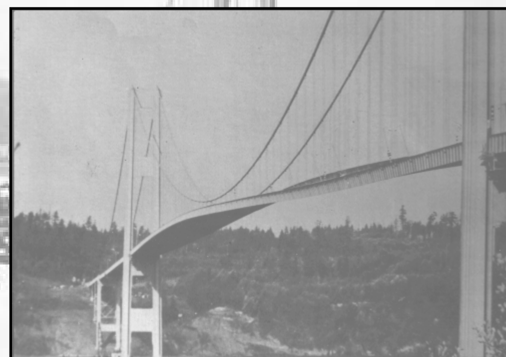
Active Control of Suspension Bridges

CE291F Term Project
Patricia Decker
3 May 2006



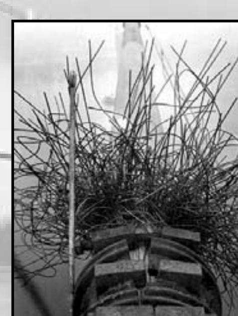
Presentation Outline

- Introduction & Motivation
- Initial Problem Formulation
  - Modeling & Equations of Motion
  - Evaluation of Flutter Instability
  - Numerical Example
- Controlled Problem Formulation
  - Revised Equations of Motion
  - Controller Design
- Results and Conclusions
- Future Work

Tacoma Narrows, During
a.k.a. Galloping Gertie

Introduction & Motivation

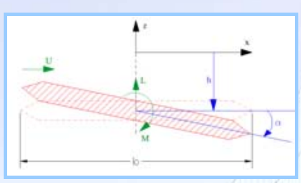
- Flutter of Suspension Bridges
  - 1940 Tacoma Narrows
- Structural Control
  - Passive, Semi-Active, Active Schemes
  - Ostenfeld & Larsen (1992)
- Research Objectives
  - Literature Review
  - Solution of “uncontrolled” problem
  - Reproduction and modification of Wilde & Fujino (1998)



“Every major bridge project is an adventure.”
–Lacey V. Murrow, former Director of State Highways

Initial Problem Formulation: Modeling

- 2DOF Flat Plate Approximation



- Equations of Motion

$$m\ddot{h} + 2m\varsigma_h\omega_h\dot{h} + m\omega_h{}^2 h = L_h$$
$$I_\alpha\ddot{\alpha} + 2I_\alpha\varsigma_\alpha\omega_\alpha\dot{\alpha} + I_\alpha\omega_\alpha{}^2\alpha = M_\alpha$$

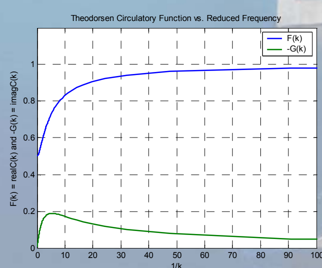
Initial Problem Formulation: Forces

- **Theodorsen Formulation**

  Thin airfoil oscillating in incompressible flow

$$L_h = \pi\rho\left(\frac{B}{2}\right)^2[\ddot{h} + U\dot{\alpha} - \frac{B}{2}a\ddot{\alpha}] + 2\pi\rho U\frac{B}{2}C(k)[\dot{h} + U\alpha + (\frac{B}{2})(\frac{1}{2} - a)\dot{\alpha}]$$

$$M_\alpha = \pi\rho\left(\frac{B}{2}\right)^2[\frac{B}{2}a\ddot{h} - U\frac{B}{2}(\frac{1}{2} - a)\dot{\alpha} - (\frac{B}{2})^2(\frac{1}{8} + a^2)\ddot{\alpha}] + 2\pi\rho U(\frac{B}{2})^2(a + \frac{1}{2})C(k)[\dot{h} + U\alpha + \frac{B}{2}(\frac{1}{2} - a)\dot{\alpha}]$$



$$C(k) = F(k) + iG(k)$$

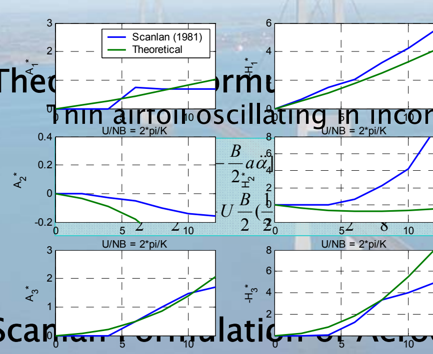
$$F(k) = -\frac{J_1(J_1 + Y_0) + Y_1(Y_1 + J_0)}{(J_1 + Y_0)^2 + (Y_1 - J_0)^2}$$

