PRINCIPLES OF ECONOMETRICS

5TH EDITION

ANSWERS TO ODD-NUMBERED EXERCISES IN CHAPTER 9

EXERCISE 9.1

(a) Subtracting and adding the term $E\left(y_{T+1}|I_T\right)$ to $E\left[\left(\hat{y}_{T+1}-y_{T+1}\right)^2 | I_T\right]$ gives

$$E\left[\left(\hat{y}_{T+1}-y_{T+1}\right)^2 | I_T\right]=E\left[\left\{\hat{y}_{T+1}-E\left(y_{T+1}|I_T\right)+E\left(y_{T+1}|I_T\right)-y_{T+1}\right\}^2 | I_T\right]$$

$$=E\left[\left\{\left(\hat{y}_{T+1}-E\left(y_{T+1}|I_T\right)\right)-\left(y_{T+1}-E\left(y_{T+1}|I_T\right)\right)\right\}^2 | I_T\right]$$

(b) Expanding the right-hand side of the equation in part (a), we have

$$E\left[\left\{\left(\hat{y}_{T+1}-E\left(y_{T+1}|I_T\right)\right)-\left(y_{T+1}-E\left(y_{T+1}|I_T\right)\right)\right\}^2 | I_T\right]$$

$$=E\left[\left(\hat{y}_{T+1}-E\left(y_{T+1}|I_T\right)\right)^2 | I_T\right]+E\left[\left(y_{T+1}-E\left(y_{T+1}|I_T\right)\right)^2 | I_T\right]$$
$$-2E\left[\left(\hat{y}_{T+1}-E\left(y_{T+1}|I_T\right)\right)\left(y_{T+1}-E\left(y_{T+1}|I_T\right)\right)| I_T\right]$$

$$=E\left[\left(\hat{y}_{T+1}-E\left(y_{T+1}|I_T\right)\right)^2 | I_T\right]+E\left[\left(y_{T+1}-E\left(y_{T+1}|I_T\right)\right)^2 | I_T\right]$$
$$-2\left(\hat{y}_{T+1}-E\left(y_{T+1}|I_T\right)\right)E\left[\left(y_{T+1}-E\left(y_{T+1}|I_T\right)\right)| I_T\right]$$

$$=E\left[\left(\hat{y}_{T+1}-E\left(y_{T+1}|I_T\right)\right)^2 | I_T\right]+E\left[\left(y_{T+1}-E\left(y_{T+1}|I_T\right)\right)^2 | I_T\right]$$
$$-2\left(\hat{y}_{T+1}-E\left(y_{T+1}|I_T\right)\right)\left(E\left(y_{T+1}| I_T\right)-E\left(y_{T+1}|I_T\right)\right)$$

$$=E\left[\left(\hat{y}_{T+1}-E\left(y_{T+1}|I_T\right)\right)^2 | I_T\right]+E\left[\left(y_{T+1}-E\left(y_{T+1}|I_T\right)\right)^2 | I_T\right]$$

Both terms in the last line of this equation are positive. The second term does not depend on $\hat{y}_{T+1}$. Thus, the mean-squared forecast error is minimized by making the first term zero which is achieved by setting $\hat{y}_{T+1}=E\left(y_{T+1}|I_T\right)$.

EXERCISE 9.3

Lagging the equation $y_t=\delta+\theta_1 y_{t-1}+\theta_2 y_{t-2}+e_t$ by one and multiplying it by ρ yields

$$\rho y_{t-1}=\rho\delta+\rho\theta_1 y_{t-2}+\rho\theta_2 y_{t-3}+\rho e_{t-1}$$

Subtracting this equation from $y_t=\delta+\theta_1 y_{t-1}+\theta_2 y_{t-2}+e_t$ gives

$$y_t-\rho y_{t-1}=\delta(1-\rho)+\theta_1 y_{t-1}+\left(\theta_2-\rho\theta_1\right)y_{t-2}-\rho\theta_2 y_{t-3}+e_t-\rho e_{t-1}$$

$$y_t=\delta(1-\rho)+\left(\theta_1+\rho\right)y_{t-1}+\left(\theta_2-\theta_1\rho\right)y_{t-2}-\theta_2\rho y_{t-3}+v_t$$

Taking expectations conditional on I_{t-1} yields

$$E\left(y_t | I_{t-1}\right)=E\left[\left(\delta(1-\rho)+\left(\theta_1+\rho\right)y_{t-1}+\left(\theta_2-\theta_1\rho\right)y_{t-2}-\theta_2\rho y_{t-3}+v_t\right)| I_{t-1}\right]$$

$$=\delta(1-\rho)+\left(\theta_1+\rho\right)y_{t-1}+\left(\theta_2-\theta_1\rho\right)y_{t-2}-\theta_2\rho y_{t-3}+E\left(v_t | I_{t-1}\right)$$

$$=\delta(1-\rho)+\left(\theta_1+\rho\right)y_{t-1}+\left(\theta_2-\theta_1\rho\right)y_{t-2}-\theta_2\rho y_{t-3}$$

If the errors e_t are autocorrelated, then $E(e_t \mid e_{t-s}) \neq 0$ for some $s > 0$. Now,

$$E(y_t \mid I_{t-1}) = \delta + \theta_1 y_{t-1} + \theta_2 y_{t-2} + E(e_t \mid I_{t-1})$$

Also, $E(e_t \mid e_{t-s}) \neq 0$ implies $E(e_t \mid I_{t-1}) \neq 0$ and thus $E(y_t \mid I_{t-1}) \neq \delta + \theta_1 y_{t-1} + \theta_2 y_{t-2}$.

EXERCISE 9.5

(a) From $e_t = \rho e_{t-1} + v_t$, we have $e_{t-s}e_t = \rho e_{t-s}e_{t-1} + e_{t-s}v_t$ and hence, and assuming stationarity,

$$E(e_{t-1}e_t) = \rho E(e_{t-1}^2) + E(e_{t-1}v_t) = \rho\sigma_e^2$$

$$E(e_{t-2}e_t) = \rho E(e_{t-2}e_{t-1}) + E(e_{t-2}v_t) = \rho(\rho\sigma_e^2) = \rho^2\sigma_e^2$$

$$E(e_{t-3}e_t) = \rho E(e_{t-3}e_{t-1}) + E(e_{t-3}v_t) = \rho(\rho^2\sigma_e^2) = \rho^3\sigma_e^2$$

$$\vdots$$

Then, $\rho_s = E(e_t e_{t-s}) / \operatorname{var}(e_t) = \rho^s\sigma_e^2 / \sigma_e^2 = \rho^s$.

(b) From $e_t = \phi v_{t-1} + v_t$, we have

$$e_t e_{t-1} = (\phi v_{t-1} + v_t)(\phi v_{t-2} + v_{t-1}) = \phi^2 v_{t-1}v_{t-2} + \phi v_{t-1}^2 + \phi v_t v_{t-2} + v_t v_{t-1}$$

$$E(e_t e_{t-1}) = E(\phi v_{t-1}^2) = \phi\sigma_v^2$$

$$e_t e_{t-s} = (\phi v_{t-1} + v_t)(\phi v_{t-s-1} + v_{t-s}) = \phi^2 v_{t-1}v_{t-s-1} + \phi v_{t-1}v_{t-s} + \phi v_t v_{t-s-1} + v_t v_{t-s}$$

For $s \geq 2$, the expectations of all terms on the right-hand side of the above equation are zero and hence, $E(e_t e_{t-s}) = 0$ for $s \geq 2$. Also,

$$\operatorname{var}(e_t) = \phi^2 \operatorname{var}(v_{t-1}) + \operatorname{var}(v_t) = (\phi^2 + 1)\sigma_v^2$$

Then,

$$\rho_1 = E(e_t e_{t-1}) / \operatorname{var}(e_t) = \phi\sigma_v^2 / (1+\phi^2)\sigma_v^2 = \phi / (1+\phi^2)$$

$$\rho_s = E(e_t e_{t-s}) / \operatorname{var}(e_t) = 0 / (1+\phi^2)\sigma_v^2 = 0 \quad \text{for } s \geq 2$$

The autocorrelations for the AR(1) error are always nonzero, but decline geometrically as the time between errors increases, eventually becoming negligible. For the MA(1) error, only errors that are one period apart have a nonzero correlation. Errors which are two or more periods apart are uncorrelated.

EXERCISE 9.7

(a) In this question, we assume all results are conditional on the sample $\mathbf{x}$, although the conditioning is not made explicit. Then,

$$\operatorname{var}(e_1^*) = (1-\rho^2)\operatorname{var}(e_1) = (1-\rho^2)\frac{\sigma_v^2}{1-\rho^2} = \sigma_v^2$$

To show that e_1^* is uncorrelated with v_t, $t = 2,3,\ldots,T$, we note that

$$e_1^* = \left(1-\rho^2\right)^{1/2} e_1 = \left(1-\rho^2\right)^{1/2}\left(v_1 + \rho v_0 + \rho^2 v_{-1} + \rho^3 v_{-2} + \cdots\right)$$

Because e_1^* depends only on current and past values of *v*, and the v_t are uncorrelated, e_1^* will be uncorrelated with all future values of v_t, $t = 2,3,...,T$.

(b) For $t = 2,3,...,T$, the transformed model is obtained by subtracting $\rho y_{t-1} = \rho\alpha + \rho\beta_0 x_{t-1} + \rho e_{t-1}$ from $y_t = \alpha + \beta_0 x_t + e_t$ to yield

$$y_t - \rho y_{t-1} = \alpha(1-\rho) + \beta_0\left(x_t - \rho x_{t-1}\right) + e_t - \rho e_{t-1}$$

or

$$y_t^* = \alpha x_{0t}^* + \beta_0 x_t^* + e_t^* \qquad \text{for } t = 2,3,...,T$$

For $t = 1$, the transformed model $y_1^* = \alpha x_{01}^* + \beta_0 x_1^* + e_1^*$ is obtained by multiplying both sides of the equation $y_1 = \alpha + \beta_0 x_1 + e_1$ by $\sqrt{1-\rho^2}$.

