PRINCIPLES OF ECONOMETRICS

5TH EDITION

ANSWERS TO ODD-NUMBERED EXERCISES IN CHAPTER 15

EXERCISE 15.1

(a) From equation (15.12) in *POE5*, we have

$$\tilde{y}_{it} = \beta_2 \tilde{x}_{it} + \tilde{e}_{it}$$

where $\tilde{y}_{it} = y_{it} - \bar{y}_i$, $\tilde{x}_{it} = x_{it} - \bar{x}_i$, and $\tilde{e}_{it} = e_{it} - \bar{e}_i$. The fixed effects estimator for β_2 is the least squares estimator applied to this equation. It is given by

$$\hat{\beta}_{2,FE} = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} \tilde{x}_{it}\tilde{y}_{it}}{\sum_{i=1}^{N}\sum_{t=1}^{T} \tilde{x}_{it}^2} = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} (x_{it} - \bar{x}_i)(y_{it} - \bar{y}_i)}{\sum_{i=1}^{N}\sum_{t=1}^{T} (x_{it} - \bar{x}_i)^2}$$

(b) The random effects estimator for β_2 is the least squares estimator applied to the equation

$$y_{it}^* = \bar{\beta}_1(1 - \hat{\alpha}) + \beta_2 x_{it}^* + v_{it}^*$$

where $y_{it}^* = y_{it} - \hat{\alpha}\bar{y}_i$, and $x_{it}^* = x_{it} - \hat{\alpha}\bar{x}_i$. This estimator is given by

$$\hat{\beta}_{2,RE} = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} \left(x_{it}^* - \bar{\bar{x}}^*\right)\left(y_{it}^* - \bar{\bar{y}}^*\right)}{\sum_{i=1}^{N}\sum_{t=1}^{T} \left(x_{it}^* - \bar{\bar{x}}^*\right)^2}$$

Now,

$$\bar{\bar{x}}^* = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} (x_{it} - \hat{\alpha}\bar{x}_i) = \bar{\bar{x}} - \hat{\alpha}\bar{\bar{x}}$$

and

$$x_{it}^* - \bar{\bar{x}}^* = (x_{it} - \hat{\alpha}\bar{x}_i) - \left(\bar{\bar{x}} - \hat{\alpha}\bar{\bar{x}}\right) = x_{it} - \hat{\alpha}\left(\bar{x}_i - \bar{\bar{x}}\right) - \bar{\bar{x}}$$

A similar result holds for $y_{it}^* - \bar{\bar{y}}^*$. Thus,

$$\hat{\beta}_{2,RE} = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} \left(x_{it} - \hat{\alpha}\left(\bar{x}_i - \bar{\bar{x}}\right) - \bar{\bar{x}}\right)\left(y_{it} - \hat{\alpha}\left(\bar{y}_i - \bar{\bar{y}}\right) - \bar{\bar{y}}\right)}{\sum_{i=1}^{N}\sum_{t=1}^{T} \left(x_{it} - \hat{\alpha}\left(\bar{x}_i - \bar{\bar{x}}\right) - \bar{\bar{x}}\right)^2}$$

(c) The pooled least squares estimator is given by

$$\hat{\beta}_{2,PLS} = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} \left(x_{it} - \bar{\bar{x}}\right)\left(y_{it} - \bar{\bar{y}}\right)}{\sum_{i=1}^{N}\sum_{t=1}^{T} \left(x_{it} - \bar{\bar{x}}\right)^2}$$

The pooled least squares estimator uses variation in x_{it} and y_{it} around their overall means; it does not distinguish between variation within and between individuals. The fixed effects estimator uses only variation from individual means, known as within variation. The random effects estimator uses both overall and between variation, weighted according to the value of $\hat{\alpha}$; between variation uses $\left(\bar{x}_i - \bar{\bar{x}}\right)$ and $\left(\bar{y}_i - \bar{\bar{y}}\right)$.

EXERCISE 15.3

(a) $\rho = \operatorname{corr}(v_{it}, v_{is}) = \dfrac{\operatorname{cov}(v_{it}, v_{is})}{\sqrt{\operatorname{var}(v_{it})\operatorname{var}(v_{is})}} = \dfrac{\sigma_u^2}{\sigma_u^2 + \sigma_e^2} = \dfrac{1}{1+1} = 0.5$

(b) If $T = 2$, $\alpha = 0.4226$. If $T = 5$, $\alpha = 0.5918$.

(c) False. The transformation parameter is $\alpha = 1 - \sigma_e \big/ \sqrt{T\sigma_u^2 + \sigma_e^2}$. As T increases the second component becomes smaller, so that α becomes larger.

(d) Case 1: $\alpha = \dfrac{1}{4} \quad \Rightarrow \quad \sigma_u^2 = \dfrac{7}{18} \cong 0.389$

Case 2: $\alpha = \dfrac{1}{2} \quad \Rightarrow \quad \sigma_u^2 = \dfrac{3}{2} = 1.5$

Case 3: $\alpha = \dfrac{9}{10} \quad \Rightarrow \quad \sigma_u^2 = \dfrac{99}{2} = 49.5$

(e) The variance of the OLS estimator is affected by the variance of the combined error, $\sigma_u^2 + \sigma_e^2$ and the correlation between random errors across time for each individual. The random effects estimator and the fixed effects estimator error variances are directly affected only by the variance of the idiosyncratic error e_{it}.

EXERCISE 15.5

(a) Using $u_i = y_{it} - 10 - 5x_{it} - e_{it}$, these values are $-0.95, 0.51, 1.75$ for $i = 1,2,3$.

(b) The LM statistic numerator includes $\sum_{t=1}^{T} \hat{e}_{it}$ for $i = 1, 2, 3$. These sums are $-9.52, 2.26, 7.26$ and their squares are $90.6304, 5.1076, 52.7076$. The sum of these squared values is $\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\hat{e}_{it}\right)^2 = 148.4456$. The sum of squared residuals is $\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{e}_{it}^2 = 86.7626$. Plugging in $N = 3$ and $T = 2$, we obtain $LM = 1.3746099$. As noted, computer software generally reports the square of this value, which is 1.8895525. The LM statistic critical values are ± 1.96. Thus, the test does not reject the null hypothesis that $\sigma_u^2 = 0$. However, the sample is too small for this result to be valid. The exercise was designed for you to understand the calculations.

(c)
$$t = \frac{b_{FE} - b_{RE}}{\left[\operatorname{se}(b_{FE})^2 - \operatorname{se}(b_{RE})^2\right]^{1/2}} = \frac{5.21 - 5.31}{\left[0.94^2 - 0.81^2\right]^{1/2}} = -0.2097$$

With critical values ± 1.96, we fail to reject the null hypothesis: x_{it} is uncorrelated with u_i.

(d) The transformation parameter is $\hat{\alpha} = 0.8107$. The fixed effects estimates arise when the transformation parameter is 1.0, and the OLS estimates are obtained when the transformation parameter is 0.0. In this case the random effects estimates should be closer to the OLS estimates than the fixed effects estimates.

(e) The estimated correlation is $\hat{\rho} \cong 0.9308$, a high value indicating a strong covariance between v_{i1} and v_{i2}. There is a large potential efficiency gain from using the random effects estimator instead of OLS.

