Hidden yet quantifiable: A lower-bound for confounding strength using randomized trials

Piersilvio De Bartolomeis, joint work with J. Abad, K. Donhauser and F. Yang

ETH zürich

Motivation: drug regulatory process

Time

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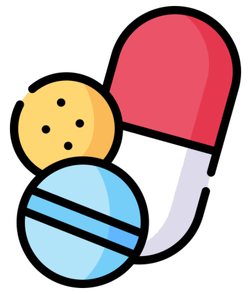
Pre-market evaluation



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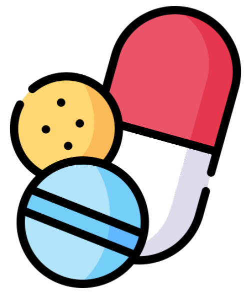

assign treatment

randomize

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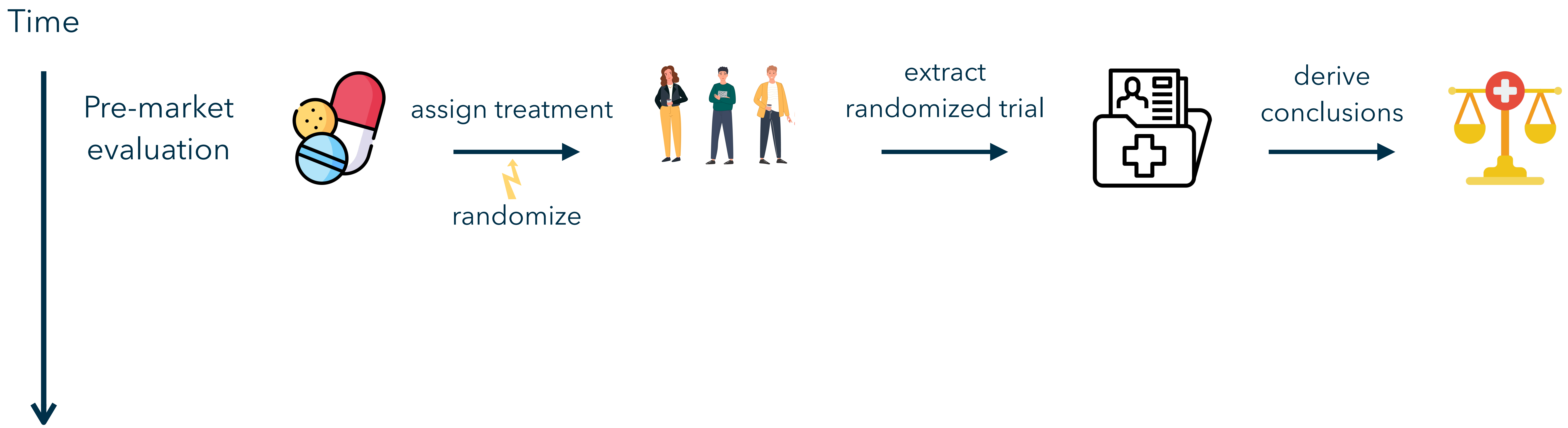

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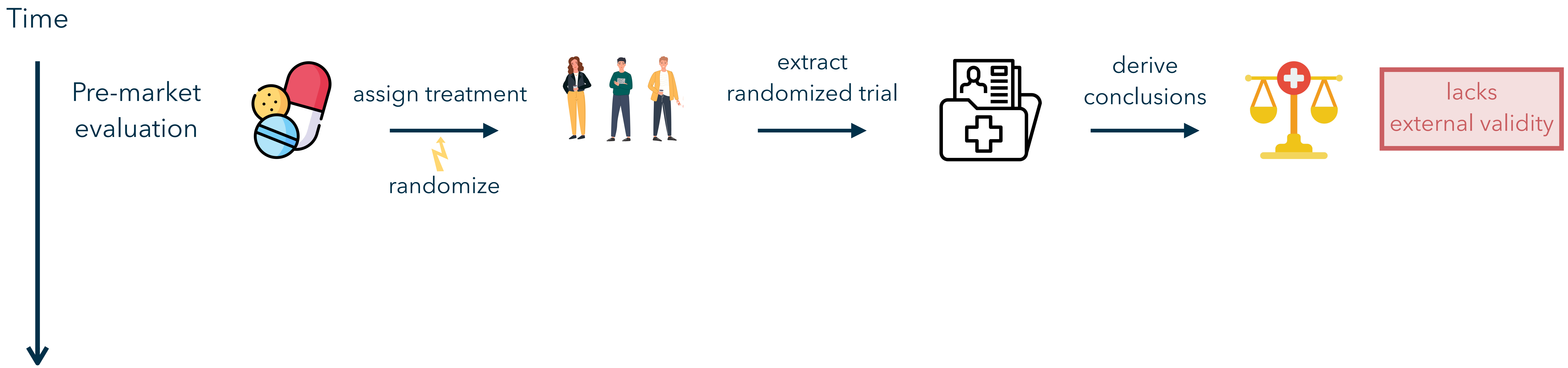

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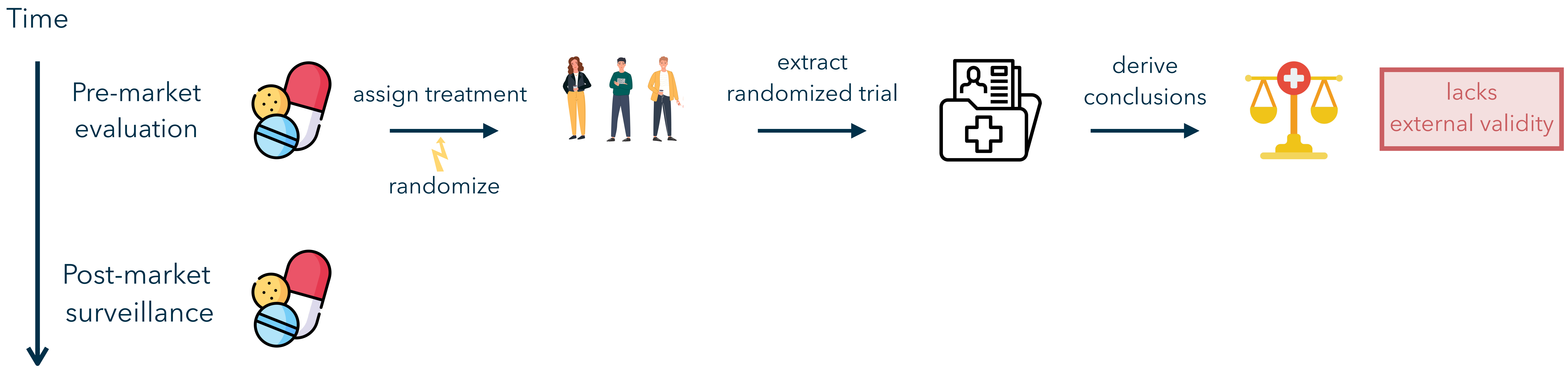

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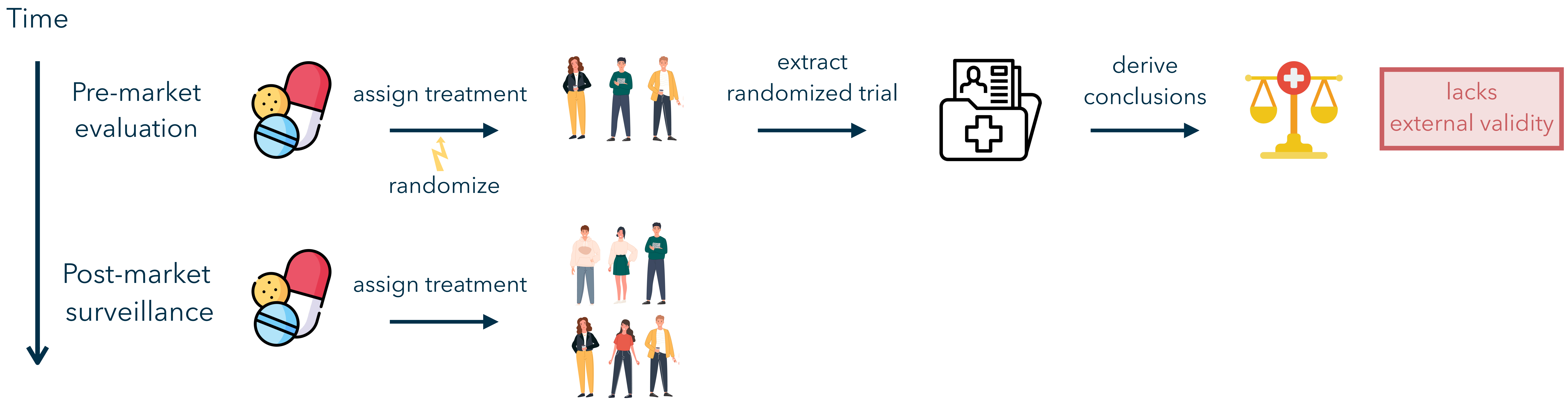

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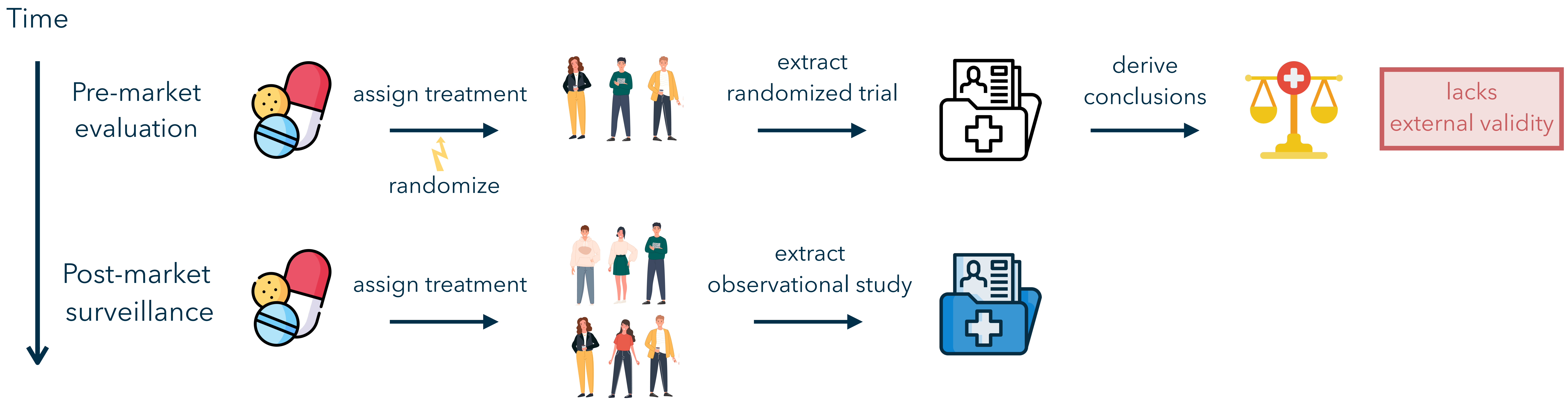

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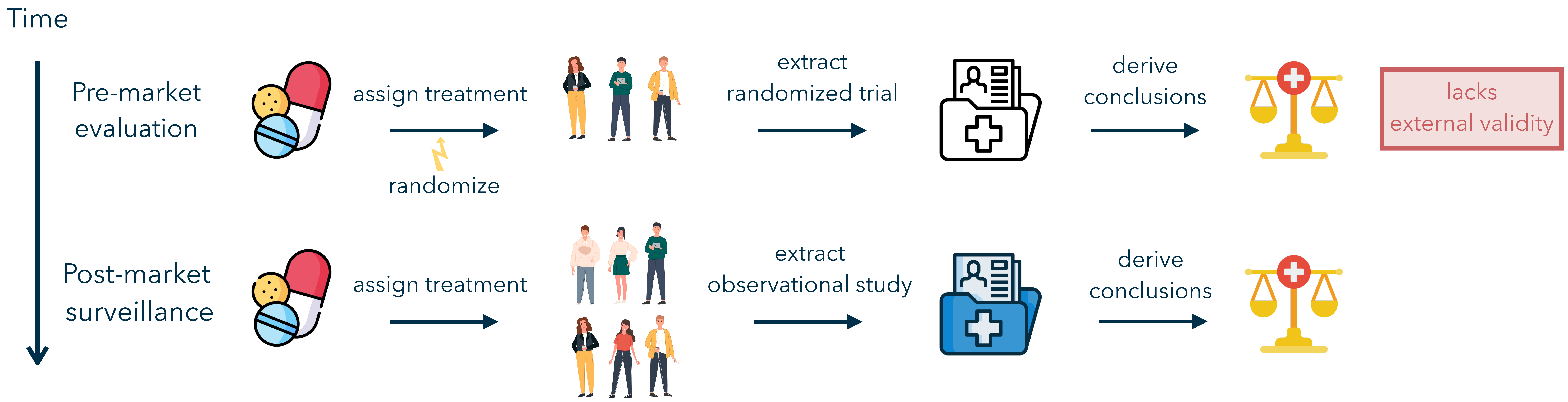