OLS applied to the transformed model for $t = 1,2,3,...,T$, will yield a minimum variance estimator because the error terms $\left(e_1^*, v_2, v_3, \ldots, v_T\right) = \left(e_1^*, e_2^*, e_3^*, \ldots, e_T^*\right)$ are uncorrelated and have minimum variance.

EXERCISE 9.9

(a) The number of degrees of freedom is $157 - 2 - 4 = 151$.

(b) Overall, advertising has a positive impact on sales revenue. There is a positive effect in the current week and in the following two weeks, but no effect after 3 weeks. The greatest impact is generated after one week. The total effect of a sustained $1 million increase in advertising expenditure is given by

$$\text{total multiplier} = b_0 + b_1 + b_2 = 1.006 + 3.926 + 2.372 = 7.304$$

(c) Relevant information for the significance tests is given in the following table. The 5% and 10% critical values for a two-tail test are $t_{(0.975,151)} = 1.976$ and $t_{(0.95,151)} = 1.655$, respectively. The 5% and 10% critical values for a one-tail test are $t_{(0.95,151)} = 1.655$ and $t_{(0.90,151)} = 1.287$, respectively. We use * to denote significance at a 10% level and ** to denote significance at the 5% level. No * implies a lack of significance. We find that b_1 and b_2 are significant for both types of test and for both significance levels; b_0 is only significant at the 10% level using a one-tail test.

Coefficient	Standard Error	*t*-Value	Two-tail *p*-value	One-tail *p*-value
b_0	0.6941	1.449	0.149	0.075*
b_1	0.8471	4.635	0.000**	0.000**
b_2	0.6865	3.455	0.001**	0.000**

(d) Using $t_c = t_{(0.975,151)} = 1.976$, the 95% confidence interval for the impact multiplier is given by

$$b_0 \pm t_c \times \text{se}(b_0) = 1.006 \pm 1.976 \times 0.6941 = (-0.366, 2,378)$$

The one-period interim multiplier is $b_0 + b_1 = 4.932$, with standard error given by $\text{se}(b_0 + b_1) = 0.7246$. The 95% confidence interval for the one-period interim multiplier is

$$\left(b_0 + b_1\right) \pm t_c \times \text{se}(b_0 + b_1) = 4.932 \pm 1.976 \times 0.7246 = (3.500, 6.364)$$

The total multiplier is $b_0 + b_1 + b_2 = 7.304$, with standard error given by $\text{se}(b_0 + b_1 + b_2) = 0.6186$. The 95% confidence interval for the total multiplier is given by

$$\left(b_0 + b_1 + b_2\right) \pm t_c \times \text{se}(b_0 + b_1 + b_2) = 7.304 \pm 1.976 \times 0.6186 = (6.082, 8.526)$$

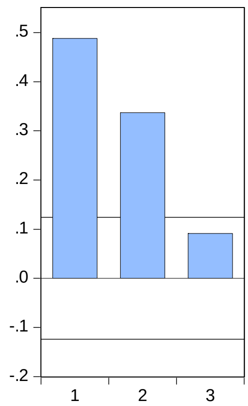
EXERCISE 9.11

(a) The first three autocorrelations are $r_1 = 0.4882$, $r_2 = 0.3369$, and $r_3 = 0.0916$.

To test whether the autocorrelations are significantly different from zero, the null and alternative hypotheses are $H_0 : \rho_k = 0$ and $H_1 : \rho_k \neq 0$, and the test statistic is given by $z_k = \sqrt{T} r_k = 15.8114 r_k$. At a 5% level of significance, the critical values are ± 1.96; thus, we reject the null hypothesis if $|z_k| > 1.96$. The test results are provided in the table below.

Autocorrelations	*z*-statistic	Critical value	Decision
$r_1 = 0.4882$	7.719	$\pm$ 1.96	Reject H_0
$r_2 = 0.3369$	5.327	$\pm$ 1.96	Reject H_0
$r_3 = 0.0916$	1.448	$\pm$ 1.96	Do not reject H_0

The significance bounds for the correlogram are $\pm 1.96/\sqrt{250} = \pm 0.124$. It leads us to the same conclusion as the hypothesis tests.



(b) The least-squares estimates for θ_1 and δ are $\hat{\theta}_1 = 0.4892$, and $\hat{\delta} = 0.8480$. The estimated value $\hat{\theta}$ is slightly larger than r_1 because the summation in the denominator for r_1 has one more squared term than the summation in the denominator for $\hat{\theta}$. The means are also slightly different.

EXERCISE 9.13

(a) Ignoring for the moment the error terms that are considered in part (e), the ARDL model can be written as $y_t = \left(1 - \theta_1 L - \theta_3 L^3\right)^{-1} \left(\delta + \delta_1 L x_t\right)$ and the IDL representation as

$$y_t = \alpha + \left(\beta_0 + \beta_1 L + \beta_2 L^2 + \beta_3 L^3 + \cdots\right) x_t$$

Equating the two formulations, we have

$$\left(1-\theta_1 L - \theta_3 L^3\right)^{-1}\left(\delta + \delta_1 L x_t\right) = \alpha + \left(\beta_0 + \beta_1 L + \beta_2 L^2 + \beta_3 L^3 + \cdots\right) x_t$$

$$\delta + \delta_1 L x_t = \left(1-\theta_1 L - \theta_3 L^3\right)\alpha + \left(1-\theta_1 L - \theta_3 L^3\right)\left(\beta_0 + \beta_1 L + \beta_2 L^2 + \beta_3 L^3 + \cdots\right) x_t$$

from which we obtain

$$\delta = \left(1-\theta_1-\theta_3\right)\alpha$$

$$\begin{aligned}\delta_1 L &= \left(1-\theta_1 L - \theta_3 L^3\right)\left(\beta_0 + \beta_1 L + \beta_2 L^2 + \beta_3 L^3 + \cdots\right)\\ &= \beta_0 + \beta_1 L + \beta_2 L^2 + \beta_3 L^3 + \beta_4 L^4 + \beta_5 L^5 + \cdots\\ &\quad - \theta_1\beta_0 L - \theta_1\beta_1 L^2 - \theta_1\beta_2 L^3 - \theta_1\beta_3 L^4 - \theta_1\beta_4 L^5 - \cdots\\ &\quad - \theta_3\beta_0 L^3 - \theta_3\beta_1 L^4 - \theta_3\beta_2 L^5 - \cdots\end{aligned}$$

Equating coefficients of like powers of the lag operator yields

$$\beta_0 = 0 \qquad \beta_1 - \theta_1\beta_0 = \delta_1 \qquad \beta_2 - \theta_1\beta_1 = 0 \qquad \beta_3 - \theta_1\beta_2 - \theta_3\beta_0 = 0$$

$$\beta_4 - \theta_1\beta_3 - \theta_3\beta_1 = 0 \qquad \beta_5 - \theta_1\beta_4 - \theta_3\beta_2 = 0$$

From which we obtain,

$$\beta_0 = 0 \qquad \beta_1 = \delta_1 \qquad \beta_2 = \theta_1\beta_1 \qquad \beta_3 = \theta_1\beta_2 \qquad \beta_s = \theta_1\beta_{s-1} + \theta_3\beta_{s-3} \;\; s \ge 4$$

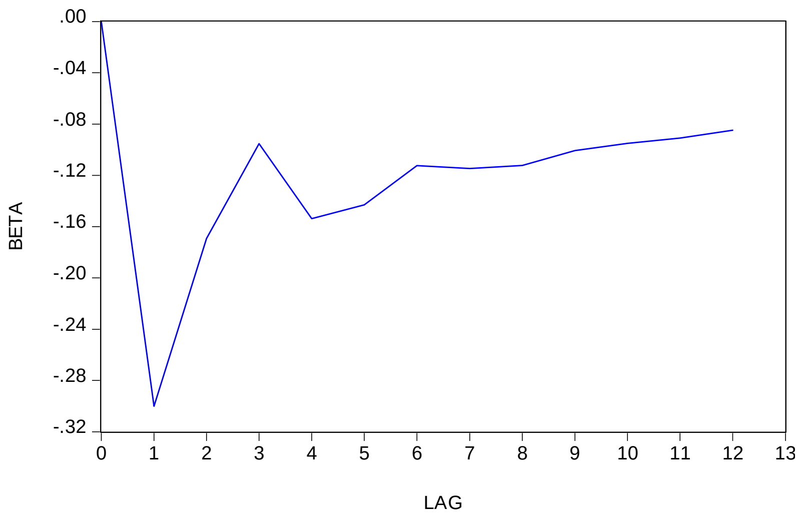
Also, from $\delta = \left(1-\theta_1-\theta_3\right)\alpha$, it follows that $\alpha = \delta/\left(1-\theta_1-\theta_3\right)$.

(b) Estimates of the first 12 lag weights are:

$\hat{\beta}_0 = 0,\ \hat{\beta}_1 = -0.3,\ \hat{\beta}_2 = -0.1692,\ \hat{\beta}_3 = -0.09543,\ \hat{\beta}_4 = -0.15372,\ \hat{\beta}_5 = -0.14304,$

$\hat{\beta}_6 = -0.11245,\ \hat{\beta}_7 = -0.11461,\ \hat{\beta}_8 = -0.11228,\ \hat{\beta}_9 = -0.10077,\ \hat{\beta}_{10} = -0.09500$

$\hat{\beta}_{11} = -0.09097,\ \hat{\beta}_{12} = -0.08486$



The greatest impact of changes in the unemployment rate occurs after one quarter, then it declines after quarters 2 and 3, increasing again at quarter 4. After quarter 4, the impact continues to decline very very slowly; the effect of a changing unemployment rate on inflation continues for a long time. If we consider more lags, we find $\hat{\beta}_{40} = -0.01522$ and $\hat{\beta}_{80} = -0.00131$.

