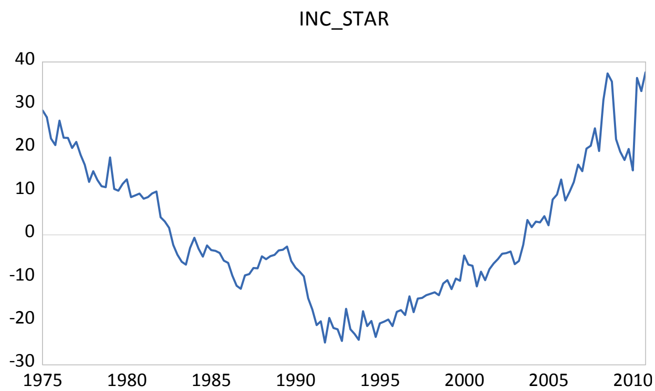
Variable	Coefficient	Std. Error	t-Statistic	Prob.
@TREND	0.007697	5.17E-05	148.7899	0.0000
QUARTER=1	4.120613	0.005621	733.1238	0.0000
QUARTER=2	4.142145	0.005654	732.5932	0.0000
QUARTER=3	4.154139	0.005688	730.3567	0.0000
QUARTER=4	4.205751	0.005722	735.0365	0.0000
R-squared	0.993854	Mean dependent var		4.705963

Dependent Variable: INC
Method: Least Squares
Sample: 1975Q1 2010Q4
Included observations: 144

Variable	Coefficient	Std. Error	t-Statistic	Prob.
@TREND	1.354345	0.031686	42.74244	0.0000
QUARTER=1	66.50581	3.442972	19.31640	0.0000
QUARTER=2	73.02514	3.463470	21.08439	0.0000
QUARTER=3	73.90771	3.484135	21.21264	0.0000
QUARTER=4	87.70389	3.504965	25.02276	0.0000
R-squared	0.930990	Mean dependent var		172.1213

(e) Plots of $LCONS^*$ and INC^* continue to display a predominance of negative values in the middle of the sample, and positive values at the ends of the sample. The seasonal component has been removed, but there is strong evidence of autocorrelation.





(f) The results for testing whether $LCONS^*$ is stationary or first-difference stationary follow. Because $LCONS^*$ has been computed as least squares residuals, it will have a zero mean. Also, the trend has been removed. Thus, a test equation without a trend or constant is appropriate. The Schwarz criterion suggests no augmentation terms are necessary. Using the 5% critical value of –3.41, found in the third row of Table 12.2, we conclude that $LCONS^*$, with test value $\tau = -2.614$, is not stationary. However, its first difference, with test value $\tau = -14.39$, is stationary. The critical value –3.41 is relevant for an equation with a trend, and hence takes into account the detrending used to create $LCONS^*$. However, it does not allow for removal of the seasonal component and hence will not be entirely suitable.

Null Hypothesis: LCONS_STAR has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.614235	0.0091
Test critical values:	1% level	-2.581233	
	5% level	-1.943074	
	10% level	-1.615231	

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LCONS_STAR)
Method: Least Squares
Sample (adjusted): 1975Q2 2010Q4
Included observations: 143 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LCONS_STAR(-1)	-0.088613	0.033896	-2.614235	0.0099

Null Hypothesis: D(LCONS_STAR) has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-14.38871	0.0000
Test critical values:	1% level	-2.581349	
	5% level	-1.943090	
	10% level	-1.615220	

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LCONS_STAR,2)
Method: Least Squares
Sample (adjusted): 1975Q3 2010Q4
Included observations: 142 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LCONS_STAR(-1))	-1.182044	0.082151	-14.38871	0.0000

For the same reasons, test equations without trend or constant are chosen for INC^*. Five augmentation terms are chosen by the Schwarz criterion. Using the 5% critical value of –3.41, found in the third row of Table 12.2, we conclude that INC^*, with test value $\tau = -0.454$, is not stationary. However, its first difference, with test value $\tau = -5.954$, is stationary. Again, the critical value –3.41 will not be entirely suitable because it does not reflect removal of the seasonal component.

Null Hypothesis: INC_STAR has a unit root
Exogenous: None
Lag Length: 5 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.453887	0.5168
Test critical values:	1% level	-2.581827	
	5% level	-1.943157	
	10% level	-1.615178	

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(INC_STAR)
Method: Least Squares
Sample (adjusted): 1976Q3 2010Q4
Included observations: 138 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
INC_STAR(-1)	-0.008881	0.019566	-0.453887	0.6507
D(INC_STAR(-1))	-0.003489	0.083696	-0.041691	0.9668
D(INC_STAR(-2))	0.029177	0.080571	0.362123	0.7178
D(INC_STAR(-3))	-0.218471	0.088851	-2.458848	0.0152
D(INC_STAR(-4))	0.304711	0.089354	3.410157	0.0009
D(INC_STAR(-5))	-0.331547	0.093159	-3.558943	0.0005

Null Hypothesis: D(INC_STAR) has a unit root
Exogenous: None
Lag Length: 4 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-5.954399	0.0000
Test critical values:	1% level	-2.581827	
	5% level	-1.943157	
	10% level	-1.615178	

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(INC_STAR,2)
Method: Least Squares
Sample (adjusted): 1976Q3 2010Q4
Included observations: 138 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(INC_STAR(-1))	-1.241767	0.208546	-5.954399	0.0000
D(INC_STAR(-1),2)	0.232208	0.194246	1.195435	0.2340
D(INC_STAR(-2),2)	0.254425	0.164027	1.551113	0.1233
D(INC_STAR(-3),2)	0.032233	0.131817	0.244526	0.8072
D(INC_STAR(-4),2)	0.334481	0.092657	3.609898	0.0004

(g) As can be seen from the output below, the estimate for β is the same from both equations, $LCONS_t = \delta t + \phi_1 D_{1t} + \phi_2 D_{2t} + \phi_3 D_{3t} + \phi_4 D_{4t} + \beta INC_t + e_t$, and $LCONS_t^* = \beta INC_t^* + e_t$. The equivalence follows from the Frisch-Waugh-Lovell theorem studied in Chapter 5. The standard errors are different because the degrees of freedom used to estimate the error variance are different. The R^2's are different because of different variation in the different dependent variables.