EXERCISE 15.7

(a) Equation (15.8) is $(y_{i2} - y_{i1}) = \beta_2 (x_{2i2} - x_{2i1}) + (e_{i2} - e_{i1})$. Estimating this equation by OLS yields the first-difference estimator. OLS consistency requires that the explanatory variable, here $(x_{2i2} - x_{2i1})$, be uncorrelated with the regression error term, here $(e_{i2} - e_{i1})$. To show this consider $E\left[(x_{2i2} - x_{2i1})(e_{i2} - e_{i1})\right] = E(x_{2i2}e_{i2}) - E(x_{2i2}e_{i1}) - E(x_{2i1}e_{i2}) + E(x_{2i1}e_{i1})$. The first and fourth terms are zero under "contemporaneous" exogeneity. However, the second and third terms involve "*x*" values from one period and random errors "*e*" from another period. The first part of (15.5a), $\operatorname{cov}(e_{it}, x_{2is}) = 0$, is much stronger than the usual sort of exogeneity assumption. It is stronger because it is more than just contemporaneous exogeneity $\operatorname{cov}(e_{it}, x_{2it}) = 0$; it says e_{it} is uncorrelated with <u>all</u> the values $x_{2i1}, x_{2i2}, \ldots, x_{2iT}$. In the case of two time periods e_{it} must be uncorrelated with x_{2is} for all values of *t* and *s* equaling 1 and 2. That is, $E(x_{2i2}e_{i2}) = E(x_{2i2}e_{i1}) = E(x_{2i1}e_{i2}) = E(x_{2i1}e_{i1}) = 0$. Then, $E\left[(x_{2i2} - x_{2i1})(e_{i2} - e_{i1})\right] = 0$, and the first difference estimator is consistent.

(b) The panel data model (XR15.6) is

$$\ln(WAGE_{it}) = \beta_1 + \beta_2 EXPER_{it} + \beta_3 EXPER_{it}^2 + \beta_4 SOUTH_{it} + \beta_5 UNION_{it} + (u_i + e_{it})$$

Adding an indicator variable $D88_t = 1$, the model becomes

$$\ln(WAGE_{it}) = \beta_1 + \delta_1 D88_t + \beta_2 EXPER_{it} + \beta_3 EXPER_{it}^2 + \beta_4 SOUTH_{it} + \beta_5 UNION_{it} + (u_i + e_{it})$$

In this model δ_1 is an intercept dummy variable. For 1988 the observations are

$$\ln(WAGE_{i,88}) = \beta_1 + \delta_1 + \beta_2 EXPER_{i,88} + \beta_3 EXPER_{i,88}^2 + \beta_4 SOUTH_{i,88} + \beta_5 UNION_{i,88} + (u_i + e_{i,88})$$

For the 1987 observations they are

$$\ln(WAGE_{i,87}) = \beta_1 + \beta_2 EXPER_{i,87} + \beta_3 EXPER_{i,87}^2 + \beta_4 SOUTH_{i,87} + \beta_5 UNION_{i,87} + (u_i + e_{i,87})$$

(c) The first differenced observations are now

$$\Delta \ln(WAGE) = \delta_1 + \beta_2 \Delta EXPER + \beta_3 \Delta EXPER^2 + \beta_4 \Delta SOUTH + \beta_5 \Delta UNION + \Delta e$$

The intercept in the differenced data model is the difference in the intercepts for the regression function in 1987 and 1988. Using the results in Table 15.11 the *t*-value is $t = -0.0774 / 0.0525 = -1.4743$. Thus, we fail to reject the null hypothesis $\delta_1 = 0$ against the alternative $\delta_1 \neq 0$ at even the 10% level of significance. There is no evidence that the intercept parameter is different in 1987 and 1988.

(d) The *t*-statistic values for *SOUTH* and *UNION* are -1.189 and -.5671, respectively. Neither coefficient is significantly different from zero, and thus based on these individual tests there is no evidence against strict exogeneity.

(e) The variables are not jointly significant at conventional significance levels. The *F*-test degrees of freedom are (2, 710), because there are two hypotheses, 716 observations and 6 parameter values. Using Statistical Table 4 the *F*-critical value is 3.00. Using statistical software the critical value is 3.0084079. We fail to reject the null hypothesis that the coefficients are jointly insignificant.

EXERCISE 15.9

(a) The percentage differences between the cluster-robust standard errors and the conventional standard errors for *LCAPITAL* and *LLABOR* are 78.3% and 73.6%, respectively. The cluster robust standard errors are substantially larger.

(b) The resulting *t*-values are 55.8 and 47.4 using conventional and robust standard errors, respectively. We strongly reject the hypothesis that $\rho = 0$. In this case, however, recall that $v_{it} = u_i + e_{it}$ and $v_{i,t-1} = u_i + e_{i,t-1}$. Even if there is no time series correlation in the idiosyncratic error component e_{it} there will still be correlation between v_{it} and $v_{i,t-1}$ if $u_i \neq 0$ because of its presence in both time periods. Thus, what we may be detecting here is the presence of unobserved heterogeneity as well as perhaps time-series error correlation.

(c) The test statistic is $t = \left(\hat{\rho} - (-.5)\right) / \text{se}\left(\hat{\rho}\right)$ where the standard error is either the conventional or robust version. The two *t*-statistic values are 5.80 and 3.00. We reject the null hypothesis that $\rho = -1/2$ and conclude there is other serial correlation of a time-series nature present. When using fixed effects we are quite justified in using cluster robust standard errors on this basis alone.

(d) The test statistic is $t = \left(\hat{\rho} - (-.5)\right) / \text{se}\left(\hat{\rho}\right)$ where the standard error is either the conventional or robust version. The *t*-statistic value in this case is 13.53. We reject the null hypothesis that $\rho = -1/2$ and conclude there is other serial correlation of a time-series nature present. When using fixed effects we are quite justified in using cluster robust standard errors on this basis alone.

EXERCISE 15.11

(a) The value *C* = 10.143333.

(b) The calculated values are 5.5708 and 9.9785. The rounded values are 5.57 and 9.98. These are identical to the LSDV estimates shown above.

(c) The residuals, rounded to two decimals, are -3.06, -6.1, 0.28, and -0.6

(d) $\hat{\tilde{y}}_{it} = 5.21\tilde{x}_{it}$.