(c) The rate of inflation consistent with a constant unemployment rate is given by the equilibrium value for *INF* when $DU = 0$. That is,

$$INF_E = \frac{0.094}{1 - 0.564 - 0.333} = 0.91$$

That is, an inflation rate of 0.91% per quarter.

(d) The test statistic for testing $H_0 : e_t = \theta_1 e_{t-1} + \theta_3 e_{t-3} + v_t$ is

$$T \times R^2 = 241 \times \left(1 - \frac{47.619}{48.857}\right) = 6.11$$

The 5% critical value is $\chi^2_{(0.95,\,2)} = 5.99$. Thus, we reject H_0; we cannot conclude that the e_t follow the specified AR(3) process. It implies that the least-squares estimates for the model in part (b) are not consistent.

EXERCISE 9.15

(a) To write the AR(1) error model in lag operator notation, we have

$$e_t = \rho e_{t-1} + v_t$$
$$e_t - \rho e_{t-1} = v_t$$
$$(1 - \rho L) e_t = v_t$$

(b) Since $(1-\rho L)(1-\rho L)^{-1} = 1$, we can show that $(1-\rho L)^{-1} = 1 + \rho L + \rho^2 L^2 + \rho^3 L^3 + \cdots$ by showing

$$(1-\rho L)(1 + \rho L + \rho^2 L^2 + \rho^3 L^3 + \cdots) = (1 + \rho L + \rho^2 L^2 + \rho^3 L^3 + \cdots) - (\rho L + \rho^2 L^2 + \rho^3 L^3 + \cdots) = 1$$

Thus, we have $(1-\rho L)e_t = v_t$, and

$$\begin{aligned} e_t &= (1-\rho L)^{-1} v_t = (1 + \rho L + \rho^2 L^2 + \rho^3 L^3 + \cdots) v_t \\ &= v_t + \rho v_{t-1} + \rho^2 v_{t-2} + \rho^3 v_{t-3} + \cdots \end{aligned}$$

EXERCISE 9.17

Estimates of the AR(1) model $G_t = \alpha + \phi G_{t-1} + v_t$ are:

Dependent Variable: G
Method: Least Squares
Sample (adjusted): 1948Q2 2016Q1
Included observations: 272 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.767330	0.101620	7.550965	0.0000
G(-1)	0.509556	0.052481	9.709371	0.0000
S.E. of regression	0.969785			

From Exercise 9.4, the point forecasts are given by

$$\hat{U}_{2016Q2} = 4.94987 \qquad \hat{U}_{2016Q3} = 5.05482 \qquad \hat{U}_{2016Q4} = 5.1716$$

Making use of the results in Exercise 9.16, the standard errors of the forecast errors are

$$\hat{\sigma}_{f1} = 0.291923 \qquad \hat{\sigma}_{f2} = 0.53638 \qquad \hat{\sigma}_{f3} = 0.7506$$

The 95% forecast intervals are

$$\hat{U}_{2016Q2} \pm t_{(0.975,267)}\hat{\sigma}_{f1} = (4.375, 5.525)$$

$$\hat{U}_{2016Q3} \pm t_{(0.975,267)}\hat{\sigma}_{f2} = (3.999, 6.111)$$

$$\hat{U}_{2016Q4} \pm t_{(0.975,267)}\hat{\sigma}_{f3} = (3.694, 6.649)$$

EXERCISE 9.19

(a) The LM test results for the two models for lags 1 to 4 are given in the following table. Using a 5% significance level, they suggest the existence of serial correlation in the errors when $p = 4$ and $q = 3$, but that serial correlation has been eliminated when (p, q) are extended to $p = 6$ and $q = 5$.

	ARDL(4,3)		ARDL(6,5)	
k	Test value	*p*-value	Test value	*p*-value
1	3.552	0.0595	0.035	0.8518
2	7.600	0.0224	1.766	0.4136
3	7.854	0.0491	2.391	0.4954
4	10.274	0.0361	4.286	0.3687

(b) The residual correlograms for the two models are displayed below. In contrast to the results from the LM tests, both correlograms are very similar. In both, there is very little evidence of serial correlation in the errors. The only autocorrelation that exceeds the significance bounds (drawn at ± 0.12) is r_8, and it's values, -0.205 and -0.236, are not very large.

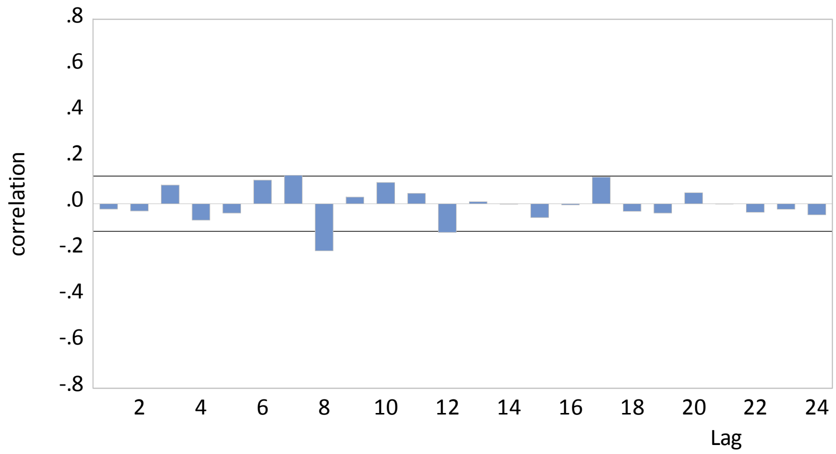


Figure 19.9(a) Residual correlogram for *p* = 4 and *q* = 3

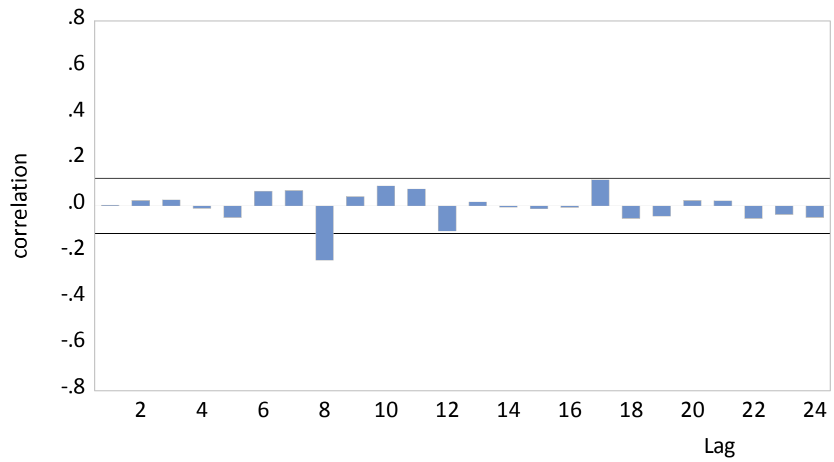


Figure 19.9(b) Residual correlogram for *p* = 6 and *q* = 5

EXERCISE 9.21

(a) In the following table, the estimates in the column headed "(XR9.21.1) NLS estimates" are those derived from the NLS estimates in Table 9.10 on page 455 of *POE5*. The derivation is provided in the question for this exercise. The estimates in the column headed "(XR9.21.2) OLS estimates" are those from direct OLS estimation of the ARDL(1,1) model in equation (XR9.21.2). The standard errors, *t*-values and *p*-values relate to these estimates. We observe that, except for the coefficient for DU_{t-1}, there is very little difference between the two sets of estimates.

Variable	(XR9.21.1) NLS estimates	(XR9.21.2) OLS estimates	Std. Error	*t*-Statistic	*p*-value
C	0.3513	0.348308	0.091337	3.813446	0.0002
INF(-1)	0.5001	0.499240	0.120583	4.140206	0.0001
DU	-0.3830	-0.372761	0.164564	-2.265142	0.0254
DU(-1)	0.1915	0.017143	0.164509	0.104209	0.9172

(b) The results from re-estimating (XR9.21.2) with DU_{t-1} dropped follow. It is reasonable to drop DU_{t-1} because the *p*-value for testing whether its estimated coefficient is significantly different from zero was 0.9172.

Dependent Variable: INF
Method: Least Squares
Sample (adjusted): 1987Q2 2016Q1
Included observations: 116 after adjustments
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.349085	0.091386	3.819905	0.0002
INF(-1)	0.497954	0.121305	4.104972	0.0001
DU	-0.363522	0.172806	-2.103643	0.0376

(c) When $INF_t^E = \delta + \theta_1 INF_{t-1} + \theta_4 INF_{t-4}$, the model becomes

$$INF_t = \delta + \theta_1 INF_{t-1} + \theta_4 INF_{t-4} + \delta_0 DU_t + e_t$$

The notation for the coefficient of DU_t has been changed from β_0 to δ_0 in line with the general ARDL notation. The estimates for this model are:

Dependent Variable: INF
Method: Least Squares
Sample (adjusted): 1988Q1 2016Q1
Included observations: 113 after adjustments
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.172202	0.078344	2.198028	0.0301
INF(-1)	0.328246	0.108646	3.021228	0.0031
INF(-4)	0.378919	0.070941	5.341327	0.0000
DU	-0.565260	0.158264	-3.571628	0.0005

(d) Inclusion of DU_{t-4} in part (c) is justified from the significance of its estimated coefficient (*p*-value = 0.0000). Also, in the residual correlogram from the model estimated in part (b), there is a significant autocorrelation at lag 4. The residual correlogram from the model in part (c) has no significant autocorrelations. In the diagrams below, the significance bounds are drawn at ± 0.18.

