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Comparison of Elo and PageRank based ranking systems for Chess

COMP4121: Advanced and Parellel Algorithms

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Contents

Abstract

Chess Grandmasters are currently ranked by the Elo rating system whereby each player has a numerical rating, and gains rating points when they defeat other players. In this report, a new rating system is developed by constructing a network between all the players based on wins, draws and losses, and by then applying both PageRank and a modified PageRank algorithm, dubbed ChessRank, to the resulting graph. The key idea behind the PageRank algorithm is that a page is highly ranked if it is linked to by other highly ranked pages. Or, in the case of chess, a player is highly ranked if they defeat other highly ranked players. Both the PageRank and ChessRank algorithms were tested on the chess games played between the top chess grandmasters from 2012-2013. The results indicate that both algorithms are less accurate than the Elo rating system currently used in practice.

Chapter 1

Introduction

PageRank is an algorithm that was developed to rank the important of pages in a network. So PageRank gives pages a high rank if they are linked to by many other pages which themselves have a high rank. The PageRank algorithm has also been applied in the sporting domain, such as with American Football [1] and with tennis [2].

In this report, the PageRank algorithm is applied to the game of chess. This is done by building a network where the worlds top chess grandmasters are the nodes and the games played between them are the links.

The original PageRank algorithm is first implemented and evaluated against the current ranking system used by the World Chess Federeation, more commonly known as FIDE (from the French acronym Fédération internationale des échecs). In comparison, the PageRank rankings are relatively inaccurate.

Then a modified PageRank algorithm is then implemented, called ChessRank, by taking into account extra information such as repeated wins and down-weighting draws, to produce another ranking. This is again evaluated against the current FIDE rankings, and while ChessRank does slightly better than the original PageRank algorithm, it is still inferior to the current FIDE rankings.

Chapter 2

PageRank

2.1 Explanation

PageRank is an algorithm that is used to rank the important of pages on the web. It is based on the idea that an important page is linked to by many other pages, who are also important.

PageRank can be explained by considering the following scenario. Let's say that a person, who we shall call the Random Surfer, picks a page on the Internet completely at random. They them click on a link that exists on this page at random. The Random Surfer then continues to do this for an extremely long time. If the number of pages on the internet is M, then we shall let the Random Surfer visit $T = 1000 * M$ pages.

The Random Surfer has been moving around the Internet randomly for a significant period of clicks, so we can then say that if a page P was important, then the Random Surfer would have ended up on that page more often that other less important pages. So we can say that $N(P)/M$ would be an accurate way to rank the pages on the web, where $N(P)$ is the total number of times the Random Surfer visiting the page P during their surf.

This has two problems:

1. What if a page happens to contain no links? What should the Random Surfer do in such situations?

2. The choice that the Random Surfer makes may influence the final ranks. For example,

consider a network that has two groups of nodes X and Y, whereby every node in X only has links to other nodes in X, and similarly for the group of nodes in Y. In such a situation, if the Random Surfer visits a node $x \in X$ then the Random Surfer will be trapped and the final PageRanks will assigned to all the nodes in X will be non-zero while every node $n \notin X$ will have a PageRank of 0. Similarly, if the Random Surfer visits a node y in Y, then all the nodes in Y will have a non-zero PageRank. It is clear that this is a problem because the PageRank is influenced on choices that the Random Surfer makes, and such a PageRank is meaningless. What we want is for the Random Surfer's surfing history to not affect the final PageRanks. That is, as $T \to \infty$ then $N(P)/T$ should converge to some value.

We solve the first problem by allowing the Random Surfer to pick another page at random when they reach a *dangling node*.

To solve the second problem, we allow the Random Surfer to jump to another random page, with some small probability. So most of the time they will follow a link, but a small percentage of the time, they will move to a random page and continue to surf.

These two modifications solves our two problems and gives us a PageRank that is independent of the surfing history of the Random Surfer and also ensures that $N(P)/T$ converges to a value, known as the PageRank.

2.2 Implementation

To implement PageRank we construct a transition matrix, G, for the Random Surfer, where the entry $g(i, j)$ is the i^{th} row and j^{th} column of G, and represents the probability of the Random Surfer moving to the j^{th} if they are current at the i^{th} page.