(e) To calculate these residuals, we must first create the within-transformed values of *y* and *x*. That is $\tilde{y}_{it} - \overline{y}_{i\bullet}$ and $\tilde{x}_{it} = x_{it} - \overline{x}_{i\bullet}$ Using the data in Table 15.9 and the sample means in (b), the values of $\tilde{y}_{it} - \bar{y}_{i\cdot}$ are 1.36, -1.36, -2.63 and 2.64, and the values of $\tilde{x}_{it} = x_{it} - \bar{x}_{i\cdot}$ are -0.03 0.03, -0.59, 0.59. The resulting residuals are 1.52, -1.53, 0.44, -0.43. [Note: the rounded computer software generated values are 1.53, -1.53, 0.44, -0.44]

(f) The within residuals in part (e) are identical to the residuals from the least squares dummy variable regression model given in the problem statement. Let $\hat{e}_{it} = y_{it} - \hat{y}_{it}$ where $\hat{y}_{it} = 5.57D_{1i} + 9.98D_{2i} + 14.88D_{3i} + 5.21x_{it}$. These residuals are analogues of the idiosyncratic errors e_{it}. The errors in part (c) are something else altogether. Call them $e_{it}^{*} = y_{it} - y_{it}^{*} = y_{it} - C - 5.21x_{it}$. To see what these residuals are, add and subtract $\hat{y}_{it}$ to the right-hand side. Then

$$\begin{aligned}
e_{it}^{*} &= \left(y_{it} - \hat{y}_{it}\right) + \hat{y}_{it} - C - 5.21x_{it} \\
&= \hat{e}_{it} + \left(5.57D_{1i} + 9.98D_{2i} + 14.88D_{3i} + 5.21x_{it}\right) - C - 5.21x_{it} \\
&= \hat{e}_{it} + \left(5.57 - C\right)D_{1i} + \left(9.98 - C\right)D_{2i} + \left(14.88 - C\right)D_{3i}
\end{aligned}$$

For *id* = 1

$$\begin{aligned}
e_{11}^{*} &= \hat{e}_{11} + \left(5.57 - C\right) = 1.52 + \left(5.57 - 10.14\right) = -3.05 \\
e_{12}^{*} &= \hat{e}_{12} + \left(5.57 - C\right) = -1.53 + \left(5.57 - 10.14\right) = -6.1
\end{aligned}$$

For *id* = 2

$$\begin{aligned}
e_{21}^{*} &= \hat{e}_{21} + \left(9.98 - C\right) = 0.44 + \left(9.98 - 10.14\right) = 0.28 \\
e_{22}^{*} &= \hat{e}_{22} + \left(9.98 - C\right) = -0.43 + \left(9.98 - 10.14\right) = -0.59
\end{aligned}$$

[Note: the rounded computer software generated values are -3.06, -6.1, 0.28 and -0.6]

EXERCISE 15.13

(a) The panel data regression is $y_{it} = \beta_1 + \beta_2 x_{2it} + \alpha_1 w_{1i} + u_i + e_{it}$. For time periods 1-3 it is

$$y_{i1} = \beta_1 + \beta_2 x_{2i1} + \alpha_1 w_{1i} + u_i + e_{i1}$$

$$y_{i2} = \beta_1 + \beta_2 x_{2i2} + \alpha_1 w_{1i} + u_i + e_{i2}$$

$$y_{i3} = \beta_1 + \beta_2 x_{2i3} + \alpha_1 w_{1i} + u_i + e_{i3}$$

Then,

$$\begin{aligned}
\Delta y_{i2} &= y_{i2} - y_{i1} = \beta_2\left(x_{2i2} - x_{2i1}\right) + \left(e_{i2} - e_{i1}\right) = \beta_2 \Delta x_{2i2} + \Delta e_{i2} \\
\Delta y_{i3} &= y_{i3} - y_{i2} = \beta_2\left(x_{2i3} - x_{2i2}\right) + \left(e_{i3} - e_{i2}\right) = \beta_2 \Delta x_{2i3} + \Delta e_{i3}
\end{aligned}$$

Upon differencing two things happen. First, the time invariant explanatory variable w_{1i} is subtracted away. Second the time invariant error u_i is also subtracted out. Thus the two differenced equations no longer contain unobserved heterogeneity.

(b) Under homoskedasticity and an assumption of no time-series error correlation the variances of Δe_{i2} and Δe_{i3} are the same. They are $\operatorname{var}\left(\Delta e_{it}\right) = \operatorname{var}\left(e_{it} - e_{i,t-1}\right) = 2\sigma_e^2$.

(c) $\operatorname{cov}(\Delta e_{i2}, \Delta e_{i3}) = E(\Delta e_{i2}\Delta e_{i3}) = E\left[(e_{i2} - e_{i1})(e_{i3} - e_{i2})\right] = -\sigma_e^2$

Then the correlation is

$$\operatorname{corr}(\Delta e_{i2}, \Delta e_{i3}) = \frac{\operatorname{cov}(\Delta e_{i2}, \Delta e_{i3})}{\sqrt{\operatorname{var}(\Delta e_{i2})}\sqrt{\operatorname{var}(\Delta e_{i3})}} = \frac{-\sigma_e^2}{\sqrt{2\sigma_e^2}\sqrt{2\sigma_e^2}} = -\frac{1}{2}$$

(d) If the errors follow a random walk process, with $e_{i2} = e_{i1} + \varepsilon_{i2}$ and $e_{i3} = e_{i2} + \varepsilon_{i3}$, where ε_{it} are independent and identically distributed with means zero and constant variances, then $\operatorname{corr}(\Delta e_{i2}, \Delta e_{i3}) = 0$.

EXERCISE 15.15

(a) The OLS estimates of the elasticities are only slightly affected by the inclusion of the year dummy variables in Model (2). On the other hand, the fixed effects estimates in Model (3) are quite different from the OLS estimates in each case. The *AREA* elasticity is much larger and the elasticities for *LABOR* and *FERT* are much smaller. The 95% interval estimate for *AREA* elasticity is approximately [0.43, 0.74], an interval for *LABOR* input elasticity is about [0.12, 0.40] and for *FERT* is [0.01, 0.18]. None of these intervals contain the OLS estimates

(b) Comparing Model (1) to (2), the *F*-test for the joint significance of the year indicators is

$$F = \frac{(SSE_R - SSE_U)/J}{SSE_U/(NT-K)} = \frac{(40.5654 - 36.2031)/7}{36.2031/341} = 5.8698$$

Using Statistical Table 4, the 95th percentile of the $F_{(7,341)}$ distribution is 2.01. Thus we reject the null hypothesis that the year indicator coefficients are jointly zero, and conclude that at least one year-indicator coefficient is not zero. Thus we prefer Model (2) to (1).

Next we compare Model (3) to Model (1). The *F*-statistic is

$$F = \frac{(SSE_R - SSE_U)/J}{SSE_U/(NT-K)} = \frac{(36.2031 - 27.6623)/43}{27.6623/305} = 3.3085$$

Using Statistical Table 4, the 95th percentile of the $F_{(43,305)}$ distribution is between 1.46 and 1.32. The computer generated value is 1.420118. The null hypothesis that the individual effects are jointly zero is rejected at the 5% level. We conclude that there are significant individual differences. We prefer Model (3) to Model (1)

Finally, we compare Model (4) to Model (3). The *F*-statistic is

$$F = \frac{(SSE_R - SSE_U)/J}{SSE_U/(NT-K)} = \frac{(27.6623 - 23.0824)/7}{23.0824/298} = 8.4468$$

Using Statistical Table 4, the 95th percentile of the $F_{(7,298)}$ distribution is 2.01. Thus we reject the null hypothesis that the year indicator coefficients are jointly zero, and conclude that at least one year-indicator coefficient is not zero. We prefer Model (4) to Model (3).