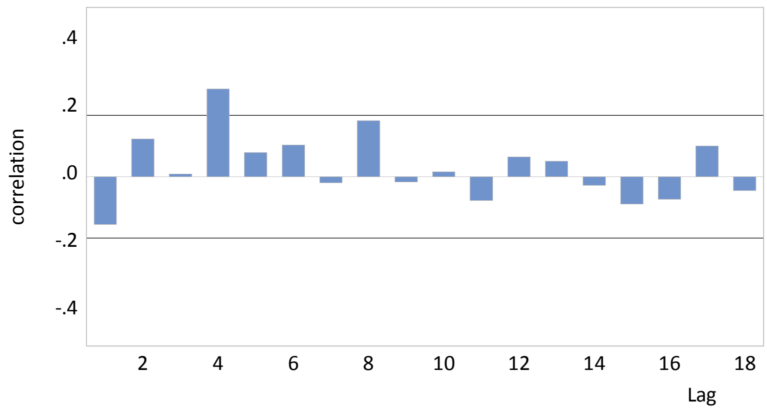


Figure 9.21(a) Residual correlogram for ARDL(1,0) model in part (b)

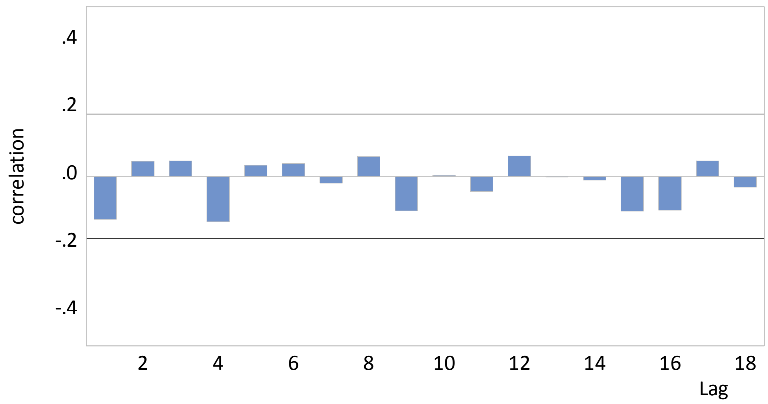


Figure 9.21(b) Residual correlogram for ARDL(4,0) model in part (c)

Having established that the model in part (c) is preferable to that in part (b), we can ask whether the model in part (c) can be improved by including the intervening lags DU_{t-2} and DU_{t-3}. Estimating this model, we obtain the following:

Dependent Variable: INF
Method: Least Squares
Sample (adjusted): 1988Q1 2016Q1
Included observations: 113 after adjustments
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.115167	0.073289	1.571414	0.1190
INF(-1)	0.252154	0.123615	2.039822	0.0438
INF(-2)	0.143427	0.094164	1.523164	0.1307
INF(-3)	0.069465	0.089130	0.779365	0.4375
INF(-4)	0.318156	0.068159	4.667828	0.0000
DU	-0.617214	0.160291	-3.850577	0.0002

The estimated coefficients for both DU_{t-2} and DU_{t-3} are not significantly different from zero (*p*-values of 0.1307 and 0.4375, respectively). Moreover, a test of the joint significance of these coefficients gives an *F*-value of 1.171, with corresponding *p*-value of 0.3139. Thus, we conclude that inclusion of DU_{t-2} and DU_{t-3} does not improve the model; the empirical evidence supports choice of the model in part (c).

EXERCISE 9.23

Estimates of the equation $INF_t = \delta + \theta_1 INF_{t-1} + \theta_4 INF_{t-4} + \delta_0 DU_t + v_t$ follow.

Dependent Variable: INF
Method: Least Squares
Sample (adjusted): 1988Q1 2016Q1
Included observations: 113 after adjustments
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.172202	0.078344	2.198028	0.0301
INF(-1)	0.328246	0.108646	3.021228	0.0031
INF(-4)	0.378919	0.070941	5.341327	0.0000
DU	-0.565260	0.158264	-3.571628	0.0005

(a) Using

$$\beta_0 = \delta_0 \quad \text{and} \quad \beta_s = \theta_1 \beta_{s-1} \text{ for } s = 1,2,3 \quad \text{and} \quad \beta_s = \theta_1 \beta_{s-1} + \theta_4 \beta_{s-4} \text{ for } s \geq 4$$

we obtain the following lag weights.

Lag(s)	Lag weight estimate (β_s)
0	-0.565260
1	-0.185544
2	-0.060904
3	-0.019992
4	-0.220750
5	-0.142767
6	-0.069940
7	-0.030533
8	-0.093669

An estimate of the total multiplier is -1.9303

(b) To test whether e_t follows the AR(4) process $e_t = \theta_1 e_{t-1} + \theta_4 e_{t-4} + v_t$, we can assume it follows a general AR(4) process

$$e_t = \psi_1 e_{t-1} + \psi_2 e_{t-2} + \psi_1 e_{t-3} + \psi_4 e_{t-4} + v_t$$

and test the hypothesis $H_0: \psi_1 = \theta_1, \psi_2 = 0, \psi_3 = 0, \psi_4 = \theta_4$. Alternatively, we can assume a priori that $\psi_2 = 0$ and $\psi_3 = 0$, and test $H_0: \psi_1 = \theta_1, \psi_4 = \theta_4$. In both cases the first step is to find the least squares residuals

$$\hat{u}_t = INF_t - \hat{\delta} - \hat{\theta}_1 INF_{t-1} - \hat{\theta}_4 INF_{t-4} - \hat{\delta}_0 DU_t$$

and then, after setting $\hat{e}_1 = \hat{e}_2 = \hat{e}_3 = \hat{e}_4 = 0$, compute recursively

$$\hat{e}_t = \hat{\theta}_1 \hat{e}_{t-1} + \hat{\theta}_4 e_{t-4} + \hat{u}_t \qquad t = 5, 6, \ldots, T$$

Then, for testing $H_0: \psi_1 = \theta_1, \psi_2 = 0, \psi_3 = 0, \psi_4 = \theta_4$, we compute the statistic $\chi^2 = (T-4) \times R^2$ from the regression

$$\hat{u}_t = \delta + \theta_1 INF_{t-1} + \theta_4 INF_{t-4} + \delta_0 DU_t + \gamma_1 \hat{e}_{t-1} + \gamma_2 \hat{e}_{t-2} + \gamma_3 \hat{e}_{t-3} + \gamma_4 \hat{e}_{t-4} + error$$

The test value is $\chi^2 = 113 \times 0.068377 = 7.727$. The 5% critical value is $\chi^2_{(0.95,4)} = 9.488$. Thus, there is insufficient evidence to reject $H_0: \psi_1 = \theta_1, \psi_2 = 0, \psi_3 = 0, \psi_4 = \theta_4$. In other words, there is insufficient evidence to reject the AR(4) error model $e_t = \theta_1 e_{t-1} + \theta_4 e_{t-4} + v_t$.

However, the opposite conclusion is reached if we assume a priori that $\psi_2 = 0$ and $\psi_3 = 0$, and test $H_0: \psi_1 = \theta_1, \psi_4 = \theta_4$. In this case the relevant regression is

$$\hat{u}_t = \delta + \theta_1 INF_{t-1} + \theta_4 INF_{t-4} + \delta_0 DU_t + \gamma_1 \hat{e}_{t-1} + \gamma_4 \hat{e}_{t-4} + error$$

The test value is $\chi^2 = 113 \times 0.064419 = 7.279$. The 5% critical value is $\chi^2_{(0.95,2)} = 5.991$. Hence, we reject $H_0: \psi_1 = \theta_1, \psi_4 = \theta_4$, and also the AR(4) model $e_t = \theta_1 e_{t-1} + \theta_4 e_{t-4} + v_t$.

EXERCISE 9.25

(a) Estimates of the two models follow:

Dependent Variable: DC
Method: Least Squares
Sample (adjusted): 1960Q1 2015Q4
Included observations: 224 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	479.1078	74.77646	6.407202	0.0000
DC(-1)	0.337633	0.060506	5.580144	0.0000
DY	0.098540	0.021963	4.486745	0.0000

Dependent Variable: DC
Method: Least Squares
Sample (adjusted): 1960Q3 2015Q4
Included observations: 222 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	683.1192	65.48353	10.43192	0.0000
DY	0.119069	0.022809	5.220230	0.0000
DY(-3)	0.063653	0.023194	2.744410	0.0066

(b)

	Model		Actual	Model	
	$DC_t = \delta + \theta_1 DC_{t-1} + \delta_0 DY_t + e_{1t}$		Actual	$DC_t = \alpha + \beta_0 DY_t + \beta_3 DY_{t-3} + e_{2t}$	
Quarter	$\widehat{DC}$	$\hat{C}$	C	$\widehat{DC}$	$\hat{C}$
2015Q4			234633		
2016Q1	1755.855	236389	236523	1237.813	235871
2016Q2	1226.257	237615	237613	789.0600	236660
2016Q3	1161.754	238777	238664	951.1129	237611

(c) For the model $DC_t = \delta + \theta_1 DC_{t-1} + \delta_0 DY_t + e_{1t}$, we have $\sum_{t=2016Q1}^{2016Q3}\left(\hat{C}_t - C_t\right)^2 \Big/ 3 = 10246$.

For the model $DC_t = \alpha + \beta_0 DY_t + \beta_3 DY_{t-3} + e_{2t}$, we have $\sum_{t=2016Q1}^{2016Q3}\left(\hat{C}_t - C_t\right)^2 \Big/ 3 = 814213$.

Using this criterion, the first model is the better model.