Dependent Variable: LCONS_STAR
Method: Least Squares
Sample: 1975Q1 2010Q4
Included observations: 144

Variable	Coefficient	Std. Error	t-Statistic	Prob.
INC_STAR	0.001339	7.81E-05	17.13285	0.0000
R-squared	0.672420	Mean dependent var		-5.11E-16

Dependent Variable: LCONS
Method: Least Squares
Sample: 1975Q1 2010Q4
Included observations: 144

Variable	Coefficient	Std. Error	t-Statistic	Prob.
@TREND	0.005884	0.000112	52.65185	0.0000
INC	0.001339	7.95E-05	16.83066	0.0000
QUARTER=1	4.031584	0.006197	650.5563	0.0000
QUARTER=2	4.044389	0.006655	607.7597	0.0000
QUARTER=3	4.055201	0.006725	602.9739	0.0000
QUARTER=4	4.088345	0.007711	530.1806	0.0000
R-squared	0.997987	Mean dependent var		4.705963

(h) The same set of residuals is obtained from both equations in part (g). Testing these residuals *EHAT_H* for a unit root, we get the output below. Comparing the test value $\tau = -5.088$ with the 5% critical value –3.42, found in Table 12.4 of *POE5*, we conclude that the residuals are stationary and hence that $LCONS_t^*$ and INC_t^* are cointegrated.

Null Hypothesis: EHAT_H has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-5.087904	0.0000
Test critical values:	1% level	-2.581233	
	5% level	-1.943074	
	10% level	-1.615231	

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(EHAT_H)
Method: Least Squares
Sample (adjusted): 1975Q2 2010Q4
Included observations: 143 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
EHAT_H(-1)	-0.338567	0.066544	-5.087904	0.0000

(i) Estimating the error correction model $\Delta LCONS_t^* = -\alpha\hat{e}_t + \delta_0 \Delta INC_t^* + v_t$ yields

Dependent Variable: D(LCONS_STAR)
Method: Least Squares
Sample (adjusted): 1975Q2 2010Q4
Included observations: 143 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
EHAT_H(-1)	-0.266084	0.059731	-4.454703	0.0000
D(INC_STAR)	-0.000117	0.000225	-0.517927	0.6053

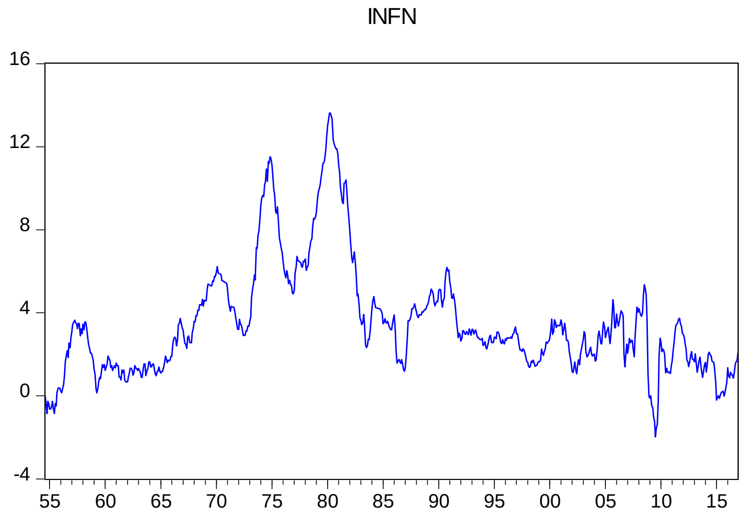
The correlogram of residuals from this model reveals a significant autocorrelation at lag 4 which is removed by estimating the model $\Delta LCONS_t^* = -\alpha\hat{e}_t + \delta_0 \Delta INC_t^* + \theta_4 \Delta LCONS_{t-4}^* + v_t$. The results from estimating this model follow.

Dependent Variable: D(LCONS_STAR)
Method: Least Squares
Sample (adjusted): 1976Q2 2010Q4
Included observations: 139 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
EHAT_H(-1)	-0.267851	0.056890	-4.708255	0.0000
D(INC_STAR)	7.52E-05	0.000220	0.342562	0.7325
D(LCONS_STAR(-4))	0.326269	0.075610	4.315160	0.0000

EXERCISE 12.15

(a) Because *INFN* is trending neither upwards nor downwards, to test whether it is stationary, we use a test equation with a constant and no trend. A plot of *INFN* and the unit root test outcome are given below. The Schwarz criterion chooses 15 augmentation terms. The test *p*-value is 0.0378. At a 5% level of significance, we reject a null hypothesis of a unit root and conclude that *INFN* is stationary. At a 1% level of significance, we conclude *INFN* is nonstationary.



Null Hypothesis: INFN has a unit root
Exogenous: Constant
Lag Length: 15 (Automatic - based on SIC, maxlag=19)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.974658	0.0378
Test critical values:	1% level	-3.439044	
	5% level	-2.865267	
	10% level	-2.568811	

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(INFN)
Method: Least Squares
Sample (adjusted): 1955M12 2016M12
Included observations: 733 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
INFN(-1)	-0.012590	0.004232	-2.974658	0.0030
D(INFN(-1))	0.383711	0.036872	10.40656	0.0000
D(INFN(-2))	0.066508	0.039532	1.682400	0.0929
D(INFN(-3))	-0.005870	0.039344	-0.149204	0.8814
D(INFN(-4))	0.053964	0.033996	1.587388	0.1129
D(INFN(-5))	0.012652	0.033749	0.374892	0.7079
D(INFN(-6))	-0.008883	0.033587	-0.264468	0.7915
D(INFN(-7))	0.066756	0.033560	1.989152	0.0471
D(INFN(-8))	-0.000594	0.033635	-0.017662	0.9859
D(INFN(-9))	0.038605	0.033599	1.148979	0.2509
D(INFN(-10))	0.086179	0.033612	2.563952	0.0106
D(INFN(-11))	0.121304	0.033840	3.584680	0.0004
D(INFN(-12))	-0.519990	0.034222	-15.19442	0.0000
D(INFN(-13))	0.109210	0.039147	2.789724	0.0054
D(INFN(-14))	0.013925	0.039126	0.355893	0.7220
D(INFN(-15))	0.138050	0.036761	3.755317	0.0002
C	0.046340	0.018417	2.516196	0.0121

(b) If, in part (a), we concluded that *INFN* is stationary, then we can say immediately that *INFN* is of order I(0). If we used a 1% significance level in part (a), and concluded that *INFN* is nonstationary, then we need to test whether the first differences of *INFN* are stationary. The *p*-value for this purpose, given in the output below, is 0.0000. Thus, in this case we conclude that the first differences are stationary and that *INFN* is of order I(1).

