PRINCIPLES OF ECONOMETRICS

5TH EDITION

ANSWERS TO ODD-NUMBERED EXERCISES IN CHAPTER 13

EXERCISE 13.1

(a) & (c) Effects of a shock to y of size σ_y on y and x:

$$t=1, \quad y_1 = \sigma_y$$

$$x_1 = 0$$

$$t=2, \quad y_2 = \delta_{11}y_1 + \delta_{12}x_1 = \delta_{11}\sigma_y + \delta_{12}0 = \delta_{11}\sigma_y$$

$$x_2 = \delta_{21}y_1 + \delta_{22}x_1 = \delta_{21}\sigma_y + \delta_{22}0 = \delta_{21}\sigma_y$$

$$t=3, \quad y_3 = \delta_{11}y_2 + \delta_{12}x_2 = (\delta_{11}\delta_{11} + \delta_{12}\delta_{21})\sigma_y$$

$$x_3 = \delta_{21}y_2 + \delta_{22}x_2 = (\delta_{21}\delta_{11} + \delta_{22}\delta_{21})\sigma_y$$

$$t=4, \quad y_4 = \delta_{11}y_3 + \delta_{12}x_3 = \delta_{11}(\delta_{11}\delta_{11} + \delta_{12}\delta_{21})\sigma_y + \delta_{12}(\delta_{21}\delta_{11} + \delta_{22}\delta_{21})\sigma_y$$

$$x_4 = \delta_{21}y_3 + \delta_{22}x_3 = \delta_{21}(\delta_{11}\delta_{11} + \delta_{12}\delta_{21})\sigma_y + \delta_{22}(\delta_{21}\delta_{11} + \delta_{22}\delta_{21})\sigma_y$$

(b) & (d) Effects of a shock to x of size σ_x on y and x:

$$t=1, \quad y_1 = 0$$

$$x_1 = \sigma_x$$

$$t=2, \quad y_2 = \delta_{11}y_1 + \delta_{12}x_1 = \delta_{11}0 + \delta_{12}\sigma_x = \delta_{12}\sigma_x$$

$$x_2 = \delta_{21}y_1 + \delta_{22}x_1 = \delta_{21}0 + \delta_{22}\sigma_x = \delta_{22}\sigma_x$$

$$t=3, \quad y_3 = \delta_{11}y_2 + \delta_{12}x_2 = (\delta_{11}\delta_{12} + \delta_{12}\delta_{22})\sigma_x$$

$$x_3 = \delta_{21}y_2 + \delta_{22}x_2 = (\delta_{21}\delta_{12} + \delta_{22}\delta_{22})\sigma_x$$

$$t=4, \quad y_4 = \delta_{11}y_3 + \delta_{12}x_3 = \delta_{11}(\delta_{11}\delta_{12} + \delta_{12}\delta_{22})\sigma_x + \delta_{12}(\delta_{21}\delta_{12} + \delta_{22}\delta_{22})\sigma_x$$

$$x_4 = \delta_{21}y_3 + \delta_{22}x_3 = \delta_{21}(\delta_{11}\delta_{12} + \delta_{12}\delta_{22})\sigma_x + \delta_{22}(\delta_{21}\delta_{12} + \delta_{22}\delta_{22})\sigma_x$$

EXERCISE 13.3

(a) To rewrite the VEC in VAR form, first expand the terms:

$$\hat{y}_t - y_{t-1} = 2 - 0.5y_{t-1} + 0.5 + 3.5x_{t-1}$$

$$\hat{x}_t - x_{t-1} = 3 + 0.3y_{t-1} - 0.3 - 2.1x_{t-1}$$

Then rearrange in VAR form:

$$\hat{y}_t = (2 + 0.5) + (1 - 0.5)y_{t-1} + 3.5x_{t-1}$$

$$\hat{x}_t = (3 - 0.3) + (1 + 0.3)y_{t-1} - 2.1x_{t-1}$$

Simplifying gives the VAR model:

$$\hat{y}_t = 2.5 + 0.5y_{t-1} + 3.5x_{t-1}$$
$$\hat{x}_t = 2.7 + 1.3y_{t-1} - 2.1x_{t-1}$$

(b) To rewrite the VAR model in the VEC form, first rearrange terms so that the left hand side is in first-differenced form:

$$\hat{y}_t - y_{t-1} = -y_{t-1} + 0.7y_{t-1} + 0.3 + 0.24x_{t-1}$$
$$\hat{x}_t - x_{t-1} = -x_{t-1} + 0.6y_{t-1} - 0.6 + 0.52x_{t-1}$$

Next recognize that the error correction term for the first equation is the coefficient in front of the lagged variable y_{t-1}, that is –0.3.