EXERCISE 9.27

(a) The estimated equation for Canada is

$$\widehat{SHARE}_t = 17.8080 - 0.106867 TAX_t - 0.174148 YEAR_t + 0.00190973 YEAR_t^2$$

$$\begin{array}{lllll} (\text{OLS se}) & (0.4751) & (0.017749) & (0.039542) & (0.00037857) \\ (\text{HAC se}) & (1.2234) & (0.040066) & (0.083959) & (0.00075884) \end{array}$$

The two 95% interval estimates for α_2, using $t_{(0.975,\,76)} = 1.9917$, are

OLS: $(-0.1422, -0.0715)$ HAC: $(-0.1867, -0.0271)$

The HAC interval is noticeably wider than that from OLS, suggesting that autocorrelated errors may be making the OLS standard errors incorrect.

(b) With pre-sample lagged residuals set to zero, the LM test with three lagged residuals gives $F = 58.7455$ with p-value $\Pr\left(F_{(3,73)} > 58.7455\right) = 0.0000$ and $\chi^2 = T \times R^2 = 56.5684$ with p-value $\Pr\left(\chi^2_{(3)} > 56.5684\right) = 0.0000$. We conclude that the errors are serially correlated, and so the use of HAC standard errors in part (a) is justified.

(c) Applying least squares to the equation $\hat{e}_t = \rho \hat{e}_{t-1} + \hat{v}_t$ yields the estimate $\hat{\rho} = 0.8477$. The critical assumption we are making when estimating this equation is that the v_t are not autocorrelated. Another way of expressing this assumption is to say that the AR(1) error assumption is adequate to capture the autocorrelation. The complete set of assumptions for v_t are

$$E\left(v_t \mid TAX_t, TAX_{t-1}, \ldots, YEAR_t, YEAR_{t-1}, \ldots\right) = 0$$

$$\operatorname{var}\left(v_t \mid TAX_t, YEAR_t\right) = \sigma_v^2$$

$$\operatorname{cov}\left(v_t, v_s \mid TAX_t, YEAR_t, TAX_s, YEAR_s\right) = 0 \quad \text{for} \quad t \neq s$$

(d) Estimating the equation by generalized least squares, we obtain

$$\widehat{SHARE}_t = 2.9900 - 0.041093TAX_t - 0.367993YEAR_t + 0.00374374YEAR_t^2$$
$$\begin{array}{lllll}(\text{GLS se}) & (0.3293) & (0.016286) & (0.108480) & (0.00111030)\\ (\text{HAC se}) & (0.6957) & (0.017190) & (0.184920) & (0.00172308)\end{array}$$

The two 95% interval estimates for α_2, using $t_{(0.975,\,75)} = 1.9921$, are

$$\text{GLS: } (-0.0735, -0.0086) \qquad \text{HAC: } (-0.0753, -0.0068)$$

These two intervals are approximately the same, a consequence of similar standard errors, which in turn suggests GLS estimation has eliminated the autocorrelation. They are very different from the intervals in part (a), largely because the estimate of α_2 has changed considerably. Also, the HAC standard error in part (a) is much bigger, reflecting the inefficiency of OLS relative to GLS.

(e) With pre-sample lagged residuals set to zero, the LM test with three lagged residuals gives $F = 2.2122$ with *p*-value $\Pr\left(F_{(3,72)} > 2.2122\right) = 0.0940$ and $\chi^2 = T \times R^2 = 6.6672$ with *p*-value $\Pr\left(\chi^2_{(3)} > 56.5684\right) = 0.0833$. At a 5% significance level, we conclude there is insufficient evidence to suggest the errors are serially correlated. There is no reason to use HAC standard errors in part (d).

(f) For the exogeneity assumption required for consistent estimation of α_2 to be satisfied, TAX_t must be uncorrelated with omitted variables whose effects are included in the error term. It is unlikely that this condition will be satisfied. Both the income share of the top 1% of earners and the tax that they pay are likely to be correlated with the growth rate of the economy whose impact will be felt through the error term. For OLS to be consistent e_t must be uncorrelated with current and past values of *TAX*. For GLS to be consistent e_t must be uncorrelated with the future value TAX_{t+1} as well as current and past values. In other words, if *TAX* is correlated with the growth rate in the previous period, GLS will be inconsistent. This is the likely cause of the discrepancy in the OLS and GLS estimates for α_2.

EXERCISE 9.29

(a) From the equation $\ln\left(AREA_t\right) = \alpha + \gamma \ln\left(PRICE^*_{t+1}\right) + e_t$, we can write

$$\ln\left(AREA_t\right) = \alpha + \gamma \ln\left(PRICE^*_{t+1}\right) + e_t = \alpha + \gamma \sum\nolimits_{s=0}^{q} \gamma_s \ln\left(PRICE_{t-s}\right) + e_t$$
$$= \alpha + \sum\nolimits_{s=0}^{q} \gamma\gamma_s \ln\left(PRICE_{t-s}\right) + e_t = \alpha + \sum_{s=0}^{q} \beta_s \ln\left(PRICE_{t-s}\right) + e_t$$

(b) Estimating the model with $q = 3$ and HAC standard errors, we obtain

Variable	Coefficient	Std. Error	*t*-Value	*p*-value
C	3.777650	0.147867	25.54767	0.0000
LN_PRICE	1.415993	0.366930	3.859030	0.0003
LN_PRICE(-1)	0.498076	0.264124	1.885762	0.0638
LN_PRICE(-2)	0.234863	0.240563	0.976304	0.3325
LN_PRICE(-3)	0.293958	0.281839	1.042999	0.3008

The estimated delay and interim elasticities are given in the following table.

Multipliers for unrestricted model

Lag	Delay	Interim
0	1.4160	
1	0.4981	1.9141
2	0.2349	2.1489
3	0.2940	2.4429

For the estimates to satisfy the a priori restriction $\gamma_0 > \gamma_1 > \cdots > \gamma_q$, we require $\hat{\beta}_0 > \hat{\beta}_1 > \hat{\beta}_2 > \hat{\beta}_3$. We find $\hat{\beta}_0 > \hat{\beta}_1 > \hat{\beta}_2$, but $\hat{\beta}_2 < \hat{\beta}_3$. The delay elasticities decline until lag 2 but that at lag 3 is greater than that for lag 2. However, the last two delay elasticities are not precisely estimated. Using one-tail tests, their estimates are not significantly different from zero at a 10% significance level.

The first four autocorrelations for the residuals are $r_1 = 0.391$, $r_2 = 0.223$, $r_3 = 0.183$ and $r_4 = -0.050$, with the first two being significantly different from zero at a 5% level.

(c) Substituting $\beta_s = \alpha_0 + \alpha_1 s$ into $\ln(AREA_t) = \alpha + \sum_{s=0}^{q} \beta_s \ln(PRICE_{t-s}) + e_t$, we obtain

$$\begin{aligned}\ln(AREA_t) &= \alpha + \sum_{s=0}^{q} (\alpha_0 + \alpha_1 s)\ln(PRICE_{t-s}) + e_t \\ &= \alpha + \alpha_0 \sum_{s=0}^{3} \ln(PRICE_{t-s}) + \alpha_1 \sum_{s=1}^{3} s\ln(PRICE_{t-s}) + e_t \\ &= \alpha + \alpha_0 z_{t0} + \alpha_1 z_{t1} + e_t\end{aligned}$$

(d) Using

$$z_{t0} = \ln(PRICE_t) + \ln(PRICE_{t-1}) + \ln(PRICE_{t-2}) + \ln(PRICE_{t-3})$$

$$z_{t1} = \ln(PRICE_{t-1}) + 2\ln(PRICE_{t-2}) + 3\ln(PRICE_{t-3})$$

and HAC standard errors, we obtain the results

Variable	Coefficient	Std. Error	*t*-value	*p*-value
C	3.781549	0.142789	26.48344	0.0000
Z0	1.145263	0.326437	3.508375	0.0008
Z1	-0.362193	0.141653	-2.556908	0.0128

In this case the estimates are significantly different from zero at a 5% level, and the coefficients for z_{t0} and z_{t1} have their expected signs, positive and negative, respectively.

(e) The new estimates for β_s, $s = 0,1,2,3$, are given by $\hat{\beta}_0 = 1.14526$, $\hat{\beta}_1 = 0.78307$, $\hat{\beta}_2 = 0.42088$, and $\hat{\beta}_3 = 0.05869$. These new weights decline with the lag length, and hence now satisfy a priori expectations. The original problem has been cured. We no longer have $\hat{\beta}_2 < \hat{\beta}_3$.

(f) The new delay and interim elasticities are given by

Multipliers for restricted model

Lag	Delay	Interim
0	1.1453	
1	0.7831	1.9283
2	0.4209	2.3492
3	0.0587	2.4079

Imposing the restriction has increased the delay elasticities at lags 1 and 2 and reduced those at lags 1 and 3. The interim elasticities were less sensitive to imposition of the restrictions.

EXERCISE 9.31

(a) Estimates of the equation $DU_t = \alpha + \beta_0 G_t + e_t$ and their two sets of standard errors follow. We estimate that an increase in the growth rate of one percentage point is associated with a decrease in the unemployment rate of 0.11 percentage points. The HAC standard error for the intercept estimate is slightly larger than the conventional standard error. However, for the coefficient of G there is very little difference in the two standard errors.