Null Hypothesis: D(INFN) has a unit root
Exogenous: Constant
Lag Length: 14 (Automatic - based on SIC, maxlag=19)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-6.582550	0.0000
Test critical values:	1% level	-3.439044	
	5% level	-2.865267	
	10% level	-2.568811	

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(INFN,2)
Method: Least Squares
Sample (adjusted): 1955M12 2016M12
Included observations: 733 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(INFN(-1))	-0.534899	0.081260	-6.582550	0.0000
D(INFN(-1),2)	-0.083315	0.079232	-1.051530	0.2934
D(INFN(-2),2)	-0.023674	0.075150	-0.315026	0.7528
D(INFN(-3),2)	-0.037895	0.069283	-0.546959	0.5846
D(INFN(-4),2)	0.013621	0.068651	0.198414	0.8428
D(INFN(-5),2)	0.022343	0.067308	0.331952	0.7400
D(INFN(-6),2)	0.008274	0.065196	0.126909	0.8990
D(INFN(-7),2)	0.069531	0.062795	1.107274	0.2685
D(INFN(-8),2)	0.063071	0.060426	1.043767	0.2969
D(INFN(-9),2)	0.094944	0.057123	1.662111	0.0969
D(INFN(-10),2)	0.174345	0.053970	3.230403	0.0013
D(INFN(-11),2)	0.287846	0.050842	5.661565	0.0000
D(INFN(-12),2)	-0.242374	0.047330	-5.120898	0.0000
D(INFN(-13),2)	-0.136239	0.043306	-3.145941	0.0017
D(INFN(-14),2)	-0.128018	0.036806	-3.478174	0.0005
C	0.001331	0.010557	0.126107	0.8997

(c) The model chosen for forecasting is likely to depend on whether *INFN* was judged to be stationary or nonstationary. Estimates for models for both cases are given on the next page.

***INFN* stationary**

The autoregressive model $INFN_t = \delta + \sum_{r=1}^{16} \theta_r INFN_{t-r} + e_t$ was estimated. Sixteen lags were included in line with the Schwarz criterion. The forecast for 2017M1 from this model is

$$\widehat{INFN}_{2017M1} = \hat{\delta} + \sum_{r=1}^{16} \hat{\theta}_r INFN_{2017M1-r} = 1.9079$$

***INFN* nonstationary**

The autoregressive model $\Delta INFN_t = \delta + \sum_{r=1}^{15} \theta_r \Delta INFN_{t-r} + e_t$ was estimated. Fifteen lags were included in line with the Schwarz criterion. The forecast for 2017M1 from this model is

$$\widehat{INFN}_{2017M1} = INFN_{2016M12} + \hat{\delta} + \sum_{r=1}^{15} \hat{\theta}_r \left(INFN_{2017M1-r} - INFN_{2017M1-r-1} \right) = 1.8760$$

Dependent Variable: INFN
Method: Least Squares
Sample (adjusted): 1955M12 2016M12
Included observations: 733 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.046340	0.018417	2.516196	0.0121
INFN(-1)	1.371121	0.037040	37.01705	0.0000
INFN(-2)	-0.317203	0.063031	-5.032508	0.0000
INFN(-3)	-0.072378	0.063879	-1.133047	0.2576
INFN(-4)	0.059835	0.059374	1.007761	0.3139
INFN(-5)	-0.041312	0.054402	-0.759383	0.4479
INFN(-6)	-0.021535	0.054305	-0.396553	0.6918
INFN(-7)	0.075639	0.054258	1.394057	0.1637
INFN(-8)	-0.067350	0.054270	-1.241015	0.2150
INFN(-9)	0.039199	0.054310	0.721760	0.4707
INFN(-10)	0.047574	0.054262	0.876756	0.3809
INFN(-11)	0.035125	0.054299	0.646889	0.5179
INFN(-12)	-0.641294	0.054566	-11.75258	0.0000
INFN(-13)	0.629200	0.059391	10.59428	0.0000
INFN(-14)	-0.095286	0.063346	-1.504217	0.1330
INFN(-15)	0.124126	0.062300	1.992391	0.0467
INFN(-16)	-0.138050	0.036761	-3.755317	0.0002

Dependent Variable: D(INFN)
Method: Least Squares
Sample (adjusted): 1955M12 2016M12
Included observations: 733 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(INFN(-1))	0.381804	0.037042	10.30735	0.0000
D(INFN(-2))	0.059645	0.039652	1.504199	0.1330
D(INFN(-3))	-0.014204	0.039431	-0.360215	0.7188
D(INFN(-4))	0.051519	0.034148	1.508707	0.1318
D(INFN(-5))	0.008735	0.033884	0.257797	0.7966
D(INFN(-6))	-0.014058	0.033702	-0.417128	0.6767
D(INFN(-7))	0.061261	0.033669	1.819516	0.0692
D(INFN(-8))	-0.006446	0.033737	-0.191078	0.8485
D(INFN(-9))	0.031880	0.033683	0.946476	0.3442
D(INFN(-10))	0.079411	0.033694	2.356807	0.0187
D(INFN(-11))	0.113515	0.033898	3.348680	0.0009
D(INFN(-12))	-0.530227	0.034211	-15.49852	0.0000
D(INFN(-13))	0.106156	0.039320	2.699807	0.0071
D(INFN(-14))	0.008221	0.039265	0.209382	0.8342
D(INFN(-15))	0.128019	0.036781	3.480578	0.0005

EXERCISE 12.17

(a) The estimated model is

$$\hat{E}\left(YIELD_t \mid RAIN\right) = -1.4200 + 0.03024t + 1.0713RAIN_t - 0.12727RAIN_t^2$$

$$\begin{array}{lllll} \text{(se)} & (0.7829) & (0.00332) & (0.3825) & (0.04559) \\ (p\text{-values}) & (0.0765) & (0.0000) & (0.0075) & (0.0077) \end{array}$$

All coefficient estimates are significantly different from zero at a 5% level of significance, except for the intercept which is significant at a 10% level. Yield is trending upwards. The response to rainfall is positive initially, but with diminishing returns, and eventually becomes negative when rainfall exceeds 4.2dm.