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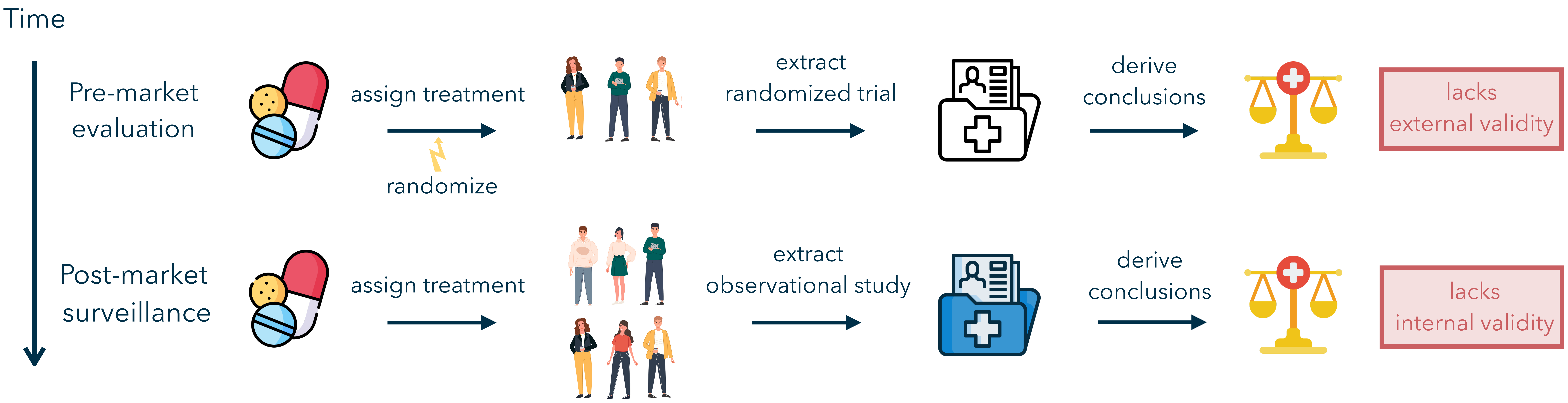
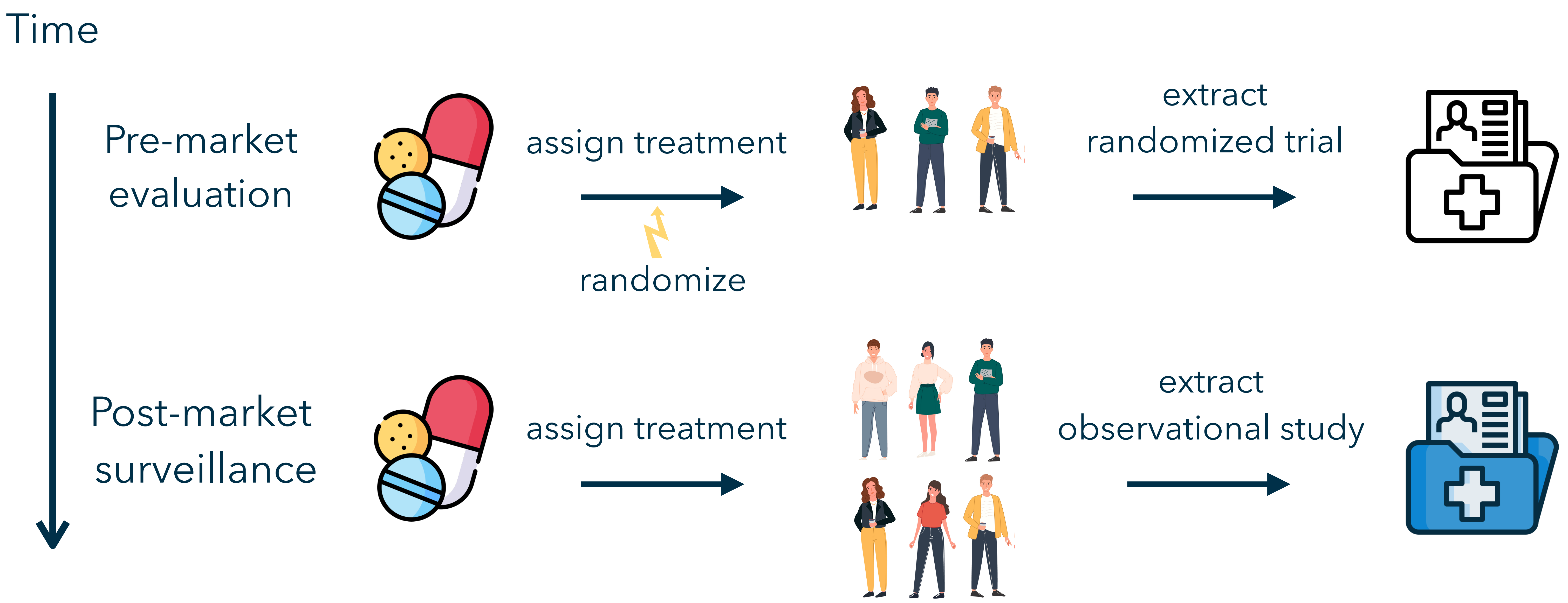


Previous works: binary tests for confounding



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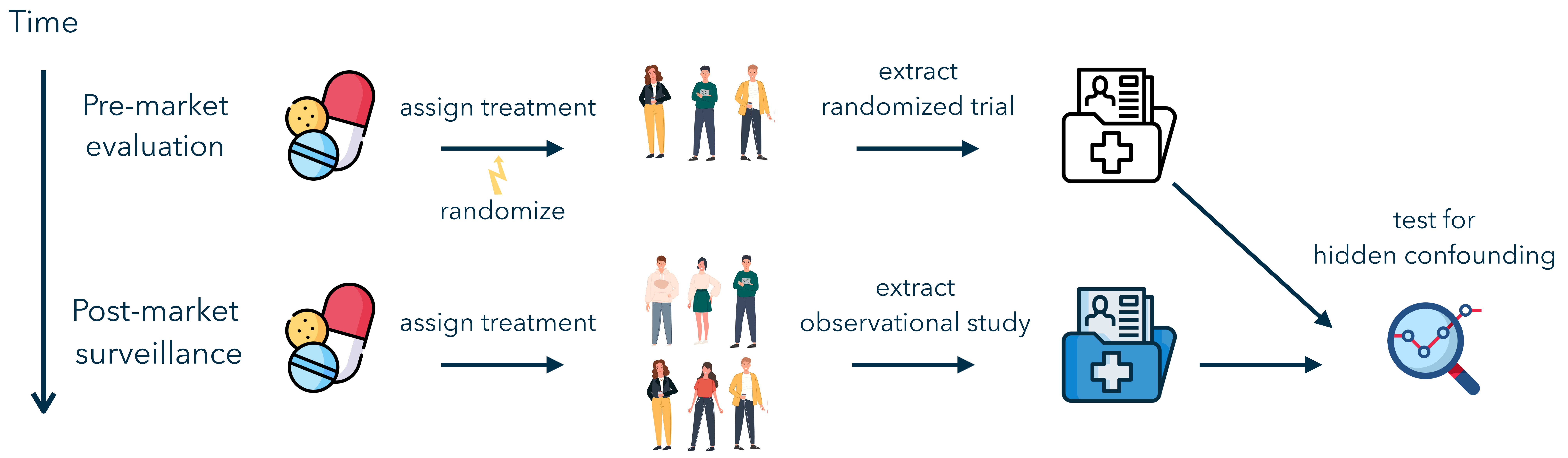

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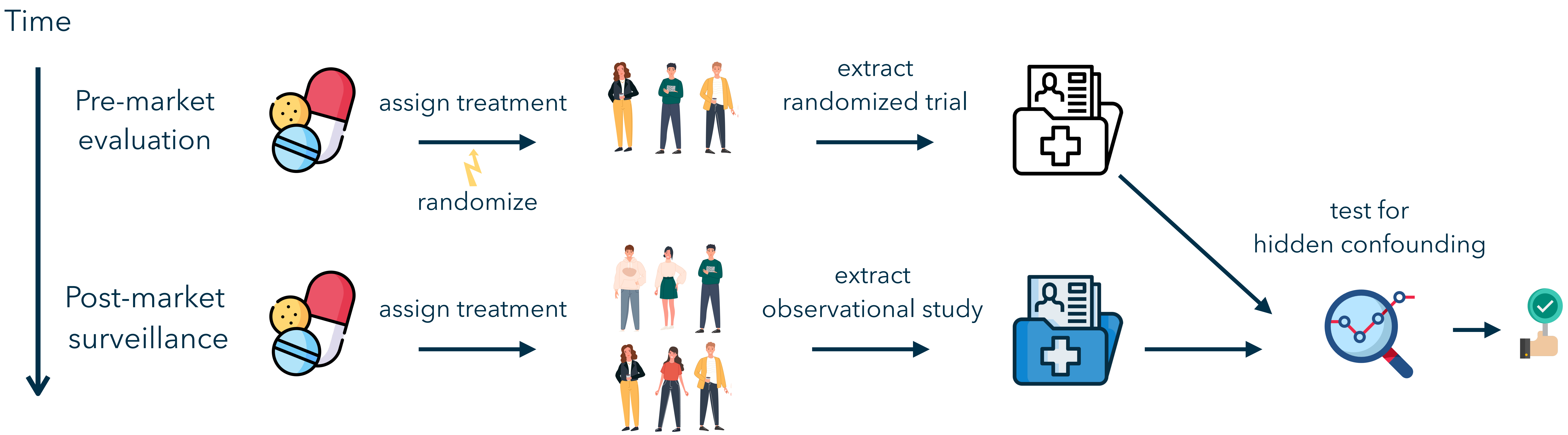

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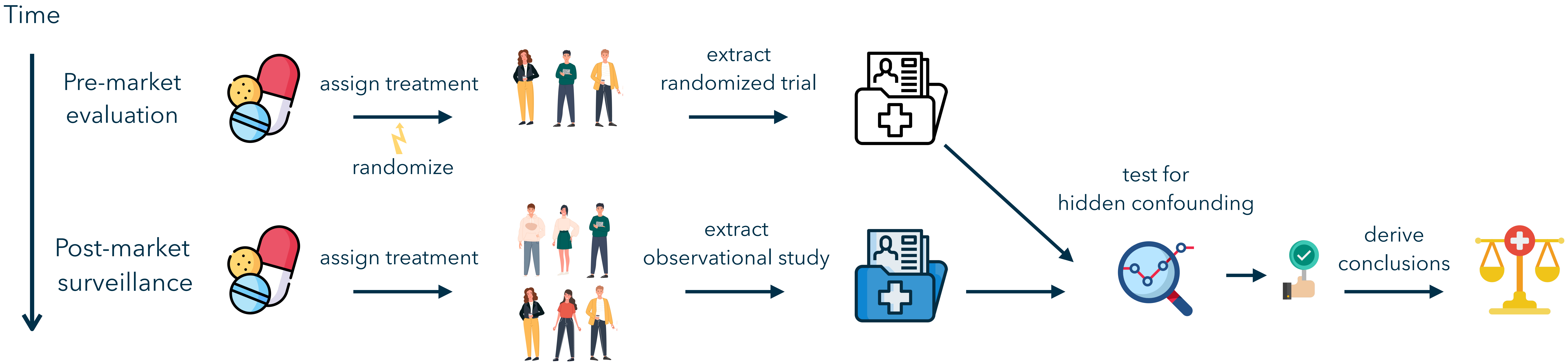

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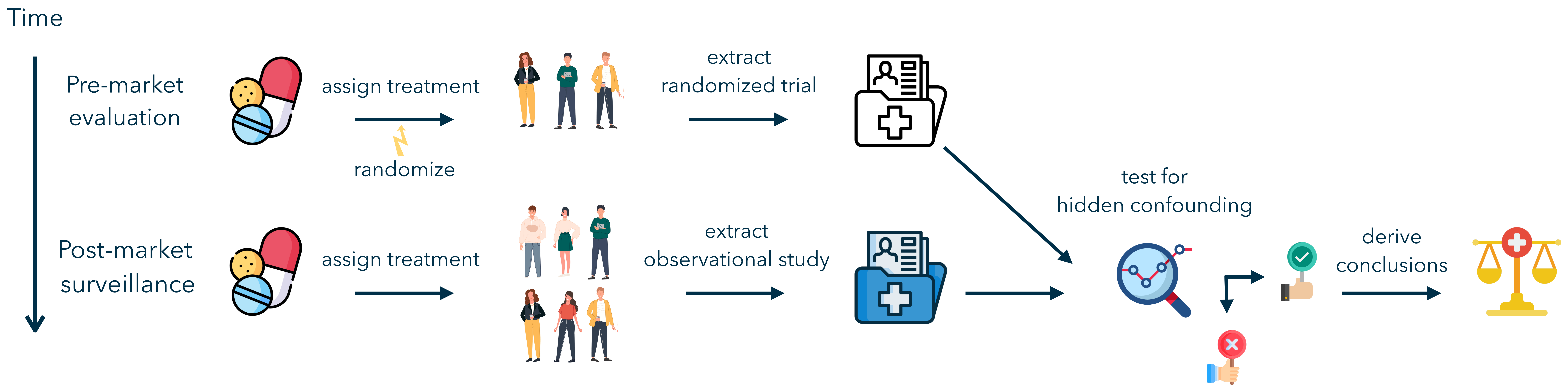

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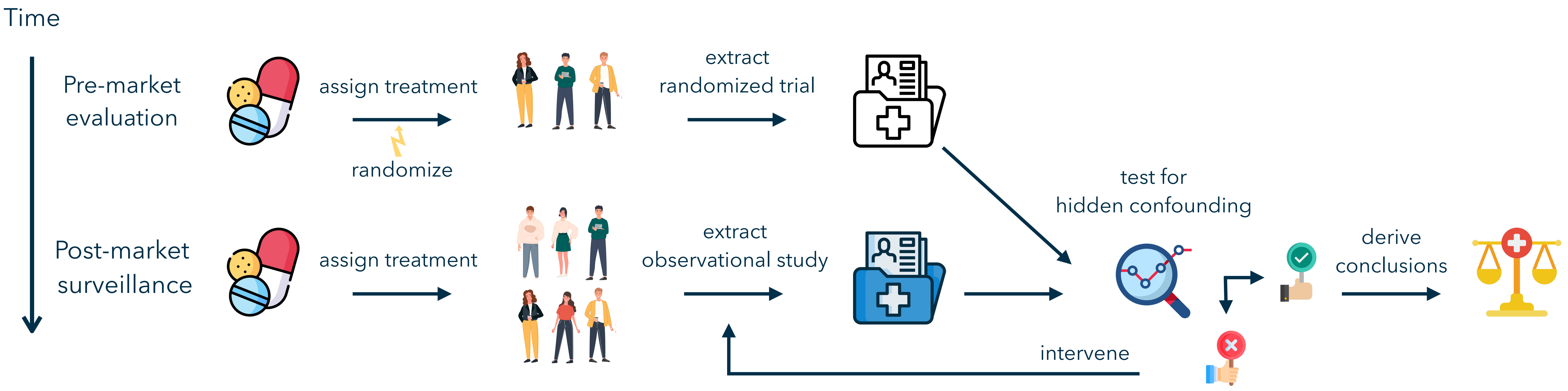