(c) Note the cluster-robust standard errors are larger than the conventional ones. First, we use the critical value 1.96 that is valid in large samples. The interval estimates for the coefficients are: for ln(*AREA*), [0.4763, 0.7723] and [0.4340, 0.8146], for ln(*LABOR*), [0.1075, 0.3749] and [0.0515, 0.4309], for ln(*FERT*), [0.0077, 0.1703] and [-0.0837, 0.2617]. Each of the intervals are relatively wide, and we come away with the assessment that we have not estimated the elasticities very precisely. In fact we cannot reject the null hypothesis that the elasticity of output with respect to *FERT* is zero, as the 5% level. When using cluster-robust standard errors the corrected degrees of freedom for the *t*-distribution should be $M - 1 = 43$. The resulting critical value is 2.0166922. The adjusted interval estimates are [0.4285, 0.8201], [0.0460, 0.4364] and [-0.0887, 0.2667]. Our conclusions remain the same.

(d) The robust *t*-value for this coefficient is $t = 0.0890/0.0881 = 1.01$. Given the large sample size the *p*-value can be calculated approximately using Statistical Table 1. The value is $p = 2\left[1-\Phi(1.01)\right] = 2\left[1-0.8438\right] = 0.3124$. However, when cluster corrected standard errors are calculated it can be shown that the "corrected" number of observations is the number of clusters, here 44, and the degrees of freedom for a *t*-test should be $M - 1 = 43$. In this case we should the *t*-distribution to compute the *p*-value, which requires computing software. The corrected *p*-value is 0.3180.

EXERCISE 15.17

(a) The estimated regression with differenced data is

$$\widehat{LIQUORD}_{it} = 0.02975 INCOMED_{it}$$
$$\text{(se)} \qquad (0.02922)$$

The 95% interval estimate of the coefficient of *INCOMED* is [-0.0284146, 0.0879082]. The interval covers zero; we have no evidence against the hypothesis that income does not affect liquor expenditures.

(b) The random effects estimates are

$$\widehat{LIQUOR}_{it} = 0.96903 + 0.02658\, INCOME_{it}$$
$$\text{(se)} \qquad (0.52101)\ (.00701)$$

The 95% interval estimate for the coefficient of *INCOME* is [0.01283, 0.04032]. We estimate with 95% confidence that for each additional \$1000 income the household will spend between \$12.83 and \$40.32 more on liquor. The random effects coefficient estimate is slightly smaller than the difference estimator coefficient, but the standard error of the random effects estimator is about 25% of the standard error of the difference estimator's standard error, yielding a statistical significance.

(c) The LM test statistic is 4.5475. If there is no unobserved heterogeneity then the test statistic has a standard normal distribution, with 5% critical values ± 1.96. We reject the null hypothesis that $\sigma_u^2 = 0$ and accept the alternative that $\sigma_u^2 > 0$, indicating that there is statistically significant unobserved heterogeneity. Your software may report $\text{LM}^2 = 20.68$. The 95th percentile of $\chi^2_{(0.95,1)} = 3.841$. We reach the same conclusion.

(d) The estimated model is

$$\widehat{LIQUOR}_{it} = 0.91633 + 0.02074 INCOME_{it} + 0.00658 INCOMEM_i$$
$$\text{(se)} \qquad\qquad\qquad\qquad\qquad\qquad (0.02220)$$

The t-value for the coefficient of *INCOMEM* is 0.30. This is statistically insignificant. There is no evidence for correlation between income and the unobserved heterogeneity based on this Mundlak test. While this was not required, the usual Hausman contrast test between the fixed effects and random effects estimates is 0.09. The 95th percentile of $\chi^2_{(0.95,1)} = 3.841$, so that we fail to reject the exogeneity of income based on this test as well. Based on these results the random effects estimator is preferred.

EXERCISE 15.19

(a) The estimates are contained in Table XR15.19a. The random effects estimates are individually significant at the 5% level or better, except for *REGULAR*, *ALCOHOL*, and *STREET*, which are insignificant. The joint test of *AGE*, *ATTRACTIVE*, and *SCHOOL* yields a Wald chi-square test statistic value of 150.84. The test has 3 degrees of freedom and the 1% critical value is 11.345 from Statistical Table 3. We conclude that at least one of these variables has a nonzero coefficient.

Table XR15.19a

	RE		FE		HT	
	Coefficient	Std. Err.	Coefficient	Std. Err.	Coefficient	Std. Err.
C	5.9104***	(0.1303)	5.4613***	(0.1303)	5.9314***	(0.1389)
AGE	-0.0258***	(0.0028)			-0.0266***	(0.0031)
ATTRACTIVE	0.2768***	(0.0602)			0.2835***	(0.0677)
SCHOOL	0.2161***	(0.0453)			0.2256***	(0.0509)
REGULAR	0.0236	(0.0162)	0.0372*	(0.0168)	0.0264	(0.0159)
RICH	0.1160***	(0.0200)	0.0826***	(0.0205)	0.1091***	(0.0195)
ALCOHOL	0.0149	(0.0250)	-0.0569*	(0.0261)	0.0031	(0.0244)
NOCONDOM	0.1390***	(0.0250)	0.1703***	(0.0258)	0.1610***	(0.0254)
BAR	0.4642***	(0.0999)	0.2985*	(0.1345)	0.4651***	(0.1026)
STREET	0.1033	(0.1011)	0.4552***	(0.1305)	0.1562	(0.1034)
N	3016		3016		3016	

Standard errors in parentheses
$^{*}\ p < 0.05$, $^{**}\ p < 0.01$, $^{***}\ p < 0.001$

(b) The approximate interval estimate is [0.0899, 0.1880]. We estimate with 95% confidence that the risk premium is between 9% and 18.8%.

(c) The value of the test statistic LM is 56.40. This statistic has a standard normal distribution if there is no unobserved heterogeneity. The LM value is greater than 1.96 so we reject the null hypothesis, and conclude that there is unobserved heterogeneity. The chi-square version of the test statistic is 3181.40 and the 5% critical value is 3.841. We reach the same conclusion.

(d) We estimate, roughly, that unprotected sex with an attractive secondary-educated sex worker will cost an extra 63.2 %, with the 95% interval estimate being 48.2% to 78.2%. The exact calculation is $100\times\left(\exp(0.63196)-1\right)=88.1\%$, with interval estimate [59.9%, 116.3%].

(e) The fixed-effects estimates and the *t*-statistic values are reported in Table XR15.19b. We see that the difference between the fixed effects and random effects coefficients are all significant at the 5% level or better, except for *BAR* which is significant at the 10% level. The usual Hausman contrast test statistic value is 155.43. The 5% critical value for this $\chi^2_{(0.95,6)}=12.592$. Using either test we reject the null hypothesis that the unobserved heterogeneity is uncorrelated with the time-varying endogenous variables. In this case the random effects estimator is inconsistent, and the fixed effects estimator is consistent, so the fixed effects estimator should be used.