Now factorize out this coefficient to obtain the cointegrating equation:

$$\Delta\hat{y}_t = -0.3(y_{t-1} - 1 - 0.8x_{t-1})$$
$$\Delta\hat{x}_t = 0.6y_{t-1} - 0.6 + (-1 + 0.52)x_{t-1}$$

For the second equation, factorize out the cointegrating equation to obtain the error-correction coefficient, 0.6. The VEC model is:

$$\Delta\hat{y}_t = -0.3(y_{t-1} - 1 - 0.8x_{t-1})$$
$$\Delta\hat{x}_t = 0.6(y_{t-1} - 1 - 0.8x_{t-1})$$

EXERCISE 13.5

(a) The data, real GDP of Australia and real GDP of the US are shown in Figure 13.1 on page 600 of *POE5*. Both series are clearly nonstationary which is confirmed by the Dickey-Fuller test with an intercept and trend. For *AUS* with no augmentation terms, we obtain $tau = -0.400$ with corresponding p-value = 0.9866. For *USA* with one augmentation term, we obtain $tau = -0.265$ with corresponding p-value = 0.9908.

(b) The estimated relationship with a constant included is

$$\widehat{AUS} = -1.072 + 1.001USA$$
$$(t) \quad (-2.66) \quad (164)$$

The test for cointegration using the residuals from this equation is

$$\Delta\hat{e}_t = -0.139e_{t-1}$$
$$(tau) \quad (-3.05)$$

The 5% critical value is -3.37. Given $-3.05 > -3.37$, there is insufficient evidence to conclude that cointegration exists.

One could argue that a negative intercept is not sensible because the real GDP for Australia will be positive even when the GDP for the US is zero, and vice versa. The cointegration equation excluding the constant term is in equation (13.7) of *POE5*. The test of stationarity in the residuals is in equation (13.8). It leads to a reversal of the above test decision.

(c) The estimated VEC model is reported in equation (13.9) of *POE5*.

EXERCISE 13.7

(a) Estimates for a VAR(3) model for $\{\Delta C, \Delta Y\}$ are as follows

Vector Autoregression Estimates
Sample (adjusted): 1987Q1 2015Q2
Included observations: 114 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	D(CONSN)	D(Y)
D(CONSN(-1))	0.182745	0.542463
	(0.09169)	(0.17497)
	[1.99306]	[3.10027]
D(CONSN(-2))	0.186586	0.243517
	(0.09340)	(0.17824)
	[1.99767]	[1.36625]
D(CONSN(-3))	0.376629	0.217420
	(0.08932)	(0.17044)
	[4.21675]	[1.27561]
D(Y(-1))	0.087481	-0.353677
	(0.05130)	(0.09790)
	[1.70520]	[-3.61263]
D(Y(-2))	-0.033967	-0.100599
	(0.05557)	(0.10605)
	[-0.61123]	[-0.94863]
D(Y(-3))	-0.087868	-0.080703
	(0.05171)	(0.09868)
	[-1.69917]	[-0.81780]
C	0.001921	0.003186
	(0.00081)	(0.00154)
	[2.38010]	[2.06846]

Lags for ΔC of orders 2 and 3 are significant at a 5% level in the consumption equation, but not beyond lag 1 in the income equation. Lags of ΔY beyond 1 are not significant in either equation.