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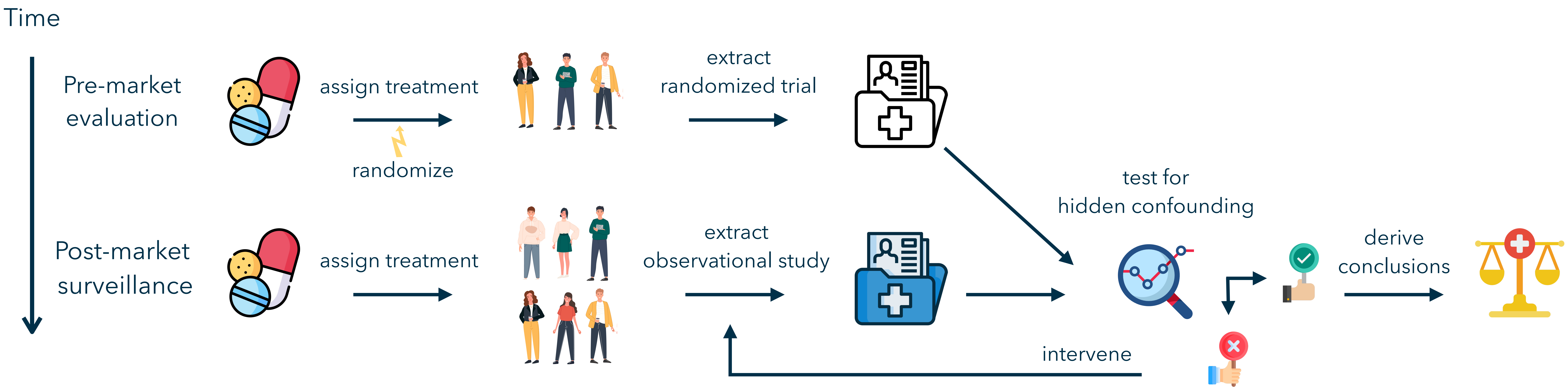
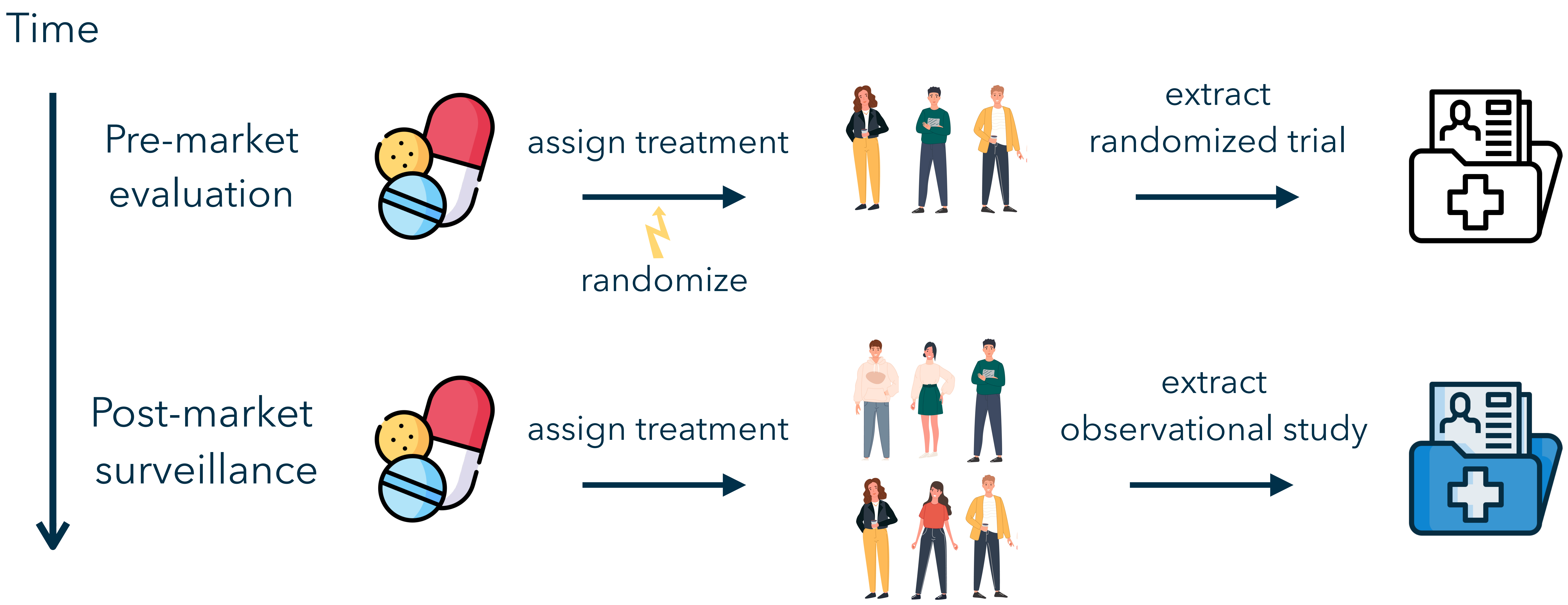


Some confounding is likely present in real-world data $\Longrightarrow$ binary tests are too sensitive

Our proposal: lower bounding confounding strength



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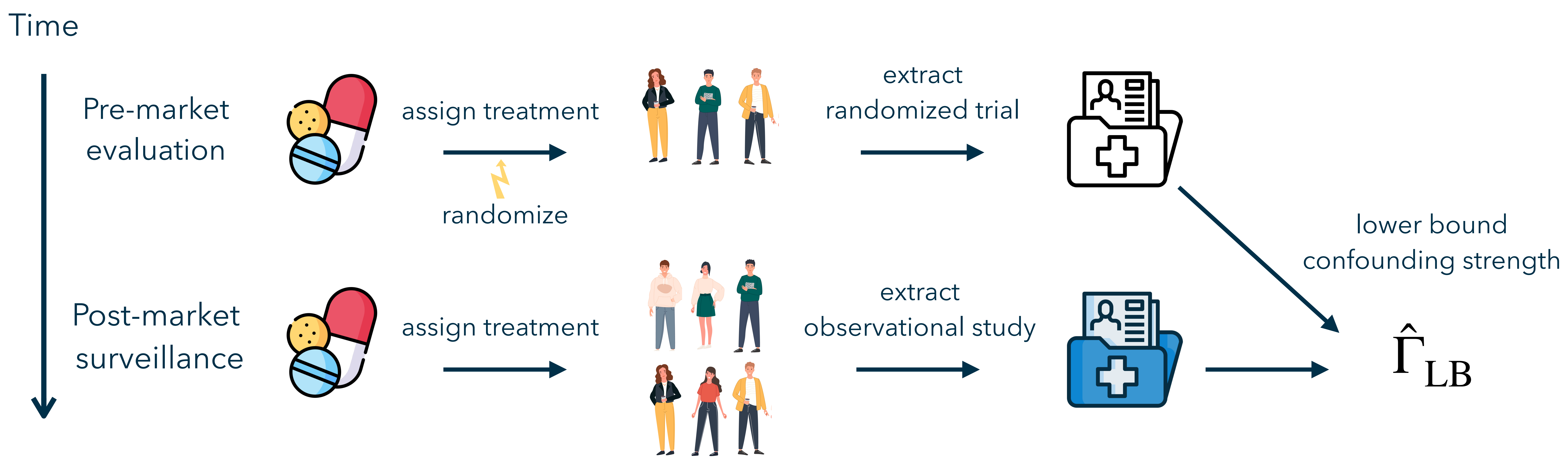

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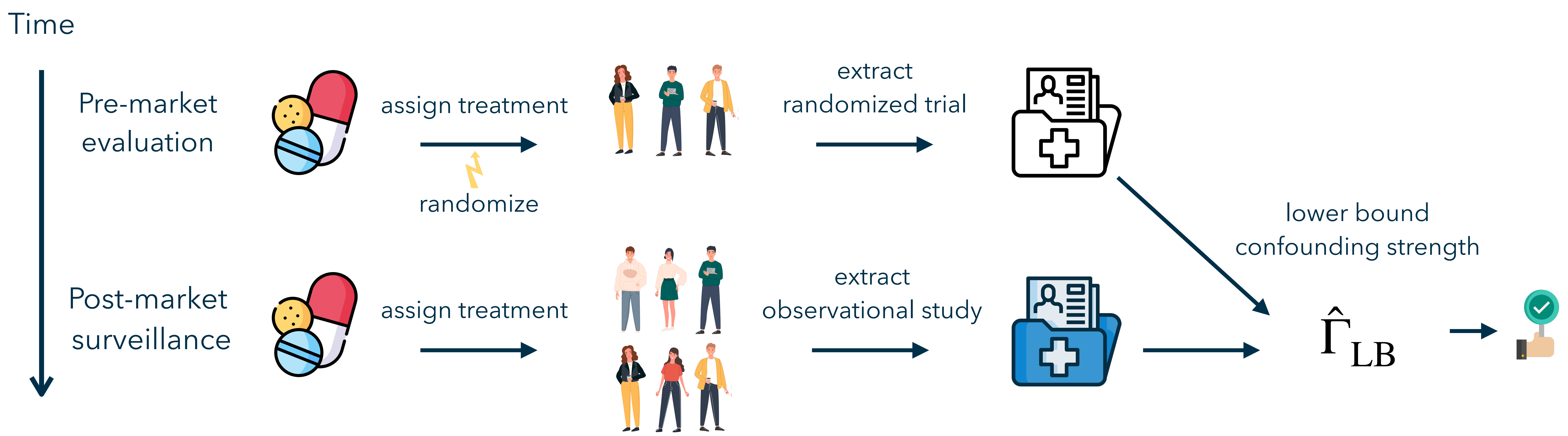

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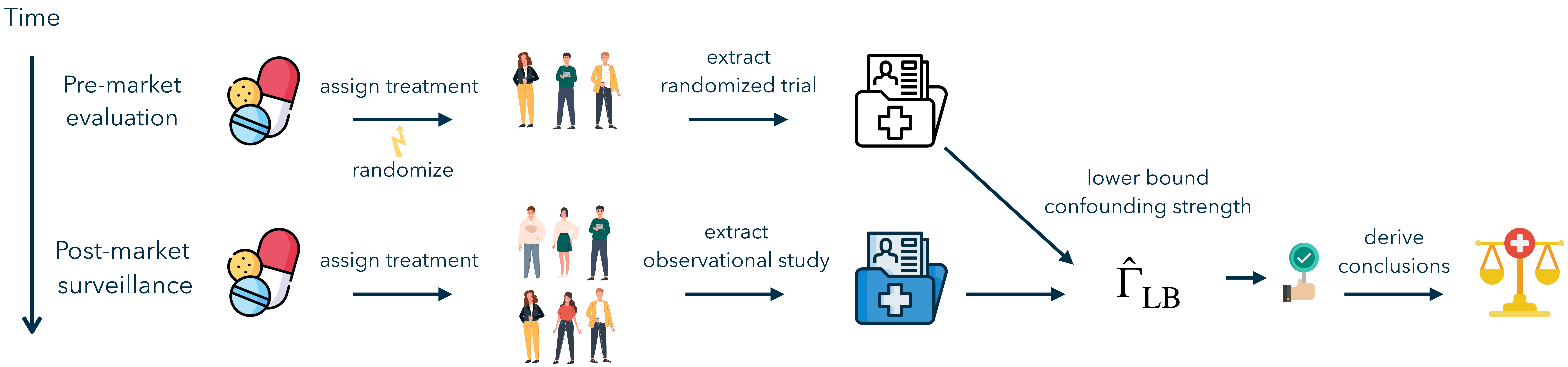

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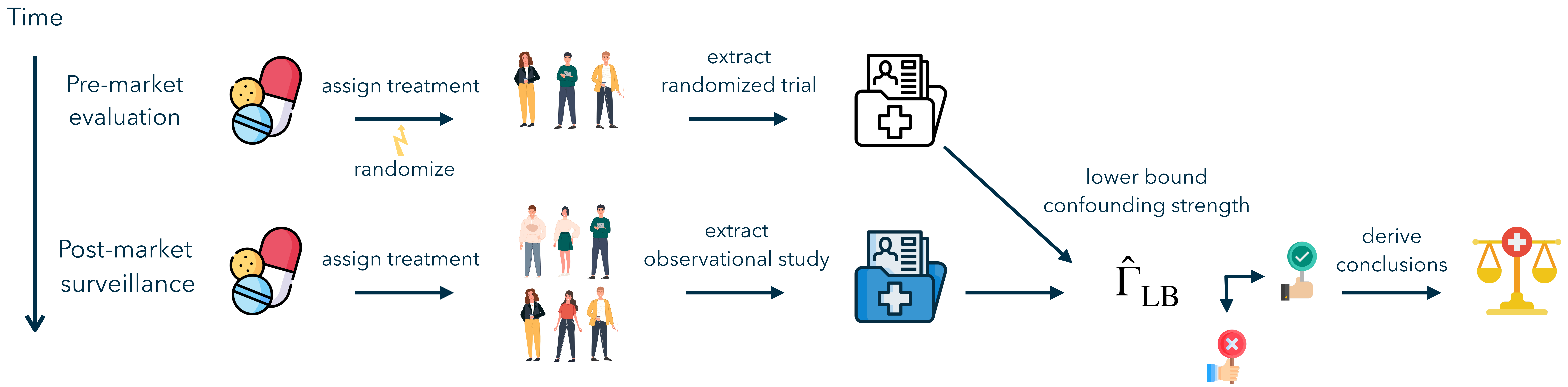

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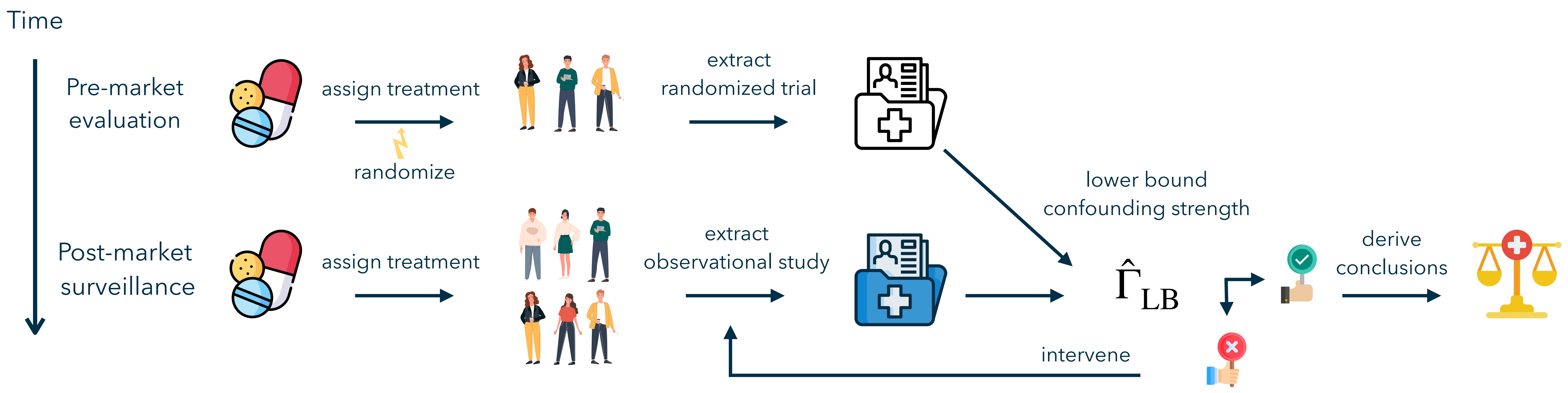