Table XR15.19b

	FE		RE		
	Coeff.	Std. Err.	Coeff.	Std. Err.	*t*-value
REGULAR	0.0236	0.0162	0.0372	0.0168	2.9012
RICH	0.1160	0.0200	0.0826	0.0205	-7.4565
ALCOHOL	0.0149	0.0250	-0.0569	0.0261	-9.2249
NOCONDOM	0.1390	0.0250	0.1703	0.0258	4.9366
BAR	0.4642	0.0999	0.2985	0.1345	-1.8423
STREET	0.1033	0.1011	0.4552	0.1305	4.2657

(f) The Hausman-Taylor estimates are reported in Table XR15.19a. We see that the signs and significance levels are the same as the random effects estimates. The rough estimate for the risk premium is 16.1% with 95% interval [11.1%, 21%]. The exact value is 17.47% with interval estimate [11.63%, 23.31%]. We estimate, roughly, that unprotected sex with an attractive secondary-educated sex worker will cost an extra 67 %, with the 95% interval estimate being 50.4% to 83.7%. The exact calculation is $100\times\left(\exp(0.67013\ )-1\right)=95.45\%$, with interval estimate [62.9%, 127.7%]/

EXERCISE 15.21

(a) The OLS estimates are in Table XR15.21a. The estimates and significance with conventional standard errors are in column (1) We see that

(i) small classes significantly increase average reading scores by 5.75 points.

(ii) We cannot reject the null hypothesis that teacher aides have no effect on average test scores.

(iii) Experienced teachers can increase average reading test scores. We estimate that each added year of teacher experience is associated with an additional 0.52 points in the average test score. But we cannot reject the null hypothesis that teacher master degrees have no effect on average test scores.

(iv) Boys are estimated to have 6.15 points lower average reading scores than girls. White or Asian students are estimated to have 4.08 point higher average scores than black students.

(v) Those receiving a free lunch are estimated to earn an average of 14.8 fewer points on the reading test. These coefficients are statistically significant using the 0.1% level of significance.

Table XR15.21a

	(1)		(2)	
	Coeff.	Std. Err.	Coeff.	Rob. Std. Err.
C	437.9559***	(1.3499)	437.9559***	(2.4958)
SMALL	5.7489***	(0.9899)	5.7489*	(2.2716)
AIDE	0.8093	(0.9528)	0.8093	(2.2522)
TCHEXPER	0.5216***	(0.0713)	0.5216**	(0.1857)
TCHMASTERS	-1.5889	(0.8614)	-1.5889	(1.9574)
BOY	-6.1533***	(0.7960)	-6.1533***	(0.8324)
WHITE_ASIAN	4.0830***	(0.9582)	4.0830*	(1.9020)
FREELUNCH	-14.7671***	(0.8901)	-14.7671***	(1.0854)
N	5766		5766	

Standard errors in parentheses
$^{*}\ p < 0.05$, $^{**}\ p < 0.01$, $^{***}\ p < 0.001$

(b) The estimates, their significance and cluster-robust standard errors are given in column (2). The cluster robust standard errors are all larger, some substantially so, than the conventional OLS standard errors. There appear to be unobserved teacher characteristics that are correlated across students. Correspondingly, some significance levels are reduced, the most noticeable being the coefficients of *SMALL* and *TCHEXPER*, two key variables in the model. The 95% interval estimate of the *SMALL* coefficient is [3.8082, 7.6896] with the conventional standard errors and [1.2799, 10.2179] using cluster-robust standard errors. The latter suggests we have a less precise estimate of the value of small classes relative to regular ones.

(c) The random effects estimates are in Table XR15.21b. Estimates, significance, and conventional standard errors are in (1) and estimates, significance, and cluster-robust standard errors are in (2). Overall the results are much the same as the OLS estimates with cluster-robust standard errors. A few of the random effects, GLS, coefficient standard errors are a little smaller. Here, we have not gained much efficiency by using GLS because the error correlation within the clusters defined by the teachers is only 0.26, that is $\hat{\rho} = \sigma_u^2 \big/ \left(\sigma_u^2 + \sigma_e^2\right) = 15.48^2 \big/ \left(15.48^2 + 26.06^2\right) = 0.26$. The LM test for the presence of random effects rejects the null hypothesis that there is no unobserved heterogeneity in the data, at the 1% level. Note that the cluster-robust standard errors are not very different from the conventional random effects standard errors. In this case there is no additional source of intra-cluster correlation, so the cluster correction has nothing really to correct. They are unnecessary.

Table XR15.21b

	(1) RE		(2) RE Rob		(3) HT	
	Coeff.	Std. Err.	Coeff.	Std. Err.	Coeff.	Std. Err.
C	437.0046***	(2.4684)	437.0046***	(2.4909)	437.0213***	(2.5260)
SMALL	5.6255*	(2.2501)	5.6255*	(2.2678)	5.4155*	(2.2928)
AIDE	0.8339	(2.3597)	0.8339	(2.2391)	0.6285	(2.4019)
TCHEXPER	0.4414**	(0.1648)	0.4414*	(0.1789)	0.4641**	(0.1686)
TCHMASTERS	-1.7080	(2.0048)	-1.7080	(1.9146)	-1.8610	(2.0429)
BOY	-5.1279***	(0.7035)	-5.1279***	(0.7633)	-5.1629***	(0.7033)
WHITE_ASIAN	6.1653***	(1.2793)	6.1653***	(1.4211)	6.0824***	(1.2874)
FREELUNCH	-14.6523***	(0.8429)	-14.6523***	(0.8599)	-14.5574***	(0.8559)
N	5766		5766		5766	

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

(d) The unobserved teacher characteristics might be related to teaching ability and student home learning resources. Some schools will be located in wealthier districts allowing them to hire better teachers, and to retain them, which might be correlated with the effect of teacher experience and teacher masters. The wealth characteristic of the teacher's school will be correlated with how many students receive free lunches, which are only offered to lower income students.

The estimated Mundlak regression results are in Table XR15.21c. These are random effects estimates of the model including the time averaged variables, denoted by an *M* at the end of the variable name. We see that the augmentation variables for *BOYM*, *WHITE_ASIANM*, and *FREELUNCHM* are significant at the 5% level. Testing the joint significance of these variables we obtain a Wald statistic of 18.39. The 5% test critical value is $\chi^2_{(0.95,3)} = 7.815$.

We reject the null hypothesis that the random effects are uncorrelated with the regressors, and thus we cannot recommend the use of the random effects estimator here.

Table XR15.21c Mundlak Regression

	Coeff.	Std. Err.
C	449.6423***	(6.2447)
SMALL	5.7542*	(2.2500)
AIDE	0.8818	(2.3591)
TCHEXPER	0.3941*	(0.1681)
TCHMASTERS	-1.3038	(2.0174)
BOY	-4.9235***	(0.7247)
WHITE_ASIAN	11.8462***	(1.5005)
BOYM	-17.1908*	(7.8569)
WHITE_ASIANM	-9.2020*	(3.7174)
FREELUNCHM	-17.3355***	(4.7254)
N	5766	

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

(e) The Hausman Taylor (HT) estimator results are in Table XR15.21b, column (3). The magnitudes, signs and significance of the coefficients is much the same as the random effects estimates. Our primary conclusions are that small classes significantly increase average reading test scores by between 5 and 6 points, and that each year of teaching experience is associated with an average reading score of between 0.4 and 0.5 points. Thus, a small class is "worth" between 12 and 13 years of teacher experience.