(b) The residuals are plotted in Figure 12.17(a) and the correlogram is displayed in Figure 12.17(b). The residuals are positively correlated. In their plot, positive residuals tend to follow positive residuals and negative residuals tend to follow negative ones. In the correlogram the first-order autocorrelation is significantly different from zero.

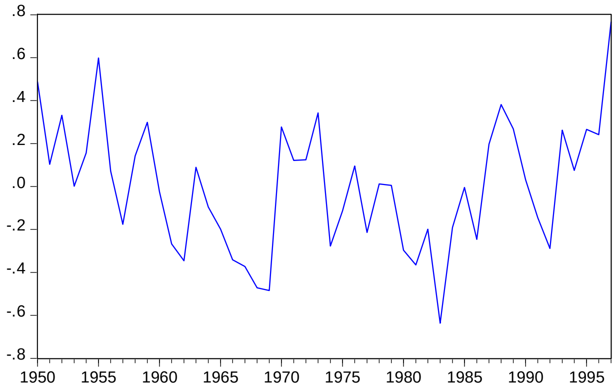


Figure 12.17(a) Residuals from linear trend model

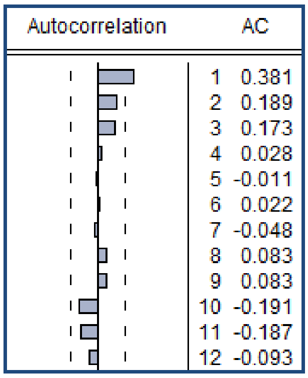


Figure 12.17(b) Correlogram from linear trend model

(c) The estimated model is

$$\hat{E}\left(YIELD_t \mid RAIN\right) = -0.2669 - 0.01902t + 0.001005t^2 + 0.7225RAIN_t - 0.08821RAIN_t^2$$

(se)	(0.6990)	(0.01120)	(0.000221)	(0.3273)	(0.03888)
(p-value)	(0.7045)	(0.0967)	(0.0000)	(0.0327)	(0.0284)

The estimated coefficients for the squared trend term and both *RAIN* terms are significant at a 5% level. The coefficient estimate for the linear trend term is significant at a 10% level. The intercept estimate is not significant, but there is no reason to discard it. The quadratic trend is such that yield declines until year 9 (1958) after which it increases at an increasing rate. Rainfall has a positive impact on yield, at a diminishing rate. At rainfalls greater than 4.1dm extra rain has a negative impact.

(d) The residuals are plotted in Figure 12.17(c) and the correlogram is displayed in Figure 12.17(d).

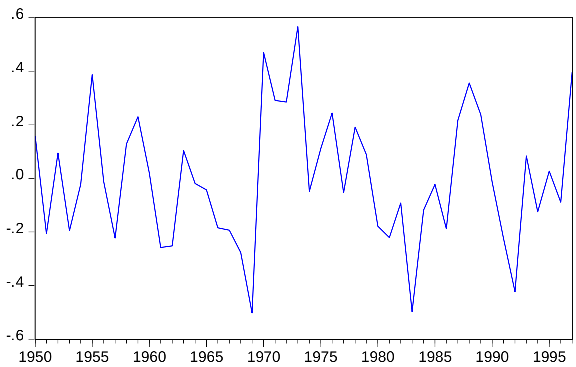


Figure 12.17(c) Residuals from quadratic trend model

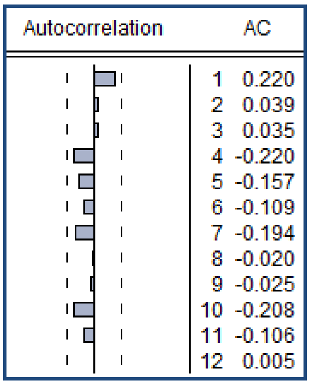


Figure 12.17(b) Correlogram from quadratic trend model

(e) To test whether these variables are trend stationary after subtracting out a quadratic trend, we fit regressions of the form

$$y_t = \alpha_1 + \alpha_2 t + \alpha_3 t^2 + u_t$$

and then test the OLS residuals $\hat{u}_t = y_t - a_1 - a_2 t - a_3 t^2$ for stationarity. In all 3 cases (*YIELD*, *RAIN* and $RAIN^2$), the Dickey-Fuller test equation was of the form

$$\Delta\hat{u}_t = \gamma\hat{u}_{t-1} + v_t$$

Intercept and trend terms, and augmentation terms, were not necessary. The τ values were

For *YIELD*: $\tau = -5.386$

For *RAIN*: $\tau = -5.931$

For $RAIN^2$: $\tau = -5.719$

Since we have eliminated a quadratic trend, not a linear trend, the critical values in the last row of Table 12.2 are not necessarily accurate. Nevertheless, the calculated τ values of less than -5 are sufficiently less than the linear trend value of -3.41 for us to safely conclude that all 3 variables are trend stationary after subtracting out the quadratic trend.

EXERCISE 12.19

Output for the error correction model, estimated with the lagged residuals from the cointegrating relationship, is given below, followed by a table that compares the two sets of estimates of like parameters.