$$G(k) = -\frac{(Y_1Y_0 + J_1J_0)}{(J_1 + Y_0)^2 + (Y_1 - J_0)^2}$$

Initial Problem Formulation: Forces

- **Theodorsen Formulation**

  Thin airfoil oscillating in incompressible flow



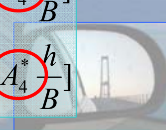
$$H_i^* = H_i^*(C(k))$$

$$A_i^* = A_i^*(C(k))$$

- **Scanlan Formulation of Aerodynamic Forces**

$$L_h = \rho U^2 B[KH_1^*\frac{\dot{h}}{U} + KH_2^*\frac{B\dot{\alpha}}{U} + K^2H_3^*\alpha + K^2H_4^*\frac{h}{B}]$$

$$M_\alpha = \rho U^2 B^2[KA_1^*\frac{\dot{h}}{U} + KA_2^*\frac{B\dot{\alpha}}{U} + K^2A_3^*\alpha + K^2A_4^*\frac{h}{B}]$$



Initial Problem Formulation: Forces

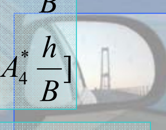
MORAL:

$$L_h = L_h(\omega)$$
$$M_\alpha = M_\alpha(\omega)$$

- Scanlan Formulation of Aerodynamic Forces

$$L_h = \rho U^2 B[KH_1^* \frac{\dot{h}}{U} + KH_2^* \frac{B\dot{\alpha}}{U} + K^2 H_3^* \alpha + K^2 H_4^* \frac{h}{B}]$$

$$M_\alpha = \rho U^2 B^2 [KA_1^* \frac{\dot{h}}{U} + KA_2^* \frac{B\dot{\alpha}}{U} + K^2 A_3^* \alpha + K^2 A_4^* \frac{h}{B}]$$



Initial Problem Formulation: Algorithm

- Determination of Flutter Speed, U_f
  - Select a value of reduced frequency, K
    - Determine values of $H_i^*(K)$ and $A_i^*(K)$
  - Assume solutions $h = h_o e^{i\omega t}$ $\alpha = \alpha_o e^{i\omega t}$

$$h_o[1 + 2i\varsigma_h X - X^2 - \frac{\rho B^2}{m}(iH_1^* + H_4^*)X^2] - \alpha_o[\frac{\rho B^3}{m}(iH_2^* + H_3^*)X^2] = 0$$

$$h_o[-\frac{\rho B^4}{I_\alpha}(iA_1^* + A_4^*)\frac{X^2}{B}] - \alpha_o[(\frac{\omega_\alpha}{\omega_h}) + 2i\varsigma_\alpha(\frac{\omega_\alpha}{\omega_h})X - X^2 - \frac{\rho B^4}{I_\alpha}(iA_2^* + A_3^*)X^2] = 0$$

where $X = \omega/\omega_h$

$$\boxed{\det|CoeffMatrix| = 0}$$

  - Complex Eigenvalue Analysis
    - Real Polynomial → $a_4X^4 + a_3X^3 + a_2X^2 + a_1X + a_0 = 0$
    - Imaginary Polynomial → $b_3X^3 + b_2X^2 + b_1X + b_0 = 0$
    - Determine (K_f, X_f) as intersection of real and imaginary roots
  - $U_f = B\omega_h X_f / K_f$

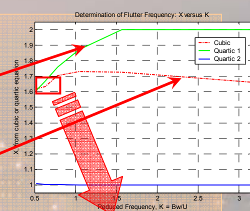
Initial Problem Formulation: Example

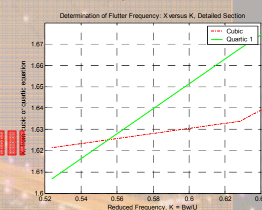
From Scanlan (1981, FHWA/RD-80-050)