EXERCISE 15.23

(a) The estimated equation is

$$\widehat{LY} = 0.3787 + 0.8624LK + 0.1373LL$$
$$\text{(se)}\ \ (0.0983)\ \ (0.00488)\ \ (0.00684)$$

Since this is a log-log equation, the coefficients represent elasticities. The coefficient of *LK* suggests that a 1% increase in capital is associated with a 0.8624% increase in GDP. The coefficient of *LL* suggests that a 1% increase in labor is associated with a 0.1373% increase in GDP.

Testing the null hypothesis $H_0 : \beta_2 + \beta_3 = 1$ (constant returns to scale) against the alternative hypothesis $H_1 : \beta_2 + \beta_3 \neq 1$ yields *F*- and *p*-values of 0.0042 and 0.9483, respectively. Since this *p*-value is much larger than the level of significance 0.05, we do not reject the null hypothesis and conclude that there is no evidence against the hypothesis of constant returns to scale.

(b) The estimated equation is

$$\widehat{LY} = 0.2995 + 0.8743LK + 0.1351LL - 0.0121t$$
$$\text{(se)}\ (0.0952)\ \ (0.0048)\ \ \ (0.00661)\ \ (0.00095)$$

The coefficient of *t* represents the growth rate of GDP, expressed in decimal form. Because it represents the growth rate not attributable to changes in capital and labor, it is often viewed as growth from technological change. These estimates suggest that the average growth rate of GDP over the period 1960-1987 is -1.21% per year. The *p*-value for testing the significance of this estimate is 0.0000; we can conclude that the coefficient is significantly different from zero at a 1% level of significance. However, we may question whether a negative growth rate is a realistic outcome. The addition of *t* to the model has very little effect on the estimates of β_2 and β_3; they are almost identical to those obtained in part (a).

(c) Substituting the restriction $\beta_2 + \beta_3 = 1$ into the model from part (b) yields

$$LY = \beta_1 + \beta_2 LK + (1 - \beta_2)LL + \lambda t + e$$

Rearranging this equation and converting it into a more familiar form,

$$\ln(Y) - \ln(L) = \beta_1 + \beta_2 \ln(K) - \beta_2 \ln(L) + \lambda t + e$$

or

$$\ln\left(\frac{Y}{L}\right) = \beta_1 + \beta_2 \ln\left(\frac{K}{L}\right) + \lambda t + e$$

$$LYL = \beta_1 + \beta_2 LKL + \lambda t + e$$

The estimated equation is

$$\widehat{LYL} = 0.4530 + 0.8731LKL - 0.0119t$$
$$\text{(se)} \quad (0.0415)\ (0.00476) \quad (0.00094)$$

The estimate of β_2 is identical to the estimate obtained in part (b) to two decimal places.

(d) The estimated equation is, with the average of the fixed effects reported as the intercept,

$$\widehat{LY} = 8.3751 + 0.5316LK + 0.1333LL + 0.00747t$$
$$\text{(se)}\ (0.5164)\ (0.0124) \quad (0.0336) \quad (0.00093)$$

To test $H_0 : \beta_{1,1} = \beta_{1,2} = \cdots = \beta_{1,82}$ against the alternative that not all intercepts are equal, we use the usual *F*-test for testing a set of linear restrictions. The calculated value is $F = 211.13$, while the 5% critical value is $F_{(0.95,81,2211)} = 1.279$. Thus, we reject H_0 and conclude that the country level effects are not all equal.

The fixed effects estimates are markedly different from those estimated in part (b). In particular, the coefficient of *t* has changed sign to positive, more in line with our expectations. The elasticity of output with respect to capital is much smaller and the standard errors of both elasticities are much larger.

(e) Testing the null hypothesis $H_0 : \beta_2 + \beta_3 = 1$ (constant returns to scale) against the alternative hypothesis $H_1 : \beta_2 + \beta_3 \neq 1$ yields *F*- and *p*-values of 107.46 and 0.0000, respectively. Since this *p*-value is smaller than the level of significance 0.05, we reject the null hypothesis and conclude there are not constant returns to scale. The outcome of this hypothesis test is clearly very sensitive to whether or not we include fixed effects.

(f) The estimated equation is

$$\widehat{LYL} = 3.1245 + 0.5435LKL - 0.000327t$$
$$\text{(se)} \quad (0.1030) \quad (0.0127) \quad (0.000551)$$

These results are very different from those in part (c). All estimates have the same sign. However, relative to the estimates in part (c), the intercept is much larger and the coefficient estimates are much smaller. Furthermore, the standard errors of this model are much larger except for $\text{se}\left(\hat{\lambda}\right)$.

The fixed effects model without the restriction for constant returns to scale is the preferred specification. It is preferred because, according to our hypothesis tests, we should allow for country fixed effects and we do not have any evidence to support the presence of constant returns to scale. Also, the trend coefficient is positive in line with our expectation that technological change should have a positive effect on output.

(g) The estimates are presented in the following table. The single time trend variable restricts the year-to-year growth rate to be the same between all years. Using time dummy variables allows the rate of growth between years to be different for each year. Since we have an intercept and then time dummies for all years except the first, each coefficient of a time dummy gives the growth rate between the year of the time dummy and the first year. The estimates for β_2 and β_3 are similar to those obtained in parts (a) and (b).

Variable	Estimate	se	Variable	Estimate	se
Intercept	0.2564	0.1024	*D14*	−0.0829	0.0561
LK	0.8741	0.0048	*D15*	−0.1011	0.0561
LL	0.1352	0.0066	*D16*	−0.1375	0.0561
D2	−0.0192	0.0560	*D17*	−0.1430	0.0561
D3	−0.0255	0.0560	*D18*	−0.1566	0.0561
D4	−0.0212	0.0560	*D19*	−0.1735	0.0562
D5	−0.0239	0.0560	*D20*	−0.1784	0.0562
D6	−0.0290	0.0560	*D21*	−0.2054	0.0562
D7	−0.0396	0.0560	*D22*	−0.2353	0.0562
D8	−0.0600	0.0560	*D23*	−0.2662	0.0562
D9	−0.0617	0.0560	*D24*	−0.2850	0.0562
D10	−0.0569	0.0560	*D25*	−0.2907	0.0562
D11	−0.0584	0.0560	*D26*	−0.2980	0.0563
D12	−0.0696	0.0561	*D27*	−0.2985	0.0563
D13	−0.0741	0.0561	*D28*	−0.2958	0.0563

EXERCISE 15.25

(a) The OLS estimates and the three types of standard errors appear in Table XR15.25. We observe that the estimated elasticities of capital and labor are about 0.10 and that for materials about 0.74. In each case the conventional standard errors are less than the heteroskedasticity robust standard errors (HCE) which are less than the cluster-robust standard errors (Cluster). This means that the interval estimates will be wider in that order as well. The 95% interval estimates for the elasticity with respect to *MATERIALS* are

(i) [0.729447, 0.7544004] using conventional standard errors

(ii) [0.7217382, 0.7621092] using White heteroskedasticity robust standard errors (HC2)

(iii) [0.7138901, 0.7699573] using cluster-robust standard errors

Interval (iii) is wider than (ii) which is wider than (i).