Dependent Variable: DBR
Method: Least Squares
Sample (adjusted): 1955M01 2016M12
Included observations: 744 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
EHAT(-1)	-0.046369	0.011840	-3.916378	0.0001
DBR(-1)	0.272374	0.037395	7.283781	0.0000
DBR(-2)	-0.242099	0.037775	-6.408904	0.0000
DFFR	0.341750	0.023909	14.29362	0.0000
DFFR(-1)	-0.105296	0.027432	-3.838367	0.0001
DFFR(-2)	0.099082	0.027250	3.636005	0.0003
DFFR(-3)	-0.065954	0.024425	-2.700319	0.0071
DFFR(-4)	0.056073	0.022687	2.471557	0.0137
R-squared	0.359525	Mean dependent var		-0.000430

Parameter	Current estimates	Estimates (12.32)
$\pm\alpha$	-0.046369	0.046381
ϕ_1	0.272374	0.272365
ϕ_2	-0.242099	-0.242108
η_0	0.341750	0.341783
η_1	-0.105296	-0.105320
η_2	0.099082	0.099059
η_3	-0.065954	-0.065975
η_4	0.056073	0.056041

The two sets of estimates are virtually identical, except for a change in sign for $\hat{\alpha}$. The change in sign occurs because, in (12.32), the equation was written with a negative sign in front of α.

EXERCISE 12.21

(a) Results from the three Dickey-Fuller tests are:

(1) Dickey Fuller test (no intercept and no trend term)

$$\widehat{\Delta CSI}_t = -0.001 CSI_{t-1}$$
$$(tau) \quad (-0.299)$$

Since the *tau* value (–0.299) is greater than the 5% critical value of –1.94, the null hypothesis of nonstationarity is not rejected. The variable *CSI* is not stationary.

(2) Dickey Fuller test (intercept but no trend term)

$$\widehat{\Delta CSI}_t = -0.051 CSI_{t-1} + 4.500$$
$$(tau) \quad (-3.001)$$

Since the *tau* value (–3.001) is less than the 5% critical value of –2.86, the null hypothesis of nonstationarity is rejected. The variable *CSI* is stationary.

(3) Dickey Fuller test (intercept and trend term)

$$\widehat{\Delta CSI}_t = -0.068 CSI_{t-1} + 5.309 + 0.004t$$
$$(tau) \quad (-3.483)$$

Since the *tau* value (–3.483) is less than the 5% critical value of –3.41, the null hypothesis of nonstationarity is rejected. The variable *CSI* is stationary.

The result of the Dickey-Fuller test without an intercept term is not consistent with the other two results. This is because it assumes that when the alternative hypothesis of stationarity is true, the series has a zero mean. This assumption is not correct (see graph in part (b)).

(b) The graph in Figure xr12.21 suggests that we should use the Dickey-Fuller test with an intercept term.

(c) Since *CSI* is stationary, it suggests that the effect of news is temporary; hence consumers “remember” and “retain” news information for only a short time.

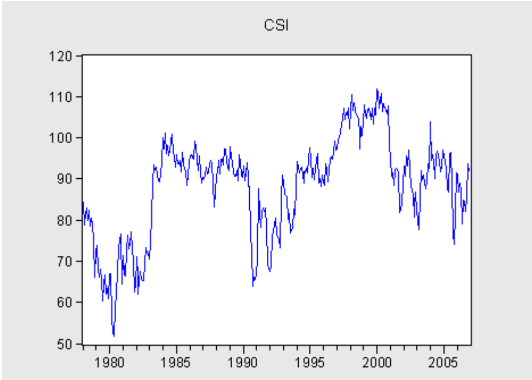


Figure xr12.21 Plot of time series *CSI*

EXERCISE 12.23

Plots from the data in *inter2.dat* are shown below. The graphs show the level Y_t, the first difference $DY = \Delta Y_t = Y_t - Y_{t-1}$, and the second difference $D2Y = \Delta^2 Y_t = \Delta Y_t - \Delta Y_{t-1}$.

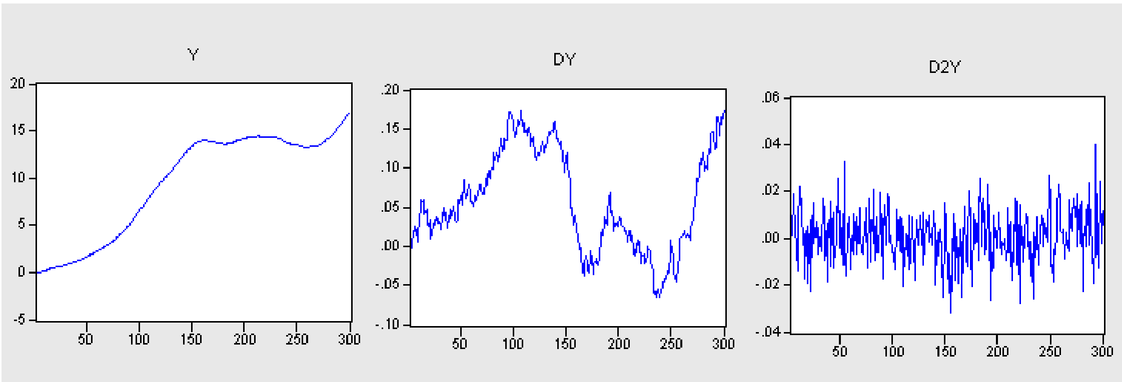


Figure xr12.23 Plots of *Y* and its first and second differences

The Dickey-Fuller unit root tests are shown below.

$$\widehat{\Delta Y_t} = -0.001Y_{t-1} + 0.001 + 0.00006t + 0.991\Delta Y_{t-1}$$
$$(tau) \quad (-3.371)$$

$$\widehat{\Delta(\Delta Y_t)} = -0.011\Delta Y_{t-1} + 0.001 \qquad \widehat{\Delta(\Delta^2 Y_t)} = -0.987\Delta^2 Y_{t-1}$$
$$(tau) \quad (-1.088) \qquad\qquad (tau) \quad (-16.940)$$

Since Y_t clearly has a trend, its Dickey-Fuller test equation includes an intercept and a trend. The *tau* value (–3.371) is greater than the 5% critical value of –3.41; thus, the null hypothesis of nonstationarity is not rejected. The variable Y_t is not stationary.