To build such a transition matrix G, we first start with a matrix G_1 that would represent what the matrix would look like assuming that we did not solve the two problems raised in the previous section.

In such a matrix G_1, there is a row of zeros if the page P_i is a dangling node and the entry $g(i, j)$ contains the value $1/\#(P_i)$ if there is a link on page P_i going to page P_j, where $\#(P_i)$ contains the total number of links on the page P_i.

We will then improve upon this matrix G_1 by solving the first problem raised, that of dangling nodes.

If the Random Surfer reaches such a page, then they jump to a random page, and since all pages are equally likely destinations, the chance of any page P_i being chosen is $1/M$.

To solve the second problem, the Random Surfer jumps to a random page with some small probability, even when the current page has outgoing links. Let $\alpha \in [0, 1]$ be the chance that the Random Surfer chooses a link on the current page, and $1 - \alpha$ be the chance that the Random Surfer chooses a random page from the entire Internet.

Now the entry $g(i, j)$ is

$$\frac{\alpha}{\#(P_i)} + \frac{1-\alpha}{M}$$

And this gives us our final transition matrix for the Random Surfer. One thing to note is that because this is a transition matrix of what the Random Surfer can do when it is on a page P_i, the sum of each row is 1, meaning that the matrix is row-stochastic. Because G is stochastic, we can think of our scenario of the Random Surfer as being a Markov Chain, where the pages in the Internet are the states and our matrix G as the stochastic matrix. This means we can use properties of Markov chains, in particular:

For any probability vector $\overrightarrow{q}^{(0)}$ the value of $\overrightarrow{q}^{(t)} = \overrightarrow{q}^{(0)} \cdot G^t$ converges to a unique stationary distribution $\overrightarrow{q}$.

This means that all we need to do to compute the PageRank is choose any probability vector $\overrightarrow{q}^{(0)}$ and iteratively multiply it against our transition matrix G until the differences between $\overrightarrow{q}^{(n)}$ and $\overrightarrow{q}^{(n-1)}$ is sufficiently small.

Chapter 3

Application to Chess

The previous chapter describes how the PageRank can be applied to pages on the Internet. The key idea is that pages are given a high rank if they are linked to by other high ranking pages. We can can use this same idea to rank chess players. A chess player is highly ranked if that chess player is linked to by other highly ranked chess players. In this sense, the links from players are based on wins, draws and losses.

A player C_i will link to a player C_j if C_i has lost to C_j in a chess match. C_j was the winner which implies that C_j is the better player, so this is why there is a link from C_i to C_j.

One must also consider the fact that the result of a chess match can be a draw. In such a case, a link is created back and forth between each player. If C_i and C_j draw, and let us say that C_i is the better player, that is, has a better rank, Then creating the two links $C_i \to C_j$ and $C_j \to C_i$ would benefit C_j more as C_j is now being linked to by better player, which is intuitively what we would want; a draw should benefit the ranking of the weaker player.

Dangling nodes in the case of chess players are players that have only ever won chess games and have never lost. This is extremely unlikely, but in such a case the method used for PageRank on webpages still seems appropriate. This player has never lost, but if they were to lose, they would have an equal chance of losing to every other player, so a row of $1/M$ is appropriate.

For the other problem of a group of players only linking to eachother, the solution used in PageRank also works. Once again, this kind of structure in the network is extremely unlikely to occur.

In PageRank, $1-\alpha$ is the chance of a random jump. In our setup, a random jump would mean that a player C_i loses to C_j. Chess is not a game of chance in any regard and it is extremely unlikely that a chess player would lose to another chess player at random, so alpha is given a relatively high value of 0.99 meaning that random jumps only occur 1% of the time.

Chapter 4

Results & Evaluation - PageRank

4.1 Elo Rating System

The current system used by the World Chess Federeation, more commonly known as FIDE (from the French acronym Fédération internationale des éche is the Elo rating system. The Elo system gives players a numerical rating based on their performance against other players.

When Player A and Player B verse each other in a match, the following formula is used to determine the expected score for Player A in the match:

$$E_A = \frac{1}{1 + 10^{(R_B - R_A)/400}}$$

Where R_A and R_B are the current ratings of Players A and B respectively. Similarly, the expected score for Player B, E_B, is:

$$E_B = \frac{1}{1 + 10^{(R_A - R_B)/400}}$$

Where $E_A + E_B = 1$. The expected score is the probability of the Player A winning plus half of their probability of drawing.