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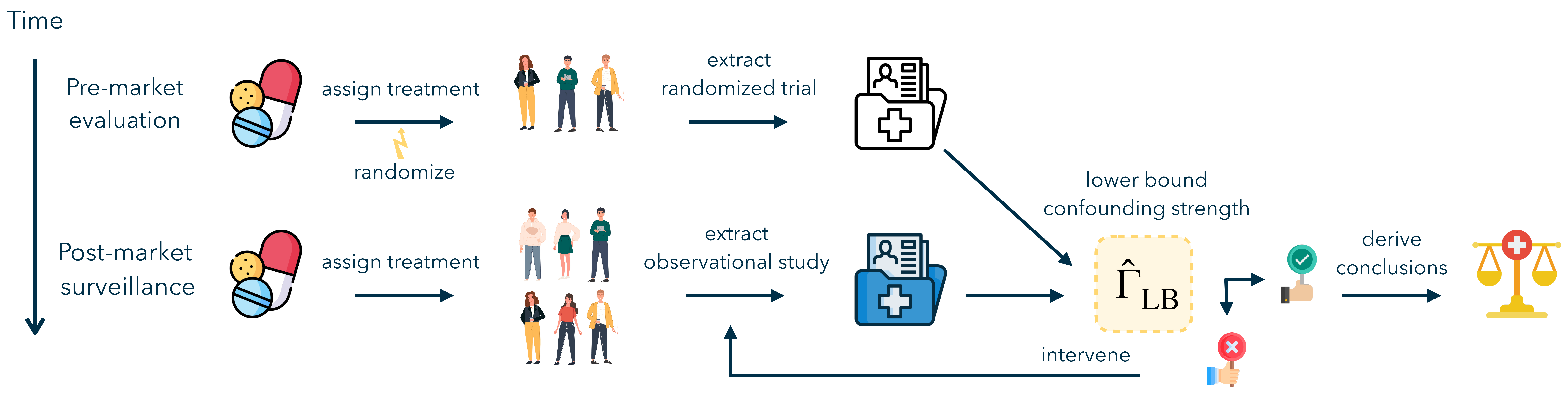

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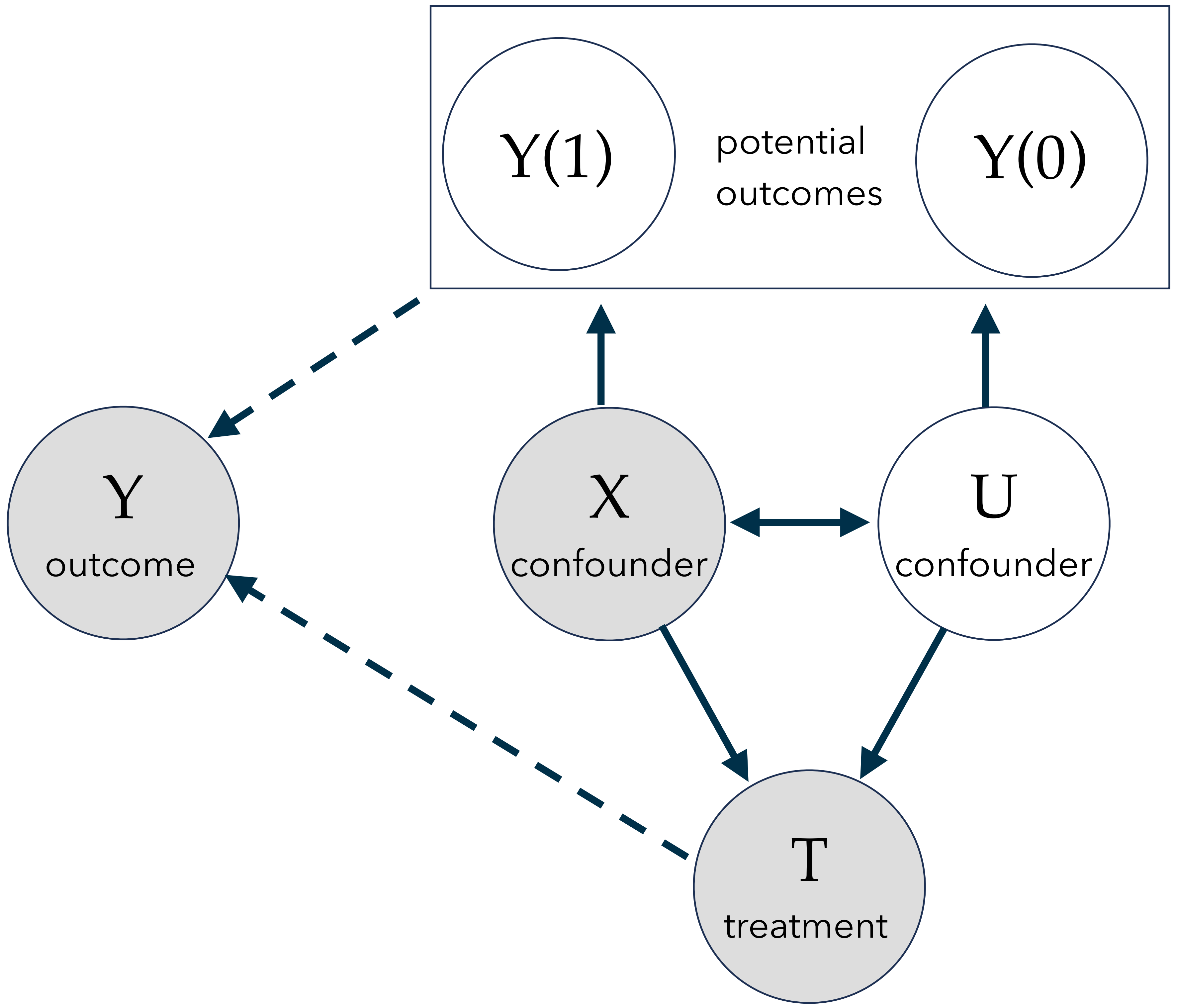

Setting: potential outcomes

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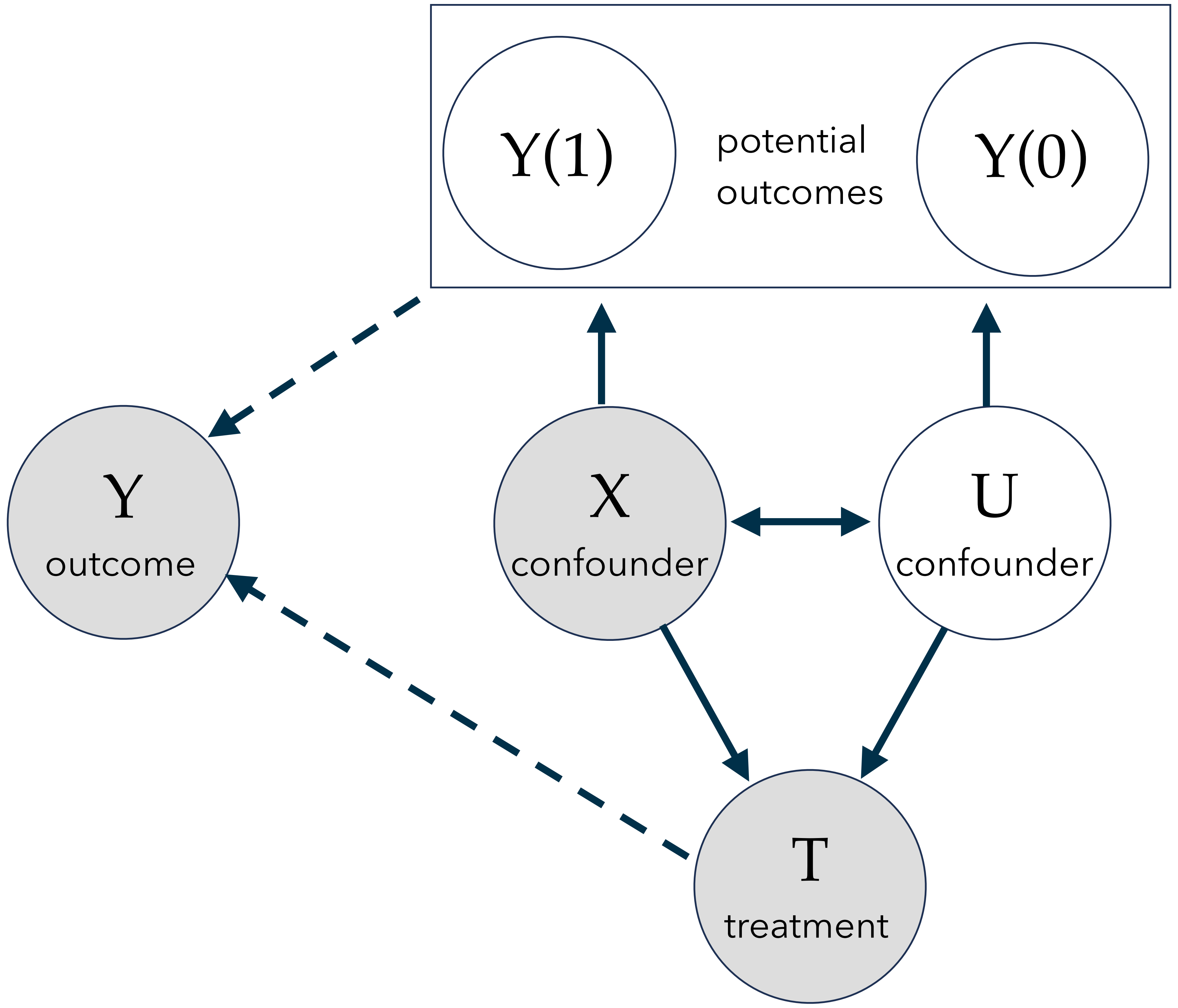

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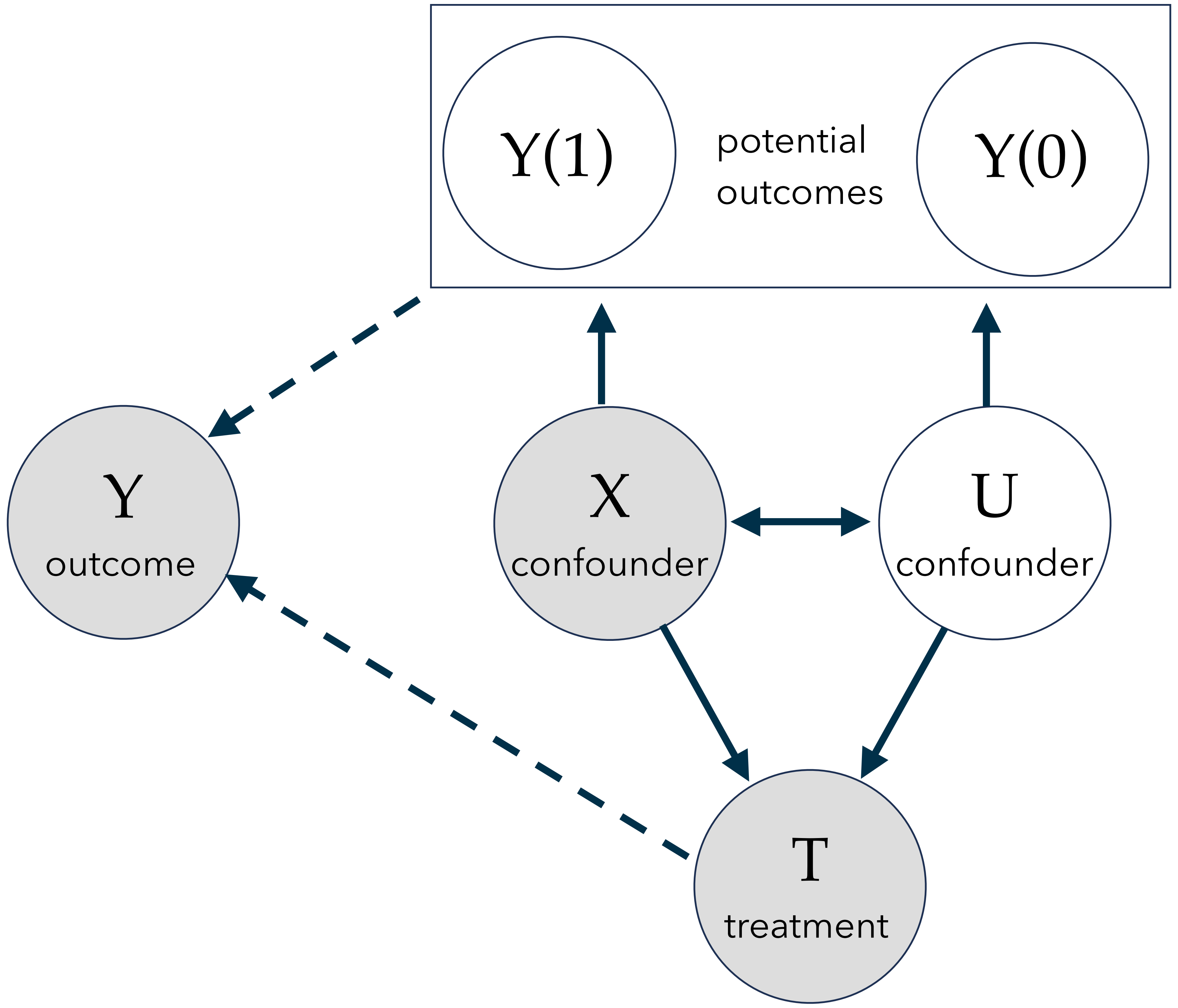

