1 Coordinate Transformations

1. Plane coordinate transformation

$$\boxed{\begin{aligned} X &= a_0 + a_1 x - b_1 y \\ Y &= b_0 + b_1 x + a_1 y \end{aligned}}$$

$$\begin{aligned} \overline{X} &= X_2 - X_1 & \overline{x} &= x_2 - x_1 \\ \overline{Y} &= Y_2 - Y_1 & \overline{y} &= y_2 - y_1 \\ a_1 &= \frac{\overline{x}\overline{X}+\overline{y}\overline{Y}}{\overline{x}^2+\overline{y}^2} & b_1 &= \frac{\overline{x}\overline{Y}-\overline{y}\overline{X}}{\overline{x}^2+\overline{y}^2} \\ a_0 &= X_1 - a_1 x_1 + b_1 y_1 & b_0 &= Y_1 - b_1 x_1 - a_1 y_1 \end{aligned}$$

2. Plane similarity transformation with overdetermination

$$\boxed{\begin{aligned} X' &= a_0 + a_1 x - b_1 y \\ Y' &= b_0 + b_1 x + a_1 y \end{aligned}}$$

i = 1 . . . n

$$\begin{aligned} X_s &= \frac{[X_i]}{n} & Y_s &= \frac{[Y_i]}{n} & x_s &= \frac{[x_i]}{n} & y_s &= \frac{[y_i]}{n} \\ \overline{X_i} &= X_i - X_s & \overline{Y_i} &= Y_i - Y_s & \overline{x_i} &= x_i - x_s & \overline{y_i} &= y_i - y_s \\ a_1 &= \frac{[\overline{x}_i\overline{X}_i]+[\overline{y}_i\overline{Y}_i]}{[\overline{x}_i^2+\overline{y}_i^2]} & & & b_1 &= \frac{[\overline{x}_i\overline{Y}_i]-[\overline{y}_i\overline{X}_i]}{[\overline{x}_i^2+\overline{y}_i^2]} & & \\ a_0 &= X_s - a_1 x_s + b_1 y_s & & & b_0 &= Y_s - b_1 x_s - a_1 y_s & & \end{aligned}$$

$$\begin{aligned} v_{xi} &= X_i - X_i' & v_{yi} &= Y_i - Y_i' \\ m_0 &= \sqrt{\frac{[v_{xi}^2+v_{yi}^2]}{2n-4}} = m_x = m_y & & \\ m_p &= m_0\sqrt{2} & & \end{aligned}$$