We can also test whether the coefficients at a given lag are jointly significant – in each equation and in both equations jointly. To describe these tests we write the VAR(3) model as follows

$$\Delta CONSN_t = \beta_{10} + \sum_{s=1}^{3} \beta_{1s} \Delta CONSN_{t-s} + \sum_{r=1}^{3} \delta_{1r} \Delta Y_{t-r} + v_{1t}$$

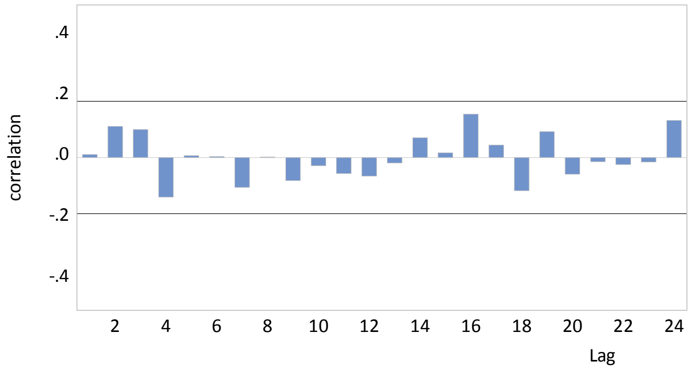
$$\Delta Y_t = \beta_{20} + \sum_{s=1}^{3} \beta_{2s} \Delta CONSN_{t-s} + \sum_{r=1}^{3} \delta_{2r} \Delta Y_{t-r} + v_{2t}$$

The results for χ^2-tests for the coefficients at each lag are as follows. See Section 6.1.5 on page 269 of *POE5* for an appreciation of the nature of these χ^2-tests. For a single equation they correspond to equation (6.14). The joint test of coefficients from both equations is more complex – it uses the covariance between the errors in the two equations.

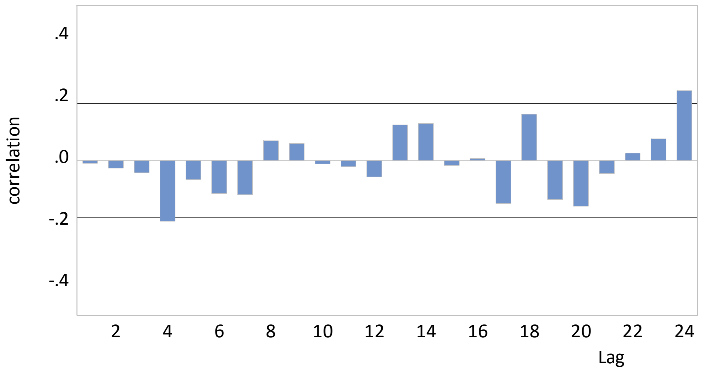
Null Hypothesis	χ^2-value	*p*-value	Description
$\beta_{11} = \delta_{11} = 0$	9.062	0.0108	Coefficients at lag 1 in *CONSN* equation
$\beta_{21} = \delta_{21} = 0$	18.305	0.0001	Coefficients at lag 1 in *Y* equation
$\beta_{11} = \delta_{11} = \beta_{21} = \delta_{21} = 0$	28.720	0.0000	Coefficients at lag 1 in both equations
$\beta_{12} = \delta_{12} = 0$	3.996	0.1356	Coefficients at lag 2 in *CONSN* equation
$\beta_{22} = \delta_{22} = 0$	2.227	0.3285	Coefficients at lag 2 in *Y* equation
$\beta_{12} = \delta_{12} = \beta_{22} = \delta_{22} = 0$	5.323	0.2558	Coefficients at lag 2 in both equations
$\beta_{13} = \delta_{13} = 0$	18.407	0.0001	Coefficients at lag 3 in *CONSN* equation
$\beta_{23} = \delta_{23} = 0$	1.930	0.3809	Coefficients at lag 3 in *Y* equation
$\beta_{13} = \delta_{13} = \beta_{23} = \delta_{23} = 0$	18.757	0.0009	Coefficients at lag 3 in both equations

The separate equation joint tests suggest that the estimates for coefficients at lags 1 and 3 are significant at a 5% level for the *CONSN* equation, but only those at lag 1 are significant in the *Y* equation. In the joint test for both equations, lag 1 and lag 3 coefficients are significant at the 5% level. Adding lags of order 4 did not lead to any significant coefficients.