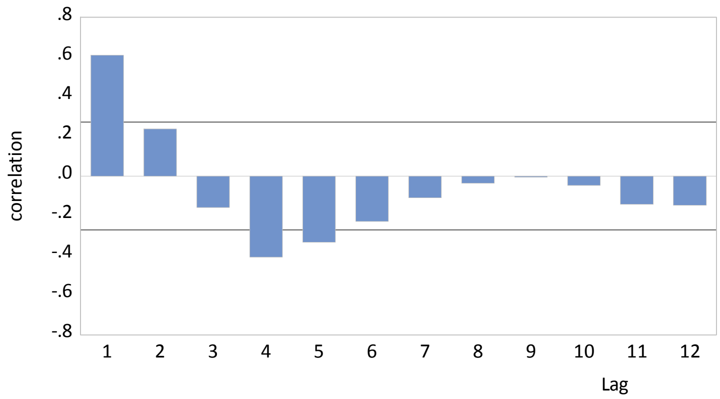
Dependent Variable: DU
Method: Least Squares
Sample (adjusted): 2000Q1 2012Q4
Included observations: 52 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.118188	0.065075	1.816171	0.0753
G	-0.111217	0.019058	-5.835642	0.0000

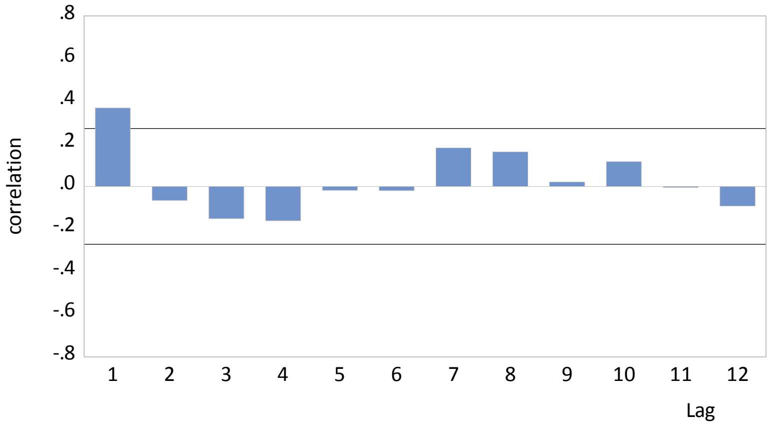
Dependent Variable: DU
Method: Least Squares
Sample (adjusted): 2000Q1 2012Q4
Included observations: 52 after adjustments
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 4.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.118188	0.089129	1.326035	0.1909
G	-0.111217	0.018381	-6.050792	0.0000

(b) For a 5% significance level and the significance bounds $\pm 1.96/\sqrt{52} = \pm 0.272$, the correlogram in the diagram below displays significant autocorrelation at lags 1, 4 and 5. Thus, there is evidence of autocorrelation in the errors.



(c) The results in parts (a) and (b) might be viewed as contradictory. Because the residuals are autocorrelated, it is reasonable to expect a noticeable difference between the HAC and conventional standard errors for $\hat{\beta}_0$. They are similar in magnitude, however. This puzzle can be resolved by examining the correlogram for $q_t = G_t \times \hat{e}_t$, displayed below with 5% significance bounds ± 0.272. From equations (9.62) and (9.63) in *POE5*, and the surrounding discussion, we see that the contribution of autocorrelation to the HAC standard errors depends on the magnitude of the term $2\sum_{s=1}^{T_0}\left(\frac{T_0 - s}{T_0}\right)\hat{\tau}_s$ where T_0 is a truncated number of lags and the $\hat{\tau}_s$ are the sample autocorrelations for $q_t = G_t \times \hat{e}_t$. From the correlogram for q_t, we see that the $\hat{\tau}_s$ are relatively small and the positive value for $\hat{\tau}_1$ is likely to be offset by the negative values for $\hat{\tau}_2$, $\hat{\tau}_3$ and $\hat{\tau}_4$.



(d) Estimates for the equation $DU_t = \alpha + \beta_0 G_t + \beta_1 G_{t-1} + \beta_2 G_{t-2} + e_t$ are given in the output below. Because $\hat{\beta}_1$ is significantly different from zero at a 5% level of significance, and, if a one-tail test is used, $\hat{\beta}_2$ is also significant at a 5% level, we conclude that there is evidence of a lagged effect of growth on unemployment.

Dependent Variable: DU
Method: Least Squares
Sample (adjusted): 2000Q1 2012Q4
Included observations: 52 after adjustments
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 4.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.229224	0.083919	2.731498	0.0088
G	-0.074765	0.021878	-3.417395	0.0013
G(-1)	-0.041937	0.018412	-2.277744	0.0272
G(-2)	-0.043167	0.022104	-1.952927	0.0567

Using $t_{(0.975,\,48)} = 2.01063$ and $\text{se}\left(\hat{\beta}_0 + \hat{\beta}_1 + \hat{\beta}_2\right) = 0.021065$, a 95% interval estimate for the total multiplier is $(-0.2022, -0.1175)$. Using $t_{(0.975,\,50)} = 2.00856$ and $\text{se}\left(\hat{\beta}_0\right) = -0.111217$, from part (a), a 95% interval estimate for the total multiplier is $(-0.1481, -0.0743)$. Allowing for lags reveals a greater effect of growth on unemployment, but also lowers the precision of estimation of this effect.

(e) Estimates of all ARDL(p,q) models $DU_t = \delta + \sum_{s=1}^{p} \theta_s DU_{t-s} + \sum_{r=0}^{q} \delta_r G_{t-r} + e_t$ for $p = 1,2,3$ and $q = 0,1,2$ are provided below. The model with the largest number of lags whose coefficients are significantly different from zero at a 5% level is the ARDL(2,0) model

$$\widehat{DU}_t = 0.0404 + 1.2234 DU_{t-1} - 0.5619 DU_{t-2} - 0.03627 G_t$$
$$\text{(se)} \quad (0.0232)\ (0.0936) \qquad (0.0913) \qquad (0.00817)$$

ARDL10 Workfile: XR9-31::Apap\

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.088708	0.042382	2.093077	0.0417
DU(-1)	0.682268	0.060720	11.23635	0.0000
G	-0.062288	0.013658	-4.560550	0.0000

ARDL11 Workfile: XR9-31::Apap\

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.099785	0.049135	2.030838	0.0479
DU(-1)	0.653240	0.071112	9.186032	0.0000
G	-0.058664	0.014725	-3.983846	0.0002
G(-1)	-0.010436	0.018369	-0.568132	0.5726

ARDL12 Workfile: XR9-31::Apap\

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.093381	0.055702	1.676443	0.1004
DU(-1)	0.665855	0.081601	8.159876	0.0000
G	-0.058038	0.014199	-4.087439	0.0002
G(-1)	-0.012009	0.016994	-0.706642	0.4834
G(-2)	0.004535	0.016616	0.272912	0.7861

ARDL20 Workfile: XR9-31::Apap\

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.040395	0.023199	1.741233	0.0883
DU(-1)	1.223381	0.093584	13.07251	0.0000
DU(-2)	-0.561912	0.091328	-6.152681	0.0000
G	-0.036271	0.008169	-4.439854	0.0001

ARDL21 Workfile: XR9-31::Apap\

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.036258	0.028870	1.255924	0.2156
DU(-1)	1.237572	0.099308	12.46201	0.0000
DU(-2)	-0.566090	0.092958	-6.089740	0.0000
G	-0.037238	0.008783	-4.239818	0.0001
G(-1)	0.003554	0.011884	0.299088	0.7662

ARDL22 Workfile: XR9-31::Apap\

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.043094	0.028054	1.536132	0.1317
DU(-1)	1.230767	0.103519	11.88924	0.0000
DU(-2)	-0.576348	0.090440	-6.372730	0.0000
G	-0.037869	0.008790	-4.308315	0.0001
G(-1)	0.005352	0.013641	0.392316	0.6967
G(-2)	-0.005393	0.012659	-0.425994	0.6722

ARDL30 Workfile: XR9-31::Apap\

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.029339	0.028815	1.018158	0.3142
DU(-1)	1.109075	0.130793	8.479627	0.0000
DU(-2)	-0.319411	0.216666	-1.474207	0.1475
DU(-3)	-0.165184	0.125897	-1.312058	0.1963
G	-0.031238	0.010682	-2.924420	0.0054

ARDL31 Workfile: XR9-31::Apap\

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.027814	0.033629	0.827102	0.4127
DU(-1)	1.115982	0.127662	8.741695	0.0000
DU(-2)	-0.323798	0.213407	-1.517282	0.1365
DU(-3)	-0.163278	0.123664	-1.320342	0.1937
G	-0.031664	0.011126	-2.845865	0.0068
G(-1)	0.001410	0.011972	0.117790	0.9068

ARDL32 Workfile: XR9-31::Apap\

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.029576	0.033585	0.880627	0.3835
DU(-1)	1.116392	0.129609	8.613547	0.0000
DU(-2)	-0.330153	0.225268	-1.465596	0.1502
DU(-3)	-0.160737	0.129821	-1.238145	0.2225
G	-0.031939	0.010686	-2.988891	0.0047
G(-1)	0.001869	0.013254	0.140993	0.8885
G(-2)	-0.001267	0.012413	-0.102096	0.9192

(f) The lag coefficients are given by

$$\beta_0 = \delta_0 \qquad \beta_1 - \theta_1\beta_0 = 0 \qquad \beta_s - \theta_1\beta_{s-1} - \theta_2\beta_{s-2} \text{ for } s \geq 2\,.$$

$$\beta_0 = \delta_0 \qquad \beta_1 = \theta_1\beta_0 \qquad \beta_s = \theta_1\beta_{s-1} + \theta_2\beta_{s-2} \text{ for } s \geq 2$$

The total multiplier estimate is -0.107141.