- Dynamic Properties
  - $\omega_h = 0.2\pi$ rad/sec
  - $\omega_\alpha = 2\omega_h$
  - $\zeta_h=\zeta_\alpha=0.01$ (% critical)
- Physical Properties
  - B = 100ft
  - L = 4000ft
  - m = 711.8 $lb_f s^2/ft^2$
  - I_α = 857,000 $lb_f s^2$
  - ρ = 0.002378 $(lb_f s^2/ft)/ft^2$
- *Flutter Speed, Tabular Data*
  - $U_f \approx$ 183.9 fps $\approx$ 125.4 mph
- *Flutter Speed, Theoretical Flutter Derivatives*
  - $U_f \approx$ 225.4 fps $\approx$ 153.6 mph (22% increase)
  - $U_f \approx$ 208.1 fps $\approx$ 141.9 mph (13% increase)

$$a_4X^4 + a_3X^3 + a_2X^2 + a_1X + a_0 = 0$$

$$b_3X^3 + b_2X^2 + b_1X + b_0 = 0$$

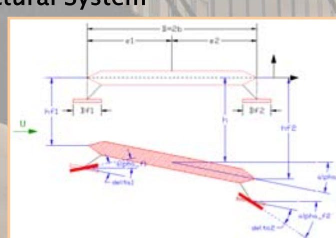




Tsing Ma
Hong Kong, 1377m

Controlled Problem Formulation

- Structural System



- Wing-Aileron System, Theodorsen (1935) and Edwards et al. (1977)

Verrazano-Narrows
USA, 1410m

Controlled Problem Formulation

- Equations of Motion

$$\overline{M}\ddot{\vec{V}}_g + \overline{C}\dot{\vec{V}}_g + \overline{K}\vec{V}_g = \vec{P} + \overline{B}_u\vec{U}$$

$$\begin{bmatrix} m & 0 & 0 & 0 \\ 0 & I_\alpha & 0 & 0 \\ 0 & I_{\delta f1} & I_{\delta f1} & 0 \\ 0 & I_{\delta f2} & 0 & I_{\delta f2} \end{bmatrix}\begin{bmatrix} \ddot{h} \\ \ddot{\alpha} \\ \ddot{\delta}_1 \\ \ddot{\delta}_2 \end{bmatrix} + \begin{bmatrix} c_h & 0 & 0 & 0 \\ 0 & c_\alpha & 0 & 0 \\ 0 & c_{\delta f1} & c_{\delta f1} & 0 \\ 0 & c_{\delta f2} & 0 & c_{\delta f2} \end{bmatrix}\begin{bmatrix} \dot{h} \\ \dot{\alpha} \\ \dot{\delta}_1 \\ \dot{\delta}_2 \end{bmatrix} + \begin{bmatrix} k_h & 0 & 0 & 0 \\ 0 & k_\alpha & 0 & 0 \\ 0 & k_{\delta f1} & k_{\delta f1} & 0 \\ 0 & k_{\delta f2} & 0 & k_{\delta f2} \end{bmatrix}\begin{bmatrix} h \\ \alpha \\ \delta_1 \\ \delta_2 \end{bmatrix} = \begin{bmatrix} L_{total} \\ M_{total} \\ M_{f1} \\ M_{f2} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

- State-Space Form

$$\dot{\vec{X}} = \overline{A}\vec{X} + \overline{B}\vec{U}$$

$$\vec{Y} = \overline{C}\vec{X} + \overline{D}\vec{U}$$

$$\vec{X}^T = \begin{bmatrix} \vec{V}_g^T & \dot{\vec{V}}_g^T \end{bmatrix}$$

$$\dot{\vec{X}}^T = \begin{bmatrix} \dot{\vec{V}}_g^T & \ddot{\vec{V}}_g^T \end{bmatrix}$$

$$\overline{B} = \begin{bmatrix} \overline{0} \\ \overline{M}^{-1}\overline{B}_u \end{bmatrix} \quad \overline{A} = \begin{bmatrix} \overline{0} & \overline{I} \\ -\overline{M}^{-1}(\overline{K} - F_{fl,5:8}) & -\overline{M}^{-1}(\overline{C} - F_{fl,1:4}) \end{bmatrix}$$

$$\overline{C} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \overline{I}_{4X4} & \overline{O}_{4X4} \end{bmatrix}$$