Correlograms for the residuals from each equation are displayed below, with 5% significance bounds drawn at $\pm 1.96/\sqrt{114} = \pm 0.184$. There are no significant autocorrelations in the consumption equation. In the income equation there are significant, but relatively small, autocorrelations at lags 4 and 24.



Consumption equation



Income equation

(b) For testing whether ΔY Granger causes ΔC, we test the null hypothesis

$$H_0 : \delta_{11} = \delta_{12} = \delta_{13} = 0$$

The test values are $F = 2.233$ and $\chi^2 = 6.698$, with p-values of 0.0886 and 0.0822, respectively. We can conclude that ΔY Granger causes ΔC at a 10% significance level, but not at a 5% level.

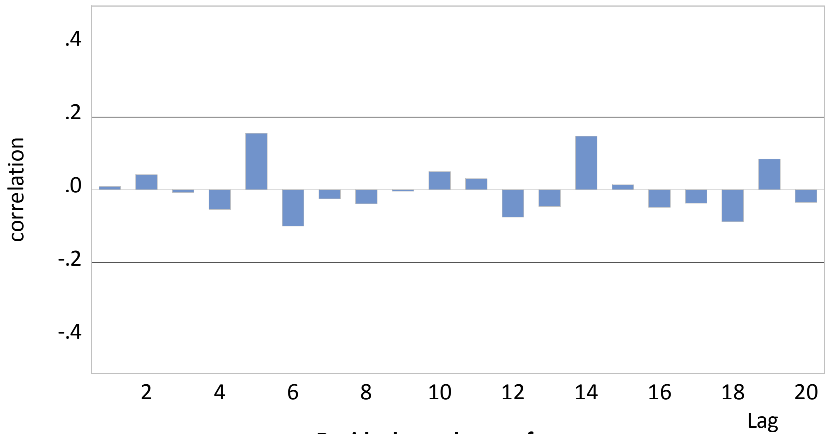
For testing whether ΔC Granger causes ΔY, we test the null hypothesis

$$H_0 : \beta_{21} = \beta_{22} = \beta_{23} = 0$$

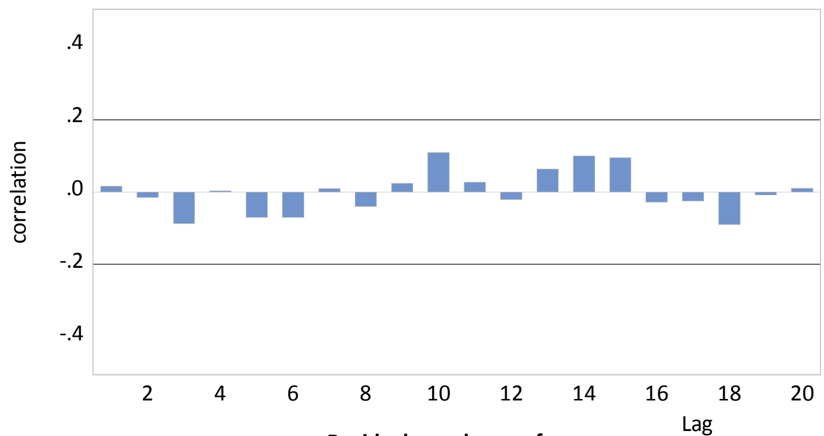
The test values are $F = 6.321$ and $\chi^2 = 18.962$, with p-values of 0.0005 and 0.0003, respectively. We can conclude that ΔY Granger causes ΔC at a 1% significance level.

EXERCISE 13.9

(a) The correlograms of the residuals from both equations are shown below with 5% significance bounds drawn at $\pm 1.96/\sqrt{98} = \pm 0.198$. Since there are no autocorrelations that exceed the significance bounds, there is no evidence of significant autocorrelation.



Residual correlogram for w



Residual correlogram for z

(b) Expressions for the impulse responses were derived in Exercise 13.1.