At the end of the match, rating points are gained or lost via the following formula:

$$R'_A = R_A + K(S_A - E_A)$$

Where S_A is the result of the match: 1 for a win, 0 for a loss and 0.5 for a draw, and K, called the K-factor, which is used to moderate how sensitive ratings are to change. A typical K-factor for grandmasters is 16. For example, if a Player A, rating of 2600, defeats Player B, rating of 2500, then $E_A = \frac{1}{1+10^{(2500-2600)/400}} = 0.64$ and $R'_A = 2600 + 16(1 - 0.64) = 2606$ for a gain of 6 points. Because the Elo rating system is zero-sum, Player B's score will decrease by the same margin of 6 points.

4.2 PageRank vs. Elo

PageRank was run on the chess games of players who were rated at 2700 or above in 2012 or 2013. This represented approximately the top 60 chess grandmasters.

Results are presented in the following table, ordered by their PageRank.

Rank	Player	PageRank
1	Karjakin, Sergey	0.03940085
2	Nakamura, Hikaru	0.03769653
3	Ivanchuk, Vassily	0.03587794
4	Grischuk, Alexander	0.03558909
5	Wang, Hao	0.03501970
6	Mamedyarov, Shakhriyar	0.03423768
7	Radjabov, Teimour	0.03346909
8	Caruana, Fabiano	0.03308060
9	Aronian, Levon	0.03307432
10	Carlsen, Magnus	0.03272312
11	Morozevich, Alexander	0.03110839
12	Leko, Peter	0.03107944
13	Ponomariov, Ruslan	0.02902584
14	Gelfand, Boris	0.02862289
15	Svidler, Peter	0.02675802
16	Giri, Anish	0.02665729
17	Topalov, Veselin	0.02561540
18	Kamsky, Gata	0.02534695
19	Adams, Michael	0.02353093
20	Kramnik, Vladimir	0.02339114
21	Fressinet, Laurent	0.02195077
22	Andreikin, Dmitry	0.02160178
23	DominguezPerez, Leinier	0.02095180

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Rank	Player	PageRank
24	Jakovenko, Dmitry	0.01803993
25	Ding, Liren	0.01771998
26	Jobava, Baadur	0.01709735
27	Tomashevsky, Evgeny	0.01682929
28	Shirov, Alexei	0.01608468
29	Bologan, Viktor	0.01566243
30	Navara, David	0.01539406
31	Naiditsch, Arkadij	0.01536454
32	Nepomniachtchi, Ian	0.01448215
33	Vitiugov, Nikita	0.01421073
34	Anand, Viswanathan	0.01400307
35	Bacrot, Etienne	0.01121875
36	Gashimov, Vugar	0.01073623
37	Malakhov, Vladimir	0.01072469
38	Wojtaszek, Radoslaw	0.01057302
39	McShane, Luke	0.00989357
40	Sasikiran, Krishnan	0.00821430
41	Movsesian, Sergei	0.00788715
42	Volokitin, Andrei	0.00754106
43	BruzonBatista, Lazaro	0.00704249
44	Le, QuangLiem	0.00702647
45	Vachier-Lagrave, Maxime	0.00666546
46	Areshchenko, Alexander	0.00593339
47	Riazantsev, Alexander	0.00557159
48	Polgar, Judit	0.00482493
49	Moiseenko, Alexander	0.00480154
50	Eljanov, Pavel	0.00394203
51	VallejoPons, Francisco	0.00385980
52	Short, Nigel	0.00257442
53	Sutovsky, Emil	0.00252970
54	Laznicka, Viktor	0.00224361
55	Cheparinov, Ivan	0.00147565
56	Grachev, Boris	0.00135787
57	Almasi, Zoltan	0.00127881
58	Korobov, Anton	0.00121622
59	Inarkiev, Ernesto	0.00016949

At first glance, it seems as though the PageRank may not be very accurate, mainly because the current world chess Champion, Magnus Carlsen, has been given a PageRank that is quite low, despite having been the top FIDE ranked player for the past few years.

We shall look at a few methods of comparing the two rankings systems in the following sections.