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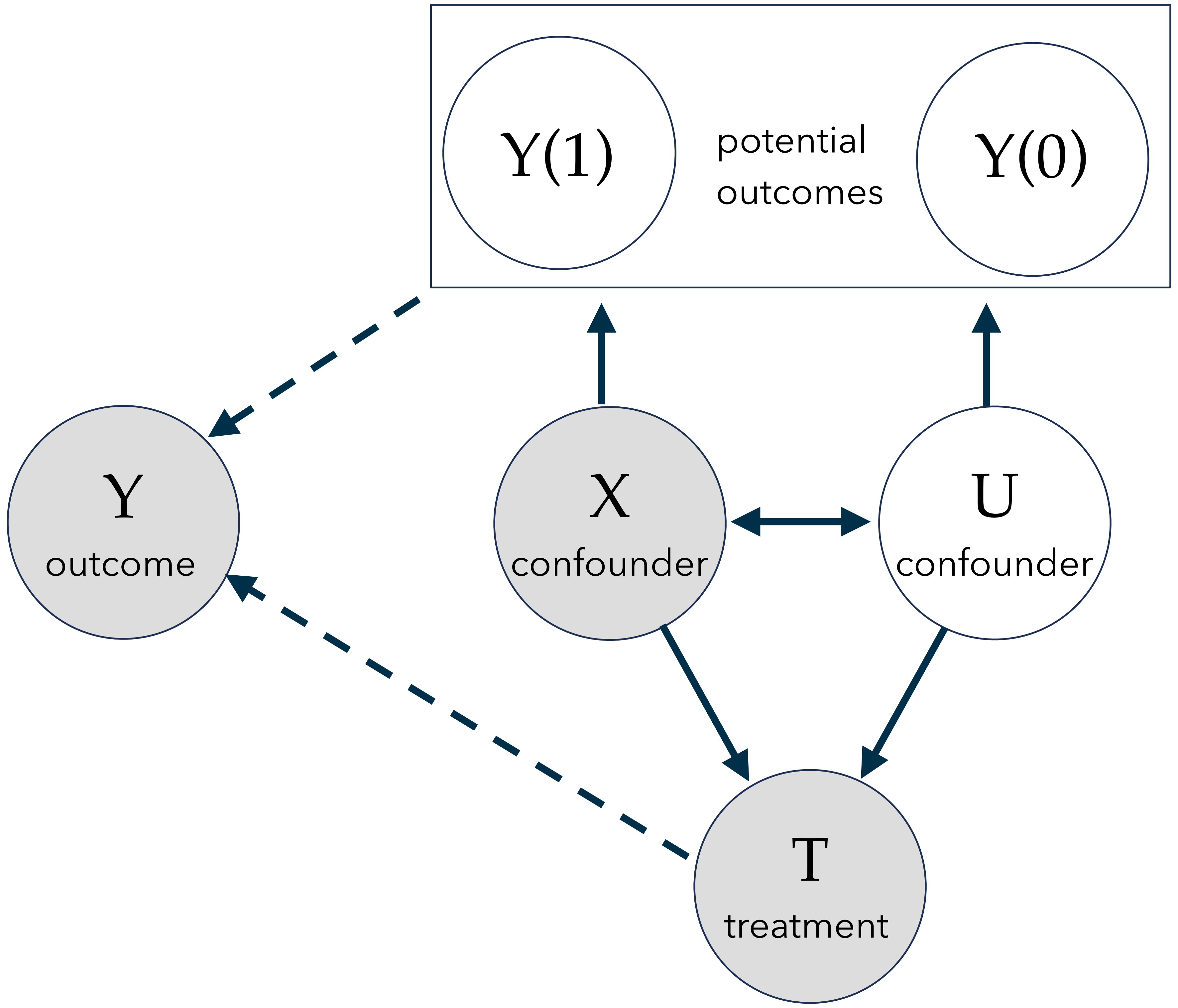

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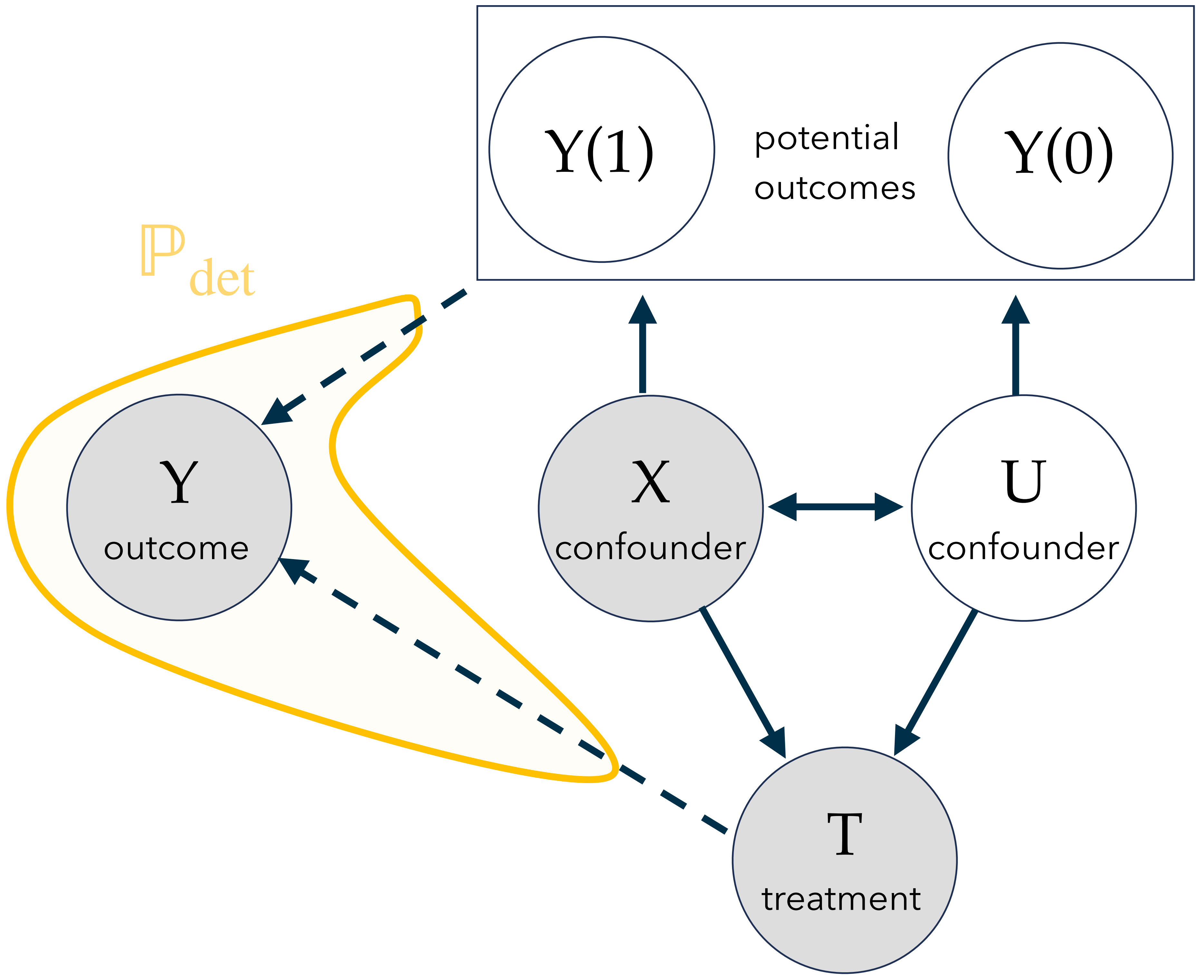

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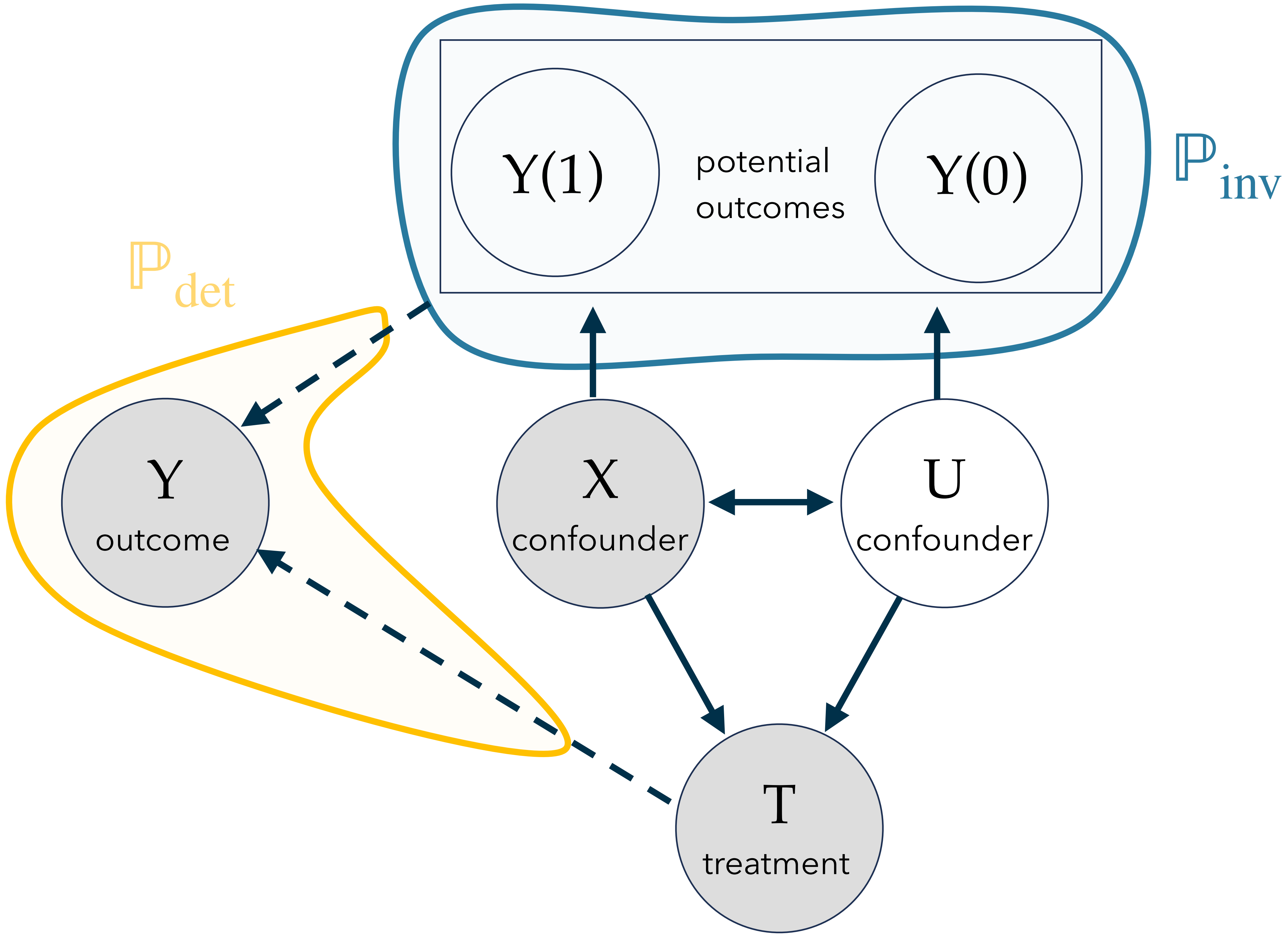

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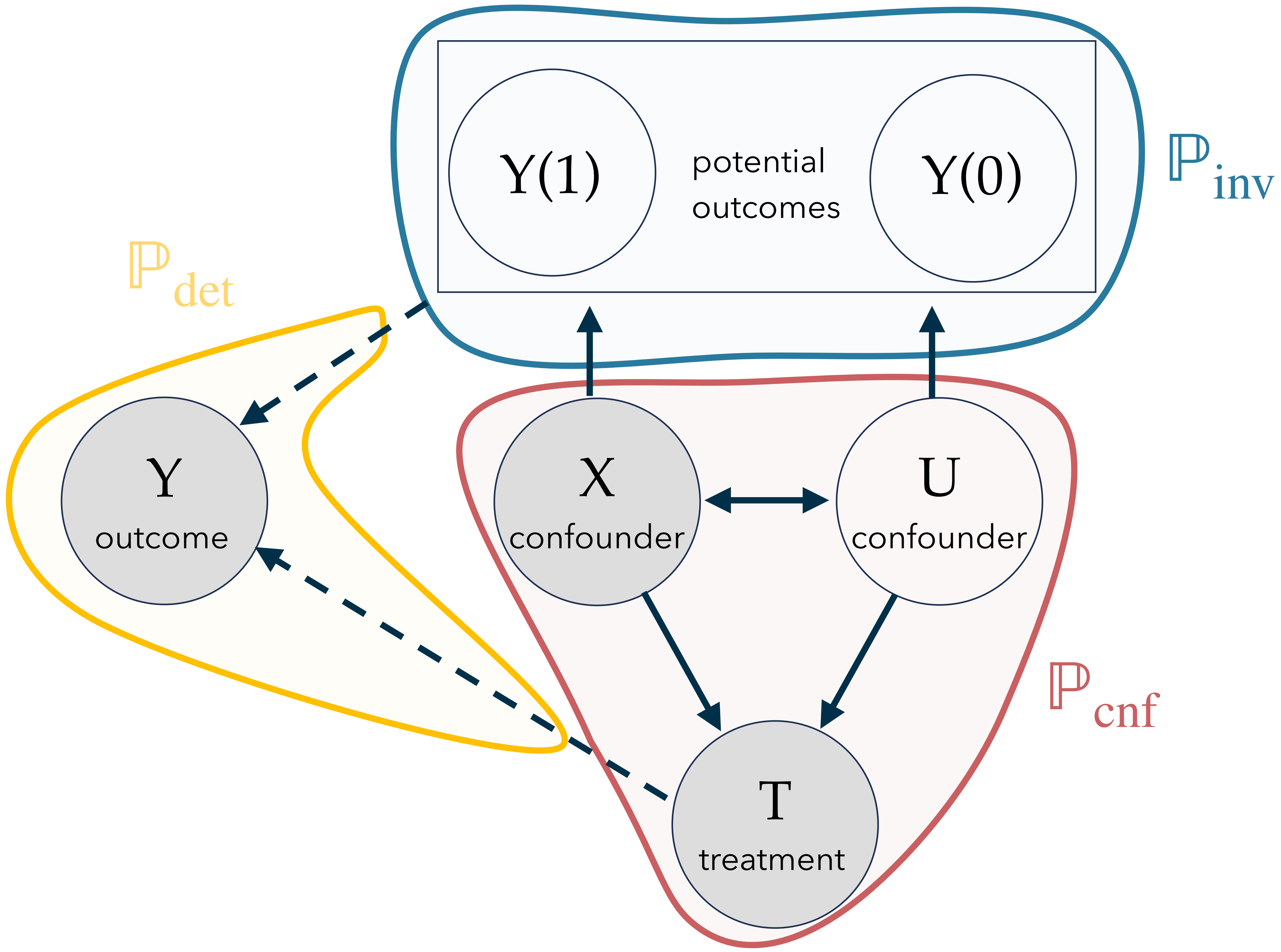