Effects of a shock to Δw of size $\sigma_{\Delta w}$ on Δw and Δz:

$$t=1,\quad \Delta w_1 = \sigma_{\Delta w} \qquad \Delta z_1 = 0$$

$$t=2,\quad \Delta w_2 = 0.743\sigma_{\Delta w} \qquad \Delta z_2 = -0.155\sigma_{\Delta w}$$

Effects of a shock to Δz of size $\sigma_{\Delta w}$ on Δw and Δz

$$t=1,\quad \Delta w_1 = 0 \qquad \Delta z_1 = \sigma_{\Delta z}$$

$$t=2,\quad \Delta w_2 = 0.214\sigma_{\Delta z} \qquad \Delta z_2 = 0.641\sigma_{\Delta z}$$

(c) Expressions for the variance decompositions were derived in Exercise 13.2.

1-step ahead forecast errors and variances:

$$FE_1^{\Delta w} = \Delta w_{t+1} - E_t[\Delta w_{t+1}] = \varepsilon_{t+1}^{\Delta w}; \qquad \text{var}(FE_1^{\Delta w}) = \sigma_{\Delta w}^2$$

$$FE_1^{\Delta z} = \Delta z_{t+1} - E_t[\Delta z_{t+1}] = \varepsilon_{t+1}^{\Delta z}; \qquad \text{var}(FE_1^{\Delta z}) = \sigma_{\Delta z}^2$$

2-step ahead forecast errors and variances:

$$FE_2^{\Delta w} = \Delta w_{t+2} - E_t[\Delta w_{t+2}] = [\delta_{11}\varepsilon_{t+1}^{\Delta w} + \delta_{12}\varepsilon_{t+1}^{\Delta z} + \varepsilon_{t+2}^{\Delta w}]$$

$$\text{var}(FE_2^{\Delta w}) = 0.743^2\sigma_{\Delta w}^2 + 0.214^2\sigma_{\Delta z}^2 + \sigma_{\Delta w}^2$$

$$FE_2^{\Delta z} = \Delta z_{t+2} - E_t[\Delta z_{t+2}] = [\delta_{21}\varepsilon_{t+1}^{\Delta w} + \delta_{22}\varepsilon_{t+1}^{\Delta z} + \varepsilon_{t+2}^{\Delta z}]$$

$$\text{var}(FE_2^{\Delta z}) = -0.155^2\sigma_{\Delta w}^2 + 0.641^2\sigma_{\Delta z}^2 + \sigma_{\Delta z}^2$$

The contribution of a shock to Δw on the 1-step forecast error variance of Δw is:

$$\sigma_{\Delta w}^2 / \sigma_{\Delta w}^2$$

The contribution of a shock to Δz on the 1-step forecast error variance of Δw is:

$$0 / \sigma_{\Delta w}^2$$

The contribution of a shock to Δw on the 1-step forecast error variance of Δz is:

$$0 / \sigma_{\Delta z}^2$$

The contribution of a shock to Δz on the 1-step forecast error variance of Δz is:

$$\sigma_{\Delta z}^2 / \sigma_{\Delta z}^2$$

The contribution of a shock to Δw on the 2-step forecast error variance of Δw is:

$$\sigma_{\Delta w}^2(0.743^2 + 1) / (0.743^2\sigma_{\Delta w}^2 + 0.214^2\sigma_{\Delta z}^2 + \sigma_{\Delta w}^2)$$

The contribution of a shock to Δz on the 2-step forecast error variance of Δw is:

$$\sigma_{\Delta z}^2(0.214^2) / (0.743^2\sigma_{\Delta w}^2 + 0.214^2\sigma_{\Delta z}^2 + \sigma_{\Delta w}^2)$$

The contribution of a shock to Δw on the 2-step forecast error variance of Δz is:

$$\sigma^2_{\Delta w}(-0.155^2)\big/(-0.155^2\sigma^2_{\Delta w}+0.641^2\sigma^2_{\Delta z}+\sigma^2_{\Delta z})$$

The contribution of a shock to Δz on the 2-step forecast error variance of Δz is:

$$\sigma^2_{\Delta z}(0.641^2+1)\big/(-0.155^2\sigma^2_{\Delta w}+0.641^2\sigma^2_{\Delta z}+\sigma^2_{\Delta z})$$

EXERCISE 13.11

(a) The coefficients (–0.046 and –0.098) suggest an inverse relationship between a change in the unemployment rate (*DU*) and a change in the inflation rate (*DP).*

(b) The response of *DU* at time $t+1$ following a unit shock to *DU* at time *t* is 0.180.