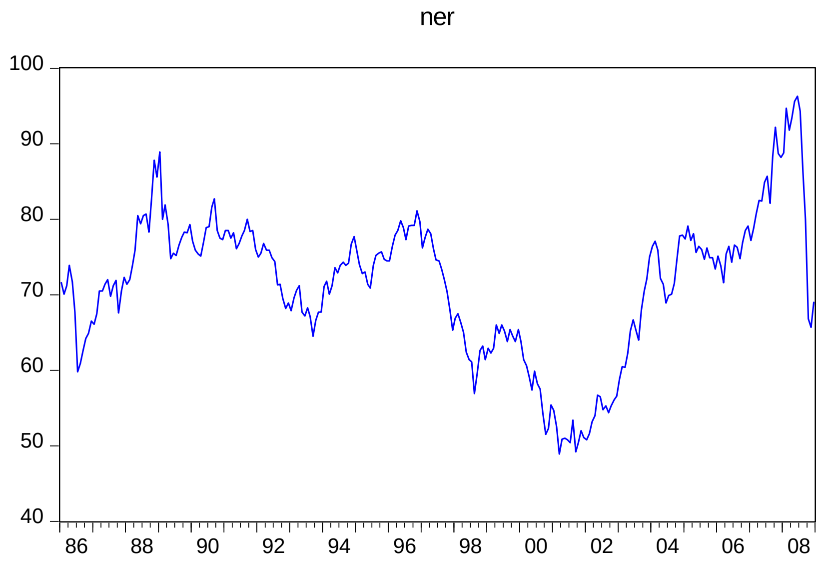
The impact multiplier estimate is $\hat{\beta}_0 = -0.036271$. The first three delay multipliers are

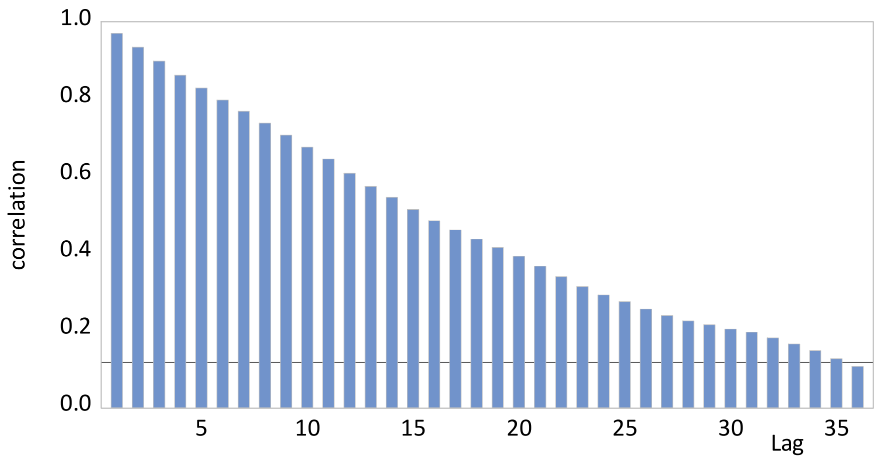
$$\hat{\beta}_1 = -0.044373 \qquad \hat{\beta}_2 = -0.033904 \qquad \hat{\beta}_3 = -0.016544$$

(g) With $t_{(0.975,\,46)} = 2.012896$, and standard error 0.021112, a 95% interval estimate for the total multiplier is $(-0.1496, -0.0646)$. Using $\text{se}\left(\hat{\beta}_0 + \hat{\beta}_1 + \hat{\beta}_2\right) = 0.020111$, a 95% interval estimate for the two-period interim multiplier is $(-0.1550, -0.0741)$. Relative to these two intervals, the interval in part (d) has overstated the impact of growth on unemployment. The intervals from the ARDL(2,0) model are more in line with the interval from part (a).

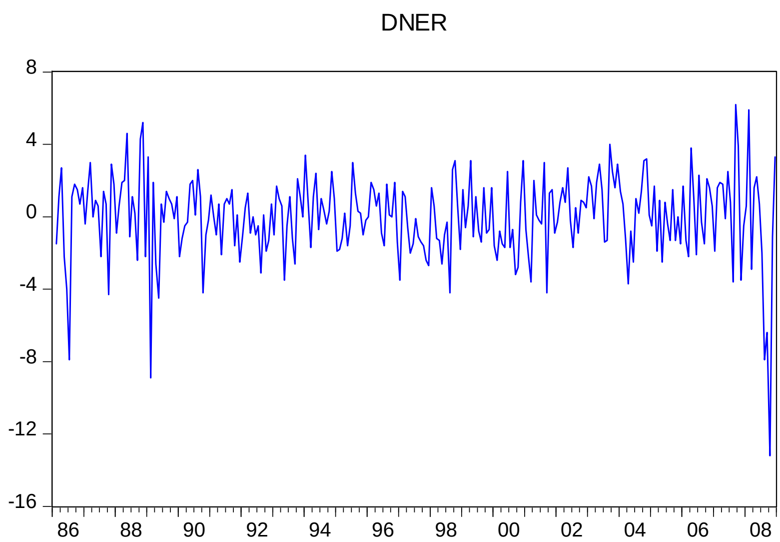
EXERCISE 9.33

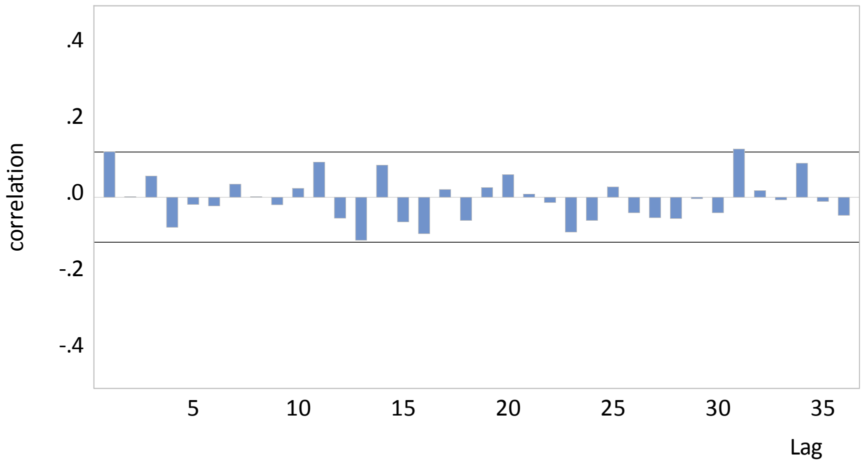
(a) The time series plot for *NER* wanders, suggesting a nonstationary series or, at best, one that is highly autocorrelated. Also, in the following correlogram, the autocorrelations exceed the significance bound $1.96/\sqrt{276} = 0.118$ for all lags up to 35. They do die out, but very slowly, suggesting a series that is not weakly dependent.





(b) A plot of the first difference $DNER_t = NER_t - NER_{t-1}$ and its autocorrelations are presented below. *DNER* does not wander; it appears to fluctuate randomly around a mean of approximately zero, suggesting a stationary series. Also, the autocorrelations are all small, suggesting a weakly dependent series.





(c) Estimates for the model $DNER_t = \alpha + \beta_0 DINF_t + \beta_1 DINF_{t-1} + \gamma_0 DI6_t + \gamma_1 DI6_{t-1} + e_t$ are given below.

Dependent Variable: DNER
Method: Least Squares
Sample (adjusted): 1986M03 2008M12
Included observations: 274 after adjustments
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 6.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.003185	0.136571	0.023323	0.9814
DINF	-0.182505	0.240329	-0.759395	0.4483
DINF(-1)	0.170398	0.184450	0.923817	0.3564
DI6	-0.054243	0.020346	-2.666007	0.0081
DI6(-1)	0.059315	0.014615	4.058450	0.0001

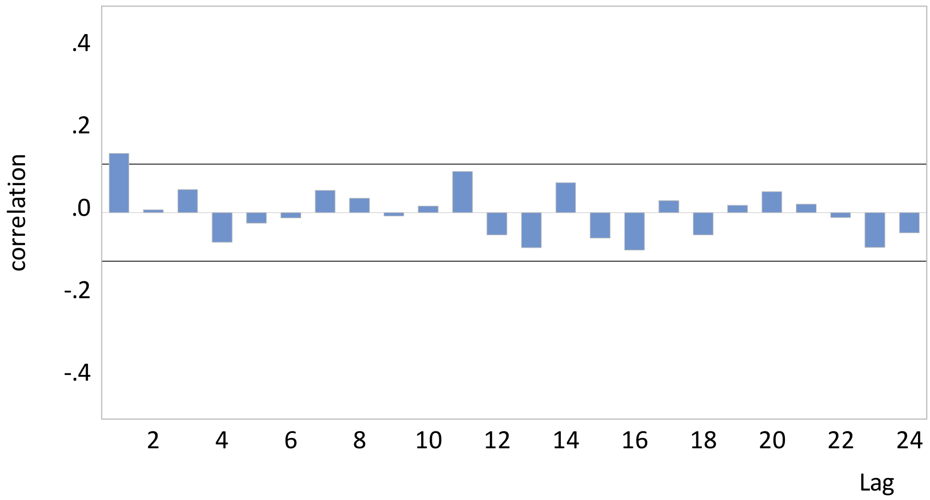
We expect $\beta_0 < 0, \beta_1 < 0, \gamma_0 > 0$ and $\gamma_1 > 0$. With these expectations, $\hat{\beta}_0$ and $\hat{\gamma}_1$ have the correct signs, but $\hat{\beta}_1$ and $\hat{\gamma}_0$ do not. Using one-tail tests and a 5% significance level, $\hat{\gamma}_1$ is the only estimated coefficient that is significantly different from zero (p-value < 0.0001); $\hat{\beta}_0$ (p-value = 0.224) , $\hat{\beta}_1$ (p-value = 0.822) and $\hat{\gamma}_0$ (p-value = 0.996) are not significantly different from zero.

(d) Dropping $DINF_{t-1}$ and $DI6_t$ from the model gives $DNER_t = \alpha + \beta_0 DINF_t + \gamma_1 DI6_{t-1} + e_t$, leading to the following estimates.