4.2.1 Method One - Average difference between rankings

One way we can compare the accuracy of PageRank to the current FIDE rankings is to compute the average distance between a players two rankings. This isn't intended to imply that the Elo rating system is perfect and completely accurate, but rather that it would be good to compare any new ranking system with the current system being used, which is known to be fairly accurate, to see if the rankings are somewhat similar to give some idea as to how accurate the new ranking system is.

Let $PR(P_A)$ denote the PageRank of Player A and $FR(P_A)$ denote the FIDE Rank of Player A, then our distance measurement, D, can be calculated as follows:

$$D = \sum_{p \in Players} |FR(p) - PR(p)|$$

Where Players are the chess grandmasters with Elo ratings of 2700 or above. This is shown in the following table comparing FIDE Ranks, PageRanks and the difference between them.

Name	FIDE Ranking	PageRank	Difference
Carlsen, Magnus	1	10	9
Aronian, Levon	2	9	7
Kramnik, Vladimir	3	20	17
Nakamura, Hikaru	4	2	2
Grischuk, Alexander	5	4	1
Caruana, Fabiano	6	8	2
Gelfand, Boris	7	14	7
Anand, Viswanathan	8	34	26
Topalov, Veselin	9	17	8
Mamedyarov, Shakhriyar	10	6	4
Karjakin, Sergey	11	1	10
DominguezPerez, Leinier	12	23	11
Svidler, Peter	13	15	2
Adams, Michael	14	19	5
Bacrot, Etienne	15	35	20
Vachier-Lagrave, Maxime	16	45	29
Vitiugov, Nikita	17	33	16
Wang, Hao	18	5	13
Eljanov, Pavel	19	50	31
Giri, Anish	20	16	4
Ponomariov, Ruslan	21	13	8
Ivanchuk, Vassily	22	3	19
Leko, Peter	23	12	11
Morozevich, Alexander	24	11	13
Naiditsch, Arkadij	25	31	6
Wang, Yue	26	not ranked	-
Tomashevsky, Evgeny	27	27	0
Nepomniachtchi, Ian	28	32	4
Jakovenko, Dmitry	29	24	5
Kamsky, Gata	30	18	12
Areshchenko, Alexander	31	46	15
So, Wesley	32	not ranked	-
Radjabov, Teimour	33	7	26
Korobov, Anton	34	58	24
Ding, Liren	35	25	10
Wojtaszek, Radoslaw	36	38	2
Andreikin, Dmitry	37	22	15
Almasi, Zoltan	38	57	19
Moiseenko, Alexander	39	49	10
Malakhov, Vladimir	40	37	3
Fressinet, Laurent	41	21	20
Rublevsky, Sergei	42	not ranked	-
Navara, David	43	30	13
VallejoPons, Francisco	44	51	7
Harikrishna, P.	45	not ranked	-
Le, QuangLiem	46	44	2
Kryvoruchko, Yuriy	47	not ranked	-
Matlakov, Maxim	48	not ranked	-
Movsesian, Sergei	49	41	8
Total Difference			476

The total difference, D, between the two rankings is 476. As the dataset used to produce the PageRanks was based on games played in 2012, players who have increased their ranks above 2700 in 2013, of which there are 7, do not have PageRanks. For the 43 remaining players who do have PageRanks, the average difference is $476/50 = 11.07$

The difference is marginally better if we only look at the top 10 players:

Name	FIDE Ranking	PageRank	Difference
Carlsen, Magnus	1	10	9
Aronian, Levon	2	9	7
Kramnik, Vladimir	3	20	17
Nakamura, Hikaru	4	2	2
Grischuk, Alexander	5	4	1
Caruana, Fabiano	6	8	2
Gelfand, Boris	7	14	7
Anand, Viswanathan	8	34	26
Topalov, Veselin	9	17	8
Mamedyarov, Shakhriyar	10	6	4
Total Difference			83

This gives us an average total difference of only 8.3, so the PageRank produces rankings more similar to the FIDE rankings when subsets closer to the top of the rankings are taken. This is likely just a reflection the dataset of games that was used, as it only contained games played between players who have a rating above 2700. And so lower ranked players who play a large number of games against players below that threshold, may not be represented accurately by the dataset and the rankings that it produces.