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Definition (informal)	Marginal sensitivity set
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If Γ is well-specified

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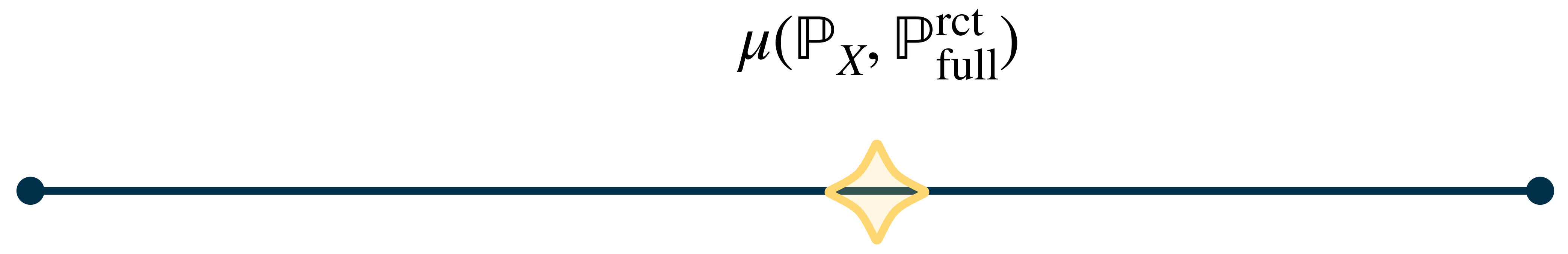

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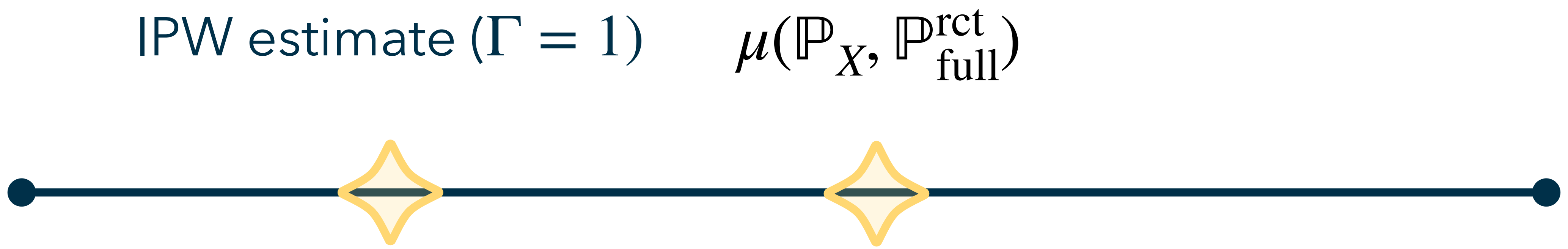

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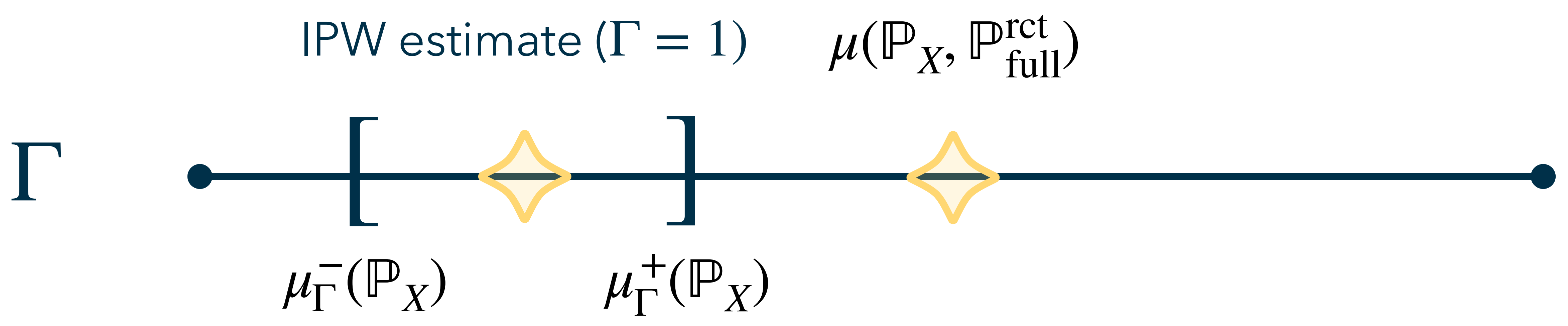

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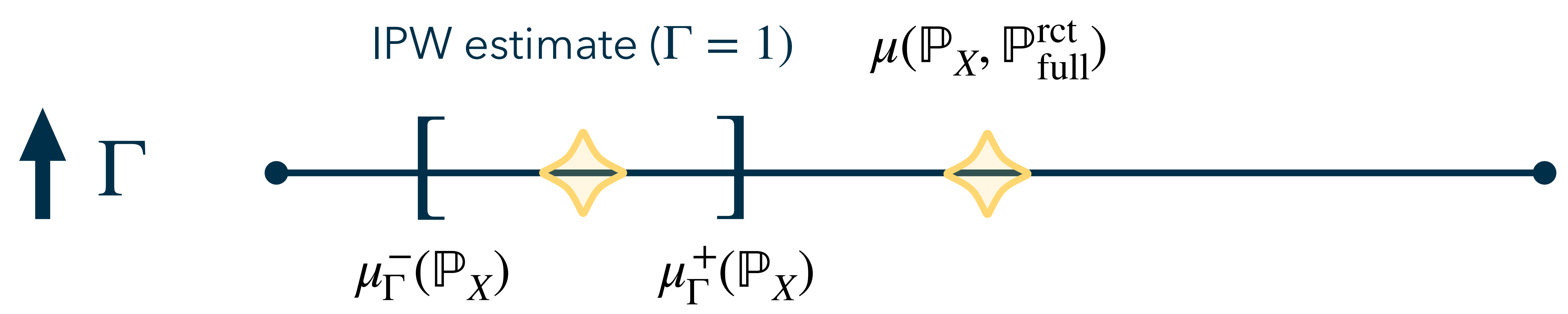
it holds that $\mathbb{P}^{\text{os}}_{\text{cnf}}$ conf. strength $\in [\Gamma^{-1}, \Gamma] \implies \mu(\mathbb{P}_X, \mathbb{P}^{\text{rct}}_{\text{full}}) \in [\mu^-_\Gamma(\mathbb{P}_X), \mu^+_\Gamma(\mathbb{P}_X)]$



Statistical test for confounding strength

- First step: build a statistical test for the null hypothesis $H_0 : \mathbb{P}^{\text{os}}_{\text{cnf}}$ conf. strength $\in [\Gamma^{-1}, \Gamma]$

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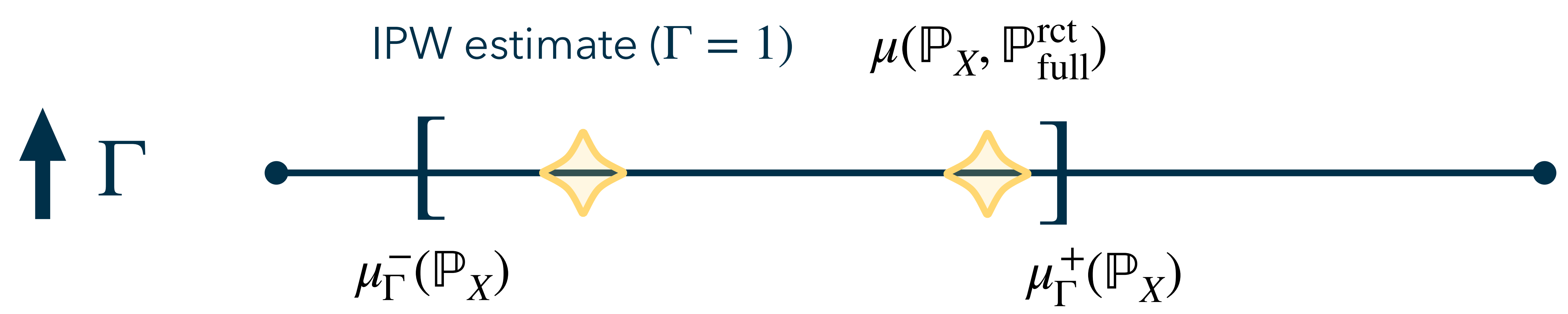

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Lemma (informal)

$\hat{\Gamma}_{\text{LB}}$ is an asymptotically valid lower-bound for $\Gamma^\star$, i.e. it holds that $\mathbb{P}(\hat{\Gamma}_{\text{LB}} \leq \Gamma^\star) \geq 1 - \alpha + o(1)$

Synthetic and semi-synthetic experiments

$\hat{\phi}_{\mathrm{rct}}$ $\hat{\phi}_{\tilde{\mathrm{o}}\mathrm{s}}$ $\phi^{\star}$ $\hat{\Gamma}^{\mathrm{rct}}_{\mathrm{LB}}$ $\hat{\Gamma}^{\tilde{\mathrm{o}}\mathrm{s}}_{\mathrm{LB}}$

Synthetic
(Gaussian)

Semi-synthetic
(Hillstrom dataset)

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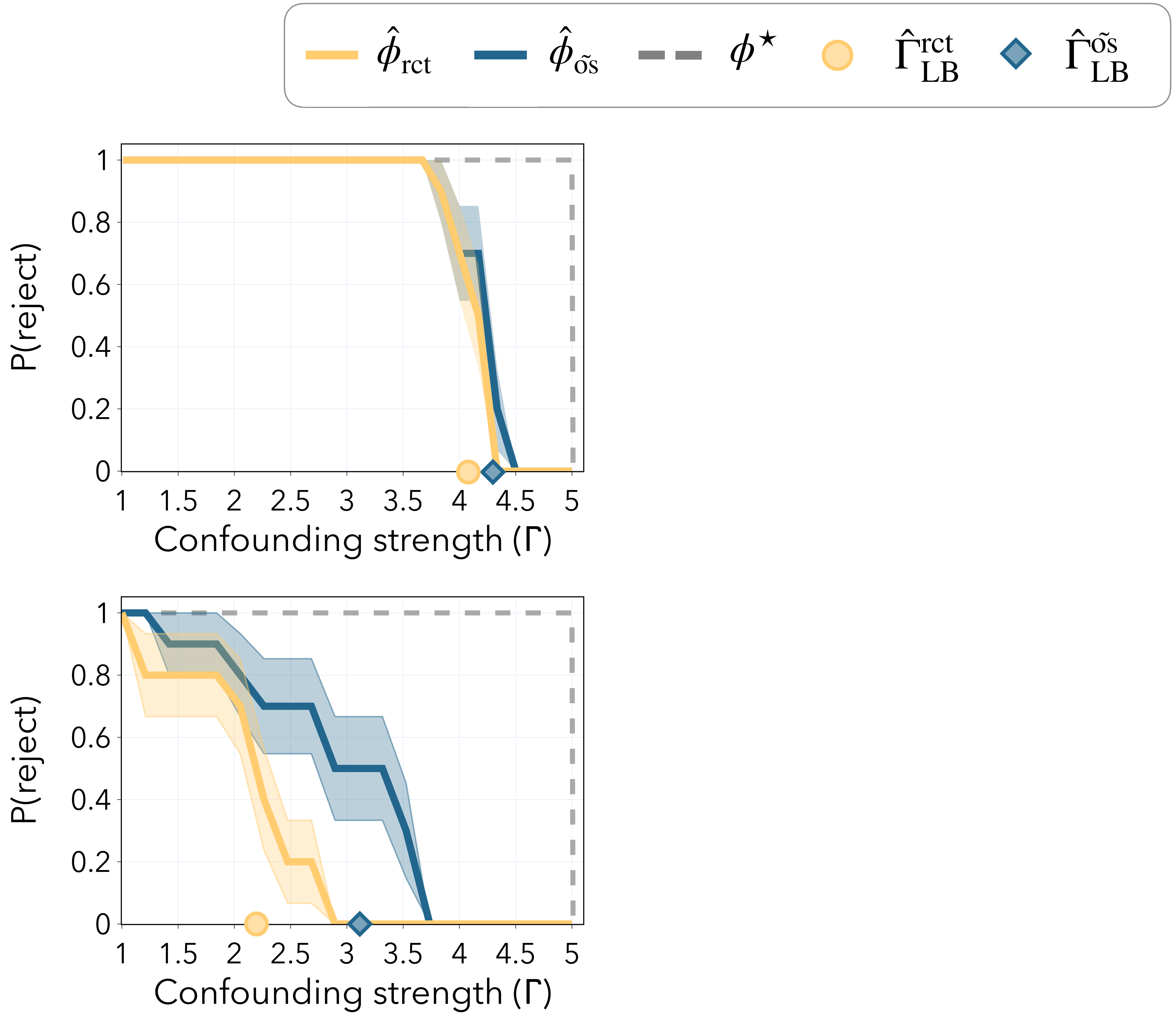