(c) The response of *DP* at time $t+1$ following a unit shock to *DU* at time *t* is -0.098.

(d) The response of *DU* at time $t+2$ is

$$DU_{t+2}=0.180DU_{t+1}-0.046DP_{t+1}=0.180\times 0.180-0.046\times -0.098=0.037$$

(e) The response of *DP* at time $t+2$ is

$$DP_{t+2}=-0.098DU_{t+1}+0.373DP_{t+1}=-0.098\times 0.180+0.373\times -0.098=-0.054$$

These results suggest, following a shock to unemployment, that *DU* increases but *DP* falls.

EXERCISE 13.13

The results for a first-order VAR and the ARDL equations are given below. Comparing the two sets of estimates, we find the coefficients of corresponding variables in the VAR and ARDL models are quite different, except for the coefficient of DV_{t-1} in the equations for *SP*. The differences should not be surprising since the coefficients in the VAR and ARDL models have are quite different interpretations. The pair of ARDL equations represents two simultaneous equations with endogenous variables SP_t and DV_t. The VAR equations are the reduced form equations from the simultaneous system. These concepts were discussed in Chapter 11. To derive the reduced form coefficients from those in the structural ARDL system, we solve the two ARDL equations simultaneously for SP_t and DV_t. The solution is

$$SP_t=\frac{\alpha_{10}+\alpha_{13}\alpha_{20}}{1-\alpha_{13}\alpha_{23}}+\frac{\alpha_{11}+\alpha_{13}\alpha_{21}}{1-\alpha_{13}\alpha_{23}}SP_{t-1}+\frac{\alpha_{12}+\alpha_{13}\alpha_{22}}{1-\alpha_{13}\alpha_{23}}DV_{t-1}+\frac{e_t^s+\alpha_{13}e_t^d}{1-\alpha_{13}\alpha_{23}}$$

$$DV_t=\frac{\alpha_{20}+\alpha_{23}\alpha_{10}}{1-\alpha_{13}\alpha_{23}}+\frac{\alpha_{21}+\alpha_{23}\alpha_{11}}{1-\alpha_{13}\alpha_{23}}SP_{t-1}+\frac{\alpha_{22}+\alpha_{23}\alpha_{12}}{1-\alpha_{13}\alpha_{23}}DV_{t-1}+\frac{\alpha_{23}e_t^s+e_t^d}{1-\alpha_{13}\alpha_{23}}$$

Deriving estimates of the reduced form coefficients from the structural coefficients estimates,

$$\hat\beta_{10}=\frac{\hat\alpha_{10}+\hat\alpha_{13}\hat\alpha_{20}}{1-\hat\alpha_{13}\hat\alpha_{23}}=3.434 \qquad \hat\beta_{20}=\frac{\hat\alpha_{20}+\hat\alpha_{23}\hat\alpha_{10}}{1-\hat\alpha_{13}\hat\alpha_{23}}=2.605$$

$$\hat\beta_{11}=\frac{\hat\alpha_{11}+\hat\alpha_{13}\hat\alpha_{21}}{1-\hat\alpha_{13}\hat\alpha_{23}}=0.3014 \qquad \hat\beta_{21}=\frac{\hat\alpha_{21}+\hat\alpha_{23}\hat\alpha_{11}}{1-\hat\alpha_{13}\hat\alpha_{23}}=0.3575$$

$$\hat{\beta}_{12} = \frac{\hat{\alpha}_{12} + \hat{\alpha}_{13}\hat{\alpha}_{22}}{1 - \hat{\alpha}_{13}\hat{\alpha}_{23}} = -0.3001 \qquad \hat{\beta}_{22} = \frac{\hat{\alpha}_{22} + \hat{\alpha}_{23}\hat{\alpha}_{12}}{1 - \hat{\alpha}_{13}\hat{\alpha}_{23}} = -0.01623$$

These estimates are identical to those obtained by directly estimating the reduced form equations. In this model, deriving the reduced form estimates from the structural least-squares estimates yields the same results as least squares estimation of the reduced form.