For example, the top FIDE ranked played, Magnus Carlsen, played 115 games in 2012, only 17 of those were against players ranked below the 2700 threshold, and so these were not included in the dataset. However, David Navara, FIDE rank 43, only has 22 games in the 2012 dataset that was used, despite playing 137 games in 2012, because all of these other games were against lower ranked opponents. As a result, we would expect the PageRank of players with more games in the dataset to be more accurate than players with fewer games in the dataset.

From these initial comparisons, it appears as though the PageRank algorithm produces a significantly different ranking than that of Elo. However this first comparison does not tell us much about the accuracy of either system. For this, we will look at another method presented in the next section.

4.3 Method Two - Win/Loss Prediction

The second method of evaluation is using the PageRanks to predict the results of matches played during 2013. We will do this by only looking at 2013 games that resulted in either a win or a loss, thereby discarding draws. We will then pick the higher ranked player as the victor of the chess game using both the Elo and PageRank systems. And finally look at the accuracy of each system in terms of how many times each ranking system correctly predicted the true outcome of the 2013 match.

A total of 415 games were predicted. These are the results:

Ranking System	Correct Predictions	Percentage Correct
FIDE	239	0.575903614458
PageRanks	202	0.486746987952

So taking the higher ranked FIDE player will give the correct outcome approximately 57% of the time, while doing the same with the PageRanks will only produce correct outcomes about 48% of the time. It appears as though the FIDE rankings are much better than the PageRanks, but what is of particular interest is that picking the player with the better PageRanks gives correct predictions less than half of the time. This means that it is more accurate to pick the player with the worse PageRank to predict the outcomes of a match, which is a terrible property of any ranking system.

So once again, it looks as though the PageRanking system is substantially less accurate than the current FIDE ranking system.

We can also look at the predictions more directly by only examining the games where the ranking systems predict different winners. There were 187 conflicting predictions, the results are shown in the following table:

Ranking System	Correct Predictions	Percentage Correct
FIDE	108	0.577540106952
PageRank	71	0.379679144385

As we can see, in cases when there is a conflict, the FIDE ranking system correctly predicts the winner more accurately than the PageRank based system.

Overall, it appears as though the PageRank system is not a considerably accurate ranking system.

Chapter 5

Improving PageRank - ChessRank

It appears as though the original PageRank algorithm did not produce desired results. This may be for a few reasons. Firstly, draws are treated as significantly as wins which may be giving players who draw more often higher PageRanks than they deserve.

Another potential issue is that the original PageRank algorithm does not consider multiple links from a page P_i to a page P_j. However, in the case of chess games, if a player P_i defeats another player P_j many times and loses only once, it would appear as though player P_i is much better than P_j, but the graph and transition matrix for this scenario will have a single link from P_i to P_j and a single link the other way. The fact that P_i has defeated P_j multiple times is not captured in the structure of the graph or the transition matrix.

We will modify the algorithm in two ways to try and solve these problems.

Firstly, we can change how multiple links are treated. In particular, we can treat every defeat as another link. So in this example above, if P_i defeats P_j many times, then when a random surfer gets to P_j in the network, they will follow the link to P_i proportional to how many times P_j has lost to P_i over how many times P_j has lost overall.

Previously, when P_j defeated P_i, the entry in the i^{th} row and j^{th} column was $\frac{1}{\#(P_i)}$ where $\#(P_i)$ was the total number of players that P_i had lost to. Let $G(i, j)$ be the entry in the i^{th} row and j^{th} column. To weight multiple wins, the entry $G(i, j)$ becomes:

$$G(i,j) = \frac{L(P_i, P_j)}{T(P_i)}$$

Where $L(P_i, P_j)$ is the number of times that P_i has lost to P_j and $T(P_i)$ is the total number of matches that P_i has lost.

Secondly, we will treat draws differently. In the traditional chess scoring systems, a win is worth a single point and a draw is worth half a point. We can use this same idea to give wins a higher weighting in our transition matrix than draws.