(b) In each case we compute the t-statistic. Using conventional standard errors $t = -8.08$. Using heteroskedasticity robust standard errors $t = -7.62$. Using cluster robust standard errors $t = -5.38$. In each case we reject the null hypothesis at the 1% or better level.

Table XR15.25 OLS estimates

	Coeff.	Std. Err.	HCE	Cluster
C	1.6415	(0.0485)	(0.0649)	(0.0916)
LCAPITAL	0.1039	(0.0068)	(0.0079)	(0.0110)
LLABOR	0.1046	(0.0099)	(0.0105)	(0.0145)
LMATERIALS	0.7419	(0.0064)	(0.0103)	(0.0143)
N	3000			

Standard errors in parentheses

(c) The $R^2 = 0.3091$ so that the LM test statistic is 618.22, compared the 5% critical value 3.841. We reject the null hypothesis of no serial correlation. The unobserved heterogeneity appears in the combined error each year, so there is serial correlation.

(d) The estimates appear in the column (1) of Table XR15.25b. The elasticity with respect to labor is a little higher and that of materials a little lower. The LM^2 test statistic is 802.0 which is very significant. Using conventional standard errors for the hypothesis test $t = -8.28$, so again constant returns to scale is rejected.

Table XR15.25b Panel Data Estimates

	(1) re	(2) fe	(3) fe rob
C	1.9475	3.4999	3.4999
	(0.0638)	(0.1664)	(0.2898)
LCAPITAL	0.1019	0.0519	0.0519
	(0.0079)	(0.0130)	(0.0157)
LLABOR	0.1302	0.1061	0.1061
	(0.0117)	(0.0205)	(0.0261)
LMATERIALS	0.6998	0.5970	0.5970
	(0.0076)	(0.0117)	(0.0287)
N	3000	3000	3000
R^2		0.5895	0.5895
SSE		120.6277	120.6277
RMSE	0.2519	0.2458	0.2007

Standard errors in parentheses

(e) The estimates are in the second column of Table XR15.25b. The coefficient estimates are quite different from the random effects and OLS estimates. The Hausman contrast test statistic is 158.39. The $\chi^2_{(3)}$ 5% critical value is 7.815. So we conclude that there is correlation between the random effect and some regressors.

(f) The estimated value of ρ is −0.3739 with a cluster-robust standard error of 0.02142. The *t*-statistic value for the null hypothesis $\rho = -1/2$ is $t = 5.89$. Thus, we conclude that the idiosyncratic errors e_{it} are correlated

(g) These estimates are in column (3) of Table XR15.25b. The standard errors are not radically different. The test of constant returns to scale is again rejected with $t = -7.01$.

EXERCISE 15.27

(a) The summary statistics for public universities are given in the first two tables. The average total cost per student rose about 46%. The number of undergraduate students increased by about 37% and graduate students by 63%. The number of tenure track faculty per 100 students fell from 4.34 to 3.98, which the number of GA's increased by almost 50% and the number of contract faculty per 100 students increased from 1.88 to 2.58. There has been a substitution of GA's and contract faculty for tenure track faculty.

1987 Public Universities	*N*	mean	sd	min	max
TC	138	26.6706	13.5162	10.4070	83.7720
CF	138	1.8826	1.5006	0.0000	8.2000
TT	138	4.3424	1.0252	0.0000	7.7300
GA	138	1.2436	1.0343	0.0000	5.1440
FTEF	138	6.5793	1.7614	2.8430	14.4410
FTESTU	138	16.6398	8.7490	2.1600	46.7040
FTGRAD	138	2.3161	2.1661	0.1650	11.7250

2011 Public Universities	*N*	mean	sd	min	max
TC	141	39.0609	25.0767	15.2210	172.4380
CF	141	2.5793	2.1584	0.3880	18.9070
TT	141	3.9751	1.2436	1.6410	10.8990
GA	141	2.2932	1.0758	0.0000	7.3420
FTEF	141	7.3097	3.2187	3.4780	31.6340
FTESTU	141	22.8458	10.7516	5.1300	64.5950
FTGRAD	141	3.7853	2.7465	0.4070	13.6870

The summary statistics for private universities are given in the following table.

1987 Private Universities	*N*	mean	sd	min	max
TC	55	61.6443	40.6732	16.6840	192.5100
CF	55	3.5408	4.4199	0.0000	22.8520
TT	55	5.6193	1.8108	0.4800	10.9330
GA	55	0.8635	1.4948	0.0000	9.8710
FTEF	55	10.0136	5.4962	1.7030	36.7020
FTESTU	55	9.5158	5.6515	2.1630	23.9970
FTGRAD	55	1.9337	1.7754	0.2110	8.2680

2011 Private Universities	*N*	mean	sd	min	max
TC	57	92.8001	73.8596	24.5320	274.3670
CF	57	6.3230	6.6921	0.0000	38.7490
TT	57	6.5796	3.4916	0.1590	17.9120
GA	57	2.4136	1.7508	0.0000	9.0500
FTEF	57	14.3224	8.9498	4.5360	49.6840
FTESTU	57	13.2421	7.9247	1.9330	37.9220
FTGRAD	57	4.9038	3.5764	0.4600	16.3290

In private universities, the average total cost per student rose about 51%. The number of undergraduate students increased by about 39% and the number of graduate students more than doubled. The number of tenure track faculty per 100 students fell from 5.62 to 6.58. The number of GA's increased by more than 100% and the number of contract faculty per 100 students increased from 3.54 to 6.32. Unlike the public universities, there has not been a substitution of GA's and contract faculty for tenure track faculty.

(b) The OLS estimates for public universities are in column (1) of Table XR15.27a. The cluster corrected standard errors are substantially larger than the usual standard errors, but the coefficients retain their significance, despite the *t*-values being cut by roughly half. The OLS estimates suggest stable ATC after 1987 for a few years, before rising dramatically in 2005.

Table XR15.27a

	(1) PUB OLS	(se)	(robust se)	(3) PUB FE	(robust se)
CONS	2.276	(0.0468)	(0.104)	2.719	(0.0939)
FTESTU	-0.0103	(0.00166)	(0.00265)	-0.00389	(0.00315)
FTGRAD	0.104	(0.00711)	(0.0132)	0.0181	(0.0116)
TT	0.141	(0.00808)	(0.0200)	0.0934	(0.0127)
GA	0.0645	(0.00669)	(0.0137)	0.00591	(0.00541)
CF	0.0742	(0.00496)	(0.0186)	0.0341	(0.00792)
D1989	0.0359	(0.0343)	(0.0181)	-0.00152	(0.0133)
D1991	-0.0531	(0.0346)	(0.0246)	-0.0877	(0.0195)
D1999	0.0544	(0.0342)	(0.0287)	0.167	(0.0179)
D2005	0.157	(0.0347)	(0.0257)	0.292	(0.0203)
D2008	0.169	(0.0344)	(0.0243)	0.314	(0.0219)
D2010	0.135	(0.0349)	(0.0304)	0.325	(0.0250)
D2011	0.189	(0.0350)	(0.0319)	0.356	(0.0260)
N	1122			1122	

Standard errors in parentheses

(c) The OLS estimates for private universities are in column (1) of Table XR25.27b. The cluster corrected standard errors are substantially larger than the usual standard errors. The coefficients retain their significance, except for *GA*, which is not significant. The indicator variable coefficient estimates suggest ATC falling relative to 1987 by large and significant amounts. Otherwise, the story told by the OLS estimates for the private schools is much the same as for public schools.