Note, however, that we are unable to derive structural estimates from the reduced form estimates. There are only 6 reduced form coefficients and 8 structural coefficients. There are multiple values of the α_{ij} that will lead to the same reduced form estimates. In the language of Chapter 11, the structural equations are unidentified.

Thus, although the contemporaneous variables (*SP* and *DV*) appear to be significant in the ARDL equations, the lack of identification means that the ARDL results should not be used to infer the **contemporaneous** role of dividends on share prices.

Vector Autoregression Estimates
Sample (adjusted): 3 91
Included observations: 89 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	SP	DV
SP(-1)	0.301399	0.357491
	(0.12119)	(0.08770)
	[2.48689]	[4.07637]
DV(-1)	-0.300147	-0.016231
	(0.15562)	(0.11261)
	[-1.92877]	[-0.14414]
C	3.434256	2.605104
	(1.77289)	(1.28289)
	[1.93709]	[2.03066]

Dependent Variable: SP
Method: Least Squares
Sample (adjusted): 3 91
Included observations: 89 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.627032	1.578864	1.030508	0.3057
SP(-1)	0.053399	0.115169	0.463655	0.6441
DV	0.693724	0.129639	5.351182	0.0000
DV(-1)	-0.288887	0.135393	-2.133686	0.0358

Dependent Variable: DV
Method: Least Squares
Sample (adjusted): 3 91
Included observations: 89 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.357627	1.140131	1.190763	0.2371
SP	0.363245	0.067881	5.351182	0.0000
SP(-1)	0.248009	0.078989	3.139810	0.0023
DV(-1)	0.092796	0.100057	0.927432	0.3563

(a) As long as v_t^s and v_t^d are serially uncorrelated, lagged values of *SP* and *DV* will be uncorrelated with v_t^s and v_t^d, and least squares estimation of the VAR yields consistent estimates. It is important to include sufficient lags to eliminate serial correlation in the errors.

(b) In the derivation above we showed that

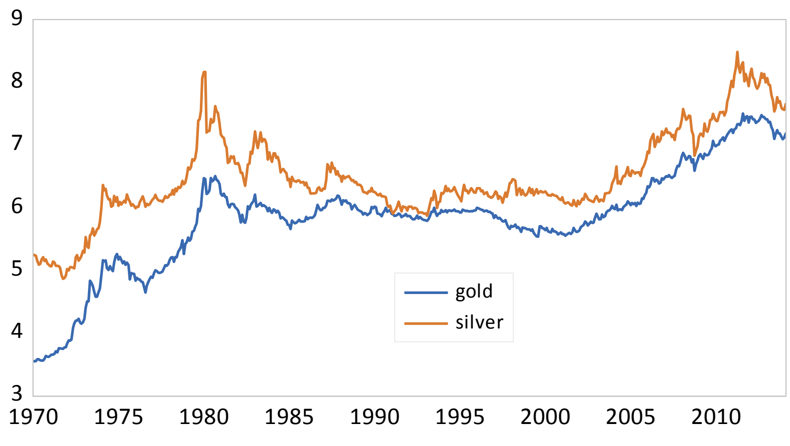
$$v_t^s = \frac{e_t^s + \alpha_{13} e_t^d}{1 - \alpha_{13}\alpha_{23}} \qquad \text{and} \qquad v_t^d = \frac{\alpha_{23} e_t^s + e_t^d}{1 - \alpha_{13}\alpha_{23}}$$

Solving these two equations for e_t^s and e_t^d shows that e_t^s and e_t^d both depend on v_t^s and v_t^d. Since SP_t and DV_t depend directly on v_t^s and v_t^d through their reduced form equations, e_t^s and e_t^d will both be correlated with SP_t and DV_t. This correlation leads least squares estimates of the ARDL equations to be inconsistent. We also have the bigger problem of structural coefficients that are unidentified.

(c) Using a 5% significance level, the VAR results show that the **lagged** rate of change in dividends has no significant influence on the rate of change in share prices, but the **lagged** rate of change in share prices has a significant effect on the rate of change in dividends.