If we let w_w be the weighting given to wins, in our case 1, and w_d be the weighting given to draws, 0.5, the entry $G(i,j)$ now becomes:

$$G(i,j) = \frac{Losses(P_i, P_j) * w_w + Draws(P_i) * w_d}{T(P_i)}$$

Where $T(P_i)$ is now calculated based on wins and draws using the following formula:

$$T(P_i) = TotalLosses(P_i) * w_w + TotalDraws(P_i) * w_d$$

Again note that these changes still produce a row stochastic matrix so we can use the properties of a Markov Chain again to compute the PageRank given these modifications. Let modified PageRank algorithm be called ChessRank.

5.1 Results

ChessRank was run on the chess games of players who were rated at 2700 or above in 2012 or 2013 representing approximately the top 60 chess grandmasters.

Results are presented in the following table, ordered by their ChessRank.

ChessRanks produced are:

Rank	Player	ChessRank
1	Karjakin, Sergey	0.06150454
2	Grischuk, Alexander	0.05331495
3	Carlsen, Magnus	0.05319353
4	Nakamura, Hikaru	0.05004709
5	Morozevich, Alexander	0.04920826
6	Ivanchuk, Vassily	0.04874709
7	Aronian, Levon	0.04475325
8	Radjabov, Teimour	0.04204825
9	Mamedyarov, Shakhriyar	0.03933584
10	Gelfand, Boris	0.03580981
11	Caruana, Fabiano	0.03510255
12	Wang, Hao	0.03497479
13	Svidler, Peter	0.03076264
14	Topalov, Veselin	0.02793112
15	Leko, Peter	0.02593708
16	Andreikin, Dmitry	0.02453672
17	Giri, Anish	0.02371730
18	Kamsky, Gata	0.02198726
19	Kramnik, Vladimir	0.02133255
20	Fressinet, Laurent	0.01687418
21	Nepomniachtchi, Ian	0.01649231
22	Anand, Viswanathan	0.01483246
23	Jakovenko, Dmitry	0.01459338
24	Ponomariov, Ruslan	0.01456103
25	Bacrot, Etienne	0.01380254
26	DominguezPerez, Leinier	0.01360244
27	Jobava, Baadur	0.01311958
28	Ding, Liren	0.01251327
29	Adams, Michael	0.01199479
30	Bologan, Viktor	0.01133846

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Rank	Player	ChessRank
31	Tomashevsky, Evgeny	0.01115340
32	Naiditsch, Arkadij	0.00923914
33	Vitiugov, Nikita	0.00863717
34	McShane, Luke	0.00821850
35	Shirov, Alexei	0.00809467
36	Wojtaszek, Radoslaw	0.00780951
37	Navara, David	0.00742856
38	Vachier-Lagrave, Maxime	0.00608207
39	Moiseenko, Alexander	0.00574723
40	Malakhov, Vladimir	0.00532919
41	Gashimov, Vugar	0.00479598
42	Movsesian, Sergei	0.00439722
43	Volokitin, Andrei	0.00386634
44	Sasikiran, Krishnan	0.00369925
45	Laznicka, Viktor	0.00354708
46	Le, QuangLiem	0.00345025
47	Areshchenko, Alexander	0.00326206
48	Polgar, Judit	0.00318732
49	BruzonBatista, Lazaro	0.00268968
50	Riazantsev, Alexander	0.00208692
51	Eljanov, Pavel	0.00189263
52	VallejoPons, Francisco	0.00167863
53	Korobov, Anton	0.00165630
54	Sutovsky, Emil	0.00109479
55	Short, Nigel	0.00093089
56	Grachev, Boris	0.00068674
57	Cheparinov, Ivan	0.00060632
58	Almasi, Zoltan	0.00059359
59	Inarkiev, Ernesto	0.00016949

We can again use the same techniques to determine the accuracy of the ChessRanks. Firstly, we can compare the difference between the ChessRanks and the FIDE ranks using

$$D = \sum_{p \in Players} |FR(p) - CR(p)|$$

Where FR is the players FIDE Rank and CR is the players ChessRank. Results are as follows:

Player	FIDE Rank	ChessRank	Difference
Carlsen, Magnus	1	3	2
Aronian, Levon	2	7	5
Kramnik, Vladimir	3	19	16
Nakamura, Hikaru	4	4	0
Grischuk, Alexander	5	2	3
Caruana, Fabiano	6	11	5
Gelfand, Boris	7	10	3
Anand, Viswanathan	8	22	14
Topalov, Veselin	9	14	5
Mamedyarov, Shakhriyar	10	9	1
Karjakin, Sergey	11	1	10
DominguezPerez, Leinier	12	26	14
Svidler, Peter	13	13	0
Adams, Michael	14	29	15
Bacrot, Etienne	15	25	10
Vachier-Lagrave, Maxime	16	38	22
Vitiugov, Nikita	17	33	16
Wang, Hao	18	12	6
Eljanov, Pavel	19	51	32
Giri, Anish	20	17	3
Ponomariov, Ruslan	21	24	3
Ivanchuk, Vassily	22	6	16
Leko, Peter	23	15	8
Morozevich, Alexander	24	5	19
Naiditsch, Arkadij	25	32	7
Wang, Yue	26	not ranked	-
Tomashevsky, Evgeny	27	31	4
Nepomniachtchi, Ian	28	21	7
Jakovenko, Dmitry	29	23	6
Kamsky, Gata	30	18	12
Areshchenko, Alexander	31	47	16
So, Wesley	32	not ranked	-
Radjabov, Teimour	33	8	25
Korobov, Anton	34	53	19
Ding, Liren	35	28	7
Wojtaszek, Radoslaw	36	36	0
Andreikin, Dmitry	37	16	21
Almasi, Zoltan	38	58	20
Moiseenko, Alexander	39	39	0
Malakhov, Vladimir	40	40	0
Fressinet, Laurent	41	20	21
Rublevsky, Sergei	42	not ranked	-
Navara, David	43	37	6
VallejoPons, Francisco	44	52	8
Harikrishna, P.	45	not ranked	-
Le, QuangLiem	46	46	0
Kryvoruchko, Yuriy	47	not ranked	-
Matlakov, Maxim	48	not ranked	-
Movsesian, Sergei	49	42	7
Total Difference			414

This gives a total difference of 9.63 per player, which is slightly more accurate than the original PageRank algorithm.

As with PageRank, if we only look at the top 10 players, the total difference drops to 54, or a difference of 5.4 on average per player, a significant improvement. So perhaps if a greater dataset was used, the ChessRanks would not be completely different from the FIDE rankings.

We shall also compare predictions using the same method as we used with the PageRank algorithm.

Ranking System	Correct Predictions	Percentage Correct
FIDE	239	0.575903614458
PageRank	208	0.501204819277

Ranking System	Correct Predictions	Percentage Correct
FIDE	101	0.590643274854
PageRank	70	0.409356725146

We can see that with the improvements made, ChessRank performs slightly better than the original PageRank algorithm as ChessRank predicted the outcome of a few more games correctly and had fewer conflicting predictions to the FIDE Ranks with only 171 compared to PageRank which had 179. Also of interest is that with these modifications, ChessRank predicts the outcome of games with greater than 50% accuracy.

But overall, while a PageRank based ranking algorithm for Chess does produce a ranking system of some accuracy, but it is not very reliable and not nearly as accurate as the current Elo system used by FIDE.

Chapter 6

Potential Improvements

The PageRank based algorithm described in this report does not appear to be as accurate as the current Elo system that is used. However some further changes could be made which may make the system more accurate.

Firstly, the FIDE rankings are based on a players entire professional career whereas the PageRank algorithm only used matches played in 2012. Some players have been playing professionally for more than 20 years. Even the youngest players in the worlds top 50 have been professional for around 5 years. If a record of all games could be obtained, perhaps a more accurate network, and in turn, a more accurate PageRank for each player could be obtained.

Secondly, a players Elo rating, and hence FIDE ranking, constantly changes based on every game they play and based upon their and their opponents current Elo rating at the time of the match. As a result, Elo ratings are more fine grained because they take into account the time the match was played and its context (the rating of each player). In PageRank, a win is a win, regardless of when it was played. Perhaps an improvement could be made to weight each win in proportion to how recently the match was played, meaning that more recent wins are worth more than a win from several years ago.

Chapter 7

Conclusion

In this report, matches played between top chess grandmasters were used to create a network of links between players. PageRank and a modified version of PageRank, named ChessRank, were run on these networks to produce a ranking of the players within that network. These rankings were compared against the current set of rankings used by FIDE. It was found that both PageRank and ChessRank do not produce very accurate rankings of players.

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