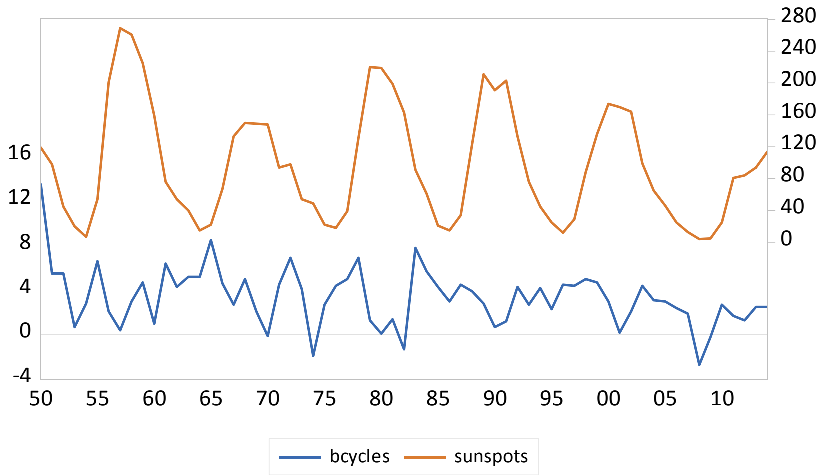
Since DY_t is fluctuating around a constant, its Dickey-Fuller test equation includes an intercept. Because the *tau* value (–1.088) is greater than the 5% critical value of –2.86, the null hypothesis of nonstationarity is not rejected. The variable DY_t is not stationary.

Since $D2Y_t$ is fluctuating around zero, its Dickey-Fuller test equation does not include an intercept. Because the *tau* value (–16.940) is less than the 5% critical value of –1.94, the null hypothesis of nonstationarity is rejected. The variable $D2Y_t$ is stationary.

Thus, Y_t needs to be differenced twice to achieve stationarity; thus Y_t is integrated of order 2.

EXERCISE 12.25

(a) The plots of the two series appear in the following graph with GDP growth measured on the left axis and sunspot activity measured on the right axis. There is some evidence that troughs in the business cycle occur when sunspot activity is greatest, but it is difficult to conclude that the business cycle tends to follow sunspot activity.



(b) Output for unit root tests for each of the series follows. In both cases the *p*-values for the tests are 0.0000, indicating that both series are stationary.

Null Hypothesis: BCYCLES has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-7.176301	0.0000
Test critical values:	1% level	-3.536587	
	5% level	-2.907660	
	10% level	-2.591396	

Null Hypothesis: SUNSPOTS has a unit root
Exogenous: Constant
Lag Length: 2 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-6.432291	0.0000
Test critical values:	1% level	-3.540198	
	5% level	-2.909206	
	10% level	-2.592215	

(c) An ARDL model to test whether sunspots can be used to predict business cycles is of the form

$$BCYCLES_t = \delta + \sum_{s=1}^{p} \theta_s BCYCLES_{t-s} + \sum_{r=1}^{q} \delta_r SUNSPOTS_{t-r} + e_t$$

To use this model to test whether sunspots can be used to predict business cycles, we need to select suitable values for p and q and then test the hypothesis

$$H_0 : \delta_1 = \delta_2 = \cdots = \delta_q = 0$$

Schwarz criterion values for $p = 0, 1, 2, \ldots, 8$ and $q = 1, 2, \ldots, 8$ are given in the following table. The minimizing values are $p = 0$ and $q = 1$. A value $p = 0$ implies lagged values of *BCYCLES* do not appear in the equation.

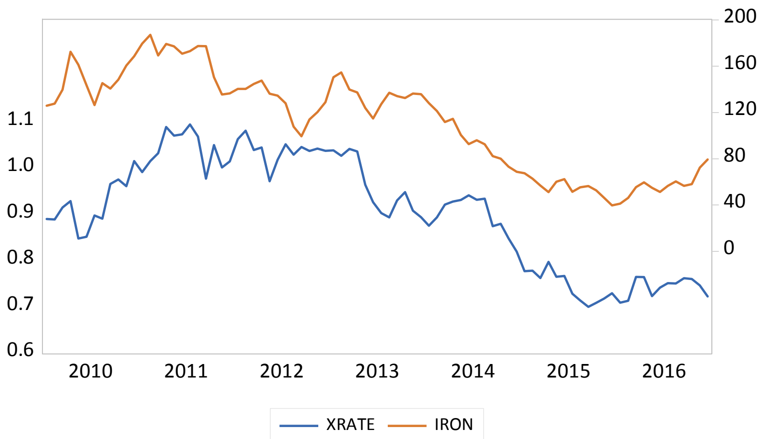
	$q=1$	$q=2$	$q=3$	$q=4$	$q=5$	$q=6$	$q=7$	$q=8$
$p=0$	**1.724003**	**1.742675**	1.778997	1.817294	1.874013	1.903025	1.963897	1.878685
$p=1$	**1.729063**	1.800063	1.848571	1.903059	1.988842	2.054519	2.100675	2.170251
$p=2$	1.765240	1.827557	1.889745	1.928191	2.010593	2.074025	2.126620	2.209258
$p=3$	1.816913	1.853814	1.918089	1.950025	2.034572	2.097677	2.153267	2.236409
$p=4$	1.853126	1.891508	1.939690	2.002594	2.087666	2.156872	2.214355	2.290579
$p=5$	1.919790	1.940930	2.000019	2.064166	2.114933	2.189382	2.238978	2.302102
$p=6$	1.934487	1.974330	2.030969	2.093376	2.155330	2.224441	2.280530	2.356183
$p=7$	1.974592	1.978672	2.043435	2.111826	2.178843	2.243109	2.298280	2.366466
$p=8$	1.915929	1.944536	2.013079	2.084007	2.154209	2.224637	2.269379	2.333242

The *F*-values for testing $H_0 : \delta_1 = \delta_2 = \cdots = \delta_q = 0$ for the 3 smallest values of the Schwartz criterion are given in the following table. In all cases we cannot reject the null hypothesis at a 5% level of significance. The results do not support Jevons' theory.

Model	*F*-value	*p*-value
$p=0, q=1$	2.529	0.1169
$p=0, q=2$	2.756	0.0716
$p=1, q=1$	1.506	0.2245

EXERCISE 12.27

(a) The two series are plotted in the figure below with *XRATE* on the left axis and *IRON* on the right axis. They have similar trends, but it is less clear whether the fluctuations around those trends move together.