EXERCISE 13.15

(a) The two series are plotted in the following diagram. The plots suggest that the two prices do move together.



(b) The first step towards developing predictive models is to find the order of integration of the two series. Unit root test results for *GOLD* and *SILVER* and their first differences follow. The unit root tests suggest both series are I(1) variables. The *p*-values for the unit root tests for *GOLD* and *SILVER* are 0.1618 and 0.4499, respectively, indicating the variables in levels are nonstationary. The tests on their first differences reject the null hypothesis of a unit root, indicating that $\Delta GOLD$ and $\Delta SILVER$ are stationary.

Null Hypothesis: GOLD has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=18)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-2.333516	0.1618

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(GOLD)
Method: Least Squares
Sample (adjusted): 1970M02 2014M02
Included observations: 529 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
GOLD(-1)	-0.007270	0.003115	-2.333516	0.0200
C	0.049264	0.018344	2.685512	0.0075

Null Hypothesis: D(GOLD) has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=18)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-22.69374	0.0000

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(GOLD,2)
Method: Least Squares
Sample (adjusted): 1970M03 2014M02
Included observations: 528 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(GOLD(-1))	-0.990337	0.043639	-22.69374	0.0000
C	0.006820	0.002605	2.618036	0.0091

Null Hypothesis: SILVER has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=18)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-1.662321	0.4499

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(SILVER)
Method: Least Squares
Sample (adjusted): 1970M02 2014M02
Included observations: 529 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
SILVER(-1)	-0.010131	0.006095	-1.662321	0.0970
C	0.070280	0.039770	1.767159	0.0778

Null Hypothesis: D(SILVER) has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=18)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-23.15548	0.0000

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(SILVER,2)
Method: Least Squares
Sample (adjusted): 1970M03 2014M02
Included observations: 528 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(SILVER(-1))	-1.010394	0.043635	-23.15548	0.0000
C	0.004622	0.004310	1.072387	0.2840

To check whether *GOLD* and *SILVER* are cointegrated, we estimate the equation

$$SILVER = \beta_1 + \beta_2 GOLD + e_t$$

and check the residual from this equation to see if they are stationary. The estimated equation is

Dependent Variable: SILVER
Method: Least Squares
Sample: 1970M01 2014M02
Included observations: 530

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.021509	0.096025	21.05188	0.0000
GOLD	0.766002	0.016300	46.99371	0.0000
R-squared	0.807046	Mean dependent var		6.489472

To test whether the residuals *EHAT* have a unit root we compare $\tau = -3.033$ with the 5% critical value -3.37 found in Table 12.4 on page 583 of *POE5*. Since $\tau = -3.033 > -3.37$, we cannot reject the existence of a unit root. We cannot treat *SILVER* and *GOLD* as cointegrated.

Null Hypothesis: EHAT has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=18)

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(EHAT)
Method: Least Squares
Sample (adjusted): 1970M02 2014M02
Included observations: 529 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
EHAT(-1)	-0.031642	0.010433	-3.032881	0.0025

Given *SILVER* and *GOLD* are not cointegrated, we estimate a VAR in their first differences.

Vector Autoregression Estimates
Sample (adjusted): 1970M03 2014M02
Included observations: 528 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	D(GOLD)	D(SILVER)
D(GOLD(-1))	-0.008610	0.077954
	(0.06034)	(0.10035)
	[-0.14269]	[0.77681]
D(SILVER(-1))	0.015915	-0.042721
	(0.03626)	(0.06031)
	[0.43887]	[-0.70835]
C	0.006874	0.004236
	(0.00261)	(0.00434)
	[2.63381]	[0.97597]

None of the lag coefficient estimates are significant, even at a 10% level of significance. The absence of lagged changes in the equations implies the best predictive models are random walks of the form $GOLD_t = GOLD_{t-1} + e_{Gt}$ and $SILVER_t = SILVER_{t-1} + e_{St}$. Because the constant is significant in the *GOLD* equation, a random walk with drift $GOLD_t = \delta + GOLD_{t-1} + e_{Gt}$ could also be considered.