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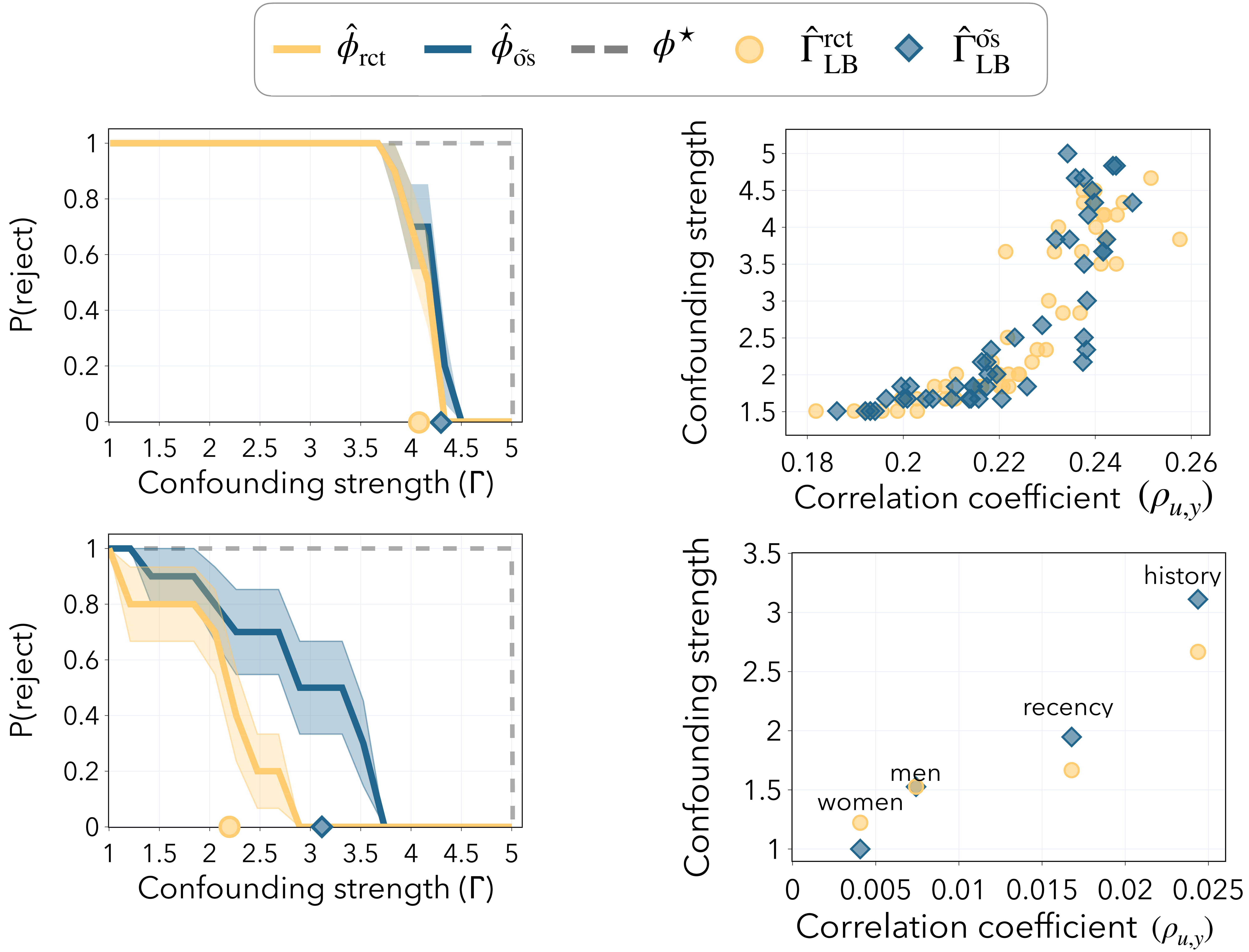
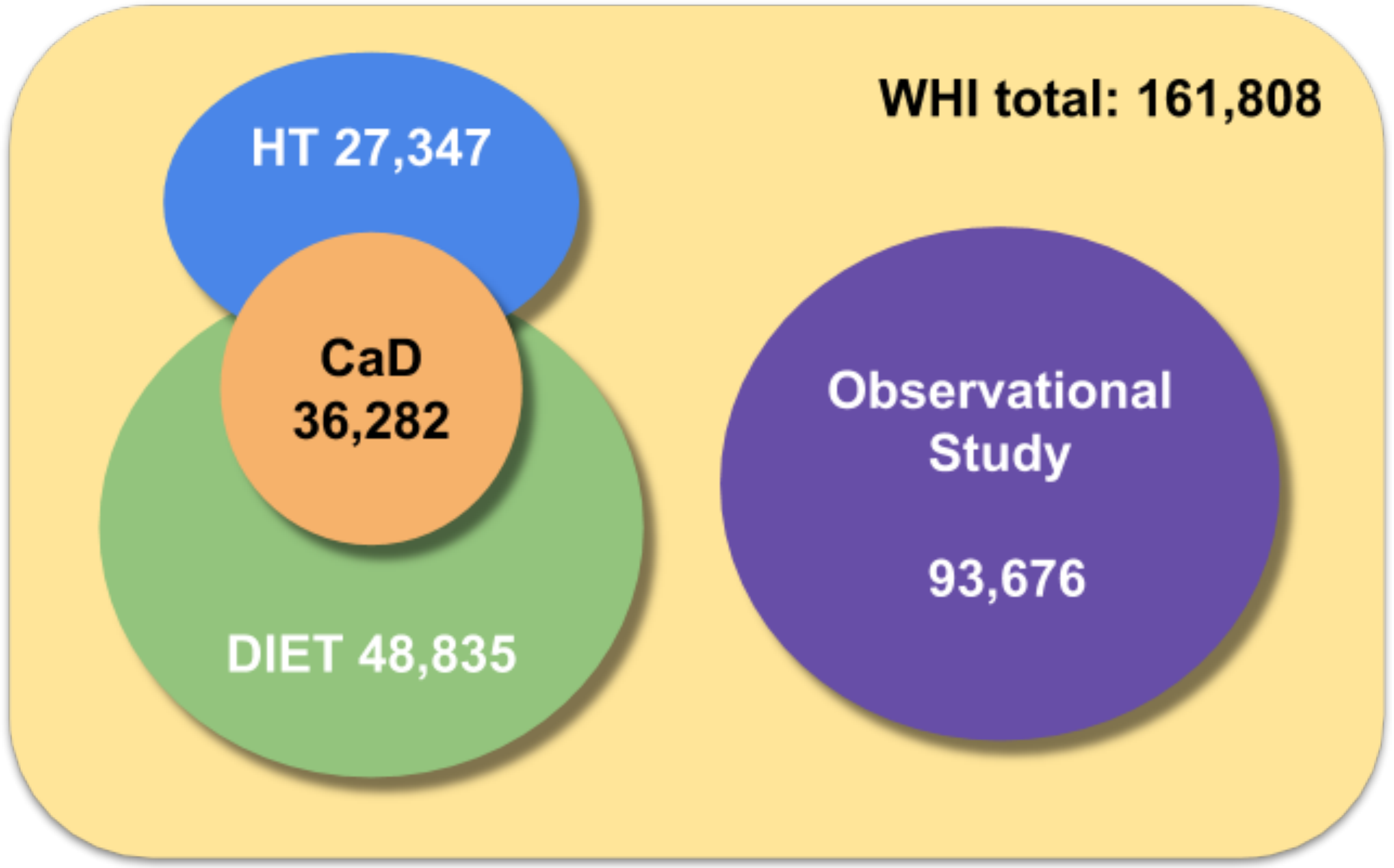
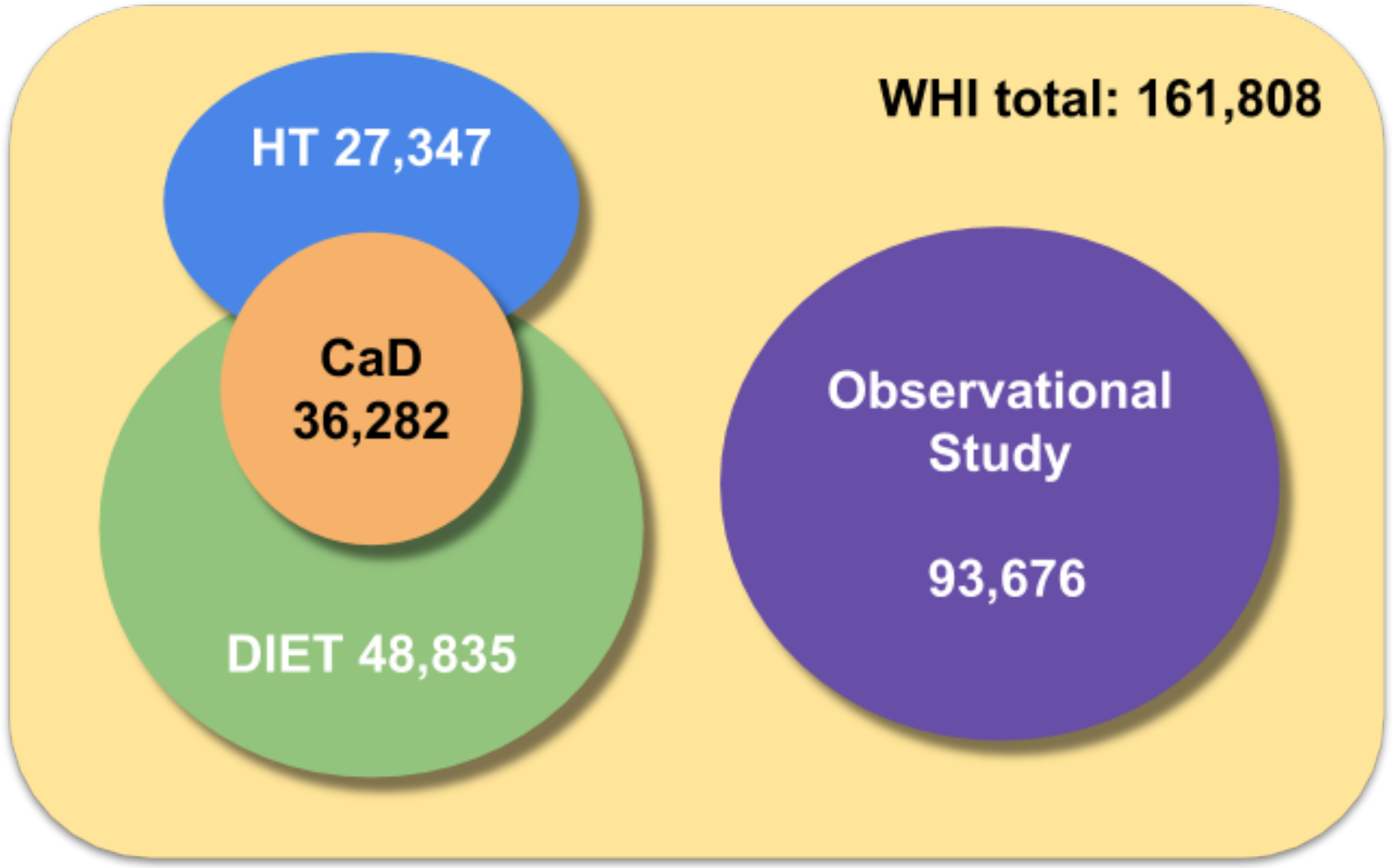


Women's Health Initiative (WHI)



Women's Health Initiative (WHI)

- Randomized trial and observational study (1993-2005)



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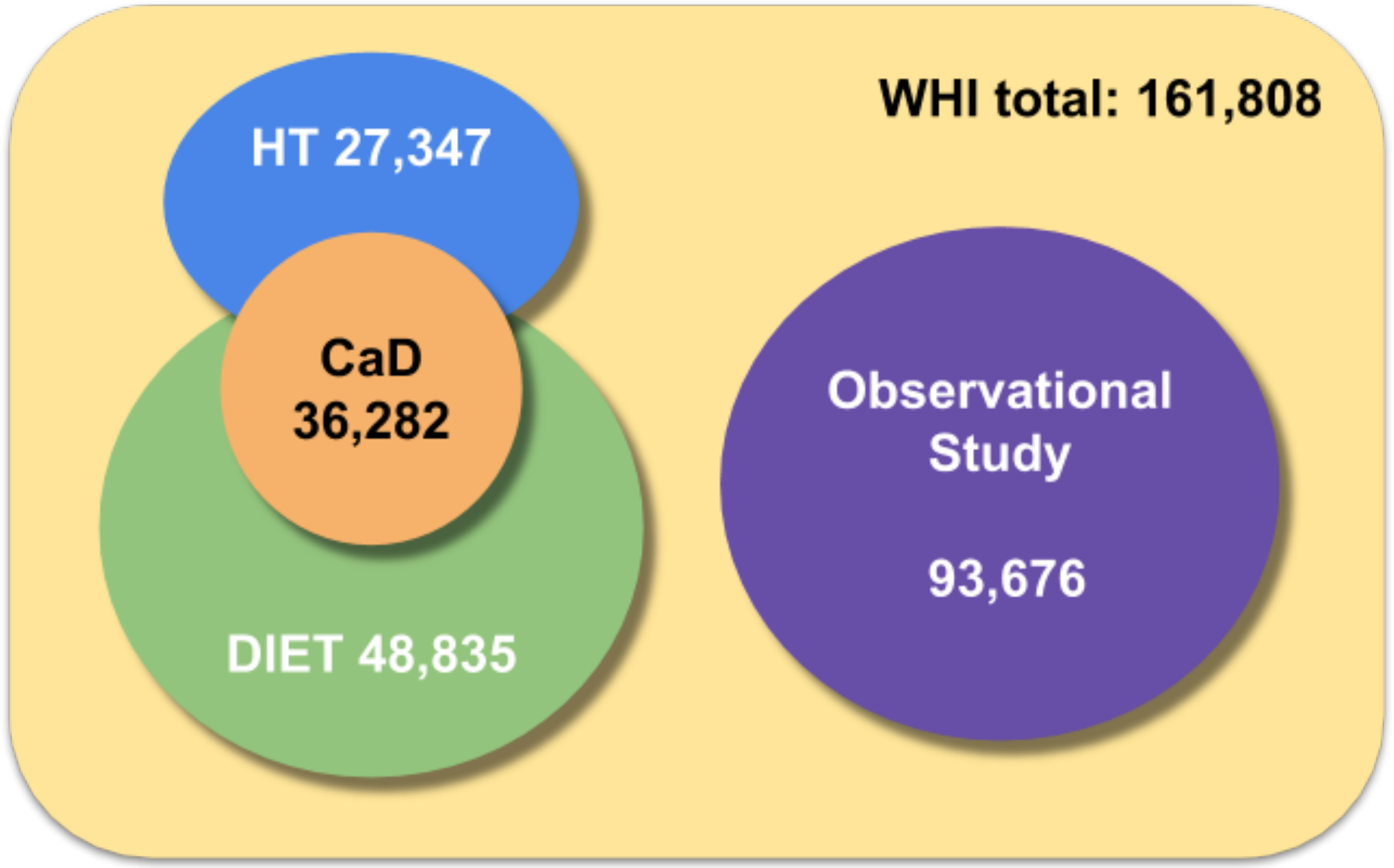