(b) In the equation to test for stationarity of the exchange rate, it is not clear whether a trend should be included. Although the overall trend of the exchange rate is downwards, it trends upwards at the beginning of the sample period, and there is no theory-based reason for it to always have a downward trend. We therefore exclude a trend and obtain the following test results.

Null Hypothesis: XRATE has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=11)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.729565	0.8328
Test critical values:	1% level	-3.511262	
	5% level	-2.896779	
	10% level	-2.585626	

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(XRATE)
Method: Least Squares
Sample (adjusted): 2010M02 2016M12
Included observations: 83 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
XRATE(-1)	-0.021654	0.029680	-0.729565	0.4678
C	0.017671	0.027217	0.649289	0.5180

From the *p*-value of 0.8328, we cannot reject the null hypothesis of a unit root. We conclude that the exchange rate is nonstationary. Accepting the null hypothesis of a unit root, and using the model chosen by the Schwarz criterion, the model that best represents the relationship between current and past exchange rates is the random walk

$$XRATE_t = XRATE_{t-1} + v_t$$

If a trend term is included in the test equation, we obtain a *p*-value of 0.3329, which again leads us to conclude that the exchange rate is nonstationary. In this case, the model that best represents the relationship would be a random walk with drift

$$XRATE_t = \delta + XRATE_{t-1} + v_t$$

(c) Output for testing whether the iron ore price is stationary follows. In the equation without a trend, we obtain a *p*-value of 0.6672, indicating that the price is nonstationary. [Including a trend gives a *p*-value of 0.0605; at the 5% level of significance we conclude the iron ore price is nonstationary.]

Null Hypothesis: IRON has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=11)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.209195	0.6672
Test critical values:	1% level	-3.512290	
	5% level	-2.897223	
	10% level	-2.585861	

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(IRON)
Method: Least Squares
Sample (adjusted): 2010M03 2016M12
Included observations: 82 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
IRON(-1)	-0.030410	0.025149	-1.209195	0.2302
D(IRON(-1))	0.229997	0.110367	2.083927	0.0404
C	2.993983	3.044766	0.983321	0.3285

(d) To investigate whether the exchange rate and the iron ore price move together, we estimate the potential long-run cointegrating relationship $XRATE_t = \beta_1 + \beta_2 IRON_t + e_t$ and test whether the residuals from this relationship are stationary. Estimates of the parameters of the equation $XRATE_t = \beta_1 + \beta_2 IRON_t + e_t$ are given below, followed by the unit root test results on the residuals. Comparing the value of the unit root test statistic $\tau = -2.724$ with the 5% critical value of –3.37, found in Table 12.4 on page 583 of *POE5*, we cannot reject the null hypothesis of a unit root in the residuals. We conclude therefore that the exchange rate and the iron ore price are not cointegrated; we cannot establish that there is a long run relationship between them.

Dependent Variable: XRATE
Method: Least Squares
Sample: 2010M01 2016M12
Included observations: 84

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.637806	0.019655	32.44943	0.0000
IRON	0.002391	0.000163	14.65255	0.0000

Null Hypothesis: EHAT has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=11)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.724406	0.0069
Test critical values:	1% level	-2.593121	
	5% level	-1.944762	
	10% level	-1.614204	

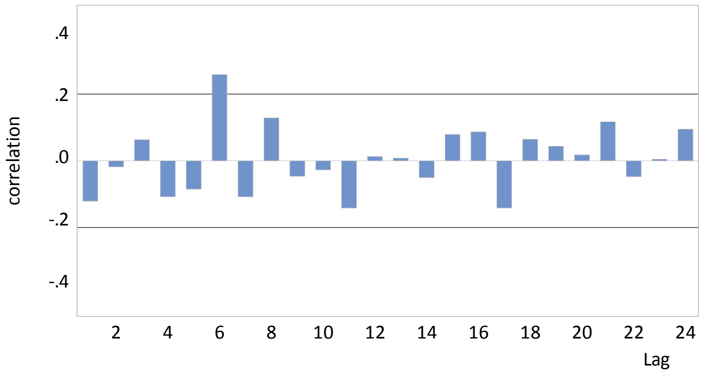
Augmented Dickey-Fuller Test Equation
Dependent Variable: D(EHAT)
Method: Least Squares
Sample (adjusted): 2010M02 2016M12
Included observations: 83 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
EHAT(-1)	-0.178214	0.065414	-2.724406	0.0079

(e) Given that *XRATE* and *IRON* are both nonstationary and not cointegrated, to investigate whether the iron price can be used to forecast the exchange rate, we estimate an ARDL model in first differences

$$\Delta XRATE_t = \delta + \sum_{s=1}^{p} \theta_s \Delta XRATE_{t-s} + \sum_{r=1}^{q} \delta_r \Delta IRON_{t-r} + e_t$$

A check of the correlogram for $\Delta XRATE_t$ reveals little if any serial correlation, suggesting its lags could be omitted from the equation, and we can focus on whether changes in the iron ore price have a significant impact on exchange rate changes. The significance bounds are drawn at $\pm 1.96/\sqrt{83} = \pm 0.215$.



Estimating $\Delta XRATE_t = \delta + \sum_{r=1}^{q} \delta_r \Delta IRON_{t-r} + e_t$ for $q = 1, 2, 3$ and 4, and testing the null hypothesis $H_0 : \delta_1 = \delta_2 = \cdots = \delta_r = 0$ in each case gives the *F*-test values and corresponding *p*-values that are displayed in the following table.

q	*F*-value	*p*-value
1	0.315	0.5763
2	0.170	0.8441
3	0.143	0.9336
4	0.123	0.9737

In all cases the null hypothesis $H_0 : \delta_1 = \delta_2 = \cdots = \delta_r = 0$ cannot be rejected. We conclude that the iron ore price cannot be used to help forecast the exchange rate.

EXERCISE 12.29

(a) The plot in Figure 12.29(a) suggests that *EAF* has an upward trend which is approximately linear. Thus, it does not appear to be stationary, nor does the logistic growth curve appear to be a suitable model for capturing changes in *EAF*.