X = State Vector
U = Input Vector
Y = Output Vector

A = State Matrix
B = Input Matrix
C = Output Matrix
D = Feedback Matrix

Controller Design

- Controller Form

$$\vec{U} \longrightarrow \boxed{\begin{aligned} \dot{\vec{X}} &= \overline{A}\vec{X} + \overline{B}\vec{U} \\ \vec{Y} &= \overline{C}\vec{X} \end{aligned}} \longrightarrow \vec{Y}$$

$$\boxed{\vec{K}_{LC}}$$

$$\boxed{\vec{U}(t) = \vec{K}_{LC}\vec{Y}(t)}$$

- Output linear feedback with time-invariant gains
- Wilde and Fujino (1998)
  - Forces estimated by Rational Function Approximation
- Proposed Solution
  - Forces calculated using theoretical flutter derivatives
  - Linear Quadratic Regulator (LQR) problem for MIMO system

Controller Design

- Performance Index → Quadratic Cost Functional

$$J = \int_0^{\infty} (\vec{X}^T \overline{Q} \vec{X} + \vec{U}^T \overline{R} \vec{U}) dt$$

Q, weighting matrix placing emphasis on states
R, weighting matrix placing emphasis on control forces

Controller Design

- Algebraic Riccati Equation

$$\overline{\mathrm{P}}\overline{\mathrm{A}} + \overline{\mathrm{A}}^T \overline{\mathrm{P}} - \overline{\mathrm{P}}\overline{\mathrm{B}}\overline{\mathrm{R}}^{-1} \overline{\mathrm{B}}^T \overline{\mathrm{P}} + \overline{\mathrm{Q}} = 0$$

P, solution to algebraic Riccati equation
Equation nonlinear in P
Existence and uniqueness guaranteed if …
(A,B) is controllable
(A,H^T) is observable, where $Q = H^T H$

- Closed-Loop System

$$\overline{K}_{LC} = \overline{R}^{-1} \overline{B}^T \overline{P}$$

$$\dot{\vec{X}} = (\overline{A} - \overline{B}\overline{K}_{LC}) \vec{X}$$

Results and Conclusions

- Matlab code for solution of flutter equations by complex eigenvalue analysis
- Flutter of suspension bridges analogous to wing-aileron problem
- State-space formulation of controlled problem
  - Controllability of system is questionable
  - Previous work by Wilde and Fujino suggests control is possible

Future Work

- Refinements
  - 3D Model/FEM
  - Time-History Analysis
- Related Problems
  - Suppression of Coupled Flutter by Tuned Pendulum Damper (Okada et al. 1998)
  - Large Amplitude Torsional Oscillations in Suspension Bridges
  - Control of Bridges with Seismic Applications

References

- **Ostenfeld, K., and Larsen, A. (1992).** "Bridge engineering and aerodynamics." *Aerodynamics of large bridges*, A. Larsen, ed., A. A. Balkema, Rotterdam, The Netherlands, 3-22.
- **Preumont, André. (1999).** *Vibration Control of Active Structures*. Kluwer, The Netherlands.
- **Scanlan, R.H., and Tomko, J.J. (1971).** "Airfoil and bridge deck flutter derivatives." *J. Engrg. Mech. Div.*, ASCE, 97(6), 1717-1737.
- **Scanlan, R.H.. (1981).** "State-of-the-Art Methods for Calculating Flutter, Vortex-Induced, and Buffeting Response of Bridge Structures." Report No. FHWA/RD-80-050. Federal Highway Administration Offices of Research and Development, Structures and Applied Mechanics Division. Washington, D.C.
- **Simiu, E., and Scanlan, R. (1986).** *Wind effects on structures*. John Wiley & Sons, Inc., New York, NY.
- **Theodorsen, T. (1935).** "General theory of aerodynamic instability and the mechanism of flutter." *NACA Rep. 496*, National Advisory Committee for Aeronautics, Washington, D.C.
- **Wilde, K., and Fujino, Y. (1998).** "Aerodynamic control of bridge deck flutter by active surfaces." *J. Engrg. Mech.*, ASCE, 124(7), 718-727.

Thank You!

Gephyrophobia: Fear of Crossing Bridges