Dependent Variable: DNER
Method: Least Squares
Sample (adjusted): 1986M03 2008M12
Included observations: 274 after adjustments
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 6.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.004906	0.145109	-0.033812	0.9731
DINF	-0.333560	0.253699	-1.314784	0.1897
DI6(-1)	0.045297	0.014726	3.076072	0.0023

The one-tail p-values for significance of $\hat{\beta}_0$ and $\hat{\gamma}_1$ are 0.095 and 0.001, respectively. Thus, at a 5% level, $\hat{\gamma}_1$ is significantly different from zero, but $\hat{\beta}_0$ is not. Testing the residuals for autocorrelation, we obtain an LM statistic value (using 1 lag and initial missing residuals set to zero) of $\chi^2 = T \times R^2 = 5.889$, with corresponding *p*-value of 0.0152. This result suggests the residuals are autocorrelated. Evidence from the correlogram below is not as strong; r_1 slightly exceeds the 5% significance bound $1.96/\sqrt{274} = 0.118$.



(e) The first step towards feasible generalized least squares (FGLS) estimation is to use the residuals $\hat{e}_t$ from the estimated equation in part (d) to estimate ρ from the equation $\hat{e}_t = \rho\hat{e}_{t-1} + \hat{v}_t$. Doing so, we obtain the following results.

Dependent Variable: EHAT
Method: Least Squares
Sample (adjusted): 1986M04 2008M12
Included observations: 273 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
EHAT(-1)	0.143831	0.060014	2.396618	0.0172

After transforming the variables, we obtain the following FGLS results, one with conventional standard errors and one with HAC standard errors.

Dependent Variable: DNER_TR
Method: Least Squares
Sample (adjusted): 1986M04 2008M12
Included observations: 273 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.011780	0.134951	-0.087292	0.9305
DINF_TR	-0.323896	0.204489	-1.583930	0.1144
DI6_TR	0.051742	0.016917	3.058547	0.0024

Dependent Variable: DNER_TR
Method: Least Squares
Sample (adjusted): 1986M04 2008M12
Included observations: 273 after adjustments
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 6.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.011780	0.129900	-0.090686	0.9278
DINF_TR	-0.323896	0.236437	-1.369904	0.1719
DI6_TR	0.051742	0.015263	3.390031	0.0008

The FGLS estimates are only slightly different from those obtained using OLS. Also, the significance of the coefficient estimates has not changed. With one-tail HAC *p*-values of 0.086 and 0.0004 for $\hat{\beta}_0$ and $\hat{\gamma}_1$, respectively, $\hat{\beta}_0$ is again not significantly different from zero at a 5% level and $\hat{\gamma}_1$ retains its significance. Similar conclusions are reached from the conventional standard errors.

(f) Estimates of the model $DNER_t = \delta + \theta_1 DNER_{t-1} + \delta_1 DINF_{t-1} + \phi_1 DI6_{t-1} + e_t$ using observations up to 2007M12 follow. The lack of significance of two of the coefficient estimates, and the very low R^2 suggest this model would be a poor one for forecasting.

Dependent Variable: DNER
Method: Least Squares
Sample (adjusted): 1986M03 2007M12
Included observations: 262 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.060817	0.124076	0.490158	0.6244
DNER(-1)	0.043757	0.061965	0.706158	0.4807
DINF(-1)	0.174223	0.199843	0.871800	0.3841
DI6(-1)	0.047346	0.019028	2.488200	0.0135

R-squared	0.028138	Mean dependent var	0.069084

(g) The one-month ahead forecasts for *NER* are

Month	$\widehat{NER}$	NER	$\lvert NER - \widehat{NER} \rvert$
2008M01	88.10022	88.80000	0.699778
2008M02	91.51812	94.70000	3.181875
2008M03	95.78876	91.80000	3.988764
2008M04	92.41902	93.40000	0.980980
2008M05	93.35701	95.60000	2.242991
2008M06	94.67551	96.30000	1.624486
2008M07	95.72220	94.30000	1.422203
2008M08	94.87491	86.40000	8.474907
2008M09	85.69087	80.00000	5.690867
2008M10	80.27388	66.80000	13.47388
2008M11	67.62575	65.70000	1.925754
2008M12	68.16406	69.00000	0.835943

The forecasts for the earlier part of the year are reasonably accurate, but the model was unable to capture the rapid fall in the exchange rate from M08 to M10. The average absolute forecast error is $\sum_{t=2008M1}^{2008M12} \left| \widehat{NER}_t - NER_t \right| / 12 = 3.712$.

EXERCISE 9.35

(a) Estimating the equation $INF_t = \delta + \sum_{s=1}^{p} \theta_s INF_{t-s} + \sum_{r=1}^{q} \delta_r INFEX_{t-r} + \sum_{j=1}^{m} \gamma_j GAP_{t-j} + e_t$ for values $p = 2$, $q = 1,2,3,4$ and $m = 1,2,3,4$, and in each case using 92 observations, leads to the Schwarz values given in the following table.

$$\text{SC} = \ln\left(SSE/92\right) + K\ln(92)/92$$

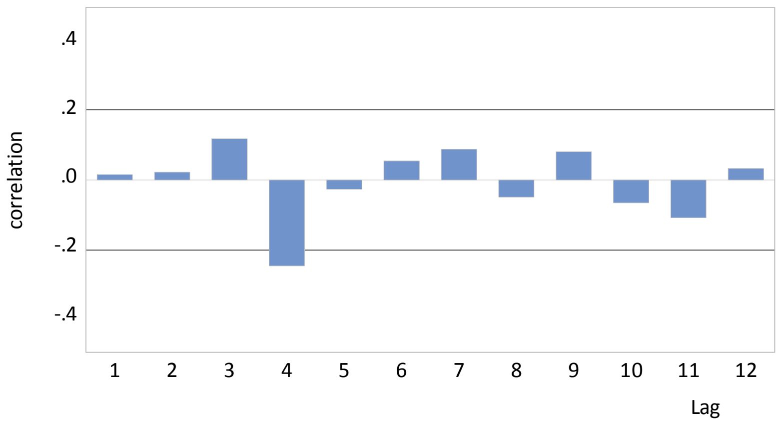
		q			
		1	2	3	4
m	1	−1.8329	−1.8393	−1.8325	**−1.8457**
	2	−1.8280	−1.8211	−1.8220	−1.8410
	3	−1.7802	−1.7730	−1.7792	−1.7964
	4	−1.7435	−1.7366	−1.7829	−1.7794

The values that lead to the smallest SC are $q=4$ and $m=1$. The estimated equation for these values is:

Dependent Variable: INF
Method: Least Squares
Sample: 1991Q1 2013Q4
Included observations: 92

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.297392	0.116387	2.555205	0.0124
INF(-1)	0.957088	0.120498	7.942787	0.0000
INF(-2)	-0.318956	0.110082	-2.897428	0.0048
INFEX(-1)	0.379311	0.239329	1.584893	0.1167
INFEX(-2)	0.220481	0.290096	0.760028	0.4494
INFEX(-3)	-0.900956	0.296265	-3.041046	0.0031
INFEX(-4)	0.475882	0.204767	2.324016	0.0225
GAP(-1)	0.063496	0.041560	1.527816	0.1303
R-squared	0.812745	Mean dependent var		1.706391

(b) Although the Schwarz criterion has selected this model, not all of its coefficient estimates are significantly different from zero. In paticular, $\hat{\delta}_1$, $\hat{\delta}_2$ and $\hat{\gamma}_1$ are not significant. In the correlogram below, there is one relatively small autocorrelation at lag 4 that is significant at a 5% level, namely $r_4 = -0.250$. Apart from this value, there is no evidence of autocorrelation in the errors.



(c) The forecast values, standard errors and 95% forecast intervals are given in the following table, along with the actual values for *INF*. We observe that the forecasts miss the mark quite badly – the forecast errors are large, but, nevertheless, the actual values do lie within the 95% forecast intervals.

				Forecast interval	
	INF	$\widehat{INF}$	se(f)	Lower	Upper
2014Q1	0.669	0.8796	0.3417	0.2002	1.5590
2014Q2	0.666	1.2666	0.4729	0.3262	2.2071
2014Q3	0.380	1.2180	0.5150	0.1938	2.2422
2014Q4	0.285	1.1983	0.5230	0.1583	2.2383

To prove that $\sigma_{f4}^2 = \left[\left(\theta_1^3 + 2\theta_1\theta_2\right)^2 + \left(\theta_1^2 + \theta_2\right)^2 + \theta_1^2 + 1\right]\sigma^2$, note that the forecast error for the 4th quarter is given by $f_4 = \theta_1 f_3 + \theta_2 f_2 + e_{T+4}$. Then, from equations (9.39) and (9.40) in *POE5*, we can write

$$f_4 = \theta_1\left[\left(\theta_1^2 + \theta_2\right)e_{T+1} + \theta_1 e_{T+2} + e_{T+3}\right] + \theta_2\left[\theta_1 e_{T+1} + e_{T+2}\right] + e_{T+4}$$

$$= \left(\theta_1^3 + 2\theta_1\theta_2\right)e_{T+1} + \left(\theta_1^2 + \theta_2\right)e_{T+2} + \theta_1 e_{T+3} + e_{T+4}$$

$$\sigma_{f4}^2 = \text{var}\left(f_4\right) = \left[\left(\theta_1^3 + 2\theta_1\theta_2\right)^2 + \left(\theta_1^2 + \theta_2\right)^2 + \theta_1^2 + 1\right]\sigma^2$$

(d) For the standard errors of the forecast errors to be valid, we need to assume the errors are uncorrelated, and have constant variance. Also, we are ignoring estimation error in the estimation of the coefficients.