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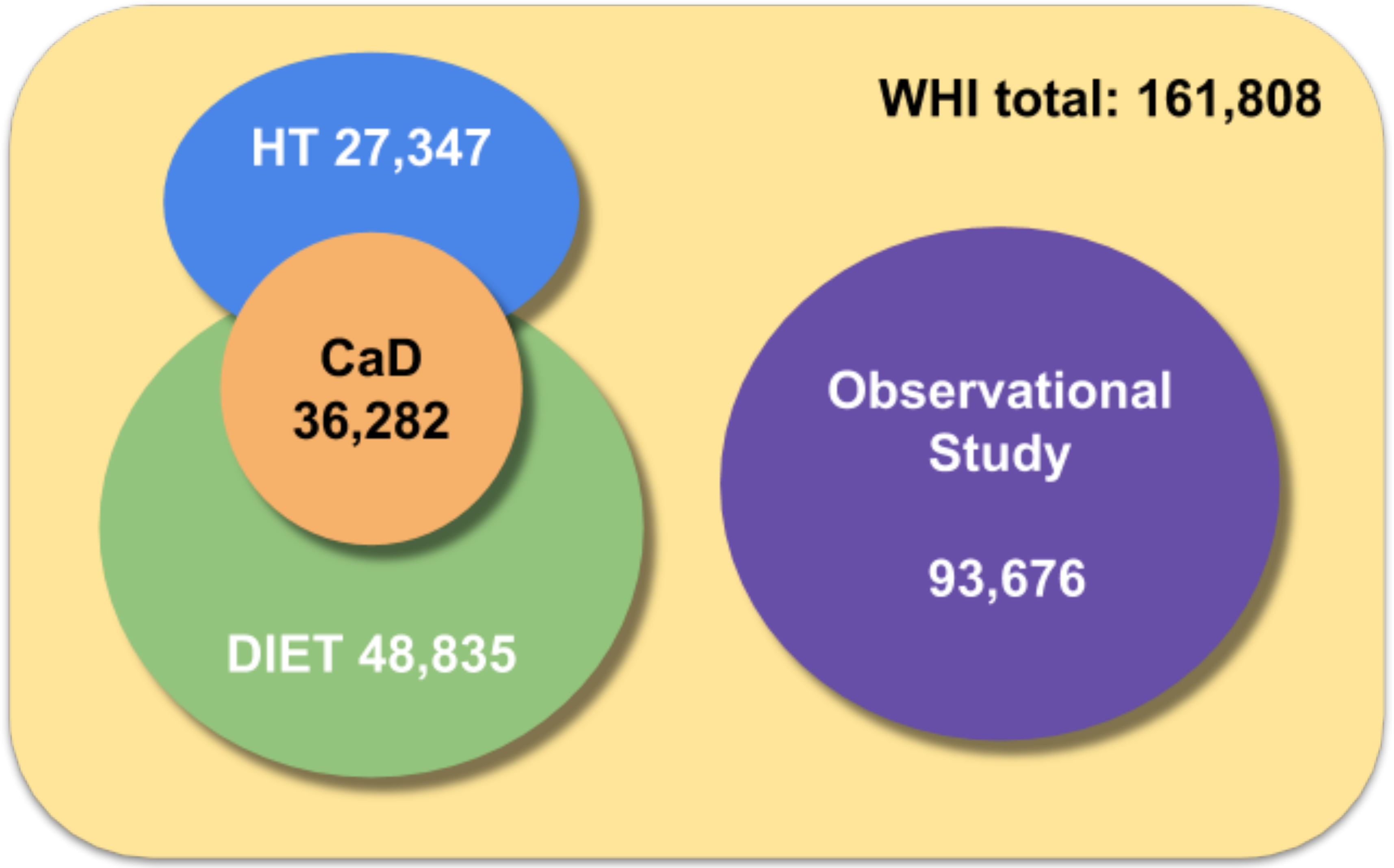

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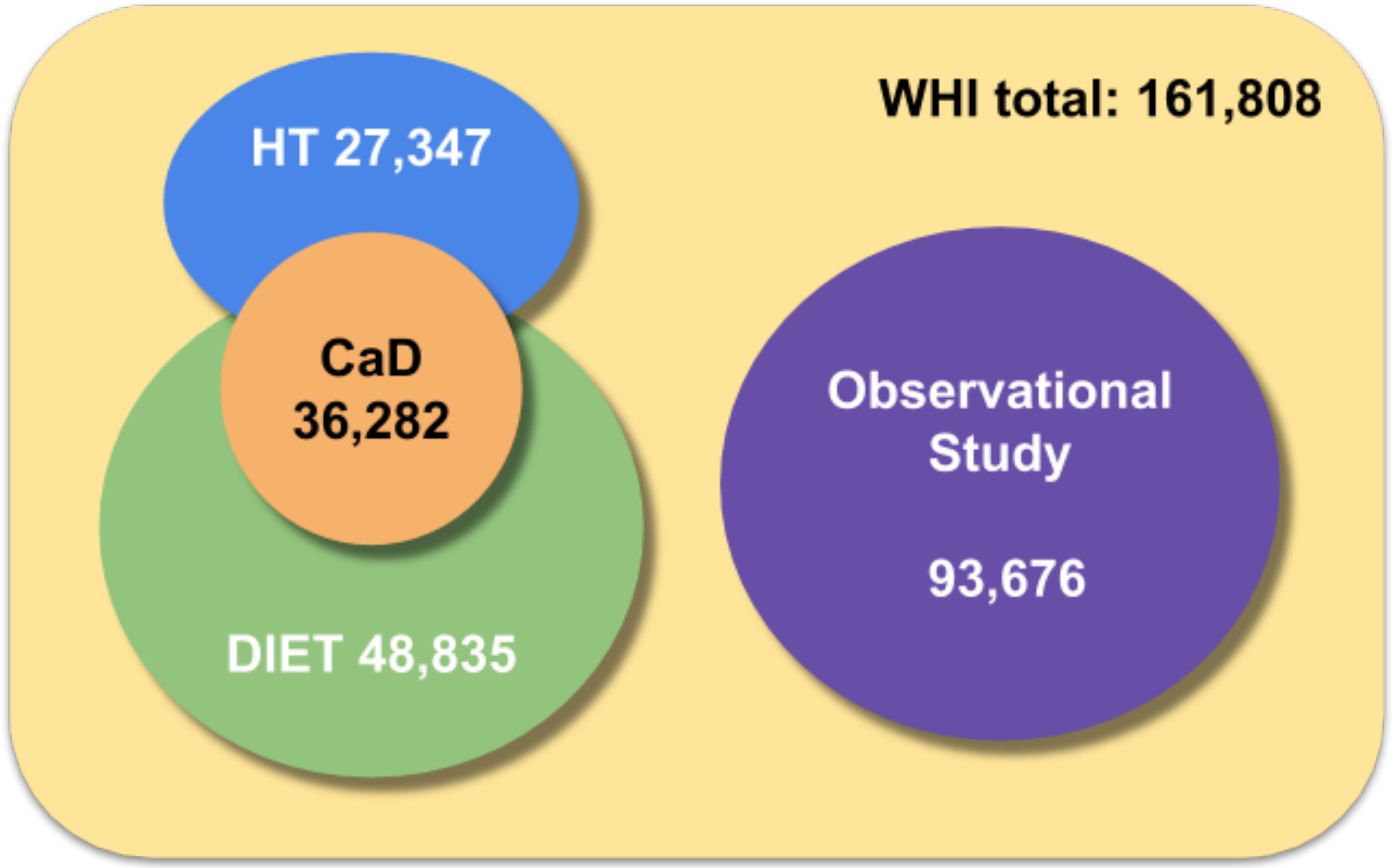
- Cohort: postmenopausal women aged between 50 and 79
- Outcomes: coronary heart disease



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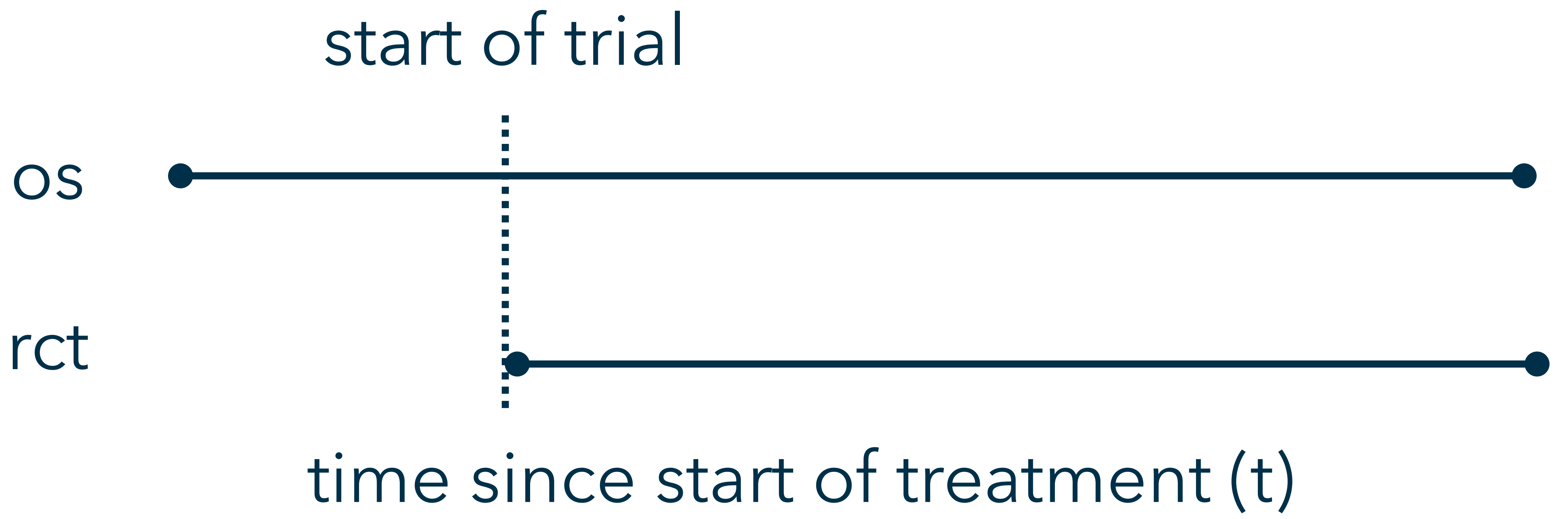

Metric	Coronary heart disease	
	$t = 0$	$t = 20$
$\hat{\Gamma}_{CT}$	1.017	1.164
$\hat{\Gamma}_{LB}$	1.009	1.224
ψ_{bin}	1	1
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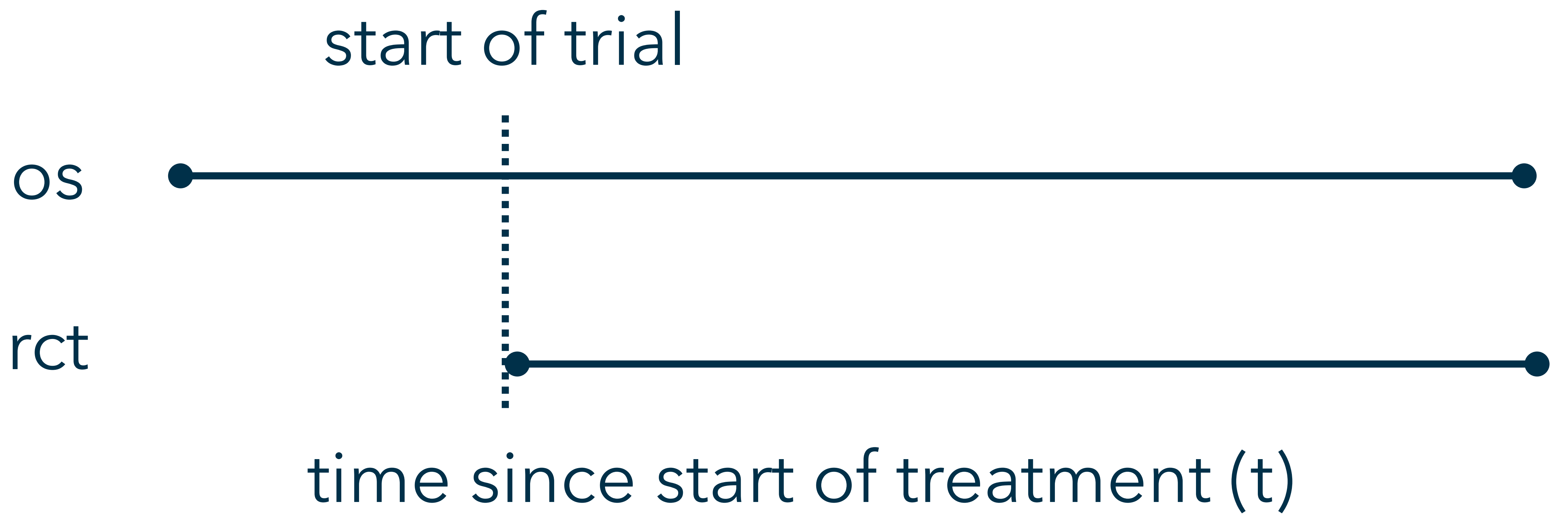

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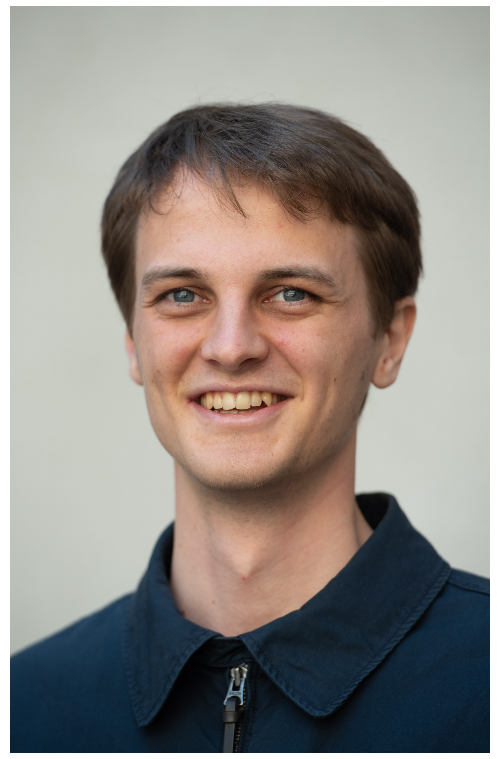
Thanks for your attention! Questions?

Pier

Javier



Konstantin

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