(d) The estimates are reported in column (3) of Table XR15.27a. The fixed effects coefficient estimates for the explanatory variables are smaller in magnitude, with *FTESTU*, *FTEGRAD* and *GA* no longer having significant coefficients. After 1991, the fixed effects estimates of the year indicator variable coefficients are much larger in magnitude than the OLS estimates.

(e) The estimates are reported in column (3) of Table XR15.27b. The coefficients of the explanatory variables are insignificant except for *TT*, tenure track faculty, which has a much smaller estimated effect than implied by the OLS estimates, about 4.8% instead of 13.0%.

The year indicator variable coefficients suggest positive increases in ATC relative to 1987, except for 1991, an outcome which is very different from what the OLS estimates suggest.

Table XR15.27b

	(1) PRI OLS	(se)	(robust se)	(3) PRI FE	(robust se)
C	2.959	(0.0700)	(0.133)	4.091	(0.253)
FTESTU	-0.0142	(0.00389)	(0.00547)	-0.0644	(0.0348)
FTGRAD	0.0918	(0.0101)	(0.0209)	0.0715	(0.0398)
TT	0.129	(0.00694)	(0.0169)	0.0476	(0.0143)
GA	0.0140	(0.0110)	(0.0228)	-0.00624	(0.0101)
CF	0.0545	(0.00393)	(0.0140)	0.0132	(0.00709)
D1989	0.0696	(0.0699)	(0.0478)	0.0859	(0.0214)
D1991	-0.0257	(0.0691)	(0.0381)	0.0291	(0.0280)
D1999	0.0664	(0.0682)	(0.0417)	0.176	(0.0370)
D2005	-0.0942	(0.0717)	(0.0621)	0.223	(0.0483)
D2008	-0.153	(0.0713)	(0.0693)	0.254	(0.0511)
D2010	-0.179	(0.0716)	(0.0751)	0.286	(0.0542)
D2011	-0.197	(0.0719)	(0.0774)	0.286	(0.0566)
N	449			449	

Standard errors in parentheses

EXERCISE 15.29

(a) The first stage results are in Table XR15.29a, in column (1). The two instruments have positive coefficients and are significant individually, with t values 3.69 and 3.33. The F-test of their joint significance is 12.31 > 10 the rule of thumb value. Thus, we can reject the null hypothesis that the IV are weak using this criterion.

(b) The two stage least squares estimates are in column (2) of Table XR 15.29a. The deterrence variables, the log of the probability of arrest (*LPRBARR*), the log of the probability of conviction (*LPRBCONV*), the log of average prison sentence (*LAVGSEN*), all have negative and significant coefficients, indicating that they are having the desired effect. The log of police per capita has a positive coefficient and is significant at the 10% level.

(c) Including the first stage residuals into the equation and estimating it by OLS we find that the first stage residuals have a coefficient of 0.1048 and a $t = 0.78$ ($F = 0.6083$ with $p = 0.4358$). Thus, based on this test we do not find evidence that *LPOLPC* is endogenous. The Sargan test of the validity of the surplus instrument yields a $\chi^2_{(1)}$ LM statistic 13.6895 with $p = 0.0002$. Thus, we reject the validity of the surplus IV, making the 2SLS results suspect at best.

Table XR15.29a

	(1) red form	(2) 2SLS	(3) red form FE	(4) 2SLS FE
C	-8.4853	-3.3274	-6.0376	1.2887
	(-19.49)	(-2.99)	(-12.97)	(1.08)
LPOLPC		0.2407		0.6723
		(1.82)		(3.82)
LPRBARR	0.1105	-0.6284	0.3038	-0.4368
	(2.02)	(-14.91)	(6.04)	(-6.15)
LPRBCONV	0.3005	-0.4697	0.3846	-0.3817
	(9.29)	(-9.98)	(13.49)	(-5.34)
LAVGSEN	0.0069	-0.1161	-0.0209	0.0306
	(0.10)	(-2.56)	(-0.51)	(1.06)
LWMFG	0.2976	0.0883	-0.0077	-0.2531
	(3.83)	(1.29)	(-0.07)	(-4.39)
WEST	0.1477	-0.5046		
	(3.22)	(-16.17)		
URBAN	0.3619	0.2932		
	(4.76)	(3.81)		
LTAXPC	0.2522		0.1382	
	(3.69)		(2.16)	
LMIX	0.1143		0.0784	
	(3.33)		(3.22)	
N	630	630	630	630
R^2	0.2253	0.7127	0.2833	
SSE	135.1568	59.2934	27.9352	13.8803
RMSE	0.4665	0.3068	0.2287	

t statistics in parentheses

(d) The results of this first stage model are in column (1) of Table XR15.29b. We have indicated the demeaned variables with the letter *W* at the end of the variable name. Both of the instruments are significant at the 5% level. Their joint test of significance yields an $F = 9.36$, with a *p*-value 0.0001, which indicates they are statistically significant. However the *F* value is less than 10, the rule of thumb for rejecting that IV are weak. The problem is that using a regression software package for the within regression miscalculates the degrees of freedom. Using software for fixed effects estimation of the reduced form yields the results in column (4) of Table XR15.29a. The IV are positive and significant; the *F*-test of their joint significance is 8.01, with $p = 0.0004$, but this is still less than 10. However, there is no guarantee that the $F > 10$ rule is useful for a panel data regression, as it was not designed for this purpose.

(e) Using software designed for IV/2SLS estimation we obtain the results in column (4) of Table XR15.29a. The deterrent variables behave as before, with the log of per capita police still positive and significant. Using standard IV/2SLS software with the within transformed data we obtain the same estimates, shown in column (2) of Table XR15.29b, but of course the standard errors are incorrect.

Table XR15.29b

	(1) red form	(2) 2SLS
LPOLPCW		0.6723
		(4.15)
LPRBARRW	0.3038	-0.4368
	(6.53)	(-6.68)
LPRBCONVW	0.3846	-0.3817
	(14.59)	(-5.79)
LAVGSENW	-0.0209	0.0306
	(-0.55)	(1.15)
LWMFGW	-0.0077	-0.2531
	(-0.08)	(-4.76)
LTAXPCW	0.1382	
	(2.34)	
LMIXW	0.0784	
	(3.48)	
N	630	630
R^2	0.2833	
SSE	27.9352	13.8803
RMSE	0.2116	0.1484

t statistics in parentheses

(f) Applying 2SLS to the within transformed data yields a Hausman test *F*-statistic of 3.269 with a *p*-value of 0.0711, using the $F_{(1,624)}$ distribution. The Sargan LM test statistic is 1.80 with a *p*-value of 0.1797. Thus, using the within data we find (weak) endogeneity of the log of police per capita and we do not reject the validity of the surplus IV. It should be noted that these tests were not designed for use with panel data.