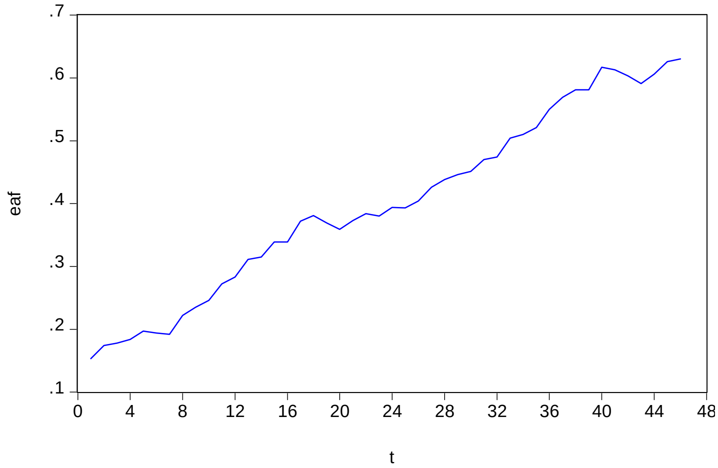


Figure 12.29(a) Share of U.S. steel produced by electric arc furnace technology

(b) Using a Dickey-Fuller test equation with a trend term and no augmentation terms gives a tau value of $\tau = -2.27$. Since this value is greater than the 5% critical value of -3.41, we cannot reject a null hypothesis that *EAF* has a unit root.

(c) Nonlinear least squares estimates are given in the following table.

	Coefficient	Std. Error	*t*-value	*p*-value
$\hat{\alpha}$	0.814375	0.051050	15.95247	0.0000
$\hat{\beta}$	-1.377672	0.056356	-24.44599	0.0000
$\hat{\delta}$	0.057223	0.004304	13.29554	0.0000

The residuals are plotted in Figure 12.29(b). They are clearly autocorrelated, but it is difficult to assess from the plot whether they are stationary. Carrying out a unit root test with no intercept and no augmentation term, we have $\tau = -2.31$ and 5% critical value -1.94. Including an intercept term gives $\tau = -2.28$, and 5% critical value -2.86. The test outcome depends on which test is chosen. Since we are testing the residuals from a nonlinear estimation, those residuals will not necessarily have a zero mean. Also, although the critical values can be used as guide, they are not necessarily the correct ones to use after a nonlinear estimation. Recognising all these facts, we conclude that it has not been possible to establish that the residuals are stationary.

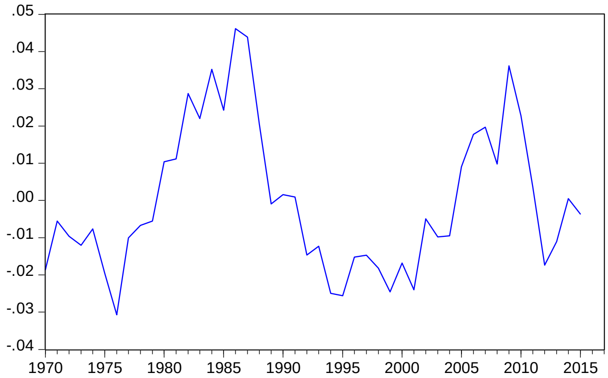


Figure 12.29(b) Residuals from nonlinear least squares estimation

(d) Using a Dickey-Fuller test equation with an intercept and no augmentation terms gives a tau value of $\tau = -7.02$. Since this value is less than the 5% critical value of -2.86, we reject a null hypothesis that ΔEAF has a unit root, and conclude that this first difference is stationary.

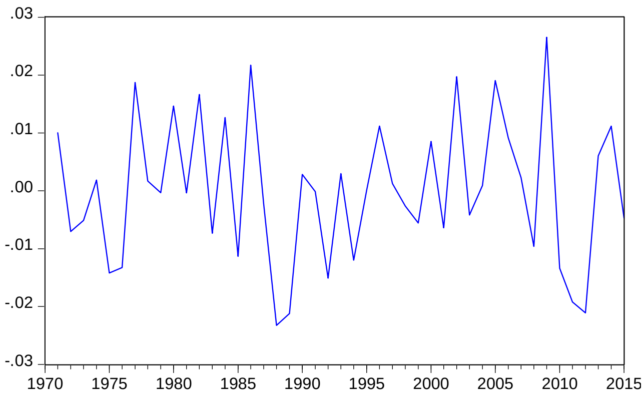
(e) A first differenced version of the model is

$$y_t - y_{t-1} = \frac{\alpha}{1+\exp(-\beta-\delta t)} - \frac{\alpha}{1+\exp\left(-\beta-\delta(t-1)\right)} + v_t$$

Estimates from this specification are given in the following table.

	Coefficient	Std. Error	*t*-value	*p*-value
$\hat{\alpha}$	1.410410	3.323493	0.424376	0.6735
$\hat{\beta}$	-0.411822	2.667195	-0.154403	0.8780
$\hat{\delta}$	0.032208	0.078014	0.412848	0.6818

The plot of the residuals below suggests that they are stationary and free from serial correlation. Carrying out a unit root test with no intercept and no augmentation term, we have $\tau = -7.13$ and 5% critical value -1.94. Including an intercept term gives $\tau = -7.04$, and 5% critical value -2.86. Once again, the critical values do not allow for the fact that the series is obtained from a nonlinear estimation, but, nevertheless, the large values for τ do suggest the residuals are stationary.



(f) The unit root tests and the vast difference in the estimates from parts (c) and (e) suggest that *EAF* is not trend stationary. The relatively small standard errors for the estimates in part (c) give the appearance that those estimates are reliable. However, all other evidence suggests otherwise. The linear plot in part (a) did not resemble a logistic curve, unit root tests suggested that *EAF* is nonstationary, and estimating a differenced version of the model changed the estimates dramatically – the saturation parameter became greater than 1, and the inflection point $-\beta/\delta$ halved from 24.1 to 12.8. The large standard errors in part (e) show that none of the parameters have been reliably estimated. We can conclude that the estimates in part (c) are spurious.